Political Disagreement and Information in Elections

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Abstract

We study the role of re-election concerns in the incentives of incumbent parties to engage in policy experimentation. In a probabilistic voting model, candidates representing two groups of voters compete for office. In equilibrium, the candidate representing the majority group wins with a probability that increases in the degree of political disagreement — the difference in expected payoffs from the policies supported by the candidates. Prior to the election, the office-motivated incumbent party (IP) can influence the degree of political disagreement through policy experimentation — i.e., a public signal about a payoff-relevant state. We show that if the IP supports the majority candidate, then it strategically designs this experiment to increase political disagreement and, hence, her victory probability. We then define conditions such that (i) the IP optimally chooses an upper-censoring experiment, which fully reveals low-disagreement states and pools high-disagreement states; and (ii) the experiment’s informativeness decreases with the majority candidate’s competence. We show that the IP uses the policy experiment to increase disagreement, even when all voters share the same payoff function, so that political disagreement is solely due to belief disagreement.

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1 Introduction

Voters and politicians are often uncertain about the possible repercussions of different policies. When candidates advocate different policies, this uncertainty plays an important role in defining electoral outcomes. Learning about the payoff consequences of policies can then change the policies that different politicians advocate, as well as electoral outcomes. Learning often occurs through policy experimentation — trying out a policy in a small-scale pilot test (see, e.g., Greenberg and Shroder, 2004). In this paper, we study the effects of reelection concerns on incumbents’ incentives to embark on policy experimentation and on which experiments they favor.

Our electoral model incorporates the empirical regularities of elections highlighted in Stokes (1963). First, voters care about multiple issues and, thus, may have to trade off the candidate who offers the highest payoff in one issue for the competing candidate who offers the highest payoff in a different, more relevant issue. Second, it is important to differentiate position issues from valence issues.\footnote{In position issues, parties advocate certain actions, and voters might have heterogeneous preferences over actions. In valence issues, parties are linked to some condition that the electorate positively or negatively values.} Third, voter behavior depends only on voters’ perception of parties’ ideological positions. Fourth, information may change this perception, thus leading a voter to revise the relative importance of the different issues. To the extent that experimentation can affect voters’ perceptions, incumbent parties have an incentive to affect voters’ learning, and to control the salience of position and valence issues — see Iyengar and Simon (2000) for a survey.

We argue that policy experimentation allows both politicians and voters to learn about the effects of different policies. We then focus on how information gleaned from experimentation shapes political disagreement, and how disagreement steers politics. More specifically, we consider an incumbent party (IP) that supports one candidate and wants to maximize her probability of victory in an upcoming election. To focus on the effect of voters’ uncertainty over policy payoffs, we assume that voters know the valence of the incumbent politician and learn the valence of the challenger during campaigning. Through its control of the government, the IP is able to influence voters’ learning about a particular position issue through
selective experimentation; the experiment’s outcome serves as a public signal about the pay-off consequences of different policies. In equilibrium, the IP designs the policy experiment to change voters’ perception of the degree of political disagreement between the candidates, which endogenously shifts the relative salience of policy and valence issues. We show that when a majority of voters share a policy view with the incumbent candidate, the IP designs the experiment with the sole purpose of increasing political disagreement, which benefits its supported candidate.\(^2\)

To illustrate the roles of political disagreement and experimentation, consider the following example. Voters care both about a position issue (policy) and a valence issue (competence). Voters are divided into two groups: a majority \(A\) that has a lower per capita income and a minority \(B\) that has a higher per capita income. One citizen-candidate from each group runs for office. The elected official must choose the rate of a proportional income tax, and the tax revenues finance a novel government program.\(^3\) Political disagreement arises since voters in low-income group \(A\) prefer a higher proportional tax than voters in high-income group \(B\). Candidates cannot commit to future policies, but voters can predict their behavior: candidate \(A\) will implement a higher tax than candidate \(B\). Consequently, candidate \(A\) has a policy advantage — a majority of voters believe that she will choose a better policy. Voters in majority \(A\) are, nonetheless, willing to vote for the minority candidate if, during the campaign, they learn that she is sufficiently more competent.\(^4\) A key observation is that voters are less willing to trade off policy for valence if voters’ perception of the degree of political disagreement is higher. That is, if the difference in voters’ expected payoffs from the policies supported by the candidates is higher. Suppose that voters are uncertain about the marginal payoff derived from the government program and that uncertainty can be partly resolved through policy experimentation. Political disagreement is low if this expected marginal pay-

\(^2\)Our IP resembles the persuaders in Downs (1957, pg. 83), who “are not interested \textit{per se} in helping people who are uncertain become less so; they want certainty to produce a decision which aids their cause.”

\(^3\)E.g., a novel after-school program to be adopted by public schools, aimed to help low-income students.

\(^4\)Voters for which the final vote goes in consonance with valence preferences, rather than with policy preferences, are dubbed “Stokes voters” by Groseclose (2001).
off is low, in which case both groups want similar low taxes. However, disagreement is high if this expectation is high, in which case group $A$ wants much higher taxes than group $B$ (see Section 4.1 for details). Therefore, experimental outcomes that lead players to believe that the program’s payoff is higher increase political disagreement. The majority group is then more likely to vote for candidate $A$, increasing her probability of victory. Consequently, experimentation can be strategically used to shape political disagreement and steer the election in favor of a candidate.

We model the strategic design of policy experiments as a persuasion game — see Kamenica and Gentzkow (2011), KG henceforth. Voters are uncertain about an underlying state that describes how different policies map into payoffs. Public policy experimentation can resolve uncertainty, although the incumbent party can shape this learning by specifying the statistical relation of the signal with the underlying state. For example, the incumbent party can commission a trial on selected public schools to study the effects of different educational policies. While the IP may not be in control of the study’s final conclusion, it can, nevertheless, shape public learning by dictating which questions the report should answer or which angle it should consider.

Our model has the following ingredients: (i) Electorate: Uninformed voters are divided into two groups, majority $A$ and minority $B$, with differing preferences about the optimal policy. (ii) Parties and Candidates: Two parties compete for office. Party $A$ runs incumbent candidate $A$, who will implement the preferred policy of group $A$ if elected; candidate $B$ from party $B$ defends the preferred policy of group $B$. Besides their supported policies, candidates also differ in a second dimension: competence. (iii) Policy Experiment: Party $A$ currently controls the government and, hence, has the authority to carry out a policy experiment that reveals information about voters’ policy payoffs. Party leaders (or bureaucrats) are purely office-motivated; thus, Party $A$ chooses an experiment that maximizes its candidate’s victory probability. (iv) Election: After observing this experiment and its results, candidates revise their beliefs and, therefore, the policies they will implement if elected, while voters update their evaluation of the candidates’ policies. Voters already know the valence of incumbent $A$. During the electoral campaign, voters also observe a noisy signal about the valence of the untried candidate $B$. Each voter then chooses candidate $A$ if she is expected to deliver a higher total payoff (valence + policy) than $B$. 

