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Manufacturers' Inventories, Sales Expectations, and the Acceleration Principle

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I. INTRODUCTION:

In his meticulous investigation of the behavior of manufacturers' inventories during the inter-war period, Moses Abramovitz considered the most elementary of accelerator models, the hypothesis that maintains [1,p.20]:

> Manufacturers and merchants are both desirous and able to maintain inventories in constant ratio to their output or sales, ... (that) inventories vary directly and proportionately with output.

Abramovitz found that this most simple concept of the accelerator was not consistent with observed behavior. Contrary to hypothesis, actual inventory investment does not lead peaks and troughs of output; inventory investment is not proportionate to the rate of change of output.

The greater variety and detail of data for the postwar period permit us to consider more complex versions of the acceleration principle. One modification involves the flexible accelerator concept originally presented by Richard Goodwin [5], a model that assumes that business firms attempt only a partial adjustment of stocks to their equilibrium level during each production period.* A second complication of the basic accelerator involves the hypothesis of Lundberg [10] and Metzler [13] that errors made by firms in forecasting future sales generate discrepancies between the actual and the desired level of inventories. In this paper both these complications are considered in investigating the appropriateness of the

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* Goodwin's flexible accelerator is related to Chenery's overcapacity principle [2].

accelerator as a mechanism explaining inventory behavior. We also consider the possibility that unfilled orders and expected price changes as well as the volume of activity affect the desired level of stocks.

Both the flexible accelerator model of Goodwin and the buffer stock type of inventory model advanced by Lundberg and Metzler explain the generation of discrepancies between actual and "equilibrium" or "desired" inventories. Estimates of the parameters of an accelerator model incorporating the complications introduced by these economists are utilized to compute a series of surplus inventories, the deviation of actual stocks from their equilibrium levels. These estimates of surpluses and deficiencies in manufacturers' inventory holdings are presented in the concluding section of this paper.

The task of investigating dynamic inventory phenomena is complicated by the difficulties involved in obtaining appropriate data based on observations collected at more frequent than yearly intervals. Since the planning horizon of the firm is surely shorter than a year for decisions involving output adjustment and inventory, annual data will not do. I have not succeeded in obtaining appropriate cross-section data on individual firms. This study is based on quarterly time series data at a fairly high level of aggregation. For total durable and total non-durable sectors of manufacturing, the Office of Business Economics, Department of Commerce, publishes data broken down by stage of fabrication into purchased materials, goods in process, and finished goods categories. But when we turn to Commerce data for individual industries we find that the finer breakdown as to industry must be paid for by sacrificing the stage of fabrication classification. Nevertheless, this is the date we must use. John Stanback of

New York University kindly provided deflated, deseasonalized sets of the Commerce inventory data for the years 1948 to 1955.* W. H. Locke Anderson

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* Stanback has published a description of the deflation procedure in his excellent study of postwar inventory behavior [15, p. 91]. For a general discussion of the difficulties involved in deflating inventories see Cobren [3]. He describes the deflation procedure utilized by the National Income Division, Department of Commerce, a procedure similar to that employed by Stanback.

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of Harvard University collaborated with me in deflating comparable sets of sales and unfilled orders data.

We will first consider two separate accelerator models of inventory behavior. The first of these is appropriately applied to stocks of purchased materials and goods in process; the second to finished goods inventory. These models are tested against the durable and non-durable sector data in the next two sections of this paper. Then, in Section IV, we combine these two models into a single equation explaining the behavior of total inventory holdings in the five component durable goods industries.

II. STOCKS OF PURCHASED MATERIALS AND GOODS IN PROCESS:

In applying the principle of acceleration to stocks of purchased materials and goods in process it seems reasonable to relate stocks at the <u>beginning</u> of the production period, S_t , to output to be forthcoming <u>during</u> the period, Q_t . If the relation is linear, the equilibrium level of stocks, S_t^e , may be represented by the equation:

$$S_t^e = \alpha + \beta Q_t . \tag{2.1}$$

The coefficient β is the "marginal desired inventory coefficient."

Only the simplest version of the accelerator hypothesis asserts that entrepreneurs always succeed in maintaining stocks at the equilibrium level. Goodwin's flexible accelerator concept provides one explanation of departures of stocks from their equilibrium level.* In order to appreciate this flexible

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* We shall regard production plans as reasonably firm at the beginning of periods so that the buffer-stock motive can be neglected in the study of purchased materials and goods in process stocks; but this alternative explanation of departures of stocks from their equilibrium level will be considered in the examination of finished goods inventory behavior.

