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OWNERSHIP AND THE PRODUCTION FUNCTION

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by

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1. Introduction

The simple concept of property, implicit in many classical models of a competitive economy, is -- we shall suggest -- an insufficiently basic representation of the phenomenon of ownership. More fundamental is the concept of an individual's operational or strategic control over certain goods or processes as subject to laws (natural or man-made) defining his rights and powers. So long as there are no possibilities of public interaction caused by private use, the simple "chattel" view of ownership may suffice; but in more complex economic situations, such as when the rate of production of A influences (e.g. through a waste by-product) the costs of B, an adequate solution may be impossible unless "legal" constraints are imposed on the individual's strategies, over and above the physical limitations of technology and the environment. The nature of the solution will depend crucially on the nature of these constraints.

We shall elaborate this thesis by considering a series of simple models based on the same technological facts, but incorporating different types of institutional constraints.

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2. A Simple Production Function

A very simple example, which has been used to illustrate increasing, constant, and then decreasing marginal returns from an input, is that of several farm laborers working a single field[1/]. Given the field, as we increase the number of laborers there is an ever increasing rise in the amount of food they produce per man until, at some point, they begin to run short of land, and even with the best of organization the added product due to added labor begins to drop off. It is conceivable that the added product might actually become negative, as when "too many cooks spoil the broth". Figure 1 illustrates the production where s is the number of laborers and $\varphi(s, 1)$ is the production function. The second variable in the production function is for land. However if the amount of land is not being varied we could write the function as $f(s)$.

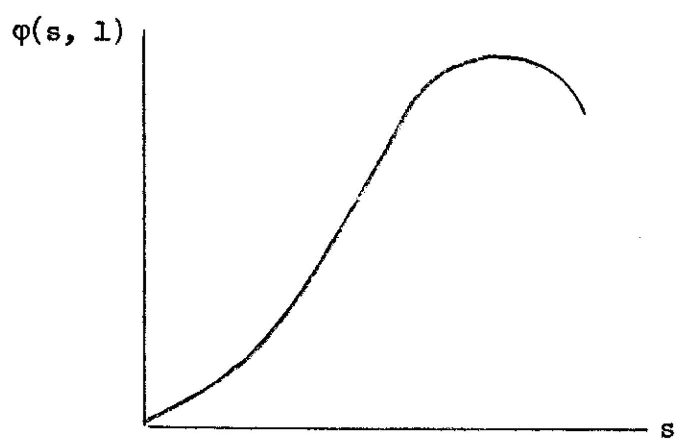

Figure 1

Taking this simple example, with the limited resource land, the variable resource labor and the output food, we can construct several different game models, to reflect different social conditions for the ownership and working of land. All of these schemes have been present in the methods employed by various societies.

In order to develop our examples we make use of the game-theoretic characteristic function, which specifies the best outcome that each subset of the "players" can achieve unaided. This has been defined more fully elsewhere[2]. However, in order to provide the relationship between the characteristic function and the production function we must offer some further specifications. Land, labor, and food we assume to be homogeneous; land and food to be infinitely divisible as well. We also assume that all the laborers have the same linear utility for food--whether they consume the food they acquire or sell it in an outside market is immaterial for our purposes. These assumptions make no conceptual difference to the examples we shall discuss, but makes them considerably simpler to handle.

The characteristic function $V(S)$ will then specify the amount of food that a set S of individuals can obtain by themselves. This function will depend upon the ownership and use conditions for the land and the degree of freedom of action allowed the individuals. The production function $f(s)$ or $\varphi(s, 1)$ on the other hand, specifies the technical optimum that can be obtained by applying s units of labor to the available lands, and has no ownership or strategic implications whatsoever.

Restricting ourselves for the moment to the production function, we must consider the meaning of permitting a negative marginal value of labor.

In work in crowded quarters it is possible for this to happen; to avoid it we must assume that there is a costless method of disposing of any unwanted surplus. This means, in our example, that the master engineer in charge of production can keep unwanted labor off the field at no cost. In a closed economy with fixed technology and fixed nonhuman resources this assumption comes up against the Malthusian fear that the productivity of added labor will not cover the added costs. In our models we shall consider both possibilities.

