# AFFECTIVE DECISION-MAKING: A THEORY OF OPTIMISM-BIAS

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# Affective Decision-Making: A Theory of Optimism-Bias *

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**Abstract**

Optimism-bias is inconsistent with the independence of decision weights and payoffs found in models of choice under risk, such as expected utility theory and prospect theory. Hence, to explain the evidence suggesting that agents are optimistically biased, we propose an alternative model of risky choice, affective decision-making, where decision weights — which we label affective or perceived risk — are endogenized.

Affective decision making ($ADM$) is a strategic model of choice under risk, where we posit two cognitive processes: the "rational" and the "emotional" processes. The two processes interact in a simultaneous-move intrapersonal potential game, and observed choice is the result of a pure strategy Nash equilibrium in this potential game.

We show that regular $ADM$ potential games have an odd number of locally unique pure strategy Nash equilibria, and demonstrate this finding for affective decision making in insurance markets. We prove that $ADM$ potential games are refutable, by axiomatizing the $ADM$ potential maximizers.

## 1 Introduction

Many of our everyday decisions such as working on a project, taking a flu shot, or buying insurance require an estimate of probabilities of future events: the probability of a project's success, of getting sick, or of being involved in an accident. In assessing these probabilities, decision-makers tend to be optimistically biased, where optimism-bias is defined as the tendency to overestimate the likelihood of favorable future outcomes and underestimate the likelihood of unfavorable future outcomes (Irwin, 1953; Weinstein, 1980; Slovic et al., 1982; Slovic, 2000). A young woman drinking at a bar thinking it would be safe for her to drive home is an example; an entrepreneur who starts a new business, confident that she is going to succeed where others have

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failed, is another. Indeed, one can argue that although statistics for these events are well documented, each of these individuals has private information concerning her tolerance for alcohol and entrepreneurial ability, respectively. Hence, each woman may have good reasons to believe that overall empirical frequencies do not apply to her. The common feature in these examples is that decision-makers have some freedom in choosing their probabilistic beliefs, and they are often optimistic — they appear to choose beliefs that are biased towards favorable outcomes.

Optimism-bias is not merely a hypothetical bias; instead it translates into both microeconomic and macroeconomic activity. For example, optimism-bias influences high-stakes decisions, such as startup investment, investment behavior, and merger decisions. It was found that 68 percent of startups' entrepreneurs believe their company is more likely to succeed than similar companies, while in reality only 50 percent of startup companies survive beyond three years of activity (Baker, Ruback and Wurgler, 2006 and references therein). Malmendier and Tate (2005) find that CEOs who are optimistic regarding their firm's future performance have a greater sensitivity to investment's cash flow, leading to distortions in investments. In their 2008 paper, Malmendier and Tate find that the optimistic CEOs are 65 percent more likely to complete mergers, are more likely to overpay for those target companies, and are more likely to undertake value-destroying mergers. On the macroeconomic level, Bob Shiller in his now classic book *Irrational Exuberance* (2000, 1st ed.) defines irrational exuberance as "wishful thinking on the part of investors that blinds us to the truth of our situation," and he makes the case that irrational exuberance contributes to generating bubbles in financial markets. Shiller points out several psychological and cultural factors that affect individuals' beliefs and consequently investment behavior, leading to real macro-level effects. Many of these factors can be summarized as optimistically biased beliefs.

Optimism-bias is inconsistent with the independence of decision weights and payoffs found in models of choice under risk, such as expected utility theory and prospect theory. Hence, to explain the evidence suggesting that agents are optimistically biased, we propose an alternative model of risky choice where decision weights — which we label affective or perceived risk — are endogenized. More specifically, we consider two systems of reasoning, which we label the rational process and the emotional process. The rational process decides on an action, while the emotional process forms perceptions of risk and in so doing is optimistically biased. The two processes interact to reach a consistent decision. This interaction is modeled as a simultaneous-move, intrapersonal potential game, and consistency between the two processes, which represents observed choice, is the equilibrium outcome realized as a pure strategy Nash equilibrium of the game.

A formulation that may be viewed as a model of the specialization and integration of brain activity considered in the recent neuroscience literature. That is, recent studies in neuroscience identify distinct brain modules that specialize in different activities. For instance, the amygdala is associated with emotions, while the prefrontal cortex is associated with higher-level, deliberate thinking (e.g., Reisberg, 2001). Our model is also consistent with the psychology literature that draws a distinction between analytical and intuitive, or deliberate and emotional cognitive activity. (Chaiken and Trope, 1999). However, in both neuroscience and psychology, behavior is thought to be a result of the different systems interacting (for example, Sacks, 1985; Damasio, 1994; Epstein, 1994; LeDoux, 2000; Gray et al., 2002; Camerer, Loewenstein and Prelec, 2004; Pessoa, 2008). Gray et al. (2002) for example conclude that "at some point of processing, functional specialization is lost, and emotion and cognition conjointly and equally contribute to the control of thought and behavior," and recently, Pessoa (2008) argues that "...emotions and cognition not only strongly interact in the brain, but [that] they are often integrated so that they jointly contribute to behavior," a point also made in the specific context of expectation formation.

Although the evidence on modular brain and the dual-processes theory cannot typically be pinned down to the formation of beliefs, given that beliefs formation is partly affected by the beliefs we would like to have, that is, by affective considerations, decision-making under risk naturally suggests the interplay between the two cognitive processes, proposed by Kahneman (2003). That is, decision-making under risk can be modeled as a deliberate process that chooses an optimal action, and an emotiona1 cognitive process that forms risk perception.

Formally, the rational process coincides with the expected utility model, where for a given risk perception (affective probability distribution), the rational process chooses an action to maximize expected utility. The emotional process forms risk perception by selecting an optimal risk perception that balances two contradictory impulses: (1) affective motivation and (2) a taste for accuracy. This model is consistent with the definition of motivated reasoning, a psychological mechanism where emotional goals motivate agent's beliefs (see Kunda, 1990), and is a source of psychological biases, such as optimism-bias. Affective motivation is the desire to hold a favorable personal risk perception — optimism — and in the model it is defined by the expected utility term. The desire for accuracy is modeled as a mental cost incurred by the agent for holding beliefs in lieu of her base-rate probabilities given her desire for favorable risk beliefs. The base-rate probabilities are the beliefs that minimize the mental cost function of the emotional process,i.e., the risk perception that is the easiest and least costly to justify; in many instances, one can think of the base-line probabilities as the empirical, relative frequencies of the states of nature.

We present an example of the demand for insurance in a world with a bad state and a good state as an application of affective decision making. The relevant probability distribution in insurance markets is personal risk; hence, the demand for insurance may depend on optimism-bias. Affective choice in insurance markets is defined as the insurance level and risk perception that constitute a pure strategy Nash equilibrium of the $ADM$ potential game.

The systematic departure of the $ADM$ model from the expected utility model is consistent with consumer research (Keller and Block, 1996), that campaigns intended to educate consumers on the magnitude of the potential loss in the unfavorable state can have the unintended consequence that consumers purchase less, rather than more, insurance. Hence, the $ADM$ model suggests that the failure of the expected utility model to explain some data sets may be due to systematic affective biases.