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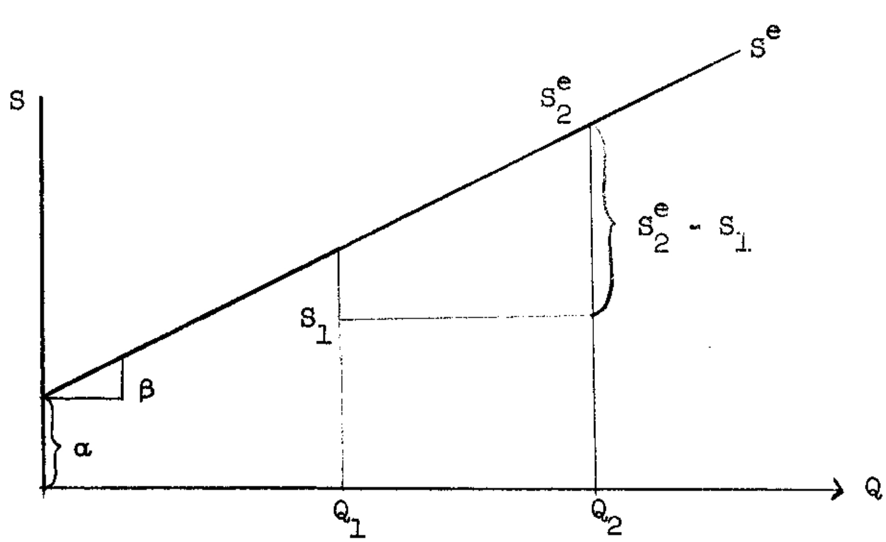
accelerator complication, plot the equilibrium relation (2.1), measuring output on the horizontal axis and stocks on the ordinate:

Figure I



The line S^e represents the equilibrium level of stocks; its positive slope implies that equilibrium stocks are larger at higher levels of output.

Suppose output is initially at Q_1 with stocks at S_1, not necessarily the equilibrium level. Suppose that output then increases to Q_2. The simple accelerator hypothesis implies that stocks will be adjusted immediately to the new equilibrium level, S_2^e, that supplies acquired will exceed immediate production requirements by $S_2^e - S_1$. But Goodwin supposes that an immediate adjustment of stocks is not attempted. Entrepreneurs are assumed to attempt only a partial adjustment of stocks to the equilibrium level each period. This may be due to costs involved in changing the level of stocks. It may also stem from problems concerned with the composition of inventories of inputs, the infrequent intervals at which certain items are ordered, and so forth. Certain categories of purchased materials may be held by the firm in relatively large stocks as a result of recent deliveries of carload orders; they will be liquidated only gradually as production demands require their utilization. On the other hand, a rapid increase in the holdings of certain items warranted by a higher level of activity would be costly if a premium has to be paid for fast delivery. Such factors as these may explain why only a partial attempt to adjust inventories to the level dictated by equation (2.1) may be attempted in any one period.

The assumption that actual inventory investment, ΔS_t, is only a fraction of that required to adjust stocks to the equilibrium level is reflected in the equation:

$$(2.2) \qquad \begin{aligned} \Delta S_t &= \delta(S_t^e - S_{t-1}) \\ &= \delta\alpha + \delta\beta Q_t - \delta S_{t-1} \quad . \end{aligned}$$

This generalization reduces to the simple accelerator hypothesis for the special case in which δ , the reaction coefficient, is unity.

The flexible version of the acceleration principle is related to Ruth Mack's notion of "passive inventory investment." She asserts [11, p. 140]:

> Passive inventory investment or disinvestment takes place in part because plans about the proper size of stocks are hardly precise figures; instead they are ranges, and variations within the range or band is a matter of indifference.

But our equation implies that while there may be a precise notion as to the appropriate size of stocks, business firms' investment in stocks is "passive" in the sense that they are not overly concerned with obtaining a rapid adjustment to that level when changes in business conditions cause stocks to depart from it.*

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* Franco Modigliani has commented upon the relation between the flexible accelerator and Ruth Mack's concept of passive inventory investment. He quite rightly suggested that the speed of adjustment may depend upon the size of the discrepancy between desired and actual inventories [14]. One might well conjecture that it would be difficult to distinguish passive from delayed adjustment inventory behavior empirically.

Within the framework of the flexible accelerator model three additional factors that may complicate the determination of the size of stocks of purchased materials and goods in process deserve consideration:

(a) Price Speculation: Manufacturers may attempt to hedge against anticipated increases in the price of inputs by adjusting their inventory position, purchsing additional stocks when price rises are expected and reducing the level of stocks when price reductions are anticipated. An adequate test of the prevalence of inventory speculation would require knowledge of expected price changes. Since information concerning price anticipations is not available, the percentage increase of actual prices in the next period will be included in the regression. This amounts to testing the hypothesis that entrepreneurs speculate successfully, accumulating stocks that are larger than would be suggested by purely non-monetary considerations in advance of actual price increases, and conversely.

(b) Changes in output: Departures of stocks from the level suggested by the simple accelerator principle may also occur when output is sharply changing. When output is increasing, orders may be placed with suppliers in an attempt to build up stocks, but they may not be delivered for a period. Consequently, there may be a tendency to temporarily deplete stocks of purchased materials when output is rising, when $\Delta Q_t > 0$, and conversely. This tendency, which is quite apart from flexible accelerator complications, may be accentuated by the need to measure output only at the final stage of the production process; output is defined as sales plus any increase in finished goods inventories.

(c) Unfilled Orders: The equilibrium level of stocks may depend on other factors besides output. In particular, we argue that unfilled orders as well as output should be included in equation (2.1). Entrepreneurs may have reasonably precise plans for production in the next period. But recognition of the fact that delivery lags and the cost of adjusting stocks to the desired level will prevent the attainment of stocks dictated by the scale of output, possibly modified by speculation may well induce firms to consider a longer planning horizon in deciding upon the adjustment of stocks to be made in the current period. If unfilled orders represent an established demand, indeed a possible committal to deliver at some future date, entrepreneurs may well consider it advisable to carry additional stocks when unfilled orders are large as a hedge against possible shortages and price committments. Unfilled orders at the beginning of the period, U_t , should be considered in the relation.*

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* Stanback observed a close correspondence between turning points for unfilled orders and stocks or purchased materials [15, p. 90].

