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PREDETERMINED PRICES AND THE ALLOCATION OF SOCIAL RISKS

by

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and

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## ABSTRACT

We propose a Walrasian explanation for the existence of fixed prices, i.e., of trades in which either the price or the quantity exchanged do not reflect all publicly available information. Such trades result in a rigid price system that facilitates the sharing of social risks; they may also cause allocative distortions which tend to increase the equilibrium price of insurance above its actuarially fair level.

The simple overlapping generations model we consider here exhibits a tradeoff between risk sharing and allocative efficiency that is familiar from the incentives literature. We demonstrate that the market for non-contingent claims is active only when the insurance "gain" from it outweighs the "cost" of allocative distortions.

Fixed price equilibria are constrained optima in this essay, i.e., they cannot be dominated by an appropriately constrained central planner.

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## I. THE ROLE OF FIXED PRICES

Macroeconomics makes extensive use of rigidities in wages and prices, often as a means by which monetary policy can affect such real variables as output and employment. The major criticism of models which rely on wage/price rigidities [Barro-Grossman (1971), Bénassy (1975), Drèze (1975), Younès (1975), Malinvaud (1977)] is the lack of a coherent explanation of how and why prices are fixed in the first place.

The purpose of this paper is to propose a Walrasian explanation for the existence of "fix price" trades. Our model will not provide an explicit basis for the Keynesian models referred to above. Instead, we shall demonstrate that the efficient sharing of social risks may lead to trades occurring at a price that does not respond to realizations of random events. In this sense our model produces the desired rigidities without appealing to the rationing mechanism needed by standard Keynesian macroeconomics. We take our cue instead from the implicit contracts literature [Baily (1974), Azariadis (1975), and Holmstrom (1983)] in which rigid wages help allocate risks but the labor market clears in the appropriate commodity space.

Before we proceed further, it may prove instructive to clarify what we mean by the adjectives "rigid" or "fixed", as well as by their putative antonym "flexible." What distinguishes flexible from fixed prices? Is it that the former clear markets and, therefore, may change frequently while the latter do not clear markets and are reset only infrequently?

We believe that only in very special cases is the frequency of price change a good proxy for price flexibility. For example, a labor contract in which wage rates are any predetermined sequence of positive

(and generally unequal) real numbers possesses the fixed price property just as much as if wages were constant over time. This is not the case if future wages are tied to future values of a cost-of-living index.

In what follows, the term "fix price trades" is used to denote an exchange in which either the price or the quantity traded fail to reflect all the information that is publicly available when the relevant commodity is delivered. Prices prevailing in spot markets are flexible in that they convey new information instantaneously. We also view contingent claims markets as flexible trades since delivery is state-contingent. Non-contingent or unconditional claims are fix price trades since payment and delivery are both independent of the state of nature.

The central question we study in this essay is whether an equilibrium exists in which fix price trades, as described above, take place. As we shall see, unconditional claims provide a mechanism for the sharing of risks; at the same time, the exchange of these claims may produce allocative inefficiencies because some traders, say producers, precommit themselves. Fix price trades emerge in equilibrium when the risk-sharing "gains" from them exceed the productive inefficiency "costs".

In section II, we outline a simple overlapping generations model consisting of a large number of agents.<sup>1</sup> Risk averse individuals live two periods, consume in old age and are endowed in youth with a stochastic amount of a single consumption good. Risk neutral agents, on the other hand, possess a deterministic endowment of leisure and a stochastic technology which converts leisure into the consumption good; for these individuals the opportunity cost of leisure is therefore stochastic. In order to

limit the formation of contingent claims markets, we assume that the randomness affecting each individual constitutes private information. Without this assumption, complete markets would obviously be the dominant market structure.

At first, we restrict agents to trade in spot markets alone. Spot market equilibrium allocates efficiently the time of risk neutral persons between leisure and work, but does not help in the sharing of risks.

In order to permit risk sharing, we introduce in sections III and IV a market for unconditional claims. This market is active when the insurance gain outweighs the cost of the productive inefficiency caused by the implied obligation of claims suppliers to produce in all states of nature, even if their marginal product falls short of their marginal disutility of work.

Welfare properties of fix price equilibria are studied in section V. We show there that, if the unconditional claims market is active, the resulting equilibrium dominates, in expected utility, the pure spot-market equilibrium. Moreover, the former equilibrium supports a constrained-optimal allocation of resources, which becomes fully optimal if productive decisions are sufficiently unresponsive to new information.

The concluding section VI sums up and discusses some extensions of the work reported here.

## II. THE PURE SPOT ECONOMY

We shall be dealing with a specific model of exchange and production in a sequence economy [Samuelson (1958), Diamond (1965), Cass and Yaari (1966), Shell (1971), and Gale (1973)]. Time is discrete

and extends from one to infinity. There is a single, homogeneous, perishable consumption good and a single store of value, money. At the beginning of each period  $t=1,2,\dots$ , a generation of  $N+M$  individuals is born.

Each generation contains  $N$  members, indexed  $i = 1, 2, \dots, N$  whom we call consumers, and  $M$  speculators, indexed  $j = 1, 2, \dots, M$ . The former are risk averse, survive for two periods and consume only in old age. Consumers have identical preferences represented by von Neumann-Morgenstern utility indices  $u : \mathbb{R}_+ \rightarrow \mathbb{R}$  which are twice differentiable, strictly increasing and strictly concave. Shortly after birth, consumers receive a stochastic endowment of the consumption good  $\omega_t$ . This is a realization of a stationary random variable  $\tilde{\omega}$  which has a lower support  $\underline{\omega} > 0$  and mean one.

Speculators live only one period. They are endowed with a unit of divisible leisure time which they can devote to the production of the consumption good. Denoting labor input by  $n$  and output by  $y$ , we assume a stationary, stochastic constant-returns-to-scale technology of the form

$$y = \theta n.$$

The average product of labor is a stationary random variable,  $\tilde{\theta}$ , with mean one, lower support  $\underline{\theta} > 0$  and cumulative distribution function  $G(\theta)$ . We assume that  $\tilde{\theta}$  and  $\tilde{\omega}$  are independent.

Speculators preferences over consumption and labor are represented by  $c-kn$ . Here the parameter  $k$  represents the marginal disutility of work; speculators are then risk neutral.

The two central assumptions of this essay are that realizations of  $\tilde{\omega}$  and  $\tilde{\theta}$  are the private information of consumers and speculators, respectively, and that  $M\theta \geq N$ . The first one rules out a full array of contingent claims market. The second one ensures that our economy has a "large" capacity to absorb social risk; under the worst possible productivity conditions, risk-neutral agents have sufficient resources in the aggregate to supply every risk-averse person with one unit of consumption, i.e., with the expected value of their commodity endowment.

These assumptions are made in order to sharpen and simplify the tradeoff between risk sharing and productive efficiency. Speculators are natural trading partners of consumers in the sharing of risks; by assuming that only the former make production decisions, we are able to examine later on the allocative distortions created by the supply of unconditional claims.