We start our analysis by studying, in Section 3, the effect that the competence $v^A$ of incumbent $A$ has on the informativeness of the IP’s optimal policy experiment. We first consider the case in which the valence distribution of untried candidate $B$ has a log-concave probability density function, such as a Normal Distribution. Then, regardless of the preferences of majority voters $A$ and minority voters $B$, the following single-crossing property holds: If an experiment does not increase incumbent’s $A$ probability of victory when her competence is $v^A$, then this experiment does not increase her victory probability if her competence is higher than $v^A$ (Lemma 1). This result implies that there are two cutoffs $v^A_1$ and $v^A_2$ in the extended real line, such that the IP finds it optimal to implement a fully informative experiment if $v^A < v^A_1$; a partially informative experiment if $v^A_1 < v^A < v^A_2$; and a completely uninformative experiment if $v^A_2 < v^A$ (Proposition 1 and Corollary 1). That is, the IP prefers to be fully transparent about policy payoffs and, thus, favors fully informative experiments when the majority candidate is sufficiently incompetent; prefers to be partially transparent for intermediate levels of competence; and prefers to be completely opaque — thus providing a completely uninformative experiment — when the majority candidate is sufficiently competent.

The single-crossing property in Lemma 1 holds for any specification of the preferences of voters in the majority group $A$ and minority group $B$. To characterize the optimal policy experiment, in Section 4, we focus on cases in which political disagreement endogenously increases in the voters’ expected state, as in the tax example we described earlier. Experimental outcomes that lead to an upward revision of the average state would then magnify political disagreement, which benefits the IP, and outcomes that produce a downward revision of the average state would reduce disagreement. In Proposition 2, we show that, under the assumption of a log-concave probability density function for the untried candidate $B$, it is optimal for the IP to use an upper-censoring experiment. Such experiments define a cutoff state, and voters learn the true state when it falls below this cutoff; otherwise, voters learn only that the state is above the cutoff. That is, an upper-censoring experiment fully reveals low-disagreement states and pools high-disagreement states. Optimality of upper-censoring obtains, as a marginal increase in the victory probability is small when disagreement is already high and is large when disagreement is low. An important implication is that the IP monotonically provides less information to the electorate through experimentation as the
incumbent’s competence improves.

We discuss in Section 5 how our above results fundamentally depend on the challenger’s valence distribution. The results are reversed if the p.d.f. of the challenger’s valence is log-convex. In the log-convex case, the single-crossing property goes in the opposite direction: lower values of the incumbent’s competence \( v^A \) induce less experimentation, while higher competence induces more experimentation. Moreover, if political disagreement increases with the expected state, then the IP would favor lower-censoring experiments if the p.d.f. of the challenger’s valence is log-convex. The sharp change in results is rooted in the change in the curvature of the incumbent’s victory probability, as a function of political disagreement and valence \( v^A \). In the log-concave case, it is as if the IP features increasing absolute risk aversion (IARA). When disagreement and the incumbent’s valence are low, the IP benefits from gambling on political disagreement — that is, from implementing a risky experiment that might increase or decrease disagreement. When political disagreement and valence are high, the IP prefers to avoid these gambles. In the log-convex case, it is as if the IP features decreasing absolute risk aversion (DARA), and the reverse results hold.

In Section 6, we extend the basic model to allow for heterogeneous prior beliefs. As Callander (2011, pg. 657) notes, “[M]uch political disagreement is over beliefs rather than outcomes” — that is, much disagreement is rooted in members of the electorate holding different views of the likely effects of various policies. To focus on the role of belief disagreement, we restrict attention to cases in which voters share the same payoff function, so that political disagreement stems solely from belief disagreement. That is, in the absence of uncertainty, all voters would agree on the optimal policy, and candidates would be judged solely on their valence. In this case, one may conjecture that public information creates consensus among voters; hence, the IP will seldom benefit from persuasion, and belief disagreement will foster opaqueness. However, we show that this view is flawed. For example, if there are more than four possible states, and political disagreement is increasing in the distance between each group’s expectation of the state, then the IP can generically design an experiment that increases political disagreement with probability one (Proposition 4).

Section 7 extends the model and Section 8 concludes. All proofs are in Appendix A, and additional results are available online in Appendix B. We next discuss the related literature.
Related Literature: Our paper is related to, and borrows from, various literatures.

Policy experimentation and electoral outcomes: A number of papers explore how policy experimentation (learning how different policies map into payoffs) can influence future policies and electoral outcomes, as well as how re-election concerns by office-motivated politicians guide the choice of policy experiments. One strand of the literature focuses on the role that experimentation plays in uninformed voters’ learning about the incumbent or the challenger’s valence (Biglaiser and Mezzetti, 1997; Majumdar and Mukand, 2004; Willens, 2013; Fu and Li, 2014; and Dewan and Hortala-Vallve, 2014). In contrast, in our setup, these learning processes are unaffected by the IP’s choice of experiment: voters perfectly observe the incumbent’s valence while in office and, prior to the election, observe an exogenous signal of the challenger’s valence. Bernecker, Boyer and Gathmann (2015) consider a model in which politicians use their choice of policy experiment to signal competence and test it with data from the 1996 US Welfare Reform. While theirs is a “signaling” model of competence, their finding that governors with high reputation are less likely to experiment is consistent with our results in Proposition 1 and Corollary 1.

Another strand considers the effect of policy experimentation on voters’ learning about policies. For instance, Callander (2011) and Callander and Hummel (2014) study the incentives of politicians to engage in trial-and-error experimentation, while Callander and Harstad (2015) consider the effect of learning spillovers on the incentives of heterogeneous districts to experiment. Millner, Ollivier, and Simon (2014) show that a policy-motivated party — in order to show to the opposite party that its belief is “wrong” and reduce belief disagreement — may over-experiment when politicians have heterogeneous prior beliefs. In contrast, in our model, the purely office-motivated IP strategically discloses information to increase belief disagreement and influence elections.

Bayesian Persuasion: Our paper relates to the recent papers on Bayesian persuasion that follow KG. In Alonso and Câmara (2015b), the goal of the incumbent party (“sender”) is also to sway elections in favor of its preferred alternative. However, the sender simply wants to convince a majority of voters that the proposal is better than the status quo. An important feature of our model is that the IP would like to convince voters from the majority group not only that its candidate supports a good policy, but also that the minority candidate supports a bad policy. That is, the “relative” expected payoff from the policies (the degree
of political disagreement) is crucial.

Kolotilin et al. (2015) study a Bayesian persuasion model with a single receiver that has private information about his type, and a sender with a payoff that is a linear increasing function of the expected state. Although their setup and focus are quite different from ours, they find (Theorem 2) that if the receiver’s type has a log-concave (log-convex) p.d.f., then it is optimal to use an upper (lower) censorship signal.\footnote{We use the term upper-censoring, since it is more common in the statistics literature.} Their proof relies on a mechanism-design approach, while our proof of Proposition 2 is closer to the concave-closure approach of KG.

Polarization and Disagreement: A number of papers argue that access to information can increase polarization and disagreement (e.g., Dixit and Weibull, 2007; Van den Steen, 2011; and Alonso and Câmara, 2015c). In most papers, a higher disagreement is a somewhat unintended side effect of the actions of individuals generating information, such as the media catering to the demand of biased voters. In our extension with heterogeneous prior beliefs (Section 6), the IP generates information with the sole purpose of increasing disagreement and benefiting its supported candidate.

## 2 Model

**Overview:** There are two parties and two groups of voters. Party $A$ represents voters in group $A$ and party $B$ represents voters in group $B$, where group $A$ is larger than $B$. In our benchmark model, party $A$ holds office at the beginning of the game (Section 7 presents the opposite case). The incumbent party (IP) strategically designs a policy experiment to influence the next election. Voters observe the experiment’s results and update their beliefs about policy payoffs. Voters then observe a (possibly noisy) signal about the valence of untried candidate $B$ — voters already know the valence of incumbent $A$. The election takes place; the elected candidate implements a policy; payoffs are realized; and the game ends.

