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THE USE OF OPTIMAL CONTROL TECHNIQUES TO MEASURE ECONOMIC PERFORMANCE

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THE USE OF OPTIMAL CONTROL TECHNIQUES TO MEASURE ECONOMIC PERFORMANCE*

by

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I. Introduction

It is a common practice in political discussions in the United States to hold a presidential administration accountable for the state of the economy that existed during its four-year period in office. Administrations are generally blamed for high unemployment rates, low real growth, and high inflation rates during their years in office and praised for the opposite. Although at first glance this seems like a natural way of evaluating the economic performances of administrations, there are at least two serious problems with it. The first is that this kind of evaluation does not take into account possible differences in the degree of difficulty of controlling the economy in different periods. The economy may be more difficult to control for one administration than for another either because of more unfavorable values of noncontrolled exogenous variables for one than for another or because of a more unfavorable initial state of the economy for one than for another (or both).

The second problem with evaluating the economic performance of an administration on the basis of the state of the economy that existed

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during its four-year period in office is that it ignores the effects of an administration's policies on the state of the economy beyond the four-year period. If, for example, an administration strongly stimulates the economy in the year of the presidential election, in, say, the belief that this might improve the chances of its party staying in power, most of the inflationary effects of this policy may not be felt until the next four-year period. Any evaluation of performance that was concerned only with the administration's four-year period in office would not, of course, pick up these effects.

The purpose of this paper is to propose a measure of economic performance of presidential administrations that takes into account both of these problems and then to estimate this measure for each of the past five administrations. The measure, which is based on the solutions of optimal control problems, requires that a specific welfare or loss function be postulated and that the economy be represented by an econometric model. The welfare or loss function must be additive across time.

The measure is discussed in Section II, and the estimates are presented in Section III. Because of a relatively small computer budget for this project, it was not possible for the work in Section III to estimate the exact measure that is proposed in Section II, and so some approximations had to be made. These approximations are discussed in Section III. The measure is presented in Section II without reference to these approximations to avoid any possible confusion between what could be computed with a larger budget and what was in fact computed for the present project.

II. The Measure

The economy is assumed to be represented by a quarterly econometric model. Denote this model as follows:

$$(1) \qquad \phi_g(y_t, y_{t-1}, z_t, x_t, \beta_g) = u_{gt} \;, \quad (g = 1, \ldots, G) \;,$$

where y_t is a vector of observations for quarter t on the G endogenous variables in the model, y_{t-1} is y_t lagged one quarter, z_t is a vector of observations for quarter t on the noncontrolled exogenous variables, x_t is a vector of observations for quarter t on the control variables, β_g is a vector of unknown coefficients in equation g, and u_{gt} is an error term corresponding to equation g for quarter t. If equation g is an identity, then u_{gt} is zero for all t. The model is assumed to be such that, for each t, given values for y_{t-1}, z_t, x_t, β_g, and u_{gt} $(g = 1, \ldots, G)$, one can solve numerically for y_t. In practice most models are solved by the Gauss-Seidel method. If the model is solved for more than one quarter and the simulation is dynamic, then the solution values of the endogenous variables for the previous quarters are used as values for y_{t-1}.

In order to simplify the present discussion, the following assumptions about the econometric model will be made. First, the longest lag in the model is assumed to be one quarter, so that, for example, y_{t-2} does not appear in the equations in (1). Second, there is assumed to be only one control variable, so that x_t is a scalar. Third, lagged values of x_t are assumed not to enter any of the equations in (1). Fourth, the values of the exogenous variables, i.e., the values in the z_t vector, are assumed always to be known by the policy makers. Fifth, all problems of data revisions are ignored. Once data become available for a given quarter, they are assumed never to be revised. It should be fairly obvious in most cases how the following discussion would be modified if these assumptions were relaxed.

Regarding the welfare or loss function that must be specified, it will be convenient for purposes of this paper to work with a loss function. The value of loss in quarter t, denoted as L_t, is assumed to be some function of y_t, y_{t-1}, z_t and x_t:

$$(2) \qquad L_t = h(y_t, y_{t-1}, z_t, x_t) .$$

Let $\mathcal{L}(k,T)$ denote the sum of L_t over the T quarters between k and $k+T-1$:

$$(3) \qquad \mathcal{L}(k,T) = \sum_{t=k}^{k+T-1} L_t .$$

Assume that one would like to see an administration that begins its term in quarter k behave in such a way as to minimize the expected value of $\mathcal{L}(k,T)$ in (3). Before presenting the measure of performance, it will be convenient to consider the question of what is the best that one can expect the administration to do towards achieving this goal, given that it has the econometric model in (1) at its disposal. The first thing to note is that if the data on the variables in the model are available quarterly, which is assumed here, then the administration can reoptimize each quarter. One can thus consider the administration to have the option of solving 16 optimization problems during its four-year period in office. The econometric model can also be reestimated each quarter, thus insuring, other things being equal, that the most efficient estimates of the coefficients in the β_g vectors are always available for use.

For the first of the 16 problems the objective is to choose the T values of x_t $(t = k, \ldots, k+T-1)$ that minimize $E\mathcal{L}(k,T)$, where E is the expected value operator. It is now computationally feasible, for the cost of no more than a few thousand dollars, to solve this problem as follows. First, given values of the exogenous and control variables through quarter $k+T-1$, given values of the endogenous variables through quarter $k-1$, and given estimates of the probability distributions of the coefficient estimates and the error terms, $E\mathcal{L}(k,T)$ can be computed numerically by means of stochastic simulation.[1] Since it is possible to compute numerically an estimate of $E\mathcal{L}(k,T)$ for each set of values of x_t $(t = k, \ldots, k+T-1)$, one can consider $E\mathcal{L}(k,T)$ to be an implicit function of these values:

$$E\mathcal{L}(k,T) = \psi(x_k, x_{k+1}, \ldots, x_{k+T-1}) .$$

Given this implicit function, the problem of finding the T values of x_t that minimize $E\mathcal{L}(k,T)$ can be considered to be a straightforward unconstrained nonlinear optimization problem, which can be solved by a

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[1] For nonlinear models and loss functions one has to be careful that this simulation is done correctly. $E\mathcal{L}(k,T)$ should be computed as the average of the individual values of $\mathcal{L}(k,T)$, where each value of $\mathcal{L}(k,T)$ is based on one set of drawings from the probability distributions. Each set of drawings corresponds to one drawing from the probability distribution of the coefficient estimates and to one drawing for each of the T quarters from the probability distribution of the error terms. Given each set of drawings, the model can be simulated dynamically over the T quarters, and then the T values of L_t can be computed from equation (2). The computed value of $\mathcal{L}(k,T)$ for this set of drawings is then merely the sum of these T values. It is unnecessary for present purposes to consider in detail how the stochastic simulation can be carried out. All that is needed here is to note that $E\mathcal{L}(k,T)$ can be computed numerically by means of stochastic simulation.

variety of algorithms that are now available. Although it is not possible to obtain analytically the partial derivatives of ψ with respect to the control variables, these derivatives can always be obtained numerically if they are needed by the particular algorithm that is being used. This way of setting up and solving optimal control problems is discussed in detail in [3], and so it will not be discussed any further here. Suffice it to say that it does appear possible to solve these kinds of problems, even in the stochastic case, for no more than a few thousand dollars.