The $ADM$ intrapersonal game is a potential game — where the potential is a penalized subjective expected utility ($SEU$) function — that defines the best response dynamic of the game. This specification has the natural interpretation of the utility function of the composite agent, or integration of the two systems. Deviations from the basic models of rational choice often raise the concern that the theory lacks the discipline imposed by a clear paradigm, and, as a result, any data set can be rationalized by such models. This concern arises in the $ADM$ model, since we allow agents to choose both actions and beliefs. We present an axiomatic characterization of $ADM$ potential maximizers that shows the model is refutable. That is, there exists data sets that cannot be rationalized by $ADM$ potential games.

The remainder of the paper is organized as follows: In section 2 we discuss the related literature, and in section 3 we present the demand for insurance in a world with two states of nature. Section 4 present an analysis of $ADM$ potential games in a world with $K$-states of nature.. In section 5 we provide a formal definition of optimistic preferences and present an axiomatic foundation of the $ADM$ potential maximizers. All proofs are in the Appendix.

## 2 Related Literature

Recent literature in economic theory recognizes the possibility that agents might choose their beliefs in a self-serving or optimistic way, such as Akerlof and Dickens (1982), Bodner and Prelec (2001), Bénabou and Tirole (2002), Yariv (2002), Caplin and Leahy (2004), Bracha (2005), Brunnermeier and Parker (2005), and Kőszegi (2006). The dual processes hypothesis, as well, was recently recognized in economic modeling. Specifically, in models of self-control and addiction such as Thaler and Shefrin (1981), Bernheim and Rangel (2004), Loewenstein and O'Donoghue (2004), Benhabib and Bisin (2005), Fudenberg and Levine (2006), and Brocas and Carrillo (2008). Existing models are restricted in the sense that choice of beliefs and choice of action are not made in tandem and the models assume that an agent chooses beliefs in a strategic manner to resolve a tradeoff between a standard instrumental payoff and some notion of psychologically based belief utility,[1] while the existing models of dual processes are restricted in that the two systems, or decision modes, are conceived as mutually exclusive.

Although there are cases where a descriptive model seems to require mutually exclusive systems, as in the case of self-control and addiction, there are other cases where a descriptive model seems to require several different processes that together determine observed choice. We provide such a formulation: one process chooses action while the other forms perceptions, and both are necessary for decision-making.

As mentioned, the $ADM$ intrapersonal game is a potential game with a potential function defined as a penalized $SEU$ model. This characterization allows us to axiomatize the set of $ADM$ potential maximizers. More importantly, the axioms suggest that $ADM$ potential games can be interpreted as representations of optimistic

[1] The axiomatic foundation for this is provided by Caplin and Leahy (2001) and Yariv (2001).

preferences. An analogous representation of optimistic preferences is the Optimal Expectations model of Brunnermeier and Parker (2005). This model considers an agent who chooses both beliefs and actions in a dynamic setting, where beliefs are chosen at period one for all future periods, trading off greater anticipated utility against the cost of poor decisions due to optimistic beliefs. Hence, optimal expectations are optimistic beliefs not constrained by reality. $ADM$, in contrast, is a static model, where beliefs and actions mutually determine observed choice, and where beliefs trade off greater anticipated utility against the mental cost of distorting beliefs — costs that are a function of reality. Having a simultaneous framework, where costs are based solely on beliefs is a parsimonious model that is consistent with cognitive dissonance. The $ADM$ potential game, is also consistent with the integration of processes in the brain, where the potential function acts as a utility function of the composite agent. Unfortunately, there is no axiomatic characterization of the Optimal Expectations model that allows an explicit comparison of the behavioral assumptions characterizing the two models.

# 3 The ADM Model of the Demand for Insurance

Affective decision making (ADM) is a theory of choice, which generalizes expected utility theory by positing the existence of two cognitive processes — the rational and the emotional process — and where observed choice is the result of their simultaneous interaction. This theory accommodates endogenity of beliefs. In this section, we present a model of affective choice in insurance markets, where probability perceptions are endogenous.

Consider an agent facing two states of the world, $B$ad and $G$ood with associated wealth levels $w_B$ and $w_G$, where $w_B < w_G$. The agent has a strictly increasing, strictly concave, smooth utility function of wealth, $u(w)$, with $\lim_{w\to-\infty} Du(w) = \infty$, $\lim_{w\to\infty} Du(w) = 0$.[2] Risk perception is defined as the perceived probability $p \in [0,1]$ of the $B$ad state occurring. For simplicity we allow the agent to purchase or sell insurance $I \in (-\infty, \infty)$ at the fixed insurance premium rate, $\gamma \in (0,1)$. The intuition and results for the case where the agent can only buy insurance are easily derived from this analysis.

The rational process chooses an optimal insurance ($I^*$) to maximize expected utility given a perceived risk $p$. Specifically, the rational process maximizes the following objective function:

$$\max_I \{pu(w_B + (1-\gamma)I) + (1-p)u(w_G - \gamma I)\}.$$

The emotional process chooses an optimal risk perception ($p^*$) given an insurance level $I$, to balance affective motivation and taste for accuracy. Specifically, the emotional process maximizes the following objective function:

$$\max_p \{pu(w_B + (1-\gamma)I) + (1-p)u(w_G - \gamma I) - J^*(p;p_0)\}.$$

[2] All qualitative results remain the same for the case of $\lim_{w\to 0} Du(w) = \infty$, $\lim_{w\to\infty} Du(w) = 0$.

Affective motivation is captured by the expected utility term — the agent would like to assign the highest possible weight to her preferred state of the world. Taste for accuracy is modeled by introducing a mental cost function $J^*(p;p_0)$ that is a nonnegative, smooth function of $p \in (0,1)$. It is strictly convex, and reaches a minimum at $p = p_0$, where $p_0$ is the base-line probability; at the boundary $p \in \{0,1\}$. If $J^*$ is strictly convex and $C^1$on $(0,1)$, where $\lim_{p\to 0} DJ^*(p) \to +\infty$ and $\lim_{p\to 1} DJ^*(p) \to +\infty$, then $\lim_{p\to 0} J^*(p;p_0) = \lim_{p\to 1} J^*(p;p_0) = +\infty$. See Figure 1.

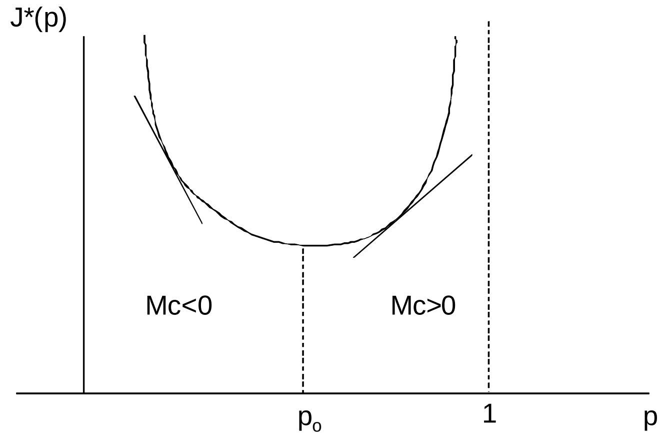


Figure 1

Why this shape? The literature in psychology argue that individuals tend to use mental strategies such as bias search through memory to find justifications for their desired beliefs (Kunda, 1990). As the desired beliefs are farther away from some base-line odds $p_0$, the odds that immediately come to mind and are easiest to justify such as the empirical, relative frequency of states of nature or available statistics like mortality tables, the search costs are likely to increase. That is, it would be increasingly more difficult to come up with justifications and find anecdotes to support the optimistic view. This is exactly what the shape of the mental cost function captures. In addition, the behavior at the extreme is a formal description of a well-known phenomenon. Namely, that decision makers assign a special quality to certain situations: getting \$100 for sure is qualitatively different from a 99 percent chance lottery to win \$100 (Kahneman and Tversky, 1979). In the current simple settings, certainty corresponds to the extreme beliefs $p \in \{0,1\}$, and the behavior at the extremes captures the dramatic difference between certain, "safe," and risky events. Hence, the psychology literature is consistent with a mental cost $J^*(p)$ that is strictly convex and essentially smooth on the interior of the probability simplex $\Delta$.

