FINANCING OF PUBLIC GOODS THROUGH TAXATION IN A GENERAL EQUILIBRIUM ECONOMY: THEORY AND EXPERIMENTAL EVIDENCE

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Financing of Public Goods through Taxation in a General Equilibrium Economy: Theory and Experimental Evidence*

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Abstract

We compare general equilibrium economies in which building and maintenance of a depreciating public facility is financed either by anonymous voluntary contributions or by taxing agents on their income from private production. Agents start with an endowment of private goods and money, while the government starts with an endowment of public good and money. All private goods produced are tendered for sale in exchange for money in a sell-all market mechanism. Agents' proceeds from sale are taxed, and they individually allocate their private goods between current consumption and investment in production for the following period. The optimal levels of supply of the public good, and tax rate to sustain it over time, are defined and calculated for infinite and finite horizons. These equilibrium theoretical predications are compared to the outcomes of laboratory economies when (1) the starting public facility is either at or below the optimal level; and (2) the tax rate is either exogenously set at the optimal level, or at the median of rates proposed by individual agents. We find that the experimental economies sustain public goods at about 70-90 percent of the infinite horizon but considerably more than the finite horizon optimum. Payoffs (efficiency) is at 90 percent of the infinite horizon equilibrium level even when the rate of taxation is determined by voting. Starting conditions play only a minor role for outcomes of the economies, as efficiency and the stock of public good adjusts to about the same level irrespective of the starting level. These results contrast with rapid decline in provision of public goods under anonymous voluntary contributions, and point to the possibility that the social institution of government enforced taxation may have evolved to address the problem of under-production of public goods through anonymous voluntary contributions.

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1. Introduction

Given the prevalence and importance of public goods in society, ways of financing their production have attracted much interest.[1] Game theoretic models suggest that egoistic individuals have little reason to finance production of public goods through individual voluntary anonymous contributions. Laboratory public good experiments tend initially to yield average contributions around 50 percent of the collective optimum, gradually declining towards 5-20 percent range. Although the averages rarely drop below 5 percent, many individuals make zero contributions. In addition to these basic results, theoretical and experimental work has explored incentive schemes for individuals to voluntarily contribute more to provision of public goods. Given the fundamentally dispersed nature of information and the difficulty of gathering such information in the hands of a central planner (Hayek 1945), devising decentralized schemes for provision of public goods through anonymous voluntary contributions has strong theoretical and intellectual appeal. However, these efforts have been only partially successful.

There is no reason for society to confine its search for efficient solutions for this pervasive problem to only anonymous voluntary contributions. Institutions may evolve to address various problems of economizing through socio-political and economic processes of adjustment, experimentation, and feedback over rules, expectations, and conventions. It is reasonable to conjecture that the scope of such social evolution includes provision of public goods.

Walker et al. (2000 appear to be the first to consider the efficiency implications of a combined common-property-with-voting allocation scheme in the laboratory. Their evidence suggests people cooperate more with perfectly enforced voting rules relative to a no-vote scheme. However, their common-property setting is quite distinct from the public good we explore here. Kroll et al. (2007) employs a more common public-goods setting and voting and reports that voting by itself does not increase cooperation; if voters can punish violators, contributions increase significantly. With perfect enforcement they find 100 percent contribution rates in most periods. While these results are useful, a tax of 100 percent is neither realistic nor desirable in the real world. We explore an economy with private and public goods where voting is used to determine the tax rate – the optimum of which is at an intermediate level, as is the consumption

[1] For a survey of the substantial pre-1995 literature on experimental gaming with public goods see Ledyard (1995). From considerable literature since then, we mention only a few. Fehr and Gächter (2000) consider public goods experiments without punishment for free riding; Gunnthorsdottir et al. (2007) investigate how individuals' cooperative disposition and experience interact; Brandts and Schram (2001) consider voluntary contribution mechanisms for public goods; Palfrey and Prisbrey (1997) consider public goods provision where the individuals have different marginal values for their private goods; Hatzipanayotou and Michael (2001) deal with public goods, tax policies and unemployment in less developed countries. Modeling in the last of these paper noted is closer to the spirit of our own emphasis on the importance of institutional structure in the economy.

rate. Furthermore in contrast we reconsider the new noncooperative equilibria when the voting mechanism is considered as a part of the new formal game.

Evolution of institutions may take decades or centuries; we do not try to capture it in our model or laboratory experiment. Instead, we modify a general equilibrium model of the economy to include government and a full process description of agents playing both economic (market behavior) and political (voting) roles. We use a process oriented strategic market game in which the provision of public goods is financed through tax on private income. The unique equilibrium solution for any tax rate yields an optimal consumption/investment policy for each individual. However a general dynamic programming analysis of our basic model enables us to solve for an optimal rate of taxation for society as a whole, using symmetry of the agents in the model. The purpose of the formal model is to illustrate, at a high level of abstraction, some fundamental features of government control of taxation to provide valued public services (see Appendix C). Note that changes in the rules of the game, such as introducing a government that can enforce tax collection, or introducing a voting mechanism, can influence the existence and number of noncooperative equilibria serving as dynamic attractors.

The dynamic character of experimental gaming requires full specification of the space of possibilities and updating mechanisms. The formal model includes symmetric agents and a government to build and maintain the public good facility. With a dynamic programming model, Karatzas et al. (2011) are able to establish the existence of an optimum size of the public good facility as is shown in Appendix C. In Appendix A we provide the explicit calculations for the special case we utilize in the experiment (for infinite and finite horizon economies). Figure 7 uses an Excel spreadsheet to illustrate the full payoff set and the optimum for consistent with the general model.

We examine the model in the laboratory under two sets of conditions: (1) when the rate of taxation is exogenously fixed at the theoretical optimal level (which is practical only in a hypothetical world of an omniscient government); and (2) when the rate of taxation can be adjusted by a democratic process that moves on a longer time scale than the day-to-day economic process (tax rate set at the median of voter proposals). Finally, we examine the performance is of otherwise identical economies in which taxation is replaced by individual anonymous voluntary contributions specified in either percent of income or as money amounts.[2]

There is a departure between the mathematical model and the laboratory game. First, the pure dynamic programming solution is based on an infinite horizon with the natural discount rate

[2] Financing the creation of a new public good facility requires considerably more capital than paying to operate and maintain of an existing facility. The two may also differ in their political feasibility, especially under uncertainty. A comparison between maintaining the old Metropolitan Opera House in New York and building a new one in Lincoln Center highlights the differences between the uncertainty and financing associated with the two options. In the first instance the annual maintenance costs and revenues are reasonably known. In the second, the expenditure is large, and the revenue is uncertain. In the present model and experiment, we confine ourselves to consideration of financing the operation and maintenance of an existing public good facility in absence of uncertainty; smooth incremental additions to stock are assumed to be feasible.

$\beta < 1$ (see Appendix C). We found it more desirable for the experiment to consider $\beta = 1$ and to construct the stationary maximum flow solution that is consistent with the underlying economic model. This is shown in Appendix A. Second, in implementing an experimental game with a finite termination we are faced with the question of how to value the stock of public good at the end of the game. With zero valuation, we expect that the maintenance of the public good facility will tend to drop off towards the end of experimental sessions. We used Excel Solver to numerically solve the dynamic program when the value of the stock of public good is zero at the end of the session. The results obtained from this optimization are labeled "finite horizon" to distinguish them from the "infinite horizon" results throughout the paper.

The equilibrium of the game without voting is replaced by a continuum of non-cooperative equilibria in a game that includes voting.[3] One of these equilibria could yield an optimal rate. We have not solved for that equilibrium, and do not know the dynamics of how it might be achieved. Our experiment may provide some evidence on whether a vote-based structure for delivery of public goods may efficiently address the challenge of coordination among individual choices.

Our 2x2 experiment has four main treatments and a supplemental treatment. In the first, we start with a given tax rate and the optimum stock of a public good facility (such as a road system) the outcome is contrasted with a suboptimal start (50 percent of optimal, presented in Treatment 2). The taxation is set exogenously at the level necessary to maintain the optimal stock of the public good facility in both cases. Information on the optimal maintenance level is not given to the agents. The basic model of tax-financed public goods is reproduced in Appendix C without proofs. In Treatments 3 and 4, the tax rate is set endogenously through voters' choice (at the median choice) once every five periods, starting with optimal and suboptimal levels, respectively, of the public good. The initial level of the public good is at the optimum (Treatment 3) and at half of the optimum level (Treatment 4).

In order to compare and contrast our results with the voluminous literature on voluntary anonymous contributions, we supplement our 2x2 experimental design with two sessions (supplemental treatment T0) which start with the optimal level of public good (as in Treatments 1 and 3 above), but taxation is replaced by anonymous voluntary contributions towards the production of public good. In one of the two sessions, the contribution is specified in units of money, and in the other it is specified as a percentage of income of individual agents.

Whether central coordination through government-determined tax rate through taxes dominates decentralized socio-political mechanism to set the tax rate through voting is an open question. We suspect that the outcome depends on not merely divergence of preferences, but on

[3] When there is voting it is straightforward to see that with a continuum of agents, if all have voted for a rate of k% then an extra voter has no influence on the rate; hence it is an NCE. If (as is the case in the experiment) each player has a finite influence then the range of the continuum of equilibria will not be as large but will be bounded by an oligopolistic influence.

the technological understanding of the costs and benefits of the public goods involved. Such knowledge is implicit in our formulation.

We find that these finite horizon experimental economies sustain public goods at about 70-90 percent of the infinite horizon optimum level, and about 90 percent efficiency even when the rate of taxation is determined by voting. Starting with the optimum level, the stock of the public good declines to around 80 percent of optimum; but starting from 50 percent of the optimum, the stock of the public good rises to about the same level. The stock of public good tends to exceed the finite horizon optimum. These results from a general equilibrium economy suggest that taxation may be the social institution that has evolved to address the problem of under-production of public goods through voluntary contributions. The paper is organized as follows: the model is presented in Section 2, followed by the experimental design, results, and a discussion in the subsequent sections of the paper.

2. General Equilibrium Model of an Economy with a Public Good

We consider a version of Samuelson's pure public good (Samuelson, 1954) embedded in a parallel dynamic programming control process that has been solved for its type-symmetric non-cooperative (rational expectations) equilibrium for any tax rate.[4] The dynamic structure also includes a government and voting. In the first two treatments of these experiments we constrain the direct choice of individuals to the private production and consumption decisions with exogenously specified taxation levels. In the third and fourth treatment, individual players have a say in setting the tax rate through a voting mechanism. This tax rate, applied to their income, determines their contribution to the public good; they do not choose the level of contribution individually.