These considerations lead to the following equation explaining the determination of manufacturers' stocks of purchased materials and goods in process:

$$S_t = \delta\ [\alpha + \beta_1 Q_t + \beta_2 \Delta Q_t + \beta_3 \left(\frac{p_t - p_{t-1}}{p_t}\right) + \beta_4 U_t] + (1-\delta) S_{t-1} + \epsilon_t . \tag{2.3}$$

The residual, ϵ_t , may be regarded as representing variables omitted from the analysis. It may be observed that an expression for investment in stocks is obtained by subtracting S_{t-1} from both sides of the equation, yielding:

$$\Delta S_t = \delta\alpha + \delta\beta_1 Q_t + \delta\beta_2 \Delta Q_t + \delta\beta_3 \left(\frac{p_t - p_{t-1}}{p_t}\right) + \delta\beta_4 U_t - \delta S_{t-1} + \epsilon_t . \tag{2.4}$$

Since the magnitude of the residual is unaffected, it is immaterial which of these two equations we fit by the method of least squares.** The regression

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** The magnitude of the regression coefficients and their estimated standard errors will be precisely the same; of course, the multiple correlation coefficient will probably be smaller for (2.4) as the variance of inventory investment is usually larger than the variance of the stock itself.

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coefficients obtained by least squares may obviously be unscrambled in order to obtain implied estimates of the model's parameters.

Do entrepreneurs speculate in stocks? Are unfilled orders a major factor influencing the inventory position of firms? Does a flexible accelerator model provide a reasonable description of observed inventory behavior during the post World War II period? These are interesting questions, but it must be emphasized that our estimates have been obtained by applying crude least squares procedures to embarassingly short time series. True, Mann and Wald [12] have demonstrated that if the residuals of a model of the form (2.4) are normally and independently distributed, the least squares procedure will yield maximum likelihood estimates of the equation's parameters. But this is of little consolation, at least for hypothesis testing, as it has been demonstrated by Leonid Hurwicz [7] that the inclusion of the predetermined variable S_{t-1} means that the estimates are biased. Quite apart from this, if the error terms ϵ_t are auto-correlated, the t distribution may not be appropriately applied in tests of hypotheses about the magnitude of the regression coefficients without adjustment* and, in

* For the case in which predetermined variables are not present Wold [16, pp. 209-213] has derived the appropriate correction procedure; the corrected value of t may be either larger or smaller than that obtained by dividing the regression coefficient by the "standard error" obtained by simple, least squares procedures. An alternative approach is suggested by Klein [9, pp.85-92].

addition, the estimating procedure is not efficient. Perhaps the most crucial limitation is that consideration of a single equation neglects the underlying, possibly simultaneous nature of the economy. These difficulties must be kept firmly in mind in examining estimates of the parameters of equations obtained by applying the least squares procedure, particularly in interpreting estimates of the standard errors of the regression coefficients.

Table I

Stocks of Purchased Materials and Goods in Process

$$S_t = \delta\alpha + \delta\beta_1 Q_{t+1} + \delta\beta_2 \Delta Q_t + \delta\beta_3 \left(\frac{p_{t+1} - p_t}{p_t}\right) + \delta\beta_4 U_t + (1 - \delta) S_{t-1} + \epsilon_t .$$

	Total Manufacturing	Total Durable	Total Non-durable
Number of Observations	29	29	29
$\delta\alpha$	5675	1412	-356.0
$\delta\beta_1$	.0632	.0528	.0230
	(.0326)	(.0187)	(.0209)
$\delta\beta_2$	-.1058	-.0803	-.0366
	(.0588)	(.0296)	(.0559)
$\delta\beta_3$	-.1108	.0385	.1481
	(.3698)	(.1732)	(.1208)
$\delta\beta_4$	.0872	.0384	.2207
	(.0150)	(.0041)	(.0514)
δ	.5865	.3628	.0974
	(.1332)	(.0338)	(.0665)
R^2	—	.994	.970
V	—	1.822	2.019
β_1	.1078	.1455	.2361

Table I presents the estimates.* It is to be observed that all the

* Least square estimates of the standard error of the regression coefficients appear below each estimate in parentheses. The coefficient of determination, R^2 , applies to the case in which total stocks are taken as the dependent variable. V , the Von Neumann coefficient is a statistic utilized in the Durbin-Watson test for serial correlation of residuals. Let $k = 5$ be the number of explanatory variables, $n = 29$ the number of observations, and $V = \Sigma(e_\tau - e_{\tau-1})^2/\Sigma e_\tau^2$, where e_τ is the observed residual. Then the hypothesis of no serial correlation is to be rejected at the 5 per cent level if $V < 1.05$ or accepted if $V > 1.84$; otherwise the situation is indeterminate. This test is based on a distribution derived only for equations which do not involve predetermined variables; consequently, it is not strictly valid for out problem. Cf. Durbin and Watson [4].

reaction coefficients, δ , are of the right sign and less than unity. This is encouraging, for one would be at a loss to explain a value of δ outside this range. Indeed, a reaction coefficient of acceptable magnitude is a prerequisite that must be satisfied if any meaning is to be attached to the other coefficients obtained in the regression. It must be admitted that the reaction coefficient for non-durable manufacturing is exceedingly small, indeed almost unreasonably so ; this implies that entrepreneurs are little concerned with adjusting inventories to the appropriate level for this sector of the economy. For all sectors the values of the reaction coefficients are sufficiently small to be inconsistent with the simple accelerator hypothesis which maintains that entrepreneurs attempt an immediate adjustment of inventories to the desired level; the low estimates of the reaction coefficients lend support to the flexible accelerator concept of Richard Goodwin.

