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THE SENSITIVITY OF FISCAL-POLICY EFFECTS TO ASSUMPTIONS ABOUT THE BEHAVIOR OF THE FEDERAL RESERVE

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I. Introduction

Most examinations of fiscal-policy effects in U.S. econometric models are based on the assumption that the behavior of the Federal Reserve (henceforth call the "Fed") is exogenous, i.e., that the behavior of the Fed is not influenced by the state of the economy. The typical procedure is to assume that the Fed has control over a particular variable in the model and then to take this variable as exogenous for purposes of the fiscal-policy experiments. An alternative procedure, if one believes that the behavior of the Fed is not exogenous, is to estimate an equation explaining Fed behavior (i.e., explaining the variable that the Fed is assumed to control), add this equation to the model, and use this expanded model to perform the fiscal-policy experiments.

The purpose of this paper is to examine within the context of a particular U.S. econometric model the sensitivity of fiscal-policy effects to alternative assumptions about Fed behavior. Five cases are considered, four in which Fed behavior is exogenous and one in which Fed behavior is endogenous. In each of the four exogenous cases the Fed is assumed to control a particular variable, which is then taken to be exogenous

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for purposes of the fiscal-policy experiments. The control variables in the four cases are: (1) the amount of government securities outstanding; (2) the money supply; (3) nonborrowed reserves; and (4) the bill rate. For the endogenous case an estimated equation explaining Fed behavior is added to the model, and the expanded model is used to perform the fiscal-policy experiments.

Section II contains a brief description of the econometric model used for purposes of this paper. The model, which is described in detail in Fair [9], is particularly suited for examining the effects of monetary and fiscal policies because it is closed with respect to the flows of funds in the system. This means, among other things, that the government budget constraint is accounted for and that the amount of government securities outstanding can be taken to be a direct policy variable of the Fed. The equation explaining Fed behavior is presented and discussed in Section III, and the results of the various fiscal-policy experiments are presented in Section IV. Some optimal control results are then reported in Section V. Given that an equation explaining Fed behavior has been estimated, it is possible to conduct optimal control experiments in which the fiscal authorities maximize an objective function taking into account the behavior of the Fed. A comparison can then be made, as is done in Section V, between how well the fiscal authorities do in this case versus the case in which the behavior of the Fed is exogenous.

The main conclusion of this paper is that fiscal-policy effects are quite sensitive to alternative assumptions about the behavior of the Fed. When, for example, the Fed behaves as estimated in Section III, the effect on real output of an increase in government expenditures on goods after eight quarters is less than half of the effect that occurs when the Fed behaves by keeping the bill rate unchanged. After twelve quarters, the effect is less than one-third. When the Fed behaves by keeping the money supply unchanged, fiscal policy is effective at all only for about the first eight quarters after the policy change. With respect to the optimal control results, when the Fed behaves as estimated in Section III, the fiscal authorities cannot achieve as low a value of loss as they can when the Fed behaves by keeping the bill rate unchanged. Also, much more fiscal stimulus is required in the case in which the Fed behaves as estimated in Section III, even to obtain the somewhat higher value of the loss, because of the need to offset the negative response of the Fed to the stimulus. When the Fed behaves by keeping the money supply unchanged, the fiscal authorities' ability to lower the value of loss at all is severely limited.

Before proceding with the discussion of the model, mention should be made about how this study relates to the literature. There have been a number of studies concerned with estimating equations to explain the behavior of the Fed. These include Dewald and Johnson [4], Goldfeld [13], Wood [22], Havrilesky [15], Christian [3], Teigen [20], Keran and Balb [16], Friedlaender [11], and Froyen [12]. The work in Section III of this paper is an addition to this literature. The main difference between the present work and previous work is the treatment in the present case of the bill rate as the policy variable of the Fed. With the exception of one set of results in Dewald and Johnson [4] and Christian [3], previous studies have not done this.[1] An argument is presented in Section III in favor of taking the bill rate to be the policy variable of the Fed

[1] See, for example, Table 3 in Froyen [12] for a list of the various policy variables used in five of the above studies.

over other possible variables that one might consider.

Work similar to that in Section IV does not appear to have been done previously. While it is well known that multipliers in models can be quite different depending on whether or not reaction functions of the monetary and fiscal authorities are postulated, there has not been to the author's knowledge any previous study in which this question is examined using an actually estimated reaction function and an actual econometric model. Goldfeld and Blinder [14], for example, in their important paper in this area use an actual econometric model in some of their work (the Moroney-Mason model [17]), but the reaction functions that they use are made up. Teigen [20] estimated an equation explaining Fed behavior within the context of a complete model, but he did not simulate the model to examine its properties. The work in Section IV thus appears to be an attempt for the first time to gauge the actual quantitative importance of endogenous Fed behavior on fiscal multipliers.

The optimal control work in Section V is perhaps most closely related to the work of Pindyck [18], although there are significant differences between the approach here and Pindyck's approach. Pindyck sets up the problem of two independent authorities with conflicting objectives as a differential game and applies Nash solution strategies to it. In the present case the problem is set up as one in which the fiscal authorities solve an optimal control problem given the behavior of the Fed. Fed behavior can be endogenous in the problem, as it is for one of the problems solved in Section V, but the Fed does not play a game with the fiscal authorities. For the case in which Fed behavior is endogenous, the reaction function of the Fed is determined first, and then the optimal control problem of the fiscal authorities is solved given this function.

Finally, it should be noted that if the results in this study are valid, they cast doubt on the properties of the MPS model and on the measures of fiscal and monetary policies reported in Blinder and Goldfeld [1]. This issue is discussed at the end of Section IV.

II. The Econometric Model

Since the econometric model that is used in this paper is described in detail in Fair [9], it will only be briefly discussed here. The model consists of 84 equations, 26 of which are stochastic. There are five sectors (household, firm, financial, foreign, and government) and five categories of financial securities (demand deposits and currency, bank reserves, member bank borrowing from the Fed, gold and foreign exchange, and an "all other" category).

For future reference it will be useful to note a few of the properties of the model. First, the bond rate has a positive effect on the price set by the firm sector. In the theoretical model of the economy [8] that was used to guide the specification of the econometric model, a higher interest rate causes a firm to contract, and the primary way that a firm contracts in the theoretical model is to raise its price, which lowers expected sales, and then to lower production, investment, and employment. The empirical work appeared to confirm this property of a higher interest rate leading to a higher price in that the bond rate turned out to be a significant explanatory variable in the price equation of the firm sector.

Second, there are no stable relationships in the econometric model between 1) the unemployment rate and the rate of inflation (i.e., no stable Phillips curve), 2) aggregate demand and the rate of inflation, and 3) real output and the unemployment rate (i.e., no stable relationship called Okun's law). It is argued in [9] that this lack of stable relationships is not surprising, since each of the variables in question is influenced by a large number of other variables in the model. There is no particular reason to expect that the combined influences on any pair of them are such as to lead to a stable relationship between the two.[2]

Finally, some optimal control experiments performed with the model indicate that it has the property for a fairly wide range of loss functions that the extra-inflation cost of increasing output (to some reasonable target level) is much less than the lost-output cost of lowering the rate of inflation. When loss functions in the level of output and the rate of inflation are minimized, the optima tend to correspond to a more closely met output target.

The most important characteristic of the model regarding the results in this paper is the fact that it is closed with respect to the flows of funds in the system. It is easiest to see this feature by considering the sixteen-equation representation of the model in Table 1. The model is almost separable into two blocks, denoted as Blocks I and II in Table 1. In Block I, equations 1-5 determine the financial saving of each sector, and equations 6 and 7 determine the demand for money of the household and firm sectors. The "saving" of a sector is the difference between the sector's receipts and expenditures; by definition the savings of all sectors sum to zero, since a receipt of one sector is a corresponding

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[2] See pages 7-9 in [9] for a discussion of the various factors that influence each variable. The unemployment rate, for example, is influenced by all the factors that influence the supply of labor of the household sector, including such variables as the interest rate and the level of assets.