The fact that the mental cost is a function solely of probability is appealing as well. It formally reflects the psychological description of reasoning and gives rise to the special and important property of our model of interaction between the rational and emotional processes — the intrapersonal game. That is, the intrapersonal game is a potential game. As such, this type of mental cost is consistent with integration of the two processes, a property supported by psychology and recent research in neuroscience.

We now consider the interaction of the two processes in decision making. We model this interaction as an intrapersonal simultaneous-move game; this choice reflects a recent view in cognitive neuroscience; namely, both processes mutually determine the performance of the task at hand (Damasio, 1994).

**Definition 1** *An intrapersonal game is a simultaneous move game of two players, namely, the rational and the emotional processes. The strategy of the rational process is an insurance level,* $I \in (-\infty, \infty)$*, and the strategy of the emotional process is a risk perception,* $p \in (0, 1)$*. The payoff function for the rational process* $g : (0, 1) \times (-\infty, \infty) \to R$ *is* $g(p, I) \equiv pu(w_B + (1-\gamma)I) + (1-p)u(w_G - \gamma I)$*. The payoff function for the emotional process* $\Theta : (0, 1) \times (-\infty, \infty) \to R$ *is* $\Theta(p, I) \equiv g(p, I) - J^*(p; p_0)$*, where* $J^*(\cdot)$ *is the mental cost function of holding belief* $p$*, which reaches a minimum at* $p_0$*.*

Proposition 2 below indicates the intrapersonal game defined above is a potential game. Potential games are a class of strategic games introduced by Monder and Shapley (1993), where all players have a common goal and therefore the game can be represented with one global common payoff function. This global payoff function is called the potential function of the game, and is used by each player to determine her best response. In the case of individual choice, since the players are decision processes and the game is a model of decision-making, the potential function has an intuitive interpretation of a utility function of the composite agent. Below is the formal proposition:

**Proposition 2** *The intrapersonal game is a potential game, in which the emotional process's objective function is the potential function for the game. Because the potential function is strictly concave in each variable (risk perception and insurance), its critical points are the pure strategy Nash equilibria of the game.*

It is straight-forward to show that the emotional process's objective function is the potential of the game, as its first order conditions with respect to $I$ and $p$ are the same as those of the rational process and emotional process, respectively. That is the potential function $\Theta(I, p) = \langle U(I), p \rangle - J^*(p)$ captures the best-response dynamics of the intrapersonal game, and this potential game is therefore a model of affective decision making.

The equilibrium notion in the potential game is pure strategy Nash equilibrium, which is a natural candidate for choice, as it reflects a mutually determined choice and consistency between the rational and emotional processes. Given that the potential is bi-concave, we have the following existence theorem (see Figure 2 for illustration).

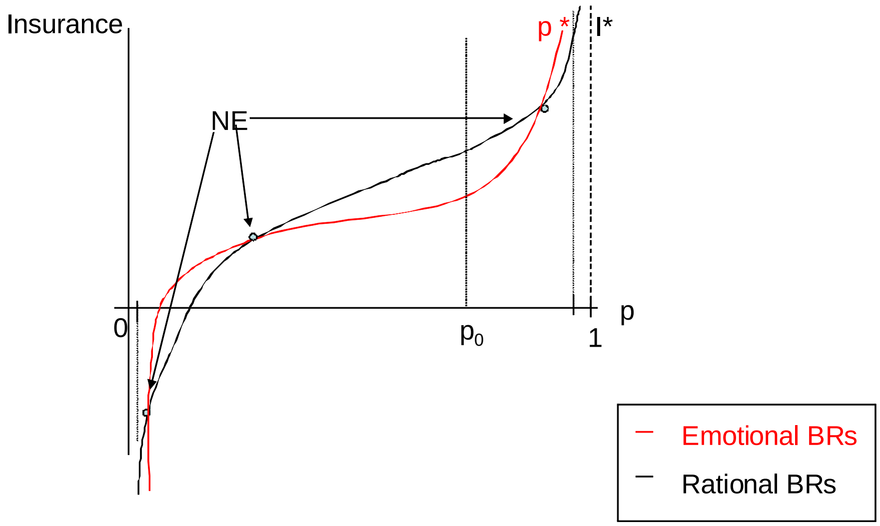


Figure 2

**Proposition 3** *The ADM intrapersonal game has an odd number of pure strategy Nash equilibria. The set of Nash equilibria is a chain in $R^2$, under the standard partial order on points in the plane.*

Note that with the $ADM$ model one captures differences in report and choice tasks, as reported in different studies (in the context of normal form games see Costa–Gomes and Weizsäcker, 2008). In the insurance context, when asked to report the probability of, say, an accident with no action to subsequently take, the agent activates only the emotional process and tends to report low chances. However, when asked to choose an action both processes are activated and together determine choice — hence the chosen action will generally be inconsistent with the reported beliefs.

Although the $ADM$ model generally has multiple equilibria, which we believe is realistic especially given framing and attentional effects, the case of a unique equilibrium $ADM$ model is attractive due to its predictive power. Sufficient conditions, due to Neyman (1997), for the uniqueness of pure strategy Nash equilibrium in a simultaneous move potential game is strict concavity of the potential function and compactness of the strategy sets. The potential is strictly concave if the Hessian of the potential is negative definite, which in this simple example reduces to:

**Proposition 4** *If the strategy sets are compact, then a sufficient condition for a unique pure strategy Nash equilibrium of the intrapersonal game is:*

$$\frac{\partial^2 J^*(p;p_0)}{\partial p^2} > -\frac{[Du(w_B+(1-\gamma)I)(1-\gamma)+Du(w_G-\gamma I)\gamma]^2}{[pD^2u(w_B+(1-\gamma)I)(1-\gamma)^2+(1-p)D^2u(w_G-\gamma I)\gamma^2]},$$

This condition is simply a statement on the relative slope of the two processes' best responses. Note that $\partial^2 J^*(p;p_0)/\partial p^2$ is the rate at which the marginal mental costs change with respect to perceived probabilities $p$, and

$$[Du(w_B+(1-\gamma)I)(1-\gamma)+Du(w_G-\gamma I)\gamma]$$

is the rate at which marginal benefits of distorting beliefs change with respect to insurance level $I$. The above condition therefore states that the ratio of change in marginal mental costs with respect to perceived risk to change in marginal mental benefit with respect to insurance is always greater than a similar ratio defined on marginal expected utility. In this case, the emotional process's best response is everywhere steeper — a change in perceived probability is accompanied there by a greater change in insurance relative to the rational process's best response — and the intrapersonal game admits a unique equilibrium. One implication of this condition is that for large mental costs the equilibrium is unique (think of $\lambda > 0$, $\hat{J}^*(\cdot) = \lambda J^*(\cdot)$), and for very large mental costs the $ADM$ model reduces to the expected utility model.[3]

Even considering a unique $ADM$ model, unless the mental costs are very large, risk perceptions are endogenous and the model systematically departs from the expected utility model. This suggests that the failure of the expected utility model to explain some data sets may be due to systematic affective biases. How exactly does affective choice in insurance markets differ from the demand for insurance in the expected utility model? Proposition 5 below shows that the expected utility outcome in the case of an actuarially fair insurance market (full insurance) falls within the choice set of the $ADM$ agent. However, if the insurance market is not actuarially fair, then this is no longer the case.