Perhaps the evidence is clearest with regard to speculation. Certainly, there is no support for the hypothesis that manufacturers successfully speculate in stocks of purchased materials or goods in process. The sign for total manufacturing is negative, contrary to hypothesis; while the coefficients are of the right sign for the durable and non-durable sectors, they are exceedingly small relative to their standard errors. We have no reason for questioning Abramovitz's tentative conclusion that price hedging is not an important phenomenon [1, pp. 127-31].

The estimates obtained for β_4 suggest that unfilled orders may be an important determinant of the level of inventories.* The importance of

* It cannot be objected that unfilled orders are only a surrogate indicating the tightness of the markets on which firms purchase their input as this would imply a relationship of opposite sign from that revealed in the regression study.

unfilled orders may in part be due to the way in which we measure output; when a backlog of unfilled orders develops as a result of a sharp rise in demand, the acceleration of production will first be felt in terms of an increase in goods in process rather than a rise in the output of completed goods, Q_t. To the extent that unfilled orders represent an established demand, entrepreneurs increase their stocks of purchased materials as output expands at earlier stages of production.

III. INVENTORIES OF FINISHED GOODS; THE BUFFER STOCK MOTIVE:

The buffer stock inventory model recognizes that the production process is time consuming. It is a modification of the basic accelerator designed to take into account errors made by firms in anticipating future sales. In the form in which the model was originally considered by Lundberg and Metzler, desired end of period inventories, I_t^d, were assumed to be linearly related to sales volume:

$$I_t^d = \alpha + \beta X_t \ . \tag{3.1}$$

But actual sales are not known by the firm in advance of output when the production decision must be made. A planned level of inventories, I_t^p , may be defined by substituting anticipated sales, X_t , into (3.1). Actual end of period inventories will differ from this planned level if sales turn out to exceed the expected volume, and conversely. Consequently, actual end of period inventories, I_t , are given by the equation:

$$\begin{aligned} I_t &= I_t^p + (\hat{X}_t - X_t) \ . \\ &= \alpha + (1 + \beta)\hat{X}_t - X_t \ . \end{aligned} \tag{3.2}$$

Lundberg and Metzler explained deviations of actual inventories from the equilibrium level as a consequence of errors in anticipating future sales.*

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* It may be objected that the buffer stock model is not appropriate in those cases in which the scale of production is determined on the basis of specific orders for finished commodities; but Stanback may well have been justified in concluding that finished goods made to specific order are shipped almost upon completion so that variations in the size of this inventory category make a negligible contribution to fluctuations in total finished goods inventory [15, p. 89]. Abramovitz has emplasized that a test of the buffer stock model is "plagued by lack of an objective standard by which to judge a surplus or deficit of stocks." He rejects the special case of the theory in which it is assumed that $\alpha = 0$, that manufacturers attempt to keep a constant ratio of inventories to sales, [1, p. 152].

The buffer-stock inventory model is readily married with Goodwin's flexible accelerator. If δ is once more the reaction coefficient, we have as the equation for the planned level of inventories:

$$I_t^p = \delta(\alpha + \beta\hat{X}_t) + (1 - \delta)I_t \tag{3.3}$$

Again, actual inventories will deviate from the planned level by any excess of anticipated over actual sales:

$$(3.4) \qquad I_t = \delta(\alpha + \beta\hat{X}_t) + (1 - \delta)I_t - \hat{X}_t - X_t , \qquad 0 \leq \delta \leq 1 .$$

This is the flexible accelerator version of the buffer stock inventory model. For the special case in which $\delta = 1$ it reduces to the more elementary equation considered by Lundberg and Metzler, (3.2) above.

Investigation of the buffer stock motive of inventory behavior necessarily involves complications concerning the nature of sales expectations. Since sales expectations are not an item in the accounting records of firms they may be measured directly only by ex ante questionaires; consequently, it is most difficult to obtain quantitative measures of manufacturers' sales expectations.*

* When data relating to expectations is accumulated, its validity may still be open to question. Albert G. Hart has made a gallant attempt to "reconstitute" Railroad Shippers Forecast data for the inter-war period into a revised series more in conformity with preconceived concepts as to how expectations should behave [6]. But how can one check the validity of such a reconstitution? One check is a pragmatic one; is the devised series useful in prediction? In connection with sales expectations, the appropriate test concerns the usefulness of the series in describing inventory behavior. The application of this test to a "sales expectations" series I devised for the postwar period from the same data utilized by Hart gave a negative answer; the procedure described in this paper for obtaining a surrogative measure of expectations provides a closer prediction of actual inventory behavior.

