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THE LONG RUN CONSEQUENCES OF MONETARY AND FISCAL POLICIES WHEN THE GOVERNMENT'S BUDGET IS NOT BALANCED

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THE LONG RUN CONSEQUENCES OF MONETARY AND FISCAL POLICIES WHEN THE GOVERNMENT'S BUDGET IS NOT BALANCED*

by

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The familiar IS-LM apparatus describes the short run impact effects on the economy of various shocks, usually changes in policy instruments such as federal expenditures or open market operations. As time passes the state variables (e.g., the stocks of money, bonds, and capital) will change and a sequence of new equilibria will evolve. Although the short run nature of IS-LM analysis has long been recognized, there was surprisingly little work done on the dynamic evolution of an economy after the initial impact of a policy change. This neglect was apparently terminated by the suspicions that were voiced by Milton Friedman (1972) and others that a short run analysis is misleading since the long run effects of policies are likely to differ qualitatively from the impact effects. Since the dynamic evolution of the economy is quite complicated and time dependent, a natural target of longer run analyses is that hypothetical distant time when the economy has converged to a steady state with replicative temporary equilibria. Unfortunately, this analysis (Blinder and Solow [1973, 1974] and Tobin and Buiter [1976]) is neither simple nor unambiguous. Tobin and Buiter conclude that "it is disturbing that the qualitative properties

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of models--the signs of important system-wide multipliers, the stability of equilibria--can turn on relatively small changes of specification or on small differences in values of coefficients" (p. 303).

These analyses have been concerned solely with stationary states in which prices and the nominal stocks of money and bonds are constant. This imposes the analytically powerful constraint that the government's budget must in the long run be balanced. If the real after tax interest payments on federal debt are considered part of government spending, then a higher level of government spending can only be sustained by a higher tax base, and hence higher national income if this is the only source of tax revenue. Monetary policies which do not change government spending can have no long run effect on national income since any change in tax revenue would imply an unbalanced budget.

Somewhat more freedom is provided if interest payments are separated from other expenditures since this latter category could be sustained by lower real interest payments (higher prices, lower interest rates, or a smaller level of federal interest bearing debt) rather than higher tax revenue. However the relevance of these limited possibilities is weakened in practice by the small size of federal interest payments. Unless the government were to become a creditor, an increase in government spending by more than the size of these interest payments must in a stationary state be financed by higher taxes.

The present paper shows that this concern with stationary states and a concomitant balanced federal budget is an artificial constraint which results from the arbitrary assumptions that the price level or the stock of money or bonds is fixed. The cost of relaxing this constraint is of course that the analysis becomes even more complicated and ambiguous.

Yet it seems preferable to candidly admit that relatively little can be said analytically about the long run effects of short run policies.

For comparative purposes I use the Tobin-Buiter model to analyze the long run equilibria in an inflationary situation. I consider both the G' policy which they analyze and a G'' policy. Like them, I find the long run equilibrium effects of fiscal policy to be expansionary. While they find monetary policy to be neutral, however, I find it to be expansionary in a G' regime, raising the capital stock and income and lowering the interest rate. When prices are endogenous, an expansionary monetary policy tends to lower the steady state rate of inflation. I would characterize my stability results as being more ambiguous and therefore less pessimistic about the stability of the model.

The notation used in this paper is described here:

Y = real net national product

C = real private consumption

K = capital stock

P = price level

π = rate of inflation, $\dot{P}/P$

π^e = anticipated rate of inflation

B = nominal government interest bearing debt

M = nominal government monetary debt

b = real government interest bearing debt, B/P

m = real government monetary debt, M/P

L = real private demand for m

r = anticipated real rate of return on debt

τ = income tax rate

$\dot{x}$ = time derivative of x

G = real government purchases of national product

G' = G plus real net debt interest, $G + (1-\tau)(r+\pi^e)b$

G'' = G' minus real capital gains on government debt, $G' - \pi(m+b)$

Steady States and the Government Budget Constraint

The government's budget constraint can be written as

$$G' - \tau Y = (\dot{M}+\dot{B})/P = \dot{m} + \dot{b} + (m+b)\pi \,. \tag{1}$$

I will assume that the government follows the deficit financing rule

$$\begin{aligned} \dot{M} &= \gamma(G' - \tau Y)P \\ \dot{B} &= (1-\gamma)(G' - \tau Y)P \end{aligned} \tag{2}$$

or in real terms

$$\begin{aligned} \dot{m} &= \gamma(G' - \tau Y) - m\pi \\ \dot{b} &= (1-\gamma)(G' - \tau Y) - b\pi \,. \end{aligned} \tag{3}$$