TABLE 1. A SIXTEEN-EQUATION REPRESENTATION OF THE MODEL IN [9]

Five Sectors: Household (H), Firm (F), Financial (B), Foreign (R), Government (G).

Notation in Alphabetic Order (Subscript t refers to quarter t ; flow variables are at quarterly rates; stocks are end-of-period stocks; a † denotes an exogenous variable; BCURT denotes that the unit of the variable is in billions of current dollars.)

$BORR_t$ = member bank borrowing from the Fed, BCURT .

BR_t = bank reserves, BCURT .

†$CURR_t$ = value of currency outstanding, BCURT .

DDB_t = value of demand deposits held in the financial sector, BCURT .

DDF_t = value of demand deposits and currency held by the firm sector, BCURT .

DDH_t = value of demand deposits and currency held by the household sector, BCURT.

†DDR_t = value of demand deposits and currency held by the foreign sector, BCURT .

†g_{1t} = reserve requirement ratio, percent.

†$GFXG_t$ = value of gold and foreign exchange held by the government sector, BCURT .

$RBILL_t$ = three-month treasury bill rate, percentage points.

†RD_t = the discount rate, percentage points.

$SAVB_t$ = saving of the financial sector, BCURT .

$SAVF_t$ = saving of the firm sector (denoted as $\overline{CF}_t$ in [9]).

$SAVG_t$ = saving of the government sector.

$SAVH_t$ = saving of the household sector.

$SAVR_t$ = saving of the foreign sector.

$SECB_t$ = net value of SEC held by the financial sector, BCURT (denoted as $LBVBB_t$ in [9]).

$SECF_t$ = net value of SEC held by the firm sector, BCURT (denoted as $-LF_t$ in [9]).

†$SECG_t$ = net value of SEC held by the government sector, BCURT (denoted as $-VBG_t$ in [9]).

$SECH_t$ = net value of SEC held by the household sector, BCURT (denoted as A_t in [9]).

$SECR_t$ = net value of SEC held by the foreign sector, BCURT .

NOTE: Z_t = vector of all predetermined variables in the model except the above exogenous variables.

TABLE 1 (continued)

Equations

BLOCK I (representing 75 of the 84 equations in the complete model):

1. $SAVH_t = f_1(RBILL_t, Z_t)$

2. $SAVF_t = f_2(RBILL_t, Z_t)$

3. $SAVB_t = f_3(RBILL_t, Z_t)$

4. $SAVR_t = f_4(RBILL_t, Z_t)$

5. $SAVG_t = -(SAVH_t + SAVF_t + SAVB_t + SAVR_t)$

6. $DDH_t = f_6(RBILL_t, Z_t)$

7. $DDF_t = f_7(RBILL_t, Z_t)$

Eq. No. in [9]	BLOCK II
62	8. $DDB_t = DDH_t + DDF_t + DDR_t - CURR_t$
45	9. $BR_t = g_{1t}DDB_t$
20	10. $BORR_t/BR_t = 0.0121 + 0.0106(RBILL_t - RD_t)$
61	11. $0 = SAVH_t - (SECH_t - SECH_{t-1}) - (DDH_t - DDH_{t-1})$
55	12. $0 = SAVF_t - (SECF_t - SECF_{t-1}) - (DDF_t - DDF_{t-1})$
64	13. $0 = SAVB_t - (SECB_t - SECB_{t-1}) + (DDB_t - DDB_{t-1}) - (BR_t - BR_{t-1}) + (BORR_t - BORR_{t-1})$
66	14. $0 = SAVR_t - (SECR_t - SECR_{t-1}) - (DDR_t - DDR_{t-1}) + (GFXG_t - GFXG_{t-1})$
69	15. $0 = SAVG_t - (SECG_t - SECG_{t-1}) + (CURR_t - CURR_{t-1}) + (BR_t - BR_{t-1}) - (BORR_t - BORR_{t-1}) - (GFXG_t - GFXG_{t-1})$
70	16. $0 = SECH_t + SECF_t + SECB_t + SECR_t + SECG_t$

expenditure of some other sector. Equations 1-7 are "quasi" reduced-form equations, and they represent most of the equations in the complete model. The vector Z_t includes all the predetermined variables in the model other than the exogenous variables listed explicitly in Table 1. Equations 1-7 are only "quasi" reduced-form equations because the bill rate, which is an endogenous variable in the model, appears on the right hand side of them.

Block II consists of equations 8-16 in Table 1. Equation 8 is a definition: the value of demand deposits held in the financial sector (DDB_t) is equal to the value of demand deposits and currency held by the household (DDH_t), firm (DDF_t), and foreign (DDR_t) sectors less the value of currency outstanding $(CURR_t)$. DDR_t and $CURR_t$ are both taken to be exogenous in the model.[3] Equation 9 relates the actual level of bank reserves (BR_t) to DDB_t. Since g_{1t} is the reserve requirement ratio and BR_t is the actual level of reserves, equation 9 reflects the assumption that the banking system holds no excess reserves in the aggregate.[4] Equation 10 explains member bank borrowing from the Fed. It is a stochastic equation in the model, and the two coefficients in the equation are estimated coefficients. The equation states that the ratio of borrowed to total reserves is a function of the difference

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[3]It should be noted that the treatment of $CURR_t$ as exogenous does not mean that the money supply is treated as exogenous. The holdings of demand deposits and currency of the household and firm sectors $(DDH_t$ and $DDF_t)$ are explained by stochastic equations in the model. What is not explained is the division of these holdings between demand deposits and currency.

[4]More precisely, equation 9 reflects the assumption that the ratio of excess reserves to total reserves is exogenous. See the discussion on pages 135-137 in [9] for more details.

between the bill rate and the discount rate.

Equations 11-15 link the saving of each sector to changes in the sector's assets and liabilities. The main category of securities in the model is the "all other" or "SEC" category, which is an aggregate of all types of securities in the economy except demand deposits and currency, bank reserves, member bank borrowing from the Fed, and gold and foreign exchange. The SEC variable for each sector is the sector's net holdings of the securities, so that if a sector is a net debtor with respect to the category, then the value of the SEC variable for the sector is negative. The household and financial sectors are net creditors with respect to the SEC category ($SECH_t > 0$ and $SECB_t > 0$), and the firm, foreign, and government sectors are net debtors ($SECF_t < 0$, $SECR_t < 0$, and $SECG_t < 0$). In what follows $-SECG_t$ will sometimes be referred to as the "amount of government securities outstanding."

Equation 11 states that any nonzero level of saving of the household sector must result in the change in either the sector's holdings of demand deposits and currency (DDH_t) or its net holdings of SEC ($SECH_t$), or both. Equations 12-14 are similar equations for the firm, financial, and foreign sectors. Equation 15 is sometimes called the government budget constraint. It states that any nonzero level of saving of the government must result in the change in at least one of the following: currency outstanding ($CURR_t$), nonborrowed reserves ($BR_t - BORR_t$), gold and foreign exchange holdings of the government ($GFXG_t$), or the government's net holdings of SEC ($SECG_t$). Equation 16 states that the sum of all SEC securities across sectors is zero: an asset of one sector is a corresponding liability of some other sector.[5] The existence

[5] In the actual model equation 16 has to be modified to allow for capital gains and losses on stocks, but this feature can be ignored for purposes of the present discussion.

of equations 11-16 in the model means that all flows of funds are accounted for.

