COWLES FOUNDATION FOR RESEARCH IN ECONOMICS

AT YALE UNIVERSITY

Box 2125, Yale Station
New Haven, Connecticut

COWLES FOUNDATION DISCUSSION PAPER NO. 283

MONEY AND INCOME: POST HOC ERGO PROPTER HOC?

James Tobin

August 22, 1969

MONEY AND INCOME: POST HOC ERGO PROPTER HOC?*

by

James Tobin

Milton Friedman asserts that changes in the supply of money M (defined to include time deposits) are the principal cause of changes in money income Y. In his less guarded and more popular expositions, he comes close to asserting that they are the unique cause.[1] In support of this position Friedman and his associates and followers have marshalled an imposing volume of evidence, of several kinds.

Historical case studies are one kind of evidence. For example, in their monumental Monetary History of the United States 1867-1960,[2] Friedman and Anna Schwartz carefully analyze and interpret the role of money and monetary policy in the important episodes of American economic history since the Civil War. Summary regressions of time series of economic aggregates are a second type of evidence. Presumed effects

---

*The research described in this paper was carried out under grants from the National Science Foundation and from the Ford Foundation. I am grateful to Milton Friedman for helpful comments on an earlier draft of this paper.

---

[1]See, for example, his column in Newsweek, January 30, 1967, p. 86, "Higher Taxes? No." He says, "To have a significant impact on the economy, a tax increase must somehow affect monetary policy--the quantity of money and its rate of growth...The Federal Reserve can increase the quantity of money by precisely the same amount with or without a tax rise. The tax reduction of 1964.....encouraged the Fed to follow a more expansionary policy. This monetary expansion explains the long-continued economic expansion. And it is the turnabout in monetary policy since April 1966 that explains the growing signs of recession."

---

[2]National Bureau of Economic Research, Studies in Business Cycles, No. 12, Princeton: Princeton University Press, 1963.

are simply regressed on presumed causes; the single equations estimated are something like the econometrician's "reduced forms". In a study with Meiselman,[3] Friedman concluded that his monetary explanation of variations in money income fits the data better than a simple Keynesian multiplier model. More recent studies in the same vein claim that monetary policy does better than fiscal policy in explaining postwar fluctuations of money income.[4]

A third type of evidence relates to timing, specifically to leads and lags at cyclical turning points. Much of the work of Friedman and his associates at the National Bureau of Economic Research has been devoted to this subject.[5] Turning points in the rate of change of money supply show a long lead, and turning points in the money stock itself relative to trend a shorter lead, over turning points in money income. A great deal of the popular and semi-professional appeal of the modern quantity theory can be attributed to these often repeated facts.

However, the relevance of timing evidence has been seriously questioned.[6] Friedman himself says, "These regular and sizable leads

---

[3]Friedman and David Meiselman, "The Relative Stability of Monetary Velocity and the Investment Multiplier in the United States 1897-1958," Commission on Money and Credit, Stabilization Policies, Prentice-Hall, Inc., Englewood Cliffs, N. J., 1963, pp. 165-268.

[4]Leonall Anderson and Jerry Jordan, "Monetary and Fiscal Actions: A Test of their Relative Importance in Economic Stabilization," Federal Reserve Bank of St. Louis Review, November 1968, pp. 11-24.

[5]See Friedman, "The Lag in the Effect of Monetary Policy," Journal of Political Economy, Vol. LXIX, October 1961, pp. 447-466. Friedman and Schwartz, "Money and Business Cycles," Review of Economics and Statistics, Feb. 1963 Supplement, pp. 32-64. Friedman, "The Monetary Studies of the National Bureau," Annual Report of the National Bureau of Economic Research 1964 .

[6]By, among others, J. Kareken and R. Solow, "Lags in Monetary Policy," Commission on Money and Credit, Stabilization Policies, Prentice-Hall, Inc., Englewood Cliffs, N.J., 1963, pp. 14-25. They pointed out that a rate of change like $\dot{M}$ will generally lead a level like Y, in the manner that a cosine series "leads" a sine series. Friedman replied in "The Lag in the Effect of Monetary Policy," loc. cit. that both $\dot{M}$ and Y have the dimension of a flow and that in any case he finds $\dot{M}$ leading $\dot{Y}$ and trend-corrected M leading Y. Kareken and Solow found little lead, if any, of $\dot{M}$ over the rate of change of the industrial production index, but they should have used a monetary rather than a real measure of business activity.

of the money series are themselves suggestive of an influence running from money to business but they are by no means decisive."[7] The apparent leads may "really" be lagged responses -- either positive or negative ("inverted") -- of money to previous changes in business activity. Friedman cautiously rejects this possibility. He finds that the M series conforms more closely to the NBER reference cycle on a positive basis with money leading than on an inverted basis with money lagging, and he regards the business-money causal nexus as very likely to be inverted. Having satisfied himself that the dominant association of M and business activity is positive, Friedman concludes, "...it is not easy to rationalize positive conformity with a lead as reflecting supply response,"[7] i.e. response of the supply of money to changes in business activity.

The purpose of the present paper is to spell out the lead-lag timing implications of alternative theoretical models of the relation between money and money income. In one model, a version of the ultra-Keynesian theory that Friedman is so often attacking, monetary developments are just a sideshow to the main events. In the other model, one of Friedman's own, monetary developments are of decisive causal significance. What kinds of observed relations between money and money income and their rates of change do the opposing models generate? Do they imply different lead-lag patterns?

In the ultra-Keynesian model, changes in the money supply are a passive response to income changes generated, via the multiplier mechanism, by autonomous investment and government expenditure. This makes it possible to see what kinds of observations of money stock M and its rate of change M would be generated in an ultra-Keynesian world. These can then be compared with the observations that would be generated by a Friedman economy.

---

[7] In "The Monetary Studies of the National Bureau", <u>loc cit</u>., pp. 13, 14.

