COWLES FOUNDATION FOR RESEARCH IN ECONOMICS
AT YALE UNIVERSITY

Box 2125, Yale Station
New Haven, Connecticut

COWLES FOUNDATION DISCUSSION PAPER NO. 105

Some Experimental Non-Zero Sum Games With Lack of Information About the Rules

Martin Shubik

January 9, 1961

Some Experimental Non-Zero Sum Games With Lack of Information About the Rules *

Martin Shubik

1. The Formalization of a Game

Two disciplines, Game Theory and Experimental Gaming have grown up in the past few years. Unfortunately a confusion exists which often causes the misinterpretation of results and constructs of the (relatively mathematical) methodology of Game Theory with the results of Experimental Gaming. Game Theory has given great impetus to work in Experimental Gaming, but often the most interesting games do not fit comfortably into the game theoretic models unless care is taken to make explicit modifications to the assumptions leading to the definition of a game in the sense of von Neumann and Morgenstern.

In this paper, the simplest type of games are treated and the modifications of the von Neumann and Morgenstern assumptions are noted when the games are used for experimental purposes.

There are two types of representation of a game employed by von Neumann and Morgenstern. They are respectively known as the <u>extensive form</u> and the <u>normalized form</u>. The first can be displayed by means of a <u>game tree</u> and the second by means of a <u>payoff matrix</u>. A simple example serves to indicate the difference. Consider the very simple parlour game where each of two players has only one move. They must make this move simultaneously without any knowledge of each other's action. Each must

---

*Research undertaken by the Cowles Commission for Research in Economics under Task NR - 047-006 with the Office of Naval Research.

select either a red or a black chip. If both players select the same color the first player wins $1, if they select different colors, the second player wins $1. In each instance the losing player must pay the $1 to his competitor.

The payoff matrices to players 1 and 2 are given below in their familiar form.

Payoff to Player 1

	Red	Black
Red	$1	- $1
Black	- $1	$1

Fig. 1

Payoff to Player 2

	Red	Black
Red	- $1	$1
Black	$1	- $1

Fig. 2

Equivalently the two matrices can be combined into one as is indicated below. The first figure in each cell is the payoff to the first player and the second figure is the payoff to the second player.

	Red	Black
Red	$1, - $1	- $1, $1
Black	- $1, $1	$1, - $1

Fig. 3

The second type of representation of this game can take two forms which are equivalent.

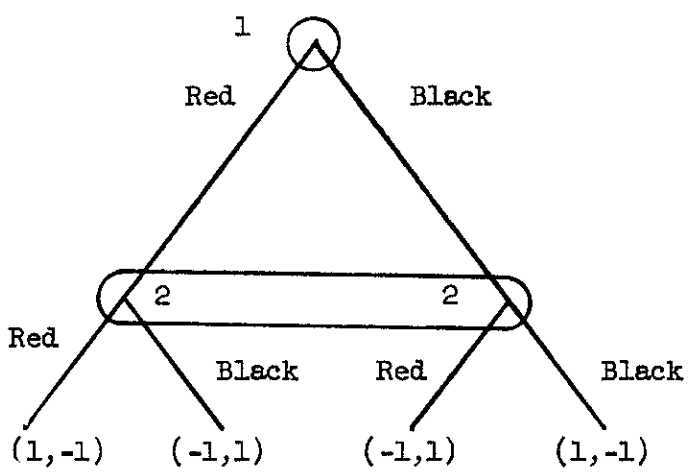

Fig. 4

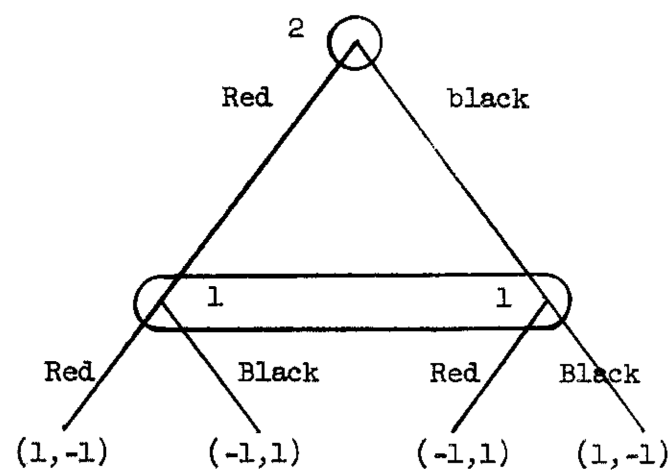

Fig. 5

In Figures 4 and 5, the vertices in the game trees represent the points at which a player must make a choice. The number next to each vertex indicates which player must make the choice. Thus in Figure 4 the topmost vertex is a choice point for the first player and the other two are choice points for the second, while in Figure 5 it is vice versa. There are four terminal vertices at the end of which are pairs of numbers which represent the payoffs to the players. The first number in each case represents the payoff to the first player and the second the payoff to the second player. Each branch in the game tree represents an alternative. In this case, red or black. In Figure 4 two of the choice points of Player 2 are encircled together and the one choice point of Player 1 has a circle around it. In Figure 5 the two choice points of Player 1 are encircled together and the one choice point of Player 2 is encircled. These enclosures represent <u>information</u> <u>sets</u>. If a set of choice points are encircled together this signifies that although the reader acting as <u>Deus</u> <u>ex</u> <u>Machina</u> or the referee can distinguish between them, the player confronted with a choice is unable to do so. The game portrayed in Figure 4 has Player 1 move first, then Player 2 moves <u>in</u> <u>ignorance</u> of what Player 1 has done. If this is so then the game is equivalent to the game in which they both move simultaneously. The game portrayed in Figure 5 has Player 2 move first, after which Player 1 makes his move <u>in</u> <u>ignorance</u> of what Player 2 has done, hence this game is also equivalent to the game in which they move simultaneously.

