INTERGENERATIONAL JUSTICE AND SUSTAINABILITY UNDER THE LEXIMIN ETHIC

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"Intergenerational justice and sustainability under the leximin ethic[1]"

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Abstract. We model an intergenerational society, with a representative agent at each date, who must deplete a renewable resource, from which he derives utility, to produce consumption goods. We adopt the intergenerational lexicographic minimum as the social welfare function. Initially, technological progress is assumed to exist exogenously. We study the technological requirements for the leximin solution to support non-decreasing welfare over time, and a non-decreasing level of the natural resource. Three utility functions are studied. With a CES utility function, possessing less substitutability than the Cobb-Douglas, the leximin solution involves increasing utilities over time and an increasing size of the natural resource, if the rate of transformation of the resource into the consumption good is greater than a computed bound. Finally we study a model with endogenous technical progress.

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1. Introduction

Let me begin by proposing that we think about intergenerational justice from the viewpoint of equality of opportunity. According to the equal-opportunity view, a person should be compensated if his welfare is low due to circumstances beyond his control, but not if it is low due to actions or choices that we (our society) thinks he should be held responsible for. The word I use to denote the second set of actions/choices is 'effort.' Consider, now, a standard economic growth model with a representative agent at each generation. The society consists of the set of all agents who will ever live, one representing each generation. Clearly, the preëminent circumstance, for an individual in this society, is the date at which he is born. So if we apply the equal-opportunity view, and if we assume that individuals are identical except for their birthdates, we would have to say that justice requires an intertemporal resource allocation which enables all individuals, regardless of their date of birth, to acquire the same level of welfare. We must, however, be interested in efficiency as well as equity, so the just and efficient allocation of resources is that one which enables all individuals, regardless of their birth date, to achieve the same level of welfare, where that level is the highest possible such level. Even this, however, may not be Pareto efficient -- it may be possible to render all individuals (weakly) better off than at that any equal-welfare distribution. And so we finally say that the just and efficient allocation of resources is that one which renders the worst-off representative agent (across generations) as well off as possible: the intergenerational maximin distribution.

I believe this is probably what intergenerational equity requires, although the short argument I have just given can surely be challenged. Note, in particular, that I do not discount the welfare of future generations. There might well grounds for such discounting if there is a probability that future generations may not exist, but my feeling is that this justification for discounting is very much overplayed by economists. We do not have much doubt that the next ten or even fifty generations will exist, and so, at least, we should not discount *their* welfare. But fifty is almost infinity.

The kind of well-being of an individual in this society with which I am concerned is a function only of her own consumption, not of the consumption of her descendents. One may think of this assumption in one of two ways: either individuals indeed *do not* worry about their descendents, or, more persuasively, the kind of well-being with which we, the impartial ethical observer, should be concerned, is the *standard of living* of individuals, rather than the kind of happiness they get when contemplating their childrens' and grandchildrens' lives, which is properly called *welfare*. *We*, the ethical observer, will take care of the children and grandchildren.

We now assume that there is a natural environment, to be thought of as a resource that can both be used in production to produce consumption goods, or can be enjoyed in its pristine state. Think of nature as a forest, which can either be harvested for timber to build houses, or used for hiking and recreation. The forest has a natural rate of growth, or regeneration, and let us assume that it is depleted only when used for timber, but not for hiking. Clearly, there is a rate at which wood can be harvested, so that the forest would remain of constant size. Call this rate the *sustainable* rate of harvesting the forest; that rate is not of any particular ethical significance, because we have not yet related it in any way to human welfare, if our ethics concern only the welfare of humans.

Sustainable has at least two meanings in the environmental literature. One meaning is anthropocentric: a path of resource use is sustainable if, over time, human welfare does not decrease. The other meaning is greener, and is the one I used just above: a path of resource use is sustainable if the stock of the resource does not decrease, or does not decrease to zero. I will be concerned with both meanings in this paper.

A more modern image than one of 'forest' and 'housing' would be to imagine the natural resource as the biosphere, which can either be enjoyed as a health- and life- giving resource, or can be depleted to produce manufactured consumption goods. I will retain the names 'forest' and 'houses' for the sake of convenience.

We will assume that the standard of living (or, for this paper, her welfare) of the individual at a given date is a function of two arguments, the consumption goods (houses) that she produces from the part of the forest she harvests as timber at that date, and of the hiking she does in the pristine forest that remains uncut. Houses depreciate fully at each generation, and so the only bequest one generation leaves to the next are the remaining forest, and its technological knowledge.

Suppose that members of our society strive to improve their lives, and to this end, they engage in research which produces technological improvements -- that is, at least under normal conditions, there will be an exogenous rate of productivity increase in the use of timber to make houses. Suppose that each generation costlessly passes down its technology to the next one. Thus, technological striving produces a positive externality for future generations.

We are interested in the allocation of use of the natural resource for this society. We may denote an allocation as a function $R:\mathbb{Z}_+ \to \mathbb{R}_+$, which specifies the size of the forest at date $t \in \mathbb{Z}_+$ that remains in its uncut state, and is used for hiking. Clearly, specifying a function R is equivalent to specifying the amount that will be harvested at each date, and used for the production of houses. The question is: What is the function R that maximizes the minimum level of welfare across all generations?

Formally, we can state this problem as follows. At date *t*, if *I* is the harvest of the forest as timber, then housing in the amount $\alpha^t I$ can be produced. The utility function of each agent is $u(R,H)$ where *R* is the recreational use of the forest and *H* is housing. We assume that the recreational value of the forest is just measured by the size of the forest left uncut. The natural rate of growth of the forest is ρ. Then we wish to solve:

$$\begin{aligned}&\max_{I(\cdot),R(\cdot)} \min_t u(R(t),\alpha^t I(t)) \\ &s.t. \quad R(t) = \rho R(t-1) - I(t), \quad t \geq 1 \qquad\qquad \text{Program (1)} \\ &\qquad\quad R(t) \geq 0 \text{ for all } t \geq 1\end{aligned}$$

The data of the problem are the sequence $\{\alpha^t \mid t = 1,...\}$, the natural growth factor of the forest ρ, and *R*(0), the initial size of the forest. I will assume that $u(0,H) = u(R,0) = u^{\min}$ for any *R,H,* and $u'(0,H) = u'(R,0) = \infty$, so that the optimal solution to program (1) must entail positive consumption of both housing and hiking at every date. Note that no resources are consumed in technological innovation.

2. The leximin solution

I must first be somewhat more precise. There may, indeed, be many maximin solutions. What we really are interested in is the lexicographic minimum solution, which I define as follows.

Consider the sequence of programs **P**(*n*), *n* = 1,2,…., defined as follows:

$$\mathbf{P}(n)\quad \begin{aligned} &\max_{I(\cdot),R(\cdot)} \min_{t\geq n} u(R(t),\alpha^t I(t)) \\ &s.t.\quad R(t)=\rho R(t-1)-I(t),\quad t\geq n \\ &\qquad\quad R(t)\geq 0 \text{ for all } t\geq n. \end{aligned}$$

A path $R^*(t)$ is a *leximin solution* if it solves $\mathbf{P}(n)$ for all $n \geq 1$. If R^* solves $\mathbf{P}(1)$ it is a maximin solution. If it solves $\mathbf{P}(2)$, it is , among all maximin solutions, one which is also maximin for generations two and beyond; and so on.