The basic model involves the maintenance of a depreciating public good facility such as a road transportation or sewage system.[5] In describing the experimental game we minimize the formal notation (see Appendix C for the details). The game has a government and n individual agents each initially endowed with a quantity of private good and money (a, m). The government is endowed with G units of the public good and M units of money at the outset. It also has the right to tax a fraction θ of individuals' income from the sale of private goods, and a production function that transforms its tax revenues into the public good government has as its control parameters an income tax rate θ on the individuals.[6]

A move by an agent i in any period t is to decide how much money to bid (b_{ti}) to buy private goods in the market and, after she receives the private good from the market, how much to consume and how much to put into production for the following period. Our theory

[4] Formally with a continuum of agents we solved for any tax level; then after solving this set with a taxation level as a parameter we solved for the optimum from the point of view of a benevolent central government.

[5] It also could be a wage-supported bureaucracy that provides a self-policing system for the economy. Although its cost could be one of the most important and earliest of costs of public goods, it is rarely mentioned in discussions of public goods.

[6] Even at this level of simplicity, given that production takes time there are accounting questions to be considered in the definition of periodic income and profits. In a stationary equilibrium the timing differences disappear.

approximates equilibrium as though the number of agents is large enough that they have no influence over price. We use n =10 in the experiment, ignoring some small oligopolistic influence that may be present.

A period begins with government in possession of taxes gathered in the preceding period in the form of money (M in period 1), and public good G from the preceding period depreciated by 10 percent. The n agents carry their after-tax money balances from the previous period (m in period 1), and the private good they produced at the end of the previous period (a in period 1). We use a sell-all market mechanism, in which individuals' entire balance of private goods is automatically offered for sale in a market (see Huber et al. 2010 for properties of the sell-all mechanism). Each individual automatically bids his total money balance b to buy the private good from the market. The government also bids all its money balance for the private good. A price p is computed as the ratio of the total money bid (by agents and the government) divided by the total number of units of private good available.

Government as well as each agent gets the money bid by each divided by this price ($k_i = b_i/p$ units for individual; units for the government) of the private good allocation. Each agent, being a producer as well as a consumer, divides the units bought between consumption and production. In addition, each agent receives the price multiplied by the number of units sold as his income in units of money. This money income is then taxed (sales tax if you will) at a uniform tax rate, either pre-set to the optimum of θ = 21.5% (in Treatments 1 and 2) or set endogenously through a vote of subjects (in Treatments 3 and 4).

The n producer/consumer agents have a private good production function $f(k)=80*k^{0.25}$ with a one period production time lag, and a payoff function of the form $u(x, G) = (x + G/4)$, with x being the consumption of private goods and G being the level of public good.

The government uses the k units of private good it buys to produce $F(k) = 2*k^{0.5}$ units of public good which is added to the capital stock at the beginning of the next period. For simplicity we assume that depreciation is a fixed percentage of the level of capital stock. The government carries the tax collected as money balance to the following period to buy goods.

In equilibrium the production of public goods precisely covers depreciation, otherwise the amount of capital stock changes. This describes one full period of the game. If the game ends at period T the terminal or "salvage value" of left over money, goods and the public good are all zero. Subjects are instructed that the session will end with 1/6th chance after period 25, 26, 27, 28, 29, or 30.

3. Experimental Design

In a laboratory session in which the players make some 10-100 moves through computer terminals, the "history" is compressed (only rarely expanded) in clock time. The clock or calendar time that separates the moves is rarely specified in economic theory or in experimental gaming. We hardly know if the specific duration of a move (denoted by , for example) represents an hour, a day, a week, a year or is valid for all possible scales of time at once. The socio-politico-economic and bureaucratic processes of providing public goods consist of many interactions among policy-makers while they watch public opinion and battle, bargain and trade with the bureaucracy. However, the latter may expect to outlast its temporary masters—whether elected representatives, hereditary lords, or generals. In laboratory experiments economists rarely consider the differences in time scales of various public good decisions. In our experiment we take a small step to address this matter by introducing "annual" economic decisions on production and consumption alongside "quintennial" politico-economic decisions for choosing the tax rate, to implement at 5:1 ratio in the two time scales.

The experiment consists of variations on tax rate and initial endowment with public goods in a 2x2 design (see Table 1). The first variation (choice of taxation) takes two values: exogenously fixed tax rate θ = 21.5% and an endogenous tax rate determined as the median of proposals from individual agents once every four periods. The second variation (initial level of public goods) also takes two values: 100 or 50 percent of optimal level of public goods as the initial condition. Table 2 shows the values of the parameters of the experiment: number of agents in each session (n) is 10; initial money endowment of each agent is 4,700; initial private good endowment of each agent is 217; their private good production function is $80*k^{0.25}$ and their single period payoff is the sum of their private consumption and 25% of the level of public good in that period. The initial endowments of the government are 13,000 in money. Public good depreciates at 10 percent each period, and the public good production function is $2*k^{0.5}$. The natural rate of discount is 1.The stationary (i.e., infinite horizon) equilibrium price is p = 27.67. Each individual should buy 170 units of the private good and consume 68.27%, i.e. 116 units, while the remaining 31.73% (54 units) are put into production to produce $80*54^{0.25}$ = 217 units for the next period. The government buys 467.7 units of the private good to produce $2*467.7^{0.5}$ = 43.25 units of the public good – just enough to offset the depreciation (10% of 427 units) of the equilibrium stock of the public good.

As shown in Table 1, Treatment 1 has an exogenous tax rate of 21.5% and initial level of public good at 100% of optimal (G = 427). In Treatment 2 the initial stock of public good is 50 percent of the optimal level (G = 213.5).

If the results for Treatment 2 show a tendency of the economy to move in the direction of optimum outcomes with respect to the stock of public good, prices, production and payoffs, the result can be interpreted to support the proposition that the institution of taxation is a viable alternative to individual contributions as an approach to providing for the appropriate level of the public good in society if the government is able to infer the taxation policy necessary to support the optimal solution.

In Treatment 3, the economy starts with the optimal level of public good but the exogenously imposed tax rate is replaced with a rate determined by quadrennial vote of the participants, i.e. a vote every five periods. Each subject submits a tax proposal and the median of the ten proposals is chosen as the rate of taxation applied to all subjects for five periods, when another vote is taken. The agents therefore have the collective freedom to increase or reduce the provision for public goods by voting. In Treatment 4, the voting mechanism is retained from Treatment 3, but the initial level of public good is changed from 100 to 50 percent of the optimal level (G =213.5).

In two supplemental sessions, labeled T0, voluntary anonymous contributions for public goods replaced taxation. In one of the two sessions, individuals specified their contributions in units of money, in the second in percent of their income.

We conducted a total of 22 independent runs, each with a different cohort of 10 subjects for a total of 220. All subjects were BA or MA students in Management or Economics. All runs were conducted at the University of Innsbruck, Austria. All sessions were carried out using a program written in z-Tree (Fischbacher, 2007) and recruitment was done with ORSEE (Greiner, 2004). Average duration of a session was approximately 60 minutes. Average earnings were 15 Euros.

4. Results from the Experiments

The main results of the experiment are organized and presented in Figures 1-6. Each of these figures shows a single measure of the outcomes of the multiple independent sessions of experimental economies with different subjects conducted in each of the four main treatments T1, T2, T3, and T4, as well as the supplemental treatment T0. In addition the figures show general equilibrium predictions for finite and infinite horizons as theoretical benchmarks to compare the experimental data with.

Stock of Public Good (Figure 1)

The top left panel in Figure 1 shows the time series of the stock of public good observed during the four independent sessions of Treatment 1 (with fixed tax rate of 21.5%, starting with the optimal level 427 of the public good in Period 1). Time paths of the stock of public good for each of the four sessions are shown in thin solid grey lines and the mean of the four paths is shown in a thick solid black line. Given the small dispersion of the four paths around the mean, the latter captures their central tendency well. Two thick broken lines chart the general equilibrium benchmarks—the horizontal line at 427 for the steady state or infinite horizon level of public good, and the curved line for the finite horizon level with expected ending stock of public good at 310.[7,8] The same conventions are used to depict data in the other panels of Figure 1 and in other figures.

[7] Since the end of the session was announced to be uncertain (1/6th chance of ending after period 25, 26, 27, 28, 29, and 30), the finite horizon equilibrium predictions are given in expected values (see Appendix A).

In all four of these sessions, the stock of public good gradually but steadily declined over the 25 rounds from the starting level (*G* = 427) to the range of 355-402 and to an average of 381, which is 46 below the steady state level, but 71 above the finite horizon optimum of 310.

The top right panel in Figure 1 shows the four runs of Treatment 2 with tax rate fixed at the optimal level (21.5%) but starting with 50 percent of the optimal level of public good (213.5). In all sessions of Treatment 2, the stock of public good rose steadily from 213.5 to the range of 357-366 and to an average of 362 (with little variation across the four sessions). As shown by the thick broken curved line, the finite horizon optimum stock of public good rises from 213.5 at the beginning of period 1 to a high of 378 in period 14, and then declines gradually to 295 at the end of the session. The average stock of public good at the end of these four sessions is 65 short of the steady state level but 67 above the finite horizon optimum of 295. So far we can conclude that with the tax rate exogenously fixed at the optimum level, the stock of the public good converges near the midpoint between the finite- and infinite-horizon levels. This happens irrespective of the starting stock of the public good.

In the bottom left panel, the six sessions of Treatment 3, with endogenously determined tax rates and starting with optimal level of public good (427), the stock of public good tended to decline over the rounds to the range of 352-404 and to an average of 371. This average is 56 below the steady state optimum but 76 above the finite horizon optimum. In the bottom right panel, the six sessions of Treatment 4, with endogenously determined tax rates but starting from half the optimal level of public good (213.5), the stock of public good tended to rise to the range of 273-406 and to an average of 344 (91 below the steady state optimum and 49 above the finite horizon optimum).

A comparison of the data in the four panels of Figure 1 suggests some differences but also strong similarities. The stock of public good shows greater dispersion across economies when tax rates are endogenous, instead of being fixed. Starting from the optimal level, the stock of public good tends to decline to the neighborhood of 370 irrespective of whether the tax rate is fixed or determined by the vote of participants. Similarly, starting from the suboptimal level, the stock of public good tends to rise gradually to the neighborhood of 360 irrespective of whether the tax rate is fixed or determined by the vote of participants. In 19 of the 20 economies, the stock of public good at the end of the sessions lies inside the range defined by steady state and finite horizon optima. The null hypothesis that the terminal stock of public good is identical across the four treatments cannot be rejected (pairwise Mann-Whitney U-test, all p-values 0.2 or higher).

It seems reasonable to infer, on the basis of these 20 independent sessions of experimental economies that the stock of public good tends towards the range of 350-375, and this range seems

[8] In an unconstrained environment, one would expect the finite horizon equilibrium stock of public good to be exhausted to zero at the end of the session. Since the stock of public good depreciates at a constant rate of 10% per period, exhaustion close to zero at the end would require lower investment in early periods. The lower payoff in those periods prevents the optimal level of public good from being driven to exhaustion at the end in finite horizon economy.

to form a domain of attraction for these economies. This domain is below the optimum (427), but above the finite horizon optima of 295-310.