Here it is necessary to express Friedman's hypothesis with more precision and simplicity than it is usually expounded. However, this can be done with the help of the model of the demand for money set forth in his article with Anna Schwartz, "Money and Business Cycles."[8]

I hasten to say that I do not believe the ultra-Keynesian model to be exhibited (nor would Keynes), any more than I believe Friedman's. I do think, nevertheless, that the exercise points up the dangers of accepting timing evidence as empirical proof of propositions about causation.[9] I shall show that the ultra-Keynesian model -- in which money has no causal importance whatever -- throws up observations which a superficial believer in *post hoc ergo propter hoc* would regard as more favorable to the idea that money is causally important than Friedman's own model. What is even more striking and surprising is that the ultra-Keynesian model implies cyclical timing patterns just like the empirical patterns that Friedman reports, while the Friedman model does not.

An Ultra-Keynesian Model

The ultra-Keynesian multiplier model has

$$Y = m(G + \mathring{K}) \tag{1}$$

where Y is net national product, G is the current rate of government expenditure, and $\mathring{K}$ is net capital accumulation, all in nominal units. (The division of cyclical fluctuations in income between real output and prices is inessential to the argument of the paper and is ignored throughout.) The multiplier m is derived routinely from

---

[8] *loc. cit.*

[9] The same methodological lesson is given by the simulations of more complicated models in William C. Brainard and James Tobin, "Pitfalls of Financial Model Building," *American Economic Review* (*Papers and Proceedings*), May 1968, pp. 99-122.

the identity:

Saving + Taxes = Government Expenditure + Net Investment

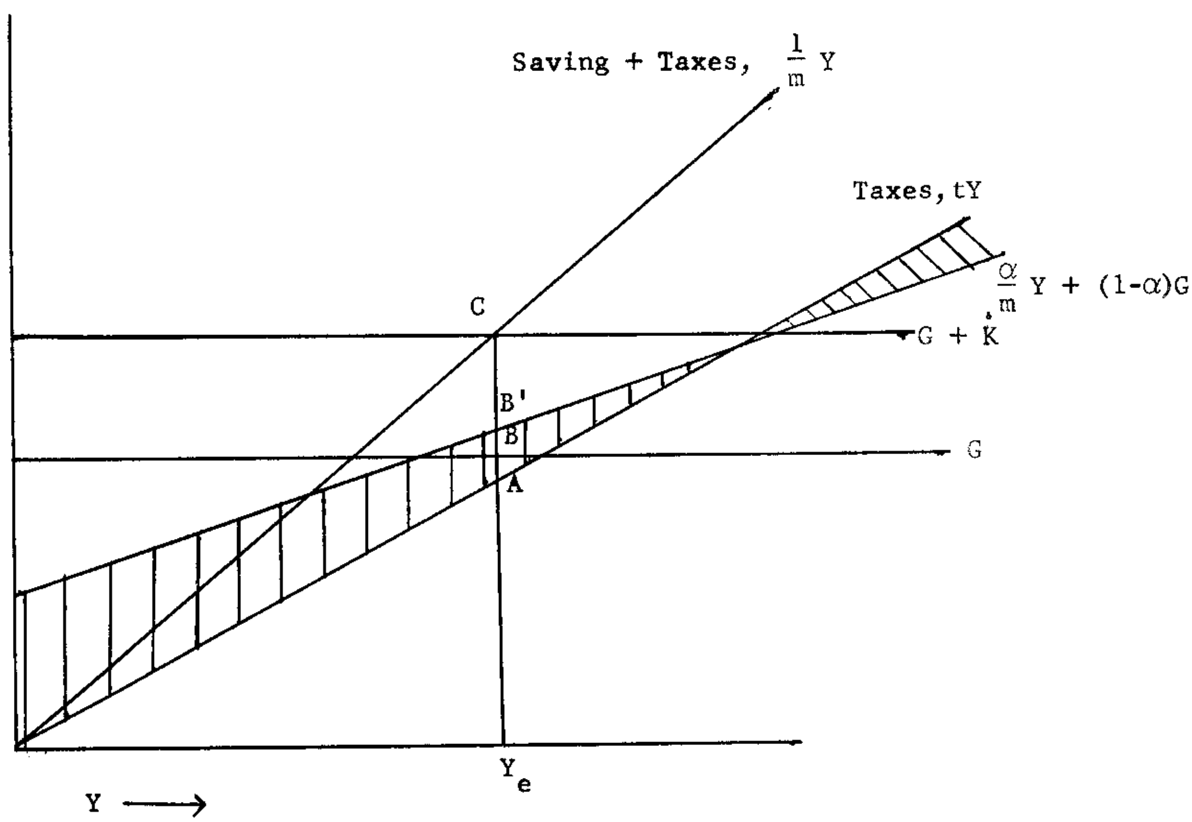
(2) $$s(1-t)Y + tY = G + \dot{K}$$

where s is the marginal propensity to save from income after taxes and t is the constant tax rate (net of transfers). Therefore the multiplier,

(3) $$m = \frac{1}{s(1-t) + t}$$

The determination of income by equation (2) is illustrated in the familiar textbook diagram, Figure 1.

FIGURE 1.



Private wealth W is the capital stock K plus the government debt D (whether monetized or not), the cumulative total of past deficits, $G - tY$. Saving, the change in private wealth is

$$(4) \qquad s(1-t)\gamma = \dot{W} = \dot{K} + G - tY = (\dot{K} + G)(1-tm)$$

In Figure 1 government deficit is AB, and net capital accumulation is BC.

The public's balance sheet is:

$$(5) \qquad W = K + D = K - L + M + B ,$$

where B is the public's holdings of the non-monetary debt (bonds) of the government, L is the debt of the public to the banking system, and M is the public's holdings of the monetary liabilities of the government and the banking system. To be consistent with Friedman's model and his empirical findings, M includes time deposits as well as demand deposits.

The portfolio behavior of the public in this ultra-Keynesian world is very primitive. Real investment is autonomous; indeed exogenous fluctuations in the pace of capital formation are the source of the business cycle. This implies that there are autonomous shifts in the proportions in which the public wishes to allocate its wealth among the available assets. During investment booms capital becomes more attractive relative to money and bonds; during investment recessions the reverse occurs.[10] By the same token borrowing from banks rises in booms and falls in recessions. Specifically, the public's debt to the banking system is taken to be a fixed proportion of the capital stock:

$$(6) \qquad L = \alpha K \qquad\qquad (0 < \alpha < 1)$$

---

[10] It might seem more Keynesian to let bonds alone bear the brunt of the autonomous shifts to and from capital. But "money" here includes time deposits.