One of the 16 equations in Table 1 is redundant because the savings of all sectors sum to zero, and so one of them can be dropped. A convenient equation to drop is equation 15, the government budget constraint. This leaves 15 independent equations. If one takes as exogenous the discount rate (RD_t), the reserve requirement ratio (g_{1t}), and the government's net holdings of securities $(SECG_t)$ and if one considers equations 11-14 as determining, respectively, $SECH_t$, $SECF_t$, $SECB_t$, and $SECR_t$, then there is one "extra" equation in the model, equation 16. One other variable must be chosen to be endogenous in order to close the model, and the obvious variable to choose in this case is the bill rate, $RBILL_t$. There is no equation in the model in which the bill rate appears naturally on the left hand side, and so when it is taken to be endogenous, it is implicitly determined. In the solution of the model each period the bill rate must be such as to make equation 16 hold. (In the actual numerical solution of the model in the endogenous bill rate case, by the Gauss-Seidel method, the structural equation for DDH_t is used to solve for $RBILL_t$, equation 11 is used to solve for DDH_t, and equation 16 is used to solve for $SECH_t$.)

The division of the model into two blocks in Table 1 allows one to see clearly how the Fed affects the economy. By changing $SECG_t$, RD_t, or g_{1t}, the Fed changes the bill rate, which in turn affects the variables in Block I. As the model is presented in Table 1, the Fed has no control over the variables in Block I except through the bill rate. This is not quite true in the actual model, however, because interest payments of the firm and government sectors in Block I are a function of $SECF_t$ and $SECG_t$. These two variables thus appear as explanatory variables in Block I, and so the actual model is not quite separable into the two blocks. The interest-payment effects are, however, fairly small. The model is also not quite separable into two blocks in a dynamic sense because the lagged value of $SECH_t$ appears as an explanatory variable in the consumption and labor-supply equations of the household sector in Block I. The Fed thus has some control over the variables in Block I other than through the bill rate through its effect on $SECH_t$ over time. The main effect of the Fed on the variables in Block I, however, is through the bill rate.

Consider for now the model as represented in Table 1, where the Fed only affects the economy through the bill rate. As just mentioned, this is a fairly good approximation. In this case there are no differences among the three primary control variables of the Fed ($SECG_t$, g_{1t} , and RD_t) regarding their effects on the variables in Block I. The Fed changes the bill rate by changing the supply of funds in the economy, and the supply of funds can be changed by changing any of the three variables. From equation 15 it can be seen that a one unit change in $-SECG_t$ is equivalent to a one unit change in BR_t and also to a one unit change in $-BORR_t$. From equation 9 a one unit change in BR_t is equivalent to a one unit change in $g_{1t}DDB_t$, and from equation 10 a one unit change in $-BORR_t$ is equivalent to a one unit change in $0.0106 \cdot BD_t \cdot BR_t$. It is thus possible to define an approximate overall measure of the (one) effective control variable of the Fed, denoted, say, as MP_t , as:

$$(1) \qquad MP_t = -SECG_t + g_{1t}DDB_t + 0.0106 \cdot RD_t BR_t \ .$$

MP_t is only an approximate measure of the effective control variable of the Fed because DDB_t and BR_t are endogenous and because the 0.0106 coefficient is only estimated as opposed to being known exactly. The main point, however, is that the Fed has only one effective control variable at its disposal, an approximation to this variable being MP_t. It matters little whether MP_t is changed by changing $SECG_t$, g_{1t}, or RD_t: the effects on the variables in Block I are the same in the three cases except for the above mentioned interest-payment effects and the effects of $SECH_{t-1}$ on the consumption and labor-supply decisions of the household sector.

With respect to the work in the next section, it is important to note the following. If the Fed though day-to-day changes in $SECG_t$, g_{1t}, and RD_t can control the bill rate, if changes in these three variables have little effect on the variables in Block I except through the bill rate, and if the Fed is interested in controlling the variables in Block I, then it should use the bill rate as its control variable. The Fed should, in other words, set the bill rate each period to whatever value it thinks is optimal for attaining its goods regarding the variables in Block I. More will be said about this in the next section.

For the empirical work in Sections IV and V the model was re-estimated through 1976II using the revised NIA data. A few small changes had to be made in the model to account for some definitional changes in the new data. The model was estimated by the two stage least squares technique as described in Chapter Three in [9]. The quantitative properties of the re-estimated model differed little from those of the original model. When, for example, the properties of the re-estimated model were examined in the manner described for the original model in Chapter Nine in [9], the new results were quite similar to the old. The reader is thus referred to this chapter for a fairly accurate description of the properties of the re-estimated model. The new coefficient estimates are available from the author upon request.[6]

III. An Explanation of Fed Behavior

For the work in this section the Fed is assumed to take the bill rate as its control variable and to choose the value of this variable each period by solving an optimal control problem. This assumption, of maximizing behavior on the part of the Fed, is analogous to the above-mentioned assumptions of maximizing behavior on the part of the banks, firms, and households in the theoretical model of the economy [8].

In order for the Fed to solve optimal control problems, it must have at its disposal some model of the economy. It will be convenient for purposes of the following discussion to write the g^{th} equation of whatever model the Fed is using as:

$$\phi_g(y_t, q_{t-1}, z_t, x_t, \beta_g) = u_{gt} , \quad (g = 1, \ldots, G) , \tag{2}$$

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[6] It is interesting to note that the two demand-for-money equations in the model (explaining DDH_t and DDF_t) do not appear to break down in the 1974-75 period. This is in contrast, for example, to the demand for money equation in the MPS model as reported in Enzler, Johnson, and Paulus [5]. The MPS equation, when estimated through 1972IV and simulated dynamically for the 1973I-1976I period, shows a percentage error (predicted minus actual) for 1976I of 14.6 percent (p. 264 in [5]). When the DDH_t and DDF_t equations in the present model were estimated through 1972IV and simulated dynamically for the 1973I-1976I period, the combined percentage error for 1976I was only 2.6 percent. When various modified versions of the MPS equation were estimated through 1974II and simulated dynamically for the 1974III-1976I period, the percentage error for 1976I ranged from 5.8 to 10.6 percent (p. 276 in [5]). This compares to a combined percentage error of 1.9 percent for the DDH_t and DDF_t equations in the present model for the same experiment.

where y_t is a G-component vector of the endogenous variables, q_{t-1} is a vector of all lagged endogenous variables (including any endogenous variables lagged more than one period), z_t is a vector of exogenous variables not under the control of the Fed (including exogenous government variables from other branches of the government), x_t is the control variable of the Fed, β_g is the vector of coefficients in equation g, and u_{gt} is an error term. If equation g is an identity, then u_{gt} is zero for all t. In what follows the β_g vectors are assumed to be known exactly, although in practice only estimates of them are available.

Assume now that the Fed at the beginning of period t is concerned with maximizing the expected value of some objective function for periods t through $t+T$. Write this function as:

$$(3) \qquad W = h(y_t, \ldots, y_{t+T}; x_t, \ldots, x_{t+T}) .$$

In principle this function can include as arguments all the endogenous variables; in practice it is likely to include only a small subset of them. The control variable is included as a possible argument of the function, since, for example, the Fed may attach some cost to large changes in the variable. The control problem for the Fed is to choose the $T+1$ values, $x_t, \ldots, x_{t+T}$, so as to maximize EW subject to the model in (2).

The success of the Fed in achieving its goals depends, among other things, on the quality of the model it uses. A good model should capture the private agents' reactions to the behavior of the Fed, including any effects of Fed behavior on people's expectations. If private agents are solving their own control problems given initial conditions and their expectations of the future, then anything that the Fed does to modify initial conditions or expectations will affect people's decisions, and a good model should capture this. The Fed is effectively solving its own control problem subject to the restriction that the private agents in the economy are solving their own control problems given the behavior of the Fed.[7]

If the model that the Fed uses is similar to the model represented in Table 1, if the endogenous variables that are included in the objective function are primarily variables in Block I of this model, and if the Fed on a day-to-day basis can control the bill rate, then, as mentioned in the previous section, the Fed should take the bill rate as its control variable for purposes of solving the above problem. In other words, the model in (2) should be taken to be the model in Block I in Table 1, and x_t should be the bill rate. The reason for this is simple. The Fed by behaving in this way is eliminating one element of uncertainty in the problem, namely the uncertainty regarding the relationship between the three primary control variables ($SECG_t$, g_{1t}, and RD_t) and the bill rate. In any given period, say a month or a quarter, this relationship is uncertain because of the random elements in the system, but as long as the Fed can make frequent changes in the primary control variables within the period, it can offset the effects of any random shocks. It is thus optimal for the Fed to take the bill rate as its effective con-

[7] In the theoretical model of the economy in [8], upon which the econometric model used in this paper is based, economic agents form their decisions by solving multiperiod optimal control problems, given initial conditions and their expectations of the future. They do not, however, have complete knowledge of the model, and so their expectations are not "rational." The model is one with maximizing agents who do not have rational expectations. See Fair [10] for an argument in favor of this type of model over the three other possible combinations: maximizing agents with rational expectations, non-maximizing agents with rational expectations, and non-maximizing agents without rational expectations.

trol variable unless the direct effects of the three primary control variables on the variables in Block I are important. Given the present model, this latter case does not seem likely, and so the bill rate was chosen here as the policy variable of the Fed.