Consider now the following general IS-LM model

$$\begin{aligned} Y + (1-\tau)(r+\pi^e)b &= C[r,\ \pi^e,\ \tau,\ Y,\ K,\ m,\ b] + \dot{K} + G' \\ m &= L[r,\ \pi^e,\ \tau,\ Y,\ K,\ m,\ b] \end{aligned} \tag{4}$$

The important assumption here is that the price level does not appear as a separate argument, due to the conventional dismissal of money illusion. Differentiating (4) with respect to time and imposing the steady state conditions $\dot{K} = \dot{Y} = \dot{r} = \dot{\pi}^e = 0$ gives

$$(5)\qquad \begin{aligned} &\frac{\partial C}{\partial m}\dot{m} + \left(\frac{\partial C}{\partial b} - (1-\tau)(r+\pi^e)\right)\dot{b} = -\dot{G}' \\ &\left(\frac{\partial L}{\partial m} - 1\right)\dot{m} + \frac{\partial L}{\partial b}\dot{b} = 0 \ . \end{aligned}$$

If $\dot{G}' = 0$,* then the solution of (5) is

$$(6)\qquad \dot{m} = \dot{b} = 0 \ ,$$

except in the special case

$$\frac{\partial C}{\partial m}\frac{\partial L}{\partial b} + \left(1 - \frac{\partial L}{\partial m}\right)\left(\frac{\partial C}{\partial b} - (1-\tau)(r+\pi^e)\right) = 0 \ ,$$

which is ruled out by the natural assumptions

$$\frac{\partial C}{\partial m} > 0 \ , \quad \frac{\partial L}{\partial b} > 0 \ , \quad \frac{\partial L}{\partial m} < 1 \ , \quad \frac{\partial C}{\partial b} > (1-\tau)(r+\pi^e) \ .$$

This steady state condition (6) and the budget constraint (1) now give the steady state government deficit

$$(7)\qquad G' - \tau Y = (m+b)\pi \ .$$

In addition, from (2), (6), and (7)

$$0 = \dot{m} = \gamma(G' - \tau Y) - m\pi = \pi(\gamma(m+b) - m)$$

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*If the government instead follows a $\dot{G} = 0$ policy, then $\dot{m} = \dot{b} = 0$ unless

$$\frac{\partial C}{\partial m}\frac{\partial L}{\partial b} + \left(1 - \frac{\partial L}{\partial m}\right)\frac{\partial C}{\partial b} = 0 \ ,$$

which is ruled out by

$$\frac{\partial C}{\partial m} > 0 \ , \quad \frac{\partial L}{\partial b} > 0 \ , \quad \frac{\partial C}{\partial b} > 0 \ , \quad \frac{\partial L}{\partial m} < 0 \ .$$

implies that

$$m = \gamma(m+b) \tag{8}$$

$$b = (1-\gamma)(m+b)$$

for any nonzero rate of inflation.

Thus a steady state requires constant real stocks of money and bonds, with the stock ratio of money to bonds identical to the flow ratio set by the government's debt financing policy unless $\pi = 0$. The rate of inflation will equal the ratio of the government deficit to the stock of debt. A stationary state with $\pi = 0$ requires a balanced government budget; a steady state inflation does not. When deficits are financed exclusively by money or bonds, a zero rate of inflation is the only steady state solution compatible with nonzero stocks of both assets.

A Model with Unemployment

Following Tobin-Buiter, I will use the specific behavioral equations

$$\dot{K} = i\left(\frac{\alpha Y}{r} - K\right)$$

$$L = \ell\left[(1-\tau)(r+\pi^e), \frac{Y}{m+b+K}\right](m+b+K) \tag{9}$$

$$C = (1-\tau)(Y+(r+\pi^e)b) - (m+b)\pi^e - s(\hat{\mu}Y - m - b - K)$$

where $s(\hat{\mu}Y - m - b - K)$ is the planned time derivative of wealth. Anticipated revelations of the capital stock do not affect consumption or the demand for money. I will assume that it is the rate of inflation rather than the price level which is exogenously fixed. The substitution of (9) into (3) and (4) gives the dynamic IS-LM model

$$i\left(\frac{\alpha Y}{r} - K\right) = s(\hat{u}Y - m - b - K) + \tau Y - G' + \pi^e(m+b)$$