The only portfolio decision left is the allocation of the remainder of the public's net worth, -- $(W - K + L)$, which is equal to $(D + \alpha K)$ -- among the two remaining assets, money (currency and bank deposits) and bonds (interest-bearing government debt). This is the choice of Keynesian liquidity preference theory. The demand for money can be written as the sum of two components, an asset demand related to the interest rate and to allocable wealth and a transactions demand proportional to income:

$$(7) \qquad M = a_0(r)(D + \alpha K) + a_1 Y$$

where r is the interest rate on bonds and the derivative $a_0'(r)$ is negative. By subtraction, public demand for bonds is $(1-a_0(r))(D + \alpha K) - a_1 Y$.

The main point of the exercise can be made by assuming that the monetary authority provides bank reserves as necessary to keep r constant, so that a_0 is a constant. The monetary system responds to the "needs of trade." With the help of the monetary authority, banks are able and willing to meet the fluctuating demand of their borrowing customers for credit and of their depositors for money. In Friedman's terms, this is a "supply response" with "positive conformity" of money to business activity. It is indeed a response which he regards as all too common in central banking, one for which he has severely criticized the Federal Reserve. If these criticisms are justified, then this endogenous response must have played an important role in generating monetary time series.

The relation among flows corresponding to (7) is:

$$(8) \qquad \dot{M} = a_0(\dot{D} + \alpha \dot{K}) + a_1\dot{Y} = a_0(G - tY + \alpha \dot{K}) + a_1\dot{Y} .$$

Using (1) converts (8) into

(9) $$\dot{M} = a_0[G(1 - \alpha) + Y(\frac{\alpha}{m} - t)] + a_1\dot{Y}$$

Thus, for given G, $\dot{M}$ is a linear function of Y and $\dot{Y}$, and these vary in response to autonomous changes in investment $\dot{K}$. The relationship to $\dot{Y}$ is, of course, positive. Consider now the relationship to Y. In Figure 1, at income level Y_e, $\dot{D}$ is represented by AB. Let $BB' + \alpha BC$, the amount of real investment covered by new indebtedness to banks. Then AB' represents $\dot{D} + \alpha\dot{K}$, the quantity which the public divides between accumulations of money and of bonds. Imagine that G is held constant, while $\dot{K}$ varies autonomously and carries Y with it. Then the vertical distance through the shaded area, of which AB' is an example, is $\dot{D} + \alpha\dot{K}$. This declines with Y, as illustrated, provided the line through B' has a slope smaller than t, i.e., that α/m is smaller than t. (For example, if the multiplier is 2-1/2 and the tax ratio is 1/5, the loan-to-investment ratio α must be smaller than 1/2.) In this case $\dot{D} + \alpha\dot{K}$ will become negative, as illustrated, at sufficiently high values of Y, where the government budget is in large surplus.

The financial operations of the government and the banks are as follows: The government and the monetary authority divide the increase of debt $\dot{D}$ between "high-powered money" and bonds in such manner as to keep the interest rate on target. Assuming no change in currency holdings by the public, the increase $\dot{M}$ in money requires an increase of $k\dot{M}$ where k is the required reserve ratio, in bank reserves. Banks' loan assets increase by $\dot{L} = \alpha\dot{K}$. The difference $(\dot{M}(1-k) - \dot{L})$ the banks allocate between excess reserves and bond holdings, in proportions that depend on the interest rate. Thus the monetary authority provides enough new high-powered money to meet increased reserve requirements and any new demand for excess reserves. The remainder of the increase in public debt $\dot{D}$ takes the form of bonds, and this is just enough to satisfy the demands of the banks and the public. This can be seen as follows: The increase in public demand for bonds is $\dot{W} + \dot{L} - \dot{K} - \dot{M} = \dot{D} + \alpha \dot{K} - \dot{M}$. The increase in the banks' demand for bonds is $\dot{M} - \dot{L} - \dot{H} = \dot{M} - \alpha \dot{K} - \dot{H}$, where $\dot{H}$ is the increase in required and excess reserves. Adding the two together, we see that the increase in demand for bonds is $\dot{D} - \dot{H}$, just equal to the supply. In short, Walras' law guarantees that if the money market is cleared, the bond market is also cleared.

A dollar increase in government spending has the same effect in raising income and tax receipts as a dollar increase in private investment. Both raise income Y by the multiplier m , and taxes by tm . However, they have different effects on $\dot{D} + \alpha \dot{K}$ and thus on $\dot{M}$. An increase in government expenditure raises $\dot{D} + \alpha \dot{K}$ by $1-tm$; an increase in private investment, by $\alpha - tm$. Since α is less than 1 , the demand for money is raised more by an increase of government expenditure. This is clear from (9). For given Y , a dollar increase in G (replacing a dollar of $\dot{K}$) increases $\dot{M}$ by $1-\alpha$.

A tax cut sufficient to create the same increase in income would entail an even larger rise in $\dot{D} + \alpha \dot{K}$ and in the demand for additional money. Our ultra-Keynesian would not be surprised to find the money supply rising especially fast in an income expansion propelled by deficit spending. He would not even be surprised if some observers of the accelerated pace of monetary expansion in the wake of a tax cut conclude that monetary rather than fiscal policy caused the boom.[11]

Let us return, however, to a model cycle generated by fluctuation in private investment $\dot{K}$, with government expenditure and the tax rate constant. The model abstracts from trends in Y and its components. However, private wealth grows over the model cycle; and this is responsible for an upward trend in M. What will be the cyclical behavior of the money supply M and of its rate of change $\dot{M}$, in reference to the cycles in money income Y and its rate of change $\dot{Y}$?