3. The Feudal System

We consider the economy consisting of n individuals, $n - 1$ of whom are landless peasants with nothing to contribute but their labor, and one who is the lord owning the land.

We must distinguish several cases. In a strict feudal relationship we do not have a true characteristic function, as there are actually no coalition possibilities available to either the lord or his fiefs. Nor do they have strategic choices; they have duties towards each other which will define the division of the total product. It is possible that product might be higher if some of the fiefs could be removed from the domaine. However this may not be feasible, hence, if, as might happen under crowded conditions, the marginal product of the serfs fails to cover their subsistence needs, an increase in numbers represents a loss in productivity to the feudal lord.

4. The Capitalist and Landless Peasants

The relationship that exists between the landlord and landless peasants of a capitalist society gives rise to a true characteristic function with the superadditivity property. Let us regard the landlord as player 1.

$$
\begin{aligned}
V(\theta) &= 0 \\
V(S) &= 0 && \text{if } 1 \text{ is not in } S \\
&= \varphi(s - 1, 1) && \text{if } 1 \text{ is in } S \text{ and } s - 1 < s^* \\
&= \varphi(s^*, 1) && \text{if } 1 \text{ is in } S \text{ and } s - 1 \geq s^*
\end{aligned}
$$

where s^* is the optimum number of individuals required to work the land. If the number of laborers is greater than s^* it is assumed that the landlord has no responsibility for them and that they can be kept off the land.

In the normalization of the characteristic function given here we have implicitly not specified the subsistence level requirements of the individuals as an input or cost to be met. Thus we ascribe a value of $V(S) = 0$ to a coalition of peasants implying that at least in the short run they are able to obtain an alternative employment to cover subsistence. Otherwise if subsistence were k we might require $V(S) = -ks$, where $s = |S|, 1 \notin S$.

In a chronically overpopulated area this assumption of no alternative employment is sufficient even to cover subsistence requirements may be reasonable, and in any attempts to introduce dynamic aspects into the relationship between the workers and landowners, the negative values for some coalitions more accurately reflect the threat potential.

In a "hacienda" dominated agricultural economy, the lack of alternative employment for the laborers has been suggested as an important factor in the stability of the system[3].

We may apply several different concepts of solution to the situation described above and obtain results which are consistent with our intuitions.

Viewed as an open market, as the number of laborers becomes large the marginal value productivity of labor approaches zero, as does its price; the price of the land factor and the rewards to the landlord rise accordingly if we impute returns to satisfy the conditions of a competitive equilibrium.

The core[4] and the value[5] solutions to this game will both show the same limiting behavior as the number of laborers becomes large; the landlord is in a position to obtain all of the gain from the economic activity.

5. A Small Landowner Capitalist Society

Suppose we conceive of an equalitarian society in which all individuals own their own land. As the population grows, say under equal inheritance for all children, we may consider that for a population of size n an individual owns a share of $\frac{1}{n}$ of the land as well as his own labor. The characteristic function for this situation is given by

$$
\begin{aligned}
V(S) &= \varphi(s, s/n) \quad \text{if} \quad s \leq s^* \\
&= \varphi(s^*, s/n) \quad \text{if} \quad s > s^*
\end{aligned}
$$

where $s^* = s^*(n)$, is the optimum number of persons who can be employed to work the amount of land s/n which is available to a coalition S. Figure 2 illustrates the characteristic function for games of this type where the population n first is fewer than, then exceeds, the smallest number needed for the technical optimum.

