ENDOGENOUS BELIEFS AND INSTITUTIONAL STRUCTURE IN COMPETITIVE EQUILIBRIUM WITH ADVERSE SELECTION

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ENDOGENOUS BELIEFS AND INSTITUTIONAL STRUCTURE IN COMPETITIVE EQUILIBRIUM WITH ADVERSE SELECTION

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Abstract

I model financial markets that structure decision-making into discrete points separating contract offers, applications, and acceptance/denial decisions. Endogenous beliefs about applicants' risk types emerge as the institutional process extracts private information allowing uninformed firms to infer risk qualities by comparing applications of *many consumers*. Endogenous beliefs and low-risk consumer behavior render truthful disclosure of transactions incentive compatible supporting a unique equilibrium robust to cream-skimming *and* cross-subsidizing deviations, even under Hellwig's "secret" policy assumption. In equilibrium each type demands low-risk's optimal pooling policy and high-risk supplement to full-coverage at fair-price. Nonpassive consumers' belief firms are sequentially rational necessary for equilibrium; lemon equilibrium with only high-risk insured possible.

Keywords: adverse selection, sequential rationality, screening, signaling, incentive compatibility, insurance pooling.

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Introduction

Markets characterized by asymmetric information generally achieve more efficient allocations if uninformed agents can extract more information from the informed. To do so, markets must provide institutional arrangements allowing informed agents willing to reveal private information to be screened efficiently by uninformed firms despite the efforts of some agents to distort the informational content of the signals. This paper shows one relevant institutional arrangement for this purpose is the sequencing of sell, buy, and underwriting decisions typically observed in financial markets where some agents trading a divisible good have incentives to keep private information private while others do not. The specific market setting examined is the canonical model of adverse-selection (Rothschild & Stiglitz, 1976; Wilson, 1977) where insurance companies compete for consumers of unobservable risk quality by designing policies to separate risk types self-selecting distinct premiums and indemnities. Famously, the model produced either no Nash equilibrium or a "separating equilibrium" inducing consumers to reveal their risk type by self-selecting low premium–low coverage or high premium-high coverage policies priced fairly for their risk class; high-risk types selecting full-coverage and low-risk types the maximum partial coverage leaving high-risk types indifferent to switching to the cheaper policy.

The paper adapts the Jaynes-Hellwig formulation of the model to a three-stage dynamic framework meant to capture the transaction structure typically found in financial markets where firms initially offer general contract terms (e.g. prices, quantity limits, etc.), accept applications eliciting applicants' private information, and only then make a final offer with an acceptance/denial decision. All agents (firms and non-passive consumers) disclose private information strategically. Although firms cannot monitor the consumer's total coverage to prevent multiple contracting, equilibrium exists even under Hellwig's (1988) nonstandard framework with firms able to offer consumers "secret policies." The three-stage framework illustrates several illuminating properties common to institutional arrangements in financial markets.

First, by differentiating the roles of a firm's policy offerings (stage 1) from its acceptance/denial (underwriting) decision (stage 3), it allows firms to evaluate consumer applications and competitors' disclosures of applications in a framework where competition extracts all the private information possible by incentivizing firms and consumers to disclose truthfully information each might prefer to conceal or even falsify. Secondly, the sequencing of actions illustrates four interactions inherent to financial markets -- interactions between a firm's acceptance/denial decision, its belief about a consumer's risk type (profitability), a consumer's application (signaling) strategy, and her belief her application will be accepted versus denied. Modeling these beliefs makes explicit a seemingly trivial but essential assumption only implicit in all previous literature (consumers believe they cannot complete a transaction if the firm believes it promises negative profit, (i.e. consumers believe firms are sequentially rational).

Because of the institutional sequencing of decisions, all agent beliefs are endogenous in the sense rational agents can deduce them from observed transaction information and agent behaviors. Endogenous agent beliefs are consistent with a unique equilibrium eliminating the embarrassment of a wealth of equilibria often found in models with adverse selection. Moreover, I argue the equilibrium obtained is the unique rational outcome for this model. Endowing agents with exogenous beliefs not consistent with the deducible beliefs naturally arising in the institutional setting could only produce untenable equilibria.

To enable firms to screen consumers via partial coverage policies, early models imposed exogenous exclusivity conditions restricting each consumer to one policy. This implicitly assumed firms share information about a client's transactions allowing them to screen each consumer with a menu of risk type revealing policies. However, Jaynes (1978) showed screening with a menu of policies is undermined by profit seeking competitors offering *undisclosed* high premium coverage that enables high-risk consumers to leverage the menu's cheapest partial coverage policy to full coverage through multiple transactions. Thus, the high-risk type's higher risk of loss and greater expected cost of incomplete coverage, no longer forces her to separate from the low-risk in order to obtain full coverage insurance. Because differences in risk fail to impose differential expected costs from holding only a partial coverage policy, in *equilibrium*, neither screening nor the signaling arising naturally from the structure of asymmetric information provides firms any information of value or affects firms' ex ante beliefs about the consumer's risk type. This is strictly a result of equilibrium, off the equilibrium path; screening and signaling play significant roles.

The mutual dependence of acceptance/denial decisions, firm beliefs about the consumer's risk type, and consumer application strategies, and beliefs about an application's acceptance suggest both firm screening and consumer signalling are inherent to transactions in financial markets. For institutional context, think of insurance information exchanges or financial markets' credit reporting agencies that receive from and provide to creditors detailed personal information describing their clients' financial transactions. Firms screening to identify risk types ask a prospective client to complete forms detailing her transaction history; the consumer, reacting to the screen strategically, divulges transaction information selectively attempting to update firm beliefs she is low-risk. Able to contract with multiple firms, consumers have incentives to conceal applications revealing them high-risk and to report those suggesting they are low-risk. This has consequences for firm and consumer behavior. If an application reveals a consumer high-risk the consumer may avoid making it for two reasons. First, if the firm's strategy is to disclose the application to other firms, and secondly, if she believes self-revealing her type will cause the receiving firm itself to deny her application. Alternatively, if an application reveals a consumer low-risk, she wants it disclosed to firms selling supplementary policies. Thus, a low-risk consumer's application and reporting strategies derive from her incentive to differentiate herself from high-risk consumers by signaling to firms her previous transactions are those preferred by low-risk types. To do this, she voluntarily reports other applications to any second firm, signaling, I chose the low-risk over the high-risk policy. Low risk incentives to reveal themselves is largely responsible for the deducibility of beliefs about applicants' risk type. High-risk consumers have no incentive to reveal their type; however, they have incentive to mimic low-risk reporting behavior regardless of their actual transactions.

It is important to observe that these results depend on modeling a market with many active consumers whose applications to firms can be juxtaposed and compared to make inferences about consumers' risk probabilities. Game theory's representative consumer selected by nature to be a specific risk type with some known probability will not do because beliefs are no longer endogenous.

The equilibrium policy configuration is the Jaynes-Hellwig-Glosten allocation (Attar & d'Aspremont, 2018); each risk-type demands the low-risk optimal zero profit pooling policy, and high-risk types supplement it with high-risk fair price insurance to full coverage. Although all firms disclose pooling policy transactions, the risk types of clients purchasing the pooling policy remain unknown because no consumer transacts with the same firm for both policies, and no firm discloses higher price transactions. Given equilibrium offers and strategies, low-risk types would only buy policies priced below their optimal pooling policy. Thus, a successful deviation must somehow screen and separate risk-types either offering two policies (low-risk cross-subsidizing the high), or one policy cream-skimming the low-risk. Either deviation threatens to leave competitors with an adverse selection of clients. However, equilibrium is robust to competitive screening because, given any deviation offer priced below the equilibrium pooling price, firms offering pooling coverage acceptance/denial strategy would deny any application for coverage that exceeds low-risk types' optimal supplementation to the deviation. Because this would induce high-risk consumers to mimic low-risk behavior by applying for the deviation targeted to the low-risk type, and the deviation policy is priced below the zero-profit pooling price, a firm offering it expects negative profit.

The paper's next sections describe the model, define equilibrium, and derive model properties needed to state the main results. The main existence result details equilibrium strategies of firms and consumers under the standard assumption firm strategies are public information. Subsequently, the equilibrium result is extended to cover Hellwig's (1988) assumption firms can hide policy offers from competitors. Equilibrium continues to exist because, equilibrium strategies render truthful disclosure of policy offers, client transactions, and consumer reports incentive compatible. A final section discusses related literature.

Screening and Strategic Signaling

I examine an insurance market with $J \geq 2$ expected profit maximizing firms accepting applications for insurance from many prospective consumers of unknown risk type $t = L$ or H whose respective probabilities of income loss p^t satisfy $1 > p^H > p^L$. Identify consumers by an index c with $1 \leq c \leq n$ and assume m^t is the proportion of consumers of risk type t. A policy is a point (γ_1, γ_2) in the nonnegative quadrant of R^2 with γ_1 the indemnity received in the event loss and γ_2 the premium paid otherwise. The unit price of a positive insurance policy γ is $q(\gamma) = \frac{\gamma_2}{\gamma_1}$ its premium per unit coverage. A contract is a policy tied to conditions restricting buyer behavior, and delineating certain actions of the policy's issuer. Possible restrictions on buyer behavior include exclusivity clauses permitting or prohibiting purchase of competitors' policies and prohibitions against giving false information. Firm strategic actions include explicit conditions determining the maximum quantity of coverage supplied to any consumer at offered prices, and whether it will disclose the transaction to competitors. Such contractual obligations turn out to be implied in firms' disclosure and offer-cum-acceptance/denial strategies, and need not be modeled separately.