There are two components of $\dot{M}$, one related to Y and one to $\dot{Y}$. The Y-component has already been discussed. Its relationship to Y is shown in Figure 2, as the downward sloping line. $\underline{Y}$ and $\bar{Y}$ are the trough and peak of the cycle. In the illustration $\dot{M}$ for stationary Y does not become negative, even at $\bar{Y}$. The second or transactions component is simply proportional to $\dot{Y}$: $a_1\dot{Y}$ in equation (8) or (9) above. This can be added to Figure 2 provided we know the relation of $\dot{Y}$ to Y. That relation is illustrated in Figure 3, on the assumption that the cycle in $\dot{K}$ and Y is a sine wave. The circle, with arrows, shows $\dot{Y}$ zero at the trough of Y, $\dot{Y}$ at its peak, Y at its peak with $\dot{Y}$ again zero, $\dot{Y}$ at its trough, etc. The ellipse within the circle represents the corresponding cycle in the second component of $\dot{M}$.

In Figure 2, this component is added vertically to the line representing the first component. The squashed ellipse in Figure 2 shows the cycle of $\dot{M}$ as income moves from $\underline{Y}$ to $\bar{Y}$ and back. The order of events

---

[11] Note Friedman's comment in Newsweek quoted above footnote 1, page 1.

in the cycle can be read by following the perimeter of the squashed ellipse clockwise. In Figure 2 there is a brief period of the cycle when $\dot{M}$ is negative. Thus M has a late peak and early trough, and grows on balance over the cycle. It can easily be imagined, however, that the ellipse in Figure 2 lies entirely above the axis, so that M grows continuously but at varying rates. Or, if the first or level component of $\dot{M}$ became negative before Y reached its peak, then M would lead Y at the peak as well as at the trough. In any case, it is clear that $\dot{M}$ not only has a long lead over Y, more than a quarter of a cycle, but also leads $\dot{Y}$.

The horizontal line through the squashed ellipse represents the average value of $\dot{M}$. The stock of money M, corrected for trend, will reach its peak and trough when actual $\dot{M}$ is equal to average $\dot{M}$. These points are also indicated in Figure 2. They precede turning points in Y but not in $\dot{Y}$.

It is easy to modify Figure 2 to allow for a rise in interest rates during expansions of money income and a decline in contractions. In an ultra-Keynesian world this "leaning against the wind" by the monetary authorities would be irrelevant to stabilization. But it might occur nonetheless, because the monetary authorities mistakenly believe in their own powers or are just operationally conservative in changing the supply of high-powered money, or because they worry about the balance of payments. Anyway, it would be represented in Figure 2 by a steepening of the central line. This would result in a still longer lead of $\dot{M}$ with respect to $\dot{Y}$ and Y.

FIGURE 2. Ultra-Keynesian Cycle

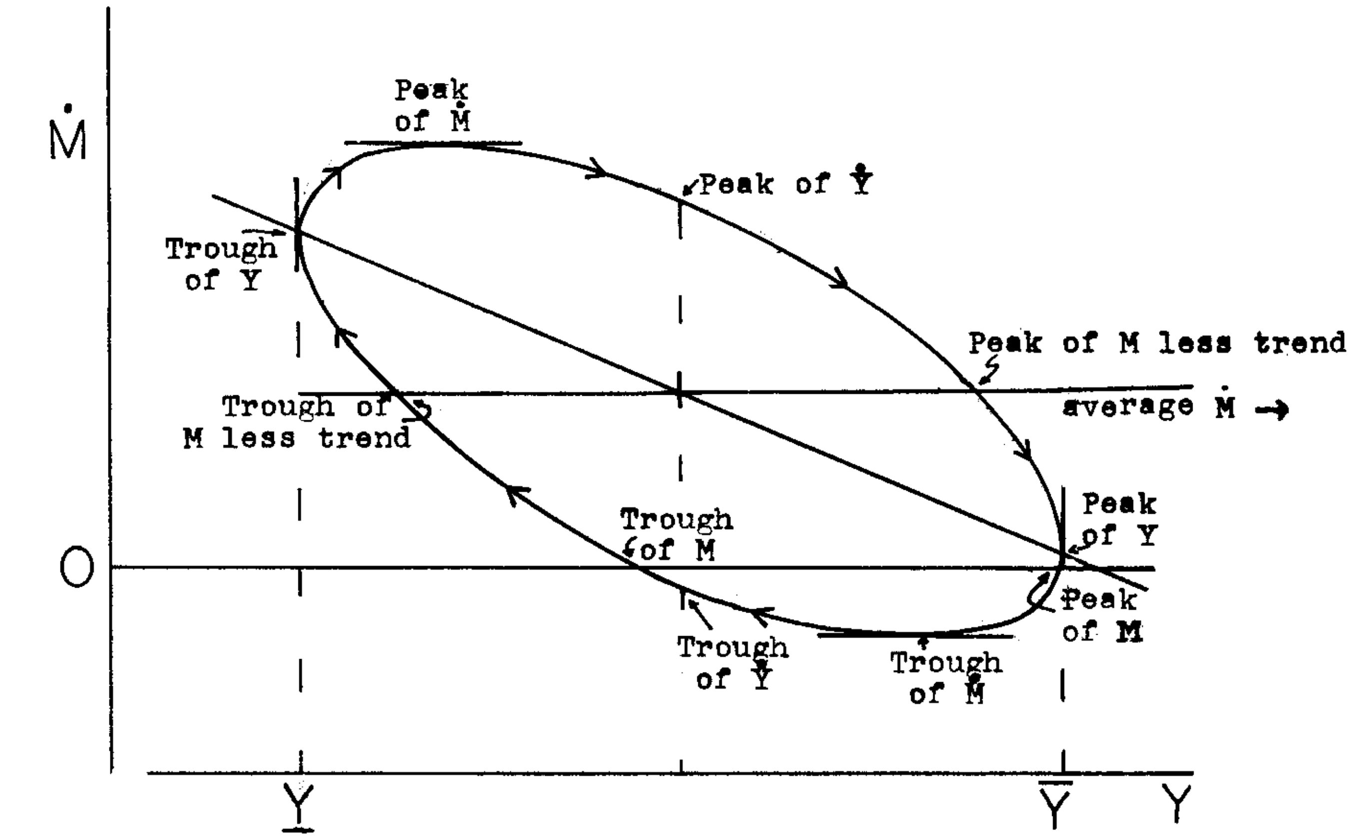
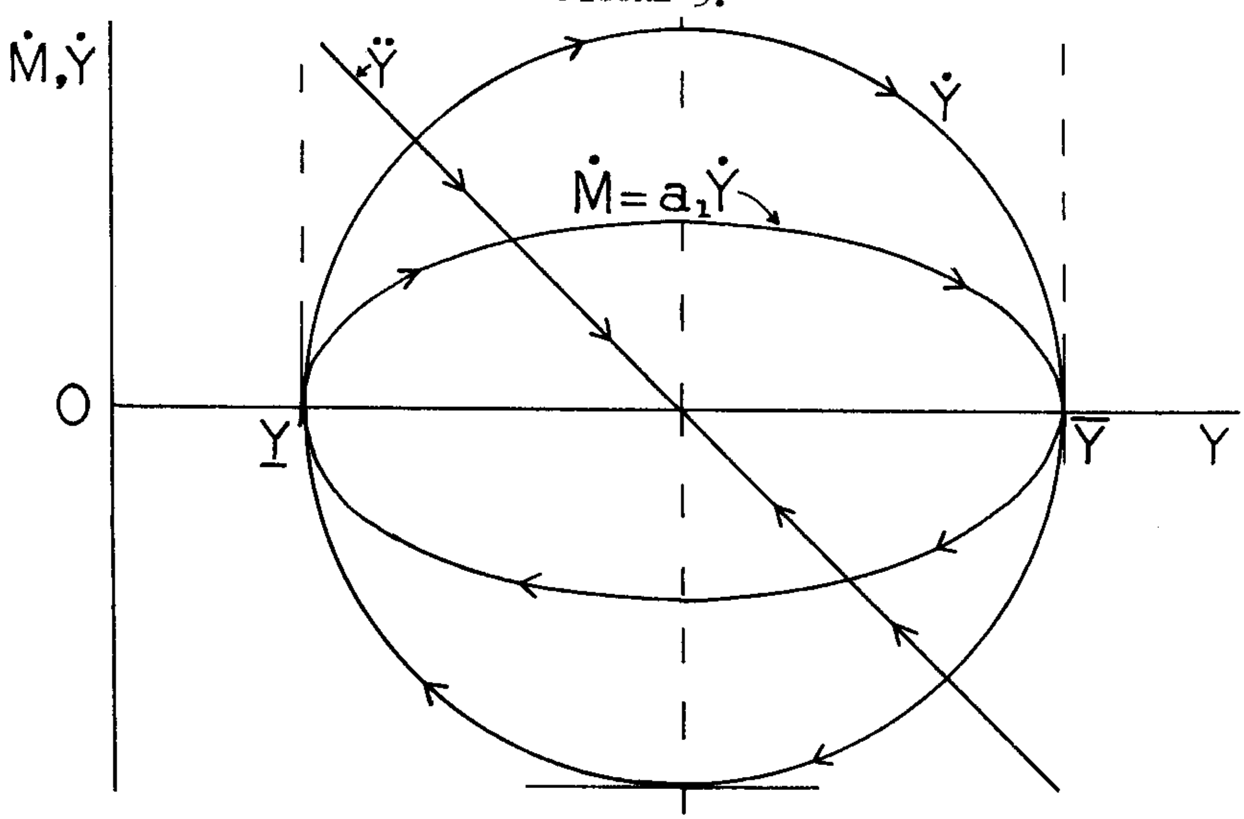