The above control problem of the Fed is not easy to solve for nonlinear models, even if the objective function in (3) is quadratic and additive across time. This is not the place, however, to discuss the various ways in which problems of this kind can be solved.[8] For present purposes it is sufficient to note that in the solution of the above problem, the optimum value of x_t, say x_t^*, turns out to be a function of q_{t-1} and all future values of z :

$$x_t^* = f(q_{t-1}, z_t, z_{t+1}, \ldots, z_{t+T}) . \tag{4}$$

For example, in the simple linear-quadratic case in Chapter 7 in Chow [2], where the model is $y_t = Ay_{t-1} + Cx_t + b_t + u_t$ and the objective function is $W = \sum_{j=t}^{t+T} (y_j - a)'K(y_j - a)$, the optimum value of x_t is:

$$x_t^* = Gy_{t-1} + g_t , \tag{5}$$

where g_t is a function of $b_t, \ldots, b_{t+T}$.[9] b_t is a vector of the non-controlled exogenous variables in the system. Equation (4) is analogous to equation (5), the only difference being that in the nonlinear-

[8] See, for example, Fair [7] and Chapter 12 in Chow [2] for a discussion of this issue.

[9] See equations (23) and (28) in Chapter 7 in Chow [2] for the determination of g_t .

model case it is generally not possible to obtain an analytic expression for x_t^* .

Many of the future values of z are not likely to be known by the Fed at the time the control problem is solved, and so the solution must be based on expectations of these values. Equation (4) can thus be interpreted as stating that x_t^* is a function of the initial state of the economy, as reflected in the q_{t-1} vector, and of the Fed's expectation of future exogenous-variable values, as reflected in the z vectors. The equation thus suggests for empirical work that one consider regressing x_t on variables representing the initial state of the economy and on variables that one believes affect the Fed's expectation of future exogenous-variable values. Any estimated equation of this kind is, of course, only an approximation to the actual behavior of the Fed, just as estimated consumption and labor-supply equations are only approximations to the actual behavior of households.

After some experimentation, the estimated equation that was chosen as the explanation of Fed behavior is the following:

$$\text{(6)} \quad RBILL_t = \underset{(2.93)}{-11.1} + \underset{(16.30)}{0.841}\, RBILL_{t-1} + \underset{(1.69)}{0.0497}\, \%PD_{t-1} + \underset{(2.97)}{0.0352}\, J_t^*$$

$$+ \underset{(1.62)}{0.0427}\, \%GNPR_t + \underset{(1.36)}{0.0188}\, \%GNPR_{t-1} + \underset{(2.10)}{0.0251}\, \%M1_{t-1} \; ;$$

$$\hat{\rho} = \underset{(2.28)}{0.229} \; , \quad SE = 0.474 \; , \quad R^2 = 0.939 \; , \quad DW = 1.82 \; ,$$

Sample period = 1954 I-1976 II ,

where $RBILL_t$ = three-month treasury bill rate, percentage points,

$\%PD_t$ = percentage change at an annual rate in the price deflator for domestic sales, percentage points,

J_t^* = a measure of labor market tightness in the model in [9],

$\%GNPR_t$ = percentage change at an annual rate in real GNP, percentage points,

$\%M_{1t}$ = percentage change at an annual rate in the money supply, percentage points.

(t-statistics in absolute value are in parentheses.)

Equation (6) states that the current bill rate is a positive function of the lagged rate of inflation, of the current degree of labor market tightness, of the current and lagged rates of growth of real GNP, and of the lagged rate of growth of the money supply. The use of the current values of labor market tightness and real GNP growth is based on the assumption that the Fed makes more than one decision regarding the bill rate within a quarter. If, for example, the Fed makes decisions regarding the bill rate once a month, then the average bill rate for the quarter is partly based on information that becomes available during the first two months of the quarter. The estimation results were not very sensitive to the use of lagged rather than current values of labor market tightness and real GNP growth, but the use of current values led to slightly better results. For purposes of estimation, J_t^* and $\%GNPR_t$ were treated as endogenous variables, and the equation was estimated under the assumption of first-order serial correlation of the error terms by the two-stage least squares technique described in Fair [6].

The behavior reflected in equation (6) is behavior in which the Fed "leans against the wind." The wind in this case is composed of the inflation rate, the degree of labor market tightness, the growth rate of real GNP, and the growth rate of the money supply. As these variables rise, so also does the bill rate. The lagged bill rate in the equation is probably best interpreted as partly picking up expectational effects and partly reflecting the desire of the Fed not to have large quarter-to-quarter changes in the bill rate.

The fit of equation (6) is fairly good, with an estimated standard error of 0.474 percentage points. The estimate of the serial correlation coefficient of 0.229 is not very large. Also, the coefficient estimates of the equation are not highly sensitive to the use of alternative sample periods. A Chow test, for example, accepted the hypothesis that the coefficients are the same for the periods before and after 1969I.[10] In other words, the test accepted the hypothesis that there was no structural change in Fed behavior with the advent of Arthur Burns. On statistical grounds, therefore, the equation does not seem too bad as an explanation of Fed behavior, although the approximate nature of any equation of this kind should be kept in mind.

Equation (6) can be easily added to the econometric model discussed in Section II. The price deflator for domestic sales, PD_t, is one of the deflators in the model, and J_t^* is the primary measure of labor market tightness used in the model. J_t^* is a detrended ratio of total hours paid for in the economy to the total population 16 and over. (Both in the overall model and in equation (6), the use of J_t^* as the measure of labor market tightness turned out to give somewhat better results than did the use of one minus the unemployment rate. The results using the two measures were, however, fairly close.) The money supply variable, M_{1t}, in equation (6) is equal to DDB_t + $CURR_t$ + a few small exogenous terms. DDB_t and $CURR_t$ are defined in Table 1. Since $CURR_t$ is always taken to be exogenous in the model, it does not matter for purposes of the results in the next two sections whether DDB_t or M_{1t} is taken as the measure

[10] The F value was 1.06, which compares to the critical F value at the 95 percent confidence level of 2.08. The Chow test is only approximate in this case because of the endogenous explanatory variables in the equation and the existence of serial correlation of the error term.

of the money supply. In any experiment, the endogenous change in M_{1t} will be the same as the endogenous change in DDB_t . In the discussion of the results in the next two sections, DDB_t will be referred to as the money supply. Real GNP, $GNPR_t$, in equation (6) does not appear explicitly in the econometric model, but a definitional equation explaining it can be easily added to the model. Real GNP is the sum of the real outputs of the firm, financial, and government sectors in the model.

IV. Simulation Results

The results of five experiments are reported in this section. Each experiment corresponds to a sustained increase in the real value of goods purchased by the government (denoted as XG in the model) of 1.25 billion dollars beginning in 1971I, a quarter that is at or near the bottom of a contraction. All flow variables in the model are at quarterly rates, so that the 1.25 billion-dollar increase is an increase of 5.0 billion dollars at an annual rate. The experiments are based on different assumptions about the behavior of the Fed.

