COWLES FOUNDATION FOR RESEARCH IN ECONOMICS

AT YALE UNIVERSITY

Box 2125, Yale Station
New Haven, Connecticut 06520

COWLES FOUNDATION DISCUSSION PAPER NO. 492

AN ANALYSIS OF THE ACCURACY OF FOUR MACROECONOMETRIC MODELS

Ray C. Fair

August 11, 1978

AN ANALYSIS OF THE ACCURACY OF FOUR MACROECONOMETRIC MODELS*

by

Ray C. Fair

I. *Introduction*

There is currently in macroeconomics a considerable difference of opinion as to what the true structure of the economy is like. One way in which this difference manifests itself is in the wide variety of macroeconometric models that are in existence. One might have thought at the beginning of large-scale model construction in the early 1950's that by the late 1970's the debate would be over fairly minor specification issues. This is, of course, not the case, as any casual glance at a number of models will reveal. There is also little sign that the range of differences is narrowing.

One reason for this lack of agreement is the difficulty of testing and comparing alternative models. Models differ, among other things, in the number and types of variables that are taken to be exogenous, and it is difficult to compare models with different degrees of exogeneity. In a recent study [8], however, I have proposed a method for estimating the uncertainty of a forecast from an econometric model that does allow comparisons across models to be made, and the primary purpose of this paper is to use this method to compare the accuracy of four models. The four models

---

*The research described in this paper was financed by grant SOC77-03274 from the National Science Foundation.

are (1) Sargent's [9] classical macroeconometric model, (2) Sims' [10] six-equation unconstrained vector autoregression model, (3) a "naive" model in which each variable is regressed on a constant, time, and its first eight lagged values, and (4) my model ([5], [6], [7]). These four models have quite different policy implications, and so comparing their accuracy provides one test of the effectiveness of government policy. In Sargent's model anticipated government actions have no effect on real output; in Sims' model and in the naive model there are no exogenous variables; and in my model even anticipated government actions can affect real output.

A secondary purpose of this paper is to discuss the estimation of Sargent's model. Sargent made a few econometric mistakes in his empirical work, and these mistakes must be corrected before his model can be compared to the others. A rational expectations model like Sargent's poses some interesting econometric issues, and one way in which this kind of model can be estimated is proposed and used in this paper.

It should be stressed at the outset that the present comparison results are highly tentative and must be interpreted with considerable caution. Sargent's model is, first of all, itself quite tentative, and an advocate of this model is likely to want to work on its specification more before rendering a final judgment about its accuracy. A second and related point is that an advocate of Sargent's or Sims' model may want to use at least slightly different data to estimate and analyze the model from what has been used in the present study. Also, the procedure that has been used to estimate Sargent's model is not the only procedure that one might try, and an advocate of this model may want to experiment with other estimation procedures. Finally, the method that is used to compare the models rests on one fairly strong assumption, and it is an open question whether the present results are sensitive to this assumption. Even given these caveats, however, the results in this paper do appear to convey some information about the relative accuracy of the models, and at the least it is hoped that this study provides an example of how models can be tested and compared in the future.

Sargent's model is discussed in Section II, and the other three models are discussed in Section III. The method and its application to the four models are described in Section IV, and the results are presented and discussed in Section V. Section VI contains a summary of the main conclusions of this study.

II. Sargent's Model

The following discussion relies heavily on the material in Tables 1-3. The model as Sargent estimated it is presented in Table 1. The main econometric mistake that Sargent made was to include variables in the regression to obtain $E_{t-1}p_t$ and in the first stage regressions of the TSLS technique that are not in the model. A related problem is that equation 5c is not identified unless one assumes that the error terms in equations 4 and 5c are uncorrelated. If this assumption is made, then R_t can be treated as predetermined in the estimation of equation 5c. Sargent did not treat R_t as predetermined in this case, and although he should not have been able to estimate the equation by TSLS, he did not encounter any problems because he used more variables in the first stage regression for R_t than he should have.

One way of dealing with the above problems would be to expand the model to include more variables. For those who are interested in this kind

TABLE 1. Sargent's Model as Originally Estimated

Eq. No.	LHS Variable	RHS Variables
1.	Un_t	$1,\ t,\ p_t - E_{t-1}p_t,\ Un_{t-i}\quad (i = 1, \ldots, 4)$
2.	nf_t	$1,\ t,\ p_t - E_{t-1}p_t,\ Un_t,\ nf_{t-i}\quad (i = 1, \ldots, 4)$
3.	y_t	$1,\ t,\ n_t,\ n_{t-i}\ (i = 1, \ldots, 4)$; filter: $(1 - .6L)^2$
4.	R_t	$1,\ t,\ R_{t-i}\ (i = 1, \ldots, 4)$
5c.	$m_t - p_t$	$1,\ t,\ R_t,\ R_{t-i}\ (i = 1, \ldots, 7),\ y_t,\ y_{t-i}\ (i = 1, \ldots, 7)$; filter: $(1 - .8L)^2$
6.	n_t	$= nf_t - Un_t + pop_t$

Notes: i) $E_{t-1}p_t$ was obtained from a regression of p_t on 1, t, 3 seasonal dummies, p_{t-i} $(i = 1, \ldots, 4)$, w_{t-i} $(i = 1, \ldots, 4)$, nf_{t-i} $(i = 1, \ldots, 4)$, Un_{t-i} $(i = 1, \ldots, 4)$.

ii) The equations were estimated by two stage least squares (TSLS) The explanatory variables used in the first stage regressions were those variables listed in i) plus pop_t, m_t, g_t, $surp_t$, and the log of current government employment. The RHS endogenous variables in the structural equations are p_t in 1, p_t and Un_t in 2, n_t in 3, and R_t and y_t in 5c.

iii) The filter $(1 - .6L)^2$ means that each variable z_t in the equation was transformed into $z_t^* = z_t - 1.2z_{t-1} + 0.36z_{t-2}$ before estimation. For the filter $(1 - .8L)^2$ the transformation is $z_t^* = z_t - 1.6z_{t-1} + 0.64z_{t-2}$.