The games portrayed in Figures 6 and 7 are very different games. In the first case the advantage is all to the second player and in the second

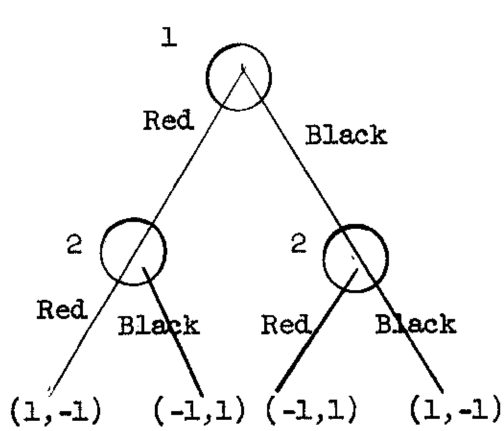

Fig. 6

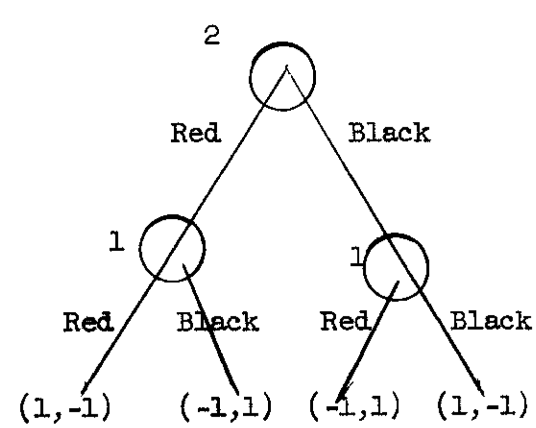

Fig. 7

case, the advantage is all to the first player. The information conditions have been changed as is exhibited in Figure 6 by the separate encirclements of the choice points for Player 2. He can now distinguish between them. This means that he knows what Player 1 has done prior to making his own move. This obviously gives him a complete advantage in this simple game. If the first player selects red, the second chooses black and wins. If the first player selects black, the second chooses red and wins. The same analysis with the roles of the players interchanged holds for the game illustrated in Figure 7. They are by no means equivalent games. The change in information conditions has had a considerable effect. The normalized forms of the two games illustrated in Figures 6 and 7 will be respectively a 2x4 matrix and a 4x2 matrix. The reader who wishes to verify this is referred to the excellent exposition by Luce and Raiffa 1/.

The type of game defined and dealt with by von Neumann and Morgenstern is a game of finite length. All the rules are fully defined to all players. All the payoffs are known to all players; and the worth of every payoff to each player is known. Furthermore the payoff comes at the end of the game.

Within the rules of the game is a complete description of what constitutes a possible choice under all circumstances. Furthermore, the state of information available to the players at any point during the game is implicitly contained within the rules. For example in chess the rules are such that at any point during the game both players are completely informed as to the disposition of all their pieces. Chess is a game with perfect information available at every move. This does not hold true for Poker. The players are not perfectly informed, they do not know the cards their competitors hold. Nevertheless the players in Poker are assumed to be totally cognizant of the rules.

One of the major confusions prevalent concerning Game Theory is the confusion caused by a failure to distinguish between a lack of knowledge of the rules of the game and the lack of knowledge of a player in a game where the rules specify that moves may have to be made without perfect information. In the first instance it would be as though one were to play Poker without knowing what defined a winning hand. In the second instance, the game of Poker where all players know the rules serves to provide an example.

In most instances of interest to the behavioral scientist a valid model of a social process is one in which the players are ignorant of the rules of the game. Thus the von Neumann and Morgenstern theories (zero-sum and non-zero sum) of how individuals should act are not usually germaine. Modifications must be made if a theory of behavior in these pseudo-games or games with lack of information about the rules, is to be developed. Much (although by no means all) of the confusion and misunderstanding in the abortive attempts to apply the concepts of the Theory of Games to political bargaining and threat phenomena [2/] springs from the misunderstanding of information conditions and the meaning of the rules of a game.

Most life processes have an uncertain point of termination. Even the inveterate gambler seated at Las Vegas playing roulette does not know when he will go broke or when he will die. With any luck the second stochastic process will get him before the first one does. Theoretically he has a chance to keep going forever. At least at any day when he is still alive and still in the chips there is usually a fair sized chance that he will be still alive and still in the chips on the morrow. It is precisely the possibility that at any point in an infinite game there is a possibility that there will be a tomorrow that makes it the natural vehicle for the study of threats and counter-threats. The players in a game without a calculable finite termination must always take into account the possibility that they will have to live with each other on the morrow. In life there are rarely once and for all settlings of debts; in finite game models there are. This is possibly why certain games of finite length provide a "mathematician's delight" in the form of many paradoxical solutions in the light of human behavior.