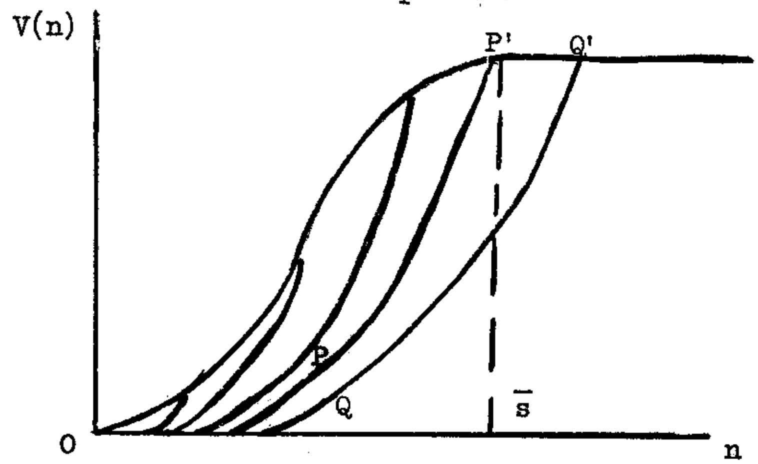

Figure 2

The characteristic function OPP' illustrates the society in which the population $n = \bar{s}$ is just sufficient to obtain a maximum product by joint effort. The shape of OPP' obeys the conditions of superadditivity required by the characteristic function, i.e. $V(S \cup T) \geq V(S) + V(T)$, and as the size of coalitions grows the shape becomes approximately linear, thereby satisfying first degree homogeneity conditions for the production function $\varphi(s, s/n)$ as both inputs are increased.

If the population n is larger than $\bar{s}$ we have OQQ' as the characteristic function. The n together are unable to obtain more than when the population was $\bar{s}$, but there are now more individuals to share the proceeds. Implicit in our assumptions is that even when there are more individuals than $\bar{s}$, these extra people are able to earn subsistence in some other employ, hence from $\bar{s}$ to n the production (but not the characteristic) function is flat. The core, competitive equilibrium and value solutions in this model lead to the same outcome. The price of land relative to labor rises and beyond a certain population size the per capita income decreases.

6. The Village Commune

Rather than being held individually, the land may be held jointly, as in a primitive village, a Eutopia or a Kibbutz. The use of the land may be decided upon by majority vote. In order to fully define such a situation we must specify the obligations of the majority and the powers of the minority. In the extreme case we may assume that the majority exercises absolute control and that once they have decided, the minority is not in a position to obstruct, abstain from ordered work or carry out other threats against the society. The characteristic function for this game is:

$$
\begin{aligned}
V(s) &= 0 && \text{if } s \leq n/2 \\
&= \varphi(s^*, 1) && \text{if } s > n/2
\end{aligned}
$$

where s^* is the optimal amount of labor for the total amount of land. Figure 3 illustrates the characteristic functions for these games for different sizes of n.

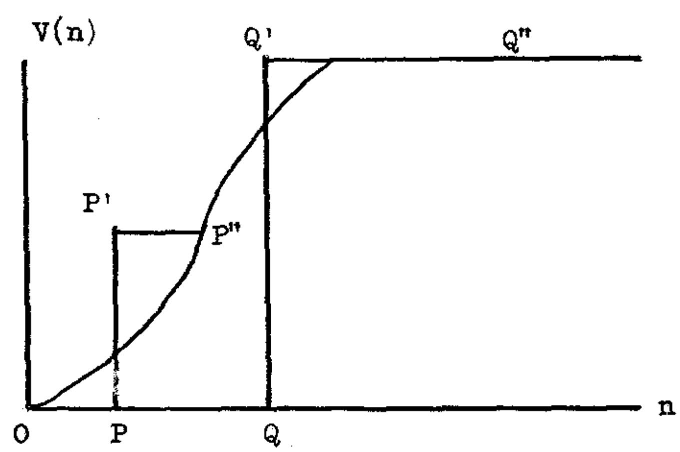

Figure 3

The production function provides us with the locus of the $V(N)$ -- the curved line in the figure. The characteristic function illustrated by $OPP'P''$ is for the simple majority game[6] where the population is still so low that the marginal value productivity of extra population would be relatively high. In the game illustrated by $OQQ'Q''$ this is no longer the case.