Variables:
Un_t = unemployment rate
nf_t = log of labor force participation rate
y_t = log of real GNP
R_t = long term interest rate (Moody's Baa rate)
m_t = log of the money supply
p_t = log of the GNP deflator
pop_t = log of population
n_t = log of employment (approximately)
w_t = log of an index of a straight-time manufacturing wage
g_t = log of government purchases of goods and services in real terms
$surp_t$ = government surplus in real terms

of model, this would be interesting future work. The aim in this paper, however, is to examine the accuracy of the current version, and so I have chosen not to expand the model. I have instead concentrated on obtaining consistent estimates under the assumption that the model as presented in Table 1 is correctly specified.

The procedure that is proposed in this paper for estimating Sargent's model is presented in Table 2. It is briefly as follows. First, the variables that Sargent used in the first stage regressions that are not in the model were excluded from consideration. Second, the error terms in equations 4 and 5c were assumed to be uncorrelated, and R_t was taken to be predetermined in the estimation of equation 5c. Third, in place of using the filters for equations 3 and 5c, the equations were estimated under the assumption of first and second order serial correlation of the error terms. Sargent's use of the filters is equivalent to constraining the first and second order serial correlation coefficients to be particular numbers, and so the approach taken in this study is less restrictive than Sargent's approach.

The most interesting and difficult question about the estimation of Sargent's model, and of rational expectations models in general, is how to treat a variable like $E_{t-1}p_t$. The first assumption that must be made in the present case is what people know at the beginning of period t about the values of the two exogenous variables, m_t and pop_t. For present purposes each of these variables was regressed on a constant, time, and its first eight lagged values; and the predicted values from these regressions, denoted $\hat{m}_t$ and $\widehat{pop}_t$, were assumed to be what people expect the values to be. Given these predicted values for each period, the model was then estimated by the iterative procedure outline in point iv) in Table 2.

TABLE 2. Sargent's Model as Estimated in This Paper

i) In place of using the filters, equations 3 and 5c were estimated under the assumption of first and second order serial correlation of the error terms.

ii) The error terms in equations 4 and 5c were assumed to be uncorrelated, and R_t was taken to be predetermined in the estimation of 5c.

iii) There are two exogenous variables in the model, m_t and pop_t. Each of these was regressed on 1, t, and its first eight lagged values, and predicted values, $\hat{m}_t$ and $\widehat{pop}_t$, from these two regressions were obtained.

iv) An iterative procedure was used to estimate the model. Given an initial guess for $E_{t-1}p_t$, equations 1, 2, 3, and 5c were estimated by TSLS. The explanatory variables used in the first stage regressions were all the predetermined variables in the model: 1, t, $E_{t-1}p_t$, Un_{t-i} $(i = 1, \ldots, 4)$, nf_{t-i} $(i = 1, \ldots, 4)$, n_{t-i} $(i = 1, \ldots, 6)$, R_t, R_{t-i} $(i = 1, \ldots, 9)$, y_{t-i} $(i = 1, \ldots, 9)$, $m_{t-1} - p_{t-1}$, $m_{t-2} - p_{t-2}$, pop_t, and m_t. Given these TSLS estimates and the OLS estimates of equation 4, the model was solved for $E_{t-1}p_t$, with $\hat{m}_t$ replacing m_t and $\widehat{pop}_t$ replacing pop_t. This new set of values for $E_{t-1}p_t$ was then used to obtain a new set of TSLS estimates, which was then used to obtain a new set of values for $E_{t-1}p_t$, and so on. The iterative procedure was stopped when successive estimates of $E_{t-1}p_t$ (for each t) were within a prescribed tolerance level.

v) Data:

Current Name	*Variable(s) in Fair [7]*
Un_t	UR_t
nf_t	$log((TLF_{1t} + TLF_{2t} - JOBGM_t)/(POP_t - JOBGM_t))$
y_t	$log\ GNPR_t$
R_t	$RAAA_t$
m_t	$log\ M1_t$
p_t	$log\ GNPD_t$
pop_t	$log(POP_t - JOBGM_t)$

This procedure constrains $E_{t-1}p_t$ to be the value of p_t predicted by the model when m_t and pop_t are replaced by $\hat{m}_t$ and $\widehat{pop}_t$. This is the value that would be computed at the beginning of period t by someone who knew the model and had as expectations for m_t and pop_t the predicted values $\hat{m}_t$ and $\widehat{pop}_t$. (Note that the predicted value of $p_t - E_{t-1}p_t$ in the Un_t and nf_t equations is always zero in this case.) Within the context of this iterative procedure, the TSLS technique was used for all but equation 4, which requires only the OLS technique.[1]

The data that were used to estimate Sargent's model were taken from the data base for my model. The matching of the variables in Sargent's model to those in mine is presented at the bottom of Table 2. The data are explained in [6] and [7]. There are at least some minor differences between the data used in this study and the data that Sargent used, and, as mentioned in the Introduction, in future work with Sargent's model one may want to use slightly different data than were used here.

---

[1]With respect to the assumption of first and second order serial correlation in equations 3 and 5c, see [4] for a discussion of the use of the TSLS technique under this assumption.

In the initial stages of the estimation work for Sargent's model, I tried substituting the LIVE technique of Brundy and Jorgenson [1] (or, in the case of equations 3 and 5c, the LIVER technique in [3]) for the TSLS technique within the context of the above iterative procedure. Since the model was being used to solve for $E_{t-1}p_t$ (with $\hat{m}_t$ and $\widehat{pop}_t$ replacing m_t and pop_t), it seemed reasonable to also use the model to solve for the instrumental variables (with the actual values of m_t and pop_t used). This attempt, however, failed. Some of the estimates after one iteration were completely unreasonable, and there was no evidence after a few iterations that the procedure was converging. A similar failure occurred when an attempt was made to estimate the model by the full-information maximum likelihood (FIML) technique using the method described in Chow and Fair [2]. Although the present attempts failed, it may be with more diligence that one can obtain LIVE and FIML estimates for Sargent's model within the context of the above iterative procedure. This is clearly a possible area for future work on the model.