Our economy begins at  $t = 1$  with an existing generation of  $N$  old consumers and a new generation of  $N$  consumers and  $M$  speculators. Each old consumer is endowed with a single unit of money; we assume that the aggregate endowment of fiat money remains unchanged over time.

Physical commodity exchange is exceedingly simple to characterize when no claims markets exist. Each period, after endowments are distributed to the young, a spot market opens: old consumers bring to this market their money balances; young consumers, who do not consume in youth, supply their commodity endowment,  $\omega_t$ , inelastically. Speculators see no reason to go to market as they live only one period.

A spot market equilibrium is characterized by a price function,  $p^*(\omega)$ , which clears the goods market in every state  $\omega$ . Equilibrium requires that the supply,  $\omega$ , of goods per young consumer equals the corresponding



demand,  $1/p$ , per old consumer, which is none other than the purchasing power of outstanding money balances. In other words,

$$p^*(\omega) = 1/\omega. \quad (1)$$

This equilibrium price function is obviously time-invariant.

In equilibrium, young consumers of generation  $t$  sell their endowment and earn money balances  $\omega_t p^*(\omega_t)$  which they give up in old age to buy goods at a price  $p^*(\omega_{t+1})$ . Hence the consumption of a member of generation  $t$  in period  $t+1$  is

$$c_{t+1} = \frac{\omega_t \cdot p^*(\omega_t)}{p^*(\omega_{t+1})} = \omega_{t+1}. \quad (2)$$

Old age consumption is risky because individuals consume in old age the endowment of the succeeding generation; the randomness of their own endowment does not influence their subsequent consumption - it is offset completely by price randomness when generations consist of identical persons.

Risk neutral agents are autarkic but still face an allocative decision: after observing their productivity shock,  $\theta$ , speculators choose  $n^*$  to maximize utility,  $(\theta-k)n$ . Clearly  $n^*$  will satisfy conditions of productive efficiency, i.e.

$$n^*(\theta) = \begin{cases} 1 & \text{for } \theta > k \\ 0 & \text{for } \theta < k \\ \text{indeterminate} & \text{for } \theta = k. \end{cases} \quad (3)$$

That is, speculators work if, and only if, the marginal product of labor

is no lower than the marginal disutility of work.

In sum, a spot market equilibrium is completely described by the equilibrium price function,  $p^*(\omega)$ ; speculators make efficient allocative decisions, but all of the endowment risk is borne by risk-averse consumers. The market structure we consider next facilitates the re-allocation of this risk to risk-neutral speculators.

### III. THE FIX PRICE ECONOMY: INDIVIDUALS

We add now to our model a market for unconditional claims on consumption: this market opens for trading at the beginning of each period, before the state of nature  $(\omega, \theta)$  is realized; therefore, neither the quantities exchanged nor the price of claims reflect that state in any way. Production takes place, goods are delivered and payments made after the state occurs; the spot market opens after the completion of all transactions associated with the claims market.

The timing of these transactions places certain restrictions on individual trades. Since the delivery of claims occurs prior to the opening of spot markets, young consumers can only sell claims; they are not endowed with balances which are needed to finance purchases. In addition, bankruptcy is disallowed: speculators who sell claims must work in every state of nature in order to honor their commitments.<sup>2</sup> This is a source of an allocative inefficiency since speculators who choose to sell claims must work even if  $\theta > k$ .

The definition of equilibrium now becomes a bit more complicated. Let  $\pi_t$  be the money price of an unconditional claim on one unit of the consumption good at the end of period  $t$  and  $p_t(\omega)$  be the spot price function. We denote by  $q_t$  the number of claims sold by a speculator;

by  $Q_t$  the number of claims bought by an old consumer; and by  $z_t$  the number of claims sold by a young consumer.

From the previous section we would expect that old consumers may desire to purchase claims from speculators to avoid the risk  $\tilde{\omega}_{t+1}$ . Young consumers should continue to sell their endowment in the spot market since their ultimate consumption was independent of  $\omega_t$ . Speculators will supply claims but only at a price which compensates them on average for the productive inefficiency, that is, for having to work in unfavorable states of nature. We formalize this intuition below.

(a) Speculators

Consider first the expected utility,  $w$ , of a speculator who sells  $q$  claims at a money price  $\pi$ , i.e., we define

$$w(\pi, q) = qE_{\omega}(\pi/p(\omega) - 1) + q\int_{\underline{\theta}}^k (1 - k/\theta)dG(\theta) + \int_k^{\bar{\theta}} (\theta - k)dG(\theta) \quad (4)$$

The first term on the right-hand side of eq. (4) is the consumption value of the speculator's expected profit from intermediation: the revenue  $\pi q$  he collects from selling claims is spent subsequently to purchase goods in the spot market at a price  $p(\omega)$ . The second and third terms correspond to the speculator's direct productive activity. The second term is a measure of the allocative distortion introduced by fixed prices: it refers to the states  $\theta \leq k$  in which the speculator is committed to producing  $q$  units of output even though the marginal disutility of labor exceeds its marginal product; in such states speculators choose to work  $n = q/\theta$  in order to minimize their utility loss (see figure 1). The final term reflects direct activity when  $\theta > k$ ; then speculators set  $n=1$  as in eq.(3).

In order to characterize the equilibrium with unconditional claims, we

define a supply price for these claims,  $\pi^S(k)$ , such that for a given  $k$ , the value of  $q$  that maximizes  $w(q, \pi)$  over the interval  $[0, \underline{\theta}]$  is indeterminate. Note that  $q \leq \underline{\theta}$  ensures the solvency of speculators in all states of nature. We define first the actuarially fair claims price  $1/\pi^* = E [1/p(\omega)]$ . Then, from (4), we have

$$\pi^S(k)/\pi^* = \begin{cases} 1, & k < \underline{\theta} \\ 1 + \int_{\underline{\theta}}^k (k/\theta - 1) dG(\theta), & k \in [\underline{\theta}, \bar{\theta}] \\ kE(1/\theta), & k > \bar{\theta} \end{cases} \quad (5)$$

The supply of unconditional claims by speculators (see also Fig. 2) is therefore  $q = 0$  if the competitive price,  $\pi$ , is less than  $\pi^S(k)$ ; it is indeterminate if  $\pi = \pi^S(k)$ ; and it is  $\underline{\theta}$  if  $\pi > \pi^S(k)$ . From equation (5) one obtains directly

Proposition 1:  $\pi^S(k)$  is a continuous, strictly increasing function of  $k$  with the property that  $\pi^S(\underline{\theta}) = \pi^*$ , and  $\pi^S(k) > \pi^*$  for  $k > \underline{\theta}$ .

The supply price  $\pi^S(k)$  summarizes completely the allocative inefficiencies involved in fix price trades. If  $k \geq \underline{\theta}$ , speculators will be forced to work in states where  $k > \theta$ ; to compensate for this inefficiency, Proposition 1 tells us, the supply price of claims will increase above the value  $\pi^*$  which corresponds to actuarially fair insurance against spot price risks. Therefore we can view  $\pi^S(k) - \pi^*$  as the measure of an allocative distortion parametrized by  $k$ .