Consider an intertemporal path R. We will say that generation t is *unconstrained on the path R* if , given $R(t-1)$ and α^t, the value of $R(t)$ is that value which is individually optimal for generation t. In other words, we say an individual in this society is unconstrained if the prescribed path at date t is just the value of the forest that agent t would decide upon were he to optimize selfishly, given his endowment, and ignoring future generations.

If an agent is not unconstrained on a path, then we say he is constrained. We immediately have:

Proposition 1. *In a leximin solution, if an agent is constrained, then he consumes less housing than is individually optimal for him, given his endowment.*

The proof is immediate. If he consumed more housing than were individually optimal, he could decrease his housing consumption to the individually optimal amount, and at the same time increase the forest endowment for future generations, which would render all future generations better off, and the solution could not be leximin.

Now denote by $u^t(R)$ the utility of generation t on a path R. Below, I will often simply write this as u^t.

We have the following:

Proposition 2. *If R is a leximin solution then*:

1. *for all* $t \geq 1, \quad u^t(R) \leq u^{t+1}(R)$.

2. *if for any t*, $u^t(R) < u^{t+1}(R)$, *then agent t is unconstrained*.

Proof:

Claim 1. Suppose $u^t > u^{t+1}$, some t. Then the value of **P**(t) is equal to the value of **P**(t+1). Now have the agent at t consume a little less timber. This raises the endowment of program **P**(t+1), and hence raises its value; it consequently raises the value of **P**(t). Hence the original solution was not leximin.

Claim 2. Suppose some agent were constrained. Then by Prop. 1, he could increase his utility by increasing I(t) a small amount. This raises the value of **P**(t), and hence the original solution was not leximin. ■

Proposition 3. *Suppose that u is strictly quasi-concave. Then there is a unique leximin solution* .

Proof:

1. Let R^* be a leximin solution. By Prop. 2, the minimum utility is achieved at t=1. This utility is the same in all leximin solutions. The budget constraint for each generation, in $(R(t),I(t))$ space, has slope –1. There are two possibilities at t=1: either the agent's indifference curve is tangent to the budget line $R(1)+I(1)=\rho R_0$ or it cuts the budget line in (at most) two points. If the latter, then we know $I(t)$ is uniquely determined by Prop. 1. If the former, then it is tangent at exactly one point, by strict quasi-concavity. Hence $R(1)$ is uniquely determined.

2. Hence the endowment of **P***(2)* is uniquely determined for all leximin solutions, and it follows that $R(2)$ is uniquely determined, as above.

3. By induction, q.e.d.

From Proposition 2, we see that leximin solutions can be grouped into three classes:

Class 1. $u^t(R) < u^{t+1}(R)$ for all $t \geq 1$

Class 2. $u^t(R) = u^{t+1}(R)$ for all $t \geq 1$;

Class 3. $u^t(R) \leq u^{t+1}(R)$, with some equalities and some inequalities.

I will say that solutions of Class 1 comprise Nirvana: for by Proposition 1, these are *intergenerationally just paths where each generation may optimize selfishly*. In other words, in a state of Nirvana, the needs of future generations place no constraint on earlier generations' use of the forest.

3. Intertemporal optimization with leximin utility: a general theorem

In the case when the intergenerational optimization problem is well-behaved (a concave problem), we have a characterization theorem for the leximin path with constant utilities at every date.

Denote the partial derivatives of a function $u(x,y)$, by u_1 and u_2.

Theorem 1 Let $\{u^t\}$ be a sequence of increasing, concave utility functions defined on $\mathbb{R}^2$, let R^0=1, and let $\{I^t$, t=1,2,…$\}$, $\{R^t$, t=1,2,…$\}$ be non-negative sequences satisfying

for all t=1,2,… $R^t = \rho R^{t-1} - I^t$.

Define $a^t = \dfrac{u_1^t(R^t, I^t)}{u_2^t(R^t, I^t)}$ and suppose for all t=1,2,…. $a^t = a \leq 1$, for some constant a, and

$u^t(R^t, I^t) = k$, for some constant k.

Then $\{I^t, R^t\}$ solves the program:

$$\begin{aligned} &\max u^1(\rho R^0 - I^1, I^1) \\ &s.t. \qquad\qquad\qquad\qquad\qquad\qquad \text{(P)} \\ &R^t = \rho R^{t-1} - I^t, \quad t = 1,2,... \\ &u^t(R^t, I^t) \geq k, \ t=2,3,.... \end{aligned}$$

Proof:

1. A *feasible point* for program (P) is a pair of non-negative sequences $\{I^t$, t=1,2,…},

$\{R^t$, t=1,2,…} satisfying the constraints of program (P), for a given, fixed value of k. Observe that the set of feasible points is convex. It follows that (P) is a concave program.

2. Suppose the claim were false, and that there is a feasible point which gives a higher value than k to the objective. Denote the investment sequence in this dominating solution by $\{I^t + g^t, t = 1,2,....\}$. Let $\{\lambda^t, t = 1,2,...\}$ be an arbitrary sequence of non-negative numbers, and define the function:

$$J(\varepsilon) = u^1(\rho R^0 - (I^1 + \varepsilon g^1), I^1 + \varepsilon g^1) + \sum_{t=2}^{\infty} \lambda^{t-1}(u^t(R^t(\varepsilon), I^t + \varepsilon g^t) - k)$$

where $\{R^t(\varepsilon)\}$ is defined recursively by:

$$\begin{aligned} R^t(\varepsilon) &= \rho R^{t-1}(\varepsilon) - (I^t + \varepsilon g^t) \\ R^0(\varepsilon) &= R^0 \end{aligned}$$

J takes values on the extended real line.

Note that J is a concave function and that $J(0)=k$, by hypothesis. According to the claim, $J(1)>k$, since $J(1)$ is the value of program (P) at the proposed solution $\{I^t + g^t\}$ plus the sum of non-negative terms. Therefore, if we can demonstrate that there exists a choice of the non-negative sequence $\{\lambda^t\}$ such that the associated function J is maximized at $\varepsilon = 0$, then in particular, $k = J(0) \geq J(1)$, a contradiction to the claim that $\{I^t\}$ is not the solution of (P).

3. By computing $R^t(\varepsilon)$ for a few terms we observe that its derivatives are:

$$\frac{dR^2(\varepsilon)}{d\varepsilon} = -\rho g^1 - g^2$$
$$\frac{dR^3(\varepsilon)}{d\varepsilon} = -\rho^2 g^1 - \rho g^2 - g^3,$$
$$etc.$$

and so we may compute the derivative:

$$J'(0) = -u_1^1 g^1 + u_2^1 g^1 + \lambda^1(u_1^2(-\rho g^1 - g^2) + u_2^2 g^2) + \lambda^2(u_1^3(-\rho^2 g^1 - \rho g^2 - g^3) + u_2^3 g^3) + \dots \quad (3.1\ )$$

The function u_j^t in this expression is of course evaluated at (R^t, I^t).