**Proposition 5** *If $\gamma = p_0$, there exists at least one Nash equilibrium $(p^*, I^*)$ with $p^* = p_0 = \gamma$, and $I^* =$ full insurance.*

*If $\gamma > p_0$, there exists at least one Nash equilibrium $(p^*, I^*)$ with $p^* < p_0$ and $I^* < I^*(p_0)$.*

*If $\gamma < p_0$, there exists at least one Nash equilibrium $(p^*, I^*)$ with $p_0 < p^*$ and $I^* > I^*(p_0)$.*

To understand the intuition behind these results, consider a standard myopic adjustment process where the processes alternate moves. If $\gamma > p_0$, at $p_0$ the rational process, similar to the expected utility model, prescribes buying less than full insurance. The emotional process, in turn, leads the decision maker to believe "this is not going to happen to me" and determines that she is at a lower risk. This effect causes a further reduction in the insurance purchase, with a result of less than full insurance, even less than what the expected utility model would predict. This proposition gives both an intuitive understanding of the effect of the emotional process in the $ADM$ model, and intuitively shows existence of pure-strategy Nash equilibrium, affective choice. In the case of a unique $ADM$ models, proposition 5 fully characterizes affective choice; in the case of multiple equilibria it points out only one equilibrium out of many possible, however, the effect of the two processes in enhancing each others initial tendency is true whether one considers the unique $ADM$ models or the entire class of $ADM$ models.

Considering such adjustment process, the $ADM$ model is consistent with two widely discussed phenomena: cognitive dissonance and attention effects. Cognitive

[3] As $J^* \to \infty$, $p^* \to p_0$ for all values of $I$. As a result, the $ADM$ model converges to the expected utility model.

dissonance is when one holds two contradicting beliefs at the same time. Hence if one thinks of the adjustment process as a process of reaching a decision, in this process the agent suffers cognitive dissonance and choice represents a resolution of it. As for attention effects — if one's attention is manipulated to first think of an action, or first think of risk beliefs, generally he or she will end up with different choices. In particular, according to our model, thinking first of probabilities of adverse events leads to greater optimism and lower insurance purchased than if the agent's attention is given to thinking of insurance first.

Note that proposition 5 also implies that, from the viewpoint of an outside observer, both optimism and pessimism (relative to $p_0$) are possible. This is due to the characteristics of insurance: if an agent purchases more than full insurance, then the "bad" state becomes the "good" state, and vice versa. Consequently, if there is no effective action, i.e., one cannot change the bad state to a good state, we would observe optimism and less-than-optimal insurance.

Here is another example of the difference between affective choice and the demand for insurance in the expected utility model. In the expected utility model, if people realize that they face a higher potential loss due to educational campaigns aimed at raising awareness of the possible catastrophe, much like smoking warning labels "Smoking Kills," campaigns against speeding that show vivid pictures of people severely injured or killed in car accidents, and flood warnings "Like Never Before," then they would purchase more insurance.[4] In the ADM model, if an agent realizes she faces higher possible loss, then she might purchase less insurance. The increased loss size affects both the emotional and the rational processes in different directions; the rational process prescribes more insurance, the emotional process prescribes lower risk belief to every insurance level (due to greater incentives to live in denial). If the emotional effect is stronger the agent will buy less insurance than previously. That is, if the loss is great, agents might prefer to remain in denial and ignore the possible catastrophes altogether, which will lead them to take fewer precautions such as buying insurance. This is consistent with consumer research showing that high fear arousal in educating people on the health hazards of smoking leads to a discounting of the threat (Keller and Block, 1996; see also Ringold, 2002 and references therein). Proposition 6 and Figure 3 below summarize the conditions for educational campaigns to produce the counter-intuitive affective result.

[4] Indeed, there may be educational campaigns aimed at increasing the base-line probability of an event occuring, or changing the mental costs directly. If one analyzes this kind of educational campaigns, it is still possible that the campaigns will back fire. It depend on the specific assumptions made as to how the margnial mental costs change with the campaign, and the type of equilibria studied.

The approach we take is because many educational campaigns such as campaigns on smoking, alcohol consumption, speeding, and natural disasters stress the outcome and not the probability of getting into a bad state. For instance, recent European anti-smoking warning label say in large letters "Smoking Kills," and in many countries anti-speeding ads show vivid pictures of people severly injured or killed in car accidents. That is, many "educational" ads do not add information on the probability of suffering from the adverse consequence, but rather draw the attention of the public to the possible dramatic consequences.

**Proposition 6** *An educational campaign result in less insurance if*

$$\frac{r(w_G - \gamma I)}{Du(w_G - \gamma I)} > \frac{r(w_B + (1-\gamma)I)}{Du(w_B + (1-\gamma)I)},$$

*where $r(\cdot)$ is the absolute risk aversion property of the utility function $u(\cdot)$.*

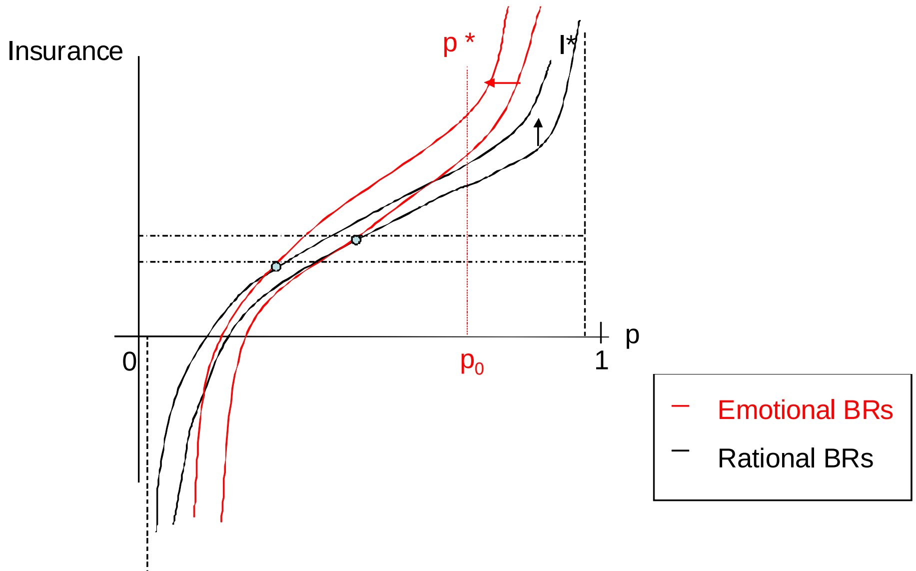


Figure 3

In Proposition 6, if the utility function $u(\cdot)$ exhibits constant or increasing absolute risk aversion, educational campaigns will lead to higher insurance purchase if and only if initially the agent buys more than full insurance. Insurees who initially buy less than full insurance will buy even less after the educational campaign. Hence, for such utility functions, educational campaigns divide the insurance market into a set of agents who purchase more insurance – the intended consequence — and a set of agents who purchase less insurance — the unintended consequence. This is true for any equilibrium, even in the presence of multiplicity of equilibria.