(b) Old Consumers

On the demand side of the claims market, we define first a price

$\pi^d$  at which risk averse old consumers are indifferent between participating in the claims market and staying out of it. Old consumers who purchase  $Q$  claims at a price  $\pi$  end up consuming

$$c = Q + \frac{h - Q\pi}{p(\omega)}, \quad (6)$$

where  $h$  denotes nominal balances carried over from youth. With the aggregate money stock held constant at  $N$  and identical consumers, each consumer begins old age with an equilibrium amount of one unit of money. The first term in (6) is claims purchased; the second one equals spot market purchases made with money balances not spent on claims.

We can now determine  $\pi^d(h)$  as the minimal claims price for which  $Q = 0$  maximizes the expected utility of consumption subject to (6), and to  $Q$  being in the interval  $[0, h/\pi]$ . Therefore,  $\pi^d$  solves

$$E_{\omega} [(1 - \pi^d/p(\omega))u'(h/p(\omega))] = 0. \quad (7)$$

Obviously,  $\pi^d(h)$  depends on the curvature of  $u$ ; if consumers were to become more risk-averse, one would expect  $\pi^d$  to rise for each  $h$  because individuals would be willing to pay more for the first unit of insurance services. We show in the appendix that<sup>3</sup>

Proposition 2A : The maximal price,  $\pi^d(h)$ , consumers will pay for unconditional claims is greater than or equal to  $1/E[1/p(\omega)]$ , and increases with their index of absolute risk aversion.

This price is a convenient measure of attitudes toward risk, just as

$\pi^s(k)$  is an index of misallocated effort.

Fix-price claims are a safe asset for old consumers, and the demand for it is simply the solution to a standard portfolio problem of maximizing expected utility over  $Q \in [0, h/\pi]$ , subject to (6) and non-negativity. From the solution, one readily demonstrates the following (see also Figure 3)

Proposition 2B: The demand  $Q(\pi, h)$  for fix-price claims by an individual old consumer carrying over nominal balances  $h$  from youth is a continuous, single-valued function with the properties that  $Q(\pi, h) = 0$  for  $\pi > \pi^d(h)$ ;  $Q(\pi, h) < h/\pi$  for  $\pi > 1/E(1/p)$ ;  $Q(\pi, h) = h/\pi$  for  $\pi \leq 1/E(1/p)$ ;  $Q$  is decreasing in  $\pi$  at  $\pi = \pi^d(h)$ .

(c) Young Consumers

How many claims young risk averters choose to sell is the outcome of a dynamic programming problem: if they carry nominal balances  $h$  into old age, their expected-utility maximizing choice is to purchase  $Q(\pi, h)$  claims. The value of  $h$  itself is determined in youth with full knowledge of the function  $Q$ ; young consumers selling  $z \geq 0$  claims in youth accumulate money balances

$$h(z) = \pi z + p(\omega)(\omega - z) , \quad (8)$$

which will support old-age consumption

$$c(z, \omega, \omega') = Q(\pi, h(z)) + [1/p(\omega')] [h(z) - \pi Q(\pi, h(z))] \quad (9)$$