Let x_k^* denote the value of x_k that results from this solution. This is the value of the control variable that the administration can use in quarter k. At the beginning of quarter $k+1$, after the actual values of the endogenous variables for quarter k have been observed, the administration can solve the second of the 16 problems in the same way as it did the first. If the length of the horizon is kept at T quarters, then the problem is to choose the T values of x_t $(t = k+1, \ldots, k+T)$ that minimize $E\mathcal{L}(k+1,T+1)$. If, on the other hand, the end quarter of the horizon is kept at $k+T-1$, then the problem is to choose the $T-1$ values of x_t $(t = k+1, \ldots, k+T-1)$ that minimize $E\mathcal{L}(k+1,T)$. It is assumed here that the length of the horizon is taken to be long enough so that any further increase in the length has a negligible effect on the solution values for the first few quarters. It thus makes no noticeable difference regarding the optimal value of the control variable for quarter $k+1$ whether the end quarter of the horizon for the second problem is kept at $k+T-1$ or increased to $k+T$. Let x_{k+1}^* denote the value of x_{k+1} that results from solving the second problem. x_{k+1}^* will, of course, not in general be equal to the value of x_{k+1} that results from solving the first problem. x_{k+1}^* is the value of the control variable that the administration can use in quarter $k+1$. At the beginning of quarter $k+2$, after the actual values of the endogenous variables for quarter $k+1$ have been observed, the administration can solve the third problem, and this procedure can be repeated for each of the remaining problems. It will be convenient to let x^* denote the 16-component vector of optimal values of the control variable that result from this procedure:

$$x^* = (x_k^*, x_{k+1}^*, \ldots, x_{k+15}^*) \ . \tag{4}$$

x^* is a vector that is feasible for an administration to compute during its term in office. To the extent that the econometric model is reestimated each quarter, the only two important problems that the above procedure has ignored are any attempt to obtain feedback equations (i.e., to solve closed-loop stochastic control problems) and any consideration of active learning. For large models it does not yet appear computationally feasible to deal with either of these problems, and so x^* is probably the best that one can currently expect an administration to achieve. In what follows an administration will be said to behave optimally if it follows the above procedure for obtaining x^* . Also, an administration that begins office in quarter k will be referred to as administration k .

The measure of performance of administration k , denoted as M_k , is defined to be the following (low values of M_k are good):

(5) M_k = actual expected loss in administration k's four-year period in office

- expected loss in this four-year period if administration k had behaved optimally

+ expected loss in the next four-year period given that administration k did not behave optimally, but assuming that administration k+16 did

- expected loss in the next four-year period if both administrations k and k+16 had behaved optimally.

The first two terms in equation (5) measure the expected loss that could have been avoided during administration k's four-year period in office had it behaved optimally. The last two terms measure the potential expected loss to administration k+16 from the fact that administration k did not behave optimally. There may also be some potential expected loss to administration k+32 from the fact that administration k did not behave optimally, but for purposes here this loss is assumed to be negligible.

M_k takes into account both of the problems mentioned in the Introduction. If the economy is difficult to control for administration k, then the second term in equation (5) will be large, which will then offset more than otherwise a large value of actual expected loss. The last two terms in equation (5) measure the effects of administration k's policies on the economy beyond its own four-year period in office, these effects being measured under the assumption that the next administration behaves optimally.

The remaining issue to consider in this section is how the four terms in equation (5) can be computed. So far the only procedure that has been discussed is how an administration could behave if it were interested in minimizing expected loss. The data that are available reflect, of course, the outcome of an administration's actual policies, not the optimal policies.

Consider first the computation of the first two terms in equation (5). Assume that the econometric model in (1) has been estimated through quarter $k-1$, and assume that x_k^* has been computed in the manner described above. Given y_{k-1} , z_k , and x_k , one can compute EL_k by stochastic simulation. This is now stochastic simulation for only one quarter, not the stochastic simulation over the entire T quarters that is needed in the process of computing x_k^* . Denote this value of EL_k as $\hat{L}_k$. Similarly, given y_{k-1} , z_k , and x_k^* , one can compute EL_k by stochastic simulation. Denote this value as $\hat{L}_k^*$. $\hat{L}_k - \hat{L}_k^*$ is then the difference between the actual expected loss and the optimal expected loss for quarter k .

Now move on to the beginning of quarter $k+1$. The model can be reestimated through quarter k , but it must be estimated on the basis of the actual outcome for quarter k , not the outcome that would have occurred had the administration behaved optimally. This is one unavoidable difference between what can be done here and what an administration could have done had it behaved optimally. At any rate, given the new set of estimates and given y_k , z_{k+1} , and x_{k+1} (the actual values), $\hat{L}_{k+1}$ can be computed by stochastic simulation. This is straightforward. What is not straightforward, however, is how to compute x_{k+1}^* and $\hat{L}_{k+1}^*$, since these computations must be based on the assumption that the administration behaved optimally in quarter k . The procedure that can be followed in this case is the following. First, one has an estimate of the actual realization of the error terms for quarter k , namely, the single-equation residual estimates for quarter k that are based on the new set of coefficient estimates. Second, one has available x_k^*, the optimal value of the control variable for quarter k. Therefore, one can solve the model for quarter k using the new set of coefficient estimates, the estimate of the actual realization of the error terms, and x_k^*. This will produce values of the endogenous variables that one can take as estimates of what would have been observed in quarter k had the administration behaved optimally. Let y_k^* denote the vector of these values. x_{k+1}^* can now be computed in the manner described above, using, however, y_k^* in place of y_k. Given y_k^*, y_{k+1}, and x_{k+1}^*, $\hat{L}_{k+1}^*$ can then be computed by stochastic simulation. $\hat{L}_{k+1} - \hat{L}_{k+1}^*$ is then the difference between the actual expected loss and the optimal expected loss for quarter $k+1$.

The process just described for quarter $k+1$ can be repeated for the remaining 14 quarters. Each quarter the model can be reestimated, the actual expected loss computed, estimates made of what the values of the endogenous variables would have been in the previous quarter had the administration behaved optimally,[2] the optimal value of the control variable computed for the quarter, and then the optimal expected loss computed. Once the process is completed for the 16 quarters, the first term in equation (5) can then be computed as $\sum_{t=k}^{k+15} \hat{L}_t$ and the second term as $\sum_{t=k}^{k+15} \hat{L}_t^*$.

[2]One small point should be noted here. When the model is reestimated through, say, quarter $k+2$, this will change the residual estimates for quarter $k+1$ from what they were when the model was only estimated through quarter $k+1$. For purposes of computing y_{k+2}^*, then, one can use either the values in y_{k+1}^*, which are based on the estimates through quarter $k+1$, or newly constructed values for these variables that are based on the estimates through quarter $k+2$. Similar considerations apply to quarters $k+3$ and beyond.

The third term in equation (5) can be computed in the same way as described above for the second term. Just substitute $k+16$ for k. The fourth term can be computed in the same way as the third term except that a different starting point is needed. In this case the values of the endogenous variables that are used for quarter $k+15$ should be the optimal values as computed in the process of computing the second term. In other words, y^*_{k+15} should be used in place of y_{k+15} to begin the computation of the fourth term.

III. Estimates of the Measure for the Past Five Administrations

In an actual policy-making situation the cost of stochastic simulation is trivial compared to the billions of dollars that might be saved from the implementation of better policies. For me, however, the cost is not trivial. It would probably require the equivalent of three or four days of computer time (on a current-generation computer), say a cost of something over \$50,000, to compute M_k in the manner described in Section II for the past five administrations. Since the computer budget for this project is considerably less than this amount, it is not feasible for me to compute M_k in the above manner. It is possible, however, to approximate M_k fairly cheaply, and the procedure that was actually used to estimate M_k in this study will now be described.

Two estimates of M_k were computed for each of the past five administrations (Eisenhower - I, Eisenhower - II, Kennedy-Johnson, Johnson, and Nixon - I) corresponding to two different loss functions.[3] These ten estimates are based on the solutions of twelve deterministic optimal control

[3] As will be discussed below, two extra estimates of M_k for Nixon - I were also computed.

problems. The econometric model that was used for these results is a new model that I have recently completed. The model is described in [2] and is based on the theoretical model in [1]. The model is quarterly, consists of 84 equations, 26 of which are stochastic, and contains 78 exogenous variables. It is nonlinear and simultaneous. Three of its important features are that it is based on solid microeconomic foundations, it accounts explicitly for disequilibrium effects, and it accounts for all flows of funds in the system. Accounting for all flows of funds in the system automatically implies that the government budget constraint is satisfied and leads to the bill rate being implicitly determined in the model. Space limitations prevent any detailed discussion of the model here. A general idea of the model and its properties can be obtained by reading the first section of Chapter 1 in [2]. There is one key feature of the model that has an important effect on the present results, and this feature will be discussed below.

The coefficient estimates that were used are the two-stage least squares estimates presented in [2]. The sample period upon which these estimates are based is 1954I - 1974II. Only one set of estimates was used for all of the results, and so this is one important difference between the procedure followed here and the procedure described in Section II that could have been followed had more computer resources been available for this project.