The basic sample period that was used to estimate the model was 1954III-1977IV, although for the application of the method below the model was estimated over 35 other sample periods as well. Coefficient estimates for the basic sample period and for two of the others are presented in Table 3, along with Sargent's original estimates. One of the other two sample periods (1954III-1968IV) is the shortest of the periods, and the other (1954III-1973III) is the period that most closely matches the period that Sargent used (1951I-1973III).

It is an unfortunate characteristic of macroeconometric models that their coefficient estimates can change substantially as the sample period changes, and this is clearly true of a number of the coefficient estimates in Sargent's model. Two of the key coefficients in the model are the coefficients for the $p_t - E_{t-1}p_t$ term in the Un_t and nf_t equations, and the estimates of these two coefficients are not particularly stable. For the Un_t equation the estimate for the basic sample period is of the wrong expected sign, and the estimates for the other two periods are not significant. For the nf_t equation the estimates for all three periods are of the wrong expected sign, although in this case not even Sargent's original estimate was significant. The coefficient estimates for the Un_t and nf_t equations are affected by the entire model, since $E_{t-1}p_t$ is generated from the model, and what the present results seem to suggest to advocates of Sargent's model is that more work on the overall model is needed before one can hope to get good estimates of the $p_t - E_{t-1}p_t$ coefficients.

TABLE 3. *Coefficient Estimates for Sargent's Model*

Sample Period	LHS Variable	RHS Variables							SE
	Un_t	1	t	$p_t - E_{t-1}p_t$	Un_{t-1}	Un_{t-2}	Un_{t-3}	Un_{t-4}	
Original (1951I-1973III)		0.0043 (2.5)	0.0000007 (0.5)	-0.287 (2.0)	1.47 (12.8)	-0.59 (2.9)	-0.03 (0.1)	0.04 (0.3)	0.00371
1954III-1968IV		0.0045 (1.68)	-0.0000302 (1.34)	-0.014 (0.14)	1.63 (12.96)	-0.80 (3.36)	-0.12 (0.50)	0.22 (1.88)	0.00268
1954III-1973III		0.0040 (2.06)	-0.0000047 (0.35)	-0.037 (0.56)	1.63 (14.39)	-0.79 (3.67)	-0.06 (0.29)	0.14 (1.37)	0.00260
1954III-1977IV		0.0012 (0.87)	-0.0000076 (0.69)	0.152 (3.97)	1.79 (18.57)	-1.10 (5.90)	0.10 (0.58)	0.18 (1.91)	0.00267

Sample Period	LHS Variable									SE
	nf_t	1	t	$p_t - E_{t-1}p_t$	Un_t	nf_{t-1}	nf_{t-2}	nf_{t-3}	nf_{t-4}	
Original (1951I-1973III)		-0.038 (1.3)	0.00004 (2.1)	0.149 (0.9)	-0.075 (1.9)	0.94 (8.2)	-0.11 (0.7)	-0.02 (0.2)	0.12 (1.0)	0.0040
1954III-1968IV		-0.084 (2.33)	-0.000037 (1.21)	-0.086 (0.68)	-0.100 (1.92)	0.88 (6.81)	-0.04 (0.26)	-0.08 (0.46)	0.06 (0.50)	0.00351
1954III-1973III		-0.012 (0.44)	0.000014 (0.68)	-0.189 (2.14)	-0.086 (2.03)	0.88 (7.45)	0.02 (0.15)	-0.04 (0.29)	0.11 (0.94)	0.00336
1954III-1977IV		0.027 (1.00)	0.000016 (0.76)	-0.087 (1.65)	-0.076 (2.08)	0.92 (8.10)	0.05 (0.36)	0.02 (0.13)	0.06 (0.54)	0.00334

Sample Period	LHS Variable										SE
	y_t	1	t	n_t	n_{t-1}	n_{t-2}	n_{t-3}	n_{t-4}	ρ_1	ρ_2	
Original (1951I-1973III)		0.35 (1.8)	0.0009 (4.5)	1.09 (3.5)	0.24 (1.0)	-0.24 (1.0)	-0.14 (0.5)	-0.02 (0.1)	1.20	-0.36	0.00964
1954III-1968IV		4.23 (1.22)	0.0103 (3.99)	1.29 (4.88)	-0.25 (1.11)	-0.57 (2.62)	0.03 (0.15)	-0.33 (1.63)	0.94 (6.10)	0.00 (0.03)	0.00653
1954III-1973III		5.85 (1.39)	0.0092 (4.91)	0.98 (3.33)	-0.19 (0.85)	-0.41 (1.83)	-0.09 (0.40)	-0.25 (1.25)	1.07 (8.23)	-0.14 (1.18)	0.00724
1954III-1977IV		4.95 (1.40)	0.0076 (4.51)	1.14 (4.68)	-0.23 (1.19)	-0.44 (2.36)	-0.13 (0.72)	-0.22 (1.24)	1.14 (9.80)	-0.17 (1.51)	0.00735

TABLE 3 (continued)