Figure 1: Supply of Labor by Speculators

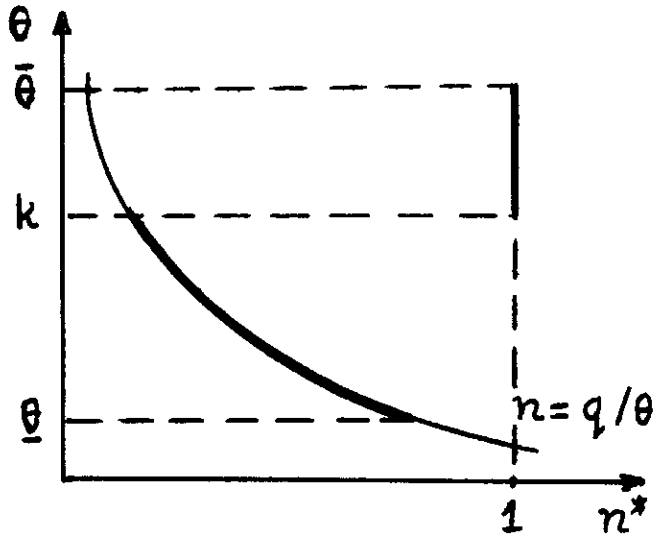


Figure 2: Supply of Claims by Speculators

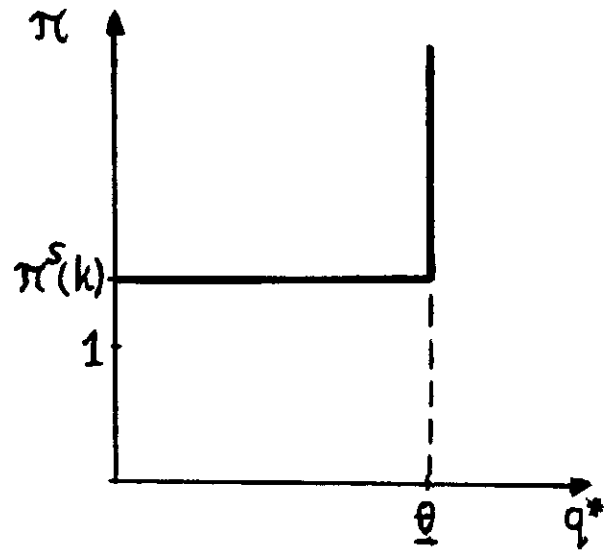


Figure 3: Demand for Claims by Old Consumers

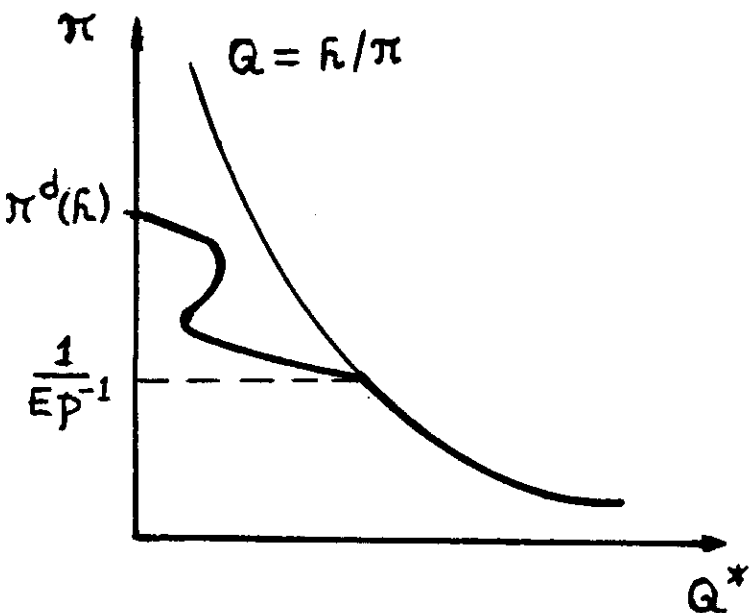
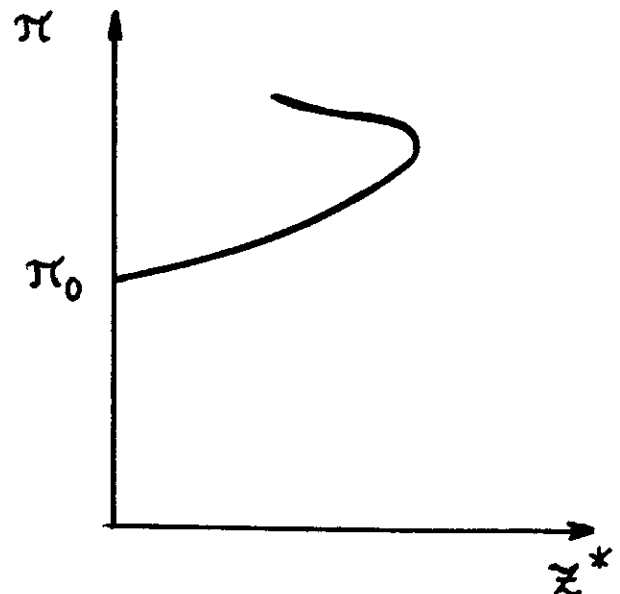


Figure 4: Supply of Claims by Young Consumers



Primed variables are endowment realizations in old age: our risk averter faces a spot price  $p(\omega)$  in youth and  $p(\omega')$  in old age, where  $\tilde{\omega}$  and  $\tilde{\omega}'$  are independent random variables.

From (8) and (9), we define the expected utility

$$V(z) = E_{\omega, \omega'} \{ u[c(z, \omega, \omega')] \}. \quad (10)$$

Given the number  $\pi$  and the function  $p$ , young consumers maximize  $V(z)$  over  $z \in [0, \underline{\omega}]$ , so that solvency is preserved in all states of nature. We state the essential property of the supply schedule  $z(\pi)$  in the following manner (see also Figure 4).

Proposition 2C: Suppose that  $1/E(1/p(\omega)) < \pi < \pi^d(h(z))$  for all  $\omega$  and any  $z \in [0, \underline{\omega}]$ . Then, young risk-aversers supply a positive number of fix-price claims if, and only if,  $\pi > \pi_0$  where

$$\pi_0 = \frac{E_{\omega, \omega'} [p(\omega)/p(\omega')] u'[c(0, \omega, \omega')]}{E_{\omega, \omega'} [1/p(\omega')] u'[c(0, \omega, \omega')]}$$

#### IV. THE FIX PRICE ECONOMY: MARKET EQUILIBRIUM

A stationary equilibrium in the economy of the preceding section is a spot price function  $\hat{p}(\omega)$ , a claims price  $\hat{\pi}$ , and trading plans  $\hat{q}$  for speculators,  $\hat{Q}$  for old consumers, and  $\hat{z}$  for young consumers, with the properties that: (i) given  $\pi$  and  $p(\omega)$ , the trading plan of each



individual maximizes his/her expected utility conditional on all available information; (ii) the consumption of every individual is non-negative in every state; and, (iii) both the spot market and the claims market clear. Spot market clearing means that, for all  $\omega \in [\underline{\omega}, \bar{\omega}]$ ,

$$\hat{p}(\omega - \hat{z}) = 1 - \pi \hat{z}. \quad (12)$$

The left hand side of (12) is the money value of goods supplied in the spot market, and the right-hand side is the corresponding demand. The claims market clears if

$$N\hat{z} + M\hat{q} = N\hat{Q}. \quad (13)$$

To describe equilibrium more fully, we consider first the case of  $k \leq \underline{\theta}$ , and next the case for  $k > \underline{\theta}$ . The former is a benchmark (first-best) equilibrium with complete insurance and no allocative distortions.

Proposition 3: If  $k \leq \underline{\theta}$ , there is a unique equilibrium satisfying

- (i)  $\hat{p}(\omega) = 1/\omega, \forall \omega$
- (ii)  $\hat{\pi} = \pi^S(\underline{\theta}) = 1$
- (iii)  $\hat{Q} = 1$
- (iv)  $\hat{z} = 0$
- (v)  $\hat{q} = N/M \leq \underline{\theta}$ .

(Proof) We assume that the price system is  $\hat{\pi} = 1$  and  $\hat{p}(\omega) = 1/\omega$ , and demonstrate that individual decisions reproduce it in equilibrium. Then we show that the equilibrium is unique.

From the definition of  $\pi^S(k)$ , we have  $\pi^S(\underline{\theta}) = \pi^* = 1/E[1/p(\omega)]$ .

Therefore speculators are indifferent with respect to the choice of  $q$ , provided the bankruptcy condition  $q \leq \underline{\theta}$  is met. Risk averse consumers will spend all their money balances in the claims market to obtain perfect insurance since  $1/\hat{\pi} = E(1/p)$ . Hence  $\hat{Q} = 1$ .

Given this decision in old age and  $\hat{\pi} = 1$ , we need to show that  $z = 0$  maximizes the function  $V(z)$  defined in eq. (10) or, by Proposition 2C, that  $\pi_0 > 1$ . Note that, at  $z = 0$ , we have  $h(0) = \omega p(\omega) = 1$  for all  $\omega$ ; hence, given prices,  $c(0, \omega, \omega') = 1$  for all  $(\omega, \omega')$ , and  $\pi_0 = E p(\omega) = E(1/\omega)$  which exceeds one by Jensen's inequality. We recall our assumption that  $E\omega = 1$ .

Given  $\hat{Q}$ , and  $\hat{z}$ , it is obvious from equations (12) and (13) that both markets clear if  $p(\omega) = 1/\omega$  for all  $\omega$ , and  $\hat{q} = N/M$ . The value of  $\hat{q}$  is consistent with non-negative consumption for speculators in every state of nature, for we have assumed that  $\underline{\theta}M \geq N$ .

To see that the equilibrium is unique, suppose that  $1/\pi > E(1/p)$ . From (4), with  $k = \underline{\theta}$ , it is clear that  $\hat{q} = 0$ , while old consumers would still demand claims thus violating (13). If  $1/\pi < E(1/p)$ , speculators will supply  $q = \underline{\theta}$  - i.e. they set  $q$  at its maximal level subject to their bankruptcy constraint. Old consumers will now demand fewer claims and therefore (13) will again be violated. Hence, conditions (i) through (v) describe the unique equilibrium.