The basic loss function that was used targets a given level of real output and a zero rate of inflation each quarter. This loss function is:

$$L_t = \gamma \left/ \frac{Y_t - Y_t^*}{Y_t^*} \right/^2 + (\%\Delta PF_t)^2 , \quad \gamma > 0 , \tag{6}$$

where

Y_t = real output of the firm sector in quarter t (at a quarterly rate),

Y_t^* = target level of Y_t ,

$$\left/ \frac{Y_t - Y_t^*}{Y_t^*} \right/^2 = \begin{cases} \left(\frac{Y_t - Y_t^*}{Y_t^*} \right)^2 & \text{if } Y_t < Y_t^* \\ 0 & \text{if } Y_t \geq Y_t^* \end{cases} ,$$

PF_t = the value of the key price deflator in the model in quarter t ,

$\%\Delta PF_t = \left(\frac{PF_t}{PF_{t-1}} \right)^4 - 1$, (percentage change in PF_t at an annual rate).

The loss function penalizes rates of inflation that are both above and below the target value of zero, but it only penalizes values of Y_t that are below the target. The target values for real output are presented in the tables below, and their construction is explained in Chapter 10 in [2]. The values are meant to correspond to high levels of economic activity. The parameter γ is the weight attached to the output target in the loss function. For the first set of estimates of M_k a value of γ of 1.0 was used, and for the second set a value of 0.1 was used. The weight attached to the output target was thus 10 times greater for the first set of estimates than for the second. The use of these two weights should provide a good indication of how sensitive the M_k estimates are to the use of fairly different loss functions.

Two variables in the model were used as control variables: the value of goods purchased by the government (in real terms), denoted as XG , and the value of government securities outstanding (in current-dollar terms), denoted as VBG . In order to lessen computational costs, it turned out to be convenient to have VBG be adjusted each quarter so as to achieve a given target level of the bill rate. The target bill-rate series is a series that has a positive trend between 1953I and 1970IV and then is flat (at 6.3 percent) from 1971I on. This treatment of VBG means that monetary policy is assumed to be accommodating in the sense of always achieving the given target level of the bill rate each quarter regardless of the value of XG chosen. Although XG is the only fiscal-policy variable used, the following results would not be changed very much if more than one variable were used. Given that the objective function targets only real output and the rate of inflation, adding, say, a tax-rate variable as a control variable would have little effect on decreasing the loss from the minimum loss that can be achieved by using XG alone. The fiscal-policy variables are collinear in this sense.

The results of solving the twelve problems are presented in Tables 1 through 6. In order to see exactly what problems were solved here, it will be useful to examine the results in Table 1 in some detail. The quarters are numbered consecutively beginning with 1953I, the first quarter of the first Eisenhower administration. The results in Table 1 are based on minimizing $\sum_{t=3}^{32} L_t$, where L_t is defined in equation (6). The first set of optimal results corresponds to the use of $\gamma = 1.0$ in the loss function, and the second set corresponds to the use of $\gamma = 0.1$. The

TABLE 1. Control Results for Eisenhower - I

t	Quarter	Actual Values						Optimal Values for $\gamma = 1.0$					Optimal Values for $\gamma = 0.1$				
		Y	100 %ΔPF	100 UR	RBILL	Target RBILL	Y*	ΔXG	ΔVBG	Y	100 %ΔPF	100 UR	ΔXG	ΔVBG	Y	100 %ΔPF	100 UR
3	1953 III	91.4	1.4	2.8	2.0	1.6	89.2	-1.6	-0.7	89.9	0.2	3.1	-1.8	-0.8	89.7	0.1	3.1
4	IV	90.0	0.1	3.7	1.5	1.6	90.0	0.4	-0.3	90.1	0.5	4.0	-0.4	-0.6	89.3	0.4	4.2
5	1954 I	89.0	4.8	5.3	1.1	1.7	90.9	1.6	0.7	90.6	5.9	5.1	0.7	0.1	89.3	5.8	5.4
6	II	88.9	0.3	5.8	0.8	1.7	91.7	2.3	1.7	91.6	1.8	5.1	1.4	1.0	90.0	1.7	5.6
7	III	90.1	1.0	6.0	0.9	1.7	92.5	1.8	2.4	92.4	2.3	5.3	0.9	1.6	90.6	2.2	5.8
8	IV	92.3	2.0	5.4	1.0	1.8	93.4	1.0	2.7	93.4	3.0	5.1	0.2	1.9	91.5	2.8	5.6
9	1955 I	95.4	0.6	4.7	1.3	1.8	94.2	-0.3	2.8	94.4	1.1	5.1	-1.0	1.9	92.5	1.0	5.6
10	II	97.3	-0.0	4.4	1.6	1.8	95.1	-0.3	3.1	95.2	-0.0	5.3	-1.0	2.3	93.3	-0.0	5.8
11	III	98.9	3.0	4.2	1.9	1.9	96.0	-0.8	3.5	95.8	2.7	5.4	-1.4	2.6	94.1	2.6	5.8
12	IV	99.9	4.1	4.2	2.3	1.9	96.9	-0.8	3.7	96.7	3.4	5.7	-1.3	2.8	95.1	3.3	6.0
13	1956 I	99.3	3.1	4.1	2.4	2.0	97.7	0.5	4.7	97.5	2.5	5.2	-0.0	3.7	96.1	2.4	5.6
14	II	99.5	2.6	4.2	2.6	2.0	98.6	0.2	5.1	98.3	2.0	5.0	-0.2	4.0	96.9	1.8	5.3
15	III	99.3	3.9	4.2	2.6	2.0	99.6	0.6	5.7	98.9	3.4	4.6	0.1	4.4	97.7	3.2	4.9
16	IV	100.8	3.9	4.1	3.1	2.1	100.5	-0.6	5.2	99.8	3.0	4.5	-1.1	3.9	98.5	2.9	4.8
17	1957 I	101.4	5.9	4.0	3.2	2.1	101.4	-0.4	5.0	100.9	5.0	4.2	-0.7	3.6	99.7	4.8	4.6
18	II	101.3	1.5	4.1	3.2	2.2	102.4	0.6	5.0	102.2	0.8	4.0	0.3	3.6	101.1	0.7	4.3
19	III	101.7	2.4	4.2	3.4	2.2	103.4	0.4	4.6	103.2	1.8	3.8	0.1	3.2	102.2	1.7	4.1
20	IV	99.9	2.5	5.0	3.3	2.2	104.5	2.4	5.0	104.3	2.2	3.9	2.1	3.5	103.1	2.0	4.2
21	1958 I	97.2	0.9	6.3	1.8	2.3	105.5	4.2	5.9	105.4	1.9	4.4	3.9	4.4	104.0	1.8	4.7
22	II	97.6	0.3	7.4	1.0	2.3	106.6	3.4	5.9	106.4	2.1	5.1	3.1	4.3	105.0	2.0	5.4
23	III	100.3	2.1	7.3	1.7	2.4	107.6	1.7	4.7	107.4	2.3	5.6	1.4	3.1	105.9	2.3	5.9
24	IV	103.0	2.2	6.4	2.8	2.4	108.7	1.1	3.5	108.5	2.0	5.4	0.8	1.9	107.0	2.0	5.6
25	1959 I	104.7	2.5	5.8	2.8	2.5	109.8	1.0	2.8	109.6	2.7	5.2	0.8	1.3	108.0	2.7	5.5
26	II	107.6	1.5	5.1	3.0	2.5	110.9	0.1	2.1	110.8	1.5	4.9	-0.1	0.6	109.3	1.4	5.1
27	III	105.9	1.5	5.3	3.5	2.6	112.0	3.2	3.0	112.0	1.4	4.5	3.0	1.6	110.5	1.3	4.8
28	IV	107.4	0.7	5.6	4.3	2.6	113.1	1.3	2.2	113.1	0.4	4.5	1.1	0.8	111.5	0.3	4.8
29	1960 I	109.8	0.6	5.2	3.9	2.7	114.2	-0.6	0.3	114.1	0.6	4.1	-0.9	-1.0	112.3	0.5	4.5
30	II	109.3	1.2	5.3	3.1	2.7	115.4	1.3	-0.1	115.1	1.6	4.1	1.1	-1.3	113.3	1.5	4.5
31	III	108.7	0.1	5.6	2.4	2.8	116.5	2.7	0.2	116.2	1.1	4.0	2.5	-1.0	114.6	0.9	4.4
32	IV	107.7	1.3	6.3	2.4	2.8	117.7	4.3	1.0	117.5	2.2	4.1	4.1	-0.2	115.9	2.0	4.5

Notes: Y = production of the firm sector (in 1958 dollars at a quarterly rate) in [2].