Sample Period	LHS Variable R_t	RHS Variables: 1	t	R_{t-1}	R_{t-2}	R_{t-3}	R_{t-4}	SE
Original (1951I-1973III)		0.15	0.0034	1.52	-0.77	0.44	-0.24	0.158
		(1.8)	(2.0)	(13.1)	(3.7)	(2.4)	(2.1)	
1954III-1968IV		0.07	0.0035	1.77	-0.70	0.30	-0.03	0.127
		(1.03)	(1.32)	(10.09)	(2.96)	(1.24)	(0.23)	
1954III-1973III		0.17	0.0047	1.44	-0.80	0.49	-0.20	0.159
		(2.09)	(2.03)	(12.74)	(4.19)	(2.57)	(1.76)	
1954III-1977IV		0.19	0.0055	1.47	-0.85	0.56	-0.26	0.165
		(2.76)	(2.43)	(14.76)	(4.93)	(3.24)	(2.65)	

Sample Period	$m_t - p_t$	1	t	R_t	R_{t-1}	R_{t-2} ... R_{t-7}	y_t	y_{t-1}	y_{t-2} ... y_{t-7}	ρ_1	ρ_2	SE
Original (1951I-1973III)		-0.22	0.0003	-0.0060	-0.0059	-0.0091...	0.45	0.16	0.19...	1.60	-0.64	0.00561
		(2.6)	(2.1)	(0.5)	(1.2)	(1.7)	(3.3)	(1.9)	(2.3)			
1954III-1968IV		5.15	0.0147	-0.0101	-0.0162	-0.0072...	0.12	0.04	0.06...	1.02	-0.04	0.00541
		(2.19)	(0.89)	(1.44)	(2.26)	(0.87)	(0.87)	(0.37)	(0.49)	(7.35)	(0.31)	
1954III-1973III		2.85	0.0026	-0.0097	-0.0078	-0.0025...	0.48	-0.06	0.03...	0.80	0.51	0.00671
		(1.73)	(0.72)	(1.91)	(1.28)	(0.38)	(3.14)	(0.50)	(0.26)	(6.60)	(1.24)	
1954III-1977IV		-4.33	-0.0127	-0.0075	-0.0045	0.0090...	1.39	-0.53	0.26...	0.58	0.15	0.01075
		(3.99)	(6.96)	(1.04)	(0.48)	(0.85)	(5.10)	(1.74)	(1.18)	(3.66)	(0.98)	

III. *The Other Three Models*

The variables and data that were used for Sims' model are presented in Table 4. For the estimation of this model each of the six variables was regressed on a constant, time, three seasonal dummy variables (although the data are seasonally adjusted), and the first four lagged values of each of the six variables. This meant that there were 29 coefficients to estimate in each equation. The naive model has already been described in the Introduction. Each variable is simply regressed on a constant, time, and its first eight lagged values. My model is described elsewhere ([5], [6], [7]), and no discussion of it will be presented here.

The sample periods that were used for these three models were the same as those used for Sargent's model except for slight differences in the beginning quarters. The beginning quarters were 1954I for Sims' model and for my model, and 1954II for the naive model. The data base began in 1952I, and slightly different beginning quarters had to be used for the models because of different lag lengths.

IV. *The Method and Its Application*

The method that is proposed in [8] for estimating the uncertainty of a forecast from a model accounts for the four main sources of uncertainty: uncertainty due to (1) the error terms, (2) the coefficient estimates, (3) the exogenous-variable forecasts, and (4) the possible misspecification of the model. It also accounts for the fact that the variances of forecast errors are not constant across time. It provides, in addition to an estimate of the uncertainty of a model's forecast from each of the four sources, a *quantitative* estimate of the degree of misspecification of the model with respect to each variable and length of forecast. Because the method

TABLE 4. Variables and Data for Sims' Model

	LHS Variable	Variable in Fair [7]
1.	log of the money supply	$log\ M1_t$
2.	log of real GNP	$log\ GNPR_t$
3.	the unemployment rate	UR_t
4.	log of the wage rate	$log\ WFF_t$
5.	log of the price level	$log\ GNPD_t$
6.	log of import prices	$log\ PIM_t$

The explanatory variables in each equation are 1, t , 3 seasonal dummies, and the first four lagged values of each of the six variables.

accounts for all four sources, it can be used to make comparisons across models.

Since the method is discussed in detail in [8], it will be only briefly described here. The uncertainty from the error terms and coefficient estimates is estimated by means of stochastic simulation. Given estimates of the relevant variance-covariance matrices, this is a fairly straightforward procedure. The uncertainty from the exogenous-variable forecasts is also estimated by means of stochastic simulation, although what is first required in this case is an estimate of the uncertainty of the exogenous-variable forecasts themselves. The procedure that was followed in this study for the 60 exogenous variables in my model and the 2 exogenous variables in Sargent's model was to regress each variable on a constant, time, and its first eight lagged values, and then to take the estimated standard error from this regression as the estimate of the uncertainty attached to forecasting the change in this variable for each quarter.

Estimating the uncertainty from the possible misspecification of the model is the most difficult and costly part of the method. It also rests on one fairly restrictive assumption, namely that the degree of misspecification of the model with respect to each variable and length of forecast is constant across time. This part of the method requires successive reestimation and stochastic simulation of the model. The estimates of misspecification are based on a comparison of estimated variances computed by means of stochastic simulation (which are based on the assumption of no misspecification) with estimated variances computed from outside sample forecast errors (which are not based on any specification assumptions).

The results for the naive model and for my model that are presented in Tables 5 and 6 below are the same as those presented in [8]. Since the same procedure was followed for the Sargent and Sims models as was followed for these other two models, little further discussion of the calculations for the Sargent and Sims models is needed here. The basic forecast period was 1978II-1981IV, and for the misspecification calculations the first of the 35 sample periods ended in 1968IV and the last ended in 1977II.