FIGURE 3.



Equation (8) would read:

$$\dot{M} = a_0(r)(\dot{D} + \alpha \dot{K}) + a_0'(r)(D + \alpha K)\dot{r} + a_1\ddot{Y} \tag{8'}$$

As r rises with Y, the decline in a_0 reinforces the decline in $\dot{D} + \alpha \dot{K}$. Assuming that $\dot{r}$ is positively related to $\dot{Y}$, and given that $a_0'(r)$ is negative, the second term contributes a negative relation of $\dot{M}$ to $\dot{Y}$. This would make the ellipse of Figure 3 flatter, as well as distorting its shape. Indeed, it could conceivably reverse the net effect of $\dot{Y}$ on $\dot{M}$ and therefore reverse the order of events in the cycle. But the central bank surely does not lean against the wind so hard as that, especially in an ultra-Keynesian world.)

The results would also be reinforced if a term in Y, with a positive coefficient, were added to the basic demand for money equation (7). The logic of such a term would be that changes in wealth are in the first instance absorbed in cash balances, with more permanent portfolio allocations following later. Thus demand for money would be especially high when income and saving are rapidly increasing. This, after all, is what one would expect of money as "a temporary abode of purchasing power," to use Professor Friedman's famous phrase.

A $\dot{Y}$ term in expression (7) for M means a $\ddot{Y}$ term in expression (8) for $\dot{M}$. In a cycle of the type illustrated in Figure 3, $\ddot{Y}$ is inversely related to Y. Therefore a $\ddot{Y}$ component of $\dot{M}$ will be high at low levels of Y and low at peak levels. Like the interest rate effect, this will increase the slope of the central line in Figure 2 and accentuate the lead-lag pattern there depicted.

There is nothing sacred about sine waves, and neither is a sine-curve cycle crucial for the timing pattern shown in Figure 2. The reader is invited to experiment with non-circular shapes of the relation of $\dot{Y}$ to Y in Figure 3. He will find it easy to change the lengths of the lags and leads, and in extreme cases to produce some coincidences and ambiguities. But the essential message of Figure 2 comes through provided that $\dot{M}$ is negatively related to Y and positively to $\dot{Y}$.