## 4 ADM Potential Games

The state-preference model of choice under risk is widely used in finance, information economics and game theory. In this section, we present a state-preference model of affective decision-making in competitive markets, where an agent maximizes her preferences subject to a budget constraint. More specifically, consider an agent who faces $K$ possible states of nature and has a utility function $u$ over outcomes. The rational process chooses an action $z$ that maps states into outcomes. Preferences over acts $z$ are represented by a convex function, $J$, of state-utility vectors $U(z)$, where $U(z)$ is the vector of utilities of outcomes for the act $z$. That is, preferences over acts

$z$ are represented by a composite utility function $J(U(z))$. The emotional process chooses a belief $p$ and the $ADM$ potential function is $\Theta(z,p) \equiv \langle U(z), p\rangle - J^*(p)$, where $J^*(p)$ is a convex function of Legendre type. That is, $J^*(p)$ is a strictly convex, essentially smooth function on the interior of the probabilty simplex $\Delta$.

The notion of a strictly convex, essentially smooth function is due to Rockafellar (1970) — see chapter 26. If $\Omega$ = the interior of the effective domain of a proper, extended real-valued convex function $f : R^K \to R \cup \{+\infty\}$, then $f$ is essentially smooth if: (i) $\Omega$ is not empty, (ii) $f$ is differentiable throughout $\Omega$, (iii) $\lim_{i\to\infty} \|\nabla f(x_i)\| = +\infty$ whenever $x_1, x_2, ...$ is a sequence in $\Omega$ converging to a boundary point $x$ of $\Omega$. He defines the class of strictly convex and essentially smooth functions to be of Legendre-type. In Theorem 26.5, Rockafellar proves that a closed convex function, $f$ is of Legendre type on the interior of the effective domain of $f$, denoted $\Omega$, if and only if the Legendre conjugate $f^*$ is of Legendre type on the interior of the effective domain of $f^*$, denoted $\Omega^*$. Moreover, the gradient mapping $\nabla f$ is a one-to-one map from the open,convex set $\Omega$ onto the open,convex set $\Omega^*$, where the gradient map is continuous in both directions and $\nabla f^* = (\nabla f)^{-1}$.

If $J^*(p)$ is a convex function of Legendre type then it follows from the envelope theorem that the affective probabilities chosen by the emotional process for the act $z$ is

$$\nabla_{U(z)} J(U(z)) = \arg\max_{p\in\Delta}\{\langle U(z), p\rangle - J^*(p)\}.$$

Moreover, for all state-utility vectors $U(z)$ and $U(y)$:

$$[\nabla_{U(z)} J(U(z)) - \nabla_{U(y)} J(U(y))] \cdot [U(z) - U(y)] > 0.$$

That is, the affective probabilities,$\nabla_{U(w)} J(U(w))$, are a strictly increasing monotone map of the state-utility vector $U(w)$. This is our definition of optimism-bias and it shows that the assumed shape of the cost function in the demand for insurance example (strictly convex and essentially smooth), reflecting the psychological characterization of affective costs, is both necessary and sufficient for optimism-bias. The definition of optimism-bias in the $K$-state world subsumes the intuitive definition of optimism-bias in the two state case, where the more favorable outcome of an act $z$ is assigned the higher probability of being realized. In Corollary 26.3.1, Rockafellar shows that if $J^*(\pi)$ is a closed proper, convex function on $R^K$, then the subgradient correspondence reduces to a one-to-one gradient map on the interior of the effective domain of $J^*(\pi)$ iff $J^*(\pi)$ is strictly convex and essentially smooth.

Is the assumed shape of the affective cost function in the demand for insurance example i.e., strictly convex and essentially smooth "typical"? That is, in some precise sense are most closed proper convex functions on $R^K$ strictly convex and essentially smooth on the interior of their effective domains? Surprisingly, the answer is yes! This result is an immediate consequence of Howe's (1982) theorem that in the uniform topology the family of strictly convex and differentiable functions on a compact, convex subset, $A$, of $R^K$ is a residual subset of the family of convex functions on $A$. Any open and bounded, convex subset of $R^K$ can be exhausted by a countable family of compact, convex subsets. Since a countable intersection of residual sets is residual, we see that in the topology of uniform convergence on compact sets (or topology of compact convergence) the family of strictly convex and differentiable functions on an open and bounded convex subset, $B$, of $R^K$ is a residual subset in the family of convex functions on $B$. If $J^*(\pi)$ is strictly convex and differentiable, then it follows from the strict monotonicity of the gradient map, that the gradient map is one-to-one. By Corollary 26.3.1, these functions are strictly convex and essentially smooth. That is, Legendre convex functions are generic.[5]

To extend the existence and local uniqueness of pure strategy Nash equilibria in the two-state $ADM$ potential game to $ADM$ potential games with $K$-states of the world , we define regular potential games. Regular potential games are potential games where the potential function $\Theta(z,p)$ is a Morse function or equivalently 0 is a regular value of $\nabla_{(z,p)}[\Theta(z,p)]$, where $z$ is an action and $p$ is a belief. That is, the Hessian of the potential function $\Theta(z,p)$ evaluated at a critical point of the potential is non-singular. See chapter 1 in Guillemin and Pollack (1974) for a discussion of Morse functions, where they show that "most" smooth functions are Morse functions. We prove that essentially smooth and strictly bi-concave potential games that are regular have an odd number of locally unique pure strategy Nash equilibria. In the proof we use the homotopy principle, which implies an algorithmic interpretation and allows for the computation of a pure-strategy Nash equilibrium.

Moreover, we show that if the potential function $\Theta(z,p)$ is essentially smooth and strictly bi-concave, then the set of pure strategy Nash equilibria is non-empty, where 0 need not be a regular value of $\nabla_{(z,p)}[\Theta(z,p)]$.

**Proposition 7** *If $\Theta(z,p)$ is the potential function for a regular potential game $G$, where the strategy-sets $Z$ and $\Pi$ are the interiors of non-empty, convex, compact subsets of $R^K$, is essentially smooth and strictly bi-concave, then $G$ has an odd number of locally unique, pure strategy Nash equilibria.*

**Corollary 8** *If $\Theta(z,p) = \langle U(z),p\rangle - J^*(p)$ is the potential function for a regular $ADM$ potential game $G$, where $Z$ is the interior of the budget hyperplane and $\Pi$ is the interior of the probability simplex, is essentially smooth and strictly bi-concave, then $G$ has an odd number of locally unique, pure strategy Nash equilibria.*

**Proposition 9** *If $\Theta(z,p)$ is the potential function for a potential game $G$, where the strategy-sets $Z$ and $\Pi$ are the interiors of non-empty, convex, compact subsets of $R^K$, is essentially smooth and strictly bi-concave, then the set of pure strategy Nash equilibria is non-empty.*

**Corollary 10** *If $\Theta(z,p) = \langle U(z),p\rangle - J^*(p)$ is the potential function for an $ADM$ potential game $G$, where $Z$ is the interior of the budget hyperplane and $\Pi$ is the interior of the probability simplex, is essentially smooth and strictly bi-concave, then the set of pure strategy Nash equilibria is non-empty.*

The conditions for uniqueness are given below, where they extend Neyman's (1997) theorem on the uniqueness of pure strategy Nash equilibrium for potential

[5] This Theorem is due to Roger Howe — personal communication, March 2010.

games with compact strategy sets to potential functions with, bounded, open strategy sets.