We adopt a strategy which enables us to analyze the buffer stock inventory model without relying upon any attempt at measuring actual sales expectations. We look for the impact of errors in forecasting upon measured inventory and sales data. We shall find that the need to consider sales expectations not only presents difficulties in the analysis of inventories; it is also rewarding in that something may be inferred concerning the nature of sales expectations as well as desired inventories from observing actual sales and inventory behavior.

One possible strategy would be to adopt the assumption of static expectations, to let $\hat{X}_t = X_{t-1}$. But we will consider a more general hypothesis concerning the nature of expectations. After all, the assumption of naive expectations is an insult to the entrepreneur; surely he can do better than this. On the otherhand, to assume perfect expectations is to attribute to him the power of the soothsayer. John Maynard Keynes makes a provocative suggestion that may help us. He states [8, p. 51]:

> "...it is sensible for producers to base their expectations on the assumption that the most recently realized results will continue except in so far as there are definite reasons for expecting a change."

If the firm's adjustment of the simple, naive projection based on definite information takes it in the right direction, the level of sales actually expected would fall between the two extremes of static and perfect forecasting. More precisely, we may hypothesize:

$$(3.5) \qquad \hat{X}_t = \rho X_{t-1} + (1 - \rho)X_t \ .$$

If $\rho = 1$ we have the extreme of static expectation; $\rho = 0$ corresponds to the opposite case of perfect forecasting.* Let us call ρ the "coefficient of anticipations."

We wish to evaluate the nature of the prediction error involved for other values of the coefficient of anticipations. Subtract X_t from both sides of the equation so as to obtain an expression for the magnitude of prediction error for period t .

$$(3.6) \qquad \hat{X}_t - X_t = \rho(X_t - X_{t-1}) \ .$$

Dividing by X_t gives an equation for the proportionate size of the error of period t .

* A perfect forecasting record may be achieved if one is willing to push goods by advertising, salesmanship, or price cutting when sales lag behind the predicted quantity and, conversely, to refuse to sell additional goods once the forecast of sales has been fulfilled; obviously, such behavior is at the expense of profits. A basic assumption underlying the buffer-stock type of model is that firms respond passively to changes in demand in the sense that they do not adjust either price or sales policy to shortrun changes in demand.

$$\frac{\hat{X}_t - X_t}{X_t} = \rho \; \frac{X_t - X_{t-1}}{X_t} \quad . \tag{3.7}$$

Now the variance of the error proportion would not be an appropriate measure of predictive accuracy if the criterion is to be utilized for making comparisons, as between different industries. After all, prediction is more difficult when sales are subject to erratic fluctuation. It seems appropriate, then, to utilize the error involved in naive prediction for purposes of normalization. A little reflection reveals that $|\rho|$, the absolute value of the anticipations coefficient, is the desired ratio of the standard deviation of the prediction error to the standard deviation of changes in sales. If $|\rho| < 1$, the prediction procedure is more accurate than a naive projection. Provided ρ is less than unity, predictions are in the right direction. If $-1 < \rho < 0$ the predictions overshoot actual developments, but are still better than naive projection.

The expression for expectations, (3.5), must be substituted into equation (3.4), the flexible accelerator buffer-stock inventory model. This yields:

$$I_t = \delta\alpha + (\delta\beta + 1)\;\rho X_{t-1} + (\delta\beta + 1)(1 - \rho)X_t - X_t + (1 - \delta)I_t \;. \tag{3.8}$$

The estimates of the paremeters of this equation were obtained by applying least squares after it had been rewritten in the form:

$$I_t = \delta\alpha + \delta\beta X_t + (\delta\beta + 1)\;\rho\;(X_{t-1} - X_t) + (1 - \delta)I_{t-1} + \epsilon_t \;. \tag{3.9}$$

It is clear that unambiguous estimates of all parameters of the model may be obtained by unscrambling the coefficients obtained from the regression.

The estimates are presented on Table II for durables and nondurables. It is to be observed that all point estimates of the reaction coefficients, the δ's, fall within the correct range; they are slightly smaller than the estimates obtained for stocks of purchased materials and goods in process. The estimates of the marginal desired inventory coefficients, all of correct sign, are larger for finished goods than for stocks in every case. But perhaps the anticipations coefficient is of greatest interest. All estimated ρ's lie between zero and unity, suggesting that there is a tendency to underestimate actual changes in sales. The smaller value of the point estimate of ρ for total durables suggests that sales expectations are more accurate for this sector than for non-durable industries.*

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* Conceivably, covariance analysis could have been applied in order to test the significance of the interindustry differences in the estimates of various parameters reported here. But our theoretical research on the dynamic implications of the type of accelerator model under consideration suggests that the hypothesis that all industries have the same values for the reaction coefficient, the marginal desired inventory coefficient, etc., is not a particularly interesting one from the point of view of studying the dynamic properties of the economy. While the extra computational effort did not seem worthwhile, we must stress that the interindustry contrasts reported here might indeed be the consequence of sampling error.

IV. TOTAL INVENTORY BEHAVIOR: A BREAKDOWN BY INDUSTRY

Evidence concerning the behavior of stocks of purchased materials and goods in process was presented in Section II; finished goods inventories were examined in Section III. But both studies were based on a high level of aggregation. Ideally, of course, one would like to utilize a cross-section approach relying on data for individual firms. Failing this, a disaggregation of the totals at least to an industry level is most useful. Such a breakdown is possible with existing data currently released by the Office of Business Economics, but only at a cost. In return for the greater detail of data as to industry, it is necessary to sacrifice the classification of inventory by stage of fabrication.