$$\ell\left[(1-\tau)(r+\pi^e),\ \frac{Y}{m+b+K}\right](m+b+K) = m$$

(10) $$\dot{K} = i\left(\frac{\alpha Y}{r} - K\right)$$

$$\dot{m} = \gamma(G' - \tau Y) - m\pi$$

$$\dot{b} = (1-\gamma)(G' - \tau Y) - b\pi\ .$$

Tobin-Buiter support the analysis of G' policies by appeals to simplicity and the interpretation of federal interest payments as a continuing demand stimulus. In an inflationary context these arguments suggest the analysis of $G'' = G' - \pi(m+b)$ policies which incorporate capital gains on federal debt. In this case the model becomes

$$\dot{K} = s(\hat{u}Y - m - b - K) + \tau Y - G'' + (\pi^e - \pi)(m+b)$$

$$\ell\left[(1-\tau)(r+\pi^e),\ \frac{Y}{m+b+K}\right](m+b+K) = m$$

(11) $$\dot{K} = i\left(\frac{\alpha Y}{r} - K\right)$$

$$\dot{m} = \gamma(G'' - \tau Y) + \pi(\gamma(m+b) - m)$$

$$\dot{b} = (1-\gamma)(G'' - \tau Y) + \pi((1-\gamma)(m+b) - b)\ .$$

The short run analysis of these models is well known. An increase in G' (or G'') raises the temporary equilibrium values of Y and r . An asset swap of m for b raises Y and lowers r . A change in the deficit financing parameter γ has no immediate effect on the equilibrium.

With a G' policy, the long run steady state equilibrium is obtained by substituting (7), (8), and $\dot{K} = \pi^e - \pi = 0$ into (10),

$$
\begin{aligned}
& G' - \tau Y = \pi(m+b) = \pi(\hat{\mu}Y - K) \\
& \ell[(1-\tau)(r+\pi),\ 1/\hat{\mu}]\hat{\mu} = m/Y = \gamma\frac{m+b}{Y} = \gamma\left(\hat{\mu} - \frac{\alpha}{r}\right) \\
& \alpha Y = rK \\
& \hat{\mu}Y = m + b + K\,.
\end{aligned}
\tag{12}
$$

Differentiating,

$$
\begin{bmatrix} 0 & -\pi\hat{\mu}-\tau & \pi & 0 \\ (1-\tau)\ell_1\hat{\mu} - \frac{\alpha\gamma}{r^2} & 0 & 0 & 0 \\ -K & \alpha & -r & 0 \\ 0 & \hat{\mu} & -1 & -1 \end{bmatrix}
\begin{bmatrix} \Delta r \\ \Delta Y \\ \Delta K \\ \Delta(m+b) \end{bmatrix}
=
\begin{bmatrix} -\Delta G' \\ \left(\hat{\mu} - \frac{\alpha}{r}\right)\Delta\gamma \\ 0 \\ 0 \end{bmatrix} .
\tag{13}
$$

The Jacobian determinant

$$
J = \left((1-\tau)\ell_1\hat{\mu}r - \frac{\alpha\gamma}{r}\right)\left(\tau + \pi\left(\hat{\mu} - \frac{\alpha}{r}\right)\right) < 0
$$

is negative for positive rates of inflation since $\hat{\mu} - \alpha/r = (m+b)/K > 0$. To keep the number of possibilities at a manageable level, I will only analyze the case $\pi > 0$ here.

Premultiplying (13) by the Jacobian inverse gives

$$(14)\quad \begin{bmatrix} \Delta r \\ \Delta Y \\ \Delta K \\ \Delta(m+b) \end{bmatrix} = \frac{-1}{|J|}\begin{bmatrix} -\left(\hat{\mu} - \frac{\alpha}{r}\right)r\left(\tau + \pi\left(\hat{\mu} - \frac{\alpha}{r}\right)\right)\Delta\gamma \\ \left(\frac{\alpha\gamma}{r} - (1-\tau)\ell_1\hat{\mu}r\right)\Delta G' + \pi K\left(\hat{\mu} - \frac{\alpha}{r}\right)\Delta\gamma \\ \left(\frac{\alpha^2\gamma}{r^2} - \alpha(1-\tau)\ell_1\hat{\mu}\right)\Delta G' + \left(\hat{\mu} - \frac{\alpha}{r}\right)K(\tau+\pi\hat{\mu})\Delta\gamma \\ \left(\hat{\mu} - \frac{\alpha}{r}\right)\left(\frac{\alpha\gamma}{r} - (1-\tau)\ell_1\hat{\mu}r\right)\Delta G' - \left(\hat{\mu} - \frac{\alpha}{r}\right)\tau K\Delta\gamma \end{bmatrix}.$$

The signs of these multipliers are displayed in Table 1. The second equation in (12) shows that for a given interest rate both the demand for and supply of money are proportional to Y . The interest rate is consequently uniquely determined by the equality of the money demand and supply. The other results are straightforward.