**Voters’ Preferences:** Voters care about the policy choice and the valence (i.e., competence) of the elected official. If elected, the candidate has to choose one policy $x$ from the compact, convex set $X \subset \mathbb{R}^d$, with a finite $d \geq 1$. For example, $X$ can represent the set of feasible governmental budget allocations across $d$ projects, the government’s policy on a left-right
Downsian model, or a proportional income tax rate. Each citizen’s payoff from policy \( x \) depends on an unknown state \( \theta \in \Theta \equiv \{ \theta_1, \ldots, \theta_N \} \), with a finite \( N \geq 2 \). To simplify presentation, let \( \Theta \subset \mathbb{R} \) and \( \theta_1 < \ldots < \theta_N \). Players share a common prior belief \( p \) in the interior of the simplex \( \Delta(\Theta) \). Citizens within each group are homogeneous, but groups differ in their policy preferences. Formally, each citizen in group \( i \in \{ A, B \} \) has preferences over policies characterized by the von Neumann-Morgenstern utility function \( u^i(x, \theta) \), where \( u^i \) is a differentiable function of \( x \). Each candidate is also endowed with a valence \( v \in \mathbb{R} \), which we discuss momentarily. For a voter in group \( i \), the total payoff from electing a politician with valence \( v \) who implements policy \( x \) when state \( \theta \) is realized is

\[
U^i(v, x, \theta) = v + u^i(x, \theta).
\]

**Political Parties:** We model each party as a primarily office-motivated institution (or, similarly, party leaders and bureaucrats as purely office-motivated individuals), with ties to the policy interests of a particular group of voters. Formally, each party receives payoff one if its candidate is elected and zero otherwise. If elected, party \( A \) implements the policy that maximizes the expected payoff of voters in group \( A \), while party \( B \) implements the best policy for voters in group \( B \).\(^6\) Consequently, the preferences of each party and those of the voters it represents are only partially aligned. Party \( A \) always strictly prefers to elect its own candidate, independently of policies and valences. However, given parties’ policies, voters in group \( A \) prefer to elect the candidate from party \( B \) if she is sufficiently more competent than the candidate from party \( A \).

**Strategic Policy Experimentation:** The IP controls the government and has the monopoly over a *policy experiment* (a public signal that is correlated with the state). By strategically designing this experiment, the party can influence voters’ beliefs and electoral outcomes. Formally, prior to the election, the IP chooses a policy experiment \( \pi \), consisting of a finite

\(^6\)We are implicitly assuming that, prior to the election, each party cannot commit to a specific policy. However, each party has long-run ties to the group of voters it represents and, once in office, will implement the policy favored by this group. For example, suppose that group \( A \) consists of poor voters and group \( B \) consists of rich voters. Although parties cannot commit to a particular tax rate, party \( A \) is expected to implement higher taxes than \( B \). The actual implemented tax will depend on the particular information available to parties about the state of the economy. See Section 4.1 for a formal optimal tax model.
realization space $S$ and a family of distributions over $S$, $\{\pi(\cdot|\theta)\}_{\theta \in \Theta}$, with $\pi(\cdot|\theta) \in \Delta(S)$. Experiment $\pi$ is “commonly understood”: $\pi$ is observed by all players who agree on the likelihood functions $\pi(\cdot|\theta), \theta \in \Theta$. Players process information according to Bayes rule, so that $q(s|\pi, p)$ is voters’ updated posterior belief after observing realization $s \in S$ of $\pi$. To simplify notation, we use $q$ or $q(s)$ as shorthand for $q(s|\pi, p)$.

Our learning technology follows important assumptions from KG: the IP has the monopoly over the experiment; it has no private information; it can choose any experiment that is correlated with the state; and experiments are costless to the IP. As in our model, Callander (2011) and Callander and Hummel (2014) consider a learning technology in which the incumbent party has the monopoly over the policy experiment and has no private information. However, they consider a different learning technology — one related to a Brownian process. In order to learn, the incumbent must implement a new policy, and all players (including the IP) incur the resulting policy payoff of this experiment. Thus, we interpret these as models of “full-scale” policy experimentation. In our setup, we view the experiment as a small-scale policy trial, that does not directly affect the payoff of the IP.\(^7\) The IP controls the informativeness of the trial by strategically designing its protocol (designing treatment and control groups, evaluation tools, etc.).

**Candidate’s Policy:** We refer to the candidates from parties $A$ and $B$ as candidates $A$ and $B$, respectively. There are no exogenous commitment devices available to politicians. However, since the candidates’ party affiliations and the experiment’s results are common knowledge, in equilibrium, voters can correctly anticipate the policy that each candidate would choose. If elected, candidate $i \in \{A, B\}$ will implement policy $x_i^*(q) \equiv \arg\max_{x \in X} \sum_{\theta \in \Theta} q_\theta u_i(x, \theta)$. We refer to $x_i^*(q)$ as the “preferred policy” of candidate $i$.

**Candidate’s Valence:** Besides the policy dimension, candidates also differ in a valence dimension. All players already know the valence $v^A$ of incumbent $A$ since they observe her performance in office. After the IP chooses its experiment, but before the election, voters observe valence $v^B$ of untried candidate $B$. Our timing assumption is rooted in the fact that it takes time to set up and implement policy experiments, while the identity (and, hence, the actual valence) of the challenger is only defined much closer to the election. Hence, at

\(^7\)In Section 7, we consider costs that increase in the experiment’s informational content.
the time that the IP chooses an experiment, there is significant uncertainty over the valence of the next challenger.

We assume that challenger’s valence $v^B$ is a random variable distributed according to the cumulative distribution function $F$, with probability density function $f$. In this paper, we focus on two cases. We first assume that:

\[(A1) \text{ } F \text{ is twice differentiable and has full support on the real numbers; and } f \text{ is log-concave.}\]

Condition (A1) holds, for example, for the normal, logistic, and extreme value distributions. See Bagnoli and Bergstrom (2005) for a discussion of the properties of log-concave density functions. In Section 5.1, we consider the case in which $f$ is log-convex and show that the main equilibrium features are reversed. We show how this sharp contrast between the two cases helps us better understand the IP’s equilibrium incentives to design the experiment.

The model is easily extended to the case in which the incumbent politician is not running for re-election. The incumbent party $\mathcal{A}$ then runs with an untried candidate, and voters simultaneously observe valences $v^A$ and $v^B$ of the untried candidates. Although we say that voters observe candidates’ “true” valences, the model can easily be reinterpreted as voters observing a noisy, exogenous signal about the valence of each candidate (e.g., information from media coverage during the campaign). In this case, variables $v^A$ and $v^B$ are interpreted as the new expected valence of each candidate, after voters observe the implicit realization of the signals about valence. See Boleslavsky and Cotton (2015) for a model of noisy information about valence.

**Election:** At the time of the election, voters can predict candidates’ policies $x^{A*}(q)$ and $x^{B*}(q)$. Voters also observe the realized valences $v^A$ and $v^B$. Thus, for a citizen in group $i$, the total expected payoff of electing candidate $j$ is

$$
\mathcal{U}^{ij}(q, v^A, v^B) = v^j + \sum_{\theta \in \Theta} q^i \cdot u^i(x^{j*}(q), \theta).
$$

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8 The full-support assumption simplifies presentation, as it avoids corner solutions in which expected victory probabilities are either zero or one. When this support is bounded, but sufficiently large, our qualitative results continue to hold if we restrict attention to preference parameters such that solutions are interior.