%ΔPF = percentage change (at an annual rate) in the price deflator PF in [2].

UR = civilian unemployment rate.

RBILL = three-month treasury bill rate.

Y* = target value of Y .

ΔXG = difference between optimal and actual values of XG .

XG = purchases of goods of the government (in 1958 dollars at a quarterly rate).

ΔVBG = difference between optimal and actual values of VBG .

VBG = value of government securities outstanding (in current dollars).

TABLE 2. Control Results for Eisenhower - II

		Actual Values						Optimal Values for $\gamma = 1.0$					Optimal Values for $\gamma = 0.1$				
t	Quarter	Y	100 %ΔPF	100 UR	RBILL	Target RBILL	Y*	ΔXG	ΔVBG	Y	100 %ΔPF	100 UR	ΔXG	ΔVBG	Y	100 %ΔPF	100 UR
17	1957 I	101.4	5.9	4.0	3.2	2.1	101.4	-0.3	-0.5	101.3	4.9	4.0	-1.1	-0.9	100.4	4.8	4.1
18	II	101.3	1.5	4.1	3.2	2.2	102.4	0.4	-0.6	102.3	1.1	3.9	-0.1	-1.1	101.5	1.0	4.2
19	III	101.7	2.4	4.2	3.4	2.2	103.4	0.3	-1.2	103.3	2.0	3.7	-0.5	-1.9	102.2	1.8	4.1
20	IV	99.9	2.5	5.0	3.3	2.2	104.5	2.3	-0.9	104.3	2.3	3.9	1.5	-1.8	103.0	2.1	4.3
21	1958 I	97.2	0.9	6.3	1.8	2.3	105.5	4.1	-0.1	105.3	2.0	4.4	3.4	-1.1	104.0	1.9	4.8
22	II	97.6	0.3	7.4	1.0	2.3	106.6	3.3	-0.3	106.4	2.1	5.1	2.5	-1.4	104.9	2.1	5.5
23	III	100.3	2.1	7.3	1.7	2.4	107.6	1.6	-1.6	107.4	2.4	5.7	0.8	-2.8	105.8	2.3	6.0
24	IV	103.0	2.2	6.4	2.8	2.4	108.7	0.9	-2.9	108.5	2.1	5.4	0.2	-4.3	106.9	2.0	5.8
25	1959 I	104.7	2.5	5.8	2.8	2.5	109.8	0.9	-3.7	109.6	2.8	5.2	0.2	-5.2	108.1	2.7	5.6
26	II	107.6	1.5	5.1	3.0	2.5	110.9	-0.1	-4.6	110.7	1.5	4.9	-0.5	-6.2	109.6	1.4	5.2
27	III	105.9	1.5	5.3	3.5	2.6	112.0	3.0	-3.8	111.9	1.5	4.5	2.8	-5.4	111.1	1.4	4.8
28	IV	107.4	0.7	5.6	4.3	2.6	113.1	1.1	-4.9	113.0	0.4	4.5	1.0	-6.5	112.5	0.4	4.7
29	1960 I	109.8	0.6	5.2	3.9	2.7	114.2	-0.7	-7.0	114.2	0.7	4.2	-1.0	-8.7	113.4	0.6	4.3
30	II	109.3	1.2	5.3	3.1	2.7	115.4	1.3	-7.4	115.3	1.7	4.1	0.9	-9.3	114.5	1.6	4.3
31	III	108.7	0.1	5.6	2.4	2.8	116.5	2.7	-7.2	116.4	1.1	4.0	2.3	-9.3	115.6	1.0	4.2
32	IV	107.7	1.3	6.3	2.4	2.8	117.7	4.1	-6.7	117.6	2.2	4.1	3.7	-8.9	116.7	2.1	4.4
33	1961 I	107.3	0.7	6.8	2.4	2.9	118.9	4.5	-6.4	118.8	1.7	4.2	4.2	-8.8	118.0	1.6	4.5
34	II	109.9	0.7	7.0	2.3	3.0	120.1	1.5	-8.7	120.0	1.7	4.6	1.2	-11.1	119.1	1.7	4.9
35	III	112.0	-0.5	6.8	2.3	3.0	121.3	1.4	-10.3	121.1	0.4	5.3	1.1	-12.8	120.2	0.4	5.5
36	IV	114.3	2.0	6.2	2.5	3.1	122.5	1.0	-11.5	122.3	2.7	5.3	0.7	-14.2	121.3	2.7	5.6
37	1962 I	116.1	0.5	5.6	2.7	3.1	123.7	1.5	-12.1	123.6	1.2	5.0	1.2	-14.9	122.6	1.1	5.2
38	II	118.0	1.3	5.5	2.7	3.2	124.9	1.2	-12.5	124.8	2.1	4.9	0.9	-15.4	123.9	2.0	5.1
39	III	119.2	0.2	5.6	2.9	3.3	126.2	1.2	-13.0	126.1	0.9	4.9	0.9	-16.0	125.0	0.9	5.2
40	IV	120.6	1.1	5.5	2.8	3.3	127.5	0.4	-14.3	127.3	1.9	5.0	0.0	-17.4	125.9	1.8	5.2
41	1963 I	121.3	0.9	5.8	2.9	3.4	128.7	1.0	-15.1	128.5	1.6	5.4	0.6	-18.2	127.1	1.5	5.7
42	II	122.4	2.0	5.7	2.9	3.5	130.0	1.0	-15.9	129.8	2.7	5.3	0.6	-19.0	128.4	2.7	5.6
43	III	124.5	0.4	5.5	3.3	3.5	131.3	-0.1	-17.6	131.1	0.8	5.3	-0.4	-20.5	129.6	0.8	5.5
44	IV	126.3	1.7	5.6	3.5	3.6	132.6	-0.5	-19.3	132.4	2.0	5.5	-0.8	-22.2	130.8	2.0	5.8
45	1964 I	128.4	1.4	5.5	3.5	3.7	134.0	-1.6	-21.8	133.7	1.7	5.8	-1.9	-24.6	131.9	1.7	6.0
46	II	130.2	1.4	5.3	3.5	3.8	135.3	-1.5	-23.8	135.1	1.7	5.6	-1.8	-26.3	133.2	1.6	5.9
47	III	131.8	1.7	5.0	3.5	3.8	136.6	-1.7	-25.7	136.4	1.9	5.6	-1.9	-28.1	134.5	1.9	5.9
48	IV	132.5	1.3	5.0	3.7	3.9	138.0	-0.8	-27.1	137.8	1.5	5.6	-0.9	-29.1	136.1	1.5	5.8

Notes: See notes to Table 1.