The experiments were performed as follows. The residuals obtained in the process of estimating each equation of the model were first added to the equations. This means that when the model is simulated using the actual values of all exogenous variables, the predicted values of all endogenous variables are equal to their actual values. In other words, a perfect tracking solution is obtained. These residuals were then used for all of the experiments. All simulations were dynamic, 12-quarter simulations for the period 1971I-1973IV. For all experiments, the reserve requirement ratio, g_{1t} , and the discount rate, RD_t , were taken to be exogenous. For the first experiment, the amount of government securities outstanding, $-SECG_t$, was also taken to be exogenous. This means, from equation 15 in Table 1, that any dissaving of the government that results from the increase in XG is financed by an increase in nonborrowed reserves, $BR_t - BORR_t$.[11] In other words, any deficit is financed by an increase in high-powered money: the Fed buys the securities that the Treasury issues to finance the deficit.

For the other four experiments, $SECG_t$ was taken to be endogenous. This requires, in order to close the model, either that one variable that was endogenous in the first experiment be taken to be exogenous or that an extra equation be added in which no new endogenous variable is introduced. For the second experiment the money supply (DDB) was taken to be exogenous; for the third experiment the level of nonborrowed reserves (BR - BORR) was taken to be exogenous; and for the fourth experiment the bill rate (RBILL) was taken to be exogenous. "Exogenous" here means that in the simulation runs the values of these variables for each period were kept unchanged from their actual (historic) values. For the fifth experiment no extra variable was taken to be exogenous, but instead the equation explaining Fed behavior from Section III was added to the model. This equation introduces no new endogenous variable, and so it meets the requirement for the model to be closed.

To summarize, the behavior of the Fed in response to the increase in XG in the five experiments is as follows: (1) The Fed allows any government deficit resulting from the increase in XG to be financed by an increase in nonborrowed reserves; (2) The Fed allows no change in the money supply to take place; (3) The Fed allows no change in nonbor-

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[11] Remember that $CURR_t$ and $GFXG_t$ in equation 15 in Table 1 are taken to be exogenous in the model.

TABLE 2. RESULTS FOR FIVE EXPERIMENTS

Effects of a permanent increase in XG of 1.25 billion dollars beginning at quarter t .

t = 1971I (bottom of a contraction).

Assumptions about Fed behavior: 1 = SECG exogenous
2 = DDB exogenous
3 = BR - BORR exogenous
4 = RBILL exogenous
5 = Fed behaves according to equation (6) in Section III.

VARIABLES	QUARTERS											
ΔRBILL	t	t+1	t+2	t+3	t+4	t+5	t+6	t+7	t+8	t+9	t+10	t+11
1	-0.86	-0.35	0.41	0.83	0.79	0.34	-0.18	-0.50	-0.54	-0.53	-0.28	-0.17
2	0.13	0.27	0.42	0.41	0.34	0.37	0.40	0.45	0.47	0.53	0.59	0.58
3	0.03	0.08	0.14	0.19	0.21	0.25	0.27	0.28	0.31	0.33	0.34	0.37
4	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
5	0.10	0.20	0.26	0.30	0.31	0.31	0.30	0.28	0.27	0.25	0.24	0.24
ΔY												
1	1.60	2.92	3.43	3.27	2.62	2.10	1.98	2.14	2.36	2.44	2.45	2.39
2	1.23	1.73	1.92	1.89	1.67	1.42	1.13	0.83	0.52	0.26	0.06	-0.09
3	1.27	1.88	2.25	2.41	2.32	2.14	1.92	1.66	1.40	1.11	0.89	0.74
4	1.28	1.93	2.38	2.66	2.73	2.75	2.72	2.62	2.47	2.24	2.04	1.91
5	1.24	1.77	2.04	2.12	1.95	1.72	1.48	1.25	1.03	0.84	0.71	0.66
100·ΔPF												
1	-0.091	-0.062	0.005	0.097	0.197	0.243	0.254	0.261	0.286	0.326	0.391	0.448
2	0.022	0.060	0.104	0.157	0.215	0.269	0.318	0.367	0.408	0.448	0.481	0.516
3	0.012	0.037	0.067	0.109	0.164	0.217	0.266	0.315	0.363	0.413	0.457	0.502
4	0.009	0.028	0.049	0.079	0.119	0.158	0.198	0.241	0.289	0.349	0.409	0.464
5	0.019	0.052	0.087	0.135	0.195	0.246	0.291	0.334	0.373	0.412	0.446	0.482
ΔUR												
1	-0.12	-0.35	-0.44	-0.41	-0.30	-0.20	-0.16	-0.18	-0.23	-0.30	-0.35	-0.39
2	-0.09	-0.19	-0.21	-0.19	-0.15	-0.10	-0.06	-0.01	0.03	0.06	0.08	0.10
3	-0.09	-0.21	-0.26	-0.26	-0.24	-0.21	-0.18	-0.14	-0.11	-0.09	-0.08	-0.07
4	-0.10	-0.21	-0.27	-0.29	-0.30	-0.30	-0.30	-0.29	-0.29	-0.30	-0.30	-0.31
5	-0.09	-0.20	-0.23	-0.22	-0.19	-0.15	-0.11	-0.08	-0.05	-0.04	-0.03	-0.04
ΔSAVR												
1	0.16	0.23	0.25	0.22	0.20	0.19	0.21	0.24	0.28	0.29	0.29	0.31
2	0.11	0.13	0.15	0.14	0.15	0.14	0.12	0.11	0.10	0.08	0.07	0.05
3	0.11	0.15	0.17	0.18	0.20	0.19	0.18	0.18	0.17	0.16	0.15	0.14
4	0.11	0.15	0.19	0.20	0.23	0.24	0.25	0.26	0.27	0.26	0.25	0.27
5	0.11	0.14	0.16	0.16	0.17	0.16	0.15	0.15	0.14	0.14	0.13	0.13

TABLE 2 (continued)

ΔSAVG	t	t+1	t+2	t+3	t+4	t+5	t+6	t+7	t+8	t+9	t+10	t+11
1	-0.62	0.18	0.41	0.30	0.06	-0.27	-0.37	-0.28	-0.13	-0.06	0.03	0.01
2	-0.61	-0.38	-0.30	-0.35	-0.42	-0.54	-0.67	-0.81	-0.97	-1.11	-1.24	-1.37
3	-0.61	-0.33	-0.17	-0.13	-0.12	-0.21	-0.32	-0.44	-0.56	-0.69	-0.80	-0.90
4	-0.61	-0.32	-0.12	-0.03	0.05	0.06	0.02	-0.02	-0.07	-0.14	-0.19	-0.25
5	-0.61	-0.37	-0.26	-0.25	-0.28	-0.41	-0.52	-0.63	-0.74	-0.84	-0.91	-0.96
ΔDDB												
1	0.95	1.42	1.36	1.04	0.67	0.73	1.23	2.03	2.65	3.22	3.49	3.87
2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
3	0.08	0.23	0.41	0.57	0.68	0.77	0.85	1.00	1.08	1.18	1.25	1.36
4	0.11	0.32	0.58	0.89	1.20	1.50	1.78	2.16	2.40	2.66	2.84	3.15
5	0.02	0.08	0.17	0.25	0.28	0.32	0.39	0.53	0.64	0.78	0.91	1.08
Δ(BR-BORR)												
1	0.62	0.44	0.03	-0.28	-0.34	-0.07	0.31	0.59	0.72	0.79	0.75	0.74
2	-0.07	-0.15	-0.22	-0.23	-0.20	-0.21	-0.21	-0.25	-0.26	-0.28	-0.36	-0.35
3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
4	0.02	0.06	0.10	0.17	0.22	0.28	0.30	0.33	0.38	0.41	0.48	0.52
5	-0.05	-0.10	-0.10	-0.12	-0.13	-0.11	-0.09	-0.07	-0.04	-0.01	0.00	0.03
-ΔSECG												
1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2	0.67	1.13	1.51	1.87	2.26	2.81	3.49	4.33	5.32	6.45	7.76	9.12
3	0.61	0.94	1.11	1.24	1.36	1.57	1.89	2.33	2.89	3.58	4.38	5.27
4	0.59	0.87	0.95	0.92	0.81	0.70	0.65	0.64	0.66	0.78	0.91	1.12
5	0.66	1.07	1.34	1.61	1.90	2.29	2.79	3.41	4.12	4.93	5.82	6.75

Notes: XG = real value of goods purchased by the government, 1972 dollars at a quarterly rate.

