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AT YALE UNIVERSITY

Box 2125, Yale Station
New Haven, Connecticut 06520

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EQUILIBRIUM WAGE DISTRIBUTIONS

Joseph E. Stiglitz

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by

Joseph E. Stiglitz

1. Introduction

The observation that different firms pay different prices for what appears to be the same commodity or pay different wages for what appears to be equivalent labor has long been explained in economics by a reference to "imperfect information." This paper is concerned with characterizing market equilibrium with imperfect information. We do not present a general theory; rather, we develop in some detail an example of importance in its own right--imperfect information in the labor market. Several properties of our example, in particular, the existence of equilibria with price (wage) dispersion, multiple equilibria and the non-optimality of some or all of the equilibria, we believe are of more general validity; other results may not be.

Two kinds of imperfect information play a role in the labor market:

(a) Whether or not individuals know about the wage distribution, they may not know the wage paid in any particular firm or whether there is a vacancy in any particular firm until they apply for a job.

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(b) There are a number of characteristics beside the wage rate associated with any job which are important to the individual. Some, like the normal length of a work week, become known when the individual applies for the job; other characteristics (e.g., personalities of colleagues) become known only after the individual has completed his training.[1]

There is one important difference between the two kinds of imperfect information: there is a return to matching individuals up with jobs that suit them in any economic system. There will be imperfect information of this sort so long as individuals and jobs differ. On the other hand, the imperfect information resulting from wage dispersion is a kind of imperfect information generated by the market itself. In particular a socialist economy could, if it chose, simply pay a uniform wage for identical labor. We are used to thinking of markets as serving a useful function in conveying information, e.g., about demands and supplies. Our analysis suggests that, under certain conditions, the market may, in effect unnecessarily "create" imperfections of information which, not surprisingly, may turn out to be quite costly.

There are two "problems" in constructing an "equilibrium" model with imperfect information: first, how do we prevent the eventual accumulation of information. If individuals were infinitely lived, and jobs

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[1]This is the analogue to the "specific information" about individuals which a firm seeks when it employs someone. See Stiglitz (1974a). Akerlof's Theory of Lemons is perhaps the earliest discussion of markets with differential information. More recently, Spence, Stiglitz (1971, 1974), Salop and Salop, Arrow (1973) and Rothschild and Stiglitz have focused on imperfect information about qualities of individuals, their productivities, turnover rates, and accident proneness.

never changed, eventually, through search,[1] everyone would find the job which most suited him. "Imperfect information" would only characterize the market in the short run. In the discussion below, we maintain a "continual flow of ignorance" through a continual flow of new entrants into the labor force and a flow of deaths out of it.[2] This "flow of ignorance" is just large enough to offset the "flow of knowledge" resulting from search behavior, and an equilibrium with imperfect information is sustained.

The second problem is, how do we induce different firms to pay different wages. That is, imperfect information may explain why individuals pay a price for a commodity which is in excess of the lowest price being charged in the market, or why they accept a wage which is below the maximum being paid in the market. But we still must explain why some profit maximizing firms charge one price or pay one wage while other firms charge another price or pay another wage.

In the discussion below, we provide two "solutions" to this problem.[3] In the first, presented in Section 2, we assume different firms face different training costs. (The training is assumed to be specific to

[1]As Rothschild has emphasized, if search were costly, individuals might stop short of obtaining "perfect information," and so price (wage) dispersions might be maintained.

[2]If there were exogeneous sources of uncertainty, e.g. technical change, so that "jobs" have a finite life, then the analysis would be similar to that contained here.

[3]Salop (1973) and Stiglitz (1974b) provide quite another explanation of wage and price dispersion in monopolistic markets: the wage and price dispersion may enable the monopolist to discriminate more effectively among his customers.

the firm.) By paying higher wages, the firm can reduce labor turnover; the importance of reducing labor turnover depends on the magnitude of the training costs. Thus firms with higher training costs pay higher wages. It should be noted that with perfect information, if firms differed in their requirements for training which is specific to the firm, the firms would absorb these training costs, but the individual in the firm with more specific training would receive the same wage as the firm with little specific training.

Somewhat more surprising, however, is the result that there can be an equilibrium with wage dispersion even in the absence of differences in firm's training costs. For this to be the case, clearly profits when viewed as a function of the wage paid must have more than one relative maxima, with the value of profits at the different relative maxima identical. This can be shown to be the case under very simple conditions.

There is an important interaction between the search for higher wage jobs and the search for jobs which "match" one's preferences. The existence of wage dispersion clearly affects the profitability of searching for a "better match." What is not so obvious, however, is that when there is imperfect information of the second type ("matching individuals to jobs") there must necessarily be imperfect information of the first type: the market equilibrium must have wage dispersion.

Previous studies have focused on firm behavior when facing individuals whose turnover rates is affected by the wage rate (Mortenson, Salop (1972)), and with individual behavior in markets with wage dispersions (Salop (1972)). But there have been no attempts to link the two sides of the market together: the quit rate of the individuals is affected by the wage distribution; the wage paid by a firm with a given training cost is determined by the quit rate function, so the wage distribution in turn is determined by the distribution of training costs. Thus, corresponding to any distribution of training costs there is an equilibrium distribution of wages. But the wage rates and quit rates associated with any given level of training costs must be such that these firms just break even (assuming free entry).

Our analysis can thus be viewed as an attempt at a simple general equilibrium formulation of the conventional search models. Such a formulation is required not only because in its absence, we are likely to be misled into formulating models in which there is not in fact wage or price dispersion (as Rothschild (1973) argues to be the case with Stigler's analysis) but also because such an analysis is required if we are to make any valid welfare economic statements about the behavior of markets with imperfect information.

The exact characteristics of the equilibrium turn out to depend rather sensitively on the exact assumptions one makes about production, search, tastes, labor supply, and information. The simplest version of the model is presented in the next section; this is then extended to a number of different directions in subsequent sections.

2. The Basic Model

2.1. Introduction

The basic model presented in this section is a simplified version of the conventional search model. The basic ingredients (described more fully below) are the following:

(a) Individuals are continually searching for a better, i.e. in our simplified model, higher paid, job. They quit when they successfully find a better job.

(b) For simplicity, we assume that there are only two commodities, one of which requires no training costs and which we choose as our numeraire. All production processes are characterized by constant returns to scale, and there is free entry. In the other industry, firms face specific training costs. By paying higher wages, they reduce their turnover rate. The optimal level of wages depends on their training costs. Firms also have a choice of technique; techniques which have higher output per man have a higher training cost.

Each firm in equilibrium is then characterized by a wage, a choice of technique, and a level of employment and output; the market equilibrium is characterized by a wage distribution[1] and a set of relative prices. In equilibrium,

(a) given the quit rate function generated by the wage distribution, each firm has chosen the technique and wage which maximizes its profits;

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[1]As we note later, there are actually three distributions which characterize the equilibrium:

(i) The distribution of individuals by the wages they receive (and the industry in which they work).
(ii) The distribution of firms by the wages they pay.
(iii) The distribution of firms by the training costs of the technique they employ.

(b) all firms make zero profits;

(c) all markets for goods and labor clear;

(d) the number of individuals quitting a firm at any moment of time equals the number of individuals who accept employment at it.

Two of these conditions require some comment: the second condition follows immediately from the assumptions of free entry and the constant returns to scale property of the technology. The fourth condition is a statement that we are concerned only with the stochastic equilibrium of the economy.[1]

In the remainder of this section, we set up and analyze the equilibrium of our simplified economy. We procede in several stages: in Section 2.2 we describe the behavior of individuals in the economy while in Section 2.3 we describe the behavior of firms, as they face a given quit rate function. Section 2.4 derives the quit rate function, given the distribution of firms by training costs. In fact, however, the distribution of firms by training costs is an endogenous variable in the system, and it is derived in Section 2.5. Finally Section 2.6 describes the determination of the relative price of the two commodities. Having thus described all the "pieces" of the economy, we put them together in Section 2.7 for an analysis of the general equilibrium of the economy.

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[1] This restriction is roughly analogous to the restriction in much of growth theory to steady state analysis, and is employed for essentially the same reason. The derivation of the optimal wage policy of the firm and the derivation of the quit rate functions out of steady state is a complicated matter which would obscure the fundamental points we wish to establish here.

2.2. Individual Behavior

We assume all individuals are identical.[1] They die exponentially at the rate μ, and they are replaced by an equal number of new workers, so that the labor supply remains constant at $\bar{L}$. Each laborer supplies one unit of labor. When individuals enter the labor force, they randomly apply for a job, which they accept. Meanwhile, they continue to search for a better, i.e. a higher paying, job. The length of time to go from one firm to another to find out its wage is a random variable described by a Poisson process; the average number of searches per unit of time is s, which is fixed. There is no cost to search or to changing jobs,[2] up to the search intensity s; more intensive search is prohibitively expensive. Individuals do not know the wage paid by any particular firm; since s is determined in effect exogenously, we need make no assumption concerning whether the individual knows the wage distribution. Later (Section 4), however, we shall have to be more explicit on this point.

2.3. The Behavior of the Firm: The Determination of the Wage, Given the Quit Rate Function

Both sectors of the economy have production processes which require no capital, and which have constant returns to scale.

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[1] In Section 5, we assume individuals differ with respect to their evaluation of the non-pecuniary characteristics associated with any firm.

[2] In Section 4, more general cost functions are introduced, and the search intensity s is determined.

Each firm chooses a wage. Clearly, if at that wage rate, it is making a profit, it will attempt to expand, if it makes a loss, it will contract or change its wage or production process. We shall be concerned with characterizing equilibria; accordingly, under the assumption of free entry, each firm makes zero profits, and so is indifferent about the scale of production. Thus, having determined a wage policy, the firm simply accepts for employment all individuals who apply. Its crucial decision is the determination of its wage rate.[1]

In the no training cost industry, the determination of the equilibrium wage is a simple matter. Let a_0 be the output per man in this sector. Letting the output of this sector be our numeraire, for the zero profit condition to be satisfied, the wage, w, must be equal to a_0:

$$w_0 = a_0 . \tag{2.1}$$

In the sector with training costs, a production process is characterized by a fixed training cost per worker, T (which is assumed to occur instantaneously upon hiring the worker), and by a quantity of output per man year, $a(T)$.

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[1]Alternatively, if firms are large, then the number of applicants will equal the number of deaths; then the firm's production process may involve capital as well as labor. In other cases, that is, for small firms with capital, after each death or quit there is a (random) period of idleness of the machine. This means that firms must worry about the percentage of time machines are idle, and individuals may apply to firms with no vacancies. This complicates but does not basically change the analysis.