To analyze the stability of these equilibria, the characteristic equation of (10) in the neighborhood of long run equilibrium is given by

$$0 = \begin{vmatrix} -s-\lambda & \pi - s & \pi - s & s\hat{\mu} + \tau & 0 \\ -(\ell - \ell_2/\hat{\mu}) & 1 - (\ell - \ell_2/\hat{\mu}) & -(\ell - \ell_2/\hat{\mu}) & -\ell_2 & -(1-\tau)\ell_1\hat{\mu}Y \\ -i-\lambda & 0 & 0 & i\alpha/r & -i\alpha Y/r^2 \\ 0 & -\pi-\lambda & 0 & -\gamma\tau & 0 \\ 0 & 0 & -\pi-\lambda & -(1-\gamma)\tau & 0 \end{vmatrix}$$

After some manipulation, this reduces to

$$0 = (\lambda+\pi)\Big\{\ell_1(1-\tau)\hat{\mu}Y\Big[is\tau + is\pi\left(\hat{\mu} - \frac{\alpha}{r}\right) + \Big(i\left(\tau + s\left(\hat{\mu} - \frac{\alpha}{r}\right)\right)$$
$$+ \pi\left(\tau + s\hat{\mu} - i\frac{\alpha}{r}\right) + \tau(s-\pi)\Big)\lambda + \left(\tau + s\hat{\mu} - i\frac{\alpha}{r}\right)\lambda^2\Big]$$
$$- \frac{i\alpha Y}{r^2}[s\gamma\tau + s\pi\ell + (\gamma\tau + (\pi+s)\ell_2 + s\hat{\mu}(\ell + \ell_2(\hat{\mu}))\lambda + \ell_2\lambda^2]\Big\}$$

TABLE 1

Long Run Multipliers with Unemployment

	G' Regime ($\pi > 0$)		G" Regime ($\pi \gtrless 0$)	
	$\Delta G'$	$\Delta\gamma$	$\Delta G''$	$\Delta\gamma$
Δr	0	-	0	-
ΔY	+	+	+	0
ΔK	+	+	+	+
$\Delta(m+b)$	+	-	+	-
stability	stable		stable for $\pi > 0$ unstable for $\pi < 0$	

which is of the form $0 = (\lambda+\pi)(-a - b\lambda - c\lambda^2)$ where a, b, and c are positive if $\tau + s\hat{\mu} - i(\alpha/r) > 0$ and $\pi^e < s$. Looking at the short run equations (10), it can be seen that the first of these sufficient conditions describes the usual assumption that an increase in Y raises saving relative to investment. The second sufficient condition describes the usual short run assumption that an increase in real government debt, either through an increase in nominal debt or a fall in prices, reduces saving relative to investment. With these assumptions then the model is stable since the coefficients of

$$-a\pi - (a+b\pi)\lambda - (b+c\pi)\lambda^2 - c\lambda^3$$

have the same signs and

$$(a+b\pi)(b+c\pi) - ac\pi = ab + b^2\pi + bc\pi^2 > 0 .$$

If the government instead follows a G'' policy, then the long run steady state equilibrium follows from substituting (7), (8), and $\dot{K} = \pi^e - \pi = 0$ into (11):

$$\begin{aligned} & G'' = \tau Y \\ & \ell[(1-\tau)(r+\pi),\ 1/\hat{\mu}]\hat{\mu} = \gamma\left(\hat{\mu} - \frac{\alpha}{r}\right) \\ (15) \qquad & \alpha Y = rK \\ & \hat{\mu}Y = m + b + K . \end{aligned}$$

These equations are recursive in Y, r, K, and $m+b$. Differentiating,

$$(16)\qquad \begin{bmatrix} 0 & -\tau & 0 & 0 \\ (1-\tau)\ell_1\hat{u} - \frac{\alpha\gamma}{r^2} & 0 & 0 & 0 \\ -K & \alpha & -r & \\ 0 & \hat{u} & -1 & -1 \end{bmatrix} \begin{bmatrix} \Delta r \\ \Delta Y \\ \Delta K \\ \Delta(m+b) \end{bmatrix} = \begin{bmatrix} -\Delta G'' \\ \left(\hat{u} - \frac{\alpha}{r}\right)\Delta\gamma \\ 0 \\ 0 \end{bmatrix} .$$