The simplest type of game is the one represented by a 2x2 matrix as is shown in Figures 1, 2 or 3. For experimental purposes it is convenient to have subjects play this type of game several times. In doing so it is important to remember that in fact the players are now playing in a "super-game" in which the individual games are sequentially arranged segments of the overall situation. Suppose we played the game illustrated in Figure 4 two times instead of once. The game tree for this new game is shown in Figure 8. It is still a finite game tree, but is much larger than before. In spite of the increase in size it is a well-defined game in the von Neumann and Morgenstern sense.

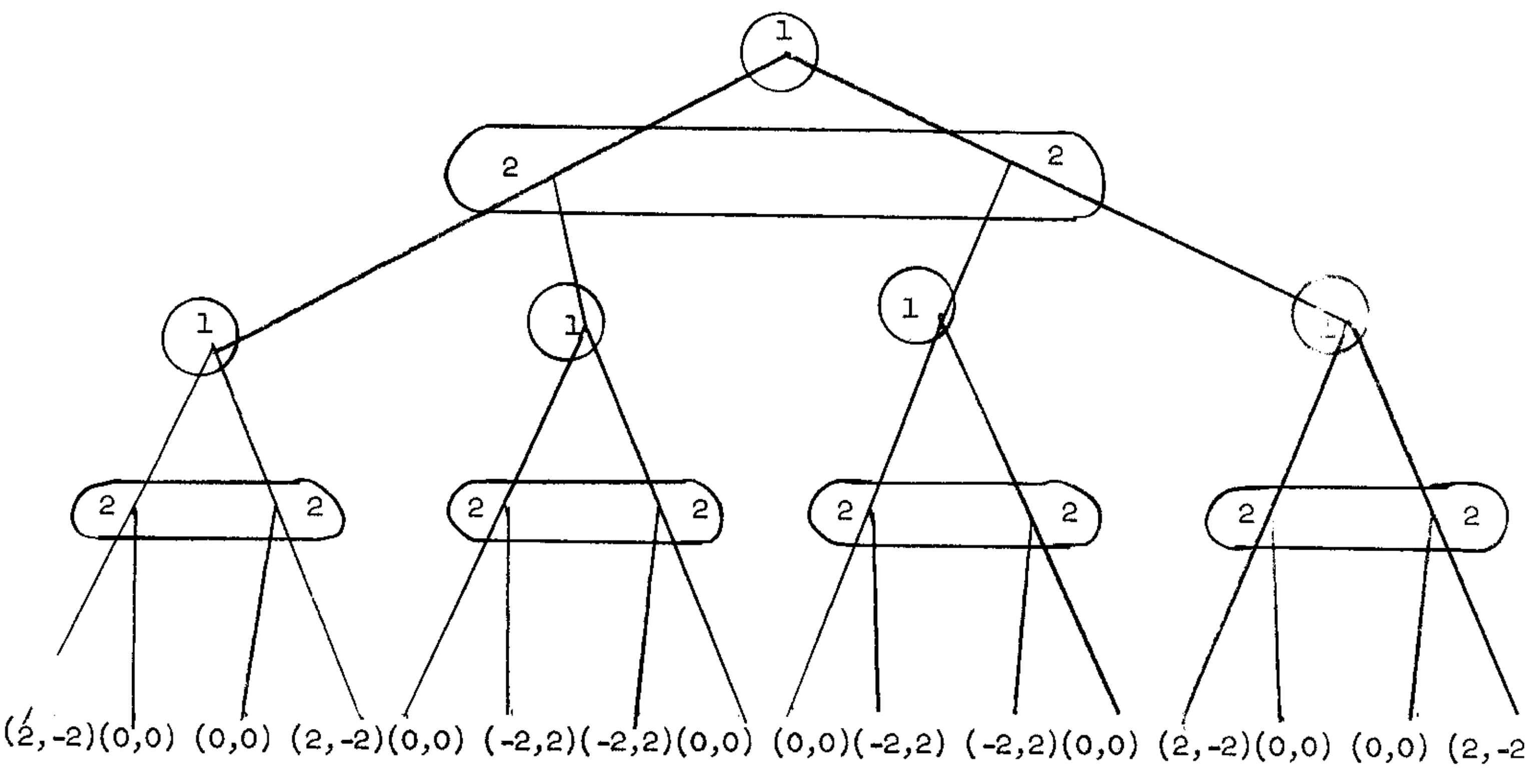

Fig. 8

If, instead of telling the players that they are to play a simple game say, 5 or n times, where n is a given fixed number the players are instructed that they will play many times but that their last play will be determined randomly, then this is no longer a game in the Von Neumann and Morgenstern sense. If the players are told the distribution of probabilities with which the game will be stopped on any trial, then it is a well-defined dynamic, stochastic or infinite game (the name for this type of game has still not been completely standardized). There exists a body of literature on this type of game 3/.

If the players are not even told how the last play is to be determined, they are then forced to supply their own assumptions concerning the termination of play. This is another form of lack of knowledge concerning the rules of the game. Attempts have been made to cope with situations where the players do not know the probabilities of certain events which are needed to well-define the game. The writings on Games Against Nature [4/] deal with this. In dynamic situations consideration must be given to learning about the rules when the players are not completely informed initially. This has given rise to investigations of game learning [5/].

2. Theories of Solutions to a Game

Section 1 dealt with the formalization of the description of a game. An economic analogy would be the formalization of the description of a market structure. Nothing was suggested about how the game will be or should be played. A theory for the solution to a game (as defined by von Neumann and Morgenstern or in a modified version) is either a normative theory or is based upon description of human behavior. The work in experimental gaming is a step in an attempt to validate theories which purport to be descriptive, or to produce descriptive theories.