Finally, the two left panels of Figure 2 show in thin chain-dotted lines the time paths of the stock of public good in two Treatment 0 economies in which taxation was replaced by individual voluntary anonymous contributions. In these two sessions, the stock of public goods declined steadily and sharply to 147 and 170 respectively, which is much lower than levels observed in any of the 20 economies with taxation. These results are consistent with those obtained in voluminous experimental literature in which public goods are provided for by voluntary contributions.

Tax Rates (Figure 2)

The two panels in the top row of this figure are empty because the tax rate was exogenously fixed at 21.5 percent in the economies of Treatments 1 and 2. In the left bottom panel for Treatment 3 (with the initial stock of public good at steady state level 427) the endogenously determined tax rate remained generally below 21.5 percent and seem to decline generally from range 17.5-22 (average 19.8) in the first vote to 5-22.5 percent (average 14.3) in the sixth and final vote. Note that the finite horizon optimal tax rate (broken thick curved line) declines from 25% to near zero because the terminal conditions assign zero value to the stock of public good at the end of the session. The endogenously determined tax rates do not decline as rapidly.

Similarly, in the right bottom panel (suboptimal stock of public good initially), the endogenously determined tax rates remained generally below the optimal level of 21.5 percent and seem to decline slightly from range 11-23 (average 18.3 percent) in the first vote to 8-23 (average 16.3 percent) in the sixth and final vote. The subjects chose higher tax rates to support a rise in the stock of public goods from a suboptimal level over time (as observed in the right bottom panel of Figure 1). In all 12 endogenous tax economies, agents vote to pay taxes higher than the finite horizon optimum in the second half of the sessions.

The chain-dotted lines in the two left panels show the voluntary anonymous contribution as a percentage of individual income in the two sessions of Treatment 0. These percentages drop steadily through time until they approach the x-axis asymptotically. This is consistent with the results of prior laboratory experiments with voluntary anonymous contributions for public goods. Voluntary contributions are almost always lower than the finite horizon optimal tax rate, thus resulting in a rapid decline of the stock of the public good, as visible in the two chain-dotted lines in the left panels of Figures 1 (stock of public good) and 2 (voluntary contribution rates).

Efficiency (Figure 3)

The total points earned by all subjects as a percent of the number of points they would have earned if the economies had achieved the infinite horizon (steady state) general equilibrium outcomes is defined as the efficiency of these economies. Results for the 20 sessions are presented in the four panels of Figure 3 which parallel the layout of panels in Figure 1 presented above for stock of public good. Note that finite horizon efficiencies (shown in thick broken curved line) can exceed 100% because agents can earn a higher but unsustainable payoff by consuming more and depleting the stock of public good towards the end of the session.

In Treatment 1 with fixed tax rate and stock of the public good starting at the optimum (top left panel) efficiencies started close to 100 percent and tended to decline gradually, albeit noisily, to the range of 67-91 percent and average of 80 percent. With suboptimal start (Treatment 2 in the top right panel) efficiencies are near 80 percent at the beginning as well as at the end of the sessions, with little drift over time. With endogenous tax rates and optimal start (Treatment 3 in the bottom left panel) efficiencies remain around the low 90s. Only with endogenous tax rates and suboptimal start (Treatment 4 in bottom right panel) do the efficiencies show a rising trend from low 70s to mid 80s over the 25 rounds.

It is reasonable to infer that 80-90 percent is the domain of attraction for the efficiency of these economies. Surprisingly, given the prior results from public good economies with individual anonymous voluntary contributions, efficiency is slightly higher with endogenous choice of tax rate. Efficiency in the last period of T3 is only marginally significant higher than in T1 or T2 (Mann-Whitney U-tests, p-value=0.09 for both tests), while there are no other significant differences between treatments.

The chain-dotted lines in the two left panels for Treatment 0 economies are hardly distinguishable from the lines for efficiency of T1 and T3 economies. While suboptimal consumption decisions tended to lower the efficiency of T0 economies, letting the stock of public good run down raised it at the expense of unobserved decline in efficiency of periods beyond 25 (the end of the sessions). These two effects on efficiency tended to cancel each other, leaving the efficiency paths of T0 to be essentially indistinguishable from T1 and T3. Overall, the null hypothesis that the efficiency of the economies under the five treatments is the same cannot be rejected (with the above mentioned exception that T3 is marginally more efficient than T1 and T2).

While the stock of public good, endogenously determined rate of taxation, and efficiency of these economies are overall measures of their performance, we can also assess the decisions made by subjects that led to these aggregate outcomes. In the following paragraphs we present analyses of these decisions.

Total Production (Figure 4)

The thick broken lines for GE production in infinite and finite economies overlap at 2170 except in the final period when the finite period production drops.

Four panels of Figure 4 show the time paths of total production of the private good in the 22 economies for the five treatments. In all economies, the level of production was near optimal at the outset (2,170), but declined over the 25 rounds to the neighborhood of 1,500 with significant amount of noise across sessions as well as across rounds. A reason for this could be the choice of the concave production function ($80*k^{0.25}$) in which the extra output from positive deviations from optimal input (54) is much smaller than the loss of output from comparable negative deviations. Thus, while the average input is closer to the optimum (44.8 units overall and 53.2 in the first 10 periods), average output is lower due to dispersion of inputs across individual subjects. Also, optimal production would fall sharply in periods 26 to 30 in the finite-horizon benchmark. Thus, the decline observed in the experiments is not unjustified.

The chain-dotted lines for T0 economies in the two left panels of Figure 4 are not distinguishable from the time paths of T1 and T3 economies.

Total Consumption as a Fraction of Total Purchase of Private Goods by Individuals (Figure 5)

The thick broken line for GE (infinite horizon) is horizontal at 68.3%. The thick broken line for GE (finite) starts lower at 68.04% in period 1, rises steadily to 79.9 in period 24, and jumps to 79.9 in period 25.

Four panels of Figure 5 show the time paths of total consumption of private good as a fraction of the private good purchased by the individuals (recall that the private goods purchased but not consumed are allocated to produce the private good in the following period). The reason for the steady decline in production of private goods (in Figure 4) becomes clear here, since subjects appear to have consumed a steadily higher proportion of their purchases of private good than the steady state general equilibrium demands. While the GE prediction is 68.3 percent, actual average of consumption of individuals ranged in 71-89 percent, with many economies exhibiting an upward spike in this consumption during the later rounds of the sessions. As suggested by the finite horizon equilibrium, subjects may consume more instead of investing in public good at the end of the session. Recall that subjects did not receive any value from the stock of public good at the end of the session and that they faced uncertainty about the length of the experiment.

Two chain-dotted lines for T0 economies in the two left panels suggest slightly higher rates of consumption in voluntary contribution economies of T0 as compared to T1 and T3.

Total Consumption as a Fraction of the General Equilibrium Consumption (Figure 6)

Figure 6 shows the total consumption of the private good as a percent of the infinite horizon GE consumption (recall that Figure 5 shows consumption as a percent of purchases). This percentage tends to be below the infinite horizon GE prediction of 100%, largely because the shortfall in production.

The rising thick broken lines in these panels show the finite period equilibrium 134.7% in period 24, and 145.6 in the final period. The chain-dotted lines for the T0 economies in the two left panels show higher rates of consumption compared to the T1 and T3 economies.

5. Discussion and Concluding Remarks

Public goods decisions are made in rich institutional settings. British history of the "tragedy of the commons" does not offer much insight into how to finance the building of a network of roads or other public facilities. States evolved over centuries by enforcing weights and measures, commercial codes, accounting rules, law and order, and tax collection. In this study we take the structure of government to be able to provide these functions as a given. We believe that economic anthropology is an important subject in its own right, but is only a distant relative to models of the economy starting with the assumption that a government with powers to tax and enforce taxation exists.

We ran experiments to explore the suitability of setting taxes through democratic voting to provide for public goods. The results of the experiment suggest that the important social problem of financing public goods can be addressed by societies through significant reliance on the institution of taxation. Total dependence on voluntary contributions among large groups may be too unreliable a basis for providing services essential to their productivity, even survival. Voluntary contribution mechanisms have the inherent appeal of being decentralized, and thus insulated from tyranny. Taxation, representing centralized power and a centralized enforcement mechanism, has historical associations with tyrannical oppression. Democratic government and taxation based on popular voting attempts to balance the consequences of centralization. These experimental results suggest that such a reasonable balance is achievable for financing of public goods and services through democratic mechanisms.

The basic intuitions behind much of the formal economic theory concerning rational conscious economic profit seeking behavior are reasonably good. However, most people know that a public good poses a socio-political choice problem that is different from a simple private good market problem. Institutional evolution favors structures that can easily be operated by the participants, and have appropriate time-lags and other stabilizing features to damp down swings and disruptions caused by transient passions. A well-functioning social institution, like its engineering cousins plumbing or electrical wiring, should be taken for granted almost all the time; continual attention to and engagement with it should be on the verge of being boring.

The present experiment examines the minimal institutional underpinnings of economic theory. Institutions carry the socio-economic processes and link the static models of much economic theory to economic dynamics. Many of the reported experiments on public goods can be interpreted as contributions to economic anthropology and social-psychology, in contrast to testing the efficacy of existing and proposed institutional designs in addressing specific problems. Evolution of institutions occurs over decades and centuries where history, context, learning, experience, tradition and innovation all matter. The extent to which one may make useful analogies, and draw valid inferences, from laboratory games lasting but an hour or two in radically different time scale and context remains open, and is worth mentioning here for caution. Contributions of experimental gaming to the development of an economic science should be considered in this institutional context.

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Table 1: Experimental Design			
		Initial Level of Public Good	
Regimes for Public Good Provision		100 percent of Optimal	50 percent of Optimal
Voluntary anonymous contributions		**Treatment 0: 2 sessions***	
Taxation	Fixed at 21.5%	**Treatment 1:4 sessions****	**Treatment 2:4 sessions**
	Tax rate set by vote	**Treatment 3: 6 sessions**	**Treatment 4: 6 sessions**

*Voluntary contributions specified in units of money in one session and in percent of wealth in the other.
**Two of these four sessions permitted 0 inputs to private good production; all other sessions of the experiment required a minimum of 1 unit of input.