**Proposition 11** *If $\Theta(z, p) = \langle U(z), p\rangle - J^*(p)$ is the potential function for a regular ADM potential game G, where the strategy-sets Z is the interior of the budget hyperplane and $\Pi$ is the interior of the probability simplex, is essentially smooth and strictly concave, then G has a unique pure strategy Nash equilibria*

In the insurance example, the affective cost function depends on base-line probabilities $p_0$. In the general $K$-states framework the analogous concept is Bregman divergence — generalizations of relative entropy, used in information theory, to measure the "directed distance" from a fixed probability distribution $p_0$ to other probability distributions $p$. See Banerjee et al. (2005) for a general discussion of Bregman divergences. Importantly, every convex function of Legendre type $J^*$ on the interior of $\Delta$ and "prior" probability distribution $p_0$ in the interior of $\Delta$ defines a Bregman divergence $D(p_0, p)$ of Legendre type, where

$$D(p_0, p) \equiv J^*(p) - J^*(p_0) - \nabla J^*(p_0) \cdot (p - p_0).$$

For our purposes, notice that (i) for all $p_0$ and $p$ in the interior of $\Delta$, $D(p_0, p) \geq 0$, and (ii) $D(p_0, p_0) = 0$. Hence $p_0$ is the minimum of $D(p_0, p)$ on $\Delta$ or the base-line probabilities in the insurance example. That is, $D(p_0, p)$ is the "directed distance" from $p_0$ to $p \in \Delta$.

If $g^*(p) = -f^*(p)$, where $f^*(p)$ is a Bregman divergence, then we define $g^*(p)$ as a dual Bregman divergence. Relative entropy, $J^*(p) \equiv \sum_{j=1}^{j=K}[p_j \lg(p_j/p_{0j})]$, is a Bregman divergence of Legendre type and is of special interest as $H^*(p) = -J^*(p)$ — its dual Bregman divergence of Legendre type — is the affective cost function in the multiplier preferences model (Hansen and Sargent, 2000). Here is an example of an essentially smooth and strictly bi-concave $ADM$ intrapersonal game.

Let

$$u(w) = w^\delta, \text{ where } \delta \in (0, 1)$$

and

$$J^*(p) = \sum_{j=1}^{j=K}[p_j \lg(p_j/p_{0j})]$$

then

$$\Theta(z, p) = \sum_{j=1}^{j=K}[z_j^\delta p_j - \sum_{j=1}^{j=K}[p_j \lg(p_j/p_{0j})].$$

# 5 Axioms for Optimistic Preferences

In this section we show that attitudes towards optimism reduce to the convexity of the utility representation of preferences over acts, and use this property to derive axioms for optimistic preferences. We show that preferences over acts, $z$, are optimistic if and only if there exists a continuous, utility function $u$ over outcomes and a continuous convex function $J$ over state-utility vectors $U(z)$, i.e., the vector of utilities of outcomes for the act $z$. The $ADM$ model, we present below, is an example of optimistic preferences over acts.

We use the Legendre–Fenchel conjugate of a continuous, convex function $J(U(z))$ to represent optimistic preferences as $ADM$ potential games. That is, the Legendre–Fenchel conjugate

$$J^*(p) \equiv \max_{U(z)\in R_+^K} \{\langle U(z), p\rangle - J(U(z))\}.$$

It follows from the biconjugate theorem that

$$J(U(z)) = \max_{p\in\Delta}\{\langle U(z), p\rangle - J^*(p)\}.$$

i.e., the double conjugate of $J$, $(J^*)^* = J$. If we assume $J^*(p)$ is a convex function of Legendre type, i.e., $J^*(p)$ is a strictly convex, essentially smooth function on the interior of the probability simplex $\Delta$, then the Legendre conjugate and biconjugate of $f$ are well defined — see Theorem 26.5 in Rockafellar (1970), stated in the previous section. The potential function for the associated $ADM$ potential game is

$$\Theta(z, p) \equiv \langle U(z), p\rangle - J^*(p).$$

Next, we derive the set of axioms characterizing optimistic preferences, and show that these axioms also characterize the $ADM$ potential maximizers. That is,

$$\arg\max_{z\in Z} J(U(z)) = \arg\max_{z\in Z, p\in\Delta} \Theta(z, p).$$

The axiomatic characterization of optimistic preferences is an amendment of the axiomatic characterization of variational preferences in Maccheroni, Marinacci and Rustichini [MMR] (2006), where: $S$ is the set of states of the world; $\Sigma$ is an algebra of subsets of $S$, the set of events; and $X$, the set of consequences, is a convex subset of some vector space. $F$ is the set of (simple) acts, i.e., finite-valued $\Sigma$-measurable functions $f : S \to X$. $B(\Sigma)$ is the set of all bounded $\Sigma$-measurable functions, and endowed with the sup-norm it is an AM-space with unit, the constant function 1. $B_o(\Sigma)$ the set of $\Sigma$-measurable simple functions is norm dense in $B(\Sigma)$. The norm dual of $B(\Sigma)$ is $ba(\Sigma)$, finitely additive signed measures of bounded variation on $\Sigma$ (see Aliprantis and Border, 1999 for further discussion). Below we present the axioms:

**A.1** (Weak Order): If $f, g, h \in F$, (a) either $g \succsim f$ or $f \succsim g$, and (b) $f \succsim g$ and $g \succsim hs \Rightarrow f \succsim h$.

*A.2* (Weak Certainty Independence): If $f, g \in F$, $x, y \in X$ and $\alpha \in (0, 1)$, then $\alpha f + (1-\alpha)x \succsim \alpha g + (1-\alpha)x \Rightarrow \alpha f + (1-\alpha)y \succsim \alpha g + (1-\alpha)y$.

**A.3** (Continuity): If $f, g, h \in F$, the sets $\{\alpha \in [0, 1] : \alpha f + (1-\alpha)g \succsim h\}$ and $\{\alpha \in [0, 1] : h \succsim \alpha f + (1-\alpha)g\}$ are closed.

**A.4** (Monotonicity): If $f, g \in F$ and $f(s) \succsim g(s)$ for all $s \in S$, the set of states, then $f \succsim g$.

**A.$\hat{5}$** (Quasi-Convexity): If $f, g \in F$ and $\alpha \in (0, 1)$, then $f \sim g \Rightarrow \alpha f + (1-\alpha)g \precsim f$.

**A.6** (Nondegeneracy): $f \succ g$ for some $f, g \in F$.

These axioms where A.$\hat{5}$ is replaced by A.5 (quasi-concavity) are due to MMR (2006).

**Theorem 12** *Let $\succsim$ be a binary order on $F$. The following conditions are equivalent:*

(1) *The relation $\succsim$ satisfies axioms A.1 − A.6.*

(2) *There exists a nonconstant function $u : X \to R$, unique up to a positive affine transformation, and a continuous, convex function $J^* : \Delta \to [0, \infty]$ where for all $f, g \in F$, $f \succsim g \Leftrightarrow W(f) \geq W(g)$, where $W(h) = J(U(h)) = \max_{p\in\Delta} \{\langle U(h), p\rangle - J^*(p)\}$ is a convex function of $U(h)$ by the biconjugate theorem.*

In the standard models of decision-making under risk such as expected utility theory and prospect theory, the decision-maker maximizes over actions, and not over both actions and beliefs. That is, $\arg\max_{z\in Z} J(U(z)) = \arg\max_{z\in Z, p\in\Delta} \Theta(z, p)$, the potential maximizers, are in general a proper subset of the set of pure-strategy Nash equilibria of the $ADM$ potential game. If the $ADM$ model has a unique pure strategy Nash equilibrium then maximizing the composite utility function $J(U(z))$ over actions and maximizing the potential $\Theta(z, p)$ over actions and beliefs rationalize the same observed choices. Hence these models are refutable. That is, not every data set can be rationalized with an $ADM$ potential game.