In addition to their option to disclose or not disclose their portfolio of applications to firms, consumers have the option to misinform firms about their transactions. Therefore, in addition to expected utility maximization over policies, consumer strategies include a communication:

a. *Signaling*, consumers report (to each transacting partner) transactions with other firms.

A firm's strategy encompasses a choice between two options: 1. Offer the market a set of policies defined by an offer-cum-acceptance/denial rule describing the conditions determining its acceptance or denial of a consumer's application, and its disclosure strategy describing the conditions determining whether it discloses an applicant and her transaction to a particular competitor; 2. Design and offer consumers take it or leave it choices consisting of two policies with contractual stipulations requiring exclusivity or not, and declaring if the policy will be disclosed to competitors. We adopt the convention of referring to the maximum coverage policy offered by a firm at a given price (whatever option chosen) as a prime policy. It turns out because there are two risk types, whichever option a firm chooses, it will offer two prime policies (although either or both could be the null policy).

For a firm choosing option two, a strategy specifies:

b. *Two policies* (either or both could be the null policy);
c. *Disclosure*, conditions describing what transactions it will disclose;
d. *Acceptance/Denial,* conditions determining acceptance or denial of applications.

In the paper's main section, we assume all strategies are observed, so firms' policy offers are published information known to all agents. In a later section, firms choosing option two can offer published and unpublished policies with the latter hidden private information known only to the firm and consumers.

The strategy of a firm choosing option one is derived below. For either option, acceptance or denial of an application will depend on the firm's expected profit from the transaction. Expected profit depends on the policy's price and the underwriter's belief concerning the applicant's risk type. In turn, that belief conditions on an applicant's specific policy application and report of other transactions, the market's aggregate policy offerings, aggregate transactions reported by the firm's applicant pool, and competitors' disclosures of their transactions with the firm's applicant pool.

Sequence of Actions

We follow Hellwig (1988), but use three stages for clarity of sequencing.

- Stage-one, firms choose strategies simultaneously.
- Stage two, consumers observe firm strategies and apply for policies reporting applications to transaction partners.
- Stage three, firms enact disclosure strategy, then accept or deny applicants.

Equilibrium: Equilibrium requires for firms and consumers a profile of strategies and beliefs $\{b^2,\ b^3\}$ with b^2 representing consumers' stage 2 belief an application will be accepted, and b^3 representing each firm's stage 3 belief any consumer it faces is low-risk, all satisfying three conditions. Given firm beliefs, communications, and offer-cum-acceptance/denial strategies, and consumer beliefs and strategies:

1. No firm can increase its expected profit by deviating from its strategy at any decision point; 2. Consumers' policies maximize expected utility at stage 2, and no deviation from reporting strategies could increase expected utility.

The Structure of Demand and Supply

Let w denote each consumer's endowment income and r the reduction in income in the event loss. If the consumer purchases the insurance policy (γ_1, γ_2), incomes in the respective events loss and no loss are $w_0 = w - r + \gamma_1,\ w_1 = w - \gamma_2$. The consumer has a twice continuously differentiable strictly concave utility function u(w) over income with Von Neumann-Morgenstern utility $p^t u(w_0) + (1 - p^t)u(w_1) = V^t(\gamma)$.

Let $z_u^t(q, \gamma)$ equal the utility maximizing demand for insurance coverage by a consumer of risk type t who has already purchased the policy γ and can buy *unrestricted* coverage at the price q, i.e.

$$z_u^t(q, \gamma) = \underset{z}{\text{argmax}}\, p^t u(w - r + \gamma_1 + z) + (1 - p^t)u(w - \gamma_2 - qz);\ qz \leq w - \gamma_2.$$

Then, since consumers are restricted to nonnegative coverage let:

$$z^t(q, \gamma) = \text{Max}\{z_u^t(q, \gamma), 0\}.$$

When γ is the null contract, $z^t(q,\ \gamma)$ is the demand for insurance at price q of a consumer of type t at the endowment income. When no confusion can arise, this endowment demand is denoted $z^t(q)$. Assume the following well-known lemmas without proof. Let q^t and $\hat{q}$ equal, respectively, odds of the event loss for risk type t, and average or pooled odds for the two risk types (i.e. the expected odds of a claim from a policy sold to representative proportions of both types).

Lemma 1a: Comparing price to the odds type t suffers income loss, demand at price q satisfies:

$$q \substack{<\\=\\>} \frac{p^t}{1-p^t} \Rightarrow z_u^t(q,\gamma) + \gamma_1 + qz_u^t(q,\gamma) + \gamma_2 \substack{>\\=\\<} r;$$

Risk averse consumers fully insure when the price of insurance equals the odds of the insured event, and under (over) insure when the price exceeds (is less than) those odds.

Lemma 1b: $z^H(q,\gamma) \geq z^L(q,\gamma)$ for all $q > 0$ and feasible γ.

Firm Disclosures and Beliefs

Firms must decide what (if anything) to do with the transaction information they receive from clients and competitors at stages two and three respectively. In deciding, firms must be aware any information received from other firms as well as from consumers could be false. In particular, information received from or about firms offering policies priced lower than a firm's own policy reveals agent actions are off the equilibrium path and should be viewed with suspicion. In what follows, I describe equilibrium strategies and beliefs.

Remark 1: Let γ_i^c denote the policy consumer i applies for to firm i with $\gamma_i^c = (0,0)$ if c does not apply to firm i. Firm i discloses applicant c and her application policy to competitor j if and only if, either the applicant does not report applying to j, or reports applying to j for a policy priced no less than γ_i^c.

Formally, if consumer c applies for the policy γ_i^c from firm i and reports to i the policy R_{ij}^c with firm j, firm i's disclosure of the transaction γ_i^c to firm j is:

$$d_{ij}^c\left(\gamma_i^c, R_{ij}^c\right) = \begin{cases} \gamma_i^c, & \text{disclose if } q\left(R_{ij}^c\right) \geq q\left(\gamma_i^c\right) \text{ or } R_{ij}^c = (0,0) \\ (0,0), & \text{otherwise do not disclose} \end{cases}$$

At stage 3, the expected profit from a policy depends on the policy's price and the applicant's risk type. To develop an offer-cum-acceptance/denial strategy, a firm must formulate a belief concerning the applicant's risk-type from the information available at stage 3. That information encompasses agent strategies including the aggregate set of policy offers and firm and consumer disclosures.

Remark 2: Screening at stage 3. It is shown below, firms never have an incentive to disclose false information to firms offering policies at the same price, and low-risk types always report transactions at equal or higher prices truthfully, and reliably report total transactions no greater than do high-risk types.

Then, a rational firm's belief about an applicant's risk type is deducible from the information it has at stage 3. Suppose every applicant to firm j submits an identical low-risk optimal report of transactions, and no same price competitor disconfirms the report from c'. The firm's posterior belief about c's type should remain unchanged from its prior, the proportion of low-risk types in the market. Alternatively, suppose this first supposition is not true. Then either some applicant c reports different total transactions to j than does c', or some same price competitor disconfirms the report of c'. In the latter case, firm j can infer c' (who has lied) is high risk; in the former case, it can infer the consumer reporting greater coverage is high risk with probability 1, and it can infer the other consumer is low risk with probabilities m^L or 1 depending on whether the firm has offered one or two policies below the high-risk price (i.e. the firm itself is screening ala Rothschild-Stiglitz.

Note the importance of modeling many active consumers. Firms can deduce beliefs about an applicant's risk type because they can compare applications from many consumers. Also note, although firms have an incentive to disclose truthfully to same price firms, firms may have incentive to mislead higher price firms. Thus, firms neither disclose transaction information to lower price firms nor utilize information received from lower price firms in their formulation of a belief about an applicant's risk type. Keeping this in mind, in order to limit use of symbols, unless clearly said otherwise, all references to firm disclosures of transactions refer to firms disclosing to others with prices no lower than the disclosed policy.

In order to describe these beliefs formally, we first develop more nomenclature. Denote the set of all policies available to consumers at stage 2 by S, the sum of low-risk optimal policies (policies in S composing low-risk types' optimal portfolio) by $\gamma^L(\mathrm{S})$, and $\bar{\gamma}^L(\mathrm{S})$ the set of low-risk optimal policies summing to the optimal policy. Recall γ_j^c denotes consumer c's policy application to firm j, R_{ji}^c denotes consumer c's report to firm j of her application to firm i, and d_{ij}^c denotes firm i's disclosure to firm j of the application consumer c made to firm i. Without loss of generality, set $\mathrm{d}_{jj}^c = \gamma_j^c = \mathrm{R}_{jj}^c$. Also, define R_j and d_j to be, respectively, the sets of all applicant reports and firm disclosures to firm j.