9 Defining the new random variable $\xi = v^B - v^A$, our assumption (A1) refers to the distribution of $\xi$. In this case, our results on changes in $v^A$ would then refer to location shifts of the distribution of $\xi$. 

To rule out uninteresting equilibria, we eliminate weakly dominated voting strategies. This implies that each voter votes for the candidate who provides him with the highest expected utility\(^{10}\). The candidate who wins the majority of the votes is elected and then implements her preferred policy. Voters in group A are decisive since the group encompasses a majority of voters. That is, a candidate wins if and only if she receives the support of the majority group.

2.1 Political Disagreement

The previous discussion implies that a voter from group \(i\) votes for the candidate from group A if and only if\(^{11}\)

\[
U^A(q, v^A, v^B) \geq U^B(q, v^A, v^B) \\
\iff \sum_{\theta \in \Theta} q_\theta \left[ u^i(x^{A^*}(q), \theta) - u^i(x^{B^*}(q), \theta) \right] \geq -(v^A - v^B). \tag{2}
\]

The RHS of (2) captures the realized valence differential. The LHS of (2) captures the degree of political disagreement between the two groups. That is, from the point of view of a voter in group \(i\), it captures the expected policy-payoff difference from electing the different candidates. Define the political disagreement from the point of view of group A voters as

\[
D(q) \equiv \sum_{\theta \in \Theta} q_\theta \left[ u^A(x^{A^*}(q), \theta) - u^A(x^{B^*}(q), \theta) \right]. \tag{3}
\]

Majority group A is decisive: after an experiment outcome that induces belief \(q\), candidate A wins the election if and only if she receives the support of voters in group A, \(D(q) \geq -v^A + v^B\). If the realized \(v^B\) is sufficiently high, then even voters from group A vote for candidate B, and vice-versa. Since \(v^B \sim F\), given \(v^A\), the majority candidate wins with probability

\[
W(q; v^A) \equiv F(D(q) + v^A). \tag{4}
\]

Therefore, candidate A wins the election with a probability that increases in the degree of political disagreement — candidate A has a “policy advantage” because a majority of voters

\(^{10}\)Voters have no private information about the state, so there is no information aggregation problem. Hence, the strategic voting considerations related to the probability of being pivotal are not relevant.

\(^{11}\)We abstract from abstentions. One could extend our model so that a citizen is less likely to abstain if his expected payoff difference between the candidates is higher, similar to Matsusaka (1995).
believe that she has the “correct” preference, and, hence, she will implement the “correct” policy.

In order to guarantee the existence of an optimal experiment and simplify notation, throughout the paper, we maintain the following assumption:

(A2) Political disagreement $D$ is upper semicontinuous in $\Delta(\Theta)$ and differentiable at the prior belief.

Condition (A2) holds for a large class of models, including the applications that we study throughout this paper. Differentiability of $F$ and (A2) imply that $W$ is upper semicontinuous in $\Delta(\Theta)$ and differentiable at the prior belief.\(^\text{12}\)

2.2 Notational Conventions

For vectors $q, w \in \mathbb{R}^J$, we denote by $\langle q, w \rangle$ the standard inner product in $\mathbb{R}^J$ — i.e., $\langle q, w \rangle = \sum_{j=1}^{J} q_j w_j$ — and we denote by $qw$ the component-wise product of vectors $q$ and $w$ — i.e., $(qw)_j = q_j w_j$.

For an arbitrary real-valued function $g$, define $\bar{g}$ as the concave closure of $g$,

$$\bar{g}(q) = \sup \{ y | (q, y) \in co(g) \},$$

where $co(g)$ is the convex hull of the graph of $g$.

We use $\pi \succ \pi'$ to denote that experiment $\pi$ is Blackwell more informative than experiment $\pi'$. Finally, $\text{card}(S)$ denotes the cardinality of the set $S$.

2.3 Party’s Expected Payoff

The incumbent party’s problem is to choose an experiment $\pi$ that maximizes the expected probability of victory $E_\pi[W(q; v^A)]$. Upper semicontinuity of $W$ ensures the existence of an

\(^{12}\text{Assumption (A2) implicitly establishes the following. Given $q$, if there are multiple optimal policies $x^{*}(q)$, then we select an optimal policy such that $D$ is upper semicontinuous. Moreover, it implicitly implies that we restrict attention to language-invariant equilibria — see Alonso and Câmara (2015a) for a discussion of language-invariant equilibria.}
optimal experiment, and choosing an optimal experiment is equivalent to choosing a probability distribution \( q \) over \( q \) that maximizes \( E_\sigma[W(q; v^A)] \), subject to the constraint \( E_\sigma[q] = p \) (see KG). That is, the supremum of the expected victory probability is

\[
W^* = \sup_\sigma E_\sigma[W(q; v^A)], \quad \text{s.t. } E_\sigma[q] = p.
\]

The following remarks follow immediately from KG:

(R1) An optimal experiment exists.
(R2) There exists an optimal experiment with \( \text{card}(S) \leq N \).\(^{13}\)
(R3) The IP’s maximum expected payoff is \( W^* = \tilde{W}(p; v^A) \).
(R4) The value of persuasion is \( W^* - W(p; v^A) = \tilde{W}(p; v^A) - W(p; v^A) \).

### 2.4 Application: Spatial Policy Model

Although we prove our main results using the general setup described above, for concreteness throughout the paper, we illustrate our results using the following application.

Consider a spatial policy model in which the state \( \theta \in \Theta \subset \mathbb{R} \) captures voters’ uncertainty over the optimal policy in a left-right dimension. Let \( X = [-\bar{x}, +\bar{x}] \), with \( \bar{x} \) sufficiently large. Voters in group \( A \) have a quadratic policy payoff \( u^A(x, \theta) = -(x - \theta)^2 \). From the point of view of majority voter \( A \), with belief \( q \), the optimal policy is linear on the expected value of the state, \( x^{A^*}(q) = E[\theta|q] \). Let \( x^{B^*}(q) \) be the optimal policy from the point of view of minority voter \( B \). Political disagreement (3) is

\[
D(q) = \sum_{\theta \in \Theta} q_\theta \left[ u^A(x^{A^*}(q), \theta) - u^A(x^{B^*}(q), \theta) \right]
= \sum_{\theta \in \Theta} q_\theta \left[ -(E[\theta|q] - \theta)^2 + (x^{B^*}(q) - \theta)^2 \right]
= (E[\theta|q] - x^{B^*}(q))^2. \quad (5)
\]

\(^{13}\)Note that, in the original setup of KG, there exists an optimal straightforward signal that directly recommends an action to the receiver. In our setup, the pivotal majority voter has a binary action space: vote for candidate \( A \) or \( B \). However, when \( N > 2 \) in our model, an optimal experiment might require more than two realizations. This is so because, from the point of view of the IP, before the valence shock is realized, the voting behavior is probabilistic rather than binary. That is, voting behavior can be interpreted ex ante as a continuous “action” (probability of electing \( A \)) in the interval \([0, 1]\) rather than a binary choice.
From (5), political disagreement translates naturally into the degree of disagreement over optimal policies, $D(q) = (x^A(q) - x^B(q))^2$.