# 6 Appendix: Proofs

**Proof. Proposition 2.** Denote the rational process's payoff function as ($R$) and the emotional process's payoff function as (E). A necessary and sufficient condition for the intrapersonal game to have a potential function (Monderer and Shapley, 1996) is $\partial^2 R/\partial p\partial I = \partial^2 E/\partial p\partial I$. This condition clearly is satisfied in the ADM model. The potential function $J(p, I)$ is a function such that (Monderer and Shapley, 1996): $\partial J/\partial p = \partial E/\partial p$, $\partial J/\partial I = \partial R/\partial I$. Because $\partial E/\partial I = \partial R/\partial I$, ($E$) can serve as a potential function. The critical points of the potential function are $\partial J/\partial p = \partial E/\partial p = 0$, $\partial J/\partial I = \partial R/\partial I = 0$. The potential function is strictly concave in each variable, so at each critical point, each process is maximizing its objective function, given the strategy of the other process. Therefore, the critical points of the potential function are the pure strategy Nash equilibria of the intrapersonal game, and all pure strategy Nash equilibria are critical points of the potential function. ■

**Proof. Proposition 3.** By having an essentially smooth cost function, we know the relationship between the emotional process and the rational process best response at the extreme beliefs $\{0, 1\}$. We know that as $p \to 0$, the rational process best response would be "higher." That is, the optimal insurance for that belief is higher than the required insurance level to support these beliefs. The relationship exactly flips when $p \to 1$. Hence, there exist a pure strategy Nash equilibria. Since the best responses are monotonically increasing, it follows that there exists odd number of Nash equilibria.

when level relative to risk perception, $0 < \underline{\beta} < \bar{\beta} < 1$, $\beta^* \in (\underline{\beta}, \bar{\beta})$, and insurance $I^* \in [I^*(\underline{\beta}), I^*(\bar{\beta})]$. Hence, all Nash equilibria will have perceived probabilities in the interval $[\beta^*(I^*(\underline{\beta})), \beta^*\left(I^*(\bar{\beta})\right)]$ where $0 < \underline{\beta} < \beta^*(I^*(\underline{\beta})) < \beta^*\left(I^*(\bar{\beta})\right) < \bar{\beta} < 1$. Define $\beta^*(I^*(\underline{\beta})) \equiv \underline{\beta}'$, $\beta^*\left(I^*(\bar{\beta})\right) \equiv \bar{\beta}'$; because all the Nash equilibria of the intrapersonal game for $\beta \in (\underline{\beta}, \overline{\beta})$ are $\in [\underline{\beta}', \bar{\beta}']$ the focus can remain on the latter probability space.The existence and chain results can be shown by defining a restricted intrapersonal game in which the insurance pure strategy space is restricted to $[I^*(\underline{\beta}), I^*(\bar{\beta})]$ and the perceived probabilities are restricted to $\beta \in [\underline{\beta}', \bar{\beta}']$, such that the equilibria points of the intrapersonal game are not altered. The restricted game is a supermodular game, and thus, these results follow from the properties of this class of games (see Topkis, 1998). To Show that the game admits odd number of equilibria, think of the geometry of the game. As $\beta \to \bar{\beta}$, the best response of the emotional process is above the best response of the rational process, while this relationship is reversed for $\beta \to \underline{\beta}$. Since the best responses are monotonically increasing, it follows that there exists odd number of Nash equilibria. ■

**Proof. Proposition 4.** The emotional process's objective function $J(p, I) = pu(\omega_B + (1-\gamma)I) + (1-p)u(\omega_G - \gamma I) - J^*(p; p_0)$ is the potential function of the game. The maximization of $(J)$ with respect to the pair $(I, \beta)$ gives rise to a pure strategy Nash equilibria of the game.$\beta \in [\underline{\beta}', \bar{\beta}']$ and $I \in [I^*(\underline{\beta}'), I^*(\bar{\beta}')]$ (see Proof of Theorem 1), hence only the restricted intrapersonal game in which both players' strategy spaces are compact need be considered. Neyman (1997), proved that a potential game with a strictly concave, smooth potential function, in which all players have compact, convex strategy sets, has a unique pure strategy Nash equilibrium. That is, the Hessian of the potential function is negative definite, as follows from the condition given above. ■

**Proof. Proposition 5.** Consider the case in which $\gamma = \beta_0$. At full insurance, there is no mental gain for holding beliefs $\beta \neq \beta_0$ but there exists mental cost. Therefore, at full insurance, the mental process's best response is $\beta = \beta_0$. Given that $\gamma = \beta_0 = \beta$, the rational process's best response is full insurance. Consequently, full insurance and $\beta = \beta_0$ is a Nash equilibrium of this case. Next, consider the case $\gamma > \beta_0$; because the insurance premium is higher than $\beta_0$, $I^*(\beta = \beta_0) < z$. Also, $\beta^* = \beta_0$ only at full insurance, where $I = z$. Therefore, at $\beta = \beta_0$ the mental process's best response falls above the rational process's best response. This relationship is reversed at the limit $\beta \to \underline{\beta}$, and both the mental and the rational best responses increase; therefore, there exists a Nash equilibrium with $\beta < \beta_0$ and less insurance than predicted by the expected utility model. A similar argument can be used to prove the result when $\gamma < \beta_0$. ■

**Proof. Proposition 6.** Define $\tilde{I}(\beta; \beta_0)$ as the inverse function $p^{*-1}$. Define $\Pi(p; p_0) = I^*(p) - \tilde{I}(p; p_0)$, $\Pi : [\underline{\beta}', \bar{\beta}'] \to R$.

Educational campaigns on impending catastrophes increase the loss size, $z$. Because $\Pi(p; p_0) = 0$ is a NE, $\partial\Pi/\partial z < 0$ represent the unintended consequence of such campaigns.

$$\frac{\partial \Pi}{\partial z} < 0 \Leftrightarrow \frac{\frac{\partial \tilde{I}}{\partial z}}{\frac{\partial I^*}{\partial z}} > 1.$$

$$\frac{\partial I^*}{\partial z} = \frac{\left[u''(w_G - z + (1-\gamma)I^*)\right]\left[u'(w_G - \gamma I^*)\right]^2}{\left[u'(w_G - \gamma I^*)\right]\left[\begin{array}{c} u''(w_G - z + (1-\gamma)I^*)u'(w_G - \gamma I^*)(1-\gamma) \\ +u'(w_G - z + (1-\gamma)I^*)u''(w_G - \gamma I^*)\gamma \end{array}\right]};$$

$$\frac{\partial \tilde{I}}{\partial z} = \frac{\left[u'(w_G - z + (1-\gamma)\tilde{I})\right]}{\left[u'(w_G - z + (1-\gamma)\tilde{I})(1-\gamma) + u'(w_G - \gamma\tilde{I})\gamma\right]} \Rightarrow \frac{\partial \Pi}{\partial z} < 0$$

$$\Leftrightarrow \frac{r(w_G - \gamma I)}{u'(w_G - \gamma I)} > \frac{r(w_B + (1-\gamma)I)}{u'(w_B + (1-\gamma)I)}, \text{where } r(x) = -\frac{u''(x)}{u'(x)}.$$