Table 2: Experimental Parameters and Design		
Parameters		
Number of Agents	*n*	10
Initial money endowment of agents	*m*	4,700
Initial pvt. good endowment of agents	*a*	217
Agents' pvt. Good production function	*f(k)*	$80*k^{0.25}$
Single period agent payoff	*u(x, G)*	*x + G/4*
Session agent payoff		Sum of period-wise payoffs
Initial government public good endow.	*G*	427 (T1, T3) or 213.5 (T2, T4)
Initial government money endowment	*M*	13,000
Government's public good prod. function	*F(k)*	$2*k^{0.5}$
Natural rate of discount	β	1
Depreciation rate (per period)	η	0.1
Terminal value of public good		0
Session termination	Announced: random between periods 25 and 30 Actual: always ended after vote in period 26	
Equilibrium Outcomes		
Price of private goods	*p*	27.67
Per capita production of pvt. good		217
Per capita purchase of pvt. good		170
Per capita consumption of pvt. good		116 (68.27% of 170)
Per capita pvt. Good into production		54 (31.73% of 170)
Production of public good		42.7

Figure 1: Stock of Public Good in Economies Grouped by Four Types of Sessions

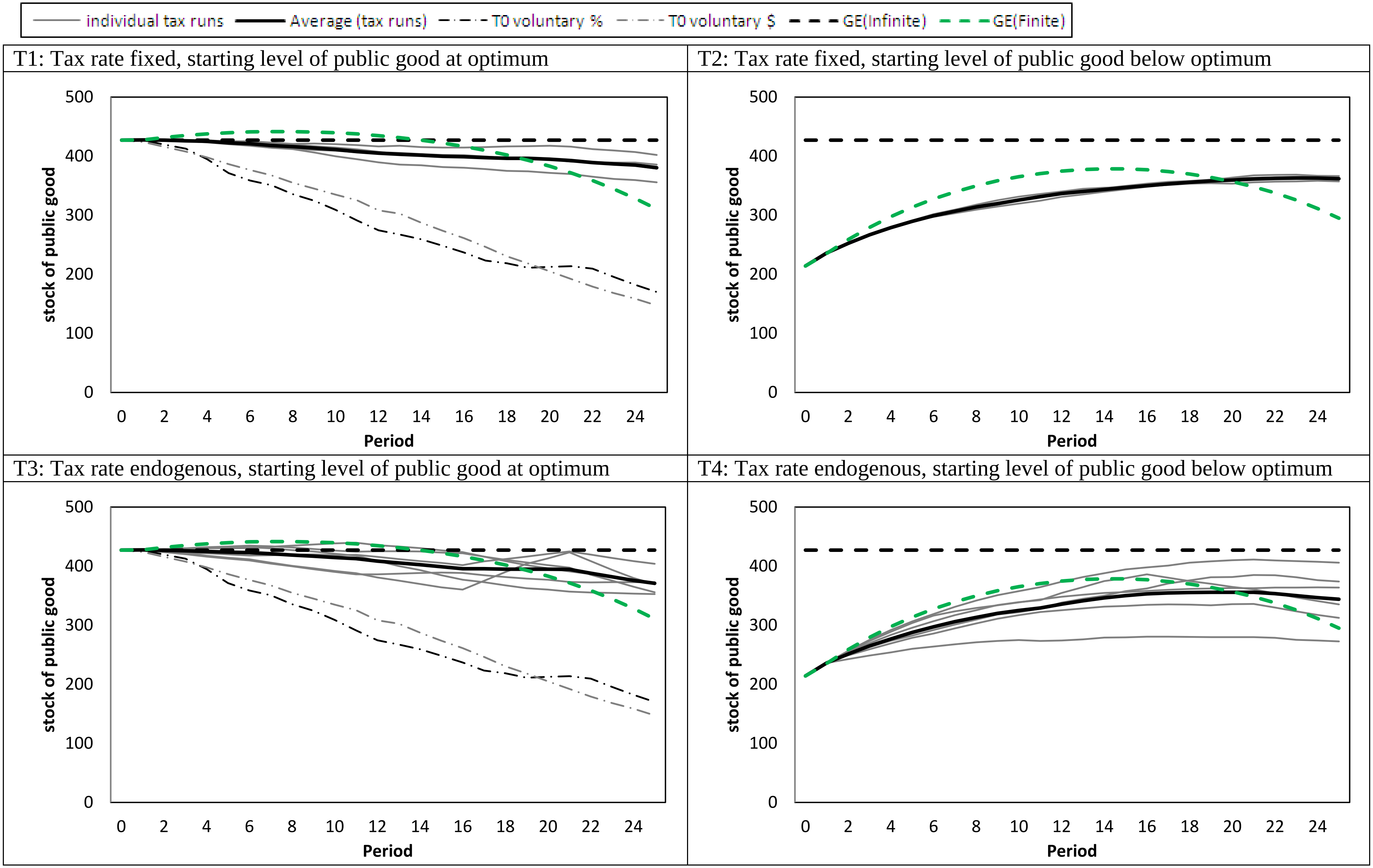
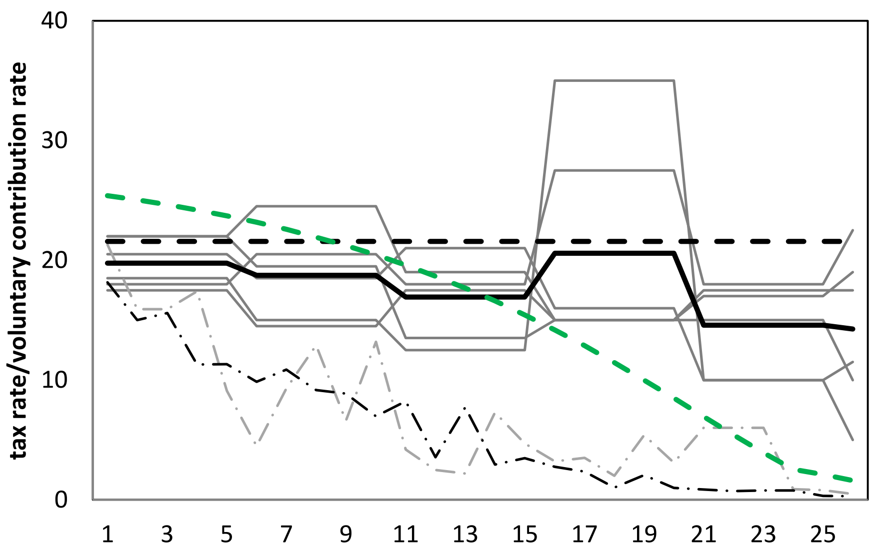


Figure 2: Tax Rates

individual tax runs — Average (tax runs) — T0 voluntary % — T0 voluntary $ — GE(Infinite) — GE(Finite)

T1: Tax rate fixed, starting level of public good at optimum	T2: Tax rate fixed, starting level of public good below optimum
Tax Rate fixed at 21.5%	Tax Rate fixed at 21.5%

T3: Tax rate endogenous, starting level of public good at optimum



T4: Tax rate endogenous, starting level of public good below optimum

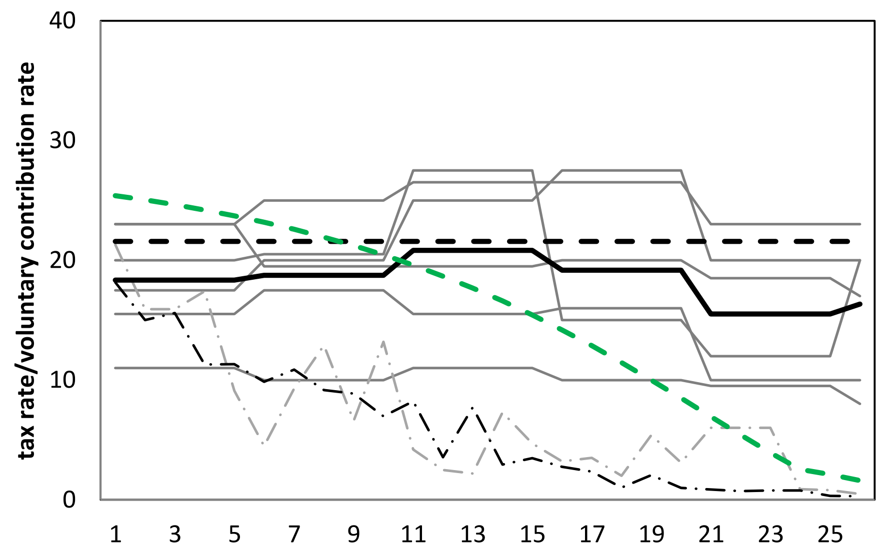


Figure 3: Efficiency of Public Good Economies with GE(Infinite) being the benchmark