TABLE 3. Control Results for Kennedy - Johnson

		Actual Values						Optimal Values for $\gamma = 1.0$					Optimal Values for $\gamma = 0.1$				
t	Quarter	Y	100 %ΔPF	100 UR	RBILL	Target RBILL	Y^*	ΔXG	ΔVBG	Y	100 %ΔPF	100 UR	ΔXG	ΔVBG	Y	100 %ΔPF	100 UR
33	1961 I	107.3	0.7	6.8	2.4	2.9	118.9	10.3	5.2	118.8	1.5	5.3	9.9	5.0	118.4	1.5	5.4
34	II	109.9	0.7	7.0	2.3	3.0	120.1	3.8	4.3	119.9	1.7	4.6	3.0	3.8	118.9	1.6	4.8
35	III	112.0	-0.5	6.8	2.3	3.0	121.3	3.5	3.9	121.1	0.4	4.8	2.6	3.1	119.7	0.3	5.1
36	IV	114.3	2.0	6.2	2.5	3.1	122.5	2.8	3.5	122.2	2.7	5.0	1.8	2.4	120.6	2.6	5.4
37	1962 I	116.1	0.5	5.6	2.7	3.1	123.7	3.3	4.0	123.5	1.1	4.7	2.5	2.8	121.9	1.0	5.2
38	II	118.0	1.3	5.5	2.7	3.2	124.9	3.2	4.7	124.7	2.0	4.7	2.4	3.4	123.4	1.9	5.1
39	III	119.2	0.2	5.6	2.9	3.3	126.2	3.3	5.7	126.0	0.9	4.8	2.7	4.2	124.7	0.8	5.1
40	IV	120.6	1.1	5.5	2.8	3.3	127.5	2.8	6.1	127.2	1.8	4.8	2.0	4.4	125.7	1.7	5.1
41	1963 I	121.3	0.9	5.8	2.9	3.4	128.7	3.4	7.0	128.5	1.5	5.2	2.7	5.2	126.9	1.4	5.5
42	II	122.4	2.0	5.7	2.9	3.5	130.0	3.4	7.9	129.8	2.7	5.0	2.7	5.9	128.2	2.6	5.3
43	III	124.5	0.4	5.5	3.3	3.5	131.3	2.4	8.1	131.1	0.8	4.9	1.8	6.0	129.5	0.7	5.2
44	IV	126.3	1.7	5.6	3.5	3.6	132.6	2.1	8.2	132.4	2.0	5.1	1.3	6.0	130.7	1.9	5.4
45	1964 I	128.4	1.4	5.5	3.5	3.7	134.0	1.1	7.8	133.7	1.7	5.3	0.4	5.5	131.8	1.6	5.6
46	II	130.2	1.4	5.3	3.5	3.8	135.3	1.2	8.0	135.0	1.7	5.1	0.5	5.6	133.1	1.6	5.5
47	III	131.8	1.7	5.0	3.5	3.8	136.6	1.0	8.1	136.4	2.0	5.1	0.5	5.8	134.5	1.9	5.4
48	IV	132.5	1.3	5.0	3.7	3.9	138.0	1.9	8.9	137.8	1.6	5.0	1.4	6.6	136.1	1.5	5.2
49	1965 I	135.9	1.1	4.9	3.9	4.0	139.4	-0.3	8.0	139.2	1.2	5.0	-0.8	5.8	137.5	1.1	5.2
50	II	137.8	2.0	4.7	3.9	4.1	140.8	0.1	8.0	140.6	2.1	5.0	-0.3	5.8	139.1	2.1	5.2
51	III	140.5	1.3	4.4	3.9	4.1	142.2	-1.1	7.3	142.0	1.4	4.8	-1.4	5.1	140.5	1.3	5.0
52	IV	143.6	1.7	4.1	4.2	4.2	143.6	-2.2	6.3	143.4	1.5	4.9	-2.6	4.0	141.7	1.4	5.1
53	1966 I	146.6	2.6	3.9	4.6	4.3	144.9	-3.2	4.7	144.7	2.1	5.0	-3.8	2.4	142.8	2.0	5.2
54	II	147.7	3.8	3.8	4.6	4.4	146.2	-2.6	3.9	146.0	3.4	5.0	-3.2	1.5	144.0	3.3	5.3
55	III	148.7	2.6	3.8	5.0	4.5	147.5	-2.5	3.0	147.3	2.0	4.9	-3.2	0.5	145.1	1.9	5.2
56	IV	150.5	4.4	3.7	5.2	4.6	148.9	-3.3	1.2	148.6	3.8	4.8	-4.1	-1.3	146.3	3.7	5.1
57	1967 I	149.7	3.7	3.8	4.5	4.7	150.2	-1.3	1.2	150.0	3.7	4.7	-1.9	-1.3	147.8	3.6	5.0
58	II	150.9	1.6	3.8	3.7	4.8	151.6	-1.9	0.6	151.3	2.0	4.5	-2.6	-1.9	149.1	1.9	4.8
59	III	152.6	3.5	3.8	4.3	4.9	153.0	-2.2	-0.8	152.7	3.3	4.5	-3.0	-3.3	150.2	3.2	4.8
60	IV	153.7	3.4	4.0	4.8	5.0	154.4	-2.0	-2.0	154.0	3.3	4.6	-2.7	-4.5	151.5	3.2	5.0
61	1968 I	155.6	2.8	3.8	5.1	5.1	155.8	-2.8	-3.5	155.4	2.6	4.6	-3.5	-6.0	152.8	2.4	4.9
62	II	158.5	3.5	3.6	5.5	5.2	157.2	-4.2	-6.0	156.8	3.0	4.6	-5.1	-8.5	154.0	2.9	5.0
63	III	160.0	3.4	3.6	5.2	5.3	158.6	-4.0	-7.8	158.3	3.1	4.8	-4.7	-10.2	155.5	3.0	5.1
64	IV	161.1	4.3	3.4	5.6	5.4	160.0	-3.6	-9.7	159.7	3.7	4.7	-3.9	-11.6	157.4	3.6	5.0

Notes: See notes to Table 1.

TABLE 4. Control Results for Johnson

		Actual Values						Optimal Values for $\gamma = 1.0$					Optimal Values for $\gamma = 0.1$				
t	Quarter	Y	100 %ΔPF	100 UR	RBILL	Target RBILL	Y*	ΔXG	ΔVBG	Y	100 %ΔPF	100 UR	ΔXG	ΔVBG	Y	100 %ΔPF	100 UR
49	1965 I	135.9	1.1	4.9	3.9	4.0	139.4	2.9	1.7	139.3	1.2	4.5	2.3	1.3	138.6	1.2	4.6
50	II	137.8	2.0	4.7	3.9	4.1	140.8	1.2	1.8	140.5	2.3	4.1	0.1	0.9	139.0	2.2	4.3
51	III	140.5	1.3	4.4	3.9	4.1	142.2	0.1	1.3	141.9	1.6	4.0	-1.0	0.2	140.0	1.4	4.3
52	IV	143.6	1.7	4.1	4.2	4.2	143.6	-1.0	0.4	143.2	1.6	4.1	-2.5	-1.3	140.6	1.4	4.6
53	1966 I	146.6	2.6	3.9	4.6	4.3	144.9	-2.0	-0.7	144.4	2.2	4.3	-3.7	-2.9	141.1	2.0	4.8
54	II	147.7	3.8	3.8	4.6	4.4	146.2	-1.2	-1.0	145.6	3.5	4.4	-3.0	-3.5	142.1	3.3	5.0
55	III	148.7	2.6	3.8	5.0	4.5	147.5	-0.9	-1.2	147.0	2.1	4.3	-2.7	-4.1	143.3	1.9	4.9
56	IV	150.5	4.4	3.7	5.2	4.6	148.9	-1.6	-2.0	148.3	4.0	4.2	-3.4	-5.1	144.5	3.7	4.8
57	1967 I	149.7	3.7	3.8	4.5	4.7	150.2	0.5	-1.1	149.8	3.9	4.0	-0.6	-4.3	146.8	3.7	4.5
58	II	150.9	1.6	3.8	3.7	4.8	151.6	0.2	-0.7	151.2	2.2	3.8	-0.8	-3.9	148.5	2.0	4.2
59	III	152.6	3.5	3.8	4.3	4.9	153.0	0.1	-0.8	152.6	3.6	3.8	-1.1	-4.3	149.9	3.4	4.2
60	IV	153.7	3.4	4.0	4.8	5.0	154.4	0.4	-0.6	154.0	3.5	3.9	-0.4	-4.2	151.8	3.4	4.2
61	1968 I	155.6	2.8	3.8	5.1	5.1	155.8	-0.2	-0.8	155.5	2.8	3.8	-0.8	-4.5	153.6	2.7	4.0
62	II	158.5	3.5	3.6	5.5	5.2	157.2	-1.5	-1.9	156.9	3.3	3.8	-2.3	-5.7	155.0	3.2	4.0
63	III	160.0	3.4	3.6	5.2	5.3	158.6	-1.2	-2.2	158.3	3.3	3.9	-2.0	-6.3	156.3	3.2	4.2
64	IV	161.1	4.3	3.4	5.6	5.4	160.0	-0.7	-2.3	159.7	3.9	3.8	-1.6	-6.6	157.7	3.8	4.1
65	1969 I	162.5	4.1	3.4	6.1	5.5	161.5	-1.0	-2.9	161.2	3.7	3.8	-1.9	-7.3	159.1	3.6	4.1
66	II	163.2	4.4	3.5	6.2	5.6	162.9	-0.5	-3.0	162.6	4.2	3.7	-1.5	-7.9	160.5	4.0	4.1
67	III	163.8	4.7	3.6	7.0	5.7	164.4	-0.1	-3.5	164.1	4.3	3.7	-1.3	-8.8	161.8	4.1	4.2
68	IV	162.9	4.4	3.6	7.3	5.8	165.9	1.8	-3.0	165.5	4.4	3.3	0.2	-8.9	162.8	4.0	3.9
69	1970 I	161.9	4.5	4.2	7.3	5.9	167.4	3.2	-1.9	166.9	4.9	3.4	1.4	-8.6	163.7	4.5	4.0
70	II	161.9	4.5	4.8	6.8	6.0	168.9	3.5	-1.3	168.4	5.0	3.5	1.6	-8.5	164.9	4.7	4.1
71	III	163.2	3.2	5.2	6.4	6.2	170.5	2.6	-1.7	169.8	3.6	3.8	0.3	-9.6	165.4	3.5	4.5
72	IV	161.2	7.2	5.8	5.4	6.3	172.0	5.8	0.4	171.3	7.9	4.2	3.3	-8.2	166.2	7.8	5.0
73	1971 I	165.6	3.6	6.0	3.9	6.3	173.6	2.1	-0.6	172.9	4.8	4.5	0.6	-8.8	168.2	4.7	5.2
74	II	166.8	4.1	5.9	4.2	6.3	175.1	4.1	0.4	174.6	4.8	4.8	3.0	-7.4	170.4	4.8	5.4
75	III	167.8	2.6	6.0	5.1	6.3	176.7	4.4	1.2	176.2	3.0	5.0	3.5	-5.9	172.1	3.0	5.4
76	IV	170.6	0.4	6.0	4.2	6.3	178.3	3.5	2.2	177.7	1.4	5.1	2.2	-4.5	173.3	1.4	5.5
77	1972 I	173.6	5.3	5.8	3.4	6.3	179.9	3.6	4.2	179.3	6.7	5.3	2.4	-2.2	174.9	6.6	5.6
78	II	177.5	1.7	5.7	3.7	6.3	181.6	2.7	5.8	181.0	2.6	5.5	1.9	0.3	176.9	2.5	5.8
79	III	180.2	2.2	5.6	4.2	6.3	183.2	3.0	8.4	182.7	2.9	5.6	2.3	3.8	178.5	2.9	5.9
80	IV	184.0	3.0	5.3	4.9	6.3	184.9	1.7	10.4	184.4	3.5	5.6	1.4	6.9	180.4	3.4	5.8