This Jacobian determinant

$$|J| = \tau\ (1-\tau)\ell_1\hat{u}r - \frac{\alpha\gamma}{r} < 0$$

is negative and the multipliers are given by

$$(17)\qquad \begin{bmatrix} \Delta r \\ \Delta Y \\ \Delta K \\ \Delta(m+b) \end{bmatrix} = \frac{-1}{|J|} \begin{bmatrix} -\tau r\left(\hat{u} - \frac{\alpha}{r}\right)\Delta\gamma \\ \left(\frac{\alpha\gamma}{r} - (1-\tau)\ell_1\hat{u}\right)\Delta G'' \\ \left(\frac{\alpha^2\gamma}{r^2} - \alpha\hat{u}(1-\tau)\ell_1\right)\Delta G'' + \tau K\left(\hat{u} - \frac{\alpha}{r}\right)\Delta\gamma \\ \left(\frac{\alpha\gamma}{r} - (1-\tau)\ell_1\hat{u}r\right)\left(\hat{u} - \frac{\alpha}{r}\right)\Delta G'' - \tau K\left(\hat{u} - \frac{\alpha}{r}\right)\Delta\gamma \end{bmatrix} .$$

Unlike G' policies, the multipliers are independent of π. The signs of these multipliers are displayed in Table 1. An increase in G'' requires a higher tax base and hence higher income. Since the interest rate is determined by the debt financing policy, the capital stock must be enlarged for the marginal product of capital to remain constant. For a given interest rate the demand for debt increases proportionately with income. A heavier reliance on money financing reduces the interest rate. With income fixed by G'', a larger capital stock is accordingly needed to reduce the marginal product of capital. The fixed income fixes desired wealth, so that the increase in K is offset by a reduced demand for government debt.

The stability of these long run equilibria can be analyzed by means of the characteristic equation for (11):

$$0 = \begin{vmatrix} -s-\lambda & -s & -s & \tau + s\hat{\mu} & 0 \\ -(\ell - \ell_2/\hat{u}) & (-(\ell - \ell_2/\hat{u}) & -(\ell - \ell_2/\hat{u}) & -\ell_2 & -(1-\tau)\ell_1\hat{u}Y \\ -i-\lambda & 0 & 0 & i\alpha/r & -i\alpha Y/r^2 \\ 0 & -\pi(1-\gamma)-\lambda & \pi\gamma & -\gamma\tau & 0 \\ 0 & \pi(1-\gamma) & -\pi\gamma-\lambda & -(1-\gamma)\tau & 0 \end{vmatrix}$$

$$= (\lambda+\pi)\left\{(1-\tau)\hat{\mu}Y\left[is\tau + \lambda\left(is\left(\hat{\mu}-\frac{\alpha}{r}\right)+i\tau+s\tau\right)+\lambda^2\left(\tau+s\hat{\mu}-\frac{i\alpha}{r}\right)\right]\right.$$

$$\left. - \frac{i\alpha Y}{r^2}[\gamma s\tau + \lambda(\gamma\tau + s\hat{\mu}\ell) + \lambda^2\ell_2]\right\} .$$

Again using the assumption $\tau + s\hat{\mu} - i\alpha/r > 0$, this is of the form $(\lambda+\pi)(-a - b\lambda - c\lambda^2) = 0$ which we've seen previously is necessary and sufficient for the stability of the system with a positive rate of inflation. With a negative rate of inflation, the system is not stable since the signs of the coefficients of $-a\pi - (a+b\pi)\lambda - (b+c\pi)\lambda^2 - c\lambda^3 = 0$ are not all the same. The source of this instability can be seen from the last two equations of (11). In long run equilibrium $G'' - \tau Y$, $\gamma(m+b) - m$, and $(1-\gamma)(m+b) - b$ will all converge to zero. Consider now if Y is at its long run equilibrium level with $G'' = \tau Y$, so that $\dot{m} + \dot{b} = 0$. If m is however slightly larger than $\gamma(m+b)$ while b is less than $(1-\gamma)(m+b)$, then m will fall and b will increase for a positive rate of inflation but the reverse will be true in a deflation.