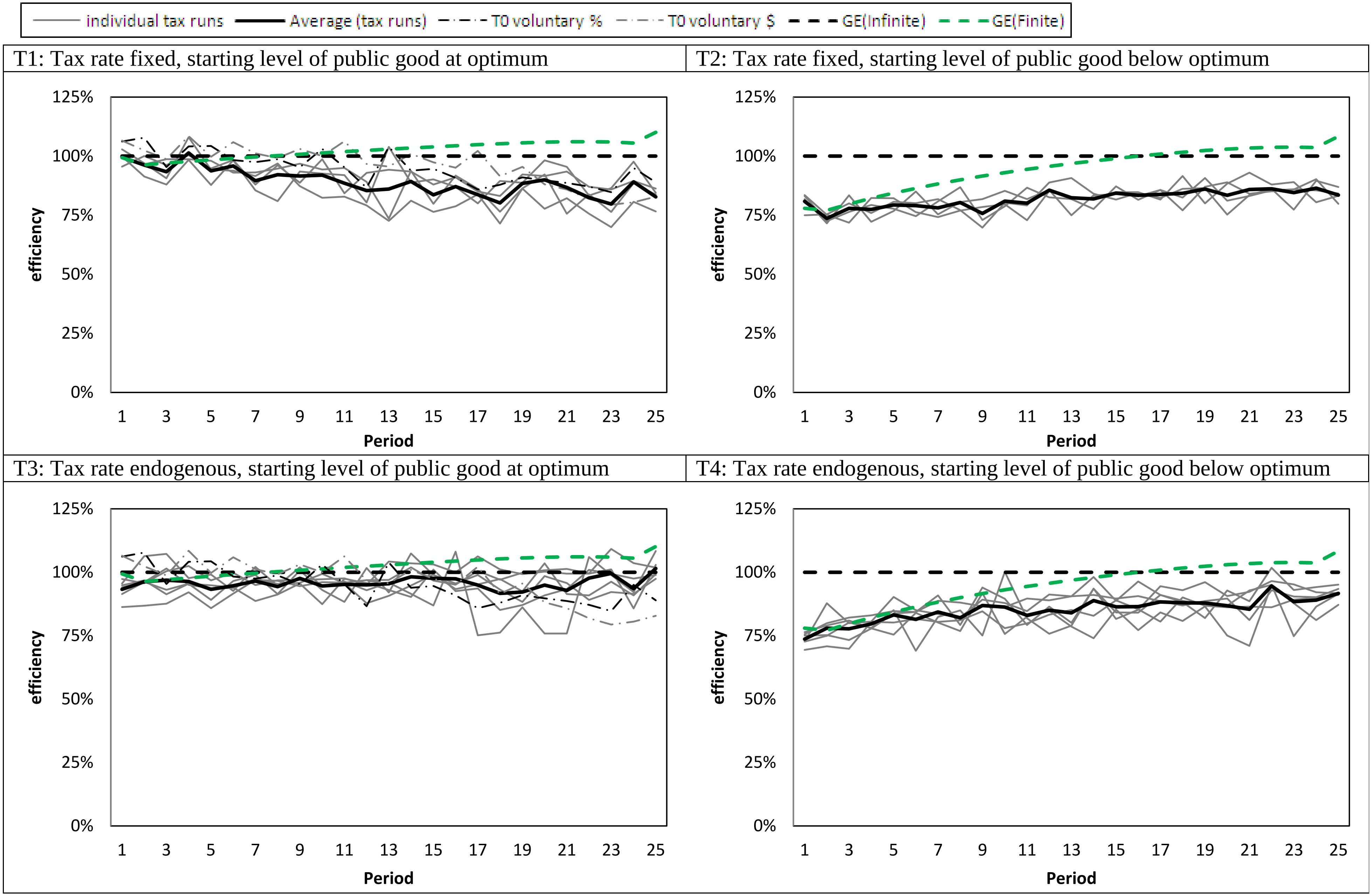


Figure 4: Total Production of Private Good in Economies

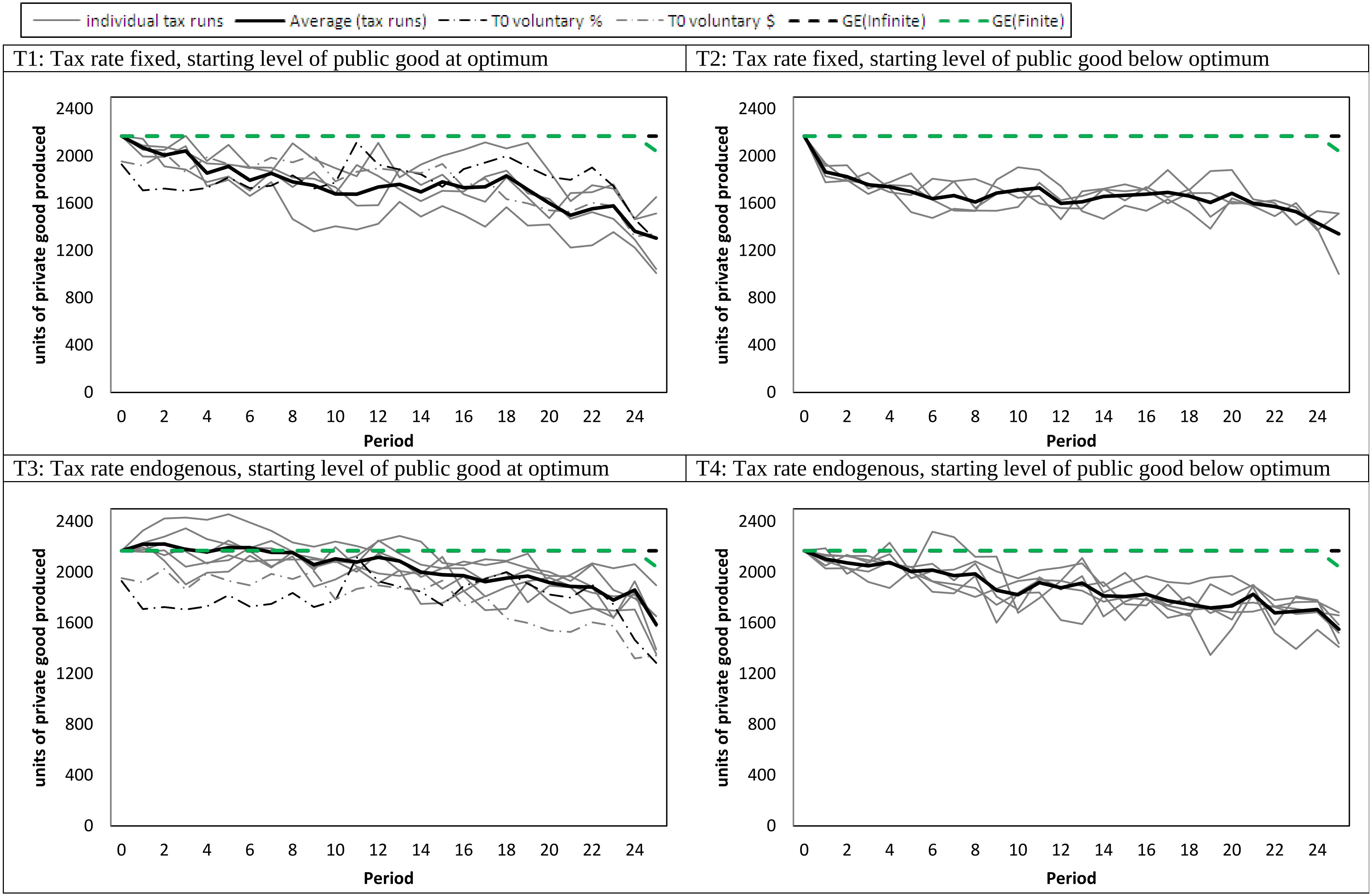


Figure 5: Total Consumption as Percentage of Total Individual Purchases of Private Good

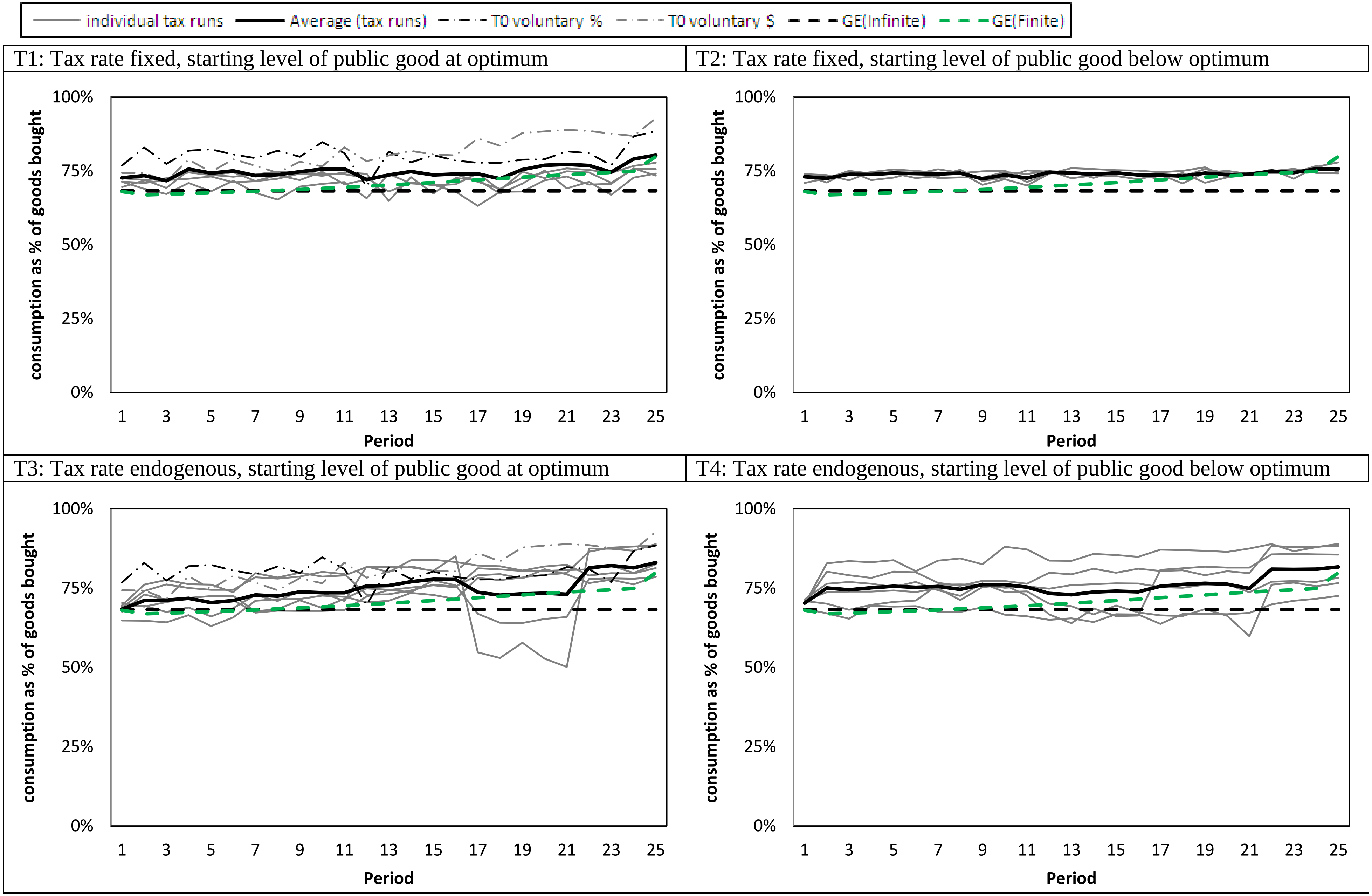
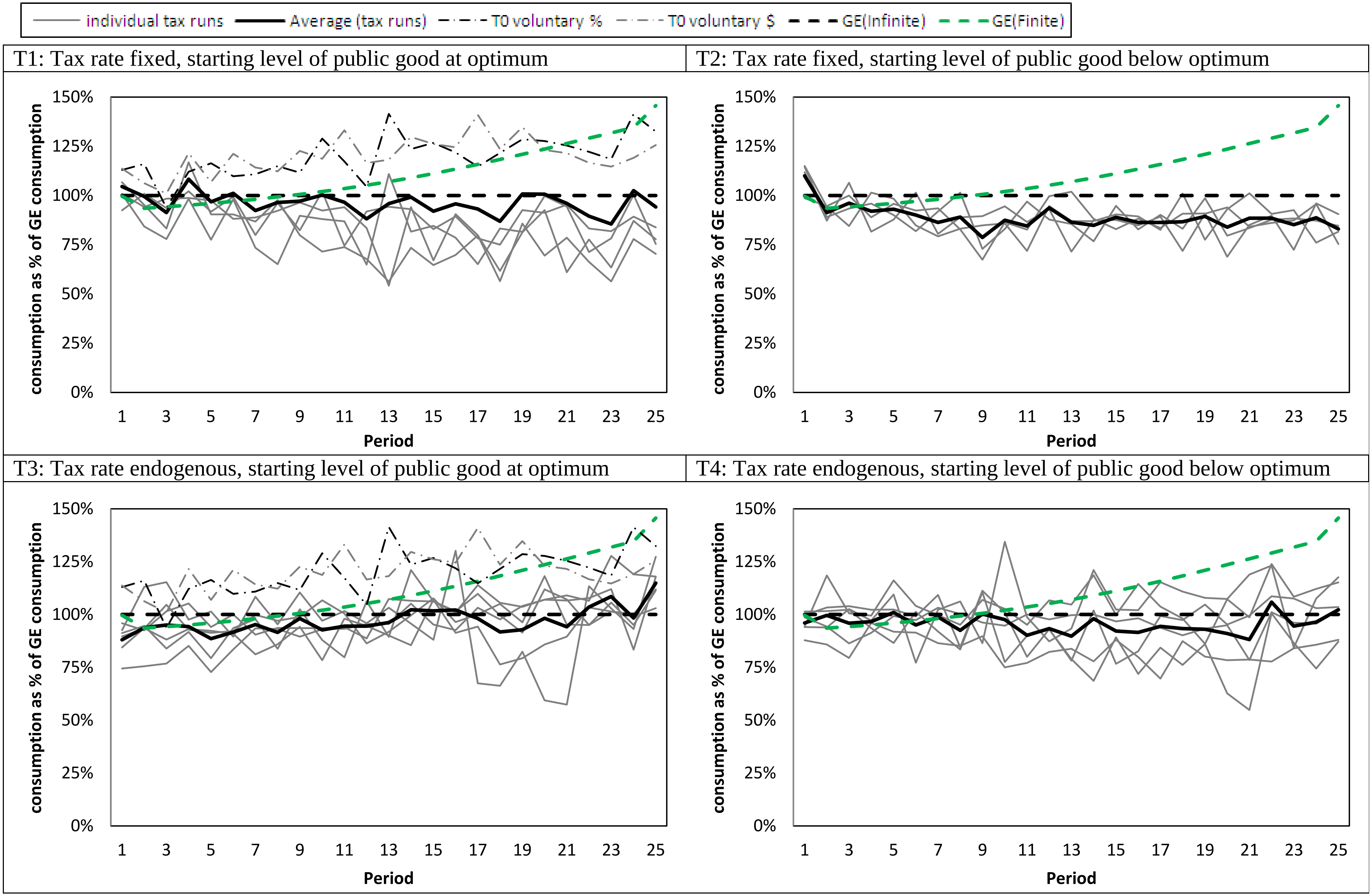