Restricting the discussion to games of finite length with the rules known by all of the players, several different concepts of solution are noted. For ease we limit ourselves to two-person games. Let $P_1(s_1, s_2)$ stand for the payoff to the first player if he employs his strategy s_1 and the second player employs his strategy s_2; and similarly $P_2(s_1, s_2)$. At this point, the distinction must be made between <u>strictly competitive</u> and not strictly competitive games. The game displayed in Figure 3 is strictly competitive. An increase in the welfare of one player implies a decrease in the welfare of the other. In this instance we observe that:

$$P_1(s_1,s_2) = -P_2(s_1,s_2).$$

This relation does not hold for the game illustrated in Figure 9. Here there is room for cooperation which will yield rewards to both. It can be observed that this is a non-constant sum game; i.e., the sum of the payoffs to both players in different outcomes is not constant. In the first game the sum is constant.

	1	2
1	5, 2	-10, -13
2	4, 1	-20, -23

Fig. 9

von Neumann and Morgenstern suggest that in a non-constant sum game the players should jointly maximize and then work out some arbitrated division of the proceeds between them. This presupposes that they are in a position to communicate with each other and are also in a position to make side-payments. They do not explicitly include the bargaining and haggling over side-payments as part of their description of the play of the game, but as something which takes place outside of it. The description of the behavior of the players in the game is given mathematically by the condition:

$$[1] \qquad \underset{s_1}{\text{Max.}}\ \underset{s_2}{\text{Max.}}\ (P_1(s_1,s_2) + P_2(s_1,s_2)).$$

This merely states that each player should select his strategy in such a manner that the sum of their payoffs is maximized. Applied to the game in Figure 9 this calls for each player to select his strategy 1 .

John Nash has suggested a theory of non-cooperative play 6/ which is a generalization of economic theories of equilibrium. His theory applies to situations where communication is limited between the players and they are not in a position to make side-payments. Nash shows that in any finite game which can be described by a set of payoff matrices (such as 1 and 2 , or using a more compact notation, 3) there will exist at least one pair of strategies s_1^* and s_2^* such that the two conditions

$$[2] \qquad \underset{s_1}{\text{Max}}\ P_1(s_1,s_2^*)$$

$$\text{and} \qquad \underset{s_2}{\text{Max}}\ P_2(s_1^*,s_2)$$

are simultaneously satisfied by the choice of s_1^* and s_2^* by the first and second players respectively. In words, if the first player believes that the second player will utilize s_2^* against him, his optimal reply (in the sense that it will maximize his own payoff) is s_1^* and vice versa.

It is possible that both players may have a very pessimistic view of the world and strive to play in a manner that minimizes the worst that can happen. In other words, on the assumption that the competitor is hostile each may assume that the other is going to minimize his competitor's payoff and each will strive to maximize his payoff on that assumption. This can be expressed as:

$$[3] \qquad \underset{s_1}{\text{Max}}\,\underset{s_2}{\text{Min}}\; P_1(s_1,s_2)$$

$$\text{and} \qquad \underset{s_2}{\text{Max}}\,\underset{s_1}{\text{Min}}\; P_2(s_1,s_2).$$

Another possibility is that the competitors adopt the attitude that it is more important to maximize the difference in gain between them than it is to maximize individual gain. This is the type of thinking prevalent in tactical calculations of damage exchange rates. This is expressed as:

$$[4] \qquad \underset{s_1}{\text{Max}}\,\underset{s_2}{\text{Min}}\; (P_1(s_1,s_2) - P_2(s_1,s_2)).$$

3. Some Experimental Games and Their Theoretical Solutions

The following six games were played by five pairs of students (Yale seniors in a class on Industrial Organization) in order to illustrate the interrelationship between behavior and structure in a market, as well as to gather data on their behavior.

Game 1

	1	2
1	6, 3	6, 7
2	10, 3	10, 7

Fig. 10

Game 2

	1	2
1	1, 3	2, 3
2	1, 1	2, 1

Fig. 11

Game 3

	1	2
1	2, 1	-1, -1
2	-1, -1	1, 2

Fig. 12

Game 4

	1	2
1	3, 3	-1, -1
2	-1 , -1	2, 2

Fig. 13

Game 5

	1	2
1	3, 3	-2, 7
2	7, -2	-1, -1

Fig. 14

Game 6

	1	2
1	5, 2	-10, -13
2	4, 1	-20, -23

Fig. 15

In Figure 16 all four solution concepts given in Section 2 have been applied to the six games given above and the resultant strategy pairs which are the solutions are noted. For example the expression (1,1) stands for the strategy pair where each player selects his first strategy, i.e., $s_1 = 1$ and $s_2 = 1$. This gives the pair of payoffs in the upper left-hand corner of the payoff matrix.

	Solution [1]	Solution [2]	Solution [3]	Solution [4]
Game 1	(2,2)	(2,2)	(2,2)	(2,2)
Game 2	(1,2)	(1,1) (1,2) (2,1) (2,2)	(1,1) (1,2) (2,1) (2,2)	(2,1)
Game 3	(1,1) or (2,2)	(1,1) or (2,2)	* {2/5,3/5} and {3/5,2/5}	(1,2)
Game 4	(1,1)	(1,1) or (2,2)	* {3/7,4/7} and {3/7,4/7}	(1,1) (1,2) (2,1) (2,2)
Game 5	(1,1)	(2,2)	(2,2)	(2,2)
Game 6	(1,1)	(1,1)	(1,1)	(1,1) (1,2) (2,1) (2,2)

Figure 16

* These both involve mixed strategies. The probabilities employed by each player are indicated in the curled brackets.