# Equilibrium in the Claims Market

Figure 5a: First-Best

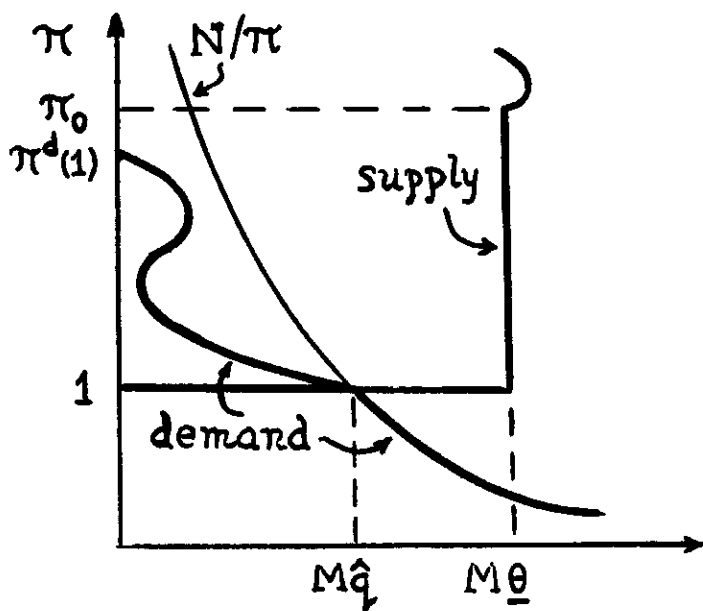


Figure 5b: Second-Best

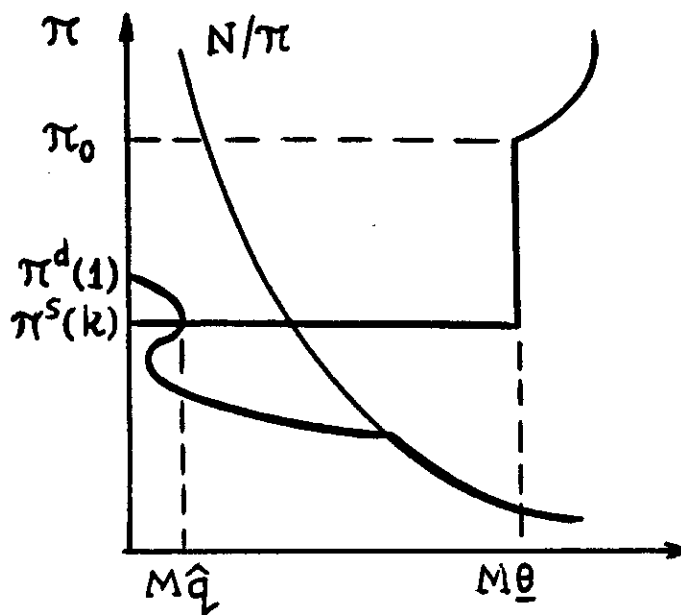
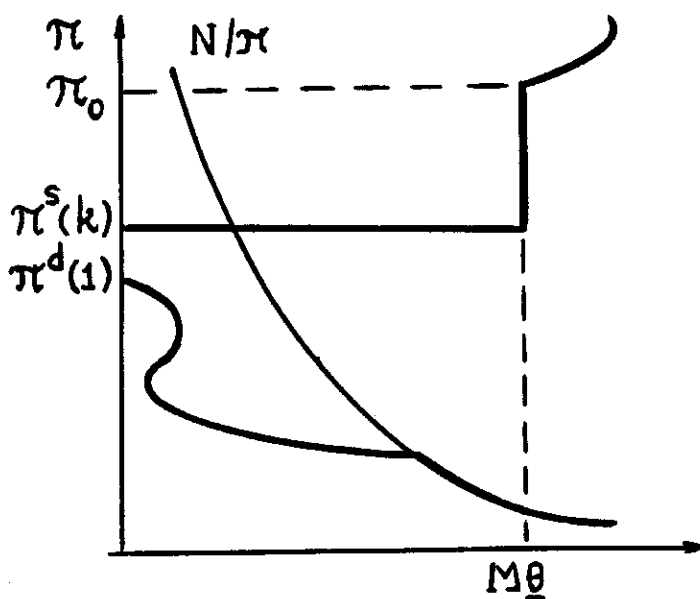


Figure 6: Inactive Market



We depict in Figure 5a the claims side of this equilibrium; we take as given the spot price function, and set

$$\pi^d(1) = Eu'(\omega)/E[\omega u'(\omega)] \quad (15)$$

and

$$\pi_0 = E(1/\omega). \quad (16)$$

Here consumers insure themselves completely in old age; they sell zero claims in youth, purchase claims at a price  $\hat{\pi} = 1$  in old age, and consume one unit of goods.

Young consumers would actually choose to purchase claims, i.e. set  $z < 0$ , if they could. At  $z = 0$ , we know that consumption is independent of  $\omega$  so that the young are locally risk neutral. From Jensen's inequality,  $E(p(\omega)) > \pi$ ; hence the young face a favorable bet as claims buyers, which they would take advantage of if they possessed any money balances.

As we shall see formally in the next section, the equilibrium is pareto optimal if  $k \leq \underline{\theta}$  as all risks are borne by speculators. This would not be possible unless  $M\underline{\theta} \geq N$ ; speculators could not offer actuarially fair insurance because of the possibility they might go bankrupt in adverse states of nature. If speculators are able to produce enough goods in the worst state  $\underline{\theta}$ , their intermediary activity will yield an actuarially fair price for claims, and complete insurance for consumers.

We consider now the case  $k \geq \underline{\theta}$ . The arbitrage activities of the speculators will ensure that  $\pi$  exceeds the actuarially fair price  $\pi^*$ . Hence the claims price consumers pay has risen to compensate speculators for the allocative distortion.<sup>4</sup> Whether or not consumers will demand

insurance at this higher price will depend on the magnitude of  $\pi^d$  relative to  $\pi^s(k)$ . We say that the claims market is active if trades occur in that market, and inactive otherwise.

Before we discuss the existence of equilibrium for  $k > \underline{\theta}$ , we recall from Proposition 3 that young consumers will set  $z = 0$  if  $\pi \leq \pi_0$ . Since the supply price of claims is, by Proposition 1, a continuous, increasing function of  $k$  that varies in the interval  $[\pi^*, \infty)$  as  $k$  changes from  $\underline{\theta}$  to infinity, we can always identify a value of  $k$ , say  $\hat{k}$ , such that  $\pi^s(\hat{k}) = E(1/\omega)$ . Then, for  $k \in [0, \hat{k}]$ , we know that young consumers set  $z = 0$ , selling their entire endowment in the spot market.

Proposition 4: If  $\hat{k} > k > \underline{\theta}$ , every equilibrium with an active claims market satisfies

- (i)  $\hat{\pi} = \pi^s(k)$
- (ii)  $\hat{p}(\omega) = 1/\omega$
- (iii)  $\hat{z} = 0$
- (iv)  $\hat{q} = NQ/M$ , and
- (v)  $\hat{Q}$  maximizes  $E u(\omega(1-\hat{\pi}Q) + Q)$   
subject to  $0 \leq Q \leq 1/\hat{\pi}$ .