Notes: See notes to Table 1.

TABLE 5. Control Results for Nixon - I

		Actual Values						Optimal Values for $\gamma = 1.0$					Optimal Values for $\gamma = 0.1$				
t	Quarter	Y	100 %ΔPF	100 UR	RBILL	Target RBILL	Y*	ΔXG	ΔVBG	Y	100 %ΔPF	100 UR	ΔXG	ΔVBG	Y	100 %ΔPF	100 UR
65	1969I	162.5	4.1	3.4	6.1	5.5	161.5	-1.2	-1.0	161.5	3.8	3.5	-1.0	-0.9	161.6	3.8	3.5
66	II	163.2	4.4	3.5	6.2	5.6	162.9	-0.4	-1.2	162.8	4.2	3.6	-1.2	-1.7	162.0	4.1	3.7
67	III	163.8	4.7	3.6	7.0	5.7	164.4	-0.0	-1.6	164.2	4.3	3.7	-1.1	-2.7	162.9	4.1	3.9
68	IV	162.9	4.4	3.6	7.3	5.8	165.9	1.9	-0.9	165.5	4.4	3.3	0.5	-2.6	163.8	4.1	3.6
69	1970I	161.9	4.5	4.2	7.3	5.9	167.4	3.3	0.3	167.0	4.9	3.3	1.8	-2.1	165.0	4.5	3.8
70	II	161.9	4.5	4.8	6.8	6.0	168.9	3.6	1.1	168.5	5.0	3.4	1.8	-2.0	166.1	4.7	3.9
71	III	163.2	3.2	5.2	6.4	6.2	170.5	2.6	0.6	169.8	3.6	3.7	0.3	-3.5	166.6	3.4	4.4
72	IV	161.2	7.2	5.8	5.4	6.3	172.0	5.6	2.6	171.1	7.9	4.2	3.2	-2.5	167.4	7.7	4.9
73	1971I	165.6	3.6	6.0	3.9	6.3	173.6	2.1	1.7	173.0	4.8	4.5	0.2	-3.7	169.4	4.7	5.2
74	II	166.8	4.1	5.9	4.2	6.3	175.1	4.3	2.9	174.8	4.8	4.8	2.7	-2.9	171.6	4.7	5.4
75	III	167.8	2.6	6.0	5.1	6.3	176.7	4.5	3.7	176.4	3.0	4.9	3.1	-2.2	173.4	3.0	5.4
76	IV	170.6	0.4	6.0	4.2	6.3	178.3	3.4	4.6	177.9	1.4	5.1	1.9	-1.6	174.9	1.4	5.5
77	1972I	173.6	5.3	5.8	3.4	6.3	179.9	3.5	6.5	179.4	6.7	5.3	2.1	-0.1	176.6	6.6	5.7
78	II	177.5	1.7	5.7	3.7	6.3	181.6	2.7	8.1	181.2	2.6	5.5	1.7	1.4	178.7	2.5	5.8
79	III	180.2	2.2	5.6	4.2	6.3	183.2	3.0	10.7	182.9	2.9	5.6	1.9	3.8	180.3	2.9	5.9
80	IV	184.0	3.0	5.3	4.9	6.3	184.9	1.7	12.6	184.5	3.4	5.6	0.4	5.3	181.7	3.4	6.0
81	1973I	188.7	3.3	5.0	5.6	6.3	186.5	-0.4	13.2	186.2	3.5	5.7	-2.1	5.4	182.6	3.4	6.1
82	II	189.6	5.7	4.9	6.6	6.3	188.2	1.3	15.9	187.8	5.5	5.7	-0.8	7.4	183.4	5.4	6.2
83	III	190.5	5.4	4.8	8.4	6.3	189.9	0.9	17.1	189.0	4.7	5.5	-2.3	7.4	183.0	4.5	6.1
84	IV	191.7	9.5	4.7	7.5	6.3	191.7	-0.3	18.1	189.6	9.3	5.3	-5.4	6.0	180.7	9.0	6.4
85	1974I	187.8	14.5	5.1	7.6	6.3	193.4	3.8	22.7	190.2	14.1	5.4	-2.6	7.8	178.4	13.8	6.8
86	II	186.9	15.5	5.1	8.3	6.3	195.1	3.6	25.2	191.3	15.0	4.8	-3.1	8.4	177.7	14.5	6.5
87	III	185.9	13.0	5.5	8.3	6.3	196.9	5.1	28.5	193.2	12.7	4.6	-0.6	11.3	179.9	12.1	6.4
88	IV	181.0	13.3	6.6	7.3	6.3	198.7	10.5	37.6	195.9	13.4	4.9	6.8	21.2	185.3	13.0	6.3
89	1975I	174.7	11.0	8.3	5.9	6.3	200.5	15.1	47.9	199.1	11.6	5.5	13.8	34.7	192.1	11.3	6.4

Notes: See notes to Table 1.

TABLE 6. Control Results for Nixon - Ford

		Actual Values						Optimal Values for $\gamma = 1.0$					Optimal Values for $\gamma = 0.1$				
t	Quarter	Y	100 %ΔPF	100 UR	RBILL	Target RBILL	Y*	ΔXG	ΔVBG	Y	100 %ΔPF	100 UR	ΔXG	ΔVBG	Y	100 %ΔPF	100 UR
81	1973 I	188.7	3.3	5.0	5.6	6.3	186.5	-2.3	-1.6	186.2	3.5	5.2	-3.2	-2.3	185.3	3.5	5.3
82	II	189.6	5.7	4.9	6.6	6.3	188.2	-0.9	-2.0	187.5	5.4	5.3	-2.3	-3.7	185.6	5.3	5.5
83	III	190.5	5.4	4.8	8.4	6.3	189.9	-1.2	-3.4	188.6	4.7	5.1	-3.8	-6.8	185.0	4.5	5.6
84	IV	191.7	9.5	4.7	7.5	6.3	191.7	-2.1	-4.8	189.3	9.3	5.1	-6.9	-11.3	182.3	9.1	5.9
85	1974 I	187.8	14.5	5.1	3.6	6.3	193.4	2.2	-2.5	189.9	14.2	5.1	-4.5	-13.0	179.2	13.9	6.4
86	II	186.9	15.5	5.1	8.3	6.3	195.1	2.4	-1.8	191.2	15.1	4.6	-5.1	-15.6	177.9	14.6	6.3
87	III	185.9	13.0	5.5	8.3	6.3	196.9	4.2	0.1	193.3	12.9	4.4	-2.3	-15.5	179.8	12.3	6.3
88	IV	181.0	13.3	6.6	7.3	6.3	198.7	9.8	7.5	196.1	13.7	4.5	5.6	-7.8	185.4	13.1	6.2
89	1975 I	174.7	11.0	8.3	5.9	6.3	200.5	14.5	16.7	199.3	11.8	5.1	13.2	4.0	192.7	11.5	6.2

Notes: See notes to Table 1.

control period began with quarter 3 in this case rather than with quarter 1 because of lack of enough earlier data.