All four solution concepts when applied to Game 1 yield the same solution pair (2,2). A closer examination of the game shows the structural reason why this is so. The players are strategically independent. Their fates are not interlinked. This game illustrates the atomistic isolation between any two competitors in a purely competitive market. Regardless of their intentions, the structure is such that their behavior is the same in all instances. A non-constant sum game in which the fates of the players are not interlinked is referred to as an <u>inessential game</u>. It is inessential in the same way as is a strictly competitive game. There is nothing to be gained by discussion, negotiation or collusion. In essence, collusion has no meaning in this context.

In Games 3, 4 and 6 the jointly maximal and the non-cooperative solutions coincide illustrating the inherent aspects of implicit collusion built into the structures of these games. In Game 4 the strategy pair (1,1) is the only non-cooperative solution in the strict sense 7/.

The solution concepts may be regarded as <u>operators</u> which act upon the payoff functions (which reflect structure) to produce <u>behavior</u>. As is illustrated here, it is possible for many different operators to produce the same behavior, depending upon the structure. The operator embodies the intent of the player.

4. A Hypothesis and the Conditions of the Experiment

Indications from other work [8] suggest that the non-cooperative concept of solution will serve as the best predictor of modal behavior of the four suggested here, in a pseudo-game situation, i.e., not a well-defined game in the von Neumann and Morgenstern sense, but one with modifications. These modifications are that the games be played under conditions of incomplete information concerning the rules. These were manifested in two ways.

The players were only informed of their own payoff functions. They did not know the payoff functions of their competitors. Partners were chosen randomly and no communication was permitted except via a monitor who transmitted the information concerning the choice of each player after every play of the subgame.

The players were initially not informed about the number of plays of the subgames. Nor were they given any probability distribution for the ending. The trial lengths were selected prior to the runs and were announced two to three trials before termination. The students were given approximately one minute per trial.

The solutions illustrated in Section 3 were for the single-period subgame. These games when played for a finite number of trials give rise to a supergame which is said to have perfect recall [9]. It has been shown that for such games it is legitimate to solve them by solving the series of subgames individually [10], thus for one of the games presented in Section 3, if it were played n times under conditions of complete information concerning the rules, the various solution concepts would call for a strategy pair in each subgame consistent with the strategy pairs predicted for a single play of the original 2x2 matrix game.

For the pseudo-games used in this experiment there exist no mathematically developed theories of solution, unless we wish to postulate that a theory of solution to a well-defined finite or infinite length game will apply them. By itself such a theory should not be enough as obviously learning must be taking place. The lengths of play were selected with consideration given to the difficulties in learning and the need for the players to learn in order for them to perform optimally in a game. This is discussed in Section 5 and the Appendix.

The data used to test the hypothesis that the non-cooperative solution theory is the best predictor among the four theories were the last five trials of each game.

The hypothesis is not fully defined until meaning is given to the word "best". Every solution concept will predict the mode correctly, have no resolution power or be incorrect. On the basis that: yes > ? > no, solutions can be ranked for all games. A vector domination can then be used for comparing solutions applied to several games. This is a rather weak condition; but is used here. For some purposes (especially if there are high stakes in the game) a loss function measure is needed.

The players were instructed to maximize their individual scores. As there was no monetary incentive involved, doubts can be raised as to the effectiveness of this instruction. In one instance in a final play, one subject played in a manner to inflict a loss on his competitor. Upon being questioned after he indicated that as it was the last trial of the last game he thought that he would teach his competitor a lesson. Beyond that, however, no end of the game pathologies were observed. The time series for the last five trials of all pairs in the fifth game are given in the Appendix.

Figure 17 gives the results for the last five trials. The frequencies encircled must be rejected for reasons indicated in the comments of the players given in the Appendix.

	Strategy Pair	Pair 1	Pair 2	Pair 3	Pair 4	Pair 5	Prediction of Solution [1]	[2]	[3]	[4]
Game 1	(1,1)	-	-	-	-	-				
	(1,2)	-	-	-	-	-				
	(2,1)	-	-	-	-	-				
	(2,2)	5	5	5	5	5	x	x	x	x
Game 2	(1,1)	-	-	-	-	-		x	x	
	(1,2)	-	-	-	1	5	x	x	x	
	(2,1)	2	-	3	2	-		x	x	x
	(2,2)	3	5	2	2	-		x	x	
Game 3	(1,1)	-	-	-	5	5	x	x	6/25	
	(1,2)	-	-	-	-	-			4/25	x
	(2,1)	-	-	-	-	-			9/25	
	(2,2)	5	5	5	-	-	x	x	6/25	
Game 4	(1,1)	5	5	5	5	3	x	x	9/49	x
	(1,2)	-	-	-	-	1			12/49	x
	(2,1)	-	-	-	-	1			12/49	x
	(2,2)	-	-	-	-	-			16/49	x
Game 5	(1,1)	-	-	-	-	-	x			
	(1,2)	-	-	-	-	-				
	(2,1)	3	1	1	-	1				
	(2,2)	2	4	4	5	4		x	xx	x
Game 6	(1,1)	5	4	5	5	5	x	x	x	x
	(1,2)	-	-	-	-	-				x
	(2,1)	-	1	-	-	-				x
	(2,2)	-	-	-	-	-				x

Figure 17

Examining Figure 16 we observe that all four theories are of equal power on Game 1 and are in complete agreement with the results. Given the nature of the game this is to be expected if the players behave rationally; hence the result can be regarded as a check of the rationality of the players.