(Proof). The proof is quite similar to Proposition 3. We first take as given the price system from (i) and (ii). Then, if  $k > \hat{k}$ ,  $z = 0$  as discussed above. With  $\pi = \pi^s(k)$ ,  $\hat{q}$  is indeterminate and can be set to clear the claims market, where demand by old consumers is determined by (v). Finally, as in (12),  $p(\omega) = 1/\omega$  clears the spot market. ||.

For  $k \in (\underline{\theta}, \hat{k})$ , equilibria with an active claims market are very similar to those of Proposition 3; see also Figure 5b. The key difference is that, for  $k \geq \underline{\theta}$ , insurance is no longer actuarially fair; old consumers choose to become only partially insured even though complete insurance is within their budget sets, corresponding to  $Q = 1/\hat{\pi}$ . The working decisions of speculators are simple:  $n(\theta) = \hat{q}/\theta = N\hat{Q}/(M\theta)$  if  $\theta > k$ , = 1 otherwise.

It is now a very simple matter to determine whether or not the claims market is working: if it is, the equilibrium looks like Figure 5 with  $\pi^d(1) \geq \pi^s(k)$ ; if it is not, the equilibrium is of the type shown in Figure 6, with  $\pi^d(1) < \pi^s(k)$ . We have thus demonstrated

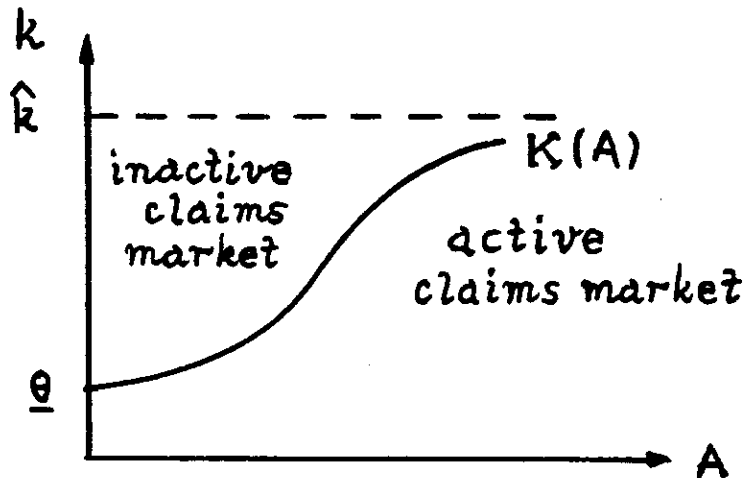
Proposition 5: Suppose that our economy has a "large" capacity to absorb risk ( $M\underline{\theta} \geq N$ ), and that the marginal value of time is not "large" relative to the expected marginal product of labor ( $k < \hat{k}$ )<sup>5</sup>. Then the claims market is active if, and only if,  $\pi^d(1) \geq \pi^s(k)$ .

Whether or not trading takes place at fixed prices is determined endogenously in our model. We know, from Proposition 1, that  $\pi^s(k)$  is an increasing function of  $k$  (with  $\pi^s = 1$ ); and from Proposition 2 that  $\pi^d(1)$  is an increasing function of  $A$ , i.e., of the consumer's Arrow-Pratt index of absolute risk aversion.

We may, therefore, define a continuous, increasing function  $K: \mathbb{R}_+ \rightarrow [\underline{\theta}, \hat{k}]$  such that  $k = K(A)$  solves  $\pi^d = \pi^s$ . We draw this function in Figure 7, which shows that fix-price trades will occur if, and only if,  $k \leq K(A)$ ; then consumers are sufficiently risk averse to benefit from such

trades even after they compensate speculators for misallocating their productive effort in adverse states.

Figure 7: Economies Supporting Fix Price Trades



#### V. WELFARE PROPERTIES OF FIX PRICE EQUILIBRIA

We investigate in this section the normative properties of the equilibria that we studied in the previous one. We are specifically interested in two questions: What, if anything, do fix price trades contribute to economic welfare? And, how close does an equilibrium with fixed prices come to some appropriately defined optimal allocation of resources ?

For the first question we compare the equilibrium allocations of a pure spot economy, in which fix price trading is arbitrarily prohibited,

with those of a fix price economy, which combines a spot market with one for unconditional claims. We have already studied these economies - the former in section II, the latter in section IV. Whenever fix price trades occur, they do so at an equilibrium price  $\pi^S(k)$  which leaves speculators indifferent, in an expected utility sense, between participation in the claims market and absence from it. Whatever that market contributes to social welfare, therefore, can be properly measured by its effect on the expected utility of risk averse consumers. This effect is clearly beneficial if  $k \leq \underline{\theta}$  for then old risk averters consume with certainty the expected value of their endowment in youth. We focus below on the case  $k > \underline{\theta}$ .

In the spot market economy, risk averse old individuals consume an amount  $c^S = \omega$  which corresponds to the endowment of risk averse young individuals ; we suppress time subscripts, and denote by  $V^S = Eu(\omega)$  the expected utility of a typical risk averter in this economy.

If  $\pi^d(1) \geq \pi^S(k)$  , then the claims market is active in the fix price economy; old risk averters consume  $c_A^F = \omega(1-\hat{\pi}\hat{Q}) + \hat{Q}$ , which yields expected utility  $V_A^F(\hat{Q}) = Eu(c_A^F)$  . We note from Proposition 4 that  $\hat{Q}$  maximizes  $V_A^F(Q)$  over the set  $Q \in [0, 1/\hat{\pi}]$  , and that  $V_A^F(0) = V_A^S$  since  $c_A^F = \omega$  for  $Q = 0$ . However, it follows immediately from Proposition 4 that  $\hat{Q} > 0$  and, hence,  $V_A^F(\hat{Q}) > V_A^S$  for all strictly concave  $u$ .

We have then shown that fix price trades do contribute to economic welfare. More formally, we have

Proposition 6: Suppose  $k \leq \min \{\hat{k}, K(A)\}$ . Then the equilibrium allocation in the fix price economy is strictly superior, in the expected utility sense, to that of the pure spot price economy.<sup>6</sup>



To study the efficiency of fix price equilibria, we compare the underlying equilibrium allocation with that achievable by a central planner. At first, we endow the planner with perfect information about the realizations of the state of nature  $(\omega, \theta)$ , and with the ability to transfer in each period resources from any individual to any other in any state.

Constrained only by the productive capacity of the economy, this planner has obvious advantages in information and economic coordination over every market structure we have considered thus far: in the economies of sections II through IV, asymmetric information rules out a complete array of contingent claims markets and thus hampers the reallocation of resources over states of nature.