A Model with Full Employment

The usual full employment interpretation of the IS-LM model assumes that output is supply determined with employment set by the equilibration (through the real wage rate) of the labor market. With an unchanging relationship between the supply of labor and the real wage, output will depend only upon the capital stock,

$$(18) \qquad Y = F[K] \; .$$

This closes the model by endogenously determining the commodity price, though the level is time dependent in an inflationary situation. It is consequently the endogenous rate of inflation π that is of interest here. Following Tobin-Buiter, inflationary expectations are modeled as

$$(19) \qquad \dot{\pi}^e = \beta(\pi - \pi^e) \; , \quad \beta > 0 \; .$$

The complexity of the analysis will force us to pay particular attention to the special cases $\beta \to 0$ (static expectations) and $\beta \to \infty$ (perfect foresight).

The short run analysis of (18) and (10) or (11) is well known.* An increase in either G' or G'' will increase both r and P. An asset swap of M for B raises P and lowers r. Income is fixed in the short run by the given capital stock.

The long run equilibrium with G' policies can be obtained by combining (12) and (18) to determine r, K, and π:

*For G' policies, I again must assume that $\pi^e < s$, so that an increase in $m+b$ (through a fall in P) reduces saving relative to investment.

$$G' - \tau F[K] = \pi(\hat{\mu}F[K] - K)$$

$$\ell[(1-\tau)(r+\pi),\ 1/\hat{u})\hat{u} = \gamma(\hat{\mu} - \alpha/r)$$

(20)

$$\alpha F[K] = rK$$

$$m + b + K = \hat{\mu}Y\ .$$

Differentiating,

$$\begin{bmatrix} 0 & \pi(1-\hat{\mu}r) - \tau r & K - \hat{u}Y & 0 \\ (1-\tau)\ell_1\hat{u} - \frac{\alpha\gamma}{r^2} & 0 & (1-\tau)\ell_1\hat{u} & 0 \\ -K & (\alpha-1)r & 0 & 0 \\ 0 & 1-\hat{\mu}r & 0 & 1 \end{bmatrix} \begin{bmatrix} \Delta r \\ \Delta K \\ \Delta\pi \\ \Delta(m+b) \end{bmatrix} = \begin{bmatrix} -\Delta G' \\ \left(\hat{u} - \frac{\alpha}{r}\right)\Delta\gamma \\ 0 \\ 0 \end{bmatrix} . \tag{21}$$

The sign of the Jacobian determinant

$$|J| = (1-\tau)\ell_1\hat{u}[(1-\alpha)r(\hat{u}Y - K) + K(\tau r - \pi(1-\hat{u}r))] - \alpha(1-\alpha)\gamma(\hat{\mu}Y - K)/r$$

depends upon the sign of $\tau r - \pi(1-\hat{\mu}r)$, which describes the effect of an increase in K on the funds available to support G' --increased tax revenue plus capital gains on a changed stock of debt. The solution of (21) is given by

$$\begin{bmatrix} \Delta r \\ \Delta K \\ \Delta\pi \\ \Delta(m+b) \end{bmatrix} = \frac{-1}{|J|} \begin{bmatrix} (1-\alpha)(1-\tau)r\ell_1\hat{u}\Delta G' - (1-\alpha)rY(\hat{u} - \alpha/r)^2\Delta\gamma \\ -(1-\tau)\ell_1\hat{u}K\Delta G' + KY\left(\hat{\mu} - \frac{\alpha}{r}\right)^2\Delta\gamma \\ (1-\alpha)\left(-(1-\tau)\ell_1\hat{u}r + \frac{\alpha\gamma}{r}\right)\Delta G' - K\left(\hat{u} - \frac{\alpha}{r}\right)(\tau r - \pi(1-\hat{u}))\Delta\gamma \\ (1-\hat{u}r)\ell_1\hat{u}K\Delta G' - (1-\hat{u}r)KY\left(\hat{u} - \frac{\alpha}{r}\right)^2 \end{bmatrix} . \tag{22}$$

If $\tau r - \pi(1 - \hat{\mu} r)$ is positive, then $|J|$ is negative and the signs of the multipliers are as displayed in Table 2. If $\tau r - \pi(1 - \hat{\mu} r)$ is negative with $|J|$ still negative, then $\Delta\pi/\Delta\gamma$ would be positive rather than negative. If $|J|$ is positive, then $\Delta\pi/\Delta\gamma$ would be negative and the signs of the remaining multiplier would be the opposite of those in Table 2. The most striking result is probably that an increased monetization of deficits raises the steady state rate of inflation only in a rather restricted region of the parameter space.