The firm must decide on what wage to pay and what technique to employ at each moment of time, so that it maximizes the present discounted value of its profits. Again, our restriction to stochastic equilibrium analysis allows us to greatly simplify the problem, for then the wage paid and the technique employed by the firm are constant over time, and are chosen to maximize:

$$\pi_i \equiv \{pa(T_i) - [w_i + (q(w_i) + r)]T\}L_i \tag{2.2}$$

where r is the rate of discount (rate of interest)

π/r is the present discounted value of profits

$q(w)$ is the quit rate function

L_i is the level of employment

p is the price of output.

The labor costs consists of the direct wage payments (wL), training costs to replace workers who quit (qTL), and interest on previous expenditures for training costs (rTL). q, the quit rate, acts essentially like a depreciation factor on "human capital" expenditures of the firm. By increasing w, the direct labor costs are increased but the turnover costs (the "depreciation rate") is reduced. The profits are maximized (labor costs per worker minimized) when

$$1 + q'(w)T = 0 .^{1} \tag{2.3}$$

[1] If the supply of laborers to the firm is a function of the wage it pays, then we have

$$1 + q'(w)T = (ap - w - (q+r)T)L'/L . \tag{2.3'}$$

If there is free entry, the RHS of (2.3') is zero, so we again obtain (2.3).

Normally, the quit rate function is drawn as in Figure 1a, as a convex function. By drawing a straight line with slope $-1/T$, we find the point where $q' = -1/T$. As we increase training costs, we increase T smoothly.

No argument, however, has been given why the quit rate function should have the given shape, rather than that of Figure 1b. There, $w \leqq \hat{w}$ for $T < \hat{T}$. As T increases above $\hat{T}$, there is a jump in the wage, and thereafter it increases smoothly with T.

In either case, however, the wage is a monotonically increasing function of training costs. In particular, this implies that

$$w_0 = w_{min} ;$$

the wage paid in the no training cost industry is less than or equal to that paid in the training cost industry.

2.4. The Determination of the Quit Rate Function, Given the Distribution of Firms by Training Costs

The crucial question then is the determination of the quit rate function. Under our assumptions, the quit rate is just the death rate, plus the probability of an individual finding a better job. The latter is equal to the average number of searches per unit of time, times the probability that one of these firms has a wage greater than the wage the individual is presently receiving. If $F(w)$ is the percentage of firms who pay a wage less than or equal to w, then the quit rate is[1]

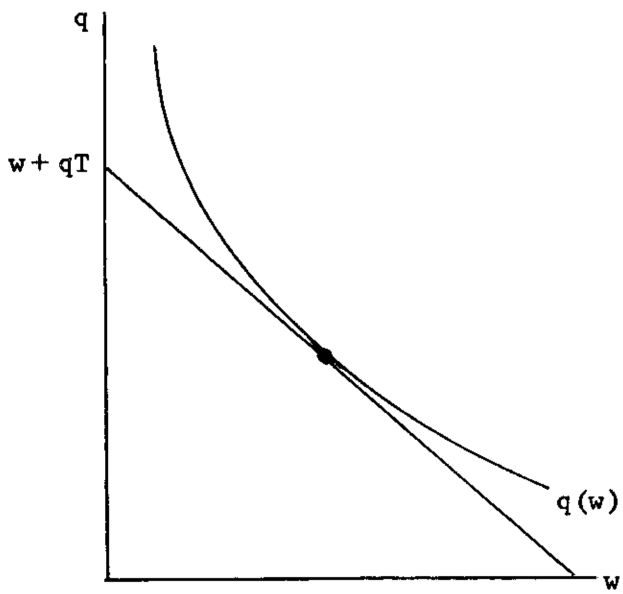

FIGURE 1a

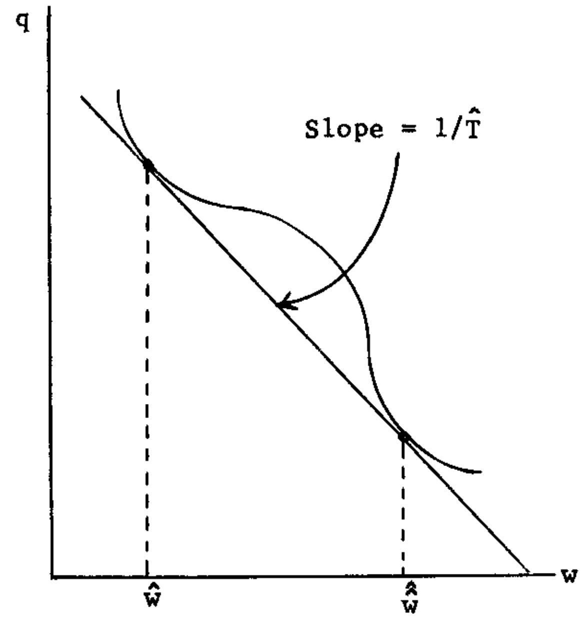

FIGURE 1b

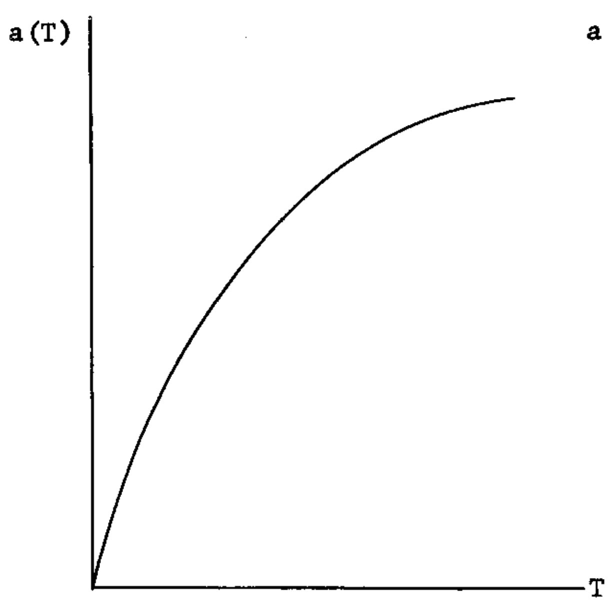

FIGURE 2a

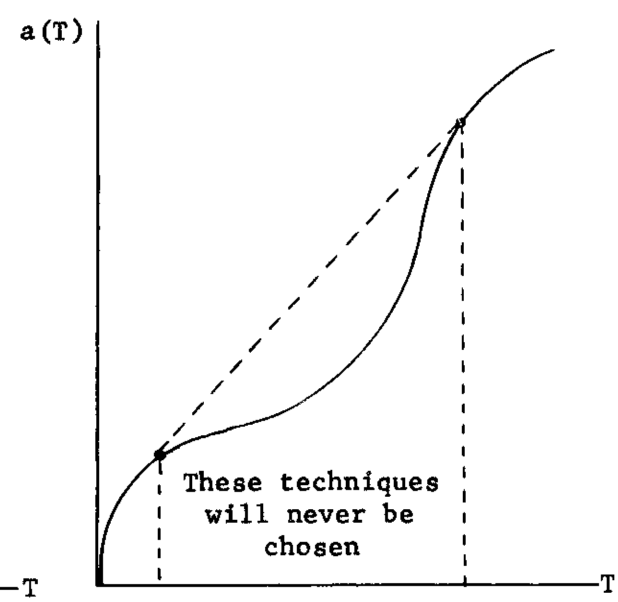

FIGURE 2b

$$(2.4) \qquad q(w) = \mu + s(1 - F(w)) \; .$$

Hence, if $F(w)$ is differentiable at w ,

$$(2.5) \qquad q'(w) = -sf(w) \; ,$$

where $f(w)$ is the density function of w . Thus, for the quit rate function to be convex, the density function must be monotonically declining; i.e. the only continuous wage density functions are those in which the density function of wage is monotonically decreasing. In particular, continuous unimodal distributions, such as the normal distribution, imply that $f' > 0$ for wages below the mode, and hence are not consistent with equilibrium.[2]

$F(w)$ is the distribution of firms by wages; let $G(w)$ be the distribution function of jobs by wages. Under our assumptions, all firms have the same number of applicants, $\bar{L}(\mu+s)/N$, where N is the number of firms and $\bar{L}$ is the total labor force. Of the applicants, $\mu\bar{L}$ are new entrants; $G(w)$ of the applicants who are presently employed accept jobs if the firm pays a wage of w . Thus the total acceptance rate is proportional to $\mu + sG(w)$.[3] But acceptances must equal quits:

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[1]If instead of assuming that the time required to sample an additional firm is a random variable we had assumed the individual makes s searches per unit time, then

$$q(w) = \mu + (1 - (F(w))^s) \; , \quad q' = -sF^{s-1}f \; .$$

The rest of the calculations must similarly be modified in a straightforward manner.

[2]Not too much emphasis should be placed on this result, since, as we shall see, it is not true in more complicated versions of the model.

[3]If there is a mass point at w , then if individuals only quit if they receive a higher wage, the acceptance rate is $\mu + s \lim_{\epsilon \to 0} G(w-\epsilon)$, $\epsilon > 0$.

$$f(\mu + sG(w))\bar{L} = L(w)[\mu + s(1 - F(w))]$$

where $L(w)$ = labor force of firms paying wage w, i.e.

$$\frac{L(w)}{\bar{L}} = g(w)$$

where $g(w)$ is the density function corresponding to G (assuming G is differentiable). Hence,

$$(2.6) \qquad g(w) = f(w)\,\frac{\mu + sG(w)}{\mu + s(1 - F(w))}$$

or

$$(2.7) \qquad \int \frac{1}{\mu + sG}\, dG = \int \frac{f(w)dw}{\mu + sF(w)}$$

which can be solved for G as a function of w.

Differentiation of (2.6) shows that

$$(2.8) \qquad \frac{g'}{g} = \frac{f'}{f} + \frac{sg}{\mu + sG} + \frac{sf}{\mu + s(1-F)}$$

so that even though f must be monotonically declining, g need not be.

Finally, we need to relate the distribution of firms by wages to the distribution of firms by training costs. Given a wage distribution F, for each value of T, we can calculate the optimal value of w.

$$(2.9) \qquad w = \psi(T;F)\ .$$

It is clear from our earlier analysis that w is a monotonic function of T; hence, we can invert H and write

(2.10) $$T = \psi^{-1}(w;F) \ .$$

Let H be the distribution function of firms by training costs. Using (2.10) we immediately obtain our distribution of firms by wages

(2.11) $$H(\psi^{-1}(w;F)) \ .$$

Equilibrium requires

(2.12) $$H(\psi^{-1}(w;F)) \equiv \hat{F}(w) = F(w)$$

for all w .

We have here then a mapping from the set of distribution functions onto the set of distribution functions. We are not concerned however with the proof of the existence of an equilibrium, only with characterizing the equilibrium if it exists.