Gathering terms, we can write the coefficient of g^1 in this expression as:

$C^1 = u_2^1 - \sum_{t=0}^{\infty} \lambda^t \rho^t u_1^{t+1}$, where $\lambda^0 = 1$.

Let C^t be the coefficient of g^t in expression (3.1). If we can find a non-negative sequence $\{\lambda^t\}$ such that C^t=0 for all t, then we will have demonstrated that, for that choice, $J'(0) = 0$. We may write the vanishing of all these coefficients as the following system:

$C^1 \qquad u_2^1 - u_1^1 = \sum_{t=1}^{\infty} \lambda^t \rho^t u_1^{t+1}$

$C^2 \qquad \lambda^1(u_2^2 - u_1^2) = \sum_{t=2}^{\infty} \lambda^t \rho^{t-1} u_1^{t+1}$

C^3 $\lambda^2(u_2^3 - u_1^3) = \sum_{t=3}^{\infty} \lambda^t \rho^{t-2} u_1^{t+1}$, etc.

Let us define $z = u_2^1 - u_1^1$. Then we may write these equations, beginning with the second one, as:

C^2 $\lambda^1(u_2^2 - u_1^2) = \dfrac{z - \lambda^1 \rho u_1^2}{\rho}$

C^3 $\lambda^2(u_2^3 - u_1^3) = \dfrac{z - \lambda^1 \rho u_1^2 - \lambda^2 \rho^2 u_1^3}{\rho^2}$, etc.

We may solve these equations sequentially for the sequence $\{\lambda^t\}$, giving:

$$\lambda^1 = \frac{z}{\rho u_2^2}, \quad \lambda^2 = \frac{z}{\rho^2 u_2^3}(1-a), \quad \lambda^3 = \frac{z}{\rho^3 u_2^4}(1-a)^2, etc.$$

Note that by hypothesis, since $a \le 1$, it follows that $z \ge 0$, and so all the λ^t are non-negative.

The only condition left to check is the equation associated with C^1 above. Substituting the derived values of the λ^t into that condition, we deduce that what must be verified is:

$$z = ?\, za + za(1-a) + za(1-a)^2 + ... \qquad (3.2)$$

If z =0, this is surely true. If z>0, then dividing (3.2) by z immediately shows that (3.2) reduces to an identity. Thus, we have produced a sequence of non-negative multipliers for which $J'(0) = 0$. ■

4. <u>Exogenous technical change</u>: <u>Special cases of utility function</u>

I will now specialize to particular cases, in order to be able to compute leximin solutions with comparative ease. We assume:

A1. The rate of technological progress is exogenous and constant: for every

$t \geq 1, \frac{\alpha^{t+1}}{\alpha^t} = \gamma$, some $\gamma > 1$.

The first case I will study is:

A2. Cobb-Douglas preferences: $u(R,H) = R^b H^{1-b}, b \in (0,1)$.

We have:

<u>Theorem</u> <u>2</u> *Let* **A1** and **A2** hold. Then:

A. If $\rho \leq \frac{1}{b\gamma^{1-b}}$, then the leximin solution is given recursively by

$$R^*(t) = \frac{\rho}{1+\delta} R^*(t-1), \; I^*(t) = \frac{\rho\delta}{1+\delta} R^*(t-1), \text{ where } \delta = \rho\gamma^{1-b} - 1.$$

Utilities are equal for all generations, and every generation is constrained iff the defining inequality of this case is strict.

B. If $\rho > \frac{1}{b\gamma^{1-b}}$, then the leximin solution is given recursively by:

$R^*(t) = b\rho R^*(t-1), \;\; I^*(t) = (1-b)\rho R^*(t-1).$ No generation is constrained, and utility increases at each date.

C. In case (A), the size of the forest decreases monotonically to zero. In case (B), the size of forest decreases to zero if $b < \frac{1}{\rho}$, and increases without bound if $b > \frac{1}{\rho}$.

The theorem tells us that sustainability in the green sense of a non-shrinking forest size occurs only if love of the forest (b) is sufficiently great. In particular, in case (B) of the theorem we have Nirvana, but nevertheless the forest may shrink to zero.

Lemma 1 Let $R^*(t)$ be a path at which each agent is unconstrained and $u^t \le u^{t+1}$ for all t. Then R^* is the leximin solution.

Proof:

u^1 cannot possibly be greater, and so R^* solves **P**(1). Hence $R^*(1)$ is determined. By induction, the path is the leximin path. ■

Proof of Theorem 2:

1. We prove part B first. We begin by asking: Is there a value γ at which, if all agents optimize selfishly, the path generated will enjoy equal utilities at all dates? If so, by Lemma 1, this is the leximin path for that process of technical change.

2. If t optimizes selfishly then

$$R(t) = b\rho R(t-1)$$
$$I(t) = (1-b)\rho R(t-1)$$

and so $I(t) = \frac{1-b}{b} R(t)$.

It follows that

$$\frac{u^{t+1}}{u^t} = \frac{(b\rho R(t))^b (\alpha^{t+1})^{1-b} ((1-b)\rho R(t))^{1-b}}{R(t)^b (\alpha^t)^{1-b} (\frac{1-b}{b} R(t))^{1-b}} = .$$

$$b\rho\gamma^{1-b} = 1 \Leftrightarrow \rho = \frac{1}{b\gamma^{1-b}}$$

Hence, if the last equation holds, the leximin solution is Nirvana, and utilities are equal at all dates.

3. It follows that if $\rho > \frac{1}{b\gamma^{1-b}}$, then individual optimization at each date generates increasing utilities, which, by Lemma 1, must be the leximin solution.

4. Now suppose $\rho < \frac{1}{b\gamma^{1-b}}$. Define:

$$u^t(R,I) = u(R,\alpha^t I).$$

Compute that the solution defined in statement A produces equal utilities at all dates, that is:

$$u^t(R^t, I^t) = \delta^{1-b} \text{ for all } t \geq 1,$$

and note that $\frac{\delta}{1+\delta} < 1-b$, which shows that I*(t) is less than the individually optimal harvest for agent t.

5. We next verify that the other premise of Theorem 1 holds. Compute that

$$a^t = \frac{u_1^t(R(t),I(t))}{u_2^t(R(t),I(t))} = \frac{b\delta}{1+\delta}.$$

What remains to be verified, then, is only that $b\delta \leq 1+\delta$. This is equivalent to the statement $b\rho\gamma^{1-b} \leq 1$, which is true by the premise of Part A.

Hence, by Theorem 1, $(I^*(t), R^*(t))$ is a maximin solution of our program. It now follows, by repeated application of theorem 1, that it is also the leximin solution. ■

Suppose that our own natural forest regenerates at a rate of 3% per annum. If a generation is 25 years, then $\rho = 1.03^{25} = 1.85$, and $\frac{1}{\rho} = 0.54$. Is b<0.54? If so, intergenerational justice and green sustainability are incompatible with this utility function.

My second exercise will involve a different utility function. One might well say: human beings require some minimal amount of forest. The idea that welfare might remain constant by continually substituting housing consumption for recreation in the forest (health) is ridiculous. So let us replace **A2** by:

A3. $u(R,H)=(R-x_0)^b H^{1-b}$, some $0<x_0<\rho R_0$.