For stability analysis the characteristic equation of (10), using (18) to replace Y and (19) to replace π, is

$$0 = \begin{vmatrix} \tau r + s\hat{\mu} r - s - \lambda & \pi - s & \pi - s & m + b & 0 \\ -r\ell_2 - (\ell - \ell_2/\hat{\mu}) & 1 - (\ell - \ell_2/\hat{\mu}) & -(\ell - \ell_2/\hat{\mu}) & -(1-\tau)\ell_1\hat{\mu}Y & -(1-\tau)\ell_1\hat{\mu}Y \\ (\alpha-1)i - \lambda & 0 & 0 & 0 & -iK/r \\ -\tau\gamma r & -\pi - \lambda & 0 & -m(1+\lambda/\beta) & 0 \\ -\tau(1-\gamma)r & 0 & -\pi - \lambda & -b(1+\lambda/\beta) & 0 \end{vmatrix}$$

$$\begin{aligned} = & -(1-\tau)\ell_1\hat{\mu}Y(\lambda+\pi) - i(1-\alpha)s(m+b) - \frac{siK}{r}(\tau r - \pi(1 - \hat{\mu} r)) \\ & + \lambda[-s(m+b) + \frac{\pi iK}{r} - i(1-\alpha)(m+b) - iK(\tau + s(\hat{\mu} - 1/r))] \\ & + \lambda^2[-m - b + iK/r] - (\lambda/\beta)(s-\pi)(m+b)((1-\alpha)i + \lambda) \\ & + \frac{iK}{r}(\lambda+\pi) \quad (1-\alpha)ms + (m + r\ell_2(m+b))\lambda + \frac{\lambda}{\beta}[(1-\alpha)ms - (m+b)\pi\ell \\ & - ((m+b)\ell_2/\hat{\mu})(\tau r + \pi(1 - \hat{\mu} r)) - \alpha\tau m/\hat{\mu} + \lambda(\ell K + (m+b)\ell_2/\hat{\mu})] \quad . \end{aligned}$$

This is formidably ambiguous, even for extreme values of β .

G'' policies are more restrictive and yield more definite results.

TABLE 2

Long Run Multipliers with Full Employment

	G' Regime ($\pi r - \pi(1 - \hat{\mu}r) > 0$)		G" Regime	
	$\Delta G'$	$\Delta\gamma$	$\Delta G''$	$\Delta\gamma$
Δr	-	-	-	0
ΔK	+	+	+	0
$\Delta\pi$	+	-	+	-
$\Delta(m+b)$	?	?	?	0
stability	ambiguous		$\beta \to \infty$: unstable $\beta \to 0$: stable if $\pi > 0$ and $\gamma \geq \ell - \ell_2/\hat{u}$	

The long run equilibrium can be described by the combination of (15) and (18):

$$
(22)\qquad
\begin{aligned}
&G'' = \tau F[K] \\
&\ell[(1-\tau)(r+\pi),\ 1/\hat{\mu}]\hat{\mu} = \gamma\left(\hat{\mu} - \frac{\alpha}{r}\right) \\
&\alpha F(K) = rK \\
&\hat{\mu}F[K] = m + b + K\ .
\end{aligned}
$$

Differentiating,

$$
(23)\qquad
\begin{bmatrix}
0 & -\tau r & 0 & 0 \\
(1-\tau)\ell_1\hat{\mu} - \frac{\alpha\gamma}{r^2} & 0 & (1-\tau)\ell_1\hat{\mu} & 0 \\
-K & (\alpha-1)r & 0 & 0 \\
0 & \hat{\mu}r - 1 & 0 & -1
\end{bmatrix}
\begin{bmatrix}
\Delta r \\ \Delta K \\ \Delta\pi \\ \Delta(m+b)
\end{bmatrix}
=
\begin{bmatrix}
-\Delta G'' \\ \left(\hat{\mu} - \frac{\alpha}{r}\ \Delta\gamma\right) \\ 0 \\ 0
\end{bmatrix} .
$$

This Jacobian determinant

$$
|J| = -r\tau(1-\tau)\ell_1\hat{\mu}K > 0
$$

is positive and the multipliers are given by

$$
(24)\qquad
\begin{bmatrix}
\Delta r \\ \Delta K \\ \Delta\pi \\ \Delta(m+b)
\end{bmatrix}
= \frac{1}{|J|}
\begin{bmatrix}
(1-\alpha)(1-\tau)\tau\ell_1\hat{\mu}\Delta G'' \\
-(1-\tau)\ell_1\hat{\mu}K\Delta G'' \\
-(1-\alpha)((1-\tau)\tau\ell_1\hat{\mu} - \alpha\gamma/r)\Delta G'' - r\tau K(\hat{\mu} - \alpha/r)\Delta\gamma \\
(1 - \hat{\mu}r)(1-\tau)\ell_1\hat{\mu}K\Delta G''
\end{bmatrix} .
$$

The signs of these multipliers are collected in Table 2. The level of G" fixes the needed tax revenue and hence K. A higher K implies a lower marginal product of capital and real interest rate; financial market equilibrium requires a higher rate of inflation to offset the lower real return. The change in debt depends upon whether the demand for debt rises or falls as the capital stock increases. Since K and r are fixed by G", a change in γ only affects the rate of inflation. An increased monetization of deficits leads to a higher stock of money, which will be held only at a lower π (and nominal yield).