Table II

FINISHED GOODS INVENTORY

$$I_t = \delta\alpha + \delta\beta X_t - (\alpha\beta + 1)\ \rho\ \Delta\ X_t + (1 - \delta)I_{t-1} + \epsilon_t$$

	Total Manufacturing	Total Durables	Total Non-Durables
Number of Observations	30	30	30
$\delta\alpha$	-258.2	-325.8	418.7
$\delta\beta$	.0419	.0550	.0058
	(.0203)	(.0143)	(.0292)
$(\delta\beta + 1)\rho$	.1315	.0970	.1695
	(.0417)	(.0283)	(.0685)
$1 - \delta$	.8479	.8171	.9351
	(.0649)	(.0523)	(.0858)
δ	.1521	.1829	.0649
ρ	.1262	.0919	.1685
β	.2755	.3007	.0894
V	1.39	1.33	1.57
R^2	.958	.966	.947

In this section we report on an investigation based on an industry breakdown for five component durable goods industries. Let the variable H_t represent total inventory stocks held at the end of the period. Since S_{t+1} represents stocks of purchased material and goods in process in the hands of producers at the beginning of period $t + 1$ while I_t stands for inventories of finished goods held at the end of the period we have the identity:

$$(4.1) \qquad H_t = S_{t+1} + I_t \ .$$

In view of this correspondence, it seems appropriate to add together the explanatory terms of equations (2.3) and (3.9), the expressions for stocks and for inventories respectively. Since our measure of quantity, Q_t , cannot be derived without knowledge as to the change in finished goods inventory, this term in equation (2.4) must be approximated by sales. In this way we obtain as the equation explaining total inventories held by each industry:

$$(4.2) \qquad H_t = \delta\alpha + \delta\beta_1 X_t + (1 - \delta)H_{t-1} - (\delta\beta_1 + 1)\ \rho\Delta X_t + \delta\beta_2 U_t + \delta\beta_3 \frac{\Delta p_{t+1}}{p_t} + \epsilon_t \ .$$

It was possible to reprocess the data for total durable, total non-durable and the total manufacturing sectors in addition to analyzing the new data for component industries. The statistics appear on Table III. A check on the effects of aggregating over stages of fabrication is provided by the total inventory estimates obtained for these sectors. Before turning to the individual industry estimates, let us test the consistency of these estimates for the aggregates with those obtained earlier.

First of all, one would surmise that the total inventory reaction coefficient obtained for each sector would be a rough average of the sector's stock and inventory reaction coefficients obtained under the separate regressions utilizing the stage of fabrication breakdown. This indeed proves to be the case, as may be seen by comparing the data on Tables I and II with the figures presented here. Again, remembering that sales are now being utilized as a proxy for output, we should find the total marginal desired inventory coefficient to be roughly the sum of the coefficient for purchased materials and goods in process stocks and that for finished goods inventory; this indeed holds, the approximation being particularly good for total manufacturing and total durable. Perhaps greatest interest centers upon the coefficient of anticipations. The estimates of the coefficient obtained when the stage of fabrication breakdown is neglected fairly closely approximates those obtained with the finished goods regression; even the largest discrepancy, that for total manufacturing, is only a contrast of 0.13 versus 0.15 in estimating the anticipations coefficient. Only the rate of changes in prices of purchased materials shows a marked change; here we find the coefficients larger in absolute value and all negative, implying that total inventories are smaller when prices of purchased materials are on the increase. With the exception of a possible distortion of the role of prices, the estimates obtained when the data on finished goods inventory are combined with the figures for stocks of purchased materials and goods in process are remarkably consistent with the earlier estimates utilizing the breakdown. It seems quite reasonable to conjecture, then, that the estimates presented for the five component durable goods industries do not differ greatly from those that would have been obtained with separate regressions if stage of fabrication data could be utilized.

Table III

Total Inventories

$$H_t = \delta\alpha + \delta\beta_1 X_t + (1 - \delta) H_{t-1} - (\delta\beta + 1) \rho \Delta X_t + \delta\beta_2 U_t + \delta\beta_3 \frac{\Delta p_{t+1}}{p_t} + \epsilon_t$$