In this situation the problem of the imputation of wealth becomes more of a socio-political problem than one that is economic. The political mechanism of the <u>vote</u> is used to decide upon the method of production and distribution of <u>joint</u> product, rather than the economic mechanism of the <u>market</u> to decide upon the combined use of <u>individually owned</u> resources.

From a model of this variety we may observe that an economic interpretation of the political process is that it is a choice mechanism for deciding the individual allocation of products obtained from a jointly owned resource.

When we apply our three solution concepts to this game, we find that the competitive equilibrium is not defined; even if we permitted the sale of votes no equilibrating price would exist. The core of this game is empty. There is no imputation which is not dominated by some group of more than half of the players. The value exists and can be interpreted as providing a price system to determine the sale price of votes[7]; however, this is on the assumption that we have no <u>a priori</u> information concerning the formation of social groups in the society.

The lack of existence of a core implies that there is no imputation of wealth which can be arrived at which is free from social pressure, in the sense that there will always exist some coalition which is effective against any imputation.

The imputation of proceeds actually observed in a situation such as this may best be explained in sociological terms. The von Neumann and Morgenstern "stable set" solution[8/] may be regarded as primarily sociologically oriented, and might be appropriate here.

7. Corporate Ownership

7.1. Model 1

Another possibility is that the land may be jointly owned, but that along with a vote for each individual and his responsibilities to the majority are attached some specific responsibilities of the majority to all individuals. For example, it may be agreed that a majority has control over how the land is to be utilized and that the minority must abide by the decision and cannot hamper the work in any manner; however each individual is to share in the total proceeds in proportion to the number of votes he possesses. Where the voting system is one vote to each individual, this of course calls for the same share for each. The characteristic function for this system is:

$$\begin{aligned} V(S) &= 0 && \text{if } s \leq n/2 \\ &= \frac{s}{n}\,\varphi(s^*, 1) && \text{if } s > n/2 \end{aligned}$$

where, as before, s^* is the optimal amount of labor for the land. Figure 4 shows the various characteristic functions for different sizes of n . Again the production function provides us with the locus

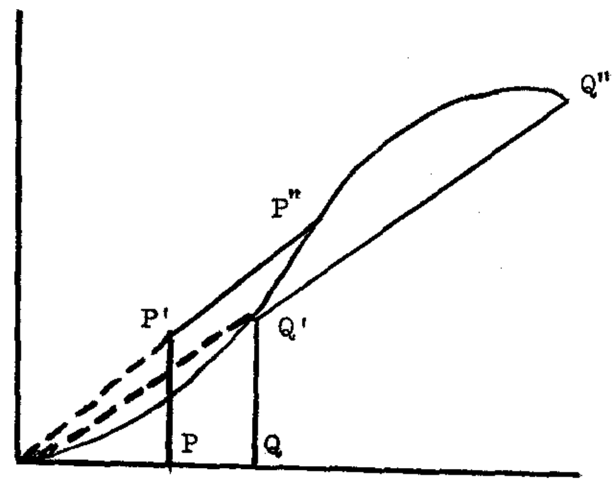

Figure 4

of the V(N). OPP'P" is the characteristic function for an economy with a sufficiently low population that there is still increasing returns to labor. OQQ'Q" is a characteristic function for an economy saturated with labor. The rays P'P" and Q'Q" pass through the origin O . This is imposed by the prorating of returns according to the number of votes held.

When we apply our three different concepts of solution the core and the value exist and as n becomes large the core approaches a single point which is the same as the value. In order to consider the competitive equilibrium we must introduce some extra conditions concerning the possibility of selling votes. In this model, the community vote decides not only on the use of the land but also upon the employment of labor. Not only does the individual not have the power to carry out an active threat, he cannot even refuse to contribute his labor. Thus if we permit a market for votes the sale carries with it the allocation of control both over the land and the individual's labor. If we permit this, then a competitive market price for votes can be established and in this case will yield the same imputation as the value and the limit of the core.