Figure 6: Total Consumption as Percentage of General Equilibrium Consumption of Private Good



APPENDIX A – Explanation of MS Excel Worksheets available online

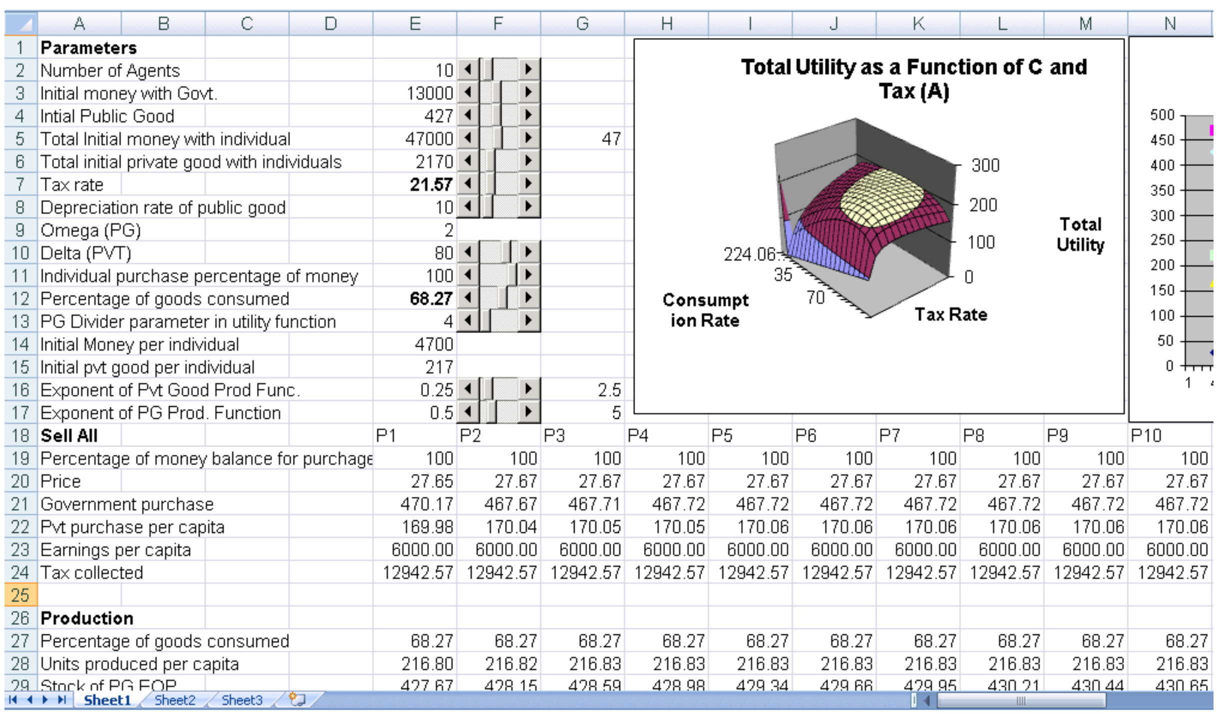
As supporting material for this paper we provide two MS EXCEL worksheets, one for infinite horizon, one for a finite horizon of 30 periods. In both worksheets all relevant input variables can be varied in cells E2 to E17. The respective notation can be found in cells A2 to A17. Especially noteworthy in the infinite horizon setting are the tax rate (E7) and the consumption rate (E12) as these are the two variables for which we optimized by use of the solver function of MS EXCEL.

In rows 19 to 24 (22 to 28 in the finite setting) the sell-all market is modeled, with period 1 in column E, and subsequent periods to the right, up to period 20 in the infinite setting and period 30 in the finite setting. Right below, are the productions of private and public goods, again from period 1 (column E) to period 20 (30 in the finite setting).

Several graphs from Columns H to AD illustrate the results and their sensitivity to variations in the input variables. Figures 7 and 8 give screenshots of part of the respective excel sheets, which would be continued in further rows down and further columns to the right.

Figure 7: MS EXCEL screenshot for model with infinite horizon (http://faculty.som.yale.edu/shyamsunder/Research/Experimental%20Economics%20and%20Finance/Presentations%20and%20Working%20Papers/Huber-Shubik-Sunder/MODEL_INFINITE_ONLINE%20MATERIAL.xls)

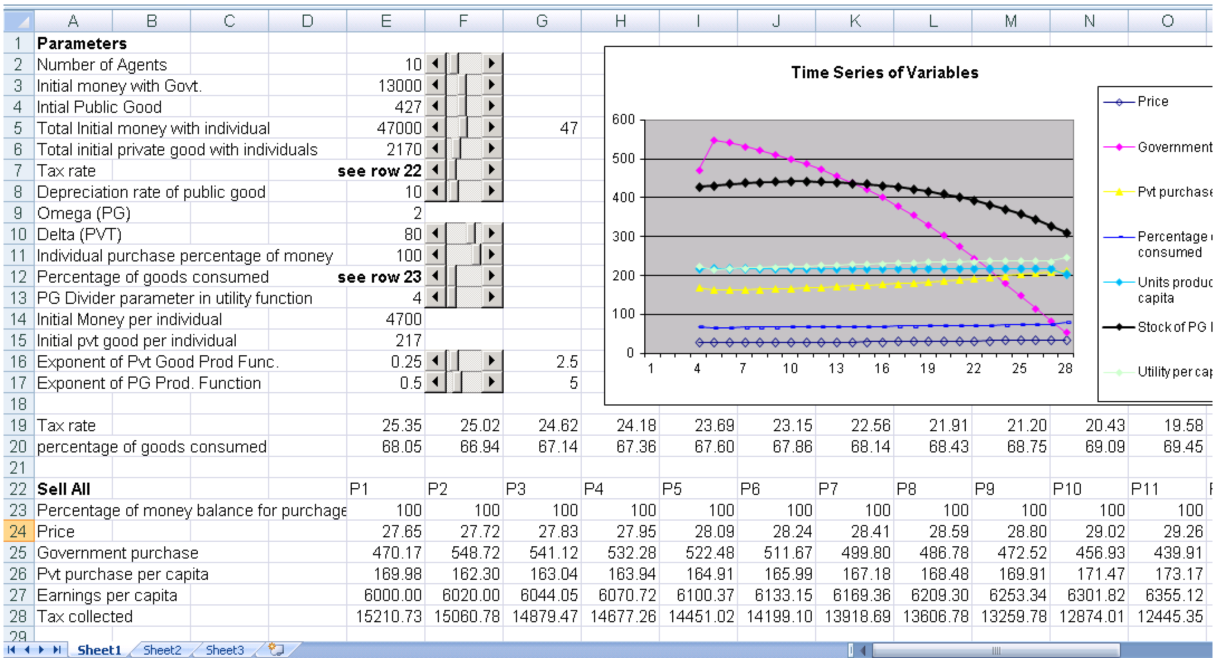
The Graph in the top rows of columns I to N shows total utility as a function of consumption rate (E12) and tax rate (E7).



	A	E	F	G	H	I	J	K	L	M	N
1	**Parameters**										
2	Number of Agents	10									
3	Initial money with Govt.	13000									
4	Intial Public Good	427									
5	Total Initial money with individual	47000		47							
6	Total initial private good with individuals	2170									
7	Tax rate	**21.57**									
8	Depreciation rate of public good	10									
9	Omega (PG)	2									
10	Delta (PVT)	80									
11	Individual purchase percentage of money	100									
12	Percentage of goods consumed	**68.27**									
13	PG Divider parameter in utility function	4									
14	Initial Money per individual	4700									
15	Initial pvt good per individual	217									
16	Exponent of Pvt Good Prod Func.	0.25		2.5							
17	Exponent of PG Prod. Function	0.5		5							
18	**Sell All**	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
19	Percentage of money balance for purchage	100	100	100	100	100	100	100	100	100	100
20	Price	27.65	27.67	27.67	27.67	27.67	27.67	27.67	27.67	27.67	27.67
21	Government purchase	470.17	467.67	467.71	467.72	467.72	467.72	467.72	467.72	467.72	467.72
22	Pvt purchase per capita	169.98	170.04	170.05	170.05	170.06	170.06	170.06	170.06	170.06	170.06
23	Earnings per capita	6000.00	6000.00	6000.00	6000.00	6000.00	6000.00	6000.00	6000.00	6000.00	6000.00
24	Tax collected	12942.57	12942.57	12942.57	12942.57	12942.57	12942.57	12942.57	12942.57	12942.57	12942.57
25											
26	**Production**										
27	Percentage of goods consumed	68.27	68.27	68.27	68.27	68.27	68.27	68.27	68.27	68.27	68.27
28	Units produced per capita	216.80	216.82	216.83	216.83	216.83	216.83	216.83	216.83	216.83	216.83
29	Stock of PG EOP	427.67	428.15	428.59	428.98	429.34	429.66	429.95	430.21	430.44	430.65

Figure 8: MS EXCEL screenshot for model with finite horizon of 30 periods (http://faculty.som.yale.edu/shyamsunder/Research/Experimental%20Economics%20and%20Finance/Presentations%20and%20Working%20Papers/Huber-Shubik-Sunder/MODEL_FINITE_ONLINE%20MATERIAL.xls)

Here the tax rate (E7) and consumption rate (E12) are no longer fixed for several periods, but instead change from period to period. The respective values are displayed in rows 19 and 20.