Stipulating that the appearance of a firm's disclosure of a transaction d_{ij}^c refers to firms i such that $\mathrm{q}(\mathrm{R}_{ji}^c) \geq \mathrm{q}(\gamma_j^{c'})$, at stage 3, considering all consumers c applying to firm j, when evaluating an application $\gamma_j^{c'}$ firm j has the following belief consumer c′ is low-risk:

$$\mathrm{b}^3(\gamma_j^{c'}, \mathrm{R}_j, \mathrm{d}_j, \mathrm{S}) = \begin{cases} \mathrm{m}^{\mathrm{L}} \text{ if for } \forall \mathrm{c}, \sum_i \mathrm{R}_{ji}^c = \sum_i \mathrm{R}_{ji}^{c'} = \gamma^L(\mathrm{S}) \text{ and } \mathrm{R}_{ji}^c = \mathrm{d}_{ij}^c\ \forall \mathrm{i} \\ 0 \text{ if } \sum_i \mathrm{R}_{ji}^{c'} \neq \gamma^L(\mathrm{S}) \text{ e.g. } \exists \mathrm{c} \text{ s.t.} \sum_i \mathrm{R}_{ji}^{c'} > \sum_i \mathrm{R}_{ji}^c \text{ and } all\ \mathrm{R}_{ji}^c = \mathrm{d}_{ij}^c; \text{ or } \mathrm{R}_{ji}^{c'} \neq \mathrm{d}_{ij}^{c'}\ \exists \mathrm{i} \\ 1 \text{ otherwise, e.g. } \exists \mathrm{c} \text{ s.t.} \gamma_j^c \notin \bar{\gamma}^{\mathrm{L}}(\mathrm{S})\ and\ \gamma_j^{c'} \in \bar{\gamma}^{\mathrm{L}}(\mathrm{S}) \end{cases}$$

Observe, the only way a firm can believe a consumer is low-risk with certainty, is to offer two policies that successfully screen self-selecting consumers into their respective risk classes. This is a choice every high-risk consumer should view with suspicion.

Strategic Disclosure and Acceptance/Denial

We are now able to describe firms' offer-cum-acceptance/denial strategy.

Remark 3: Firms' offer-cum-acceptance/denial strategy is conditioned on the expected profitability of the prospective transaction. At stage 1, offer each consumer pooling price coverage limited to the amount optimal for a low-risk type reporting the consumer's stage 2 report of outside applications. At stage 3, if a policy transaction is believed profitable, accept it; if believed unprofitable, deny pooling price coverage and offer coverage at the high-risk fair price. Denote, the expected profit on a policy γ sold to risk type t by $\pi^t(\gamma) = (1 - p^t)\gamma_2 - p^t\gamma_1$. Then, noting, $\sum_{i \neq j} R_{ji}^c$ is consumer c's report to firm j of her aggregate policy application to other firms, the offer-cum-acceptance/denial is:

$$\alpha(\gamma_j^c, R_j, d_j, S) = \begin{cases} z^L\left(\hat{q}, \sum_{i \neq j} R_{ji}^c\right) \text{ if } b^3 \pi^L\left(z^L\left(\hat{q}, \sum_{i \neq j} R_{ji}^c\right)\vec{\hat{q}}\right) + (1 - b^3)\pi^H\left(z^L\left(\hat{q}, \sum_{i \neq j} R_{ji}^c\right)\overrightarrow{)\hat{q}}\right) \geq 0; \\ z^H\left(q^h, \sum_{i \neq j} R_{ji}^c\right) \text{ at price } q^h \text{ otherwise} \end{cases}.$$

Remark 4: Note, the only information used to decide whether to disclose an application is the price of the applied for policy and the consumer reported price of the firm receiving the disclosure. Alternatively, the information used to formulate firm beliefs about a consumer's type and the decision to accept or deny at the pooling price is the above plus all reports received from the firm's applicants (true or false), and all same price firms' application disclosures.

Chosen simultaneously, firm strategies are independent at stage 1 and consistent with firm information sets at all decision points. At stage 3 (observing consumer reports from stage 2), firms disclose applicants then deny or accept applications.

Consumer Behavior

The foundation of consumer behavior is their belief firms are sequentially rational. That is, (at stage 3) an application will be accepted if and only if the firm believes it promises nonnegative expected profit, and therefore, any application believed to promise negative expected profit will be denied. It is important to observe that consumers not endowed with this belief would naively maximize expected profit applying for policies and truthfully reporting transactions without regard to any effects their transactions have on firms' beliefs about their risk type and the profitability of the proposed transaction. Instead, I model consumers who formulate their stage two belief that a policy application will be accepted at stage 3 based on the conjecture all agents behave rationally. That is, at stage 2, each consumer assumes other consumers report low-risk optimal policies and firms disclose according to stage 1 announced strategies. This conjecture determines b^3 as a function of a consumer's own strategy and the aggregate set of policies. Then, b^2, consumers' stage 2 belief application $\gamma_j^{c'}$ will be accepted at stage 3 is determined by the conjectured value for b^3:

$$\mathrm{b}^2(\gamma_j^{c'}, \mathrm{R}_{ji}^{c'}, \mathrm{d}_{ij}^{c'}, \mathrm{S}) = \begin{matrix} 1 \text{ if } \mathrm{b}^3\pi^{\mathrm{L}}(\gamma_j^{c'}) + (1-\mathrm{b}^3)\pi^{\mathrm{H}}(\gamma_j^{c'}) \geq 0 \\ 0 \text{ otherwise} \end{matrix}.$$

If consumer type t's optimal choice of policies is a sum of policies supplementing some prime policy γ_j^t with coverage at price q, let $\mathrm{v}^t[\gamma_j^t]$ denote the attained *optimal* utility so $\mathrm{v}^t[\gamma_j^t] = V^t(\gamma_j^t + (z^t(q,\gamma_j^t), qz^t(q,\gamma_j^t))$. Now suppose type t has applied for her most preferred prime policy γ_j^t and firms limit additional coverage at the zero-profit pooling price $\hat{q}$ to the quantity α for any consumer reporting (or disclosed) to have applied for γ_j^t. The quantity α is firms' offer-cum-acceptance/denial quantity and is determined endogenously as described in Remark 3. Thus, at stage 3, the firm will believe any consumer applying for pooling coverage exceeding α is high-risk, promises negative expected profit, and should be denied, i.e., for any pooling coverage application γ_j^c, if

$$\gamma_j^c > \alpha = z^t(q,\gamma_j^t), \mathrm{b}^2(\gamma_j^c, (z^t(q,\gamma_j^t), qz^t(q,\gamma_j^t)),, d_{ij}^c, \mathrm{S}) = 0,$$

and no expected utility maximizing consumer would exceed α in an application for pooling coverage.

Given consumers' belief, the supplementary $\hat{q}$ coverage effectively demanded by consumer type t with the reported or disclosed γ_j^t is $\mathrm{D}^t(\hat{q}, \gamma_j^t, \alpha) = \mathrm{Min}\,\{\mathrm{z}^t(\hat{q}, \gamma_j^t);\ \alpha\}$. Let $\vec{\mathrm{q}}$ denote the vector [1,q] so, $\mathrm{z}^t(\mathrm{q})\vec{\mathrm{q}}$

equals the policy with coverage $z^t(q)$ at unit price q. Then, if a consumer reports applying for γ_j^c and coverage at price $\hat{q}$ is restricted to a maximum α, but unrestricted at price $q^h > \hat{q}$:

$$v^t[\gamma_j^c] = E[V^t(\gamma_j^c + D^t(\hat{q}, \gamma_j^c, \alpha)\vec{\hat{q}}) + z^t(q^h, \gamma_j^c + D^t(\hat{q}, \gamma_j^c, \alpha)\vec{\hat{q}})\overrightarrow{q^h}];$$

Where the expectation is understood to be taken with respect to consumer's $b^2(\ldots)$ beliefs equal to 1 for each applied for policy.

The forgoing shows this consumer maximization produces a specific portfolio of t-risk optimal policies and an aggregated sum of policies. Let S/γ_j^t denote the set S without γ_j^t, and designate $\{\gamma^L,\ \gamma^H\}$ to equal respectively, low-risk type's preferred policy in S and high-risk type's preferred policy in S/γ^L. Then, given an aggregate offer set S and firm strategies, risk-type t, optimizes expected utility.

Recalling $\gamma^t(S)$ is the sum of type t's optimal policies, and letting $\bar{\gamma}^t(S)$ denote the optimal set of policies for which type t applies, it follows, if consumers can apply for restricted and unrestricted coverage at prices $\hat{q}$ and q^h :

$$\bar{\gamma}^L(S) = \left\{\gamma^L,\ D^L(\hat{q}, \gamma^L, \alpha)\vec{\hat{q}}, z^L(q^h, \gamma^L + D^L(\hat{q}, \gamma^L, \alpha)\vec{\hat{q}})\overrightarrow{q^h})\right\}$$

$$\bar{\gamma}^H(S) = \begin{matrix}\{\gamma^L, D^L(\hat{q}, \gamma^L, \alpha)\vec{\hat{q}},\ z^H(q^h, \gamma^L + D^L(\hat{q}, \gamma^L, \alpha)\vec{\hat{q}})\overrightarrow{q^h}\})\text{ if } v^H[\gamma^L] \geq v^H[\gamma^H] \\ \{\gamma^H,\ D^H(\hat{q}, \gamma^H,, \alpha)\vec{\hat{q}},\ z^H(q^h, \gamma^H + D^H(\hat{q}, \gamma^H, \alpha)\vec{\hat{q}})\overrightarrow{q^h})\}\text{ otherwise}\end{matrix}$$

$$\gamma^L(S) = \gamma^L + D^L(\hat{q}, \gamma^L, \alpha)\vec{\hat{q}} + z^L(q^h, \gamma^L + D^L(\hat{q}, \gamma^L, \alpha)\vec{\hat{q}})\overrightarrow{q^h})$$

$$\gamma^H(S) = \operatorname{argmax}_{v^H[\gamma^H]}^{v^H(\gamma^L)}\ .$$

Thus,

$$\max_{\gamma \in S} v^t[\gamma]] = V^t(\gamma + D^t(\hat{q}, \gamma, \alpha)\vec{\hat{q}}) + z^t(q^h, \gamma + D^t(\hat{q},, \gamma, \alpha)\vec{\hat{q}})\overrightarrow{q^h}$$

For some set of policies $\bar{\gamma}(S)$ with $b^2(\gamma \ldots) = 1$ for each γ in $\bar{\gamma}^t(S)$.