Consider the solution of the control problem for $\gamma = 1.0$. The aim here is to try to pick a set of control values that closely approximates the set that the first Eisenhower administration would have chosen had it behaved in the manner described in Section II. Since it was too costly to do any stochastic simulation and to reoptimize each quarter, what was done instead was to convert the stochastic control problem into a deterministic control problem and then to solve this problem only once for the 30-quarter period.

A standard way of converting stochastic control problems to deterministic control problems is to set all of the error terms in the model equal to their expected values, usually zero. An alternative way, however, and the one followed here, is to set the error terms equal to their historic values, i.e., to their estimated values. The procedure of setting the error terms equal to their historic values before solving assumes that an administration has more knowledge than it actually has. An administration clearly does not know all future values of the error terms. The procedure of setting the error terms equal to their expected values before solving (and solving only once), on the other hand, assumes that an administration has less knowledge than it actually has because it can continually adjust to past error terms by reoptimizing each quarter. The procedure of setting the error terms equal to their historic values was chosen here on the grounds that it seemed likely to lead to a set of optimal values that more closely approximates the set of values that would result if an administration behaved optimally as defined in Section II.

The optimal values of the endogenous variables in the six tables

$(Y, \%\Delta PF, UR)$ are obtained from a dynamic simulation of the model using the optimal values of the two control variables and the historic realization of the error terms. These values can be compared directly to the actual values of the endogenous variables because when the model is simulated using the actual values of the control variables and the historic realization of the error terms, the solution values of the endogenous variables are just the actual values.

The results in Tables 2 through 6 are based on minimizing, respectively, $\sum_{t=17}^{48} L_t$, $\sum_{t=33}^{64} L_t$, $\sum_{t=49}^{80} L_t$, $\sum_{t=65}^{89} L_t$, and $\sum_{t=81}^{89} L_t$. The length of the control horizon is thus 30 quarters in Table 1, 32 quarters in Tables 2 through 4, 25 quarters in Table 5, and 9 quarters in Table 6. These lengths are, of course, somewhat short relative to what would have been desirable had there been more data and a larger computer budget for this project. Only observations through 1975I were available at the time that the data were collected for this project.

The twelve deterministic control problems were solved by the method described in [3] using a gradient algorithm. See Chapter 10 in [2] for a detailed discussion of the solution of deterministic optimal control problems using the present model. The solution of each of the twelve problems required about 8 minutes of computer time on the IBM 370-158 at Yale. Although the solutions obtained from this work will be referred to as optimal solutions, it should be stressed that there is no guarantee that the algorithm actually found the true optimum in each case. The solutions should thus be interpreted as being likely to be close to the true optima, but not necessarily exactly the true optima.

One important characteristic of the results in the six tables is that the output targets are generally much closer to being achieved than are the inflation targets. The model has the property that output can be increased to some reasonable target value (from a lower value) without having too serious an effect on the rate of inflation. It is not, however, generally possible to decrease the rate of inflation to, say, zero percent (from a higher rate) without having serious effects on the level of output. Consequently, when a loss function like (6) is minimized, with equal weights attached to the output and inflation targets, the optimum tends to correspond more closely to the output targets being achieved than it does to the inflation targets being achieved. Even when the weight on the output targets is only one-tenth of the weight on the inflation targets, it is still the case that the inflation targets of zero percent are generally not close to being achieved in the tables.

Given the results in the six tables, it is now possible to describe how M_k was estimated for each administration. Because of data limitations for Nixon - I, it was necessary in order to make all of the estimates comparable to modify the third and fourth terms in equation (5) to be the loss in the next two-year period rather than in the next four-year period. Consider the estimate of M_k for Eisenhower - I. The first term in equation (5) can be estimated by taking the actual values of Y and $\%\Delta PF$ in Table 1 for each of the first 14 quarters, substituting them into equation (6) to compute a value of L_t for each quarter, and then summing the 14 values of L_t to estimate the first term. 14 rather than 16 quarters have to be used here because of data limitations. The second term in equation (5) can be estimated by doing the same thing for the optimal values of Y and $\%\Delta PF$ in Table 1 for the first 14 quarters. The third term in equation (5) (for two years now, rather than four) can be estimated by taking the optimal values of Y and $\%\Delta PF$ in Table 2 for each of the first 8 quarters (quarters 17 through 25), substituting them into equation (6) to compute a value of L_t for each quarter, and then summing the 8 values of L_t to estimate the third term. Finally, the fourth term in equation (5) can be estimated by doing the same thing for the optimal values of Y and $\%\Delta PF$ in Table 1 for quarters 17 through 24.

The estimate of M_k for Eisenhower - I thus requires the use of both Tables 1 and 2. The optimal values in Table 1 for quarters 3 through 16 are interpreted as being approximations to what Eisenhower - I could have achieved had it behaved optimally; the optimal values in Table 2 for quarters 17 through 24 are interpreted as being approximations to what Eisenhower - II could have achieved had it behaved optimally, but Eisenhower - I did not; and the optimal values in Table 1 for quarters 17 through 24 are interpreted as being approximations to what Eisenhower - II could have achieved had both it and Eisenhower - I behaved optimally. The estimates of M_k for the other administrations are computed in an analogous way. The estimate of M_k for Nixon - I, for example, requires the use of both Tables 5 and 6.

The estimates of M_k, denoted as $\hat{M}_k$, for the five administrations and the two values of γ are presented in Table 7. Since the loss function is additive in output and the rate of inflation, it is possible to break up $\hat{M}_k$ into two parts: a part due to the output performance, denoted as Q in the table, and a part due to the inflation-rate performance, denoted as P in the table. The values of $\hat{M}_k$ are the values in the last column of Table 7. Two estimates of M_k are actually presented in the table for Nixon - I for each value of γ. The first estimate for each value of γ is based on the results in Tables 5 and 6. The results

TABLE 7. Estimates of M_k for the Five Administrations

a = estimated actual expected loss in administration k's four-year period in office ($3\frac{1}{2}$ years for Eisenhower-I).

b = estimated expected loss in the four-year period ($3\frac{1}{2}$ years for Eisenhower-I) if administration k had behaved optimally.

c = estimated expected loss in the next two-year period given that administration k did not behave optimally, but assuming that administration k+16 did.

d = estimated expected loss in the next two-year period if both administrations k and k+16 had behaved optimally.

$\hat{M}_k = a - b + c - d$.

Q = output part of loss.

P = inflation-rate part of loss.