	A	E	F	G	H	I	J	K	L	M	N	O
1	**Parameters**											
2	Number of Agents	10										
3	Initial money with Govt.	13000										
4	Intial Public Good	427										
5	Total Initial money with individual	47000		47								
6	Total initial private good with individuals	2170										
7	Tax rate	**see row 22**										
8	Depreciation rate of public good	10										
9	Omega (PG)	2										
10	Delta (PVT)	80										
11	Individual purchase percentage of money	100										
12	Percentage of goods consumed	**see row 23**										
13	PG Divider parameter in utility function	4										
14	Initial Money per individual	4700										
15	Initial pvt good per individual	217										
16	Exponent of Pvt Good Prod Func.	0.25		2.5								
17	Exponent of PG Prod. Function	0.5		5								
18												
19	Tax rate	25.35	25.02	24.62	24.18	23.69	23.15	22.56	21.91	21.20	20.43	19.58
20	percentage of goods consumed	68.05	66.94	67.14	67.36	67.60	67.86	68.14	68.43	68.75	69.09	69.45
21												
22	**Sell All**	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P11
23	Percentage of money balance for purchage	100	100	100	100	100	100	100	100	100	100	100
24	Price	27.65	27.72	27.83	27.95	28.09	28.24	28.41	28.59	28.80	29.02	29.26
25	Government purchase	470.17	548.72	541.12	532.28	522.48	511.67	499.80	486.78	472.52	456.93	439.91
26	Pvt purchase per capita	169.98	162.30	163.04	163.94	164.91	165.99	167.18	168.48	169.91	171.47	173.17
27	Earnings per capita	6000.00	6020.00	6044.05	6070.72	6100.37	6133.15	6169.36	6209.30	6253.34	6301.82	6355.12
28	Tax collected	15210.73	15060.78	14879.47	14677.26	14451.02	14199.10	13918.69	13606.78	13259.78	12874.01	12445.35

APPENDIX B: Instructions

Dear participant: Welcome to the experiment. Please do not talk to any other subject for the duration of the experiment.

You are one of ten subjects populating a small economy with money and two kinds of goods: one private and one public good. As subjects, you will produce, sell, buy, and consume the private good. The government (played by the experimenter) will tax the income of subjects (from sale of the private good) and use the proceeds to buy some of the private good, to be used to produce the public good. The tax rate will be either fixed, or determined by the vote of the ten subjects once every five periods. Your earnings for each period depend on the quantity of private good you consume, and the quantity of the public good provided by the government for benefit of all in that period.

Money and Goods

There is money and two kinds of goods in the economy:

- A private good produced, sold, bought and consumed by the participating subjects; some the private good is also bought by the government and used to produce the public good.
- The public good (e.g., a public facility) which depreciates at the rate of 10 percent per round. The government uses tax collected from subjects to replenish the depreciating stock of public good.

In round 1 each subject starts with 4,700 units of money and 217 units of the private good. The government starts with 13,000 units of money and 427 (*213.5 in half of the runs*) units of the public good.

At the beginning of each round, all private good produced in (and carried over from) the preceding round) is sold in a market. Thus, the initial private good endowment of 217 units in the hands of each subject (for a total of 2,170) is sold at the start of round 1.

Money serves only a means of exchange in this economy, but it has no role in savings, etc. An amount of money is given to you at the beginning of the session, and any balance left over at the end of the session has no value to you. Each round all money you have (either initial endowment or earned from sale of goods the round before) is spent for the purchase of goods at the start of each round. No borrowing is possible.

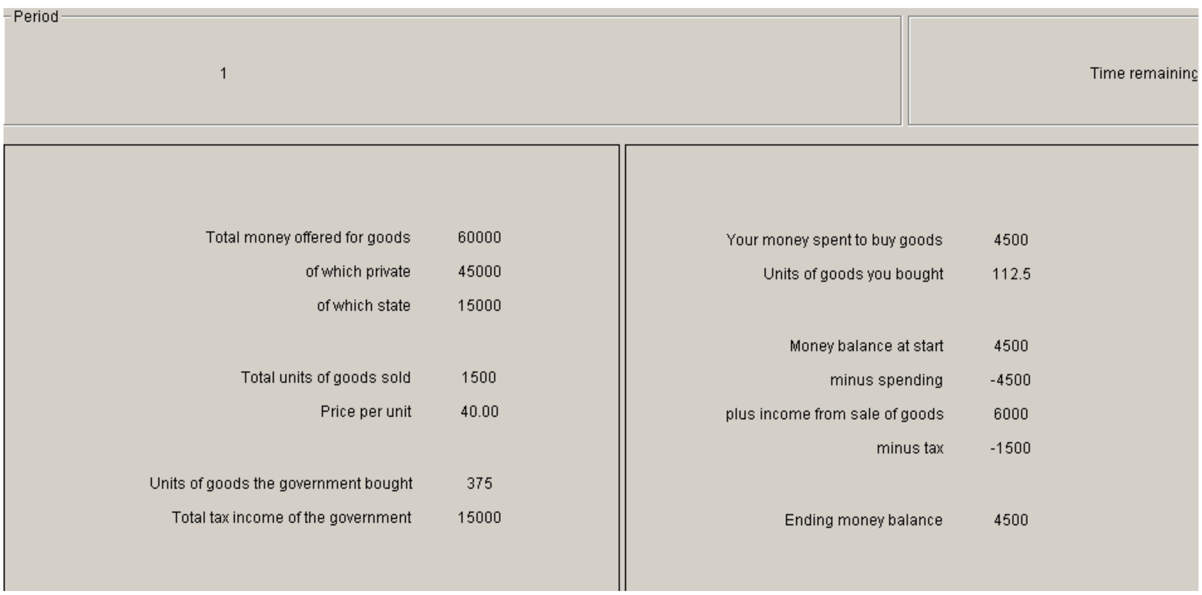
At the start of a period all money held by the government and individuals is tendered to buy units of the private good. In the first period 2,170 units are sold for a total of 60,000 units of money.

Total agent and government bids in money = 60,000/2,170 (total number of units of private good) = 27.65. These numbers will change in subsequent rounds.

Each individual buys 170 units and earns 217*27.65=6,000 units of money. Your first decision is how many of these 170 units you invest into production for the next period, with the remainder being consumed this period. Your money income (6,000 in the first period) is taxed by the government at a rate set by all subjects through a vote (see details below).

On the left side of the Screen 2 you learn the total money bid for private good, the resulting price, the units bought by the government, and government's tax revenue (all of which is spent to buy private goods in the following round). On the right side of Screen 2 you see how many units you bought, your spending, income, tax, and the initial and final money balances (the latter to be carried over to the following round).

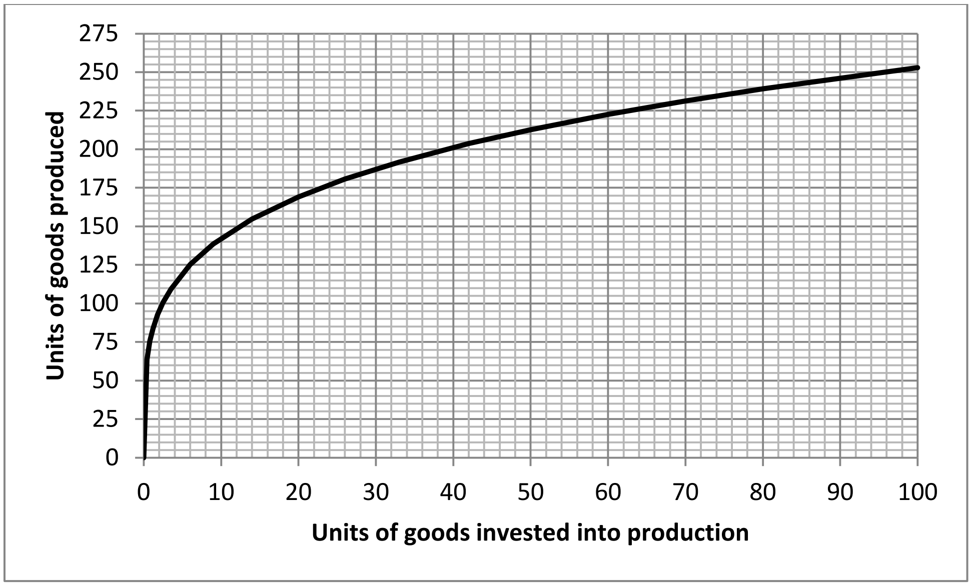
Screen 2



Out of the units of private good you bought, you have to decide on how many you wish to consume, and how many you wish to invest to produce private goods to be sold during the next round. The following equation and chart show the relationship between the units you invest and the units produced:

UNITS OF THE PRIVATE GOOD PRODUCED = 80*(UNITS INVESTED)$^{0.25}$.

Note, for example, that investing 1 unit produces 80 units; investing 40 units produces 201.19 units.



Public Good

The government starts with a stock of 427 units of the public good. This stock depreciates by 10% each round, like, for example, roads deteriorate. To maintain or upgrade the public good the government taxes the subjects' income (from sales of goods) at the selected rate. All tax receipts are used to buy the private good and all private goods are used to produce new units of the public good according to the following function:

UNITS OF THE PUBLIC GOOD PRODUCED = 2*(UNITS OF PRIVATE GOOD INVESTED)$^{0.5}$.

Taxes

All individual income (proceeds from sale of private good) will be taxed at a flat tax rate (which is either fixed by the experimenter in advance, or is set by the vote of ten subjects). In the latter case, every five rounds (i.e., at the beginnings of rounds 1, 6, 11, 16, etc.) each subject is asked to submit his/her suggested percent rate of taxation to be applicable to all ten subjects. You are free to suggest any integer number between zero (no tax) and 100 (everything taken by the government) as the percent tax rate. The computer collects the suggested tax rates from the ten subjects, sorts them from highest to lowest, and sets the median (average of the 5^{th} and the 6^{th} suggested rates) as the tax rate for all subjects. The selected tax rate is announced, and it remains in effect for five rounds until the next tax rate is determined though another vote. (*In half of the treatments the tax rate was fixed at 21.5. percent and no vote was carried out*)

Points earned

The points you earn in each round are calculated as:

POINTS = CONSUMPTION OF PRIVATE GOOD + PUBLIC GOOD/4.

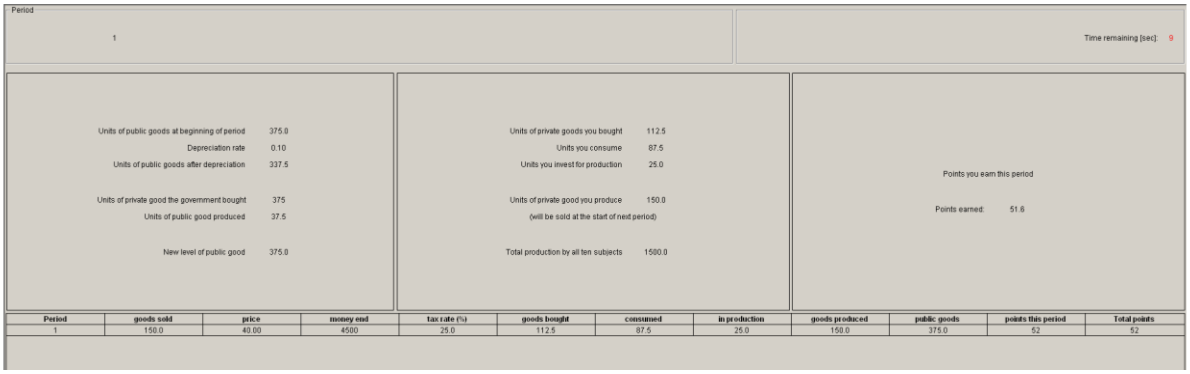
For example, if you consume 60 units of private good and the government provides 200 units of public good, you earn 60 + 200/4 = 110 points in that period. Both higher private good consumption as well as higher stock of the public good increase your earnings. Chart 1 and Table 1 show the number of points resulting from various combinations of private good consumption and public good provision by government.