A Friedman Model

I turn now to the cyclical pattern implied by Friedman's own "permanent income" theory of the demand for money. For present purposes this may be expressed as follows:

$$\ln M = A' + \delta \ln Y_p' \tag{10}$$

Here M is the same quantity of money as in the ultra-Keynesian model; Y_p' is permanent income; δ is the elasticity of the demand for money with respect to permanent income, estimated by Friedman to be of the order of 1.8. Income and permanent income grow secularly at an exponential rate β. As above, we abstract from this trend of income and consider the deviations from trend Y_p and Y. Since $\ln Y_p = \ln Y_p' - \beta t - C$, equation (10) can be restated as

$$\ln M = A + \delta \ln Y_p + \delta\beta t \tag{11}$$

For rates of change, (11) implies

$$\dot{M}/M = \delta(\dot{Y}_p/Y_p) + \delta\beta \tag{12}$$

Permanent income, corrected for trend, is a weighted geometric average of current and past actual incomes, also corrected for trend, with the weights receding exponentially. Thus when actual and permanent income differ, the public changes its estimate of permanent income by some fraction of their relative difference. Specifically,

$$(13) \qquad \dot{Y}_p/Y_p = w(\ln Y - \ln Y_p), \quad \text{or} \quad \ln Y = \frac{1}{w}(\dot{Y}_p/Y_p) + \ln Y_p$$

Friedman has estimated, mainly in connection with his work on the consumption function, that revision of permanent income estimates eliminates about one third of its deviation from actual income within a year. In other words, the weight of the current year's income is one third, and the weights of past years' incomes two thirds, in the calculation of permanent income. If the revision is taken to be continuous, as in (13), rather than discrete, these weights imply a value of .40 for w.

In this model the supply of money and its rate of change are autonomous. The demand for money must adjust to the supply at every point of time. Permanent income is the only variable involved in the demand for money, so it must do the adjusting. But much of permanent income is past history; the only part that can adjust is current income. Roughly speaking, Friedman's numerical estimates imply that permanent income must rise .55 percent to absorb a 1 percent increment in the supply of money. But in the short run money is much more powerful. Current year's income must rise by 1.65 percent to make permanent income rise .55 per cent. Thus in a cyclical boom, in which the supply of money is accelerating, current income must rise even faster. In this way the theory explains why the velocity of money moves up and down with income in business cycles and reconciles this observation with Friedman's finding that secularly velocity declines as income rises.

An explicit relation of income to money supply can be obtained from (13) by using (11) to express $\ln Y_p$ in terms of $\ln M$ and (12) to express $\dot{Y}_p/Y_p$ in terms of $\dot{M}/M$.

$$(14) \qquad \ln Y = \frac{\dot{M}/M}{\delta w} + \frac{\ln M}{\delta} - \beta t - \frac{\beta}{w} - \frac{A}{\delta}$$

$$(15) \qquad \dot{Y}/Y = \frac{\dot{g}_M}{\delta w} + \frac{g_M}{\delta} - \beta ,$$

for convenience letting g_M denote $\dot{M}/M$ and $\dot{g}_M$ its time derivative. Equation (15) will be used for the analysis of cyclical timing patterns. It relates the rate of change of income, abstracting from trend, to the rate of change of the money stock and to the change in that rate. Note that if g_M is held steady at $\delta\beta$ then $\dot{Y}/Y$ will be zero and income will be on trend.

This exposition is based on Friedman's theory as set forth in his article with Anna Schwartz.[12] I have used continuous rather than discrete time, and I have related money demand to money income, ignoring the complication that real income and price level enter Friedman's formula somewhat differently. These simplifications do not impair the essential

---

[12]"Money and Business Cycles," loc. cit., pp. 56-59.

message of the theory for the present purpose.[13]

Consider a business cycle generated by a sine wave in g_M. What will be the resulting movement of $\dot{Y}/Y$? This is, according to (15), the sum of two components, one linear in g_M itself, the other proportional to $\dot{g}_M$. The first is indicated by the positively sloped line in Figure 4. The trough and peak of g_M are indicated by $\underline{g}_M$ and $\overline{g}_M$. The average value of g_M over the cycle is positive, specifically $\delta\beta$, while the average value of $\dot{Y}/Y$ is of course zero. These average values are shown as point Q in Figure 4. To show the second component on the same diagram we must use the relationship between $\dot{g}_M$ and g_M, depicted by the circle in Figure 5. The large ellipse in which the circle is inscribed is $\dot{g}_M/\delta w$, where $1/\delta w$ exceeds one in keeping with Friedman's theory and numerical estimates. It is this which must be added vertically to the line of Figure 4 to exhibit the total change in income $\dot{Y}/Y$. As in the case of Figure 2, the order of events in the cycle may be read by following the perimeter of the misshapen ellipse in Figure 4 clockwise.

In this monetary model of business fluctuations, $\dot{M}/M$ <u>lags</u> $\dot{Y}/Y$ and has only a short lead over Y itself. The money stock itself lags Y at peak and trough. However, as in the other model, there might be no cycle in M at all: $\dot{M}/M$ might never be negative. This would be shown in Figure 4 by moving the vertical axis entirely to the left of the ellipse. If it were moved part way, the trough in M might precede the trough in Y. But the major conclusions remain.

As in Figure 2, it is also possible to indicate in Figure 4 the peak and trough in the deviation of the money stock from trend. The

---

[13] Elsewhere, with Craig Swan, I have considered the permanent income theory in full detail and tested the model and Friedman's numerical estimates of the parameters against post-war U.S. data. See J. Tobin and Craig Swan, "Money and Permanent Income: Some Empirical Tests," <u>American Economic Review</u>, Vol. LIX, May 1969, pp. 285-295.