A few points about the estimation of the variance-covariance matrices for Sargent's model should be noted. First, for the estimation of these matrices for the coefficient estimates, no account was taken of the fact that $E_{t-1}p_t$ is estimated along with the coefficients. In other words, for purposes of estimating the matrices, the $E_{t-1}p_t$ series was treated as if it were known with certainty. With respect to the estimates of the first and second order serial correlation coefficients in equations 3 and 5c, on the other hand, the correlation between these estimates and the other coefficient estimates in the equation was taken into account in the estimation of the variance-covariance matrices. The procedure that was followed in this case is analogous to the procedure that was followed for my model for those equations in which the first order serial correlation coefficient was estimated. (See [8].) It should finally be noted that for the estimation of the variance-covariance matrix of the error terms for Sargent's model, the error term in equation 4 was assumed to be uncorrelated with the other four error terms in the model.

The number of trials used for each of the a, b, and c rows in Table 5 below was 2000 for the Sargent, Sims, and naive models and 1000 for my model. For each of the 35 stochastic simulations that were used for the misspecification estimates (the d rows in Table 5), the number of trials was 500 for the Sargent, Sims, and naive models and 100 for my model. The results in Table 5 are thus based on 77,000 solutions of the models. For the a, b, and c row calculations, each solution is a dynamic simulation of the model over the fifteen quarters, 1978II-1981IV; and for the d row calculations, each solution is a dynamic simulation of the model over differing eight-quarter periods.[2]

The method that I have proposed is meant to replace the traditional way of estimating the accuracy of a model by computing root mean squared errors (RMSEs). Computing RMSEs does not account for the fact that the variances of forecast errors vary across time, and it does not account for the uncertainty due to the exogenous-variable forecasts. Since computing RMSEs is such a widespread procedure, however, I have for purposes of this study also computed RMSEs for the four models. These results are presented in Table 6 below. They are based on the same 35 sets of estimates and sample periods that were used for the d row calculations in Table 5. The forecasts upon which these results are based are all outside-sample forecasts and use actual values of all the exogenous variables.

---

[2]At the risk of being pedantic, it should perhaps be noted what is meant by a dynamic simulation for Sargent's model. First, the values of m_t , pop_t , $\hat{m}_t$, and $\widehat{pop}_t$ for all t are taken to be exogenous, where $\hat{m}_t$ and $\widehat{pop}_t$ are static predictions from the two regressions mentioned above. (It would not be appropriate to take for $\hat{m}_t$ and $\widehat{pop}_t$ the dynamic predictions from the two regressions because of the exogeneity of m_t and pop_t themselves.) Given these values, the model is first solved for the initial quarter of the prediction period, say quarter t , using $\hat{m}_t$ and $\widehat{pop}_t$ in place of m_t and pop_t . This produces a value for $E_{t-1}p_t$, namely, the predicted value of p_t from this solution. (The term $p_t - E_{t-1}p_t$ in the Un_t and nf_t equations does not play any role in these calculations because its solution value in this case is alway zero.) The model is then solved again for quarter t using this value for $E_{t-1}p_t$ and the actual values of m_t and pop_t . This completes the predictions for quarter t . The model is next solved for quarter $t+1$ using the (second) predicted values of the lagged endogenous variables for period t and $\hat{m}_{t+1}$ and $\widehat{pop}_{t+1}$ in place of m_{t+1} and pop_{t+1} . This produces a value for E_tp_{t+1} . The model is then solved again for quarter $t+1$ using this value for E_tp_{t+1} and the actual values of m_{t+1} and pop_{t+1} . This process continues through the end of the prediction period.

V. *The Results*

In order to simplify the discussion, it will be useful to concentrate on the four- and eight-quarter-ahead results in Table 5. For real GNP the estimated standard errors of the four-quarter-ahead forecast taking into account all four sources of uncertainty are (in percent terms) 3.45 for the naive model, 3.72 for Sargent's model, 4.04 for Sims' model, and 1.96 for my model. The corresponding eight-quarter-ahead errors are 4.74, 5.10, 7.79, and 2.27. For the GNP deflator the four-quarter-ahead errors are 2.25, 4.70, 2.07, and 1.87; and the eight-quarter-ahead errors are 6.20, 8.53, 6.26, and 3.48. For the unemployment rate the four-quarter-ahead errors are (in percentage points) 1.48, 1.57, 1.30, and 0.82; and the eight-quarter-ahead errors are 2.19, 1.88, 2.23, and 0.71. The results for these three variables thus show that my model is the most accurate for all three. For the unemployment rate the other three models are about the same. For real GNP the naive model and Sargent's model are about the same and are better than Sims' model. For the GNP deflator the naive model and Sims' model are about the same and are better than Sargent's model. The Sargent and Sims models are thus either about the same or not as good as the naive model.

For the money supply the naive model is better than my model and Sims' model, and Sims' model is slightly better than mine. The eight-quarter-ahead errors are 3.70 percent for the naive model, 6.79 for Sims' model, and 7.50 for my model. (The money supply is exogenous in Sargent's model.) For the nominal wage rate the naive model is better than my model and Sims' model, and my model is better than Sims'. The eight-quarter-ahead errors are 2.04 for the naive model, 5.69 for Sims' model, and 4.16 for my model. (The wage rate is not a variable in Sargent's model.) The naive model is thus better than both Sims' model and my model for the money

TABLE 5. Estimated Standard Errors of Forecasts

a = uncertainty due to error terms.
b = uncertainty due to error terms and coefficient estimates.
c = uncertainty due to error terms, coefficient estimates, and exogenous-variable forecasts.
d = uncertainty due to error terms, coefficient estimates, exogenous-variable forecasts, and possible misspecification of the model.
e = estimate of the degree of misspecification of the model *(e = d-c)*.

Forecast Period = 1978II-1981IV.

For the unemployment rate, the errors are in percentage points. For the other variables, the errors are expressed as percentages of the forecast means (in percentage points).