The shape of the disagreement function $D$ depends fundamentally on the nature of preference misalignment between the two groups. We next present three examples, using different payoff functions for group $B$. In Example 1, disagreement endogenously becomes a strictly convex function of beliefs; therefore, any experiment $\pi$ increases the expected political disagreement, $E_\pi[D(q)] \geq D(p)$. The opposite is true in Example 2: since disagreement is strictly concave, information, on average, decreases disagreement. In Example 3, disagreement is neither concave nor convex. In these examples, we consider a binary state space $\Theta = \{0, 1\}$, and let $q_2$ be the probability that the state is $\theta = +1$. Formally,

**Example 1** — Suppose that $u^B(x, \theta) = -(x - \frac{1}{2}\theta)^2$. Then, $x^B(q) = \frac{1}{2}E[\theta|q]$, and disagreement (5) becomes $D(q) = \frac{1}{4}E[\theta|q]^2$.

**Example 2** — Suppose that $u^B(x, \theta) = -(\frac{x^2}{2} - \theta)^2$. Then, $x^B(q) = \sqrt{2E[\theta|q]}$, and disagreement (5) becomes $D(q) = \left(E[\theta|q] - \sqrt{2E[\theta|q]}\right)^2$.

**Example 3** — Suppose that $u^B(x, \theta) = -(x - \theta)^3$. Then, $x^B(q) = \frac{q_2 - \sqrt{q_2(1-q_2)}}{2q_2-1}$, and disagreement (5) becomes $D(q) = \left(q_2 - \frac{q_2-\sqrt{q_2(1-q_2)}}{2q_2-1}\right)^2$.

Figure 1 illustrates these examples. The three figures on the top contrast the optimal policy $x^A(q) = q_2$ (dashed lines) and the different optimal policies $x^B(q)$ (solid lines). The three figures on the bottom depict the corresponding political disagreement.

### 3 Valence and Information

In this section, we show that the incumbent party’s gain from any given experiment $\pi$ has a single-crossing property with respect to the incumbent’s valence. This property leads to a monotone behavior of the informativeness of optimal experiments: as we increase the incumbent’s competence $v^A$, her party does not benefit from providing a more-informative experiment.
3.1 Single-Crossing

In our model, the incumbent party seeks to maximize its candidate’s chances of re-election. Following (4), the likelihood that candidate $A$ wins the election increases in the degree of political disagreement — a larger $D$ implies that, in the eyes of group $A$ voters, the minority candidate $B$ is expected to implement a much “worse policy” than $A$. As the outcome of the experiment can change the policy championed by each candidate, as well as voters’ expected payoff from these policies, it follows that policy experimentation can change the degree of political disagreement. As a result, the IP’s choice of an experiment is driven by its desire to uncover information that increases political disagreement.

As the underlying state $\theta$ is independent of both candidates’ valences, the IP’s choice of experiment cannot affect the distribution of the challenger’s valence. Nevertheless, if the IP has access to an experiment that, on average, increases disagreement, as in the example in Figure 1(d), then it is not clear why the IP would not gain from this experiment independently of $v^A$. The next lemma shows that, for any experiment $\pi$, this gain actually satisfies a single-crossing condition: If the IP prefers not to experiment rather than provide experiment
\(\pi\) when its candidate’s valence is \(v^A\), then the IP continues to find no experimentation better than experiment \(\pi\) for any higher valence \(v^{A'} > v^A\).

**Lemma 1** Suppose that \((A1)\) and \((A2)\) hold. Consider any experiment \(\pi\) and incumbent’s valence \(v^A\). If, for the IP, no experimentation is better than experiment \(\pi\) when the incumbent has valence \(v^A\), then no experimentation continues to be better for all higher valences. That is, if \(E_\pi[W(q; v^A)] \leq W(p; v^A)\), then \(E_\pi[W(q; v^{A'})] \leq W(p; v^{A'})\) for all \(v^{A'} > v^A\).

To understand Lemma 1, note that the effect of changing disagreement by an amount \(\Delta\) is that it changes the probability of victory by \(F(z + \Delta) - F(z)\), with \(z = D(p) + v^A\). If \(\Delta > 0\), then the benefit in increasing victory probability, relative to the likelihood that the challenger’s valence induces \(z\), is given by

\[
\frac{F(z + \Delta) - F(z)}{f(z)} = \int_0^\Delta \frac{f(z + s)}{f(z)} ds. \tag{6}
\]

If \(\Delta < 0\), then the cost of decreasing victory probability relative to \(f(x)\) is

\[
\frac{F(z) - F(z + \Delta)}{f(z)} = \int_{\Delta}^0 \frac{f(z + s)}{f(z)} ds. \tag{7}
\]

Lemma 1 then follows from the fact that, for log-concave probability density functions, the ratio \(f(z + \Delta)/f(z)\) decreases in \(z\) if \(\Delta > 0\), but increases in \(z\) if \(\Delta < 0\). That is, the relative benefit (6) of increasing victory probability decreases in \(z\) — hence, in the incumbent’s competence \(v^A\) — while the relative cost (7) increases in \(z\). Integrating over all possible realizations of \(\Delta\) generated by experiment \(\pi\), we then have that the relative gain from an experiment \(\pi\) weakly decreases in the incumbent’s competence. In other words, if the IP does not gain from experiment \(\pi\) when the incumbent’s valence is \(v^A\), this is still true for an incumbent candidate of higher valence. Notice that this property is satisfied irrespective of whether, in the absence of the IP’s experiment, the incumbent is expected to win the election \((F(z) > 1/2)\) or the minority candidate is the frontrunner \((F(z) < 1/2)\).

The next proposition builds upon Lemma 1 to show that, if we increase the competence of the majority candidate, then the IP does not benefit from providing a more-informative experiment.

**Proposition 1** Suppose that \((A1)\) and \((A2)\) hold. Suppose, also, that \(\pi^*\) is an optimal experiment given incumbent’s valence \(v^A\). Then, for any higher valence, experiment \(\pi^*\) is
weakly better than any Blackwell more informative experiment. That is, for every \( v^{A'} > v^A \) and every \( \pi' > \pi^* \), we have

\[
E_{\pi^*}[W(q; v^{A'})] \geq E_{\pi^0}[W(q; v^{A'})].
\] (8)

In the proof of the proposition, we first rewrite the Blackwell more informative experiment \( \pi' \) as a payoff equivalent grand experiment. In this grand experiment, voters first observe realization \( s \) of \( \pi^* \), and then they observe an additional experiment \( \pi_s \) conditional on \( s \). When the incumbent’s valence is \( v^A \), optimality of \( \pi^* \) implies that the IP does not benefit from disclosing any additional information \( \pi_s \) after each realization \( s \) of \( \pi^* \). We then apply Lemma 1 to each posterior belief \( q^* \) in the support of \( \pi^* \): if the IP does not benefit from disclosing information in addition to \( \pi^* \) when the incumbent’s valence is \( v^A \), then the IP does not benefit from disclosing any information in addition to \( \pi^* \) when the incumbent’s valence is higher.\(^{14}\)

Next, we apply Proposition 1 to characterize the relationship between the IP’s optimal level of transparency and the incumbent’s valence.

**Corollary 1** Suppose that (A1) and (A2) hold. There are cutoffs \( v^A_1 \) and \( v^A_2 \) in the extended real line, with \( v^A_1 \leq v^A_2 \), such that:

(i) a fully informative experiment is optimal if \( v^A < v^A_1 \);

(ii) a partially informative experiment is optimal if \( v^A_1 < v^A < v^A_2 \); and

(iii) an uninformative experiment is optimal if \( v^A_2 < v^A \).