Remark 5 (optimal reporting): Both risk types choose a prime policy and possibly supplement it to maximize expected utility. Low-risk types truthfully report any application γ_j^c to firms to which they apply for an equal or higher price policy. High-risk types truthfully report any application that would be reported by a low-risk type, but do not report policies not low-risk optimal.

To formalize remark 5, if consumer c is type t, transacts with firm j and γ_i^c is her transaction with firm i, R_{ji}^c her report to j of her transaction with firm i is:

$$R_{ji}^c(\gamma_i^c, \gamma_j^c, S) = R_{ji}^t = \begin{matrix} \gamma_i^c \text{ if } \gamma_i^c \in \bar{\gamma}^L(S) \textit{ and } q(\gamma_i^c) \leq q(\gamma_j^c), t = L, H \\ (0,0) \textit{ otherwise} \end{matrix}.$$

In equilibrium, the low-risk types truthfully report all transactions, while the high-risk types withhold only those not low-risk optimal.

It is now possible to describe the equilibrium policies for each risk type. Observe, $\gamma^{L*} = [z^L(\hat{q}), \hat{q}z^L(\hat{q})]$ maximizes low-risk utility at the zero profit pooling price, and $\gamma^{H*} = z^H(q^h, \gamma^{L*}),\ q^h z^H(q^h, \gamma^{L*})$ is the policy maximizing high-risk utility when supplementing γ^{L*} at the high-risk fair price. By lemma 1a, γ^{L*} provides incomplete coverage, and $\gamma^{L*} + \gamma^{H*}$ provides full coverage. Below we show these are the types' respective equilibrium allocations, and all references to deviations from equilibrium refer to deviations from these allocations. It will also turn out that letting S^* denote the equilibrium aggregate policy offering, $\gamma^L = \gamma^{L*}$ and $\gamma^H = \gamma^{H*}$. Until relaxed, assume $\gamma^{L*} > (0, 0)$.

Equilibrium

Proposition 1: Let consumers and firms hold the respective beliefs $b^2(\gamma_i^c, R_{ji}^c, d_{ij}^c, S)$ and $b^3(\gamma_i^c, R_j, d_j, S)$. If firms adopt the offer-cum-acceptance/denial and disclosure strategies $\alpha(\gamma_i^c, R_j, d_j, S)$ and $d_{ij}^c(\gamma_i^c, S)$, and consumers follow the transaction strategy $\gamma^t(S)$ and report transactions according to $R_{ji}^c(\gamma_i^c, S)$, strategies and beliefs support an equilibrium. In equilibrium, each firm j offers consumers two nonexclusive divisible prime policies $\{\gamma_j^L, \gamma_j^H\} = \{\gamma^{L^*}, \gamma^{H^*}\}$, and therefore $S^* = \{\gamma | \gamma = \lambda\gamma^{L^*} + \mu\gamma^{H^*}$ for $\exists\lambda$ and $\exists\mu$ each $\in [0, 1]\}$, i.e. the convex hull of the two prime policies. Furthermore, $\gamma^L(S^*) = \gamma^{L^*}$ and $\gamma^H(S^*) = \gamma^{L^*} + \gamma^{H^*}$.

Proving Proposition 1 requires showing each risk-type's portfolio of policies and communications is optimal at their decision point; that no firm could increase its expected profit by deviating from its strategy (offer-cum-acceptance/denial or, communication) at its decision points. The task is accomplished in two steps. First, taking the strategies of Proposition 1 as given, I show those strategies induce a portfolio of policy purchases for each risk type conforming to Proposition 1. Second, I demonstrate strategies are sequentially rational (i.e. no alternative feasible strategy could earn any consumer type greater expected utility or any firm positive expected profit).

Consumer and Firm Behavior on Equilibrium Path

Figure 1 depicts consumers' trading opportunities in indemnity-premium space (γ_1, γ_2). Letting q^t denote the fair odds of a claim for type t, the rays $\gamma_2 = \hat{q} \cdot \gamma_1$; $\gamma_2 = q^L \cdot \gamma_1$; $\gamma_2 = q^h \cdot \gamma_1$ represent the loci of policies earning zero profit if purchased by respectively, the pooled, low-risk, and high-risk consumers. At stage 2, the consumer's choice set includes the null policy, the pooling policy γ^{L*} and its derivatives, and the high-risk fair odds priced policies. The filled bold line depicts the set of policies providing any coverage γ_1 at minimum cost.

The policy γ^{L*} maximizes low-risk consumers' expected utility on the zero-profit pooling portion of the minimum cost line. According to consumer strategies, any low-risk consumer c applying to some firm j for $\gamma^{L*} = \gamma_j^c$ truthfully reports total policy applications (0,0) with other firms, and high-risk types do the same. Therefore, all client reports of transactions to firm j are identical and low-risk optimal, and for any c, $R_{ji}^c = d_{ij}^c = (0,0)$ all $i \neq j$, and $R_{ii}^c = \gamma_i^c = \gamma^{L*} = z^L(\hat{q}, \sum_{i \neq j} R_{ji}^c)$. Therefore, the probability each such firm j believes any client is low-risk is m^L, and firm j believes the transaction is profitable. Thus, $\alpha(\gamma_j^c, R_j, d_j, S^*) = \gamma^{L*}$ implying

$$D^L(\hat{q}, \gamma^{L*}, \alpha)\vec{\hat{q}} = D^L(\hat{q}, \gamma^{L*}, \gamma^{L*})\vec{\hat{q}} = 0 = z^L(q^h, \gamma^{L*} + D^L(\hat{q}, \gamma^{L*}, \gamma^{L*})\vec{\hat{q}})\overrightarrow{q^h}$$

and low-risk consumers' optimal policy is:

$$\gamma^L(S^*) = \gamma^{L*} + D^L(\hat{q}, \gamma^{L*}, \gamma^{L*})\vec{\hat{q}} + z^L(q^h, \gamma^{L*} + D^L(\hat{q}, \gamma^{L*}, \gamma^{L*})\vec{\hat{q}})\overrightarrow{q^h} = \gamma^{L*}.$$

Although every high-risk type would prefer greater coverage at the pooling price, applying for more coverage than γ_1^{L*} either requires exceeding the pooling limit $\alpha(\gamma_j^c, R_j, d_j, S^*) = \gamma^{L*}$ in an application to one firm j or exceeding this limit through multiple applications and falsely reporting the low-risk optimal total to each firm. Given agents' reporting strategies, either action would lead firms to believe the consumer high-risk, thus, high-risk consumers believe doing either would result in denial of their pooling applications producing suboptimal expected utility. The high-risk optimal transaction strategy is:

$$\gamma^H(S^*) = \gamma^{L*} + D^L(\hat{q}, \gamma^{L*}, \gamma^{L*})\vec{\hat{q}} + z^H(q^h, \gamma^{L*} + D^L(\hat{q}, \gamma^{L*}, \gamma^{L*})\vec{\hat{q}})\overrightarrow{q^h} = \gamma^{L*} + \gamma^{H*}.$$

The policy $\gamma^{L*} + \gamma^{H*}$ maximizes high-risk expected utility on the minimum cost line with each applying for γ^{L*} from some firm j then applying to another firm i for insurance priced at high-risk fair odds trading up hh the price line of slope q^h through γ^{L*} in Figure 1. Sold policies have zero expected profit.

Remark 6: Derivatives of γ^{L*} and γ^{H*} are *latent* policies -- unsold in equilibrium, these latent policies perform a strategic function analogous to an oligopoly pre-committing stage 1 sunk costs to credibly signal a potential entrant it will earn negative profit.

The next section of the paper shows existence of equilibrium leans heavily on these latent policies. If the latent policies were not available for trade, firms would be able to design alternative policies that cream-skim low-risk consumers for positive expected profit.

Consumer and Firm Behavior off Equilibrium Path

The demonstration that firm strategies are sequentially rational begins with their offer-cum-acceptance/denial strategy. I first show no deviation designing policies different from the equilibrium offer promises positive expected profit. Then I show no deviation from the offer-cum-acceptance/denial strategy could either. Afterwards, firm and consumer disclosure and reporting strategies are shown to be sequentially rational. Given the proposed equilibrium, any deviation attracting only the high-risk type promises negative profit, and because any alternative policy desired by low-risk types must be priced below $\hat{q}$, it would promise negative profit if purchased by both risk types. Thus, any offer capable of promising positive expected profit to upset equilibrium must separate risk types either by offering two policies with the low-risk's preferred policy subsidizing the high-risk's, or by offering one policy that cream-skims the low-risk leaving the high-risk buying from competitors at the higher prices.