	1978			1979				1980				1981			
	II	III	IV	I	II	III	IV	I	II	III	IV	I	II	III	IV
Real GNP:															
Naive															
a	0.61	1.02	1.34	1.64	1.84	1.94	2.01	2.03	2.04	2.03	2.04	2.04	2.03	2.03	2.03
b,c	0.67	1.13	1.53	1.90	2.20	2.38	2.50	2.59	2.64	2.68	2.74	2.77	2.81	2.84	2.87
d	1.09	1.93	2.72	3.45	4.01	4.32	4.58	4.74							
e	(0.42)	(0.80)	(1.19)	(1.55)	(1.81)	(1.94)	(2.08)	(2.15)							
Sargent															
a	0.82	1.35	1.72	1.91	2.09	2.19	2.27	2.38	2.45	2.51	2.62	2.67	2.69	2.72	2.77
b,c	0.98	1.59	2.05	2.39	2.64	2.86	3.04	3.25	3.49	3.79	4.11	4.37	4.61	4.83	5.09
d	1.30	2.23	3.00	3.72	4.23	4.61	4.93	5.10							
e	(0.32)	(0.64)	(0.95)	(1.33)	(1.59)	(1.75)	(1.89)	(1.85)							
Sims															
a	0.64	0.91	1.07	1.29	1.53	1.72	1.93	2.12	2.25	2.33	2.35	2.36	2.35	2.33	2.36
b,c	0.88	1.32	1.64	2.13	2.72	3.23	3.68	4.02	4.32	4.58	4.73	4.91	5.03	5.14	5.25
d	1.30	2.29	3.04	4.04	5.26	6.24	7.13	7.79							
e	(0.42)	(0.97)	(1.40)	(1.91)	(2.54)	(3.01)	(3.45)	(3.77)							
Fair															
a	0.65	0.88	1.03	1.15	1.25	1.30	1.35	1.34	1.36	1.40	1.43	1.44	1.47	1.46	1.43
b	0.67	0.95	1.19	1.38	1.49	1.59	1.66	1.69	1.77	1.81	1.82	1.84	1.88	1.88	1.94
c	0.74	1.09	1.37	1.63	1.76	1.94	2.04	2.08	2.15	2.18	2.22	2.30	2.34	2.36	2.43
d	0.80	1.23	1.54	1.96	2.27	2.51	2.48	2.27							
e	(0.06)	(0.14)	(0.17)	(0.33)	(0.51)	(0.57)	(0.44)	(0.19)							

TABLE 5 (continued)

	1978			*1979*				*1980*				*1981*			
	II	*III*	*IV*	*I*	*II*	*III*	*IV*	*I*	*II*	*III*	*IV*	*I*	*II*	*III*	*IV*
GNP Deflator:															
Naive															
a	0.20	0.36	0.53	0.71	0.90	1.08	1.24	1.37	1.49	1.58	1.65	1.71	1.76	1.80	1.83
b,c	0.24	0.45	0.70	1.00	1.36	1.73	2.10	2.48	2.84	3.18	3.52	3.85	4.17	4.48	4.80
d	0.45	0.94	1.53	2.25	3.12	4.05	5.10	6.20							
e	(0.21)	(0.49)	(0.83)	(1.25)	(1.76)	(2.32)	(3.00)	(3.72)							
Sargent															
a	1.02	1.34	1.67	1.84	2.05	2.24	2.46	2.71	2.95	3.15	3.33	3.43	3.52	3.59	3.72
b	1.32	1.80	2.29	2.62	3.03	3.36	3.71	4.15	4.58	5.06	5.54	5.94	6.33	6.69	7.09
c	1.58	2.20	2.75	3.13	3.56	3.92	4.29	4.73	5.17	5.54	5.92	6.28	6.68	7.00	7.38
d	2.00	2.95	4.00	4.70	5.54	6.20	7.32	8.53							
e	(0.42)	(0.75)	(1.25)	(1.57)	(1.98)	(2.28)	(3.03)	(3.80)							
Sims															
a	0.22	0.30	0.42	0.56	0.70	0.84	0.96	1.05	1.13	1.19	1.26	1.30	1.36	1.42	1.49
b,c	0.28	0.42	0.63	0.88	1.16	1.44	1.74	2.03	2.31	2.58	2.86	3.12	3.39	3.66	3.92
d	0.50	0.77	1.34	2.07	2.87	3.89	5.01	6.26							
e	(0.22)	(0.35)	(0.71)	(1.19)	(1.71)	(2.45)	(3.27)	(4.23)							
Fair															
a	0.28	0.35	0.42	0.47	0.51	0.55	0.59	0.61	0.64	0.65	0.65	0.65	0.66	0.67	0.68
b	0.31	0.47	0.58	0.71	0.83	0.93	1.02	1.10	1.19	1.28	1.37	1.44	1.50	1.57	1.63
c	0.44	0.67	0.84	1.04	1.21	1.36	1.49	1.62	1.75	1.88	1.98	2.09	2.23	2.35	2.43
d	0.53	0.93	1.37	1.87	2.33	2.74	3.15	3.48							
e	(0.09)	(0.26)	(0.53)	(0.83)	(1.12)	(1.38)	(1.66)	(1.86)							
Unemployment Rate:															
Naive															
a	0.28	0.55	0.77	0.94	1.02	1.08	1.12	1.14	1.15	1.16	1.16	1.16	1.15	1.15	1.16
b,c	0.29	0.58	0.84	1.04	1.17	1.27	1.34	1.40	1.44	1.48	1.52	1.55	1.59	1.63	1.66
d	0.36	0.74	1.12	1.48	1.73	1.91	2.07	2.19							
e	(0.07)	(0.16)	(0.28)	(0.44)	(0.56)	(0.64)	(0.73)	(0.79)							
Sargent															
a	0.26	0.55	0.78	0.93	1.01	1.05	1.08	1.09	1.13	1.16	1.20	1.22	1.24	1.25	1.27
b,c	0.28	0.58	0.85	1.05	1.16	1.22	1.26	1.31	1.37	1.45	1.54	1.62	1.69	1.75	1.81
d	0.42	0.83	1.23	1.57	1.76	1.82	1.88	1.88							
e	(0.14)	(0.25)	(0.38)	(0.52)	(0.60)	(0.60)	(0.62)	(0.57)							
Sims															
a	0.21	0.35	0.43	0.49	0.56	0.63	0.69	0.75	0.82	0.87	0.90	0.91	0.90	0.89	0.90
b,c	0.28	0.48	0.59	0.68	0.84	1.01	1.16	1.28	1.38	1.48	1.55	1.61	1.65	1.68	1.71
d	0.44	0.85	1.19	1.30	1.45	1.64	1.96	2.23							
e	(0.16)	(0.37)	(0.60)	(0.62)	(0.61)	(0.63)	(0.80)	(0.95)							
Fair															
a	0.27	0.45	0.57	0.64	0.71	0.77	0.80	0.82	0.82	0.85	0.90	0.92	0.93	0.96	0.97
b	0.36	0.58	0.76	0.92	1.03	1.12	1.16	1.23	1.28	1.34	1.38	1.42	1.50	1.56	1.62
c	0.36	0.60	0.80	0.95	1.08	1.17	1.24	1.31	1.35	1.41	1.47	1.50	1.55	1.59	1.64
d	0.35	0.60	0.77	0.82	0.85	0.83	0.77	0.71							
e	(−0.01)	(0.00)	(−0.03)	(−0.13)	(−0.23)	(−0.34)	(−0.47)	(−0.60)							