It is reasonable then to expect that an omniscient and omnipotent planner could improve, in the expected utility sense, on the allocations supported as equilibria in a fix price economy. Formally the planner will choose three functions  $c_A^*(\omega, \theta)$ ,  $c_N^*(\omega, \theta)$  and  $n^*(\omega, \theta)$  which correspond to consumption for old risk averters, consumption for speculators and work for speculators, to solve the following problem:

$$\begin{aligned}
 (P^*) \quad & \text{maximize } E_{\omega, \theta} [\lambda N u(c_A) + (1-\lambda) M(c_N - kn)] \\
 & \text{subject to:} \\
 & N c_A + M c_N \leq N \omega + M \theta n \quad \forall (\omega, \theta) \\
 & 0 \leq n \leq 1, c_A \geq 0, c_N \geq 0 \quad \forall (\omega, \theta)
 \end{aligned} \tag{17}$$

Here  $\lambda$  is an arbitrary number in the interval  $[0, 1]$  which parametrizes the expected utility possibility frontier. Any solution to  $(P^*)$ , termed a full-information optimum, satisfies the productive

efficiency requirements of equation (3) ; if the solution is an interior one, it also requires that all risk be borne by speculators. We thus have

Proposition 7: The equilibrium in the fix price economy is a full-information optimum if , and only if,  $k \leq \underline{\theta}$ .

(Proof) Directly from Propositions 3 and 4. If  $k \leq \underline{\theta}$ , then both time and risk are allocated efficiently; if  $k > \underline{\theta}$  neither is.  $\parallel$ .

A more relevant way to judge the efficiency of equilibria involving fixed prices is to compare them with the allocations achieved by a central planner who is constrained "as much as" the individuals in our model. Hart (1975) and Grossman (1977), for instance, argue that we must restrict the planner's ability to move resources across states so that it parallels the corresponding ability of the market structure we are evaluating. Moreover, the planner's actions must be consistent with the timing of our economy in which the unconditional claims market opens before the state  $(\hat{\omega}, \hat{\theta})$  is realized. This implies that central planning should proceed sequentially: certain allocative decisions are made ex ante, that is, before the state of nature is known; others are made ex post and may be conditioned on the state that has occurred.

A convenient way to formalize such imperfect coordination in central plans is to suppose there is a great number of planners. One of them, the ex-ante planner, allocates resources, in a way that parallels a market for unconditional claims: he picks a number  $q$  in the interval  $[0, \underline{\theta}]$  which corresponds to the minimal output each speculator is directed to transfer

to the hands of old risk averters; assuming the latter are treated equally, each of them is guaranteed a minimal level  $Mq/N$  of consumption.

For each state of nature  $(\omega, \theta)$  we appoint an ex-post planner whose allocative procedure parallels the spot market. Given  $q$  and the state, each ex post planner picks two numbers  $h(\omega, \theta|q)$  and  $c(\omega, \theta|q)$  representing, respectively, the amount of work each speculator must perform after satisfying the instructions of the ex-ante planner, and the amount of consumption each old consumer is assigned beyond the  $(M/N)q$  units set aside for him by the ex-ante planner.

With these instructions, old risk averters consume in state  $(\omega, \theta)$  an amount

$$c_A = (M/N)q + c(\omega, \theta|q) ; \quad (18)$$

speculators, on the other hand, work

$$n = q/\theta + h(\omega, \theta|q), \quad (19)$$

and share equally in consuming the aggregate amount of goods left over after risk averters have claimed their allotments. Using equations (18) and (19) in conjunction the aggregate resource constraint embodied in equation (17), we find that the consumption of each speculator in state  $(\omega, \theta)$  is

$$c_N = \theta h(\omega, \theta|q) + (N/M)[\omega - c(\omega, \theta|q)] . \quad (20)$$

Following Grossman (1977), we suppose that each planner seeks to achieve the best possible allocation of resources with the instruments he

has, taking as given the actions of all other planners, observing the resource constraints of the economy as well as non-negativity constraints on the consumption of every agent in each state. The outcome is a constrained optimum which Grossman calls a social Nash optimum.

Given  $q \in [0, \underline{\theta}]$ , and some arbitrary  $\lambda \in [0, 1]$ , ex-post planner  $(\omega, \theta)$  chooses the numbers  $c$  and  $h$  to solve the following problem:

$$\begin{aligned}
 (P_{\omega, \theta, q}) \quad & \text{maximize} \quad \lambda N u(c_A) + (1-\lambda) M(c_N - kn) \\
 & \text{subject to:} \\
 & \text{equations (18), (19) and (20)} \\
 & 0 \leq n \leq 1 \quad \quad \quad V(\omega, \theta) \\
 & c \geq 0, h \geq 0, c_N \geq 0 \quad \quad \quad V(\omega, \theta)
 \end{aligned}$$

The solution here is that  $h(\omega, \theta | q) = 0$  if  $\theta < k$ ,  $= 1 - q | \theta$  otherwise; and  $c(\omega, \theta | q)$  is any function that satisfies the constraints. In other words, planner  $(\omega, \theta)$  tells speculators to work no more than instructed by the ex-ante planner in adverse states of productivity, and as much as they are physically able in favorable states of productivity. But the ex-post planner cannot increase the consumption of any person without decreasing that of another.

We conclude that no ex-post planner can improve on the equilibrium allocations of a fix price economy, unless the ex-ante planner orders each speculator to subsidize old risk averters by an amount different from what is specified in Propositions 3 and 4, that is, different from  $N/M$  if  $k \leq \underline{\theta}$  or from  $(N/M)\hat{Q}$  if  $k \in (\underline{\theta}, \hat{k})$ .

To see whether the ex-ante planner can improve on the fix price economy, let us suppose first that ex-post planners choose to subsidize

each old consumer with the function

$$c(\omega, \theta | q) = \omega [1 - (M/N)\hat{\pi}\hat{q}] \quad (21)$$

which corresponds to the consumer's spot market purchases in the fix-price economy. As before,  $\hat{\pi}$  is the equilibrium price of unconditional claims, and  $\hat{q}$  is the equilibrium supply of these per speculator. We note that this function satisfies all the constraints of the problem  $(P_{\omega, \theta, q})$ .

Given equation (21) and some selection of  $q$  (not necessarily equaling  $\hat{q}$ ) by the ex-ante planner, consumption bundles are given by equations (18) through (20), viz.,

$$c_A = (M/N)q + \omega [1 - (M/N)\hat{\pi}\hat{q}] \quad (22)$$

$$n = q/\theta \text{ if } \theta < k, = 1 \text{ otherwise} \quad (23)$$

$$c_N = \hat{\omega}\hat{\pi}\hat{q} \text{ if } \theta < k, = \hat{\omega}\hat{\pi}\hat{q} + (\theta - q) \text{ otherwise} \quad (24)$$

It is straightforward, and rather tedious, to demonstrate now that the ex-ante planner can do no better than the fix price economy: if he were to maximize the expected utility of consumption for risk averters subject to (22), (23), (24) and to giving speculators as much expected utility as they obtain from the fix price economy, he would set  $q = \hat{q}$ , i.e., duplicate exactly the market allocation. We restate the outcome in

Proposition 8: The equilibrium of the fix price economy is a social Nash optimum.