In game 2 solutions 1 and 4 must be rejected, while solutions 2 and 3 have no power of resolution and hence are at least not inconsistent with the data. In this game there appears to be an attraction to the lower right hand corner of the matrices which could be due to geometric considerations. It would be desirable to rerun this game with the rows or columns interchanged.

In game 3, solutions 1 or 2 account for all the data. Solutions 3 and 4 must be rejected.

In game 4, solutions 1 and 2 are consistent with the data of four pairs (the data from pair #1 had to be rejected). The data from pair #5 shows some variability. The interview material in the Appendix indicates that the first player in this pair was attempting to damage his opponent even at cost to himself. This stresses the desirability of adequate incentives. Solutions 3 must be rejected and 4 has no resolution power.

In game 5 the players indicated that they had difficulty in learning. Solution 1 must be rejected; all the other solutions are acceptable.

In game 6 solutions 1, 2 and 3 are in complete agreement with the data; solution 4 has no power of resolution.

We can see that for at least one game, in the strict sense of the testing of the hypotheses that any of these theories of solution is consistent with the data, all must be rejected as frequencies appear when none are predicted. A learning (and teaching) modification needs to be introduced. Section 5 discusses the perceptions of the players for their opponents' payoff matrices; this gives some indication of the state of learning and teaching by the end of the game. For the weaker hypothesis concerning the prediction of mode, the table below presents the data analysis. A " ? " indicates that the solution has no resolution power.

Games	Solutions 1	2	3	4
1	Yes	Yes	Yes	Yes
2	No	?	?	No
3	Yes	Yes	?	No
4	Yes	Yes	?	?
5	No	Yes	Yes	Yes
6	Yes	Yes	Yes	?

5. Some Aspects of Learning

At the end of the games, the subjects were required to rank the entries in their opponents' matrices for each game. The table for the perceived matrices in Game 1 is given below. The entries are ranked $a > b > c > d$. The letter in the upper left hand corner is the value in the actual matrix. The five entries below are the perceived entries.

Game 1

Matrix of 1 perceived by 2

b b,c,b,b,b	b b,c,b,b,b
a a,b,a,a,a	a a,a,a,a,a

Matrix of 2 perceived by 1

b b,b,b,b,b	a b,a,a,b,a
b a,b,b,a,b	a a,a,a,a,a

We note that the players' perceptions of each others' matrices were accurate in eight out of ten cases. In the two instances in which the first player attributed an incorrect structure to his opponent's matrix (pairs 1 and 4), the misperception would make no difference to non-cooperative behavior. The players had nothing to gain by correcting their misperceptions.

Game 2

Matrix of 1 perceived by 2

b b,d,b,b,d	a b,a,c,c,a
b a,c,d,d,c	a a,b,a,a,b

Matrix of 2 perceived by 1

a b,b,b,d,b	a a,b,b,a,a
b b,a,a,c,c	b a,a,a,b,b

In Game 2 in only two instances out of ten the structure of the competitor's matrix was "guessed" by a player (the second players in pairs 2 and 5). The game was designed so that no information concerning an individual's payoffs was provided by knowing his move.

Game 3

Matrix of 1 perceived by 2

a / b,a,a,b,b	c / c,c,c,b,a
c / c,c,c,c,d	b / a,b,b,a,c

Matrix of 2 perceived by 1

b / b,a,b,c,a	c / b,c,c,d,b
c / a,c,c,b,c	a / a,b,a,a,b

In Game 3 three players correctly perceived their opponents' matrix. Four others misperceived the structure in a manner peculiarly in their favor! This game was selected to have two equilibrium points, one of which favored one player and the other, the other player. The misperceptions of four of the players were based on the belief that the strategy pair which resulted in an equilibrium favoring them also contained the most desirable outcome for their competitors.

Game 4

Matrix of 1 perceived by 2

a / a,a,b,a,b	c / c,c,c,-,d
c / c,c,c,-,a	b / b,b,a,-,c

Matrix of 2 perceived by 1

a / a,a,b,a,a	c / -,c,a,c,b
c / -,c,c,b,d	b / -,b,c,d,c

In Game 4, seven players correctly surmised that the upper left entry was the best for both themselves and their competitors. In two instances the players made no further exploration and the information concerning one entry in their opponent's matrix was sufficient for playing, no other estimate of the structure was given.

Game 5

Matrix of 1 perceived by 2

b / -,c,b,d,b	d / -,c,c,b,d
a / -,b,a,a,a	c / -,a,d,c,c

Matrix of 2 perceived by 1

b / b,b,a,c,c	a / a,a,c,a,b
d / a,d,d,d,d	c / a,c,b,b,a

Game 5 was the classical "Prisoner's Dilemma" game. The sole equilibrium point is jointly minimal. Two players, (one from pairs 2 and 3) had an accurate perception of the competitor's matrix; six were able to locate the opponent's maximum, and five, the minimum.