	Total Manufacturing	Total Durable	Total Non-Durable	Stone, Clay and Glass	Primary Metal	Transportation Equipment	Machinery	Other Durables
Number of Observations	30	30	30	33	30	31	31	29
$\delta\alpha$	3066.	1032.	-661.0	27.38	-172.8	266.0	751.9	32.77
$\delta\beta_1$	.1672	.1256	.0355	.1085	.0631	.0827	.0347	.1221
	(.0331)	(.0368)	(.0383)	(.0197)	(.0309)	(.0318)	(.0389)	(.0298)
$1 - \delta$	.5699	.6756	.9263	.7333	.9446	.6840	.7010	.8055
	(.0514)	(.0448)	(.0653)	(.0508)	(.0533)	(.0542)	(.0432)	(.0500)
$(\delta\beta + 1)\rho$	+.1697	+.1043	+.1706	+.2343	+.0432	+.0306	+.0697	+.1347
	(.0471)	(.0538)	(.0746)	(.0503)	(.0321)	(.0459)	(.0653)	(.0484)
$\delta\beta_2$	.0618	.0374	.3291		.0179	.0319	.0593	
	(.0076)	(.0062)	(.0823)		(.0115)	(.0049)	(.0067)	
$\delta\beta_3$	-1.6852	-.4991	-.6180	.0025	-.0355	.0057	.0288	
	(.4101)	(.3311)	(.1847)	(.0200)	(.0346)	(.0842)	(.0668)	
δ	.4301	.3244	.0737	.2667	.0554	.3160	.2990	.1945
β_1	.3887	.3872	.4817	.4068	1.1390	.2617	.1161	.6278
ρ	.1454	.0927	.1648	.2114	.0406	.0283	.0674	.1200
R^2	.993	.991	.981	.978	.939	.990	.991	.960
V	1.98	1.46	2.23	1.29	1.72	1.13	1.49	0.92

Now let us turn to the data for the five component durable goods industries. It is to be observed that all reaction coefficients are of the correct sign and of reasonable magnitude. The point estimate of the reaction coefficient for the transportation equipment industry is largest, implying that firms in this industry attempt the most rapid adjustment of stocks to the desired level; firms in primary metal appear to be the slowest. Observe that the total durable reaction coefficient is larger than that for any component industry rather than a rough average of the estimates for different industries: this implies that at least some distortion is involved when data limitations require aggregating over component industries in order to work with data for a major sector of the economy.

All marginal desired inventory coefficients appear to be of correct sign. The considerable range in the value of these coefficients may be partially due to sampling errors as well as to interindustry differences in the extent to which the optimal level of inventories is actually related to the volume of sales. The point estimate is lowest for machinery; the largest coefficient, that for primary metal, is almost ten times the figure for machinery. The figure obtained from the aggregate regression for total durables was about three times the smaller figure.

The coefficients of anticipations are all positive and less than unity. This suggests that firms are successful in attempting to adjust the simple naive projection of sales in the direction of actual developments. They do not overshoot, but rather tend to underestimate the actual change in sales by a surprisingly small amount. In the transportation equipment industry particularly accurate forecasts are suggested by the point estimates. But even in stone, clay and glass-the least accurate industry considered -- almost four-fifths of the actual change in sales is anticipated. The regressions for individual industries confirm the conclusion based on the data for durable and non-durable totals that manufacturers are quite accurate estimators of future sales volume.

In sum, the parameter estimates obtained utilizing the industry data are reasonably consistent with the values one would expect on the basis of the data that is aggregated over industries but broken down by stage of fabrication. While the estimates are clearly inconsistent with the simple, naive accelerator hypothesis rejected by Abramovitz, they are in conformity with more complicated versions of the accelerator which incorporate the flexible principle of Goodwin and the buffer stock concept of Lundberg and Metzler.

V. SURPLUS INVENTORIES

Discrepancies between the desired or equilibrium level of inventories and the actual size of stocks are explained by the complicated version of the acceleration principle developed in this paper, a model incorporating the flexible accelerator principle of Goodwin and the buffer-stock concept of Lundberg and Metzler. Estimates of the parameters of the model were provided on Table III. In this section these estimates of the model's parameters are utilized to construct a series for the surplus inventory holdings of five durable goods industries for the years 1948 to 1955.

The equilibrium level of inventories is defined as that level which entrepreneurs would work to obtain on the basis of the current level of sales and the backlog of unfilled orders if they were not disturbed by dynamic factors.*

* A periodic survey currently conducted by the Office of Business Economics, Department of Commerce, inquires as to whether the firm's current inventory position is above, below, or approximately equal to the volume of stocks that the firm would like to hold on the basis of current sales and the backlog of unfilled orders, but a figure as to the size of the surplus or deficit is not requested. Lawrence Klein utilized residuals from an equation predicting actual inventories as a measure of undesired stocks in one of his econometric investigations [9a, p. 102].

The equation for estimating the desired level of inventories is obtained from (4.2) by eliminating the dynamic disturbances of price changes and errors in anticipating future sales, by setting $\frac{\Delta p_{t+1}}{p_t} = \Delta X_t = 0$, and then equating current with last periods sales, $X_t = X_{t-1}$,

$$H_t^d = \alpha + \beta_1 X_t + \beta_2 U_t \; . \tag{5.1}$$

Here are the estimates of the parameters of the equilibrium level of inventory equation for each of the five durable goods industries:

	α	β_1	β_2
Stone, Clay and Glass	102.7	.4068	———
Primary Metal	3119.1	1.1390	.3231
Transportation Equipment	841.7	.2617	.1009
Machinery	2514.9	.1161	.1983
Other Durable	168.5	.6278	———

If equation (5.1) is subtracted from (4.2) one obtains an equation for estimating surplus inventories:

$$H_t - H^d = (\delta - 1)\alpha + (\delta - 1)\beta_1 X_t + (1 - \delta)H_{t-1} - (\delta\beta_1 + 1)\, \rho \, \Delta X_t + \delta\beta_2 U_t + \delta\beta_3 \frac{\Delta p_{t+1}}{p_t} \; . \tag{5.2}$$

The estimates of the parameters of this equation for the five durable goods industries appear on Table IV.