With **A3**, we have a Stone-Geary utility function, with a minimal necessary consumption of forest. The upper bound on x_0 simply assures that it is possible for the first generation to satisfy its minimal need of hiking with the forest that it inherits.

We now have:

Theorem 3 Normalize by setting $R(0)=1$. Assume **A3**.

A. If $b\rho+(1-b)x_0<1$ then there is a unique process $\{\hat{\alpha}^t\}$ of technical progress at which the leximin solution yields constant utilities and unconstrained agents at all dates. Under this process, the size of the forest decreases over time and approaches the value $\frac{x_0(1-b)}{1-b\rho}$ in the limit.

B. If $b\rho+(1-b)x_0<1$ and technical progress is more rapid than in $\{\hat{\alpha}^t\}$ at all t, then all agents are unconstrained, utilities increase over time, and the limit size of the forest is as in A.

C. If $b\rho+(1-b)x_0>1$ then under *any* process of technical progress the leximin solution has agents unconstrained at every date, and utilities increase over time. If $b\rho<1$ then the size of forest converges to the limit in A; if $b\rho>1$, the size of the forest grows without bound.

Proof:

1. We attempt to compute a value of γ at which selfish optimization at each date produces equal utilities at all dates. Selfish optimization now entails:

$$R(t) = b(\rho R(t-1) - x_0) + x_0 \qquad (1)$$
$$I(t) = (1-b)(\rho R(t-1) - x_0) \qquad (2)$$

Note that $I(t) = \dfrac{1-b}{b}(R(t) - x_0)$.

Hence

$$\frac{u^{t+1}}{u^t} = \frac{(b(\rho R(t) - x_0))^b (\alpha^{t+1})^{1-b}((1-b)(\rho R(t) - x_0))^{1-b}}{(R(t) - x_0)^b (\alpha^t)^{1-b}(\frac{1-b}{b}(R(t) - x_0))^{1-b}} = \qquad (3)$$

$$\frac{b(\rho R(t) - x_0)\gamma^{1-b}}{R(t) - x_0} = 1$$

$$\Leftrightarrow$$

$$R(t)\{1 - b\rho\gamma^{1-b}\} = x_0(1 - b\gamma^{1-b})$$

For the last equation to hold there are two possibilites: either R(t) is constant over time at the required value, or $1 = b\rho\gamma^{1-b}$ *and* $x_0 = 0$. In the first case, utilities would increase with time, because of technical progress, which contradicts our hypothesis; and in the second case, we are back in the Cobb-Douglas world. So, if $x_0 > 0$, there is no constant rate of technical progress that will engender the solution we seek.

2. Returning to equation (3), and relaxing the assumption of a constant rate of technical progress, we require:

$$(\gamma^{t+1})^{1-b} = \frac{R(t) - x_0}{b(\rho R(t) - x_0)} \qquad (4)$$

A path where every agent optimizes and (4) holds will have equal utilities. By Lemma 1, this will be the leximin path.

3. We study the process $\{\alpha^t\}$ that would produce (4) when each generation optimizes selfishly. From (1) and (4) we have:

$$(\gamma^{t+1})^{1-b} = \frac{R(t) - x_0}{R(t+1) - x_0}. \qquad (4')$$

Consequently, a process of technical progress (with $\alpha^{t+1} > \alpha^t$) necessarily involves $R(t+1) < R(t)$.

4. By the recursion (1), compute that

$$R(t) = (b\rho)^t + x_0(1-b)(1 + b\rho + ... + (b\rho)^{t-1}) \qquad (5)$$

From (5), compute that

$$R(t+1) - R(t) = (b\rho)^{t+1} - (b\rho)^t + (b\rho)^t x_0(1-b) < 0$$
$$\Leftrightarrow b\rho + (1-b)x_0 < 1. \qquad (6)$$

So if (6) holds, we can define uniquely a process of technical progress for which (4'), and hence (4), holds. Because a fortiori $b\rho < 1$ in this case, we compute from (5) that

$$\lim R(t) = \frac{x_0(1-b)}{1-b\rho} \qquad (7)$$

This proves statement A.

[5. An aside: the formula (5) only describes R(t) if at each date $\rho R(t) > x_0$. Failing this inequality, the agent would have to consume less than x_0 in hiking. Since the sequence R(t) described in step 4 decreases monotonically to the value in (7), it suffices to verify the inequality $\frac{\rho x_0(1-b)}{1-b\rho} \geq x_0$, which is true.]

6. Statement B follows immediately. If every generation optimizes selfishly, then (1) holds, but now $(\gamma^{t+1})^{1-b} > \frac{R(t) - x_0}{R(t+1) - x_0}$ for all *t,* and so utilities increase at every date. This is the leximin solution by Lemma 1.

7. Statement C. If agent 1 optimizes then $R(1) = b(\rho R(0) - x_0) + x_0 = b\rho + (1-b)x_0 > 1$.

Hence agent 2 has a greater forest endowment and a better technology than agent 1, and so if he optimizes selfishly, he will be better off than agent 1. We know that, if every agent optimizes then $R(t+1) > R(t)$ (step #4), and so an increasing sequence of utilities is generated. By Lemma 1, this is the leximin solution, proving the first part of statement C. Since the size of the forest at date *t* is given by (5), we see that it approaches $\frac{x_0(1-b)}{1-b\rho}$ if $b\rho < 1$ and becomes infinitely large if $b\rho > 1$. ■

The hopeful part of Theorem 2 is part C. It says that if x_0 is fairly close to ρR_0, then the leximin solution entails Nirvana for *any* process of technical progress. This is a bit surprising: one might have thought that is x_0 is 'large,' we would be in a situation of scarcity, which would have bad implications for the forest. However, the size of the forest will increase to a finite limit, unless *b* is large.

The Cobb-Douglas utility function has an elasticity of substitution of unity. Finally, I study the CES utility function where there is more complementarity between hiking and housing than in the Cobb-Douglas function. We might hope that, with these preferences, citizens will not be so willing to deplete the forest. Here is a theorem:

<u>Theorem</u> <u>4</u> Let $u(R,H) = (bR^r + (1-b)H^r)^{1/r}, \quad r < 0$. Let $\{\alpha^t\}$ be any process of technical progress, and define $p^t = \frac{1-b}{b}(\alpha^t)^r$ and $\varphi(t) = \frac{(p^t)^{\frac{1}{r-1}}}{1+(p^t)^{\frac{1}{r-1}}}$.

A. If the law of motion of technical change is given by

$$\varphi(t+1) = \rho^{\frac{r}{1-r}} \varphi(t)^{\frac{1}{1-r}} \qquad (8)$$

then we have Nirvana with constant utilities. We have

$$\lim \varphi(t) = \rho^{-1}, \lim \alpha^t = \alpha^* \equiv (\frac{b}{1-b})^{\frac{1}{r}} (\rho - 1)^{\frac{1-r}{r}} \quad \text{and } \lim \frac{R(t+1)}{R(t)} = 1.$$

B. Let $\{\alpha^t\}$ be a process of technological progress such that for large *t*, $\alpha^t > \alpha^*$. Then $\varphi(t) > \frac{1}{\rho}$ and the forest increases in size at every date. Henceforth, all agents optimize selfishly, and utilities increase.