Differentiating (2.12), we obtain

(2.13) $$f(w) = - \frac{h(T)}{sf^2} f'$$

where $dH/dT = h(T)$ is the density function of T .

Thus, given any continuous density function of firms by training costs, we can find a distribution of firms by wages.

Notice that regardless of the density function $h(T)$, $f' < 0$.

An example may be instructive. Assume $h(T)$ is uniformly distributed over the unit interval $[1,2]$.

$$h(T) = \begin{cases} 1 & 1 \leq T \leq 2 \\ 0 & T > 2\ ,\quad T < 1\ . \end{cases}$$

Then from (2.13)

$$(2.14)\qquad f(w) = \left(\frac{1}{w - w_{min} + s}\right)^{1/2}\left(\frac{1}{2s}\right)^{1/2}\ ,\quad w_{min} \leq w \leq s + w_{min}\ .$$

2.5. Determination of the Distribution of Firms by Training Costs

We now must determine the distribution of firms by training costs, $h(T)$. If all firms are to be making zero profits,

$$(2.15)\qquad pa(T) = w + (\mu + s(1 - F(w))T\ .$$

Given p and w , each firm chooses its technique to maximize profits:

$$(2.16)\qquad pa' = q = \mu + s(1 - F(w))$$

$$(2.17)\qquad pa'' \leq 0\ .$$

Points along the part of the $a(T)$ curve for which $a'' > 0$ will never be chosen. (For the remainder of this section we assume $a'' \leq 0$ everywhere.)

From (2.16) and (2.15) we obtain

$$(2.18)\qquad w = p(a - Ta') \equiv pb(T)$$

where $$b(T) = a - Ta' \ ; \quad b' = -Ta'' > 0 \ ;$$

the real product wage is an increasing function of T . Since (2.15) must hold for all w actually paid,

$$pa'' = -sf\,\frac{dw}{dT} = -\,\frac{sf^3}{f'} = -h(T) \ . \tag{2.19}$$

Attention needs to be paid to certain boundary value problems.

(a) Determination of highest training cost technique used, $\bar{T}$. Note that if

$$pa'' + h(\bar{T}) < 0$$

profits at $\bar{T}$ will be lower than at $T < \bar{T}$. Hence $h(\bar{T}) = 0$. From (2.16),

$$\bar{T} = a'^{-1}(\mu/p) \ . \tag{2.20}$$

(b) Determination of lowest training cost technique used. Let $\underline{T}$ be the lowest training cost technique used and $\underline{w} \equiv w(\underline{T})$ be the lowest wage paid in this sector. If $w(\underline{T}) > w_{min}$, then there must be a mass point of firms at $w(\underline{T})$. Let $\tilde{T}$ be the solution to

$$\max_{\{T\}} pa(T) - w_{min} - sT(1 - F(w_{min})) - \mu sT \ . \tag{2.21}$$

If a firm were to pay a wage less than $w(\underline{T})$, it would clearly have the same quit rate in the interval $(w_{min}, w(\underline{T}))$ and so would pay w_{min} . (2.21) gives the technique with $T \leq \underline{T}$ with maximal profits. This clearly must be less than profits at $\underline{T}$, i.e.

$$pa(\underline{T}) - w_{min} - \underline{T}(\mu + s(1 - F(w_{min}))) \leq pa(\tilde{T}) - w_{min} - \tilde{T}(\mu + s(1 - F(w_{min})))$$

$$\leq pa(\underline{T}) - w(\underline{T}) - \underline{T}(\mu + s(1 - F(w(\underline{T})))$$

so

$$F(w(\underline{T})) - F(w_{min}) \geq \frac{w(\underline{T}) - w_{min}}{s\underline{T}} > 0 \; . \qquad (2.22)$$

Thus the quit rate function must look like either Figure 3a or 3b.

2.6. Determination of Relative Price

Finally, we must determine p. For simplicity we assume all individuals have identical, homothetic indifference maps. This means that if Q_0 is the output of our numeraire, Q_1 the output of the industry requiring specific training[1] we can describe equilibria in the goods markets by

$$Q_1 = D(p, \int w dG(w)) = \alpha(p)\int w dG(w) \; , \qquad (2.23)$$

where $\alpha(p)p$ is the proportion of income spent on good 1 when its price is p. Price is determined, as usual, to make demands equal supplies. The relative supplies in turn are determined by the wage distribution.

[1] Implicitly, we have assumed that output of sector 0 is the sole resource required for training. This is an inessential assumption which can easily be modified.

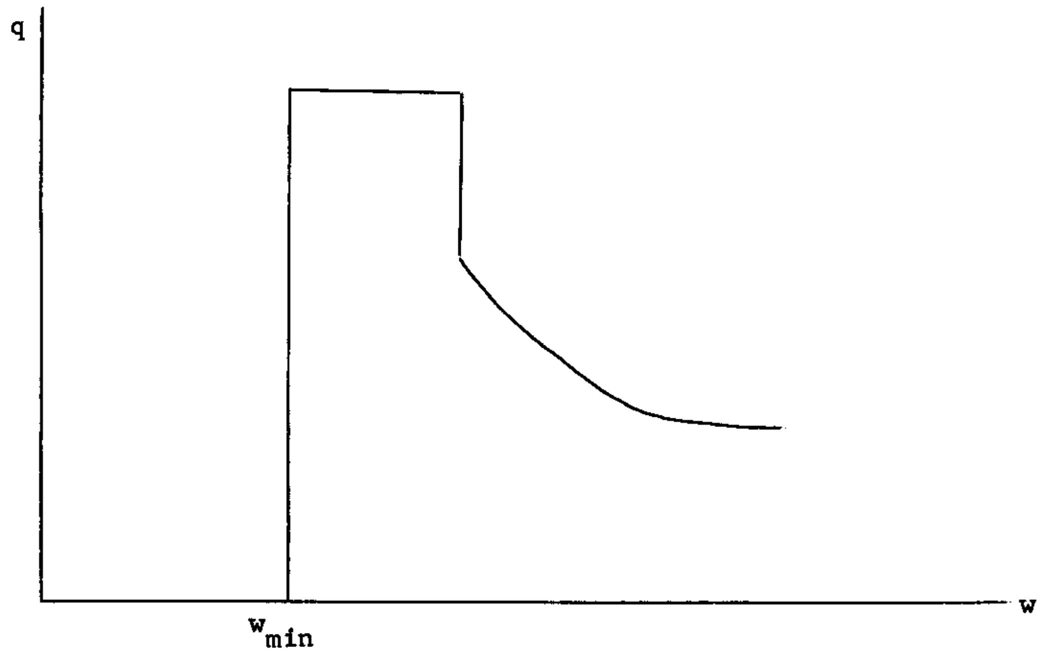

FIGURE 3a

$w(\underline{T}) > w_{min}$

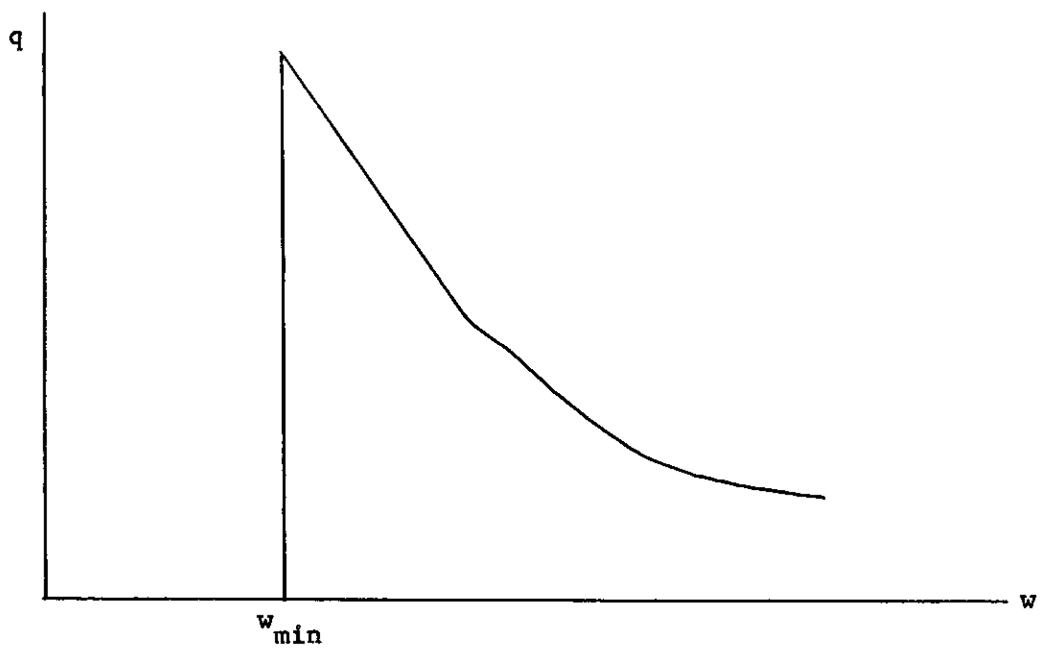

FIGURE 3b

$w(\underline{T}) = w_{min}$

2.7. The Full Solution

We can now "solve" for the equilibrium as follows. Consider any set of values of $\underline{w}$ and p. Consider first the case of $\underline{w} = w_{min}$. From (2.18) we immediately have

$$\underline{T} = b'(\underline{w}/p), \ \frac{d\underline{T}}{d\underline{w}/p} > 0 , \tag{2.18'}$$

and from (2.20)

$$\overline{T} = a'^{-1}(\mu/p), \ \frac{d\overline{T}}{dp} > 0 . \tag{2.20'}$$

We can immediately solve (from (2.19)) for $H(T)$ and hence for $F(w)$ up to a constant of integration. We need to know the distribution of individuals by jobs; to know this, we must know the percentage of individuals working at the minimum wage, $G(w_{min})$. Given $G(w_{min})$, we can solve for $G(w)$ for $w \geq \underline{w}$. Let $G(w;\, p,\, \underline{w},\, G(w_{min}))$ be the solution. We then solve for the demand for labor by firms paying wages in excess of w_{min},

$$\overline{L}\int_{w > w_{min}} dG(w;\, p,\, \underline{w},\, G(w_{min})) . \tag{2.24}$$

We require

$$1 - G(w_{min}) = \int_{w > \underline{w}} dG(w;\, p,\, \underline{w},\, G(w_{min})) . \tag{2.24'}$$

It is easy to show that for any value of $\underline{\mu}$ and p there must exist at least one solution to (2.24'). Let any such solution be denoted by

$G_{min}(\underline{w}, p)$, and we can now express $G(w)$ simply as a function of $\underline{w}$ and p . The supply of output can thus be written

$$(2.25) \qquad Q_1 = \bar{L}\int_{w>\underline{w}} a(T(w))dG(w;\ \underline{w},\ p) + a(\underline{T})(1-\rho)G_{min}(\underline{w}, p)$$

where ρ is the proportion of individuals working at the minimum wage who work in industry 0 . Substituting (2.25) into (2.22) yields a "demand price"

$$(2.26) \qquad p^d = \alpha^{-1}\left(\frac{Q_1}{\int wdG(w)}\right) .$$

Any value of p which solves (2.26) is an equilibrium. At a finite value of p , given by the solution to

$$(2.27a) \qquad \underline{p} = \frac{\mu}{a'(T)}$$

$$(2.27b) \qquad \underline{w} = \underline{p}(a(T) - a'(T)T)$$

$$\underline{T} = \bar{T} , \quad \text{so} \quad p^d > \underline{p} .$$

On the other hand, let $\bar{p}$ be the solution to $G_{min}(\underline{w}, \bar{p}) = 0$. Then $Q_0 = 0$, and $p^d < \bar{p}$. Since the functions are continuous, there must exist a solution $\underline{p} < p^* < \bar{p}$, for any value of 0 sufficiently large.