Y = real output of the firm sector, 1972 dollars at a quarterly rate.

PF = price deflator for total firm sales less farm output, 1972 = 1.0.

UR = civilian unemployment rate, percentage points.

See Table 1 for a definition of the other variables.

From equation 15 in Table 1, $\Delta SAVG_t - \Delta SECG_t + \Delta(BR - BORR) = 0$ except for rounding. ($CURR_t$ and $GFXG_t$ in equation 15 are exogenous.)

rowed reserves to take place; (4) The Fed allows no change in the bill rate to take place; (5) The Fed behaves as estimated in Section III.

The results for the five experiments are presented in Table 2. The effects on nine endogenous variables in the model are presented. Each number in the table is the difference between the predicted value of the endogenous variable for the quarter and the actual value. The variables are defined at the bottom of the table: Y is the key output variable in the model; PF is the key price variable; UR is the civilian unemployment rate; and SAVR , the saving of the foreign sector, is the negative of the U.S. balance of payments on current account.

The results in Table 2 clearly show that fiscal-policy effects are sensitive to assumptions about the behavior of the Fed. The most expansionary experiment with respect to real output changes is the first, where Y is 2.39 billion dollars higher in quarter t+11 than it was historically, and the least expansionary is the second, where Y is actually 0.09 billion dollars lower in quarter t+11 than it was historically. For purposes of explaining the different effects in the table, it will be useful to concentrate on the results for the bill rate for the first quarter. In the first experiment, the bill rate fell in the first quarter as a result of the increase in XG . This is because the dissaving of the government $(-\Delta SAVG_t)$ of 0.62 was financed by an increase in nonborrowed reserves. The government action in this case effectively increases the amount of funds in the system, which causes the bill rate to fall. Experiment 1 is thus doubly expansionary in the sense that sales are higher because of the increased purchase of goods by the government and the bill rate is lower because of the increase in funds in the system.

Consider now the fourth experiment, where the Fed prevents the bill rate from changing. In the first quarter the Fed prevents the bill rate from falling (as it did in experiment 1) by selling government securities. -SECG increased by 0.59 in this case, which met all but 0.02 of the dissaving of the government of 0.61. (The 0.02 was met by an increase in nonborrowed reserves of this amount.) Experiment 4 is thus less expansionary than is experiment 1 because the bill rate is not allowed to fall: fewer funds are released to the system.

Even fewer funds are released to the system in experiment 2, where the money supply is prevented from changing. In this case the bill rate must rise to choke off any increase in the demand for money caused by an increase in income, the tise being 0.13 percentage points for the first quarter. This required an increase in -SECG of 0.67. Over time, the higher bill rates in experiment 2 had a gradual contractionary effect on the economy until, as mentioned above, by quarter t+11 real output was actually lower.

Experiment 3, where nonborrowed reserves are prevented from changing, is more expansionary than is experiment 2. Keeping nonborrowed reserves unchanged allows the money supply to increase through an increase in bank borrowing. The money supply is linked directly to BR , and BR can increase to the extent that BORR does while still keeping unborrowed reserves unchanged. There are thus somewhat more funds allowed in the system in experiment 3 than in experiment 2. The increase in -SECG in the first quarter was 0.61 in experiment 3, compared to 0.67 in experiment 2.

In experiment 5, where the behavior of the Fed is endogenous, the Fed responds to the increase in economic activity by increasing the bill rate. This experiment is thus less expansionary than is experiment 4, where the bill rate remained unchanged. It is, however, more expansionary than is experiment 2: the money supply does increase some in experiment 5. The results for this experiment are closest to the results for experiment 3, where the level of nonborrowed reserves remained unchanged. Comparing experiments 4 and 5, it can be seen that the effects on real output are about the same for the first three or four quarters, and then they start to diverge as the contractionary effects of the higher past bill rates in experiment 5 begin to be felt.

A few more points about the results in Table 2 should be made. First, the bill rate has a positive effect on the bond rate, and the bond rate in turn, as mentioned in Section II, has a positive effect on the price level. This explains the fall in PF in the first two quarters in experiment 1 in Table 2. In this case the negative effects of the decrease in the bill rate were large enough to offset any positive effects of an increase in aggregate demand on the price level. This is a good illustration of the fact that there is no stable relationship between aggregate demand and inflation in the model. Second, the fact that SAVG did not fall by the full current dollar[12] amount of the increase in XG is explained by endogenous government expenditures and tax receipts. When the economy expands, tax receipts increase and some transfer payments decrease. Government interest payments also change as interest rates change and as SECG changes. The net result of all of these effects is that SAVG fell in the first quarter by only about half of the increase in the value of goods purchased by the government.

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[12]For the simulation period the current-dollar increases in XG were close to the constant dollar increase of 1.25 billion dollars. Constant dollars are 1972 dollars in the newly revised NIA data.

Finally, it should be noted that unemployment rate is actually higher by quarter t+8 in experiment 2 even though real output is still higher. As mentioned in Section II, there are many factors that affect the unemployment rate in the model, and there is no particular reason to expect there to be a stable relationship between real output and the unemployment rate. Experiment 2 is a good illustration of this fact. One of the reasons for the higher unemployment rates in experiment 2 is the higher level of bill rates. The bill rate has a positive effect on the mortgage rate, which has a positive effect on labor force participation, which in turn has a positive effect on the unemployment rate.

In summary, then, the results in Table 2 show that fiscal-policy effects are quite sensitive to assumptions about the behavior of the Fed. The most expansionary case is where the government deficit is financed by an increase in nonborrowed reserves, and the least expansionary case is where the Fed keeps the money supply unchanged. This latter case is in fact expansionary only for about the first eight quarters. The second most expansionary case is where the Fed keeps the bill rate unchanged. The case in which the Fed behaves as estimated in Section III is considerably less expansionary after the first few quarters than is the case in which the bill rate is kept unchanged. The Fed as estimated in Section III responds to the increase in XG by increasing the bill rate. By quarter t+11 the increase in Y is only 0.66 billion dollars in this case compared to 1.91 billion dollars in the constant bill-rate case. The case in which nonborrowed reserves are kept unchanged is similar to the case in which the Fed behaves as estimated in Section III.

Other experiments could have been performed using different fiscal-policy variables, but for present purposes there is no real need to do this. Different fiscal-policy variables in the model do have different effects on the economy, as can best be seen by examining the results in Table 9-6 in [9],[13] but these effects will all be sensitive to assumptions about the behavior of the Fed. In this respect the effects of changing other fiscal-policy variables are not different from the effects of changing XG.

Before concluding this section it should be noted that the results obtained here are in sharp contrast to some results reported in Blinder and Goldfeld [1]. In their analysis of the effects of monetary and fiscal policies in the MPS model, Blinder and Goldfeld found that the "interaction effects" between the two types of policies were negligible (p. 792). Given the results in the present paper, this is puzzling, for it seems to imply that fiscal-policy effects in the MPS model are not sensitive to assumptions about the behavior of the Fed. Unfortunately, Blinder and Goldfeld do not state explicitly what they are assuming about the behavior of the Fed for their computations of the effects of fiscal policies, but their brief discussion of the lack of interaction effects does seem to indicate that fiscal-policy effects are not sensitive to assumptions about Fed behavior.