<u>Proof</u>:

1. If a generation optimizes selfishly, then compute that:

$$R(t) = \varphi(t)\rho R(t-1), \quad I(t) = (1-\varphi(t))\rho R(t-1).$$

2. Hence, if each generation optimizes selfishly then:

$$(\frac{u^{t+1}}{u^t})^r = \frac{b(\varphi(t+1)\rho R(t))^r + (1-b)(\alpha^{t+1}(1-\varphi(t+1))\rho R(t))^r}{bR(t)^r + (1-b)(\alpha^t \frac{1-\varphi(t)}{\varphi(t)} R(t))^r} =$$

$$= \frac{\rho^r [\varphi(t+1)^r + p^{t+1}(1-\varphi(t+1))^r]}{1 + p^t (\frac{1-\varphi(t)}{\varphi(t)})^r}.$$

By inverting the defining equation of φ, compute that $p^t = (\frac{1-\varphi(t)}{\varphi(t)})^{1-r}$, and substituting this into the last expression, we compute that $\frac{u^{t+1}}{u^t} = 1 \Leftrightarrow \frac{\varphi(t+1)^{1-r}}{\varphi(t)} = \rho^r$ which is equation (8) in statement A.

3. Note that $\varphi(t) \in (0,1)$, and so a limit of $\varphi(t)$ exists. It follows from the recursive definition of φ that $\lim \varphi(t) = \varphi^* = 1/\rho$. The expression in statement A for $\lim \alpha^t$ follows immediately from the definition of φ and p.

4. $\frac{R(t+1)}{R(t)} = \rho\varphi(t) \to \rho\varphi^* = 1.$

5. If $\alpha^t > \alpha^*$ then we have:

$$\begin{aligned}
&(\alpha^t)^r < \frac{b}{1-b}(\rho-1)^{1-r} \Rightarrow \\
&p^t < (\rho-1)^{1-r} \Rightarrow \\
&(p^t)^{\frac{1}{r-1}} > \frac{1}{\rho-1} \Rightarrow \varphi(t) > \frac{1}{\rho} \Rightarrow \\
&R(t) = \varphi(t)\rho R(t-1) > R(t-1)
\end{aligned}$$

Thus, selfish optimization yields an increasing size of forest. Hence we have achieved Nirvana with increasing utilities. ■

This is our most hopeful result. With CES utility functions, in which r<0, there is a finite value of the technological coefficient α, which if exceeded, will permit all generations to optimize selfishly along the just path, well-being will increase at every date, and the forest increases in size.

Thus, our preliminary investigation suggests that the most hopeful scenario for consistency between the green view and justice à la equality-of-opportunity is that hiking and consumption of produced commodities are more complementary than they are in Cobb-Douglas preferences.

For instance, if r=-0.5 and b=0.5 and ρ=1.85, then α^*=1.628.

5. Endogenous technical change

In this section, we introduce endogenous technical change.

We would like to study the program

$$
\begin{aligned}
&\max \quad k \\
&s.t. \\
&u(R(t),H(t)) \geq k, \quad t = 1,2,... \\
&R(t) = \rho R(t-1) - I(t), \quad t = 1,2,... \\
&H(t) = (1-\lambda(t))^{a} \alpha(t) I(t)^{1-a} \\
&\alpha(t) = (1+\hat{\gamma}\lambda(t))\alpha(t-1)
\end{aligned}
$$

where $\alpha(0), R(0), \gamma, \rho$ are given. ρ is the natural growth factor of the forest. Think of $\lambda(t)$ as the fraction of the labor force assigned to the R&D industry at date *t*, the rest working in the housing industry. Production in the housing industry is Cobb-Douglas: in the R&D industry, the single input is labor time, and production is linear in labor[3]. Thus the rate of technical change, if all labor were used in R&D, is $\hat{\gamma}$. Please note that $\hat{\gamma}$ of this section corresponds to $\gamma - 1$ of previous sections.

The intergenerational leximin solution solves this program. Moreover, we will show that the solution to this program must be the leximin solution.

The endogeneity of technical progress makes this a non-concave program. (Just check that if (λ, R, I) and $(\hat{\lambda}, \hat{R}, \hat{I})$ are two feasible paths, their convex combinations are not generally feasible.) This is a general feature of endogenizing technical change in growth models. Uzawa (1965) solves an intertemporal problem with endogenous technical change (though not in the sustainability framework and not in the leximin framework) by demonstrating that there is a *unique* path which satisfies the necessary conditions for a maximum. We will not, however, attempt to solve this non-concave problem here. Most growth theorists who study endogenous technical change do not solve the general non-concave problem: they restrict themselves to a subset of the set of all feasible paths which is convex (e.g., constant growth paths). This, for instance, is the strategy of Lucas (1988).

There is an important externality here: if generation *t* invests in R&D, it reaps the benefits from that investment, and the new technological knowledge is passed on for free to the future. This feature is embodied in the constraint defining $\alpha(t)$. This externality gives us the possibility of supporting *increasing utilities* on the leximin path – which we study below.

It is worth nothing that it is not immediately apparent how one might capture this externality if one modeled the problem in continuous time In the continuous case, the technology constraint becomes $\dot{\alpha}(t) = \hat{\gamma}\lambda(t)$. No amount of investment in R&D at time *t* can increase the value of α at *t*: all it does is increase the rate at which α increases for people in the future. Hence, generation *t* will have no selfish motive to invest in R&D in the continuous model. Thus, as we are interested in this kind of externality, it appears that we must use the discrete-time model.

I do not here study the general non-concave problem stated above. I study a simpler program where the R&D industry is constrained to employ a constant *fixed* fraction of the labor force: $\lambda(t) = \lambda$ for all *t*.

We solve this program, and derive *k* as a function of λ. We then may choose $\lambda \in [0,1]$ to maximize *k*. Of course, this does not solve the general non-concave program, which may well involve varying the fraction of the labor force employed in R&D.

---

[3] Note that the model of the previous sections is a special case of this one, where we take $a = 0$, $b=c$, and all labor is assigned to the R&D industry, generating a rate of growth of $\alpha(t)$ equal to $\hat{\gamma}$.

Normalize by setting $R(0)=1$. Our program is:

$$
\begin{aligned}
&\max \quad k \\
&s.t. \\
&\left.\begin{aligned}
&u(R(t),(1-\lambda)^a\alpha(t)I(t)^{1-a}) \geq k \\
&R(t)=\rho R(t-1)-I(t) \\
&\alpha(t)=(1+\hat{\gamma}\lambda)^t\alpha_0
\end{aligned}\right\} P(\lambda)
\end{aligned}
$$

A *point* is a feasible path *{I(t),R(t)}*. Note this is a concave program for λ fixed.