The only modification required for $\underline{w} > w_{min}$ is that inequality (2.21) must be satisfied as well. For sufficiently large $\underline{w}$ it is easy to establish that (2.21) cannot hold. But there exists an interval $w_{min} \leq \underline{w} < w_{max}$, such that for any $\underline{w}$ in that interval, firms are able to "pass along" the higher wages they pay to consumers.

Among the equilibria is one in which all firms pay the same wage, in which case the quit rate function appears as in Figure 4. Clearly, regardless of the training cost, the wage paid will be w_{min} . The firms choose the technique to maximize profits, given the price of output,

$$\max_T [pa - \mu T - w_{min}]$$

since the only turnover is due to the exogenous "death" rate μ . Thus

$$pa' = \mu \ .$$

In equilibrium, the maximum value of profits must be zero, so

$$p = (\mu T + w_{min})/a(T) \ .$$

2.8. Welfare Implications

Thus, the set of equilibria yielded by an economy with imperfect information is much richer than that generated by essentially the same economy with perfect information. The different equilibria in the economy with imperfect information differ not only in the distribution of wages, but also in total expenditure on training and relative prices between commodities. Clearly, average wlefare is lower in the economy with wage dispersion than in that without. Although some individuals are better off and some worse off, *ex post*, from an *ex ante* point of view, that is, before the individual enters the labor market *everyone's expected utility is lower in the economy with wage dispersion*. To see this observe that, because of concavity of a ,

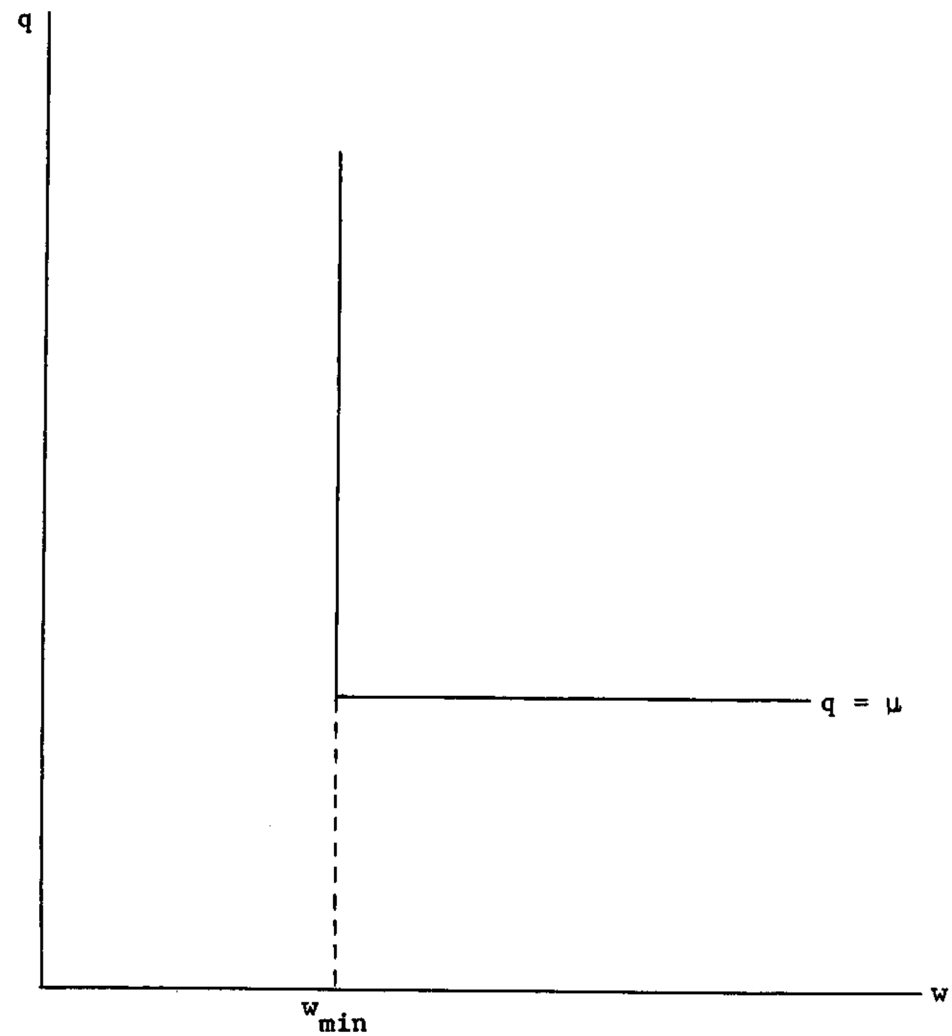

FIGURE 4

Single Wage Equilibria

$$Q_1 \leq L_1 a(ET) \equiv \hat{Q}_1 \tag{2.28}$$

where ET is the mean value of T . Let

$$U(C_0, C_1)$$

be the utility function; let $\overline{C}_0$ and $\overline{C}_1$ be the average levels of consumption of the two commodities. Then

$$\overline{C}_0 \leq Q_0 - \mu L_1 T \equiv \hat{Q}_0$$

$$\overline{C}_1 = Q_1$$

$$U(\overline{C}_0, \overline{C}_1) < U(Q_0 - L_1 \mu T, Q_1) \leq U(\hat{Q}_0, \hat{Q}_1) \leq U(C_0^*, C_1^*)$$

where C_0^* , C_1^* are the levels of consumption with no price dispersion. (The first inequality follows from the fact that some resources are used in training from turnover for reasons other than death, the second inequality from (2.28), the third inequality from the fact that if a single technique is used in each sector and the only turnover is from death, then (C_0^*, C_1^*) is the optimal level of consumption of the two commodities.) Let $V(p,w)$ be the indirect utility function, w^* the equilibrium wage with no wage dispersion. Then

$$V(p^*, w^*) = U(C_0^*, C_1^*) , \quad V(p, Ew) = U(\overline{C}_0, \overline{C}_1) ;$$

but from the concavity of V ,

$$EV(p,w) \leq V(p, Ew) \leq V(p^*, w^*) .$$

2.9. Concluding Comments

In this section, we have constructed a very simplified model in which we have characterized the equilibrium, established the existence of multiple equilibria with wage dispersion, and shown that, in an *ex ante* welfare sense, these are unambiguously worse than the equilibrium with no wage dispersion.

Unfortunately, at this level of generality, that is as far as we can go. For a more detailed examination of the welfare economics and the comparative static analysis of the effects of a change in search intensity, we turn in the next section to a simplified version of the analysis presented here.

3. Equilibria with Only One Technique

Although the existence of a wage distribution when different firms employ techniques with different training costs seems not unreasonable, it turns out that there may be a wage distribution even when there is only one technique of production, or only one "best" technique of production (as may be the case even when there are a continuum of techniques, but when $a'' > 0$).

The model, although somewhat stark, can be generalized to make it look more "reasonable," and this is done in subsequent sections. There are, however, advantages to seeing how the model works in its most simple form. In particular, the analysis of this section will help clarify the reasons for multiplicity of equilibria.

The model is the same as in the previous section except now there is no choice of technique in the training cost industry. We choose our units so that its output per unit labor is one and its training cost per worker is T .

3.1. The Single Wage Equilibrium

The single wage equilibrium is characterized, as before by

$$w = w_{min} \tag{3.1}$$

$$p = w_{min} + \mu T \ . \tag{3.2}$$

At that wage, firms make zero profits, and at higher wages the firm makes a loss. Hence w_{min} is the equilibrium wage.

3.2. Two Wage Equilibria

On the other hand, there are an infinite number of equilibria with two groups of firms. Let w_1 be the wage paid by the high wage firm, $w_1 > w_{min}$. If w_1 and w_{min} are both cost minimizing for the industry with training costs, and π is the percentage of firms which pay $w = w_1$

$$\frac{s\pi}{w_1 - w_{min}} = \frac{1}{T} \tag{3.4}$$

i.e.

$$w_1 = w_{min} + s\pi T \ . \tag{3.5}$$

If

$$w_1 < w_{min} + s\pi T \tag{3.6}$$

then all firms in the industry with training costs pay wage w_1.

Profits must be zero; since if both wages are paid, labor costs are the same, profits must be the same. Hence we require

$$p = w_1 + \mu T \ . \tag{3.7}$$

In equilibrium, quits must equal acceptances,

$$(1-\pi)\mu\bar{L} = L(w_{min})(\mu + s\pi) \tag{3.8}$$

where $L(w_{min})$ is the total number of workers at wage equal w_{min}. Then, normalizing the total labor supply at unity we have

$$Q_1 = 1 - \frac{(1-\pi)\rho\mu}{\mu + s\pi} \ , \tag{3.9}$$

where, as before, ρ is the proportion of workers at w_{min} working in industry 0 . Demand for Q_1 can be written

$$Q_1^D = Q_1^D(p, \ (w_1\pi + w_{min}(1-\pi))) \ . \tag{3.10}$$

Equilibrium requires supply to equal demand, so, using (3.5), (3.7), (3.9) and (3.10) we obtain

$$Q_1^D(w_{min} + (\mu + s\pi)T), \ (w_{min} + s\pi^2 T) = 1 - \frac{(1-\pi)\rho\mu}{\mu + s\pi} \ , \quad \text{if} \quad \rho < 1 \tag{3.11a}$$

$$Q_1^D(w_1 + \mu T, \ w_1\pi + w_{min}(1-\pi)) = 1 - \frac{(1-\pi)\mu}{\mu + s\pi} \ , \quad \text{if} \quad \rho = 1 \ . \tag{3.11b}$$

Any solution to (3.11a) satisfying the inequalities $0 \le \rho \le 1$, $0 \le \pi \le 1$, or to (3.11b) satisfying the inequalities $w_1 < w_{min} + s\pi T$, $w_1 > w_{min}$, $0 \le \pi \le 1$ is an equilibrium to the system.[1] (See Figure 5.)