7.2. Model 2

A variant of corporate ownership separates the corporate ownership of land and individual control over labor. We consider that a majority has control over the use of the land, but by law it is constrained to pay a uniform wage to all labor used. Furthermore all individuals can only accept or reject the offer for work at the price named. After wages have been paid to all, the group in control must prorate the remainder to all stockholders in proportion to the number of shares held. The characteristic function for this system is:

$$
\begin{aligned}
V(s) &= 0 && \text{if } s \leq n/2 \\
&= \frac{s}{n}\,\varphi(s, 1) && \text{if } n/2 < s \leq s^* \\
&= \frac{s}{n}\,\varphi(s^*, 1) && \text{if } s^* < s
\end{aligned}
$$

where s^* is the optimal amount of labor for the land and we have assumed $s^* \geq n/2$. The most that a coalition can guarantee for itself is to have an optimum number of its members work the land at price zero. Figure 5 shows the different characteristic functions for various sizes of n.

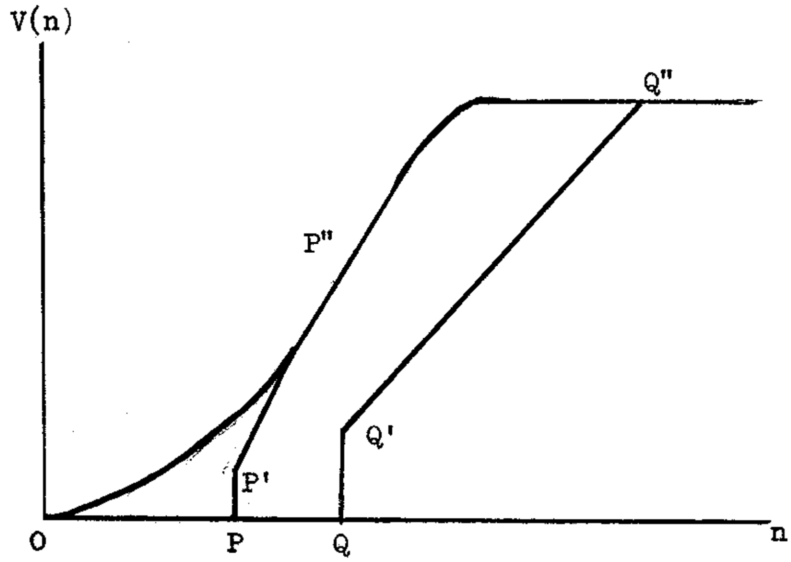

Figure 5

When we apply our three solutions to this game, the core exists and as n grows it approaches the same limit as the value. If we permit the selling of votes then a price system will exist both for the votes and for labor. These latter models are related to the models used by Arrow and Debreu[9] and others to handle the existence of joint stock companies in a competitive economy. The strategic freedom of the individuals in these joint ownership situations is limited in such a manner by the "rules of the game" -- i.e., the legal system -- that even in the mixed voting-economic game there will exist at least one undominated imputation.

8. Threats and Joint Product and Ownership

In the previous sections we have provided examples with the same economic background but different legal and socio-political structures of ownership. Critically important to all of this is the threat potential of the individual; and this is explicitly or implicitly included in the legal structure. It is easy to see that we could construct many other variants of the games we have considered by allowing minority groups to obstruct the production plans of the rest.

Tied in closely with the problem of threats is that of joint product, external economies and diseconomies. These are all part of the same phenomenon where the activities of individuals who are not members of a coalition can strategically affect the payoff to those who are. For example, in chronic overpopulation the presence of the extra population, if they have any strategic choice whatsoever, affects the payoffs to any coalition. In the classical cases of external economies and diseconomies, for instance where the production of A influences the costs of B, then the payoff to a coalition consisting of A varies continuously with the threats of B. This is not true in classical economic trading games and in all the examples given here. There appear to be at least two properties necessary for the establishment of a price system, they are the absence of variable threats and the presence of a core. The design of price system economies must involve the introducting of rules or laws which limit threat possibilities and ensure the presence of a core.

FOOTNOTES

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