Consequently, by the variational method used in the proof of Theorem 1, we can compute shadow prices. (Define the appropriate function $J(\cdot)$ as in the proof of that theorem.) We here search for the characterization of a solution that entails constant utility across time. If, at a point *{I(t),R(t)}* at which $u^t = k$ for all *t,* there are non-negative sequences $\{x^t\},\{y^t\}$ such that:

$$
\begin{aligned}
&1=\sum_1^\infty x^t \\
&0=x^t u_1^t - y^t + \rho y^{t+1}, \quad t=1,2,... \\
&0=u_2^t(1-\lambda)^a\alpha(t)(1-a)I(t)^{-a}x^t - y^t, t=1,2,...
\end{aligned}
$$

then the point is a solution of the program.

Now we specialize to the case $u(R,H)=R^cH^{1-c}$, so

$u_1^t = c(H/R)^{1-c}, \quad u_2^t=(1-c)(R/H)^c$. But $R^cH^{1-c}=k$, and so $u_1^t=\dfrac{ck}{R(t)}, \quad u_2^t=\dfrac{(1-c)k}{H(t)}$.

Thus the stated conditions can be written:

(1) $1 = \sum x^t$

(2) $0 = \frac{ckx^t}{R(t)} - y^t + \rho y^{t+1}$

(3) $0 = \frac{x^t(1-c)k(1-a)}{I(t)} - y^t$

$\Rightarrow x^t = \frac{I(t)y^t}{(1-c)k(1-a)}$

Substituting into (2) we have

(2') $\rho y^{t+1} = y^t(1 - \frac{I(t)}{R(t)}\frac{c}{(1-c)(1-a)})$.

We now try for a solution where $R(t) = A^t$, some $A > 0$. Repeated use of the 'budget' constraint gives

$I(t) = A^{t-1}(\rho - A)$, t=1,2,...

Therefore $\frac{I(t)}{R(t)} = \frac{\rho - A}{A}$ and so (2') becomes

$$\rho y^{t+1} = y^t(1 - \frac{\rho - A}{A}\frac{c}{(1-c)(1-a)}).$$

Consequently we must check that:

$$1 \geq \frac{\rho - A}{A}\frac{c}{(1-c)(1-a)} \quad (T1)$$

to guarantee the non-negativity of the y^t.

Denote $Q = \frac{1}{\rho}(1 - \frac{\rho - A}{A}\frac{c}{(1-c)(1-a)})$. Then $y^t = Q^{t-1}y^1$ for $t = 1,2,\ldots$ since $\frac{y^{t+1}}{y^t} = Q$. Therefore $x^t = \frac{Q^{t-1}y^1A^{t-1}(\rho - A)}{(1-c)k(1-a)}$ for all t, from (3). So (1) requires that

$$1 = \frac{y^1(\rho - A)}{(1-c)k(1-a)} \sum_{t=1}^{\infty} (QA)^{t-1} \text{ or}$$

$$1 = \frac{y^1(\rho - A)}{(1-c)k(1-a)} \frac{1}{1-QA} \quad (4)$$

For this series to sum as stipulated, we must have

$$QA < 1 \qquad \text{(T2)}$$

But we have

$$k = R(t)^c [\alpha(t) I(t)^{1-a} (1-\lambda)^a]^{1-c}$$

$$= [A^c (1+\hat{\gamma}\lambda)^{1-c} A^{(1-c)(1-a)}]^t \alpha_0{}^{1-c} (\frac{\rho - A}{A})^{(1-a)(1-c)} (1-\lambda)^{a(1-c)}$$

which implies that the term in square brackets is unity:

$$A^{c+(1-c)(1-a)} (1+\hat{\gamma}\lambda)^{1-c} = 1 \text{ and so}$$

$$A = (1+\hat{\gamma}\lambda)^{\theta} \text{ where } \theta = \frac{c-1}{c+(1-c)(1-a)}.$$

Note that $\theta < 0$, and so we have $A<1<\rho$.

This gives us the formula:

$$k = \alpha_0^{1-c} \left(\frac{\rho - A}{A} \right)^{(1-a)(1-c)} (1-\lambda)^{a(1-c)} \quad , \qquad (5)$$

which defines the function $k(\lambda)$.

From (4), we now define:

$$y^1 = \frac{(1-QA)(1-c)k(1-a)}{\rho - A}.$$

Because $A < 1 < \rho$ we have $\rho - A > 0$. Consequently we have solved for non-negative sequences $\{x^t, y^t\}$ that satisfy conditions (1)-(3), subject to verifying (T1) and (T2).

We check (T1) and (T2). (T1) reduces to the statement:

$$\rho < \frac{c\text{-}1}{c\theta}\,(1+\hat{\gamma}\lambda)^{\theta} \; .$$

On the other hand, (T2) reduces to the statement

$A(1+\dfrac{c}{(1-c)(1-a)}) < \rho(1+\dfrac{c}{(1-c)(1-a)})$, which is true. Therefore we have the following:

<u>Proposition</u> <u>Let</u> <u>$u(R,H)=R^{c}H^{1-c}$.</u> <u>Let</u> <u>$\rho < \frac{c\text{-}1}{c\theta}\,(1+\hat{\gamma}\lambda)^{\theta}$</u> . <u>Then</u> <u>the</u> <u>solution</u> <u>to</u> <u>P(λ)</u> <u>entails</u> <u>$u^{t}=k$</u> <u>for</u> <u>all</u> <u>t,</u> <u>where</u>

$$R(t)=A^{t}$$
$$I(t)=A^{t-1}(\rho-A)$$
and $A=(1+\hat{\gamma}\lambda)^{\theta},$ where $\theta=-\dfrac{1-c}{c+(1-c)(1-a)}.$

k is given by eqn. (5).

I began with a feasible point at which $u^{t}=k$ for all t. The variational method shows that this point is a solution to the program $P(\lambda)$, under the premise of the proposition, which is a maximin program. One can see that this must be the leximin solution. This entails looking at the maximin program which begins at date 2 with the endowment $\rho A=\rho R(1)$. The same argument shows that the solution of that program entails $u^{2}=k$. By induction on the date, we have that the maximin solution at each date, along this path, taking at each date the endowment from the previous date along the path, yields constant utilities. But this means the solution is the leximin solution.

The solution of the Proposition is not Nirvana: at every date, the agent would like to consume more of the forest as housing, but he is obliged not to, for the sake of future generations. We next study the conditions for the Nirvana solution to hold, where each generation maximizes selfishly on the leximin path. If selfish choice generates a non-decreasing sequence of utilities, then it is the leximin solution.