---

[1]For (3.11a), we have

$$(3.12) \qquad - \frac{d\rho}{d\pi} \quad \frac{\left(\frac{\partial Q_1}{\partial p} + \frac{\partial Q_1}{\partial I} 2\pi\right) sT - \frac{\mu(\mu+s)\rho}{(\mu+s\pi)^2}}{\frac{(1-\pi)\mu}{\mu+s\pi}} .$$

But since

$$(3.13) \qquad \pi < 1 - \frac{(1-\pi)\rho\mu}{\mu+s\pi}$$

from the Slutsky equation

$$\frac{\partial Q_1}{\partial p} + \pi \frac{\partial Q_1}{\partial I} < \left(\frac{\partial Q_1}{\partial p}\right)_{\bar{U}} < 0 .$$

Using this and (3.9), it can be shown that (assuming unitary income elasticities)

$$\frac{\partial \rho}{\partial \pi} > 0 .$$

As expected, an increase in high wage firms increases the proportion of low wage firms producing in sector 0 .

For (3.11b), we have

$$- \frac{dw_1}{d\pi} = \frac{\partial Q_1^D}{\partial I}(w_1 - w_{min}) - \frac{\mu(\mu+s)}{(\mu+s\pi)^2} \Big/ \frac{\partial Q_1^D}{\partial p} + \pi \frac{\partial Q_1^D}{\partial I}$$

with homotheticity, the numerator can be written as

$$\frac{Q_1}{I}\left[\frac{w_1 - w_{min}}{(w_1 - w_{min})\pi + w_{min}} - \frac{\mu}{(\mu+s\pi)\pi}\right] < 0 \quad \text{for} \quad w_1 - w_{min} \quad \text{small.}$$

As before, $$\frac{\partial Q_1^D}{\partial p} + \pi \frac{\partial Q_1^D}{\partial I} < 0 .$$

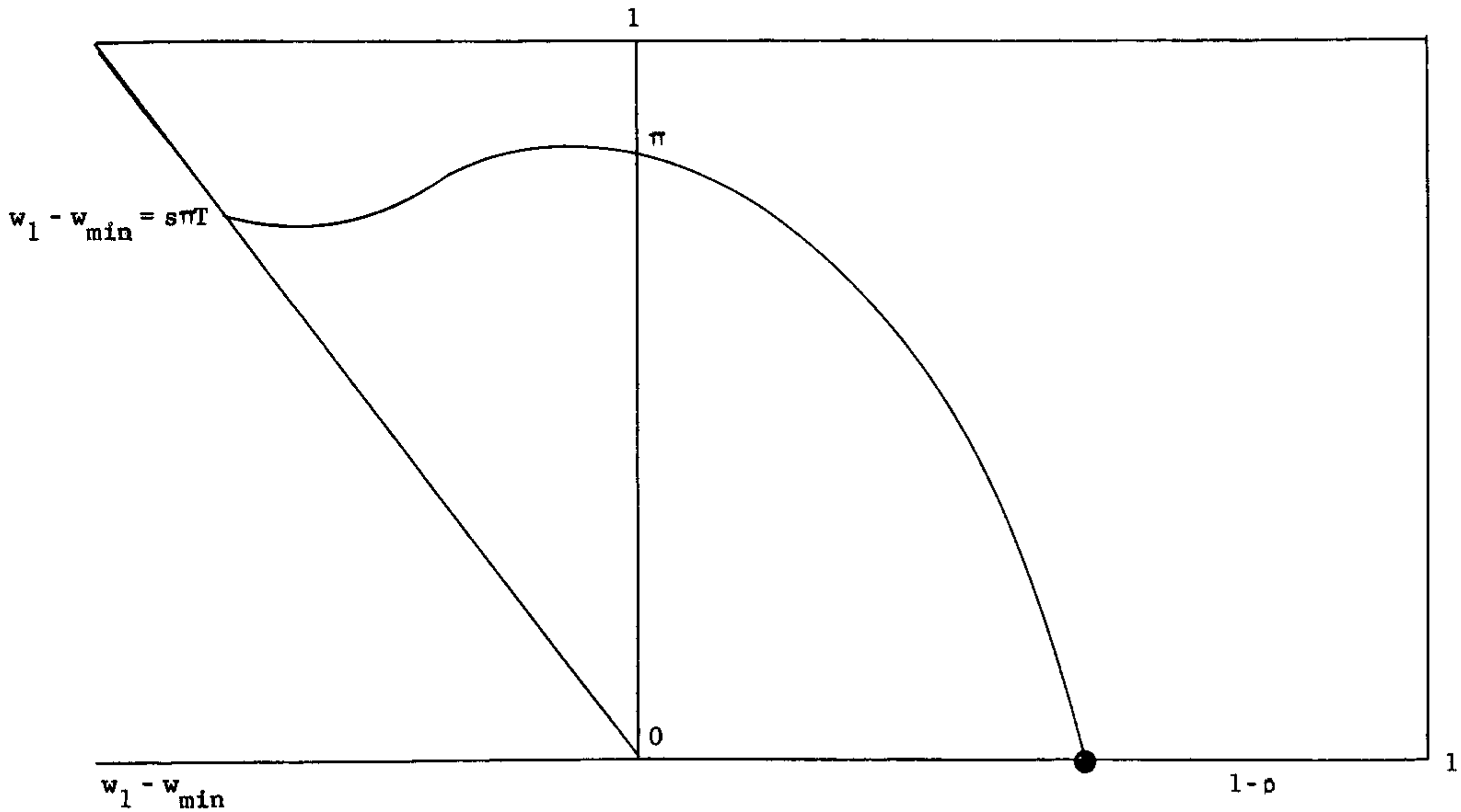

FIGURE 5

Two Wage Equilibria

3.3. Three Wage Equilibria

Even when all the firms in the high training cost industry may pay higher wages than the firms in the low training cost industry, there may be dispersion among the wages they pay. Consider the simplest case where there are two such groups of firms, paying wages (w_1, w_2), $w_1 > w_2 > w_{min}$. There is then a multiplicity of equilibria even for a given price of the output of the high training cost industry, p. Assume $p = p^*$. Then

(3.14) $$w_1 = w_1^* \equiv p^* - \mu T \ .$$

Let π_i be the fraction of firms paying wage w_i : If w_1 and w_2 are both cost minimizing,

(3.15) $$\frac{s\pi_1}{w_1 - w_2} = \frac{1}{T}$$

or

(3.16) $$w_2 = w_1 - s\pi_1 T$$

and

(3.17) $$\frac{s\pi_2}{w_2 - w_{min}} > \frac{1}{T}$$

or

(3.18) $$w_2 < w_{min} + s\pi_2 T \ .$$

Substituting (3.16) into (3.18), we obtain

$$(3.19) \qquad \pi_1 + \pi_2 > \frac{w_1 - w_{min}}{sT} .$$

For demands to equal supplies, letting $\pi_1 + \pi_2 \equiv \pi$, and using (3.16), we require

$$(3.20) \qquad Q_1^D(p^*, w_1^*\pi - s\pi_1\pi_2 T + w_{min}(1-\pi)) = 1 - \frac{(1-\pi)\mu}{\mu + s\pi} .$$

Any set of values of (π_1, π_2) satisfying (3.19) and (3.20) is an equilibrium. For instance, assuming unitary income elasticities, (3.20) may be rewritten

$$(3.20') \qquad sT\pi_1\pi_2 = w_1^*\pi + w_{min}(1-\pi) - \frac{(\mu+s)\pi}{\mu + s\pi}\frac{1}{\alpha(p^*)} .$$

where $\alpha(p)p$ is the proportion of income spent on good one. (3.20') is plotted in Figure 6. Any value of π between $\underline{\pi}$ and $\bar{\pi}$ generates an equilibrium.[1]

---

[1]
$$\frac{d\pi_1}{d\pi} = \frac{w_1^* - w_{min} - \dfrac{(\mu+s)\mu}{(\mu+s\pi)^2\alpha} - sT\pi_1}{sT(\pi - 2\pi_1)}$$

The denominator is positive or negative depending on whether $\pi_1 \lessgtr \frac{1}{2}\pi$, i.e. whether $\pi_1 \lessgtr \pi_2$. The numerator is negative or positive depending on whether