Cross-subsidizing Policies: Two policies exhibit cross-subsidization if the risk-types separate, one policy promises negative, the other positive profit, and total expected profit is nonnegative.

Cream-Skimming Policies: A policy cream-skims if the low-risk type prefers it to γ^{L*}, the high-risk type does not, and it promises positive expected profit if bought only by the low-risk type.

Cross-subsidization and cream skimming cover all relevant deviations from the equilibrium strategies. To examine their properties, it is useful to distinguish "large" versus "small" deviations.

Large and Small Policy Deviations

Definitions: A large policy deviation is a single policy γ satisfying $\frac{\gamma_2}{\gamma_1} \leq \hat{q}$ and $V^L(\gamma) \geq V^L(\gamma^{L*})$. A small policy deviation is a single policy γ satisfying $\frac{\gamma_2}{\gamma_1} < \hat{q}$ and $V^L(\gamma) < V^L(\gamma^{L*})$, but $v^L(\gamma) > v^L(\gamma^{L*})$.

Cream skimming requires a large policy deviation because the low-risk type *substitutes* a large policy deviation for γ^{L*}. In contrast, the low-risk type will not substitute a small policy deviation for γ^{L*} but would use it to construct a new portfolio if enough coverage at price $\hat{q}$ remains available. Therefore, she applies for the small policy deviation and activates latent derivatives of γ^{L*} to supplement the deviation.

Cream-Skimming and Cross-Subsidizing Deviations

Suppose some firm considers unilaterally offering $\{\gamma_d^L, \gamma_d^H\}$ deviating from equilibrium. At stage 2, each consumer would face a choice set including the $\{\gamma_d^L, \gamma_d^H\}$ policies, pooling policies, the high-risk priced policies, and the null policy. Limited to purchasing one deviation policy, a consumer would have the option of purchasing any feasible combination to construct one of two portfolios comprising respectively either deviation policy supplemented with any coverage available at the pooling and high-risk prices. The demonstration that no deviation strategy could promise positive expected profit begins by first disposing of the possibility of breaking equilibrium with a single cream-skimming policy.

Begin by demonstrating possible cream-skimming policies exist. In Figure 1, consider *any* policy γ satisfying three conditions: it lies below the low-risk indifference curve (LL) tangent to the pooling price line at γ^{L*}, on or above the q^h price line hh through γ^{L*}, and above the low-risk zero-profit line. By construction, γ is a large policy deviation with positive expected profit if only purchased by low-risk consumers. If an applicant cannot supplement it with pooling coverage, the low-risk consumer strictly prefers any such γ to γ^{L*} but the high-risk type does not. To see this last point, note the high-risk equilibrium allocation is on indifference curve HH through policy h* and tangent to the q^h price line hh through γ^{L*}. Since γ lies on or above this q^h price line through γ^{L*}, the (not shown) budget line of slope q^h through the cream-skimming γ defining the policies available to a consumer supplementing γ with high-risk insurance is also on or above the parallel q^h price line through γ^{L*}. Trading up the budget line from γ would either take the high-risk consumer to her equilibrium policy h* or to a best total policy left of the high-risk equilibrium indifference curve. This shows for high-risk consumers to prefer γ to γ^{L*} they would have to be able to supplement γ with pooling coverage as well as q^h price coverage. A firm offering γ as an exclusive disclosed transaction hopes to prevent high-risk consumers from purchasing it and supplementing with enough pooling coverage to make γ also preferred to γ^{L*} by high-risk consumers.

Lemma 2. The offer-cum-acceptance/denial strategy is optimal at stage 1.
The demonstration has three parts.

Lemma 2a: No cream-skimming deviation has positive expected profit.

Proof: Any cream-skimming deviation has form $\{\gamma_d^L, (0,0)\}$. Suppose γ_d^L is a large deviation policy lying below the low-risk indifference curve through γ^{L*} and on or above the q^h price line through γ^{L*} in Figure 1. Since $\gamma_d^L \neq \gamma^{L*}$, we have $\frac{\gamma_{d2}^L}{\gamma_{d1}^L} < \hat{q}$. To offer positive expected profit, the prospective deviation must only attract the low-risk. However, a deviating firm would offer the market's least price policy and would receive no reports or disclosures of its clients' other transactions, and would have no exclusivity basis for denying applications. Hence consumers may apply for γ_d^L and supplemental pooling coverage without fear of denial based on violating exclusivity. For any consumer c applying to a pooling seller j, this means $\alpha(\gamma_j^c, R_j, d_j, S^* \cup \gamma_d^L) = z^L(\hat{q}, \sum_{i\neq j} R_{ji}^c)$ if firm j believes the transaction promises nonnegative expected profit at the pooling price and $\alpha(\gamma_j^c, R_j, d_j, S^* \cup \gamma_d^L) = z^h(q^h, \sum_{i\neq j} R_{ji}^c)$ otherwise. For firm j to believe transacting at the pooling price would promise nonnegative expected profit, the consumer must report low-risk optimal transactions. Thus, firms would deny any application for pooling coverage that fails to report the consumer applied for γ_d^L, and the *only* policies available to any consumer are γ_d^L, total pooling coverage to $z^L(\hat{q}, \gamma_d^L)$ (conditional on reporting γ_d^L) and the q^h price policies. It follows, $\gamma^t(S) = \gamma_d^L + z^L(\hat{q}, \gamma_d^L)\vec{\hat{q}} + z^t\left(q^h, \gamma_d^L + z^L(\hat{q}, \gamma_d^L)\vec{\hat{q}}\right)\overrightarrow{q^h}$ for t = L and H. A firm considering this deviation would expect to face a pool of applicants all reporting the same transactions without being disconfirmed by any competitor hence believes each applicant low-risk with probability m^L. Since $\frac{\gamma^L_{d2}}{\gamma^L_{d1}} < \hat{q}$, and γ^L_d attracts both risk types, it forecasts negative expected profit. Alternatively, firms selling pooling coverage offer $\alpha(\gamma^c_j, R_j, d_j, S^* \cup \gamma^L_d) = z^L(\hat{q}, \gamma^L_d)$ coverage to each consumer for zero expected profit. But then, based on stage 3 beliefs, the prospective deviator must expect to find itself facing negative profit at stage 3 leading it to deny all applications which implies no consumer would apply for the policy at stage 2. Thus, a firm contemplating deviating with a large cream-skimming policy could at best expect zero profit. A deviation offering a small policy deviation γ^L_d triggers the same argument showing no small policy deviation promises positive expected profit either.

Lemma 2b: No cross-subsidizing deviation has positive expected profit**.**

Proof: First observe, equilibrium policies provide enough pooling coverage and coverage at the high-risk fair price to sate low-risk and high-risk types respectively. If both deviation policies are demanded, it must be true that $\frac{\gamma^L_{d2}}{\gamma^L_{d1}} < \hat{q}$ and $\frac{\gamma^H_{d2}}{\gamma^H_{d1}} < q^h$. If the latter inequality were not true, since high-risk types can obtain all the undisclosed q^h coverage they desire from non-deviating firms, γ^H_d could not attract them. Moreover, $\frac{\gamma^H_{d2}}{\gamma^H_{d1}} < q^h$ implies $\pi^H(\gamma^H_d) < 0$ and γ^H_d is the subsidized policy. Since γ^L_d and γ^H_d separate risk types, the deviating firm d would observe for any pair of consumers c and c' with $\gamma^c_d = \gamma^H_d = R^c_{dd}$ and $\gamma^{c'}_d = \gamma^L_d = R^{c'}_{dd}$ that c reports an application not low-risk optimal. This implies $b^3(\gamma^H_d, R_j, d_j, S^* \cup \gamma^H_d \cup \gamma^L_d) = 0$ and $b^3\pi^L(\gamma^H_d) + (1 - b^{3)}\pi^H(\gamma^H_d) < 0$. Therefore, $b^2(\gamma^H_d, R^c_{jd}, d^c_{dj}, S^* \cup \gamma^H_d \cup \gamma^L_d) = 0$. Since consumers believe an application for the high-risk preferred policy γ^H_d promises a denial at stage 3, no consumer applies for it. This implies cross-subsidizing deviation policies are equivalent to offering a cream-skimming or small deviation policy that by lemma 2a would have negative expected profit.

Lemma 2c. No firm could increase expected profit by changing the price or coverage of its prime policy offers leaving the acceptance/denial structure intact.

Given competitors' strategies, it is obvious no firm could increase expected profit by raising its price on the high-risk policy. Alternatively, lowering the high-risk fair price toward the pooling price would attract only the high-risk consumer lowering expected profit. Moreover, firms already supply enough coverage at the high-risk price to sate consumers. Similarly, if a firm were to raise its price on its pooling policy (between the pooling and high-risk prices) it would expect to lose its low-risk consumers to firms selling at the pooling price, and could not increase its expected profit. Any firm lowering the price of its pooling policy would expect to sell to the pooled risk for a reduction in profit. Finally, since low-risk consumers optimize for price $\hat{q}$ at $z^L(\hat{q}, \sum_{i \neq j} R_{ji}^c)\vec{\hat{q}}$, a firm unilaterally accepting some maximum coverage $\alpha' > \alpha(\gamma_j^c, R_j, d_j, S) = z^L(\hat{q}, \sum_{i \neq j} R_{ji}^c)\vec{\hat{q}}$ or $\alpha' < \alpha(\gamma_j^c, R_j, d_j, S) = z^L(\hat{q}, \sum_{i \neq j} R_{ji}^c)\vec{\hat{q}})$ could not expect to increase sales to low-risk clients, thus, could not raise expected profit.