If generation 1 maximizes selfishly, then it chooses R to

$$\max R^c[(1-\lambda)^a(1+\hat{\gamma}\lambda)\alpha_0(\rho-R)^{1-a}]^{1-c}$$

which is equivalent to solving

$$\max R^{\frac{c}{c+(1-c)(1-a)}}(\rho-R)^{\frac{(1-c)(1-a)}{c+(1-c)(1-a)}}$$

whose solution is

$$R(1)=\frac{c}{c+(1-c)(1-a)}\rho, I(1)=\frac{(1-c)(1-a)}{c+(1-c)(1-a)}\rho.$$

This gives a value of utility at the first date of:

$$u^1=\beta^c(1-\beta)^{(1-c)(1-a)}\rho^{c+(1-c)(1-a)}(1-\lambda)^{a(1-c)}[(1+\hat{\gamma}\lambda)\alpha_0]^{1-c} \quad (6)$$

where $\beta=\frac{c}{c+(1-c)(1-a)}$. Now the available forest at date 2 is $\beta\rho^2$, and it follows by the same reasoning that if the date 2 agent maximizes selfishly, she chooses:

$$R(2)=\beta^2\rho^2, I(2)=(1-\beta)\beta\rho^2.$$

By substituting these values into her utility function, we can compute that on this path:

$$\frac{u^2}{u^1}=\beta^{c+(1-c)(1-a)}\rho^{c+(1-c)(1-a)}(1+\hat{\gamma}\lambda)^{1-c}. \quad (7)$$

Indeed, this is the ratio for the utilities at any two consecutive dates along the selfish path. We require, then, that this ratio be at least unity, which is equivalent to the statement:

$$\beta\rho\geq(1+\hat{\gamma}\lambda)^{\theta},$$

which in turn says that:

$$\rho\geq\frac{c-1}{c\theta}(1+\hat{\gamma}\lambda)^{\theta}.$$

In summary:

Theorem 5 (Cobb-Douglas utility) Let $\lambda \in [0,1]$. If $\rho < \frac{c-1}{c\theta}(1+\hat{\gamma}\lambda)^{\theta}$, then the leximin solution entails constant utilities at each date, with the path given in the Proposition. If $\rho \geq \frac{c-1}{c\theta}(1+\hat{\gamma}\lambda)^{\theta}$, then the leximin path entails selfish maximization by the agent at each date, and if this inequality is strict, then utilities increase at each date[4].

We now consider the issue of choosing among the leximin paths associated with the various values of λ. Define the function $\rho(\lambda) = \frac{c-1}{c\theta}(1+\hat{\gamma}\lambda)^{\theta}$. Note that this function is decreasing. Define the number λ* as the solution of the equation

$$\rho = \rho(\lambda^*), \text{ that is, } \lambda^* = \frac{\left(\frac{c\theta\rho}{c-1}\right)^{1/\theta} - 1}{\hat{\gamma}}.$$

Then the theorem tells us that:

- If $\lambda^* \leq 0$, then for every λ<1, the leximin path is Nirvana;
- if $\lambda^* \geq 1$, then for every λ, the leximin path has constant utilities;
- if $0 < \lambda^* < 1$, then for $0 \leq \lambda \leq \lambda^*$, the leximin path has constant utilities, and for $\lambda^* < \lambda < 1$ the leximin path is a Nirvana path.

We now find the leximin path among all Nirvana paths, should they exist. This is the path among these that maximizes utility at the first date. This utility is given in eqn. (6); hence,

[4] The reader may check that Theorem 1 is a special case of Theorem 5, where we let *a*=0 and λ=1.

we must choose λ to maximize $(1-\lambda)^a(1+\hat{\gamma}\lambda)$. The solution of the FOC of this function

(after taking logarithms to render it concave) is $\lambda = \frac{\hat{\gamma}-a}{\hat{\gamma}(1+a)}$. We therefore have:

Among Nirvana paths, the leximin solution is given by

$$\lambda_1^{lex} = \begin{cases} \lambda^*, \text{ if } \frac{\hat{\gamma}-a}{\hat{\gamma}(1+a)} \le \lambda^* \\ \frac{\hat{\gamma}-a}{\hat{\gamma}(1+a)}, \text{ otherwise.} \end{cases}$$

Among constant-utility paths, the leximin path maximizes $k(\lambda)$, which is to say that it maximizes

$$(\rho(1+\hat{\gamma}\lambda)^{-\theta}-1)^{1-a}(1-\lambda)^a .$$

The derivative of the logarithm of this concave function is

$$-\left(\frac{a}{1-\lambda}+\frac{(1-a)\theta\rho\hat{\gamma}(1+\hat{\gamma}\lambda)^{-(1+\theta)}}{\rho(1+\hat{\gamma}\lambda)^{-\theta}-1}\right).$$

If this expression is somewhere zero in the interval $[0,\lambda^*]$, then that value of λ generates the leximin path among all constant-utility paths. If this expression is everywhere positive in this interval, then the leximin path among constant-utility paths occurs at $\lambda=\lambda^*$; if it is everywhere negative in the interval, then the leximin path occurs at $\lambda=0$.

Finally, the *overall* leximin path, among all paths with constant λ, is found by comparing the two leximin paths just computed[5]. The one which gives a higher utility at date 1 is the overall leximin path. (If they give the same utility at date 1, then the Nirvana path, with increasing utilities, is the leximin path.)

---

[5] Of course, *pace* section 1, the 'overall' leximin path is not the *true* universal leximin path which may well require varying the labor employed over time in the two industries.

By looking at the formula for λ*, we see that condition $\lambda^* \le 0$ is equivalent to the condition

$$1 \le c(1-\rho\theta) \text{ or } \rho \ge 1 + \frac{(1-c)(1-a)}{c}. \qquad (8)$$

If this is true, then at every $\lambda < 1$ the leximin solution is Nirvana, so the overall leximin solution is Nirvana. It is interesting to note that (8) does not involve $\hat{\gamma}$.

We note an interesting fact concerning the fate of the forest along the leximin path. Note that, in the constant-utility paths, $R(t) = A^t$, and since $A<1$, the forest size approaches zero. However, in the Nirvana paths, we have $R(t) = (\beta\rho)^t$ and so the forest size increases without bound, stays the same size, or decreases to zero, as $\beta\rho$ is greater than, equal to, or less than one, respectively. More precisely, we have:

Theorem 6 (Cobb-Douglas utility) Fix a value $0 < \bar{\lambda} < 1$. We have:

A. The forest sizes increases without bound on the leximin path at $\bar{\lambda}$ if an only if it increases without bound on the $\lambda -$leximin path *for every* $\lambda \in (0,1)$.

B. If the forest sizes increases without bound on the $\bar{\lambda}$-leximin path then utilities increase without bound on the $\lambda -$leximin path at every λ.

Proof:

1. We have noted, just above, that the condition for the forest's increasing without bound on the $\bar{\lambda}$-leximin path is that the path be Nirvana *and* that $\beta\rho > 1$. This is equivalent to $\bar{\lambda} > \lambda^*$ and $\dfrac{c\theta}{c-1} > 1$. But the last inequality implies that $\lambda^* < 0$, and so the inequality $\bar{\lambda} > \lambda^*$ is redundant. Therefore, if the forest increases without bound at

$\bar{\lambda}$, then the $\lambda-$leximin path is Nirvana for every λ (since $\lambda^* < 0$) and hence the size of the forest increases without bound on every $\lambda-$leximin path (because the inequality $\frac{c\theta}{c-1} > 1$ is independent of λ). The converse of statement A is trivial.

2. The premise of statement B now tells us that $\beta\rho > 1$ and the $\lambda-$leximin path is Nirvana for all λ. Note that the formula for $\frac{u^2}{u^1}$ given in equation (7), which is also the formula for $\frac{u^{t+1}}{u^t}$ on any Nirvana path, says that on Nirvana paths, utility increases without bound if and only if $\beta\rho > (1+\gamma\lambda)^\theta$. This is surely true if $\beta\rho > 1$, which proves statement B. ■

We note, from the last part of the proof, that there may be Nirvana paths at which utilities increase without bound but the forest goes to zero size: the converse of statement B is not generally true. This happens when we have Nirvana with $1 > \beta\rho$. Nirvana implies that $\lambda > \lambda^*$ which is equivalent to $\beta\rho > (1+\hat{\gamma}\lambda)^\theta$.