$$\pi_1 \gtrless w_1^* - w_{min} - \frac{(\mu+s)\mu}{\alpha(\mu+s\pi)} .$$

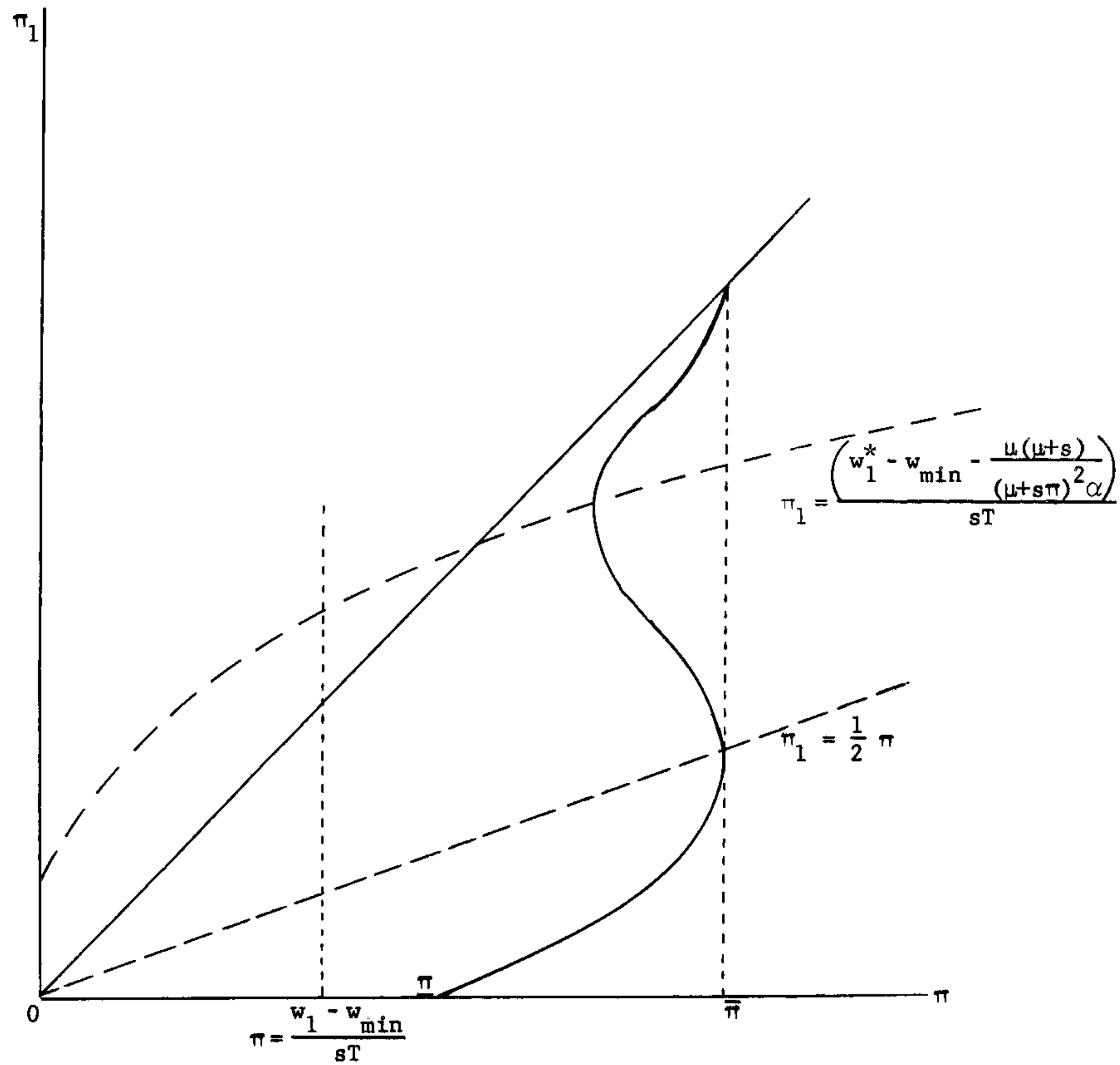

FIGURE 6

Three Wage Equilibria

3.4. Other Equilibria

It is easy to construct equilibria with any number of wage rates, including a continuum. (This requires that the distribution of firms by wages be the uniform distribution.)

3.5. Comparative Statics

What is the effect of an increase in s, the search intensity, e.g. as a result of a reduction in the cost of search? It has been conjectured that this should reduce the dispersion in wages. Clearly, if search were costless (s were infinite) the only equilibria would be the conventional equilibrium of economics with no wage dispersion at all; if s is finite there may be a wage dispersion, and it is natural to assume then that as s gets larger the dispersion decreases.

Our analysis makes clear that this need not be the case. Notice that in much of the above analysis s and π appear multiplicatively. Consider the two wage equilibrium described above. An increase in s may be associated with a proportionate reduction in high wage firms, i.e. the wages paid in low wage firms and wages paid in high wage firms remain the same; the proportion of firms paying low wages increases. Hence the mean wage is reduced. Prices remain unchanged: the presumed advantages of extra search are completely abosrbed in increased training costs.[1]

---

[1]From (3.12) it is clear that the consequence of this is to change ρ .

Alternatively, the percentage of firms in the two categories could remain unchanged, but the wage paid by the high wage firm increased. In that case the price rises.

4. Search Costs and the Determination of Search Intensity

If there are costs associated with undertaking search, then a rational individual, in deciding how much search he should undertake, would compare the expected benefits at each search intensity with the costs. It is natural to assume that the costs are an increasing, convex function of search intensity, i.e. if $C(s)$ is the cost per unit of time of search at intensity s,

$$C' \geq 0 \;, \quad C'' \geq 0 \;. \tag{4.1}$$

The correct calculation of the benefits, is however, not so simple a matter. We must know what the individual's knowledge about the distribution of wages is, his attitudes towards risk, as well as his expectations concerning the duration which he will keep any job. For instance, if the individual does not know the wage distribution, not only does search yield a direct return in the possibility of finding a better job, but it yields an indirect return in enabling the individual to know better the wage distribution and hence to make a "better" decision with respect to search intensity. For simplicity, assume the individual is risk neutral, and has perfect information about the distribution of wages. The maximum expected present discounted value of net income of the individual who is presently receiving a wage w is (in discrete time)

$$(4.2)\quad V(w) = \max_{s}\left\{ w - C(s) + \frac{1}{(1+\mu)(1+r)}\left[(1 - s(1 - F(w))V(w) + s\int_{w}^{w_{max}} V(w)dF(w)\right]\right\}$$

Hence s is chosen so that

$$(4.3)\qquad C'(s) = \frac{\int_{w}^{w_{max}} V(w)dF(w) - (1 - F(w))V(w)}{(1+\mu)(1+r)}$$

where the second order condition implies that

$$(4.4)\qquad C'' > 0 .$$

From (4.3) we obtain the search intensity as a function of the wage paid (given the distribution $F(w)$). Since

$$(4.5)\qquad \frac{ds}{dw} = - \frac{(1 - F(w))V'}{C''(1+\mu)(1+r)} ,$$

$$(4.6)\qquad \frac{dq}{dw} = - \frac{(1-F)^2 V'}{C''(1+\mu)(1+r)} - sf , \quad \text{and}$$

$$(4.7)\qquad \frac{d^2q}{dw^2} = -sf' + \left[\frac{3(1-F)fV'}{C''} - (1-F)^2\left(\frac{V''}{C''} - \frac{V'C'''}{(C'')^2}\right)\right]\frac{1}{(1+\mu)(1+r)} .$$

Thus the convexity of the quit rate function depends on the third derivative of the search-cost function; clearly there is no necessity for the quit rate function to be convex even when f is monotonically declining.

The extension of the analysis of the previous section follows in a straightforward manner. Consider the three wage equilibria. We first need to calculate the slope of the quit rate function at the highest wage. This, it can easily be shown, is equal to[1]

$$(4.8) \qquad \left(\frac{\partial q}{\partial w}\right)^{+}_{w=w_1} = 0 \, , \quad \left(\frac{\partial q}{\partial w}\right)^{-}_{w=w_1} = - \frac{\pi_1^2}{C''(0)(\mu + r + s\pi_1)}$$

---

[1]The $V(w)$ function can easily be calculated for that case. If an individual at wage $w_2 < w < w_1$ searches at the intensity s, the probability that he will find a firm paying wage w_1 at t (but not before) is $s\pi_1 e^{-s\pi_1 t}$. Once he finds a firm paying a wage w_1, he quits searching and $V(w_1) = \frac{w_1}{\mu+r}$. The present discounted value of his net wages from time 0 on, if he finds a firm of wage w_1 at t is

$$\frac{w - C(s)}{\mu+r}\left(1 - e^{-(\mu+r)t}\right) + \frac{w_1 e^{-(\mu+r)t}}{\mu+r} \, .$$

Thus expected PDV is just

$$V(w) = \frac{w - C(s)}{\mu+r} + \frac{(w_1 - (w-C))}{\mu+r} \frac{s\pi_1}{\mu + r + s\pi_1} \, .$$

Hence s is chosen so that

$$C' = \frac{(w_1 - (w-C))\pi_1}{\mu + r + s\pi_1}$$

$$\frac{ds}{dw} = \frac{\pi_1}{\mu + r + s\pi_1} \Big/ C'' \, .$$

Assume for simplicity $C''(s) = \text{constant} > 0$. Then if

$$(4.9) \qquad \frac{\pi_1^2}{(\mu + r + s\pi_1)C''(0)} = \frac{1}{T}$$

the high training cost firms will be indifferent to paying any wage between w_2 and w_1 (Figure 7); in particular, w_1 and w_2 are cost minimizing wages; by appropriately selecting π_2 , the percentage of firms paying w_2 , we can construct an equilibrium as before.

There is one situation in which a problem does arise: if the search cost appears as in Figure 8, then for the highest paying firm $s = 0$, and as is clear from (4.8), the quit rate function is flat at $w = w_{max}$. Hence, no firm with positive training costs would pay w_{max} . Accordingly, the wage distribution becomes degenerate: the only equilibrium wage distribution is that where all firms pay the same wage, equal to w_{min} .

Although the cost function depicted in Figure 8 may be considered pathological, even in that case there may exist a multiple wage equilibrium if the non-pecuniary benefits resulting from search are taken into account.

5. Non-Pecuniary Returns

As we noted in the introduction, jobs differ not only in the wages they pay, but in certain non-pecuniary characteristics. Some individuals will prefer the characteristics associated with one firm, others those associated with another.