FIGURE 4. Friedman Cycle

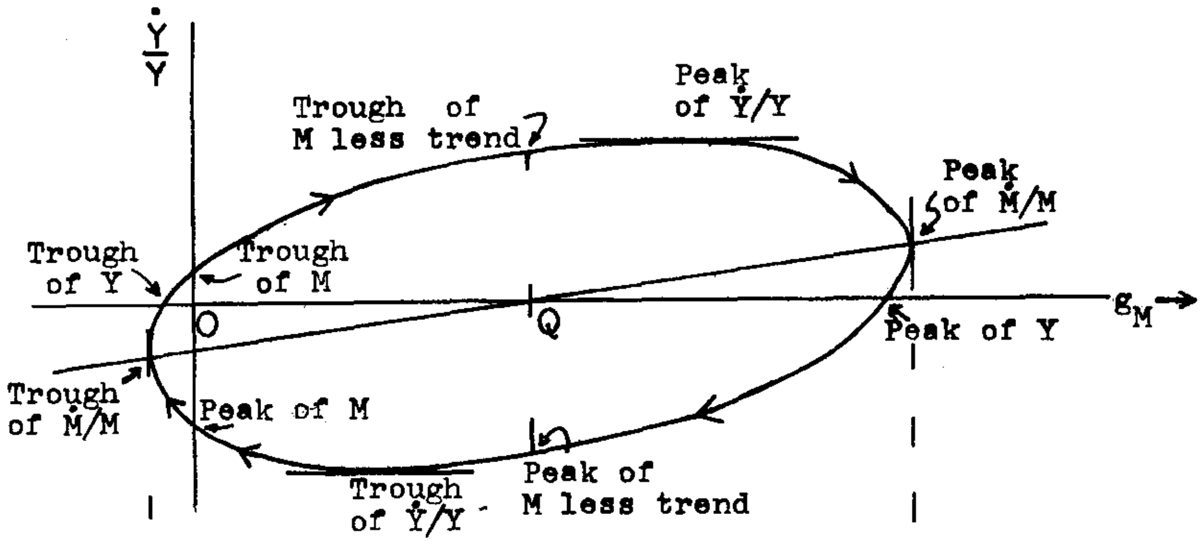
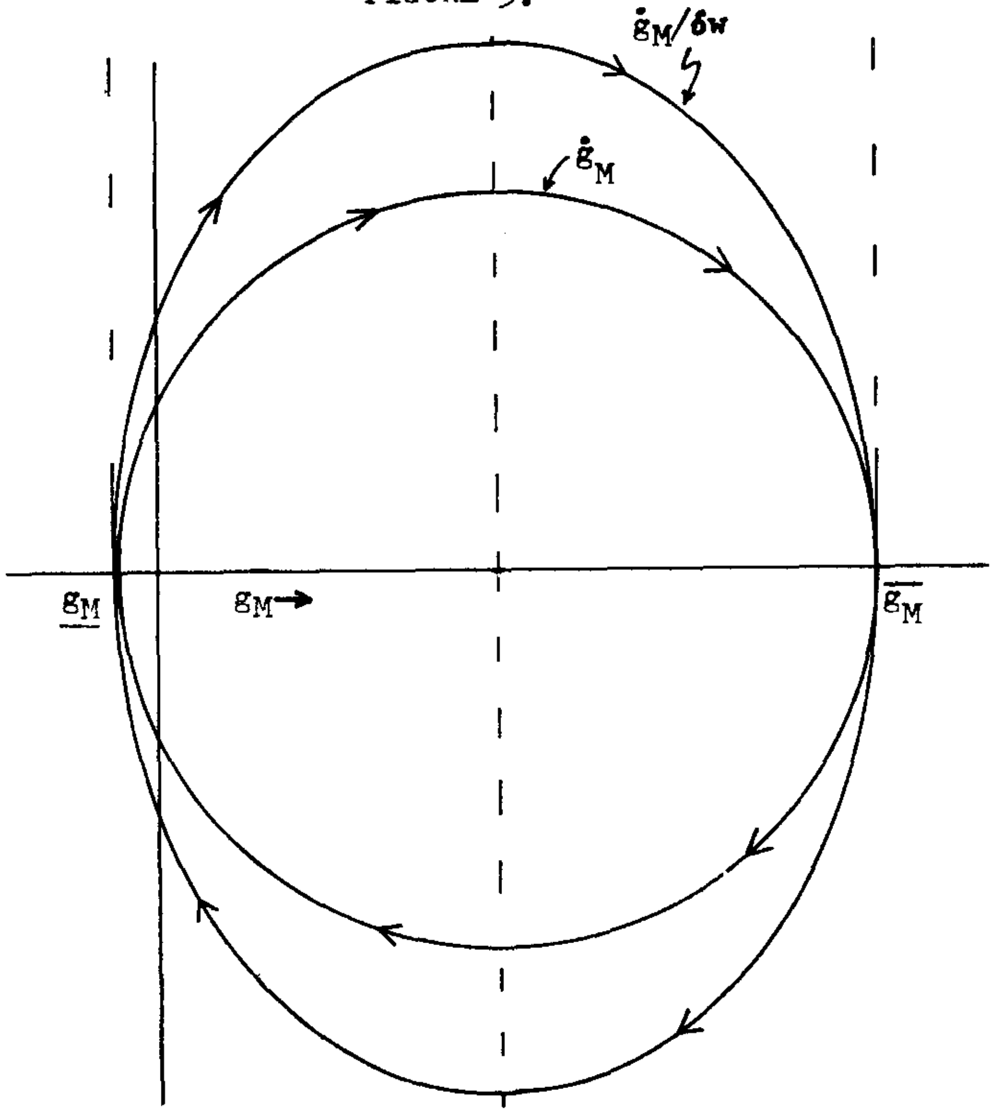


FIGURE 5.



average level of $\dot{M}/M$ is shown by the dashed vertical line through Q . When actual $\dot{M}/M$ equals this average, trend-corrected M reaches its peak and trough. Figure 4 shows that these turning points lag the corresponding turning points in Y .

As in the case of the ultra-Keynesian model, the cycle does not need to be a sine wave in order to produce the basic order of events over the cycle.

Comparisons of Timing Implications

In Table 1, I have summarized the timing implications of the two models, as indicated in Figures 2 and 4.

Clearly the monetary-causal model implies a much less impressive lead of money over business activity than its opposite.

Consider now the empirical evidence. The cyclical timing patterns reported by Friedman and Schwartz[14] are as follows:

(a) For "mild depression cycles" they find no cycle in M .

(b) For "deep depression cycles" they find a cycle in M , mildly lagging the NBER reference cycle, with which money income is roughly coincident, at peaks.

(c) They find that the rate of change of the money stock leads at peaks and troughs. This lead is dramatically long, so much so "as to suggest the possibility of interpreting the rate of change series as inverted, i.e., as generally declining during reference expansion and rising during reference contraction."