The failure of Blinder and Goldfeld to be clear about Fed behavior may be more a fault of the MPS model than of their own analysis. As is the case with most macroeconometric models, the MPS model is not closed with respect to the flows of funds in the system. This means, among other things, that the government budget constraint is not explicitly accounted

[13]The meticulous reader will note that the results in line 3 in Table 9-6 in [9] are similar to the results in line 1 in Table 2 of the present paper. Similar experiments have been performed in the two cases.

for. The MPS model, in other words, does not completely account for equations like 11-16 in Table 1. Because of this, the direct purchase and sale of government securities by the Fed cannot be considered in the model, and so it is not surprising that Blinder and Goldfeld have failed to be explicit about Fed behavior. Nevertheless, the results in this paper indicate that the failure of the MPS model (and other models) to account for all flows of funds may be a serious omission and that the Blinder-Goldfeld measures of fiscal and monetary policies may not be very trustworthy.[14]

V. Optimal Control Results

For the present model it is obvious from the results in the previous section that the performance of the fiscal authorities with respect to maximizing some objective function will depend significantly on the behavior of the Fed. In order to indicate the possible quantitative effects of this dependency, the results of solving three control problems for the fiscal authorities are presented in this section. The problems correspond to three different assumptions about the behavior of the Fed. The control period is 1969I-1976II, for a total of 30 quarters.

The objective function that was used targets a given level of real output and a zero rate of inflation for each quarter. It is easiest to consider the objective function to be a loss function that is to be minimized. This loss function is:

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[14] In this regard it is interesting (or perhaps discouraging) to note that even the monthly money market model developed at the Fed is not closed with respect to the flows of funds. See, for example, Thomson, Pierce, and Parry [21] or Pindyck and Roberts [19].

$$(7) \qquad L = \sum_{t=1969I}^{1976II} \left[\left\backslash \frac{Y_t - Y_t^*}{Y_t^*} \right\backslash^2 + (\%PF_t)^2 \right] ,$$

where Y_t^* = target level of Y_t ,

$\%PF_t$ = percentage change in PF_t at an annual rate,

and

$$\left\backslash \frac{Y_t - Y_t^*}{Y_t^*} \right\backslash^2 = \begin{cases} \left(\frac{Y_t - Y_t^*}{Y_t^*}\right)^2 & \text{if } Y_t < Y_t^* \\ 0 & \text{if } Y_t \geq Y_t^* \end{cases} .$$

The loss function penalizes rates of inflation that are both above and below the target value of zero, but it only penalizes values of Y_t that are below the target. The target values for real output are presented in Table 3 below, and their construction is explained in Chapter Ten in [9]. The values are meant to correspond to high levels of economic activity.[15]

The three assumptions about the behavior of the Fed are: (1) the Fed behaves as estimated in Section III, (2) the Fed allows no change in the bill rate, and (3) the Fed allows no change in the money supply. By "no change" here is meant that the value of the variable for each quarter was not changed from its historic value when solving the control problem. For all three problems SECG is endogenous. XG was used as the control variable of the fiscal authorities. The error terms in the model were set equal to their estimated values, and the resulting deterministic

[15]The values for Y_t^* presented in Table 3 differ from those presented in [9] because of the use in the present study of the revised NIA data.

TABLE 3. THE OPTIMAL CONTROL RESULTS

	Actual Values				Optimal Values When Fed Behaves Endogenously					
Quarter	RBILL	Y	100%PF	Y*	RBILL	Y	100%PF	ΔXG	-ΔSECG	ΔDDB
1969 I	6.1	231.2	4.9	232.3	6.2	231.6	5.0	0.4	0.2	0.0
II	6.2	232.1	4.5	234.3	6.4	233.4	4.8	1.3	0.8	0.1
III	7.0	232.8	5.0	236.3	7.3	234.9	5.5	2.0	1.5	0.2
IV	7.3	231.0	4.2	238.4	7.9	236.9	5.4	6.1	4.1	0.6
1970 I	7.3	230.5	4.3	240.4	8.3	238.6	5.4	7.4	6.9	1.1
II	6.8	230.5	5.1	242.4	8.1	240.6	6.3	8.7	10.1	1.8
III	6.4	232.1	3.0	244.5	8.0	242.8	3.9	8.3	12.7	2.6
IV	5.4	229.3	6.3	246.5	7.4	243.3	7.5	11.6	17.0	3.3
1971 I	3.9	235.2	4.0	248.5	5.9	245.7	5.3	6.7	18.2	3.7
II	4.2	237.2	5.6	250.5	6.2	247.9	6.7	8.9	21.7	4.2
III	5.1	238.8	3.5	252.6	7.3	251.7	4.5	11.1	26.4	5.0
IV	4.2	241.2	1.9	254.6	6.6	253.8	3.4	10.7	31.1	5.8
1972 I	3.4	246.1	5.6	256.6	5.7	256.2	7.3	9.5	35.3	6.2
II	3.7	251.5	1.9	258.7	6.2	263.2	3.6	13.1	42.6	6.8
III	4.2	255.2	1.4	260.7	6.7	266.5	3.0	12.9	49.9	7.6
IV	4.9	260.8	4.3	262.7	7.1	267.3	5.4	8.8	55.0	9.0
1973 I	5.6	267.8	3.1	264.8	7.4	269.2	3.4	5.4	59.1	9.6
II	6.6	267.7	5.0	266.8	8.1	269.0	5.2	7.4	65.9	10.5
III	8.4	269.2	6.3	268.8	9.5	267.1	5.9	3.5	70.4	10.9
IV	7.5	270.9	10.1	270.9	8.1	265.0	9.2	0.3	73.4	11.7
1974 I	7.6	267.8	11.6	272.9	8.1	264.2	10.7	3.4	79.5	11.9
II	8.3	265.6	15.4	274.9	8.8	263.7	14.8	3.8	85.9	12.3
III	8.3	263.4	13.8	276.9	9.0	265.1	13.5	6.5	94.1	13.1
IV	7.3	258.4	14.0	279.0	8.7	267.6	14.3	12.1	106.2	14.5
1975 I	5.9	250.5	11.7	281.0	8.4	272.8	12.7	21.4	123.8	16.2
II	5.4	254.6	5.4	283.0	8.6	277.5	6.8	14.4	131.8	19.8
III	6.3	262.5	4.8	285.1	9.3	279.2	5.8	7.3	131.6	22.2
IV	5.7	264.8	4.6	287.1	8.5	280.4	5.8	10.1	138.1	23.9
1976 I	5.0	271.2	6.7	289.1	7.5	282.9	7.6	7.3	142.4	25.3
II	5.2	274.4	3.3	291.2	7.9	289.4	4.2	13.0	153.8	26.8

TABLE 3 (continued)