Administration \ $\gamma = 1.0$	a Q	a P	a Q+P	b Q	b P	b Q+P	c Q	c P	c Q+P	d Q	d P	d Q+P	$\hat{M}_k$ Q	$\hat{M}_k$ P	$\hat{M}_k$ Q+P
Eisenhower-I	0.225	1.036	1.261	0.013	1.035	1.048	0.002	0.534	0.536	0.005	0.510	0.515	0.209	0.025	0.234
Eisenhower-II	4.774	0.738	5.512	0.003	0.750	0.753	0.002	0.216	0.218	0.001	0.236	0.237	4.772	-0.033	4.739
Kennedy-Johnson	5.753	0.247	6.000	0.004	0.483	0.488	0.008	0.497	0.505	0.002	0.449	0.451	5.755	-0.188	5.567
Johnson	0.127	1.470	1.597	0.012	1.393	1.406	0.006	1.927	1.933	0.007	1.940	1.947	0.113	0.064	0.178
Nixon-I	1.972	2.623	4.595	0.011	3.209	3.220	0.146	9.328	9.474	0.135	9.141	9.276	1.972	-0.400	1.573
Nixon-I*	1.972	2.623	4.595	0.021	2.881	2.902	0.160	9.872	10.032	0.135	9.876	10.011	1.977	-0.262	1.714
$\gamma = 0.1$															
Eisenhower-I	0.022	1.036	1.058	0.043	0.959	1.002	0.015	0.498	0.513	0.017	0.473	0.489	-0.023	0.102	0.080
Eisenhower-II	0.477	0.738	1.215	0.022	0.695	0.717	0.012	0.195	0.207	0.007	0.224	0.230	0.461	0.014	0.475
Kennedy-Johnson	0.575	0.247	0.823	0.030	0.438	0.468	0.041	0.435	0.476	0.017	0.415	0.432	0.570	-0.191	0.399
Johnson	0.013	1.470	1.483	0.065	1.261	1.326	0.020	1.790	1.810	0.041	1.778	1.819	-0.074	0.221	0.147
Nixon-I	0.197	2.623	2.820	0.048	3.045	3.093	0.286	8.684	8.970	0.317	8.543	8.860	0.117	-0.281	-0.164
Nixon-I*	0.197	2.623	2.820	0.040	2.734	2.774	0.295	9.250	9.545	0.331	9.239	9.570	0.121	-0.100	0.021

Notes: *These claculations are based on the solutions of the optimal control problems in which the target bill rates are taken to be the historic rates.
All numbers in the table have been multiplied by 100. The numbers may not add because of rounding.

in Tables 5 and 6, as in Tables 1 through 4, are based on the solutions of the optimal control problems in which the bill rate each quarter is constrained to be equal to a given target rate. The target rates are presented in Tables 1 through 6. The second estimate of M_k for each value of γ for Nixon - I is, on the other hand, based on the solutions of the optimal control problems in which the bill rate each quarter is constrained to be equal to the historic rate. In other words, for these problems the target bill rates were merely taken to be the historic rates. To conserve space, the results of solving the four extra optimal control problems that were needed to estimate the two extra values of M_k for Nixon - I are not presented here.

The reason for the two extra estimates of M_k for Nixon - I is as follows. First note from Table 5 that the bill rates in the last eight quarters of the Nixon - I administration (1971I - 1972IV) are considerably less than the target rates of 6.3 percent. In the model the bill rate has, other things being equal, a positive effect on the rate of inflation. Consequently, constraining the optimum to correspond to the target bill rates causes the rate of inflation in these and later quarters to be higher than it would have been had, say, the target bill rates for the eight quarters been the actual rates. This constraint was severe enough to cause the estimate of M_k for $\gamma = 0.1$ to be negative for Nixon - I. From columns Q+P under a and b in Table 7 it can be seen that the actual loss for the period of the Nixon - I administration (2.820) is less than the optimal loss (3.093). The periods of the Nixon - I and Nixon - Ford administrations are the only periods in which there are substantial differences between the target bill rates and the actual rates, and so it was decided to reestimate M_k for Nixon - I under the assumption that the target rates are the actual rates. As can be seen in Table 7, the second estimates of M_k are only slightly larger than the first. For $\gamma = 1.0$, $\hat{M}_k$ increased from 1.573 to 1.714, and for $\gamma = 0.1$, $\hat{M}_k$ increased from -0.164 to 0.021. The second estimate of M_k for $\gamma = 0.1$ is, however, positive.

Consider now the rankings of the five administrations in Table 7. The results for both values of γ show that Eisenhower - II and Kennedy - Johnson did relatively poorly and that Eisenhower - I and Johnson did relatively well. Nixon - I was the best of the five for $\gamma = 0.1$, but was only average for $\gamma = 1.0$. Except for Nixon - I, the results in Table 7 indicate that the relative evaluation of the administrations is not very sensitive to the use of the two quite different weights on the output targets in the loss function.

In order to see better what lies behind the estimates of M_k in Table 7, it will be useful to examine the results for Nixon - I and Kennedy - Johnson in more detail. Note first that for both values of γ, Nixon - I does not do well regarding the actual expected loss during its four-year period in office (column Q+P under a). For $\gamma = 0.1$, it is in fact the worst of the five (Q+P = 2.820). Most of this loss, however, is due to the inflation loss (e.g., P = 2.623 for $\gamma = 0.1$). Since the model has the property that it is expensive (in terms of lost output) to lower the rate of inflation, Nixon - I does not get penalized for the fact that there was a lot of inflation during its term in office. Most of the actual expected loss for Kennedy - Johnson, on the other hand, is due to the output loss. Since the model has the property that it is not expensive (in terms of extra inflation) to increase output to some high-activity level, Kennedy - Johnson gets penalized heavily for not doing so.

It would thus be quite misleading in the present context to evaluate an administration on the basis of the actual expected loss during its term in office. The evaluation of an administration depends crucially on whether this loss is primarily output loss or inflation loss.

The value c-d in Table 7 measures for each administration the expected loss in the next two-year period from the fact that it did not behave optimally. These values are generally quite small and in a few cases are actually negative.[4] The results thus indicate that none of the five administrations left its successor with a particularly bad state of the economy in the sense defined here. In particular, the results indicate that Johnson did not leave Nixon - I with a bad state of the economy, although it is commonly alleged that he did so. (Remember that by a bad state here is meant a state from which it is difficult for an optimally behaving administration to recover.)

Two final points about the results in Table 7. First, because the c-d estimates are small and because actual output loss is penalized much more heavily than actual inflation loss, a fairly close approximation to $\hat{M}_k$ in Table 7 is merely actual output loss (column Q under a in the table). This may, of course, not always be the case, but it is for the most part for the five administrations considered here. Second, note for Kennedy - Johnson that the results in Table 3 say that the administration should have increased government spending (XG) by a huge amount in the first quarter of its term (an increase in XG of 10.3 or 9.9 billion dollars at a quarterly rate). This may be a bit extreme to expect of an administration, but even if the administration had been allowed to

[4]There is nothing in the procedure followed in this section that guarantees that c-d will be positive.

spread this amount out over, say, four or six quarters with no cost, it still would have done poorly in the present context. The bad output performance of the Kennedy - Johnson administration, and hence the bad value of $\hat{M}_k$, is not due solely to the first few quarters of its term. Its output performance is fairly weak over the entire 16 quarters.

IV. Summary and Conclusion

The measure of economic performance proposed in this paper takes into account the difficulty of controlling the economy and the problem of leaving one's successor with a bad state of the economy. The results of estimating this measure for the past five administrations show that Eisenhower - I and Johnson did well, that Eisenhower - II and Kennedy - Johnson did poorly, and that Nixon - I did well according to one loss function and average according to the other. The results also show that none of the five administrations left a particularly bad state of the economy to its successor (bad in the sense defined here). Finally, the results show that the measure of performance of an administration is not closely correlated with the actual expected loss during the administration's four-year period in office, but that it is closely correlated with the output part of this loss.

A key property of the econometric model used here is that it is expensive in terms of lost output to lower the rate of inflation, but that, conversely, it is not expensive in terms of extra inflation to raise the level of output. For a model with the opposite property, the results in Section III would be quite different. It does seem to be the case, however, that most models have the property of the present model,[5] which adds some support to the results in Section III. Nevertheless, the sensitivity of the results in Section III to this property should be stressed. Given this property, the results in Section III are not very sensitive to the use of quite different weights on the output targets in the loss function.

The estimates of M_k in Section III may be sensitive to the computational approximations that had to be used, but until more money is available and more experiments performed, this will have to remain an open question.

Finally, an obvious point. If an administration wants to carry out a certain policy and the Congress or Federal Reserve prevents it from doing so, it should not necessarily be blamed for what happened. For simplicity, this paper was written as if an administration should be completely praised or blamed for what happens, but the word "administration" could obviously be changed to, say, "government" or "policy makers." It is beyond the scope of this paper to allocate praise or blame to particular people or groups. If, however, the measure proposed in this paper does become widely used as a way of evaluating the economic performances of particular people or groups, I have a perfect name for it.

[5] This property, for example, was much in evidence in the optimal control results for eight models (including the present model) that were presented at an AEA session in Dallas on December 28, 1975. The title of this session was "Economic Fluctuations and Stabilization Policy 1965-75: Some Econometric Evidence."

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