Game 6

Matrix of 1 perceived by 2

a / a,a,b,a,a	c / c,b,a,-,a
b / b,d,c,-,b	d / d,c,d,-,b

Matrix of 2 perceived by 1

a / a,a,b,a,a	c / -,d,d,d,c
b / -,b,c,d,b	d / -,d,d,d,c

In the sixth game, eight players out of ten correctly anticipated that the optimum for their competitor coincided with their own. A ninth player (pair 3, the second player) predicted a matrix whose structure would result in the same behavior in spite of his misperception.

As can be seen from some of the comments (given in the Appendix) there is not only an individual learning process, but a signalling process taking place as well. These signals are closely connected to the as yet rather unsatisfactory concept of threat strategy.

6. Conclusions

There are great difficulties to be faced in attempting to use experimental games in vitro to learn about human processes in vivo. It is the belief of this writer that there may be many types of learning which are worth distinguishing from each other. The environment of the simple experimental game probably destroys many of the factors contributing to the acceptance of stable standards of behavior in societies or even in industries and trade associations. Nevertheless the games appear to provide a promising tool to help to separate out variables in the study of bargaining, threats and other aspects of competition and cooperation.

In particular the relationship among theories of games, learning and gaming need further clarification. Many of the proponents and opponents both of the theory of games and of gaming have failed to appreciate the need for extreme care in interpreting and relating the axioms behind the various theories of games to the experimental conditions.

- - - - - -

APPENDIX

I . Formats and Time Series

The games were played for 10, 12, 15, 17, 22 and 10 trials each. Games 1 and 6 require the least learning; game 5 the most. The following two tables are the actual formats used for game 5. In order to save space, the sheet for Player 1 has been truncated and the time series for the last five moves of the five sets of players have been entered in the table for this game. The untampered format is given for Player 2.

Player 1

GAME 5

My CHOICE	1	2
1	3	-2
2	7	-1

TRIAL NUMBER	MY MOVE					HIS MOVE					MY.PAYOFF
1	2	1	1	2	1	1	2	2	2	2	
2	2	2	2	2	2	1	2	2	2	2	
3	2	2	2	2	2	1	2	2	2	2	
4	2	2	2	2	2	2	2	2	2	2	
5	2	2	2	2	2	2	2	2	2	2	

Player 2

GAME 5

My CHOICE

	1	2
1	3	7
2	-2	-1

TRIAL NUMBER	MY MOVE	HIS MOVE	MY PAYOFF
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			
13			
14			
15			
16			
17			
18			
19			
20			
21			
22			
23			
24			
25			

II. Comments of the Players

The players were required to comment on all games after they had played all six. These comments are given below. The comments marked with an * occasionally make life for the experimenter an unrewarding experience.

Pair 1, Player 1

Game 1: I started out hoping for the best and it turned out all right.

Game 2: I hoped I could get to the intent of 1 . I felt the opposition had a rather simple matrix, with no great feeling between one or the other. I decided to stick with strategy 2 and let him settle on something.

Game 3: Seeing my opponent's dislike of strategy 1 , I shifted to strategy 2 to take a steady profit of 1 .

*Game 4: The proctor told me to stick with 1 , so I did and the game turned out very well indeed.

Game 5: Not quite able to figure this out. Stuck with two after unsuccessful attempts to learn something. Sought to minimize loss but wound up slightly ahead.

Game 6: Stuck with the best choice 1 and it turned out as I had hoped -- No thought on opponent's strategy except that I must have been best.

Pair 1, Player 2

Game 1: My opponent's strategy could also have a reason other than the matrix I gave as my guess: a matrix which looks like the one of game 6. Analogous reasoning for game 6.

Game 2: This game seems to be biased in favor of player one.

Game 3: His strategy 2 seems to offer the same as my strategy 2: possible losses less than possible gains.

Game 4: We were in agreement: no need to change strategy, our best point seemed to be 1/1 for both of us.

Game 5: 2/2 is presumably his best point.

Pair 2, Player 1

Game 1: Both maximize at 2 -- no problem-- his best move is 2 as is mine.

Game 2: Again we both maximize at 2 -- move indifferent.

Game 3: We both maximize on 2 although my best move is 1 .

Game 4: Easy -- both same matrix and we do the same.

Game 5: Very difficult -- my best move is 1 , his best is 2 ; we minimize our losses at 2 each.

*Game 6: Easy -- same matrix, same choice, + 1 each; I screwed him on the last move to make him lose much, me little.

Pair 2, Player 2

Game 1: In agreement from the start -- I played my best and he played his.

Game 2: He played one presumably because it was his best but he had two: one in mind; when I played 1 he switched to 2 because he felt it was his best in that situation. He then found me switching and played 2:2 as better than 1:2.

Game 3: We seemed to jibe pretty well after the start. I think his payoffs were the reverse of mine.

Game 4: Hit it off at the start. 1:1 was his best or he would have switched at least once.

Game 5: Thoroughly confused.

Game 6: Hit it off at the start

Pair 3, Player 1

Game 1: 2 was my best move and 2 was also his best move --result immediate accord.

Game 2: We soon settled on 2 and 2 as best for us both, but he displayed a touch of irrationality (or just "playing around") at the end.

Game 3: 1 my best -- he displayed irrationality again (it seems) after staying on 1 for 3 turns, he suddenly switched to 2 and wouldn't be budged -- strange.

Game 4: We soon settled on a saddle point with 1.1 -- no trouble.

Game 5: He wouldn't budge from 2 (no matter how hard I tried to "force" him) so I had to content myself with minimizing my losses.