TABLE 5 (continued)

	1978			1979				1980				1981			
	II	III	IV	I	II	III	IV	I	II	III	IV	I	II	III	IV
Money Supply:															
Naive															
a	0.54	0.67	0.81	0.88	0.97	1.02	1.10	1.15	1.22	1.28	1.33	1.40	1.45	1.49	1.51
b,c	0.62	0.81	1.03	1.21	1.36	1.53	1.72	1.93	2.12	2.34	2.51	2.73	2.96	3.19	3.42
d	1.38	1.64	1.95	2.32	2.62	2.91	3.30	3.70							
e	(0.76)	(0.83)	(0.92)	(1.11)	(1.26)	(1.38)	(1.58)	(1.77)							
Sims															
a	0.69	0.86	1.02	1.17	1.29	1.46	1.56	1.71	1.82	1.94	2.00	2.09	2.15	2.21	2.31
b,c	0.92	1.19	1.56	1.92	2.33	2.72	3.08	3.44	3.80	4.19	4.57	4.93	5.28	5.66	6.05
d	1.37	1.53	2.29	3.29	4.18	5.23	6.07	6.79							
e	(0.45)	(0.34)	(0.73)	(1.37)	(1.85)	(2.51)	(2.99)	(3.35)							
Fair															
a	0.83	1.09	1.29	1.47	1.62	1.76	1.84	1.92	1.98	2.03	2.11	2.13	2.17	2.19	2.23
b	0.91	1.31	1.63	1.87	2.13	2.36	2.56	2.79	2.96	3.16	3.35	3.55	3.72	3.94	4.15
c	0.91	1.33	1.69	1.98	2.34	2.68	3.06	3.45	3.79	4.14	4.51	4.88	5.28	5.63	5.97
d	1.29	2.16	2.95	3.75	4.62	5.50	6.49	7.50							
e	(0.48)	(0.83)	(1.26)	(1.77)	(2.28)	(2.82)	(3.43)	(4.05)							
Wage Rate:															
Naive															
a	0.30	0.40	0.48	0.53	0.59	0.61	0.67	0.72	0.76	0.81	0.85	0.88	0.91	0.95	0.98
b,c	0.36	0.48	0.59	0.75	0.86	0.97	1.15	1.29	1.46	1.64	1.81	1.99	2.19	2.39	2.59
d	0.63	0.84	1.04	1.26	1.41	1.56	1.81	2.04							
e	(0.27)	(0.36)	(0.45)	(0.51)	(0.55)	(0.59)	(0.66)	(0.75)							
Sims															
a	0.47	0.62	0.71	0.77	0.82	0.92	0.95	1.02	1.10	1.15	1.17	1.22	1.26	1.30	1.34
b,c	0.63	0.90	1.08	1.23	1.35	1.50	1.65	1.85	2.05	2.25	2.42	2.59	2.75	2.95	3.12
d	1.23	2.20	3.02	3.71	4.26	4.70	5.12	5.69							
e	(0.60)	(1.30)	(1.94)	(2.48)	(2.91)	(3.20)	(3.47)	(3.84)							
Fair															
a	0.60	0.77	0.88	0.89	0.96	1.01	1.03	1.05	1.07	1.10	1.08	1.07	1.08	1.04	1.05
b	0.70	0.93	1.12	1.34	1.52	1.65	1.76	1.82	1.94	2.04	2.15	2.27	2.35	2.45	2.51
c	0.67	0.95	1.16	1.35	1.53	1.66	1.80	1.94	2.08	2.20	2.32	2.40	2.52	2.61	2.69
d	0.65	1.06	1.45	2.01	2.53	3.07	3.59	4.16							
e	(-0.02)	(0.11)	(0.29)	(0.66)	(1.00)	(1.41)	(1.79)	(2.22)							

supply and the nominal wage rate.

The same qualitative conclusions emerge from the RMSE results in Table 6 except that my model is now more accurate than Sims' model with respect to the money supply. This difference is explained in part by the fact that the calculations in Table 6 do not take into account the uncertainty from the exogenous-variable forecasts, whereas those in Table 5 do. There are no exogenous variables in Sims' model, unlike in my model, and so any comparison of the two models that does not take into account the exogenous-variable uncertainty is likely to be biased in favor of my model.

The a, b, c, and d row comparisons in Table 5 are self-explanatory, and so no discussion of these results is needed here.[3] One can determine from these comparisons how much of the total error is due to each of the four sources, which may be useful information regarding possible future work on the model.