(Insert Chart 1)

History screen:

After all subjects have entered their consumption/investment decisions, computer carries out all the calculations, and a history screen provides a round-by-round overview of the results (the accounting of public goods on the left, your consumption and production of goods in the middle, the points you earn during the round on the right, and the summary of the round at the bottom.

History Screen



Final payment:

There is 1/6 chance that the experiment will last for 25, 26, 27, 28, 29, or 30 rounds. The actual number of rounds in the session will be determined randomly before we start, but will not be announced to you until the session ends.

The points earned during all rounds are added up (column "Total points" in the History Screen). Your take-home payment in euro is TOTAL POINTS / 200. For example, if the experiment ends in round 28 and you earned a total of 3,000 points during these 28 rounds, your take-home payment is 3,000/200 = 15 Euros.

(Appendix C to be inserted in PDF file here)

APPENDIX C

The formal model presented here is based on Karatzas et al. (2011) which also includes proofs and further details. There is a continuum of producer-consumer agents $\alpha \in I$ who hold cash $m_n^\alpha \geq 0$ and goods $q_n^\alpha \geq 0$ at the start of each period n.

The government provides a quantity $G_n \geq 0$ of a generic public good in every period n. We assume that the government spends all its tax revenues on the production and maintenance of the public good. The tax rate is θ and $\bar{\theta} = 1 - \theta$.

Each agent α makes a cash bid b_n^α to purchase the sole producer-consumer good for sale in period n where

$$b_n^\alpha \in \left[0, m_n^\alpha + \frac{\bar{\theta}\hat{p}_n q_n^\alpha}{1+\rho}\right], \tag{1}$$

The second term in the expression above is a lending limit imposed by the central bank, if the agents choose to borrow. The $\hat{p}_n$ is the bank's estimate of the price p_n of the private good, and, in a rational expectations equilibrium, $\hat{p}_n = p_n$, thus in equilibrium the individual will be able to pay back her loan. This appears as a partially "secured" loan. In actuality banks handle this feature by adding a "haircut". Although default is possible out of equilibrium, we do not model it explicitly.

Concerning the possibility that agents wish to deposit and earn interest this is implicitly covered in the selection of b_n^α, it is as though there is a sweep program that gathers any spare cash of the individuals. In the theory below and experiments above with $1 + \rho = 1/\beta$ there is no need for borrowing or depositing in equilibrium.

In period n each agent α sells her goods q_n^α in a market at price p_n and pays taxes in the amount $\theta p_n q_n^\alpha$, (where α denotes a single agent of measure zero whose holdings are indicated by a density).

The government spends its total tax revenues

$$B_n^G = \int_I \theta p_n q_n^\alpha \, d\alpha = \theta p_n \int_I q_n^\alpha \, d\alpha = \theta p_n Q_n \tag{2}$$

in the private goods market. Thus the price p_n is formed as

$$p_n = \frac{B_n + B_n^G}{Q_n}, \tag{3}$$

where

$$B_n = \int_I b_n^\alpha \, d\alpha$$

is the total bid by the agents. Each agent α purchases the quantity b_n^α / p_n of goods and inputs $k_n^\alpha \in [0, b_n^\alpha / p_n]$ for production and consumes the rest.

The government acquires the quantity

$$k_n^G = \frac{B_n^G}{p_n}$$

of the private good all of which is used for the production or maintenance of the public good.

Suppose $G_n \geq 0$ is the quantity of the public good available at the beginning of period n and that $\eta \in (0, 1]$ is the depreciation rate. Then the amount of the public good in the next period is

$$G_{n+1} = (1 - \eta)G_n + F(k_n^G), \tag{4}$$

where $F(\cdot)$ is the government's production function for the public good.

The utility of an agent α in period n is

$$u(x_n^\alpha, G_n),$$

a concave increasing function of the agent's private consumption

$$x_n^\alpha = \frac{b_n^\alpha}{p_n} - k_n^\alpha$$

and the public good G_n provided by the government. For fixed values of G, we assume that $u(\cdot, G)$ is concave, increasing, and differentiable. As before each agent α seeks to maximize her total discounted utility, where $0 \leq \beta < 1$.

$$\sum_{n=1}^{\infty} \beta^{n-1} u(x_n^\alpha, G_n).$$

The government is regarded as a controller who moves first in the game. Its actions are specified by its selection of its control variables, the interest rate ρ and the tax rate θ. The tax rate determines the income level of the government and thereby also determines its production of the public good. Furthermore the goal or the objective function of the government is assumed known to the agents. For example, the government may wish to supply some level of the public good subject to some condition on inflation.

An equilibrium for the Model

A type-symmetric equilibrium is constructed in which money and prices may inflate, but consumption and production remain constant.

Suppose that all agents begin with the same amount of cash $m^\alpha = M$ and goods $q^\alpha = Q = f(k) + y$. Assume each agent makes the same bid $b^\alpha = b = aM$ and inputs the same amount $k^\alpha = k$ for production. Since the government bids its total income, namely $\theta p Q$, the price of the private good is given by

$$p = \frac{\int_I b^\alpha \, d\alpha + \theta p Q}{Q} = \frac{aM}{Q} + \theta p$$

and hence

$$p = \frac{aM}{\bar{\theta} Q}.$$

The amount of private good consumed by each agent in the period is

$$x^\alpha = \frac{b^\alpha}{p} - k^\alpha = \frac{aM}{p} - k = \bar{\theta} Q - k. \tag{5}$$

The government inputs

$$k^G = \frac{\theta p Q}{p} = \theta Q$$

for production of the public good and thus produces the quantity $F(\theta Q)$. Assume that the government holds the quantity G_n of the public good in each period n equal to a constant G. Then

$$G = (1-\eta)G + F(\theta Q),$$

and thus

$$G = \frac{1}{\eta} F(\theta Q) \tag{6}$$

is the amount of the public good provided by the government in every period. The utility received by each agent in every period is

$$u(x^\alpha, G) = u(\bar{\theta} Q - k, \frac{1}{\eta} F(\theta Q)). \tag{7}$$

Theorem 1 *Suppose that every agent $\alpha \in I$ begins with cash $m^\alpha = M > 0$ and goods $q^\alpha = Q = f(k_1)$, where $f'(k_1) = (1+\rho)/(\beta\bar{\theta})$. Assume also that the government initially provides the quantity $G = F(\theta Q)/\eta$ of the public good. Then there is an equilibrium in which all agents bid the proportion*

$$a = \frac{(1+\rho)(1-\beta)}{\rho} \tag{8}$$

of their cash and input k_1 for production in every period. In this equilibrium, the government inputs $k^G = \theta Q$ for production of the public good in every period thereby holding $G_n = G$ for all n.

The proof given in Karatzas et al. (2011).

In this equilibrium money and prices inflate at the rate $\tau = \beta(1+\rho)$. To see this, suppose that all the agents begin at $m^\alpha = M$ and $q^\alpha = Q$ and play the strategy of the theorem. Then at the next stage, each agent has cash

$$\begin{aligned} \tilde{M} = \tilde{m}^\alpha &= (1+\rho)(m^\alpha - a m^\alpha) + \bar{\theta} p q^\alpha \\ &= (1+\rho)(M - aM) + \bar{\theta} \cdot \frac{aM}{\bar{\theta} Q} \cdot Q \\ &= \tau M. \end{aligned}$$

The price at the next stage is

$$\tilde{p} = \frac{a\tilde{M}}{\bar{\theta} Q} = \tau \frac{aM}{\bar{\theta} Q} = \tau p.$$

A control problem for the government

A benevolent but controlling government will try to maximize the welfare of the agents through its choice of the values of its control variables, the interest rate ρ and the tax rate θ. Observe that, in the equilibrium the total discounted utility of every agent is,

$$\sum_{n=1}^{\infty} \beta^{n-1} u(x_n^{\alpha}, G_n) \;=\; \frac{1}{1-\beta} \cdot u(\bar{\theta}Q - k_1, \frac{1}{\eta} F(\theta Q)),$$

where

$$k_1 = (f')^{-1}\left(\frac{1+\rho}{\bar{\theta}\rho}\right) \qquad \text{and} \qquad Q = f(k_1).$$

Thus the utility of an agent can be written as a function $\varphi(\theta, \rho)$ of the the government's control variables θ and ρ.

We illustrate the government's optimization with an example.

Suppose $f(k) = 2\sqrt{k}, F(k) = k, \beta = 1/2, \eta = 1$,

$u(x, G) = \log(xG)$. Then

$$f'(k_1) = \frac{1}{\sqrt{k_1}} = \frac{1+\rho}{\beta\bar{\theta}} = \frac{2(1+\rho)}{\bar{\theta}}$$

so the equilibrium values are

$$k_1 = \frac{\bar{\theta}^2}{4(1+\rho)^2}, \qquad Q = f(k_1) = \frac{\bar{\theta}}{1+\rho},$$

and

$$G = F(k^G) = k^G = \theta Q = \frac{\theta\bar{\theta}}{1+\rho}.$$

The utility to maximize is $\varphi(\theta, \rho)$, where

$$\begin{aligned} \varphi(\theta, \rho) \;&=\; \frac{1}{1-\beta} \cdot u(\bar{\theta}Q - k_1, \theta Q) = \\ &\frac{1}{1-\beta} \cdot u\left(\frac{\bar{\theta}^2}{1+\rho} - \frac{\bar{\theta}^2}{4(1+\rho)^2}, \frac{\bar{\theta}\theta}{1+\rho}\right). \end{aligned}$$

Since $u(x, G) = \log(xG)$, simple algebra shows that

$$\varphi(\theta, \rho) = 3\log\bar{\theta} + \log\theta + \log(3+4\rho) - 3\log(1+\rho) + C,$$

where C does not depend on θ or ρ. So

$$\frac{\partial\varphi}{\partial\theta}(\theta, \rho) = \frac{-3}{1-\theta} + \frac{1}{\theta} = 0$$

if $\theta = 1/4$. This gives the optimal tax rate at 25%. It's also easy to see that

$$\frac{\partial\varphi}{\partial\rho}(\theta, \rho) < 0$$

for all $\rho > 0$.

The example is fairly typical in that there is an optimal tax rate that is an interior point of (0,1). This is intuitively obvious since a tax rate of 0 results in no production of the public good and a rate of 1 means that the agents have no incentive to produce the private good.