Lemma 3. The offer-cum-acceptance/denial strategy is optimal at stage 3.

Given strategies and belief $b^3(\gamma_j^c, R_j, d_j, S)$ at stage 3, on or off the equilibrium path, firm j maximizes expected profit with $\alpha(\gamma_j^c, R_j, d_j, S)$. Deviating from this strategy (e.g. by denying a transaction believed profitable or accepting a transaction believed unprofitable could not be optimal.

Lemma 4. Communication strategies are sequentially rational

At stage 1, suppose firm j switched its disclosure strategy promising to disclose clients to firms offering a policy at a lower price. At stage 2, on or off the equilibrium path, if the disclosure reveals a consumer high-risk to firms offering policies below q^h, high-risk consumers would not apply to the disclosing firm for q^h coverage, but a disproportionately high number of such types would now be expected to apply to firm j for pooling coverage lowering its profit. Thus, such a disclosure strategy could not be optimal for a firm's high-risk policies. Now consider a firm's pooling policies. Clearly, on the equilibrium path, there are no lower price firms to disclose to. Alternatively, off the equilibrium path, at stage 2, consumers believe j's later disclosure to a lower price firm would reveal a consumer violated the exclusivity restriction on a lower price deviation policy, and no consumer would apply to j for pooling coverage implying j could not increase expected profit. Alternatively, if at stage 3, the firm were to renege on its stage 1 strategy and disclose clients, the change would allow lower-price sellers to deny the disclosed clients but would not change firm j's expected profit. Now suppose firm j unilaterally switches its client disclosure strategy and does not disclose to same price firms. Since low-risk consumers are sated at price $\hat{q}$, changing at stage 1 not to disclose clients along the equilibrium path could only attract more high-risk types and reduce expected profit. Similarly, off the equilibrium path, if firm j announced an altered disclosure strategy at stage 1, the change could only attract disproportionate high-risk types and could not raise expected profit. If firm j were to renege on its contractual promise and disclose clients reporting a $\gamma_{\mathrm{d}}^{\mathrm{L}}$ cream-skimming policy at stage 3, the disclosure could enable the firm offering the exclusive $\gamma_{\mathrm{d}}^{\mathrm{L}}$ to deny the disclosed clients, but could not increase firm j's expected profit. Similar arguments demonstrate optimality of consumer reporting strategies.

Possibility of a Lemon Equilibrium

Until now, we assumed low-risk types' most preferred zero profit pooling policy is positive. Suppose the low-risk types' optimal zero profit pooling policy is the null contract. From Proposition 1:

Lemon Effect: Corollary to Proposition One. Suppose $\gamma^{L*} = (0, 0)$. Firm beliefs and strategies described in Proposition 1 support equilibrium with the low-risk consumer entering no transactions, and the high-risk transacting for complete coverage at the high-risk fair price.

Under the conditions of the corollary, the policy set available to consumers is described as in Proposition One. Given Proposition One's strategies, low-risk consumers demand optimal pooling policy $\gamma^{L*} = (0,0)$ (thus, never apply for insurance) and high-risk supplement γ^{L*} with the full-coverage policy priced at high-risk fair odds. Equilibrium follows from the lemmas proving Proposition 1.

This corollary implies the intuitive result that adverse selection can produce a lemon effect driving the low-risk type from a market where trade in policies priced below high-risk fair odds cannot occur. The result obtains because the low-risk type will not buy positive coverage at or above the zero-profit pooling price and the high-risk type dissimulates mimicking low-risk behavior to render any policy the low-risk would purchase unprofitable. The lemon equilibrium will occur whenever (depending on the absolute risk aversion of the utility function) risk type probability spreads are "large" and/or the low-risk proportion m^L is "small" and/or incomes in the two events loss/no loss are "close."

Equilibrium and Hidden Policies

This section demonstrates equilibrium continues to exist even under Martin Hellwig's (1988) nonstandard assumption allowing a firm to deviate by offering a "secret" policy unobserved by its competitors. The basic intuition behind equilibrium's robustness to this assumption is, although a deviating firm and its high-risk clients may have incentives to hide a policy from the firm's competitors, its low-risk clients (preferring to signal they are low-risk) do not. Low-risk consumers' incentive to reveal transactions prevent a firm deviating from equilibrium from keeping its offers hidden from competitors.

If firms offer unpublished policies, the model becomes a search model (albeit one assuming zero consumer and prohibitive firm search costs). Consumers should sample every firm to ascertain if there exists a desired hidden offer. Thus, at stage 2, consumers observe published and unpublished policies but firms only observe published policies and consumer communications of transactions.[2] To show the equilibrium described below in the corollary to Proposition 1 exists, Lemma 5 shows truthful disclosure of offers and transactions remains incentive compatible for firms and consumers, respectively. Thus, firm

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[2] Alternatively, it could be assumed each firm simply receives its fair share of consumers of both types and there is no search. It will be apparent that the results obtain no matter which of the assumptions is made.

and consumer information sets are identical at each stage whether firms can offer unpublished policies or not, and the corollary follows from the arguments demonstrating Proposition 1.

Publishing All Offered Policies is Optimal

Equilibrium is established by showing optimal low-risk strategy, firm beliefs, disclosure, and offer-cum-acceptance/denial strategies induce competitors to publish their true offers and consumers to report low-risk optimal transactions truthfully. Since this leaves all agents' information sets identical at each decision point whether policies can be hidden or not, arguments establishing equilibrium in the latter case apply. Incentive compatibility occurs for two reasons: given firm strategy, low-risk consumers always have incentive to report true information, and in the event of a deviation from equilibrium, low-risk reports are identifiable because the amount of coverage they demand and report is reliably less than the high-risk.

Claim 1: Truthful reporting of all transaction policies priced equal to or below the pooling price is incentive compatible for both risk types even if hidden policy offerings are possible.

This claim is demonstrated in three parts. First for the two cases when some firm's deviation would take action off the equilibrium path, and then for the equilibrium path case to show consumers have no incentive to mislead firms into believing they are off the equilibrium path. Observe any possible profitable deviation must still be either a cream-skimming offer or a cross-subsidizing pair. Because a cream-skimming offer could only promise positive expected profit if only sold to low-risk types, a deviating firm can achieve that result only if high-risk consumers have access to a policy they prefer to the cream-skimming deviation. To see why this cannot occur, first, suppose at the otherwise equilibrium set of offers a firm deviating were to offer a hidden cream-skimming policy requiring exclusivity under penalty of denial. The deviating firm hopes its competitors (unable to observe its offer) will continue offering all pooling policies attracting all high-risk consumers with γ^{L*} while it sells the hidden cream-skimming policy to low-risk consumers. This deviation would fail because, given firms' beliefs, and disclosure and offer strategies, both risk types would prefer to apply for the cream-skimming policy, report it to the deviating firm's competitors, and apply for supplementary pooling coverage.

Claim 1a: Consumers truthfully report any hidden cream-skimming deviation to firms selling at pooling price and no cream-skimming deviation has positive expected profit.

Proof: First observe if every consumer prefers the cream skimming policy supplemented with pooling coverage to γ^{L*}, all would apply for γ_d^L and it will earn negative profit. This is so because any consumer, reporting the deviation policy γ_d^L to some pooling price firm j, knows the deviating firm (having the lowest price, $q(R_{jd}^c) < \hat{q}$ will not receive transaction disclosures from other firms, thus cannot enforce any exclusivity condition. Therefore, consumers may apply for the cream-skimming policy without fear of being denied if they supplement it. Thus, establishing the claim requires showing *all* consumers would prefer supplementing γ_d^L so higher price sellers (i.e. pooling firms) would deny applications not reporting γ_d^L. Pursuing her equilibrium reporting strategy, if low-risk consumer c applies for supplemental pooling coverage from firm j, she reports $R_{jd}^c = \gamma_d^L$ to firm j. Given firm beliefs, $\alpha(R_{jd}^c, R_j, d_j, S^* \cup \gamma_d^L)\vec{\mathrm{q}} = z^L(\hat{q}, \gamma_d^L)\vec{q}$, and by definition, $V^L[\gamma_d^L + z^L(\hat{q}, \gamma_d^L)\vec{q}] \geq V^L[\gamma_d^L]$ with equality holding only if $z^L(\hat{q}, \gamma_d^L) = 0$. Note equilibrium could be broken only if this last condition obtains implying no low risk type will apply for pooling coverage so the deviation remains hidden from pooling policy sellers. In that case, pooling firms believe they are on the equilibrium path, and high-risk types can obtain γ^{L*} (which by construction they prefer to the cream-skimming policy) from pooling sellers.