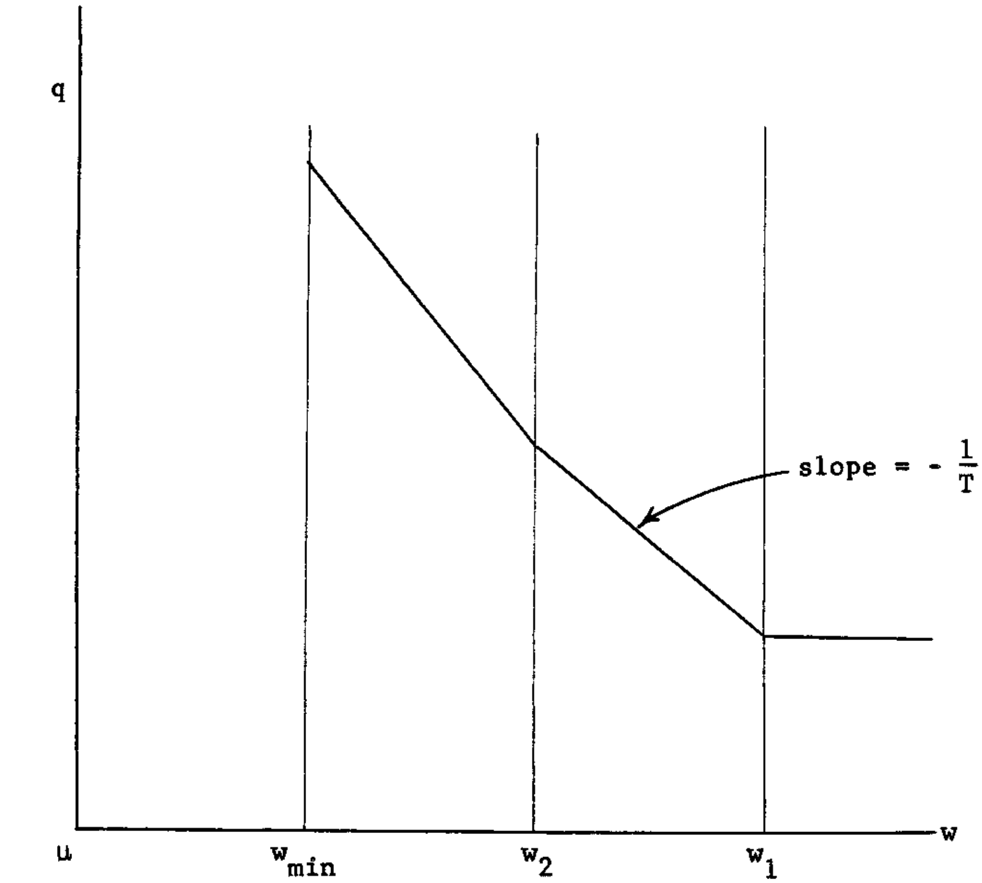

FIGURE 7

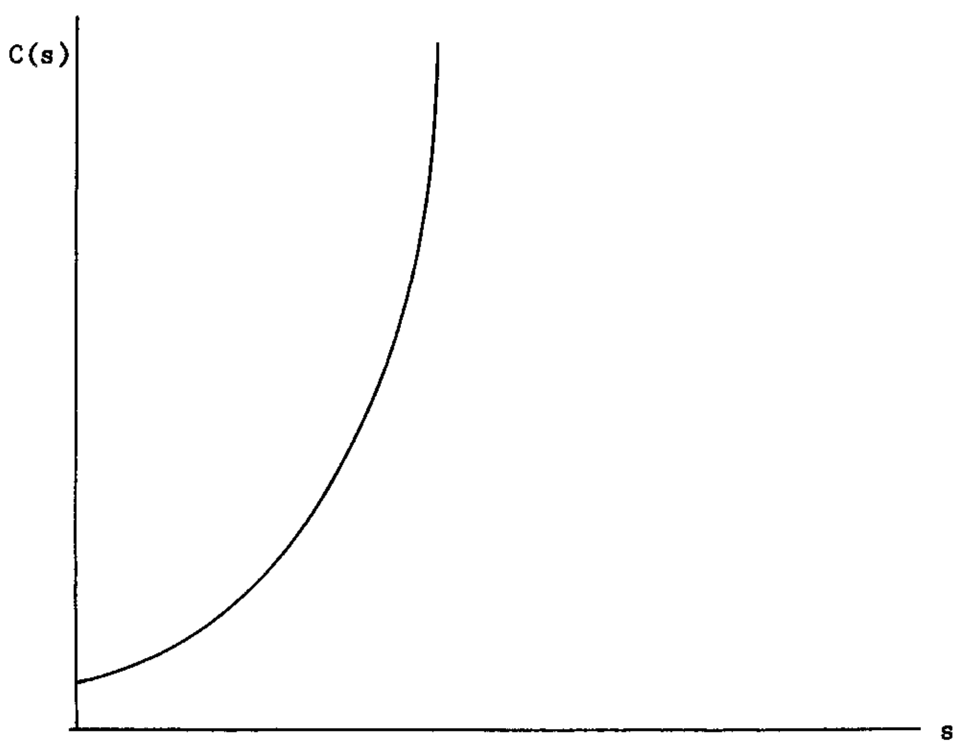

FIGURE 8

We assume, for simplicity, that the individual does not know these characteristics until after completing training, thereupon he knows them fully. The value, in "consumption equivalents," of the non-pecuniary characteristics we denote by θ. Without loss of generality, we let $E\theta = 0$.[1] The distribution of evaluations of the non-pecuniary characteristics of any firm we denote by $M(\theta)$; that is, in a random sample of individuals arriving at the given firm, $M(\theta)$ will discover that their evaluation of its non-pecuniary characteristics is less than or equal to θ. We assume moreover that the evaluation of firm i is independent of his evaluation of firm j; and for simplicity, this distribution is the same for all firms. The density function we denote by $m(\theta)$. A risk neutral individual at a job with wage w and whose non-pecuniary characteristics he values at θ accepts a job if its wage, $\hat{w}$, satisfies

$$\hat{w} \geqq w + \theta . \tag{5.1}$$

Thus the quit rate function is simply

$$q(w) = \mu + \int s(w+\theta)(1 - F(w+\theta))m(\theta)d\theta \tag{5.2}$$

so

$$q' = -\int sf(w+\theta)m(\theta)d\theta + \int s'(1 - F(w+\theta))m(\theta)d\theta . \tag{5.3}$$

---

[1] If most individuals agree that one firm has more desirable characteristics than another, we can simply add the mean value of θ onto the wage.

The important implication of (5.3) is that even the firm which pays the highest wage can, by further increments in its wage, reduce its turnover rate, and, if it attempted to reduce its wage, it would increase its turnover rate, even though there are costs of search. Thus, the problem encountered in the previous section is, in this sense, resolved. There are, however, several interesting properties of the equilibrium in this case, which we can analyze in the simplified version of our equilibrium model presented in Section 3. Thus, we assume that the high training cost firms have only one technique, there is no cost to search and search intensity is fixed at s .

5.1. Impossibility of Single Wage Equilibrium

If all firms paid the wage w_{min} , then all individuals who discovered that $\theta < 0$, would search for a better job and indeed, since they do not know the characteristics of the job before accepting it and going through training, would accept the first offer (since all firms pay the same wage).

The quit rate of a firm (in steady state) which decides to pay, say a wage $w > w_{min}$ can be calculated as follows: all those with $\theta < w_{min} - w$ will quit.

Those who do not quit make up a proportionately larger fraction of each firm's labor force. Those with $\theta < w_{min} - w$ have an average duration on the job of $1/\mu+s$ while those with $\theta \geq w_{min} - w$ have an average duration of $1/\mu$. Thus the average quit rate is just

$$(5.5) \qquad q = \frac{1}{\dfrac{M(w_{min} - w)}{\mu+s} + \dfrac{1 - M(w_{min} - w)}{\mu}}$$

with

$$(5.6) \qquad q' = \frac{-q^2 sm(w_{min} - w)}{\mu(\mu+s)} .$$

For there to be a single wage equilibrium (assuming $M(0) = 1/2$)

$$(5.7) \qquad -q'(w_{min}) = \frac{4sm(0)\mu(\mu+s)}{(2\mu+s)^2} = \frac{1}{T}$$

which will not in general be the case.

5.2. The Two Wage Equilibrium

As before, let π be the proportion of firms paying $w_1 > w_{min}$. The quit rate function can now be written as

$$(5.8) \quad \frac{1}{q} = \frac{M(w_{min} - w)}{\mu+s} + \frac{M(w_1 - w) - M(w_{min} - w)}{\mu + s\pi} + \frac{1 - M(w_1 - w)}{\mu}$$

so

$$(5.9) \quad q' = -q^2 \left\{ m(w_{min} - w)\left(\frac{1}{\mu + s\pi} - \frac{1}{\mu+s}\right) + m(w_1 - w)\left(\frac{1}{\mu} - \frac{1}{\mu + s\pi}\right)\right\} .$$

In equilibrium, we require

(a) The slope of the quit rate function to equal $-1/T$:

$$(5.10) \qquad -q'(w_1) = \frac{q(w_1)^2}{\mu + s\pi}\left\{\frac{m(w_{min} - w_1)s(1-\pi)}{\mu+s} + \frac{m(0)}{\mu} s\pi\right\} = \frac{1}{T} .$$

(b) Quits equal acceptances

$$(5.11)\quad q(w_1)L(w_1) = \pi\mu\bar{L} + L(w_{min})s\pi q(w_{min})\left(\frac{M(0)}{\mu+s} + \frac{M(w - w_{min}) - M(0)}{\mu + s\pi}\right)$$
$$+ L(w_1)q(w_1)s\pi\left(\frac{M(w_{min} - w)}{\mu+s} + \frac{M(0) - M(w_{min} - w_1)}{\mu + s\pi}\right)$$

so

$$(5.12)\qquad L(w_1) = \varphi(w_1, \pi) \ .$$

(c) Demand for each commodity must equal supply. Since

$$p = Tq + w_1 = p(w_1, \pi)$$

market clearing requires (assuming for simplicity unitary income elasticities)

$$(5.13)\qquad \alpha(p(w_1, \pi))(w_1\pi + w_{min}(1-\pi)) = L(w_1) = \varphi(w_1, \pi) \ .$$

A set of values of (w_1, π) satisfying (5.13) and (5.10) is an equilibrium; Figure 9 depicts one possible equilibrium.[1]

---

[1]The functions defined by (5.10) and (5.13) appear to be quite complex and we have found no simple conditions ensuring, for instance, monotonicity. Limiting values may however easily be calculated. Let $y = w_1 \quad w_{min}$. First consider (5.10). Then, when $y = 0$, $1/q(w_1) = \frac{1}{2}\frac{2\mu + s}{\mu(\mu+s)} \equiv A/2$. Thus

$$-q' = \frac{4}{A^2}\frac{m(0)s}{\mu(\mu+s)} \ .$$

For y very large, the only quits are to other high wage paying firms i.e.

$$\frac{1}{q(w_1)} = \frac{1}{2}\left(\frac{1}{\mu} + \frac{1}{\mu + s\pi}\right) = \frac{1}{2}\,\frac{2\mu + s\pi}{\mu(\mu + s\pi)}$$

$$-q'(w_1) = \frac{m(0)s\pi}{\mu(\mu + s\pi)}\,\frac{4\mu^2(\mu + s\pi)^2}{(2\mu + s\pi)^2} \quad .$$

Hence, as $y \to \infty$

$$\pi \to \pi^* \;, \quad 0 < \pi^* < 1 \;, \quad \text{if} \quad \frac{4}{A^2}\,\frac{m(0)s}{\mu(\mu+s)} > \frac{1}{T} \;.$$

Similarly, as $\pi \to 0$

$$\frac{1}{q(w_1)} \to \frac{1}{\mu}\,\frac{M(-y)s}{\mu(\mu+s)}$$

$$-q'(w_1) \to \frac{M(-y)s\mu(\mu+s)}{(\mu + s - sM(-y))^2}$$

as $y \to y^* > 0$.