---

[14] Friedman and Schwartz, "Money and Business Cycles," loc. cit., especially Charts 2, 4, and 6, and p. 36.

TABLE 1

Order of Events in Model Cycles

Ultra-Keynesian	Friedman
trough of Y	trough of Y or [trough of M]
peak of $\dot{M}$	[trough of M] or [trough of Y]
peak of $\dot{Y}$	trough of M less trend
peak of M less trend	peak of $\dot{Y}$
peak of Y or [peak of M]	peak of $\dot{M}$
[peak of M] or peak of Y	peak of Y
trough of $\dot{M}$	peak of M less trend
trough of $\dot{Y}$ or [trough of M]	trough of $\dot{Y}$ or [peak of M]
[trough of M] or trough of $\dot{Y}$	[peak of M] or trough of $\dot{Y}$
trough of M less trend	trough of $\dot{M}$
trough of Y	trough of Y or [trough of M]

Note: events in brackets [ ] need not occur at all.

(d) He shows a generally pro-cyclical behavior of velocity Y/M, but with some tendency for velocity to start declining before the reference peak.

Friedman has also summarized the evidence in an earlier article,[15] as follows:

> "...peaks in the rate of change of the money stock precede reference cycle peaks by 16 months (on the average) ... peaks in the deviation of money stock from its trend do so by five months ... such absolute peaks as occur in the money stock precede reference cycle peaks by less than five months and may even lag ... peaks in the rate of change of income precede such peaks as occur in the stock of money ... they probably also precede peaks in the deviation of the money stock from its trend ... they probably also follow peaks in the rate of change of money."

In comparing these findings with the patterns of Figures 2 and 4, it is helpful to recall that 16 months is roughly 3/8 and five months roughly 1/8 of a complete cycle. Figure 2 agrees with the empirical summary not only in order of events but also in the lengths of these leads or lags.

<u>Every single piece of observed evidence Friedman reports on timing is consistent with the timing implications of the ultra-Keynesian model, as depicted in Figure 2</u>. This evidence actually contradicts his own "permanent income" theory and lends support to the ultra-Keynesian model.

As the quotation in (c) above indicates, Friedman himself has worried whether the very long lead of M over Y and the reference cycle may not prove altogether too much. It might be a lag instead of a lead. "An inverse relation," he says elsewhere, "with money lagging would be much easier to rationalize in terms of business influencing money than of money influencing business ..."[16]

---

[15]"The Lag in the Effect of Monetary Policy," <u>loc. cit.</u>, p. 456.

[16]"The Lag in the Effect of Monetary Policy," <u>loc. cit.</u>, p. 458.

It is only fair to notice, however, that there are two Friedmans when it comes to describing the causal mechanism from money to money income. One is the Friedman of the permanent income hypothesis, with the implications set forth above. The logic is that the demand for money is quite insensitive to current income, because current income has only a fractional weight in permanent income. This has the virtue of explaining why the monetary multiplier in the cyclical short run is so large and why velocity varies pro-cyclically. But the cost of this explanation, as we have seen, is that it implies an immediate response as well as a powerful response. What is gained from the hypothesis in explaining amplitude is lost in explaining timing.

Friedman recognizes some of the limitations of the permanent income model. He sees that it cannot be applied without modification to quarterly as well as annual data. Since the current quarter of income experience has presumably even less weight in determining permanent income, and thus the demand for money, than the current year of income experience, the money multiplier should be much larger (three to four times as large) on a quarter-to-quarter application of (15) than on a year-to-year application.[17]

---

[17] When the model is formulated in discrete rather than continuous time, equation (14) becomes (here interpreting M, as well as Y, as trend-corrected):

$$\ln Y(t) = \frac{1}{\delta w} \ln M(t) - \frac{(1-w)}{\delta w} \ln M(t-1) - \text{const.}$$

Since w, the weight of current period income, varies inversely with the length of the period, the multiplier of $\ln M(t)$ is larger the shorter the period.

A formulation free of this paradox would relate trend-corrected "permanent money balances" to trend-corrected "permanent income":

Faced by this sort of reductio ad absurdum, Friedman says:

> "In generalizing to a quarterly basis, it will no longer be satisfactory to suppose that actual and desired money balances are always equal. It will be desirable to allow instead for a discrepancy between these two totals, which the holders of balances seek to eliminate at a rate depending on the size of the discrepancy. This will introduce past money balances into the estimated demand equation not only as a proxy for prior permanent incomes [as in (14) and (15)] but also as a determinant of the discrepancies in the process of being corrected."[18]

The second Friedman explains the money-income causal nexus, and the reason it takes some time to operate, in much more conventional and less controversial terms. This description relies heavily on discrepancies of the type just discussed. Excessive money balances, for example, are not immediately absorbed by mammoth spurts of money income. They are gradually worked off--affecting interest rates, prices of financial and physical assets, and eventually investment and consumption spending.[19] This account, though not yet expressed with the precision of the permanent income hypothesis, can doubtless be formulated so as to be consistent with the observed evidence on timing. But at a cost. It cannot attribute to money a large short-run multiplier or explain the pro-cyclical movement of velocity. Indeed it leaves room for interest rates and other variables to affect velocity. Therefore it cannot have those clearcut implications regarding monetary and fiscal policy with which Professor Friedman has so confidently identified himself.

---

[17] cont'd.

$$\ln M_p(t) = \sum_{-\infty}^{t} v(1-v)^{t-\tau} \ln M(\tau) = \delta \sum_{-\infty}^{t} w(1-w)^{t-\tau} \ln Y(\tau) = \delta Y_p(t)$$

$$v \ln M(t) + (1-v) \ln M_p(t-1) = \delta w \ln Y(t) + \delta(1-w) \ln Y_p(t-1)$$

$$\ln Y(t) = \frac{v}{\delta w} \ln M(t) + \frac{w-v}{\delta w} \ln M_p(t-1)$$

Since v/w is presumably independent of the time period chosen, this formulation avoids the reductio ad absurdum. But it also has different implications both for policy and for estimation.

[18] "Money and Business Cycles," loc.cit., p. 59.

[19] Passages describing this mechanism may be found in each of the Friedman articles previously cited.