The proofs of propositions 7 and 9 use the homotopy principle– see chapters 1, 2 and 22 in Garcia and Zangwill (1981). The homotopy principle admits an algorithmic interpretation that can be used to compute a pure strategy Nash equilibrium of the potential game — see chapter 2 in Garcia and Zangwill. ■

**Proof. Proposition 7.** Consider the following homotopy: $H(t, \theta, \theta_0) = (1-t)(\theta - \theta_0) + t\nabla_\theta[P(\theta)]$, where $\theta = (z, \pi)$, $\theta_0 = (z_0, \pi_0)$ and $t \in [0,1]$.0 is a regular value of $H(0, \theta, \theta_0)$, since $[\partial H(0, \theta, \theta_0)/\partial\theta] = I_{2K}$, the identity matrix on $R^K x R^K$. 0 is also a regular value of $H(1, \theta, \theta_0)$, since $[\partial H(1, \theta, \theta_0)/\partial\theta] = \nabla_\theta[P(\theta)]$ and $P(\theta)$ is a Morse function. For $t \in (0,1)$, $[\partial H(t, \theta, \theta_0)/\partial\theta_0] = -I_{2K}$. Hence 0 is a regular value of $H(t, \theta, \theta_0)$ for all $t \in (0,1)$ by the transversality theorem (parametric Sard's theorem). That is, 0 is a regular value of $H(t, \theta, \hat{\theta}_0)$ for almost all $\hat{\theta}_0 \in \Theta$, where $\Theta \equiv Kx\Pi$ – see chapter 2 in Guillemin and Pollack for a proof of the transversality theorem. The assumption that $P(\theta)$ is essentially smooth, i.e., $\|\nabla_\theta[P(\theta_n)]\| \to \infty$, as $\theta_n \to \text{bdry}(\Theta)$ implies that the homotopy is boundary-free. Hence by the homotopy principle, $\nabla_\theta[P(\theta)]$ has an odd number of regular points — see the proof of Theorem 3.2.3 in Garcia and Zangwill. Since $P(\theta)$ is strictly bi-concave, it follows that $P(\theta)$ has an odd number of locally unique, pure strategy Nash equilibria. ■

**Proof. Corollary 8.** Proof is immediate. ■

**Proof. Proposition 9.** If the set of pure strategy Nash equilibria is empty, then the set of critical points is empty and 0 is a regular value, contradicting proposition 7. Hence there exists at least one singular critical point. That is, there exists at least one pure strategy Nash equilibrium. ■

**Proof. Corollary 10.** Proof is immediate. ■

**Proof. Proposition 11.** If $P(\theta)$ is strictly concave, then it has at most one critical point, but by proposition 7, $P(\theta)$ has an odd number of critical points. Hence there exists a unique pure strategy Nash equilibrium. ■

**Proof. Theorem 12.** Axioms 1–4 are used in MMR to derive a nonconstant utility function, $u$, unique up to a positive affine transformation, over the space of consequences, $X$. $u$ is extended to the space of simple acts, $F$, using certainty equivalents. That is, $U(f) = u(x_f) \in B_o(\Sigma)$ for each $f \in F$, where $x_f$ is the certainty equivalent of $f$. This is lemma 28 in MMR, where $I(f) = U(f)$ is a niveloid on $\Phi = \{\varphi : \varphi = u(f) \text{ for some } f \in F\}$. Niveloids are functionals on function spaces that are monotone: $\varphi \leq \eta \Rightarrow I(\varphi) \leq I(\eta)$ and vertically invariant: $I(\varphi + r) = I(\varphi) + r$ for all $\varphi$ and $r \in R$ — see Dolecki and Greco (1995) for additional discussion. $\Phi$ is a convex subset of $B(M)$ and by Schmeidlers's axiom 5, $I$ is quasi-concave on $\Phi$.

We also assume axioms 1–4, so lemma 28 in MMR holds for the niveloid $J$ in the $ADM$ representation theorem. By axiom $\hat{5}$, $J$ is quasi-convex on $\Phi$. MMR show in lemma 25 that $I$ is concave if and only if $I$ is quasi-concave. Hence $J$ is convex if and only if $J$ is quasi-convex, since $J$ is convex(quasi-convex) if and only if $-J$ is concave(quasi-concave). MMR extend $I$ to a concave niveloid $\hat{I}$ on all of $B(\Sigma)$ — see lemma 25 in MMR. Epstein, Marinacci and Seo [EMS] (2007) show in lemma *A.5* that niveloids are Lipschitz continuous on any convex cone of an AM-space with unit and concave(convex) if and only if they are quasi-concave(convex). Hence, since $B(\Sigma)$ is a convex cone in an AM-space with unit, $\hat{I}$ is Lipschitz continuous. It follows from the theorem of the biconjugate for continuous, concave functionals that $I(\varphi) = \inf_{p \in ba(\Sigma)}\{\int \varphi dp - \hat{I}^*(p)\}$, where $\hat{I}^*(p) = \inf_{\varphi \in B_o(\Sigma)}\{\int \varphi dp - \hat{I}(\varphi)\}$ is the concave, conjugate of $\hat{I}(\varphi)$ — see Rockafellar (1970, p. 308) for finite state spaces. MMR show on page 1476 that we can restrict attention to $\Delta$, the family of positive, finitely additive measures of bounded variation in $ba(\Sigma)$. Hence $I(\varphi) = \min_{p\in\Delta}\{\int \varphi dp - \hat{I}^*(p)\} = \min_{p\in\Delta}\{\int u(f)dp + c(p)\}$, where $\varphi = u(f)$ and $J^*(p) = -\hat{I}^*(p)$. $J^*(p)$ is convex since $\hat{I}^*(p)$ is concave.

Extending $-J$ to $-\hat{J}$ on $B(\Sigma)$, using lemma 25 in MMR, it follows from the theorem of the biconjugate for continuous, convex functionals that

$$J(\varphi) = \max_{p \in ba(\Sigma)} \left\{ \int \varphi dp - \hat{J}^*(p) \right\}$$

where

$$\hat{J}^*(p) = \max_{\varphi \in B_o(\Sigma)} \left\{ \int \varphi dp - \widehat{J}(\varphi) \right\}$$

is the convex, conjugate of $\hat{J}(\varphi)$ — see Rockafellar (1970, p. 104) for finite state spaces and Zălinescu (2002, p. 77) for infinite state spaces. Again it follows from MMR that

$$J(\varphi) = \max_{p\in\Delta} \left\{ \int \varphi dp - \hat{J}^*(p) \right\} = \max_{p\in\Delta} \left\{ \int u(f)dp - J^*(p) \right\} = W(f),$$

where $\varphi = u(f)$ and $J^*(p) = \hat{J}^*(p)$. $J^*(p)$ is convex since $\hat{J}^*(p)$ is convex. ■

$$f \succsim g \Leftrightarrow J(u(f)) \geq J(u(g)) \Leftrightarrow W(f) \geq W(g).$$

Hence $\arg\max_{f \in F, p\in\Delta}\{\int u(f)dp - c(p)\} \subseteq$ set of pure strategy Nash equilibria of the $ADM$ intrapersonal game, where $u(\cdot)$ is the Bernoulli utility function of the rational process and $\hat{J}^*(\cdot)$ is the cost function of the emotional process. It follows that the axioms for ambiguity-seeking preferences also characterize the $ADM$ potential maximizers : $\arg\max_{f \in F, p\in\Delta}\{\int u(f)dp - J^*(p)\}$.

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