Game 6: We hit the saddle on the first try and never deviated.

Pair 3, Player 2

Game 1: Can't lose if I move 2, win same regardless of opponent's move, opponent can't lose if play 2,win same regardless of opponent's move. Me - 2 always, opponent - 2 always.

Game 2: Game beyond me. His safest 2, his best 2, 2.

Game 3: He wanted me to play 1 for 1, his 1 together is his best, second best is 2,2. I psyched him,although losing for a while to take second best, in end he came to me where 2,2 was my best.

Game 4: 1+ (on assumption either losses on any other combination would be same or proportional) was a matter on who could convince other to play their way. I won.

Game 5: Convinced him I'd never play 1. He took third best, me 2, him 1. For long run game both lost by not compromising.

Game 6: You both win a great deal by playing 1 no matter what opponent does. Both lose by playing 2 no matter what opponent does.

Pair 4, Player 1

Game 1: Choice 2 was equally profitable whether he chose 1 or 2. I stayed at 2 and maximized my return.

Game 2: My object was to try to elicit a response of 2 from opponent. He bounced equally with mine alternating 1,2. Seeing this I reversed his pattern getting a response of 2,2 on the last two moves.

Game 3: The choice 1,1 was my most profitable and 1,2 was my least loss. I stayed on 1 most of the time. He moved around trying to get me to change. I did move but settled down. It appeared that he settled down to minimize losses.

Game 4: We settled on 1,1 and no variation proving it was probably most mutually profitable point.

Game 5: I failed in trying to get him to give a response of 1. I got a few 1 in the beginning but when I went to 2 he went to 2 and would not be deterred.

Game 6: We settled on 1,1, both fearing any variation.

Pair 4, Player 2

Game 1: 2 is optimum move for me regardless of what opponent does. Since he made no attempt to see what I would do if he chose 1, he also must have enjoyed 2.

Game 2: My strategy here was to get player 2 to choose 1. I had no preference as to what I chose. He seemed to prefer choosing 2. I don't see much rhyme or reason to him.

Game 3: Player 2 only chose 2 twice, each time I think he expected me to play 1.

Game 4: Player 2 seemed perfectly happy with 1,1. We never tried anything else.

Game 5: Neither liked 2,2 or 1,1. This amounted to various probing until we figured out that the best either of us could do was 2,2.

Game 6: Again, 1,1 was the universal choice.

Pair 5, Player 1

Game 1: I believe his matrix is something like this; my payoff was 100.

4	8
4	8

Game 2: Matrix here is like this:

-1	4
2	-1

and my payoff is 21.

Game 3: Matrix 3:

2	1
-1	1

and my payoff is 9.

Game 4:

6	2
-2	1

My payoff was 21. Here I tried to make my opponent move from (1,1) because I figure that was his best position. I could have stayed with (1,1) if I had wished, with my payoff a steady 3.

Game 5:

-1	4
-2	5

Due to some of my opponent's mistakes, my nose stayed slightly above water in this fixed matrix. Payoff 3.

Game 6:

5	-1
3	-1

My payoff 50.

Pair 5, Player 2

Game 1: Both possibilities in choice 2 for my opponent were better than either possibility in choice one. The same was true for me, so we were locked into 2,2. We were cooperating for our mutual gain.

Game 2: Again we were cooperating after we found common ground. The first four tries were experimental and again we settled on one set of moves mutually beneficial.

Game 3: Cooperation again after solution was found. Neither of us was getting maximum profits but both were winning nevertheless.

Game 4: A battle all the way. He was continually trying to get me to play one so that he could play 2. 1,1 was suitable for me but he had a chance for great returns on 1,2 and so he was continually trying to get it.

Game 5: 2,2 was a slight loss to him. 1,1 was great gain so he was trying to get me to play 1 so that he could also.

Game 6: 1,1 was mutually agreeable although it may have not been maximum profit for him it was better than risking great loss.

FOOTNOTES

1/ Luce, R.D., and H. Raiffa, Games and Decisions, (New York: Wiley, 1957), Ch. 3.

2/ For example see the incorrect normalizations of games presented in: Schelling, T. C., The Strategy of Conflict (Cambridge: Harvard University Press, 1960).

3/ Dresher, M., A. W. Tucker and P. Wolfe (eds.) Contributions to the Theory of Games, Vol. III, (Princeton: Princeton University Press, 1958).

4/ Savage, L. J., The Foundations of Statistics, (New York: Wiley, 1954).

5/ Flood, M. M., "On Game-Learning Theory and Some Decision Making Experiments," R. M. Thrall, C. H. Coombs and R. L. Davis (eds.) in Decision Processes, (New York: Wiley, 1954); and more recently the work of S. Seigel and other experimental psychologists.

6/ Nash, J. F., Jr. "Non-Cooperative Games," Annals of Mathematics, LIV (September, 1951), pp. 286-295.

7/ Luce, R. D., and H. Raiffa, Op. Cit., p. 107.

8/ The work referred to here is a Princeton Ph.D. Thesis by David Stern, (1959); "An Experimental Business Game" by Austin Hoggatt in Behavioral Science, Vol. 4, 1959; The business game of George Feeney and some preliminary investigations of quantity-variation Duopoly by L. Fouraker, M. Shubik and S. Siegel.

9/ Luce, R. D., and H. Raiffa, Op. Cit., p. 161.

10/ Ibid 6, p. 162.