Table IV

Coefficients of Surplus Inventory Equations

	Stone, Clay and Glass	Primary Metal	Transportation Equipment	Machinery	Other Durables
$(\delta - 1)\alpha$	75.3-	2946.3	575.7-	1762.9-	135.7-
$(\delta - 1)\beta_1$	.2983-	1.0759-	.1790-	.0814-	.5057-
$(\delta - 1)\beta_2$	——	.3052-	.0690-	.1390-	——
$\delta\beta_3$	.0025	.0355-	.0057	.0288	——
$-(\delta\beta_1 + 1)\rho$	.2343-	.0432-	.0306-	.0697-	.1347-
$(1 - \delta)$	.7333	.9446	.6840	.7010	.8055

These last coefficients were applied to the same data utilized in the original regression in order to obtain time series representing surplus inventory holdings for each of the durable goods industries. For each quarter the surplus holdings of the five industries were summed so as to provide an estimate of total surplus inventory holdings for the durable manufacturing sector of the economy. The data appears on Table V together with a constant dollar series of actual durable manufacturing inventories. The provisional character of the derived series of surplus inventories must be emphasized; their validity depends upon the accuracy of our specification of the equation explaining actual inventory behavior; they are obviously subject to sampling error. While the evidence presented should be considered as most tentative, it nevertheless provides an interesting interpretation of the behavior of durable goods inventories during a period of military mobilization.

Table V

Surplus Inventories in Durable Goods Industries, (Millions of 1947 Dollars)

Years and Quarters	Total Durable: Actual (Inventories)	Total Durable: Surplus (Inventories)	Surplus Inventories by Industry: Stone, Clay and Glass	Primary Metal	Transportation Equipment	Machinery	Miscellaneous Non-Durable
48-1	13,739		37-		258	180	
2	13,796	836-	26-	502-	219	140	667-
3	13,826	611-	22-	437-	193	268	613-
4	13,835	876-	28-	548-	61	271	632-
49-1	13,900	748	22	132-	162	212	484
2	13,517	1,633	17	494	198	294	630
3	13,001	1,604	15	772	118	207	492
4	12,591	1,959	4-	1,330	131	51	451
50-1	12,463	184-	59-	197	30-	31	323-
2	12,765	1,688-	108-	575-	215-	176-	614-
3	12,878	3,021-	138-	1,077-	474-	434-	898-
4	13,656	3,322-	133-	1,218-	524-	567-	880-
51-1	14,102	3,540-	101-	1,149-	629-	844-	817-
2	15,475	4,018-	79-	1,735-	673-	1,057-	474-
3	17,023	2,717	3-	1,613-	428-	831-	158
4	17,708	2,254-	28	1,402-	321-	694-	135
52-1	18,312	1,549-	18	969-	250-	564-	216
2	18,319	640-	54	2-	286-	446-	40
3	18,381	1,143-	17	173-	316-	376-	295-
4	18,719	2,674-	16-	1,139-	595-	469-	455-
53-1	19,122	2,091-	32-	937-	549-	320-	253-
2	19,741	1,788-	10-	995-	431-	149-	203-
3	19,865	534-	3-	594-	149-	124	88
4	19,461	1,391	14	488	206	384	299
54-1	18,893	1,872	24	938	205	556	149
2	18,274	1,914	16-	985	186	604	155
3	17,724	1,535	41-	853	151	504	68
4	17,560	1,047	51-	748	104	402	156-
55-1	17,493	187-	91-	40	33-	232	335-
2	17,799	967-	106-	432-	10	115	554-
3			93-	857-	95	116	

At the end of 1949 durable manufacturing firms held considerable excess stocks; more than one-fifth of their total inventory holdings were surplus. Three months later stocks were deficient. They remained below the equilibrium level until the end of 1953. From midyear 1950 to midyear 1951 durable goods manufacturing firms accumulated some two and one-half billion dollars of inventories, measured in 1947 prices. But this dramatic rate of inventory investment was not sufficient to prevent the deficiency of stocks from enlarging considerably. Indeed, actual inventory accumulation during the first year of the Korean emergency was only slightly more than one-half the change in the desired level of stocks. Although the deficiency in inventory holdings reached its peak at the end of June 1951, the additional stocks necessary to eliminate the gap between desired and actual stocks were not accumulated until two more years had elapsed.

It may well be that inventory investment would have been much larger during the period of military mobilization except for two basic factors. First, of all, firms in manufacturing follow a flexible inventory policy, attempting only a partial adjustment of actual inventories to the desired level during each production period. Second, our estimates of the coefficients of anticipations all imply a tendency for manufacturers to underestimate actual increases in sales. It is tempting to hypothesize that during the Korean crisis these two factors, by reducing actual inventory investment, served to limit effective demand during a period of inflation. This would imply that inaccurate expectations and a flexible inventory policy may at times serve to stabilize the economy. This is a conjecture concerning the behavior of the economy for alternative, hypothetical values of the parameters of the equation determining inventory behavior. But it must be observed that if the parameters of the equations explaining inventory investment in certain sectors of the economy had been different, the level of sales and, possibly the backlog of unfilled orders might well have been affected; consequently, inventory behavior would have been different from that implied from considering the actual level of sales. The interdependent nature of the economy means that considerable difficulty is involved in appraising the effects of alternative inventory practices.

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