I show $z^L(\hat{q}, \gamma_d^L) > 0$ must be true, so low-risk types applying for the cream-skimming policy find it optimal to report it to a pooling seller for supplementary coverage, and for $t = L \text{ or } H$,

$$v^t[\gamma_d^L + \alpha(\gamma_d^L, \dots)\vec{q}] > v^t[\gamma^{L*}].$$

Recall (if high-risk types are to prefer γ^{L*}) in Figure 1, any potential cream-skimming policy γ_d^L must lie on or above (left) of the q^h price line through γ^{L*}. I show $\gamma_d^L + z^L(\hat{q}, \gamma_d^L)\vec{\hat{q}}$ lies below (right) of this q^h price line through γ^{L*} in Figure 1 so high-risk consumers can supplement γ_d^L with $\hat{q}$ coverage $\alpha(\gamma_d^L, \dots)\vec{\hat{q}} = z^L(\hat{q}, \gamma_d^L)\vec{\hat{q}}$ to reach a q^h price line beyond the q^h price line through γ^{L*} to obtain a full-coverage policy preferred to the one reached by supplementing γ^{L*} with q^h price coverage. This implies both risk-types would prefer γ_d^L with supplementary pooling coverage. The key to this last claim is:

Lemma 5: *Income consumption curves at price q^* (AA for low-risk in Figure 1) have negative slope.*

The proof of lemma 5 is purely technical and is given in the appendix.

In Figure 1, the curve AA is the income-consumption curve for price $\hat{q}$ of a low-risk consumer. Each policy on AA is an optimal total policy $\gamma + z^L(\hat{q}, \gamma)$ demanded by a low-risk consumer able to supplement some limited policy γ with additional coverage at the fixed price $\hat{q}$. As drawn in the figure, AA is negatively sloped through γ^{L*}, and lies right of the positively sloped q^h price line through γ^{L*} for all γ on this price line satisfying $q(\gamma) < \hat{q}$. Any cream-skimming policy γ_d^L must lie on or above the q^h price line through γ^{L*}, and $\gamma_d^L + z^L(\hat{q}, \gamma_d^L)$ lies on AA showing $z^L(\hat{q}, \gamma_d^L) > 0$. Moreover, from $\gamma_d^L + z^L(\hat{q}, \gamma_d^L)$ consumers reach a q^h price line right of the one through γ^{L*}. Because the income-consumption curve of a high-risk consumer must lie right of AA, both risk types would prefer supplementing any cream-skimming policy under these circumstances. Hence, the hidden cream-skimming deviation would collapse to the conditions of a published cream-skimming deviation, and a firm contemplating such a deviation would expect no consumer would apply for it because its negative expected profit would lead to denial, see lemma 2a.

Claim 1a establishes a "hidden" cream-skimming policy will be revealed to competitors at stage 2 and be unprofitable. Suppose the deviating firm attempts to circumvent this problem by making the type of cross-subsidizing offer proposed by Hellwig (1988). The firm publishes its offer of the equilibrium pooling policy γ^{L*} and offers a hidden exclusive cream-skimming policy preferred to γ^{L*} only by the low-risk.

Claim 1b: No hidden cross-subsidizing policies can earn positive expected profit.

Proof: Since the deviation's two policies separate risk types, given their beliefs, no consumer would reveal herself high-risk to the deviating firm by applying for the non-low-risk optimal γ^{L*}. This means if high-risk are to buy γ^{L*} it must be from one of the deviating firm's competitors. However, that means the situation is identical to the previous case of a lone cream-skimming policy that must only attract low-risk types. However, the argument establishing Claim 1a shows both risk-types would prefer to purchase and supplement the deviation intended for the low-risk only. Furthermore, the argument shows no two cross-subsidizing policies with the low-risk intended policy hidden could have positive expected profit, and it is clear a deviation policy can remain hidden at stage 2, only if it is uniquely preferred by high-risk consumers with an incentive not to report it. However, to achieve this result and anticipate positive expected profit, the deviating firm must still separate self-selecting risk types with cross-subsidizing policies. In this case, given consumer beliefs, the high-risk, expecting denial of their application at stage 3, would not select the high-risk revealing policy, and the offer could not earn the deviating firm positive expected profit.

The preceding arguments demonstrate equilibrium remains robust to deviations from firms unilaterally designing alternative policy offerings. It does not, however, establish robustness to deviations in consumer strategy. If firms can offer hidden policies, it must be shown no consumer type (observing the equilibrium offer set) could increase her expected utility by falsely reporting she applied for a hidden cream-skimming policy. That is, it must be shown:

Claim 1c: Neither risk type has incentive to deviate from the equilibrium path even if hidden policy offerings are possible.

Proof: The demonstration begins by showing the claim is true for low-risk types. Suppose at the equilibrium allocation (with all policy offers published), some low-risk consumer c (who applied to firm j for γ_j^c pooling coverage) also applies to firm i for pooling coverage γ_i^c falsely reporting to firm i she applied for a hidden cream-skimming policy γ_d^L from j. Observe that irrespective of its effects on firm i's beliefs concerning consumer c's risk-type, this false report could at best induce firm i to accept consumer c with pooling coverage $\alpha\left(\gamma_d^L, R_i, d_i, S^* \cup \gamma_d^L\right)$. However, since consumer c falsely reported the cream-skimming policy, any additional pooling coverage from i merely adds to the γ_j^c she seeks from firm j. Since γ^{L*} optimizes her demand for pooling coverage, she could have made $\gamma_j^c = \gamma^{L*}$ showing the false report gives her no gain. Therefore, assume the market is not in equilibrium and γ_j^c is in fact a hidden cream-skimming policy, but low-risk consumer c falsely reports to firm i she applied for another hidden policy γ_d^c. Again, we may conveniently ignore any effects this strategy has on firm i's beliefs about c's risk type because such a false report could not possibly increase the low-risk type's expected utility. The best utility level she can attain is to supplement the actual cream-skimming policy from j with supplementary pooling coverage $\alpha(\gamma_d^c, R_i, d_i, S^* \cup \gamma_d^c)$ that at best provides her expected utility $V^L(\gamma_j^c + z^L(\hat{q}, \gamma_d^c)\vec{\hat{q}})$ which is maximized when she truthfully reports $\gamma_j^c = \gamma_d^c$. This shows low-risk consumers always optimize by truthfully reporting their transactions with same price pooling and lower-price deviating firms.

Therefore, assume that at the equilibrium allocation, consumer c is high-risk and considering deviating from the equilibrium by falsely reporting to firm i she applied for a hidden cream-skimming policy (γ_d^c) with firm j when she really applied for pooling coverage γ_j^c. Since consumer c desires more coverage than γ^{L*}, if successful, the subterfuge would increase her expected utility. If consumer c fails to report a low-risk optimal demand $\gamma_d^c + z^L(\hat{q}, \gamma_d^c)$ to i, she will be inferred high-risk. Because we are on the equilibrium path, i receives from any other client c′, a report $\sum_j R_{ij}^{c'} = (0,0) + \gamma_i^{c'}$ with $\gamma_i^{c'} = \gamma^{L*}$. As shown earlier, Lemma 5 implies the total coverage in the fabricated low-risk optimal report must exceed γ^{L*} and $\sum_i R_{ij}^{c} > \sum_i R_{ij}^{c'}$ for any other applicant to j. Given j's beliefs, she would accurately infer applicant c to be high-risk and deny her application. It follows, consumer c optimizes by applying to one firm only for γ^{L*}, and truthfully reporting transactions.

Discussion

The Jaynes-Hellwig-Glosten equilibrium policy configuration was first obtained in Jaynes [1978], and subsequently shown to be an equilibrium outcome under a robust set of modeling conditions (Beaudry and Poitevin, 1993, 1995; Attar, Mariotti, & Salanie', 2014; Dubey & Geanakoplos, 2002). Under the assumptions of the model, it is obvious and trivial to show that the equilibrium is unique. No previous paper endows both uninformed and informed agents with beliefs. However, as remarked earlier, the consumers' belief that firms are sequentially rational is implicitly assumed in all other papers discussing similar issues. Otherwise, consumer behavior (especially of high-risk agents) cannot be rationalized. Furthermore, consumer's belief that firms are sequentially rational and would deny an applicant who has revealed herself high-risk by self-selecting a policy only the high-risk would accept is logically implied by the model and apparently necessary to prevent a deviation with cross-subsidizing policies from breaking the equilibrium. In this regard, it is interesting that Rothschild and Stiglitz (1973), writing without a multistage framework, and prior to the widespread use of the concept sequential rationality, originally believed an equilibrium policy set could not include policies with negative expected profit because a rational firm would want to withdraw such policies. They ultimately abandoned this view because of the possibility of cross-subsidizing policies, the treatment of which remained open until this paper. I argue R-S initial intuition was correct. In a modeling framework with each firm coordinating its offer and acceptance/denial decision within the common timing sequence modelled in this paper, sequential rationality requires no firm sell (accept) a policy with negative expected profit at stage 3. In effect offers off the equilibrium path that look too good to be true, are. However, it requires introducing both the multi-stage dynamic framework and forward-looking consumers with rational beliefs to model this point appropriately.