	Optimal Values When Bill Rate is Kept Unchanged						Optimal Values When Money Supply is Kept Unchanged					
Quarter	RBILL	Y	100%PF	ΔXG	-ΔSECG	ΔDDB	RBILL	Y	100%PF	ΔXG	-ΔSECG	ΔDDB
1969 I	6.1	231.7	5.0	0.5	0.2	0.0	5.2	225.7	4.1	-6.0	-3.1	0.0
II	6.2	233.4	4.7	1.2	0.7	0.2	4.8	225.8	3.2	-5.6	-5.1	0.0
III	7.0	235.0	5.4	2.1	1.3	0.4	5.4	227.6	3.8	-4.6	-6.6	0.0
IV	7.3	237.0	5.3	5.9	3.5	1.0	5.9	228.7	3.6	-2.5	-7.3	0.0
1970 I	7.3	239.0	5.2	7.2	5.7	1.9	6.3	230.4	4.3	-1.2	-7.9	0.0
II	6.8	241.1	6.1	8.2	7.9	3.1	6.3	222.6	5.3	0.1	-8.4	0.0
III	6.4	243.3	3.6	7.2	9.1	4.5	6.5	236.1	3.4	1.2	-8.8	0.0
IV	5.4	244.8	7.1	11.1	12.0	6.1	6.0	234.8	6.9	2.1	-9.1	0.0
1971 I	3.9	247.1	4.7	4.9	10.6	7.7	4.4	240.5	4.7	1.5	-10.3	0.0
II	4.2	249.4	6.2	6.6	11.0	9.1	4.9	242.3	6.3	1.7	-11.0	0.0
III	5.1	252.1	3.9	7.2	11.6	10.4	6.0	244.2	4.2	2.3	-11.2	0.0
IV	4.2	254.3	2.7	6.7	11.7	12.0	5.0	245.6	2.5	1.7	-11.7	0.0
1972 I	3.4	256.5	6.5	4.5	10.7	13.3	3.8	247.9	6.1	0.0	-12.9	0.0
II	3.7	260.1	2.5	3.8	9.8	14.2	3.9	251.2	2.1	-0.8	-13.9	0.0
III	4.2	262.5	1.7	3.3	9.5	14.9	4.0	252.5	1.3	-2.2	-15.3	0.0
IV	4.9	264.2	4.4	0.0	7.5	16.0	4.2	255.1	3.9	-4.3	-17.5	0.0
1973 I	5.6	266.3	2.8	-3.6	4.0	15.6	4.4	260.4	2.3	-5.2	-19.9	0.0
II	6.6	267.7	4.7	0.2	4.3	15.5	4.8	259.5	4.0	-6.1	-22.8	0.0
III	8.4	268.2	6.0	-1.4	3.0	15.0	5.7	260.2	5.0	-7.5	-27.0	0.0
IV	7.5	267.8	9.6	-3.2	0.9	15.1	4.7	260.9	8.6	-9.0	-32.0	0.0
1974 I	7.6	267.2	11.2	0.0	1.4	14.7	4.7	259.3	10.3	-8.0	-36.7	0.0
II	8.3	266.5	15.2	0.7	1.9	14.6	5.4	259.7	14.4	-6.5	-41.0	0.0
III	8.3	267.4	13.9	3.4	4.1	14.8	5.9	263.1	13.3	-2.5	-43.1	0.0
IV	7.3	270.2	14.5	9.9	10.6	16.2	6.8	266.0	14.4	2.9	-42.0	0.0
1975 I	5.9	275.3	12.5	18.4	20.6	18.8	7.5	266.7	12.9	8.4	-39.1	0.0
II	5.4	279.6	6.5	10.5	20.1	23.5	8.5	275.1	7.1	8.6	-39.3	0.0
III	6.3	282.2	5.7	3.9	12.1	26.4	11.1	282.2	6.8	6.7	-42.6	0.0
IV	5.7	284.3	5.7	7.3	10.5	28.7	9.7	280.5	6.5	6.0	-44.4	0.0
1976 I	5.0	286.8	7.4	3.5	5.1	30.9	7.9	282.9	8.2	4.6	-47.0	0.0
II	5.2	290.2	3.9	5.8	3.2	32.5	7.6	282.5	4.5	3.8	-47.6	0.0

Notes: ΔXG = difference between the optimal and actual values of XG (the real value of government expenditures on goods).

-ΔSECG = difference between the optimal and actual values of -SECG (the amount of government securities outstanding).

ΔDDB = difference between the optimal and actual values of DDB (the money supply).

See Table 1 and the text for the definitions of the variables.

control problem was solved as described in [7].[16] The results by quarters are presented in Table 3; a summary of them is as follows:

	Actual	Optimal When Fed Behaves Endogenously	Optimal When Bill Rate is Kept Unchanged	Optimal When Money Supply is Kept Unchanged
Value of Loss Function	0.2279	0.1785	0.1685	0.1901
Average Value of the Bill Rate over the First 24 Quarters	6.1	7.5	6.1	5.2
Sum of Y over the First 24 Quarters	5946.3	6085.3	6092.8	5910.1
Average Rate of Inflation over the First 24 Quarters (annual rate)	6.0	6.6	6.3	5.7
Sum of ΔXG over the First 24 Quarters	0.0	169.9	86.3	-58.5

Note: Sum of Y^* over the first 24 quarters = 6135.0.

The following discussion will concentrate on the results for the first 24 quarters of the 30-quarter horizon. The optimal values for the last few quarters may be heavily influenced by the fact that there is no tomorrow after 30 quarters, especially since in the model inflation responds with a longer lag to current stimulative measures than does output. It is thus best to ignore the results for the last few quarters.

[16] See also Chapter Ten in [9], pp. 198-203, for a discussion of the solution of optimal control problems using the present model.

The fiscal authorities do best in the case in which the Fed keeps the bill rate unchanged. The optimum in this case corresponds to more output and more inflation than existed historically. Inflation is, however, only slightly higher than existed historically, whereas output is much higher. The output target is in fact close to being met. This example reflects one of the characteristics of the model mentioned in Section II: the extra-inflation cost of increasing output is generally less than the lost-output cost of lowering inflation for loss functions like (7). Note from Table 3 that by 1974IV money supply was 16.2 billion dollars higher in this case than it was historically.

The fiscal authorities do not do as well when the Fed behaves as estimated in Section III. The Fed responds to stimulative measures by increasing the bill rate, which, other things being equal, has a positive effect on the rate of inflation. The optimum in this case corresponds to slightly less output and somewhat more inflation than existed in the unchanged bill-rate case. It is also interesting to note that the increase in XG is substantially larger in this case than it is in the unchanged bill-rate case. The higher bill rates that the Fed sets when it behaves endogenously cause a contraction in private demand, which, since output is nearly the same in the two cases, the fiscal authorities nearly completely offset by increasing XG . The optimum in the endogenous Fed case thus corresponds to a substantially larger government sector than does the optimum in the unchanged bill-rate case. This difference would, of course, not exist if, say, the personal income tax rate were used as the control variable of the fiscal authorities instead of XG . The amount of government securities outstanding increased substantially in the endogenous Fed case, which, loosely speaking, is partly to finance the large increase in XG (for an unchanged bill rate) and partly to cause the bill rate to rise.

The fiscal authorities do worst in the unchanged money-supply case. Output is not only lower in this case than it was in the other two cases, but it is also slightly lower than existed historically. Inflation is, however, also lower. The optimal strategy in this case was for the fiscal authorities to lower on average XG, which, with an unchanged money supply, causes the bill rate to fall. The lower bill rate offsets to some extent the lower output caused by the fall in XG, and it also leads to somewhat less inflation. The main point of this example is not, however, that the optimal level of output is slightly lower than existed historically; with less weight on inflation in the loss function this result is likely to be reversed. The main point is that when the Fed keeps the money supply unchanged, the fiscal authorities have little room to maneuver. They can increase output by increasing XG, but only at the expense of a substantually higher bill rate and thus higher inflation. They can lower the bill rate and thus inflation by decreasing XG, but the net result of this policy is also to lower output. The optimal policy may go either way, but except for small changes, the fiscal authorities can do little about changing the output path once the money-supply path is fixed.

The results in this section are thus as expected, given the results in Section IV. The optimal performance of the fiscal authorities depends significantly on the behavior of the Fed. When the Fed behaves endogenously, the fiscal authorities do not do as well as when the Fed behaves by keeping the bill rate unchanged. They do not do as well in terms of lowering the value of loss, and the optimal policy also calls for about twice as much fiscal stimulus to offset the increases in the bill rate by the Fed.

It should finally be noted that given the model and the loss function in (7), it would not be reasonable to solve an optimal control problem in which the fiscal authorities and the Fed cooperate, i.e., in which, for example, both XG and SECG were used as control variables. With only the two arguments in the loss function, the optimal values of XG and the bill rate would probably be close to zero. Since lower bill rates decrease inflation, the optimum is likely to correspond to a very low bill rate and a value of XG low enough to offset any "undesired" increase in output caused by the low bill rate's positive effect on private demand. One would need other arguments in the loss function, such as a target size of the government sector, before it would be reasonable to use both XG and SECG as control variables.

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