Turning to (5.13), observe that as $y \to \infty$, that individuals only leave the high wage paying firm as a result of death, while all individuals who "find" a high wage job accept it, so

$$\mu L(w_1) = \pi(\mu L + s(L - L(w_1)))$$

i.e.

$$\frac{L(w_1)}{L} = \frac{\pi(\mu+s)}{\mu + s\pi} \;.$$

Under the restriction that $\lim_{p\to\infty} \alpha(p)p < 1$, as $y \to \infty$,

$$\pi \to 0 \;.$$

Finally, as $y \to 0$,

$$\frac{L(w_1)}{L} \to \pi$$

so

$$\pi \to \pi^{**} \equiv \alpha\left(\frac{2T}{A} + w_{min}\right) w_{min} \;.$$

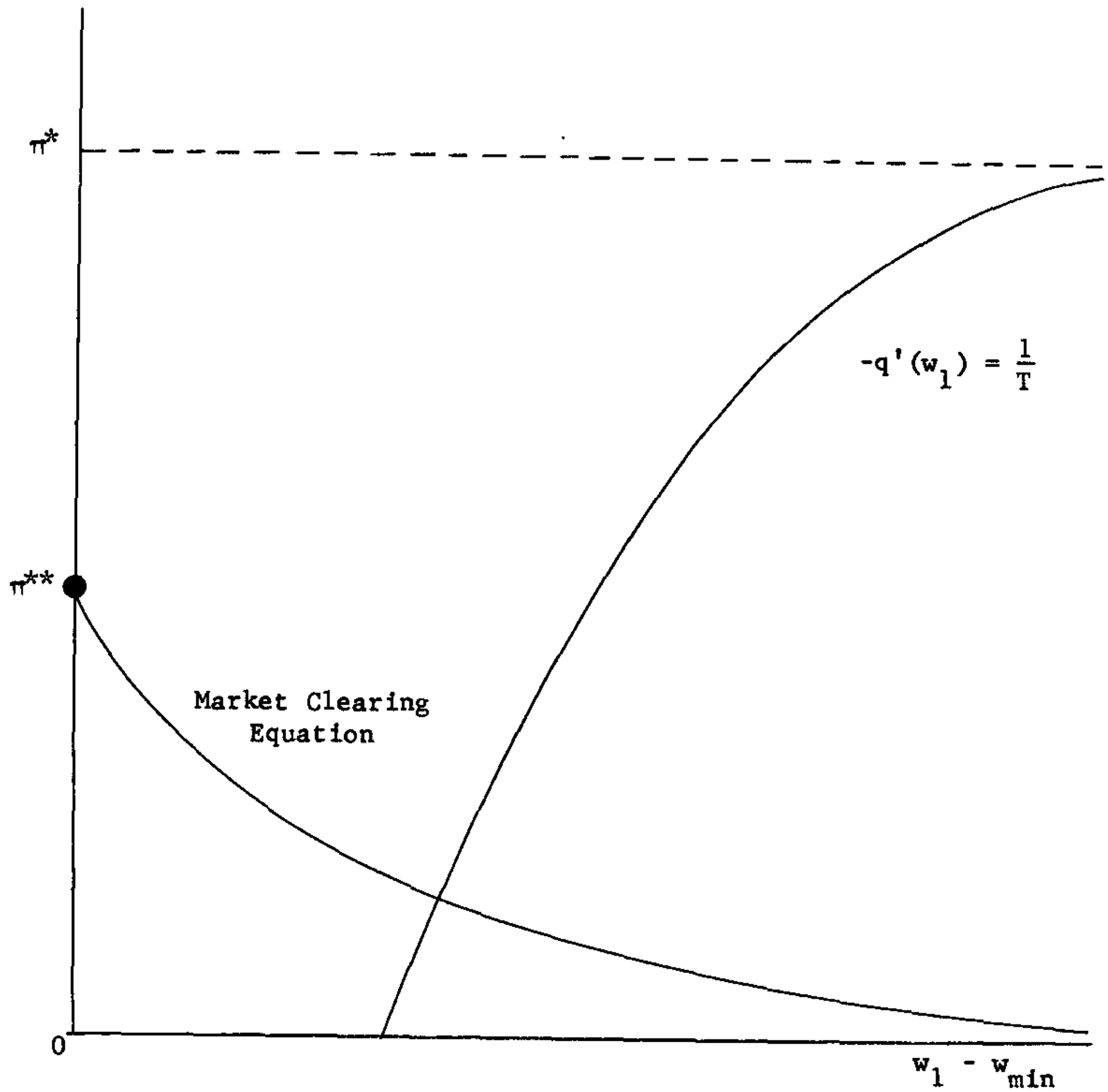

FIGURE 9

6. Welfare Economics of Search

In the very nature of search and information there are important externalities which firms will not take into account.

First, as has long been recognized (see, e.g. Arrow (1959)), imperfect information means that all firms have, as it were, some degree of monopoly power; they can obtain workers even though they pay less than the "market wage." They quickly loose workers to other firms, but in the meanwhile, they are able to exploit the absence of information, provided the costs of turnover are not too high. But what has not been sufficiently recognized is that this exercise of monopoly power is, in some sense, itself the cause of imperfect information; that is, it is the exercise of this monopoly power which results in the wage differentials, in the absence of which search would be unnecessary. There is thus a cost, in addition to the direct loss of consumer surplus usually associated with the exercise of monopoly power, in the additional search and turnover costs.

There is the further externality imposed on the low wage firm by an increase in the wage of a high wage firm: although it reduces its own quit rate--which at equilibrium results in a reduction in turnover costs just sufficient to compensate for the increased wage--it increases the quit rate of other firms and the firm fails to take account of the additional turnover costs of the other firms.

The consequences of these externalities is seen most clearly in the example formulated in Section 3. There, we constructed several equilibrium at the same set of relative prices of the two commodities, such that in one, some individuals were unambiguously worse off than in the other and no one was better off: that is, one equilibrium was Pareto Inferior to another equilibrium.

In the more general model presented in Section 5 above, the contrast between the market and optimal equilibrium may be seen as follows. Since all individuals are assumed *ex ante* to be the same, it seems reasonable to take as our objective function the maximization of the average level of utility. For simplicity, we shall assume a cardinalization which implies "risk neutrality," i.e. the indirect utility function is linear in income, $U = a(p) + b(p)I + \theta$. We shall assume the government is constrained to making each firm pay for itself; the only difference between the "government" solution and the "market" solution is associated with the determination of wages in the high training cost industry. This means that the government must choose a (w_1, π) combination lying along the curve $\pi(w)$ defined above by equation (5.13) (the market clearing curve). The government thus chooses w_1 so that

$$\frac{\partial U}{\partial p}\left[\frac{\partial p}{\partial w_1} + \frac{\partial p}{\partial \pi}\frac{\partial \pi}{\partial w_1}\right] + \frac{\partial U}{\partial I}\left\{\pi + (w_1 - w_{min})\frac{\partial \pi}{\partial w_1}\right\} + \frac{d\bar{\theta}}{dw_1} \leq 0$$

with $w_1 = w_{min}$ when the inequality holds. The contrast between the government and private solution is clear; the private solution entails

$$\left(\frac{\partial p}{\partial w}\right)_{w=w_1} = 1 - Tq^2\left\{\frac{m(w_{min} - w_1)s(1-\pi)}{(\mu + s\pi)(\mu + s)} + \frac{m(0)s\pi}{(\mu + s\pi)\mu}\right\} = 0 \ .$$

Firms overestimate the effect of an increase in w on turnover, for when all firms change their wages together

$$-\frac{\partial q}{\partial \bar{w}_1} = q^2 \frac{m(w_{min} - w_1)s(1-\pi)}{(\mu + s\pi)(\mu+s)} < -\left(\frac{\partial q}{\partial w}\right)_{w=w_1} .$$

This means that the private solution will entail too much wage inequality, although the effects of this are mitigated by the positive sign of the second, third, and fifth terms; in particular the competitive firm ignores the gain in terms of the better matching resulting from more search.

Indeed, constrained optimality always entails some inequality, since[1]

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[1]We use the fact that

(a) $$-\frac{\partial U}{\partial p} = Q_1 \frac{\partial U}{\partial I}$$

(b) $$Q_1 = \frac{L(w_1)}{L} = \pi \quad \text{when} \quad w_1 = w_{min}$$

(c) $$\frac{\partial p}{\partial w_1} = 1 + T \frac{\partial q}{\partial w_1}$$

(d) $$\frac{1}{q(w_1)} = \frac{M(w_{min} - w_1)}{\mu+s} + \frac{\frac{1}{2} - M(w_{min} - w_1)}{\mu + s\pi} + \frac{1}{2\mu}$$

so

$$\left(\frac{\partial q(w_1)}{\partial \pi}\right)_{w_1=w_{min}} = 0$$

(e)

$$\bar{\theta} = q(w_1)\pi\left[\frac{\int_{-\infty}^{w_{min}-w_1} \theta dM(\theta)}{\mu+s} + \frac{\int_{w_{min}-w_1}^{0} \theta dM(\theta)}{\mu + s\pi} + \frac{\int_{0}^{\infty} \theta dM(\theta)}{\mu}\right]$$

$$+ q(w_{min})(1-\pi)\left[\frac{\int_{-\infty}^{0} \theta dM(\theta)}{\mu+s} + \frac{\int_{0}^{w_1-w_{min}} \theta dM(\theta)}{\mu + s\pi} + \frac{\int_{w_1-w_{min}}^{\infty} \theta dM(\theta)}{\mu}\right]$$

so

$$\left(\frac{d\bar{\theta}}{dw}\right)_{w_1=w_{min}} = 0 .$$

$$\left(\frac{\partial U}{\partial w_1}\right)_{w_1=w_{min}} = \frac{U_I 4\mu^2 m(0)sT}{(2\mu + s)^2(\mu + s\pi)} > 0 .$$

It should be noted that we have just characterized the optimal two wage equilibrium. The full analysis would require finding the optimal distribution of wages. This would take us beyond the scope of this paper.

7. Concluding Comments

In this paper we have investigated the implications of imperfect information for the equilibrium wage distribution. We have shown how, even in a steady state equilibrium, identical labor may receive different wages; the competitive forces which we normally think of as eliminating such differences in the long run may not do so when there is imperfect information. Indeed, this may be the case even when the only source of "imperfect information" is the inequality of wages which the market is perpetuating. When there are information imperfections arising from (symmetric) differences in non-pecuniary characteristics of jobs and preferences of individuals, there will always be wage dispersion in equilibrium. Although it was established that (constrained) optimality involved some wage dispersion, there was a presumption that the market solution entailed more inequality than was optimal.

Once again, we see that the assumption of perfect information is not the innocuous assumption we sometimes make it seem to be: equilibrium in markets with imperfect information appears to differ in fundamental ways from that in markets with perfect information.

Although we have not explicitly introduced unemployment into the model, it would be an easy matter to do so, e.g., by requiring a period of unemployment between job trainsitions, by assuming that search could only be undertaken while the individual is unemployed, or, more generally, by assuming that search may be carried on "less expensively" while unemployed. The equilibrium would then be characterized by an unemployment rate as well as a wage distribution. The results derived above concerning the non-optimality of the search equilibrium, as well as our earlier study of labor markets with unemployment equilibria (Stiglitz, 1974b) lead to the presumption that both those who argue that the existence of unemployment implies a zero shadow price of labor as well as those who argue that since the unemployment is "competitively determined" it must be "efficient" in some sense, and that therefore the shadow price of labor is the market wage[1] are likely to be incorrect. But the formalization of such a model and the calculation of the shadow price of labor in it is a matter which we shall have to leave for another occasion.

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[1]Since homogeneous labor is receiving different wages, it is not clear what we mean by the market wage. This simply illustrates the difficulties of conventional welfare analysis in this class of models.

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