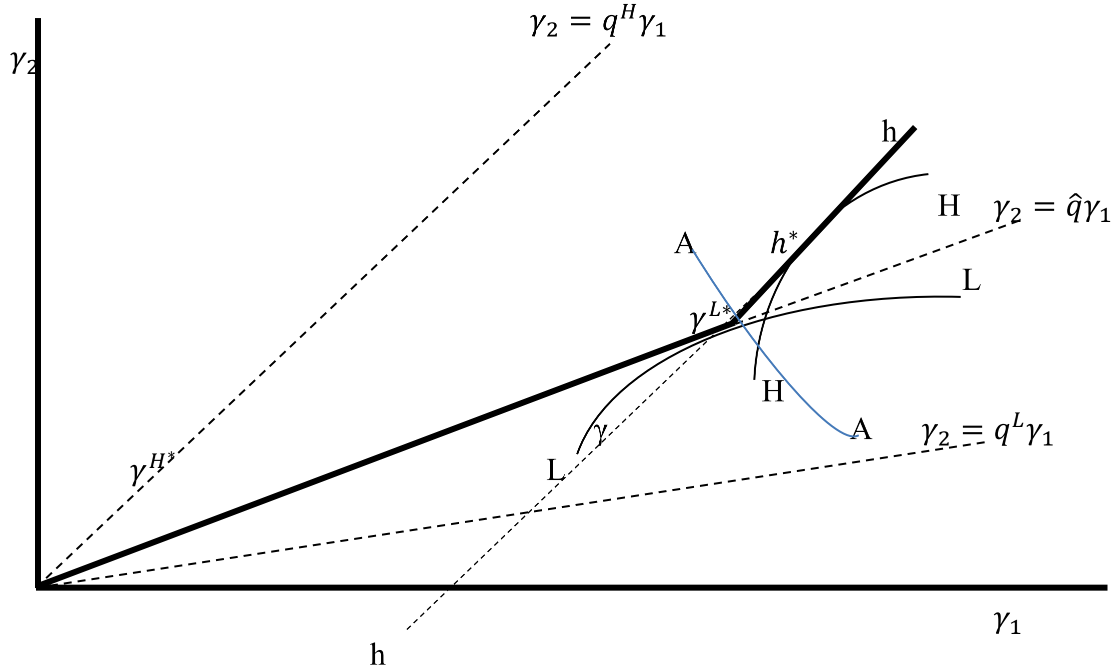
This paper shows, the logic of asymmetric information in markets with a divisible product (e.g. financial markets) requires firms share information. The introduction of non-passive consumers is an extension of Hellwig's (1988) insightful point that sharing information involves sending *and* receiving information. In addition to what information to send and to who, firms must decide what to do with the information they receive. They should also attempt to extract information from consumers directly by extending communication strategies to consumers as well as firms. Optimal firm strategy should attempt to elicit important information such as consumer transactions or competitors' policies from consumers (especially if the latter could be hidden).

It is also important to observe that sharing information between firms is strategic and therefore endogenous in this model. Each firm (and consumer) decides what information and to whom it will share. The Lemon equilibrium is a good way to clarify the power of this endogeneity. In an interesting discussion of its public policy implications, Ales and Maziero (2014), derive the lemon equilibrium of our corollary to Proposition 1 under the assumptions that low-risk types prefer their endowment to all positive coverage zero profit pooling policies, and trades are nonexclusive, i.e. firms do not share client information. Their result is the special case of the corollary to proposition 1 when low-risk types' most preferred zero profit pooling policy is the null policy they receive in equilibrium, while the high-risk supplement it to full coverage at the high-risk fair price. Note, however, the result obtained here is more general than the Ales and Maziero result on two counts. First, once low-risk types are assumed to desire no positive pooling policy, the equilibrium of Proposition One follows, and second, high-risk policy sellers' non-disclosure strategies need not be assumed because they can be derived endogenously. In the lemon equilibrium, if firms offering high-risk price coverage disclose consumers, a competitor (now able to deny the disclosed high-risk types) could upset the equilibrium by offering lower price positive profit coverage to low-risk consumers only. Hence, nondisclosure is optimal for sellers of high-risk policies, and contrary to Ales' and Maziero's suggestion otherwise, strategic communication (among firms) is necessary to their result. It just happens the strategic communication choice is to not communicate.

Another way to see how the institutional arrangements of the present paper support its equilibrium is to compare it to an interesting adverse selection model where a single informed agent of two possible risk-types could trade both positive and negative contracts with many uninformed agents Attar et al (2014). Their model covers market activities involving financial transactions (e.g. mortgage backed securities, credit default swaps) where an agent can be on either side of the transaction. Explicitly assuming contracts are nonexclusive (i.e. firms share no transaction information), they find equilibrium may not exist, but when it does, it depends crucially on the assumption agents can trade negative as well as positive contracts. To interpret their model as including adverse selection insurance as a special case, a positive contract means the insured party receives a payment when the insured event occurs and pays a premium when it does not. A negative contract means the insured party pays a premium when the event occurs and receives a payment if it does not.

Attar et al find "there is no equilibrium in which both types of the consumer trade non-trivial quantities on the same side of the market." Thus, no equilibrium has both types purchasing positive insurance of the kind typically found in casualty, health, and life insurance where the prime motivation for buying is the insured's need for income if the event occurs. To reach equilibrium, the model must have risk types on different sides of the market. To understand further the different results between the present paper and theirs, observe their explicit modelling of non-exclusive contracts has two crucial implications concerning the flow of information: inter-agent communication is precluded by assumption and therefore neither of the informed agent type's trades can be numerically limited. Indeed, the reason why equilibrium with both risk types trading positive insurance policies exists with strategic communication but not under the Attar et al assumptions is strategic communication of transactions allows pooling firms sharing client transaction information to place a limit on individual agent's *total* coverage *at the pooling price* even when policies are nonexclusive. In the Attar et al model, uninformed agents cannot enforce coverage limitations at the pooling price. Nevertheless, if both types trade positive policies, equilibrium would still require a pooling policy; but non-exclusivity and its implicit assumption of nondisclosure of client transactions prevents equilibrium with pooling contracts because the high-risk type would want to trade several causing negative profit for pooling contracts. Moreover, offer of the high-risk fair contracts only might not support equilibrium either. Suppose only the high-risk fair contracts are offered. If the low-risk type will trade some pooling contract, a single uninformed agent deviating with an offer priced above the pooling price becomes its sole seller and enjoying a temporary monopoly can limit total transactions. Thus, the deviating agent expects to earn positive profit even though further entry will prove its expectation incorrect. Finally, the JHG allocation with one uninformed agent offering the zero-profit pooling contract and several others offering the high-risk fair price contracts could not support equilibrium in the Attar et al model either because given static expectations, the sole supplier of the pooling contract would want to raise its price. As they conclude, the model could only have both types trading in an equilibrium only if one type trades a negative contract and the other a positive contract.

Figure 1



$\gamma_2 = q^H\gamma_1$
γ_2
h
H
$\gamma_2 = \hat{q}\gamma_1$
A
h^*
L
γ^{L*}
H
γ
A
$\gamma_2 = q^L\gamma_1$
γ^{H*}
L
γ_1
h

Appendix

Lemma 5: *Both risk types' income consumption curves at price q^* have negative slope.*

Let γ be any cream-skimming policy. Consider a consumer who has applied for γ, and (minimizing notational cumbersome) denote the set of policies (α, β) she could now obtain by supplementing γ with coverage at price q^* by $B(q^*, \gamma)$. This budget set contains all nonnegative policies on the budget line $\beta = \gamma_2 + q^*(\alpha - \gamma_1); 0 \leq \beta \leq w - \gamma_2$. The supplementary q^* coverage *desired* by risk-type t is $z^t(q^*, \gamma)$ making the total policy desired by the low-risk $(\gamma_1 + z^L(q^*, \gamma),\ \gamma_2 + q^* z^L(q^*, \gamma))$. Allowing γ to vary, for q^* constant, AA, the locus of optimal total purchases in Figure 1, is the income-consumption curve for risk-level L.[3] Constraining the consumer to one of the γ budget lines and allowing a consumer to choose an amount of coverage α, gives a FOC defining an income consumption curve for risk-level t implicitly:

$$p^t u_0'(w - r + \gamma_1 + \alpha) - (1 - p^t) u_1'(w - \gamma_2 - \beta) q^* = 0.$$

Let A_i denote the Arrow-Pratt measure of absolute risk aversion computed for income event i, and u_i' marginal utility in the same event. Differentiating the FOC implicitly with respect to α and noting the FOC implies $\frac{p^t}{1-p^t} = \frac{u_1'}{u_0'} q^*$, shows the slope of these curves $\frac{d\beta}{d\alpha} = -\frac{A_0}{A_1}$. Therefore, both risk-classes' income-consumption-curves have negative slopes as exhibited for AA in Figure 1.

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[3]Let $\gamma^{L^*} = (\alpha^*, \beta^*)$, so $q^* = \frac{\beta^*}{\alpha^*}$. At $\gamma = (\alpha^*, \beta^*)$ or $\gamma = (0,0)$, the budget line ($\beta = \gamma_2 + q^*(\alpha - \gamma_1)$ becomes $\beta = q^* \alpha$ and the income-consumption and price-consumption curves intersect. The price-consumption curve is the locus of policies where indifference curves are tangent to different budget lines emanating from the origin and with slopes equal to prices. As $q = \frac{\beta}{\alpha}$ varies, the t-risk price-consumption curve is the locus of contracts $(z^t(q), q z^t(q))$ in α-β space. We also note, the low-risk income consumption curve intersects the q^* price line at (α^*, β^*) where $\gamma = (\alpha^*, \beta^*)$ and $z^l(q^*, \gamma) = 0$. Moreover, because the optimal total purchase is identical for all γ lying on the same budget line, and (α^*, β^*) and (0,0) lie on the same budget line, the income consumption curve also passes through (α^*, β^*) when $\gamma = (0,0)$ and $z^l(q^*, 0,0) = \alpha^*$.

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