Let us try some parameter values: I suggest

$a = 0.75$ (labor's share is about 75%)

$c = 0.5$ (people value the environment and commodities equally)

$\hat{\gamma}$ = 13. (productivity growth at 2 % per annum implies that over a generation of 25 years, the growth factor is 1.64. Assuming employment in R&D is 5% of the labor force, this gives $.05\,\gamma = 0.64$)

$\rho=1.64$. (assume a 2% per annum rate of regeneration)

With these values, we compute that $\lambda^* = -.02$ and so we have Nirvana at all λ .

If we think of the biosphere as necessary for health, then *c* should be quite large. If *a* and ρ are given the above values, then $\lambda^* \leq 0$ as long as $c \geq 0.32$. This seems fairly optimistic.

Suppose, then, that $c = 0.333$. Then with *a* and $\hat{\gamma}$ as above, the overall leximin path is given by $\lambda = .55$, a far cry from what we see. This suggests that a weakness in the model is the assumption that the rate of technical change is linear in labor: in reality, there must be fairly strongly decreasing returns to R&D in labor, due to the high level of education needed to be productive in that sector. Perhaps a more realistic model would postulate a limited supply of labor capable of working in the R&D sector. Endogenizing *that* supply would require putting an education sector into the model.

6. Concluding contemplations

Let me conclude with some conjectures and remarks.

1. Perhaps the most important conclusion is that the leximin social welfare function does not automatically relegate us to a world with no human progress, that is, no increase in well-being over time. Clearly this can be the case with exogenous technical change: but it holds even if technical change is endogenous and costly. If the technology that transforms labor into technical progress is sufficiently productive (the value of *k* in theorem 5), then leximin appears to imply increasing well-being over time – at least on the domain of constant growth paths.

The most general formulation of the conditions under which leximin involves an increase in welfare over time is given in Silvestre (2002).

I believe this is a potentially important observation. Intergenerational maximin is often associated with the view that utilities must be constant over time, and hence, in the welfare sense, there is no progress. Because many consider this to be an unacceptable outcome (is it really?), the maximin or leximin social welfare function is not taken as seriously as it should be. We may, however, live in a world where the conditions hold where intergenerational leximin implies increasing utilities over time.

2.. An important assumption, which I believe is responsible for the somewhat pessimistic results in the Cobb-Douglas case, is that technological progress occurs in the housing industry but not in the hiking industry. This, I think, is a realistic assumption. The attraction that Nature has for us is, I believe, that we enjoy it unmediated by sophisticated technology. If we interpret 'nature' as the services of the biosphere that sustain life, then the appropriate assumption is that our capacity to create consumption commodities using the biosphere as a resource improves more rapidly than our capacity to create health from the biosphere.

3. Other sentient beings. Our discussion does not take into account the welfare of other sentient beings that use the natural resource. Of course, doing so could radically alter our conclusions about forest use. The 'green' definition of sustainability is perhaps a quick reduced form for modeling the welfare of animals who also use the forest for life. Even if, today, our society will not reach a consensus to include the welfare of other sentient beings in the calculus of equity, we might wish to retain the flexibility on this matter.

Our possible concern with animals is one which would cause us to alter the arguments of the intergenerational utility function, to include more than one representative agent at each date. Indeed, we may wish to represent different agents as being of different sizes.

3. If we relax the assumption of the single representative agent at each date, there are plausible ways of doing so short of admitting animals as citizens. We should recognize that there are different types of humans, in particular, humans who have different wealth and income levels. The equality–of-opportunity approach directs us to consider the case of types of people who live simultaneously and who have different welfare levels because of circumstances beyond their control -- for instance, because they were born in different countries, or in different families in the same country, or with different genetic dispositions in the same family. All these differences among people are circumstances, and arguably, an equal-opportunity ethic would declare that it is unjust that these differences should entail different welfare levels for the individuals in question.

Thus, the intergenerational equity problem would here be to maximize the welfare level of the worst-off person who ever lives, assuming that we do not complexify the model further by allowing people to expend different degrees of voluntary effort. It will now be appropriate to have intra-generational transfers through taxation, and so the optimization problem is more difficult: there are at least two controls, the use of the natural resource and intra-generational taxation. As well, if there are poor and rich families, we probably should take into account the process of formation of human capital, and hence investment will take the form not only of building houses, but of education.

Roberto Veneziani and I (2004) have recently studied a version of this problem, but without the natural resource: we are concerned only with the nature of educational investment and taxation to make the worst-off person who ever lives as well off as possible, in a society with two dynasties, distinguished by their initial endowments of human capital. Discussing this problem is beyond my scope here. Let me just offer a remark. Our attention is naturally focused, in such a problem, on the poor , low human-capital people at each date: they are the ones who are the worst-off. Because demand for hiking in the forest is a normal good, it will be optimal to consume more of the forest as investment than if everyone were well-off.

This is, perhaps, one of the most important ways that our model fails to capture what is ethically important in today's world. Those of us at this conference have demands for preservation of the natural environment that are associated with having incomes in the top 1% or so of world income. But intergenerational equity, of the equal-opportunity kind that I have been discussing, probably will focus upon the welfare of those who live in South Asia and Africa, and perhaps Latin America: they are the worst-off today, and will be for some time to come. It may be that the optimal solution entails running down the natural environment until a point that technological advance has become sufficient that it is no longer necessary to do so; but perhaps this path can be avoided with sufficient transfers from the north to the south.

The problem is interesting, because, to some extent at least, the natural environment is a commons, from which all countries can harvest timber. If the citizens of China or Brazil or the Sudan are close to their minimal consumption of housing, then one can only expect them to consume from the natural commons at a rate faster then citizens of the rich countries would like. The travesty is not the overconsumption of the poor countries *per se*, but rather the refusal of the advanced countries to transfer more resources to them to substitute for timber. Such transfers would increase the welfare off the worst-off while allowing them to deplete the natural resource at a slower rate. It is difficult to fault the poor countries for their use of the forest given the constraints that they face, and an equal-opportunity intergenerational ethic.

The equal-opportunity ethic, as I have here described it, ignores national boundaries. This is a contentious move, and is an instance of what is currently called the cosmopolitan view. A number of political philosophers who are quite egalitarian in the context of the single nation, are not so when it comes to the international community. (See, for example, Rawls(1999) and Nagel[6](2005).) My own view is that, in the next millennium, when people look back on our time, they will find the most egregious inequalities to be those in income per capita across nations. When even the most humanitarian countries transfer only about 1% of their national income to other countries in aid, we may say that human beings have scarcely begun to view themselves as citizens of a world, rather than of a nation-state.

The most challenging problem, then, is to study the issue of sustainability when we address not only inter-generational equity, but intra-generational and international equity as well.

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[6] Nagel (2005) advocates the view that the proper relations among nations at this time are ones governed by bargaining, not by justice. If and when a supranational state comes into being, then justice would become the appropriate currency.