---

[3]One point about the c row calculations for Sargent's model should be noted. If, as was assumed for the present calculations, the uncertainty attached to forecasting m_t is the same as that attached to forecasting $\hat{m}_t$, then the predicted values of Un_t, nf_t, and y_t are unaffected by this uncertainty. In other words, if in the stochastic simulations the draws for the m_t and $\hat{m}_t$ errors are the same, then $E_{t-1}p_t$ and the "second" predicted value of p_t are affected in the same way, and so these draws have no affect on the real variables. Only the predictions of p_t are affected by the uncertainty in forecasting the money supply. Also, the uncertainty attached to forecasting pop_t is so small as to be negligible, which means that there is in effect no exogenous-variable uncertainty in forecasting the real variables in Sargent's model. Consequently, the b and c rows in Table 5 for real GNP and the unemployment rate have not been listed separately for Sargent's model.

TABLE 6. Root Mean Squared Errors of Outside-Sample Forecasts

Variable	*Model*	*Number of Quarters Ahead* 1	2	3	4	5	6	7	8
Real GNP (percent)	*Naive*	1.11	1.96	2.76	3.51	4.09	4.42	4.70	4.91
	Sargent	1.31	2.26	3.04	3.77	4.27	4.59	4.89	5.00
	Sims	1.42	2.54	3.54	4.79	6.34	7.79	9.36	10.98
	Fair	0.79	1.26	1.63	2.12	2.59	2.97	3.24	3.52
GNP Deflator (percent)	*Naive*	0.47	0.98	1.59	2.36	3.26	4.23	5.35	6.52
	Sargent	2.89	4.93	7.02	8.81	10.58	12.22	14.05	15.87
	Sims	0.54	0.90	1.53	2.39	3.36	4.56	5.90	7.42
	Fair	0.50	0.93	1.43	1.97	2.49	2.95	3.43	3.83
Unemployment Rate (percentage points)	*Naive*	0.36	0.75	1.13	1.49	1.73	1.89	2.03	2.14
	Sargent	0.44	0.87	1.30	1.66	1.87	1.94	2.01	2.00
	Sims	0.47	0.91	1.30	1.46	1.64	1.89	2.31	2.74
	Fair	0.36	0.60	0.75	0.80	0.79	0.79	0.77	0.77
Money Supply (percent)	*Naive*	1.41	1.71	2.10	2.52	2.89	3.23	3.64	4.09
	Sims	1.47	1.95	3.05	4.40	5.80	7.41	9.04	10.82
	Fair	1.34	2.12	2.93	3.81	4.74	5.72	6.82	7.97
Wage Rate (percent)	*Naive*	0.67	0.94	1.21	1.47	1.71	2.01	2.40	2.83
	Sims	1.33	2.37	3.32	4.15	4.91	5.60	6.37	7.36
	Fair	0.78	1.25	1.71	2.31	2.89	3.49	4.06	4.65

Notes:

i) These results are based on 35 sets of estimates of each model.

ii) Each eight-quarter outside-sample forecast began two quarters after the end of the estimation period. The first estimation period ended in 1968IV, and the last (the thirty-fifth) ended in 1977II. Data through 1977IV were used, which allowed 35 one-quarter-ahead errors to be computed for each variable, 34 two-quarter-ahead errors, and so on.

iii) The predicted values for these calculations were the mean values from the 35 stochastic simulations that were generated for the d-row results in Table 5.

VI. *Summary and Conclusions*

The main results of this study are easy to summarize. Sargent's model and Sims' model are no more accurate than the naive model, and for some variables they are less accurate. My model is more accurate than the other models for real GNP, the GNP deflator, and the unemployment rate. It is less accurate than the naive model for the money supply and the nominal wage rate, and it is slightly less accurate than Sims' model for the money supply.

The tentative nature of the results in this paper has been mentioned in the Introduction, and this point should again be stressed. Clearly, advocates of the Sargent and Sims models are likely to want to work more on the models before reaching a final judgment about their accuracy. In addition, one may want to try other estimation techniques for Sargent's model from the one used in this study. The present results do indicate, however, that the Sargent and Sims models are not very accurate, and so the present prognosis for these two models is not very encouraging.

REFERENCES

[1] Brundy, James M., and Dale W. Jorgenson, "Efficient Estimation of Simultaneous Equations by Instrumental Variables," *The Review of Economics and Statistics*, LIII (August 1971), 207-224.

[2] Chow, Gregory C., and Ray C. Fair, "Maximum Likelihood Estimation of Linear Equation Systems with Auto-Regressive Residuals," *Annals of Economic and Social Measurement*, II (January 1973),17-28.

[3] Fair, Ray C., "Efficient Estimation of Simultaneous Equations with Auto-Regressive Errors by Instrumental Variables," *The Review of Economics and Statistics*, LIV (November 1972), 444-449.

[4] __________, "A Comparison of Alternative Estimators of Macroeconomic Models," *International Economic Review*, XIV (June 1973), 261-277.

[5] __________, *A Model of Macroeconomic Activity, Volume I: The Theoretical Model* (Cambridge: Ballinger Publishing Co., 1974).

[6] __________, *A Model of Macroeconomic Activity, Volume II: The Empirical Model* (Cambridge: Ballinger Publishing Co., 1976).

[7] __________, "The Fair Model as of April 15, 1978," mimeo.

[8] __________, "Estimating the Expected Predictive Accuracy of Econometric Models," Cowles Foundation Discussion Paper No. 480, January 14, 1978, revised August 1978.

[9] Sargent, Thomas J., "A Classical Macroeconometric Model for the United States," *Journal of Political Economy*, LXXXIV (April 1976), 207-237.

[10] Sims, Christopher A., "Macro-Economics and Reality," paper presented as the 1977 Fisher-Schultz Lecture, Vienna, December 1977, revised November 1977.