This result should not surprise. Ex-post planning cannot improve fix price equilibrium because it cannot reallocate over states of nature the consumption of a single physical commodity; ex-ante planning does not help either because the fix price economy balances efficiently the gains from insurance against the losses from the misallocation of effort.

## VI. SUMMARY AND EXTENSIONS

We have examined the role of fix price trades in the sharing of real risks. Such trades confer the insurance benefits of an additional securities market and, at the same time, cause other allocative distortions as they reduce the responsiveness of trading plans to new information.

In the specific model of this essay, insurance benefits and allocative costs are correctly reflected in the equilibrium price of these trades: the fix price economy generally supports constrained optimal allocations of resources, and yields a full optimum when distortions are absent.

There are two possible extensions of our work we do not consider here - contingent claims and changes over time in the aggregate money endowment. Since the state of nature  $(\omega, \theta)$  is never observed publicly, a full array of contingent claims markets cannot be established. However, anyone who observes the spot price and knows that  $p(\omega) = 1/\omega$  may infer precisely what  $\tilde{\omega}$  was realized. It does seem then that individuals could trade claims contingent on the spot price, a possibility discussed by Svensson (1981).

This possibility does not bear fruit in our model as the trade of price-contingent claims tends to destroy the very information on which

it is conditioned.<sup>7</sup> This is easiest to understand if one realizes that the purpose of price-contingent claims is to mimic the equilibrium supported by claims directly contingent on  $\omega$ . If  $\omega$  itself were directly in the public domain, a full-information optimum could be achieved by supplementing an ordinary spot market with a complement of Arrow-Debreu securities [see Arrow (1954)], that is, with  $\omega$ -contingent inside "money". In the equilibrium of this "monetary" Arrow-Debreu economy, consumers insure themselves fully by arranging to hold a positive amount of securities when  $\omega$  is small, a negative amount when  $\omega$  is large; the spot price turns out to be completely deterministic, conveying no information whatsoever.

Another natural extension of the work reported here would be to allow changes in the aggregate money endowment. Assuming for the moment that any such change occurs in proportion to existing holdings<sup>8</sup>, it is easy to see that the equilibria defined in Propositions 3 and 4 prevail essentially intact if changes in monetary aggregates are public knowledge.

For instance, deterministic variations in the stock of money are equivalent to redefining the unit of account each period. Prices would vary proportionally with money in both markets while quantities remained invariant; the price of unconditional claims, in particular, would become predetermined rather than fixed for ever.

A similar outcome will emerge if money is a random variable whose realizations are public knowledge. Then a market for claims conditional on the money stock will internalize all nominal disturbances in its price structure, leaving quantities again unchanged in equilibrium.

The interesting case that deserves attention, it seems, is the one studied by Lucas (1972): What equilibria obtain if nominal disturbances

are not in the public domain? Then, of course, the spot price is an imperfect signal that conveys partial information on both  $\omega$  and the stock of money. It is no longer obvious that this signal will be extinguished by the trading of price-contingent claims; if it is not, we shall have an interesting equilibrium to investigate. Price contingencies are, after all, a form of indexation and a pervasive empirical regularity of considerable importance in macroeconomics.<sup>9</sup>



APPENDIX: PROOF OF PROPOSITION 2A

(Proof.) It is obvious that  $\pi^d > E(1/p)$ . Now, for  $\pi^d$  to be the maximal price consumers will pay for insurance, it must be the case that consumer demand equals zero at  $\pi > \pi^d$ . We denote by  $Q(\pi)$  the consumer demand function for claims given the consumer's money balances. It is easy to show (see Proposition 2B) that  $Q(\pi)$  is single-valued, continuous, and decreasing at  $\pi^d$ . Hence  $Q(\pi) = 0$  for  $\pi > \pi^d$ .

To show that  $\pi^d$  increases with risk aversion, assume  $v(x) = g[u(x)]$  where  $g$  is increasing and concave. Assuming that  $v$  is a concave transformation of  $u$  means that the preferences represented by  $v$  are more risk averse than those of  $u$  by the Arrow-Pratt measure of absolute risk aversion; see Pratt (1964) and Ross (1981).

We replace  $(1/p(\omega))$  by  $\hat{x}$  to simplify the notation; furthermore, we denote by  $\pi_v^d$  the claims price for the  $v(x)$  function and  $\pi_u^d$  the price for the  $u(x)$  function. We want to show that  $\pi_v^d > \pi_u^d$  when  $v$  is a concave transformation of  $u$ .

Setting  $h=1$  in (7) we obtain

$$\pi_v^d = E(g' u') / E(xg' u') \quad (A1)$$

and

$$\pi_u^d = E u' / E(xu') \quad (A2)$$

Defining  $\hat{x} = 1/\pi_u^d$ , we wish to show that  $\hat{x} > 1/\pi_v^d$ . This is equivalent to proving that

$$E[(x-\hat{x})g'(u(x))u'(x)] < 0. \quad (A3)$$

From the concavity of  $g(\cdot)$ ,  $x < \hat{x}$  is equivalent to  $g'(u(x)) > g'(u(\hat{x}))$ . Hence

$$E[(x-\hat{x})g'(u(x))u'(x)] < g'(u(x))E[(x-\hat{x})u'(x)]. \quad (A4)$$

From the definition of  $\hat{x}$ , we have  $E[(x-\hat{x})u'(x)] = 0$  so that (A4) implies (A3) ||.

FOOTNOTES

1. The overlapping generations model is not essential here. Similar results would obtain in other intertemporal models with incomplete markets. See Danthine (1978) for a partial equilibrium approach to the hedging and information transfer properties of futures markets.
2. This would not necessarily be true if speculators lived for many periods and held inventories. In addition, if speculators could purchase goods in the spot market prior to meeting their claims commitments, this distortion would be reduced. However, actuarially fair insurance would still not be feasible so that our assumptions on the timing of trades only serve to highlight the distortions created by the provision of non-contingent claims.
3. We are grateful to Richard Kihlstrom for assistance with the proof.
4. A similar price increase would occur if the economy faced a "shortage" of risk-bearing capacity, i.e., if  $M\theta < N$ .
5. If one of these assumptions fails, the equilibrium price of claims is no longer equal to  $\pi^S(k)$ . In particular, if  $\theta > k > \hat{k}$ , the price of claims will be high enough for young consumers to supply claims; Proposition 5 will still hold but, with  $\hat{z} > 0$ , the equilibrium no longer satisfies Proposition 4.
6. Kihlstrom and Laffont (1983) prove a similar result in an implicit contract setting. Welfare properties of such contracts are studied

extensively by Holmstrom (1983).

7. This paragraph is based on Cooper (1982), ch. 1.
8. This assumption is no more than a convenient benchmark case, albeit a time-honored one, in monetary theory; it is generally violated when the money stock changes in response to open-market operations, money-financed government purchases, and a host of other policy actions which we do not aim to study here.
9. Gray (1975) and Fischer (1977) have developed indexation theories based on transactions costs rather than on insurance.

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