Corollary 1 defines partitions on the expected competence of the majority candidate. When the incumbent party’s candidate is sufficiently incompetent, it prefers to be completely transparent about policies, and engages in fully informative experimentation; the IP is partially transparent for intermediate levels of competence and is completely opaque (forgoes experimentation) when its candidate is sufficiently competent.

\(^{14}\)Although Lemma 1 holds for any experiment, the result in Proposition 1 is deeply rooted in the endogenous properties of optimal experiments. In general, two Blackwell-ordered experiments do not enjoy this single-crossing property. If \( \pi' \succ \pi \) for some non-optimal pair of experiments, then it might be the case that the IP prefers the less informative \( \pi \) when valence is low and prefers the more informative \( \pi' \) when valence is high: \( E_\pi[W(q; v^A)] > E_{\pi'}[W(q; v^A)] \) and \( E_\pi[W(q; v^{A'})] < E_{\pi'}[W(q; v^{A'})] \) for some \( v^{A'} > v^A \). See Section B.6 in online Appendix B for details.
Corollary 1 does not guarantee that cutoffs $v_1^A$ and $v_2^A$ are finite. Proposition B.1 in online Appendix B provides sufficient conditions so that $v_1^A$ and $v_2^A$ are finite.

3.2 Examples

We next provide some examples to illustrate the effects of the incumbent’s valence $v^A$ on the IP’s payoff function $W$ and on the optimal experiment.

Recall that $W(q; v^A) = F(D(q) + v^A)$. Figure 2 illustrates how a higher $v^A$ increases $W$ for each $q$ and changes the overall curvature of $W$. It assumes that $F$ follows a Normal Distribution and uses the political disagreement $D$ from the spatial policy model in Figure 1(d).

Recall that we can derive the optimal experiment from the concave closure of $W$ (see KG for details). In particular, whether $W$ is concave or convex is important to define whether or not the IP benefits from implementing an informative experiment. Although in Figure 1(d) disagreement $D$ is strictly convex, the resulting payoff $W$ might be locally concave or locally convex, depending on belief $q_2$ and on valence $v^A$. Log-concavity of $f$ implies that $F(D(q) + v^A)$ is locally concave for sufficiently high values of $D(q) + v^A$ and locally convex for sufficiently low values. The red solid lines in Figure 3 depict the concave closure of $W$. We next use Figure 3 to derive an optimal experiment.

First, suppose that $v^A$ is sufficiently low, as in Figure 3(a). The IP’s payoff $W$ is everywhere strictly convex; hence, any optimal experiment must be fully informative, independently of the prior belief.

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15E.g., in Example 2 from Section 2.4, if the prior belief already maximizes disagreement, $p_2=0.5$, then no information disclosure is optimal for all values of $v^A$, so that $v_1^A = v_2^A = -\infty$.  

---

Figure 2: Effects of $v^A$ on victory probability $W$, using disagreement $D$ from Figure 1(d).
Now suppose that $v^A$ is intermediate, as in Figure 3(b). The concave closure $\tilde{W}$ is given by a straight line in the set of beliefs $q_2 \leq \tilde{q}$, and by $W$ itself for $q_2 \geq \tilde{q}$. Consequently, no experimentation is optimal for all priors $p_2 \geq \tilde{q}$. When $p_2 \geq \tilde{q}$, although any informative experiment increases average disagreement ($D$ is strictly convex), any informative experiment is strictly worse for the IP than no information disclosure. Signal realizations that increase political disagreement increase victory probability by only a small amount, while signal realizations that decrease political disagreement decrease victory probability by a relatively large amount. Now suppose that $p_2 \leq \tilde{q}$. Since in this set $\tilde{W}(q; v^A) > W(q; v^A)$, policy experimentation is valuable. Every optimal experiment is partially informative and induces exactly two posterior beliefs, $q_2 = 0$ and $q_2 = \tilde{q}$. Finally, for each prior belief $p_2 \in (0, 1)$, optimal experiments are less informative in Figure 3(b) than in Figure 3(a).

As we further increase $v^A$, the cutoff $\tilde{q}$ decreases to $\tilde{q}'$ — see Figure 3(c). Therefore, no experimentation is optimal for a larger set of prior beliefs. Moreover, for the prior beliefs in the set $p_2 \leq \tilde{q}'$, every optimal experiment is supported only on the posterior beliefs $q_2 = 0$ and $q_2 = \tilde{q}'$. Consequently, the partially informative experiment in Figure 3(c) is less informative than the partially informative experiment in Figure 3(b).

What if political disagreement is everywhere strictly concave, as in Figure 4? Figures 5 and 6 use a normally distributed $v^B$ to illustrate the corresponding victory probability $W$, which might be locally concave or convex, depending on the incumbent’s valence. See online Appendix B (Section B.2.1) for a detailed discussion of this example.

We conclude by highlighting that the IP might find it optimal to experiment even when its candidate is the frontrunner and might find it optimal not to experiment even when its candidate is the underdog. For example, in Figure 3(b), if the prior belief is $p_2 = 0.8$, then,
without experimentation, the majority candidate wins with a very high probability, above 90%. Nevertheless, it is optimal for the incumbent party to provide a partially informative experiment, because it increases its candidate's expected victory probability even further. In Figure 6(a), if the prior belief is $p_2 = 0.45$, then, without information disclosure, the majority candidate wins with a very low probability, around 26%. Nevertheless, any informative experiment decreases the candidate’s expected victory probability even further.

![Figure 4: Strictly Concave Political Disagreement.](image1)

![Figure 5: Effects of $v^A$ on victory probability $W$, using disagreement $D$ from Figure 4.](image2)

![Figure 6: Concave closure of $W$ from Figure 5.](image3)
4 Disagreement as a Function of the Expected State

To derive a sharper characterization of optimal experiments, in this section, we focus on models in which political disagreement is a strictly increasing function of the expected value of some unknown state. Formally, we assume:

\[(A2')\] Political disagreement takes the form \[D(q) = H(E[\theta|q]),\] where \(H\) is twice differentiable and strictly increasing. Moreover, the ratio \[\frac{H''}{H'^2}\] is non-increasing.

Assumption \((A2')\) holds in many important cases. For example, it holds if disagreement is a power function of expectation \[D(q) = \gamma E[\theta|q]^{\rho},\] with \(\gamma > 0, \theta_1 \geq 0\) and \(\rho \geq 1\). The spatial policy model of Section 2.4 satisfies \((A2')\) when voters have quadratic payoffs \[u^A(x, \theta) = -(x - \beta^A\theta)^2\] and \[u^B(x, \theta) = -(x - \beta^B\theta)^2,\] where \(\beta^A\) and \(\beta^B\) are known preference parameters and \(\theta_1 \geq 0\). In this case, disagreement is proportional to the square of the expectation of the state, \[D(q) = (\beta^A - \beta^B)^2 E[\theta|q]^2.\] Later in this section, we study two other relevant applications in which \((A2')\) holds (optimal tax and the relative importance of policy dimensions).