The characteristic equation of (11), using (18) and (19), is

$$0 = \begin{vmatrix} s(\hat{\mu}r - 1) + r\tau - \lambda & -s & -s & -(m+b)\lambda/\beta & 0 \\ -\ell_2 r - (\ell - \ell_2/\hat{\mu}) & 1 - (\ell - \ell_2/\hat{\mu}) & -(\ell - \ell_2/\hat{\mu}) & -(1-\tau)\ell_1\hat{\mu}Y & -(1-\tau)\ell_1\hat{\mu}Y \\ (\alpha-1)i - \lambda & 0 & 0 & 0 & -iK/r \\ -\tau\gamma r & -(1-\gamma)\pi - \lambda & \gamma\pi & 0 & 0 \\ -\tau(1-\gamma)r & (1-\lambda)\pi & -\gamma\pi - \lambda & 0 & 0 \end{vmatrix}$$

$$= (\lambda+\pi)(1-\tau)\ell_1\hat{\mu}Y\frac{iK}{r}[rs\tau + \lambda(r\tau + s(\hat{\mu}r - 1)) - \lambda^2] - (\lambda+\pi)\frac{m+b}{\beta}[i\tau K(-\gamma + \ell - \ell_2/\hat{\mu})$$

$$- \lambda iK(r\ell_2 + \ell - \ell_2/\hat{\mu})/r + (1-\tau)\ell_1\hat{\mu}Y(i(1-\alpha)\lambda + \lambda^2)] .$$

With perfect foresight $(\beta \to \infty)$, this reduces to

$$0 = (\lambda+\pi)(1-\tau)\ell_1\hat{\mu}Y\frac{iK}{r}[rs\tau + \lambda(r\tau + s(\hat{\mu}r - 1)) - \lambda^2] ,$$

which is an equation of the form

$$0 = (\lambda+\pi)(-a - b\lambda + \lambda^2)$$

$$= -a\pi - (a+b\pi)\lambda + (\pi-b)\lambda^2 + \lambda^3$$

where a is positive and b ambiguous. This is clearly not stable with $\pi > 0$. For $\pi \leq 0$, $\pi-b \geq 0$ implies $b \leq \pi \leq 0$, which implies $a+b\pi > 0$ so that the model is again unstable.

With static expectations $(\beta \to 0)$ the characteristic equation is dominated by

$$0 = (\lambda+\pi)(m+b)\{i\tau K(\gamma - \ell + \ell_2/\hat{\mu}) + \lambda[-i(1-\alpha)(1-\tau)\ell_1\hat{\mu}Y$$

$$+ iK\ell_2 + iK(\ell - \ell_2/\hat{\mu})/r] - \lambda^2(1-\tau)\ell_1\hat{\mu}Y\},$$

which is of the form

$$0 = (\lambda+\pi)(a + b\lambda + c\lambda^2)$$

$$= a\pi + (a+b\pi)\lambda + (b+c\pi)\lambda^2 + c\lambda^3$$

where b and c are positive and a is proportional to $\gamma - (\ell - \ell_2/\hat{\mu})$, the difference between the fraction of a deficit that is money financed and the marginal demand for money out of an increase in wealth. If $\gamma \geq \ell - \ell_2/\hat{\mu}$, then the model is stable for $\pi > 0$ and unstable for $\pi < 0$. The model will be unstable if $\gamma < \ell - \ell_2/\hat{\mu}$, since $a\pi$ will be negative if $\pi > 0$ and $a + b\pi$ will be negative if $\pi \leq 0$.

REFERENCES

Blinder, A. S. and R. M. Solow, "Does Fiscal Policy Matter?" Journal of Public Economics (1973), 319-337.

Blinder, A. S. and R. M. Solow, The Economics of Public Finance (Washington, D.C.: Brookings, 1974).

Tobin, J. and W. Buiter, "Long-Run Effects of Fiscal and Monetary Policy on Aggregate Demand," Chapter 4 of Monetarism, Studies in Monetary Economics, North-Holland, 1976.