Given \((A2')\), political disagreement increases if voters learn that the realized state is “high,” which benefits the incumbent, and disagreement decreases if they learn that the state is “low.” One could then conjecture that the incumbent party would prefer to hide information about low-disagreement states, and to fully disclose information about high states. However, Proposition 2 shows that the opposite is true. Borrowing from the statistics literature, we define an upper-censoring experiment (or right-censoring experiment) as one that fully reveals low-disagreement states and pools high-disagreement states. Formally:

**Definition:** Experiment \(\pi\) is upper-censoring at cutoff state \(\theta_k\) if it has a realization space \(S = \{s_1, \ldots, s_k, s_{\text{pooling}}\}\) and the following holds. For each \(n < k\), state \(\theta_n\) induces signal realization \(s_n\) with probability one. For each \(n > k\), state \(\theta_n\) induces signal realization \(s_{\text{pooling}}\) with probability one. Cutoff state \(\theta_k\) induces realization \(s_{\text{pooling}}\) with some probability \(\alpha_k \in [0, 1]\) and induces realization \(s_k\) with probability \(1 - \alpha_k\).

**Proposition 2** Suppose that \((A1)\) and \((A2')\) hold. Then, there exists an optimal experiment \(\pi^*\) that is upper-censoring at some cutoff state \(\theta_k\). Moreover, cutoff state \(\theta_k\) weakly decreases with the incumbent’s valence \(v^A\).
In the proof of Proposition 2, we show that for each optimal experiment \( \pi^* \), there exists a payoff-equivalent upper-censoring experiment. The intuition behind the result is the following. Under our assumptions, given \( v^A \), the IP’s payoff \( W(q; v^A) = F(H(E[\theta|q]) + v^A) \) is concave if \( E[\theta|q] \) is high and strictly convex if \( E[\theta|q] \) is low. Strict convexity implies that the incumbent party always strictly benefits from providing additional information if the initial experiment yields a non-degenerate belief corresponding to a low expected state. Therefore, outcomes under optimal experiments that indicate the state to be low must be fully revealing. Conversely, concavity of the incumbent’s payoffs implies that the IP cannot be made worse off by an experiment that pools all outcomes corresponding to high expected states into a single realization. That is, the incumbent then (weakly) gains from bundling all states in the concave (high-disagreement) region: they all induce signal pooling with probability one, resulting in a single posterior belief \( q^+ \) and a high expectation \( E[\theta|q^+] \).

While the IP does not gain from designing an experiment that pools together only states in the convex region, it may gain from “hiding” some low-disagreement states, such that these states induce signal pooling with positive probability. Of course, pooling low-disagreement states would make \( s_{\text{pooling}} \) more likely but would reduce expected disagreement if \( s_{\text{pooling}} \) occurs. Still, the incumbent must decide which disagreement states should be pulled in \( s_{\text{pooling}} \). Suppose that \( \theta_l \) and \( \theta_h \) are in the convex region, with \( \theta_l < \theta_h \). Should \( \theta_l \) or \( \theta_h \) be the incumbent’s first choice to be mixed with the high-disagreement signal \( s_{\text{pooling}} \)? The IP now faces an important tradeoff. One the one hand, pooling \( \theta_h \) leads to a lower reduction in posterior disagreement resulting from \( s_{\text{pooling}} \). On the other hand, disclosing \( \theta_l \) to voters is worse than disclosing \( \theta_h \); thus, “hiding” \( \theta_l \) by pooling it with \( s_{\text{pooling}} \) is more important than hiding \( \theta_h \). The proof of Proposition 2 shows that, given (A1) and (A2'), the first effect always dominates: the IP’s optimal decision must be a cutoff on \( \bar{\theta} \), independent of prior beliefs, the incumbent’s valence, and the other parameters of the model — these values are relevant only for defining the actual cutoff state.

Finally, the cutoff state defined by Proposition 2 monotonically decreases with the incumbent’s valence \( v^A \). This implies that the set of optimal upper-censoring experiments that we construct are Blackwell-ordered: they become less Blackwell-informative as the majority candidate become more competent.

It is important to note that the logic behind the proof of Proposition 2 applies to a broad
class of models. Consider a Bayesian persuasion game between a sender and a receiver, as in KG. Suppose that the sender’s payoff can be written as a twice differentiable, strictly increasing function of the expected state. If the derivative of the sender’s payoff function is single-peaked, then there exists an optimal experiment that is upper-censoring. In our model, conditions (A1) and (A2') simply imply that this derivative is log-concave, hence single-peaked. See Proposition A.1 in Appendix A for details.

We next present two additional applications of our model, in which political disagreement endogenously becomes a function of the expected state, and Proposition 2 applies.

### 4.1 Application: Optimal Tax

Consider the following model, in which the elected politician must choose a proportional income tax \( x \in [0, 1] \). Voters care about the consumption of a private good and a public good. Each voter in group \( i \in \{A, B\} \) is endowed with income \( \beta^i > 0 \), where \( \beta^A \neq \beta^B \). Given the implemented tax rate \( x \), voter \( i \) consumes \((1-x)\beta^i\) units of the private good. The government uses all tax revenues to produce the public good. The production technology is such that the government produces \( x^{\psi} \) units of the public good, where \( \psi \in (0, 1) \) is a known technology parameter.\(^{16}\) Voters’ policy payoff is \( u^i(x, \theta) = (1-x)\beta^i + \theta x^{\psi} \), where state \( \theta \) represents the unknown marginal value of the public good, with \( 0 \leq \theta_1 < \ldots < \theta_N < \max\{\beta^A, \beta^B\}^{1/\psi} \).

Given belief \( q \), the optimal tax rate of voter \( i \) is \( x^i*(q) = \left( \frac{\psi E[\theta|q]}{\beta^i} \right)^{\frac{1}{1-\psi}} \). Both groups want higher taxes if the marginal value of the public good is higher. However, voters agree on the optimal tax if and only if the expected marginal value of the public good is zero. Political disagreement increases with \( E[\theta|q] \):

\[
D(q) = \left( 1 - x^{A*}(q) \right) \beta^A + E[\theta|q](x^{A*}(q))^{\psi} - \left( 1 - x^{B*}(q) \right) \beta^A - E[\theta|q](x^{B*}(q))^{\psi} = \gamma E[\theta|q]^\rho,
\]

where \( \gamma \equiv \psi \frac{1}{1-\psi} \left\{ (1-\psi)(\beta^A)^{\frac{1}{1-\psi}} + \beta^A \psi (\beta^B)^{\frac{1}{1-\psi}} - (\beta^B)^{\frac{1}{1-\psi}} \right\} > 0 \) and \( \rho \equiv \frac{1}{1-\psi} > 1 \).

Independently of whether the majority group is richer or poorer than the minority group \((\beta^A \text{ is higher or lower than } \beta^B)\), disagreement is a power function of the expected state.

\(^{16}\)Without loss of generality, let \( m^A \beta^A + m^B \beta^B = 1 \), where \( m^i \) is the number of voters in group \( i \). Total tax revenue is then \( x \).
and satisfies the conditions of Proposition 2. To maximize the majority candidate’s victory probability, the IP’s optimal experiment either partially reveals that the public good is sufficiently important, or fully reveals that the public good has a low marginal value.

4.2 Application: The Relative Importance of Policy Dimensions

There is a single policy issue in the spatial policy model of Section 2.4 and in the optimal tax model of Section 4.1. Moreover, information about \( \theta \) induces politicians to reevaluate their beliefs and choose a new policy. In other important cases, the policy issue is multidimensional, and voters and politicians are convinced about what the optimal policy is, but they are uncertain about the relative importance of different policy dimensions.

To study these cases, consider the following model. There are \( d \geq 2 \) policy dimensions (e.g., public education, public health, national defense, etc.). A policy is a \( d \)-dimensional vector \( x = (x_1, \ldots, x_d) \). The preferences of voter \( i \in \{A, B\} \) are captured by the preference vector \( \beta^i = (\beta^i_1, \ldots, \beta^i_d) \) and by the loss function \( l \), with \( l(0) = 0 \) and \( l' > 0 \). Voter’s policy payoff is

\[
    u^i(x, \theta) = \sum_{j=1}^{d} -\lambda_j(\theta)l(|x_j - \beta^i_j|),
\]

where each function \( \lambda_j(\theta) \) captures the relative importance of policy dimension \( j \) given state \( \theta \), with \( \sum_{j=1}^{d} \lambda_j(\theta) = 1 \) and \( \lambda_j(\theta) > 0 \). Note that voters’ preferred policies are independent of beliefs about \( \theta \), \( x^*_j(q) = \beta^i_j \). Although voters know their preferred policy for education and national defense, they are uncertain about which policy issue will be more important during the next term.

The degree of political disagreement, from the point of view of voters in group \( A \), is simply the expected (weighted) loss from the policy of candidate \( B \),

\[
    D(q) = E \left[ \sum_{j=1}^{d} -\lambda_j(\theta)l(|\beta^A_j - \beta^B_j|) \right] - \sum_{j=1}^{d} -\lambda_j(\theta)l(|\beta^B_j - \beta^A_j|) \]

\[
    = E \left[ \sum_{j=1}^{d} \lambda_j(\theta)l(|\beta^B_j - \beta^A_j|) \right].
\]

To apply Proposition 2, rewrite the unknown state as follows. For each \( \theta \in \Theta \), compute \( \theta' \equiv \sum_{j=1}^{d} \lambda_j(\theta)l(|\beta^B_j - \beta^A_j|) \). Define a new state space \( \Theta' \) as the collection of \( \theta' \). We can then rewrite disagreement simply as the expected value of \( \theta' \) and apply Proposition 2.
In summary, voters have a fundamental disagreement over the optimal policy but are uncertain about how important each policy dimension will be. For instance, suppose that there are only two issues. Voters disagree relatively more on national defense and less on education (that is, $|\beta^B_j - \beta^A_j|$ is larger for the national defense dimension). The incumbent’s optimal experiment pools together states that attach more weight to national defense and fully reveals states that attach more weight to education. That is, the optimal experiment either reveals that the controversial national defense issue will be “sufficiently important” in the upcoming years, or it fully reveals that the more agreeable education issue will be more important.

5 Discussion

5.1 Log-convex Valence Distribution

The results in Sections 3 and 4 depend fundamentally on the assumption that the p.d.f. of the challenger’s valence distribution is log-concave. The results are reversed if we change (A1) so that $f$ is log-convex.\(^{17}\) In the log-convex case, the single-crossing property goes in the opposite direction: lower values of the incumbent’s competence $v^A$ induce less experimentation, while higher competence induces more experimentation. Moreover, suppose that political disagreement is a strictly increasing function of the expected state, as in Section 4. With a log-convex $f$, if we change assumption (A2') so that the ratio $\frac{H''(H')^2}{(H')'}$ is non-decreasing,\(^{18}\) then the optimal experiment is lower-censoring at some cutoff state $\theta_k$. Furthermore, this cutoff state decreases with the incumbent’s valence $v^A$; hence, the experiment becomes more informative.

The reason for the sharp change in results is rooted in the change in the curvature of the incumbent’s victory probability as a function of political disagreement and valence $v^A$. Loosely speaking, in the log-concave case, it is as if the IP features increasing absolute risk aversion (IARA). This is so because the coefficient of absolute risk aversion $\frac{u''}{\frac{u}{u''}}$ in our setup takes the form $-\frac{f''}{f}$, which is increasing if $f$ is log-concave. When disagreement and the

\(^{17}\)With a log-convex p.d.f., we can no longer assume full support on the real numbers. To simplify presentation, we want to avoid corner solutions — that is, cases such that the victory probability $F(D(q) + v^A)$ equals one or zero. To this end, we assume that the support of the challenger’s valence $v^B$ is large, and restrict attention to valence $v^A$ and disagreement $D$ such that $F(D(q) + v^A) \in (0, 1)$ for all $q \in \Delta(\Theta)$.

\(^{18}\)For example, if disagreement is a power function $D(q) = \gamma E[\theta]_q^\rho$, with $\gamma > 0$, $\theta_1 \geq 0$ and $\rho \in (0, 1]$. 
incumbent’s valence are low, the IP benefits from gambling on disagreement. That is, the IP benefits from implementing a risky experiment that might increase or decrease disagreement. When disagreement and valence are high, the IP prefers to avoid these gambles. In the log-convex case, it is as if the IP features DARA, and the reverse results hold.

5.2 Information and Voter Welfare

In many voting models, interested parties strategically provide information to voters. In some cases, this information can adversely affect voters’ equilibrium welfare — voters’ payoff would be higher if they made uninformed choices. For instance, in Alonso and Câmara (2015b), the information provided by the IP always weakly decreases the expected payoff of a majority of voters under a simple majority voting rule. This is so because the optimal experiment has signal realizations targeting different winning coalitions of voters.

In our model, the IP cannot target different winning coalitions because voters in group $A$ are representative. Nevertheless, the next proposition shows that the IP’s optimal experiment may hurt all voters in majority group $A$.

**Proposition 3** Suppose that $(A1)$ holds. Consider the spatial policy model from Section 2.4, with payoffs $u^i(x, \theta) = -(x - \beta^i \theta)^2$. If either $\beta^A \beta^B < 0$ or $|\beta^B| > 2|\beta^A|$, then there exists a finite cutoff $\tilde{v}^A$ such that, for any $v^A < \tilde{v}^A$, the IP’s optimal experiment strictly decreases the expected payoff of all voters in majority group $A$.

The result follows from the different interests of the IP and voters $A$, and from the fact that the IP benefits from promoting disagreement. The IP’s goal is to elect candidate $A$, not to ensure that the elected candidate implements a good policy for voters $A$. Candidate $A$ is more likely to win when information leads candidate $B$ to adopt a new policy that voters $A$ consider worse. This worse policy benefits the IP, but it hurts voters $A$ when candidate $B$ is elected.

Under the conditions of the proposition, the IP implements a fully informative experiment, and the information (on average) leads candidate $B$ to implement a worse policy for voters $A$. This is the case here, as preference misalignment is sufficiently severe: If $\beta^A \beta^B < 0$ (candidates want policies with opposing signals) or $|\beta^B| > 2|\beta^A| \geq 0$ (from $A$’s point of view,
candidate B “overreacts” to information), then voter A strictly prefers an elected candidate B not to have access to any informative experiment. When \( v^A \) is sufficiently low, candidate A is unlikely to win (the benefit of providing information to candidate A is small), while candidate B is likely to win (the loss of providing information to candidate B is large). Hence, voters in group A are strictly worse off because of the IP’s experiment. They would prefer no experiment over the IP’s equilibrium experiment.

### 5.3 Policy-Motivated Parties

In this paper, we focus on a purely office-motivated incumbent party, whose primary concern is to be re-elected. Consider, now, the opposite case: a purely policy-motivated party. Suppose that each party has the same payoff function as the voter it represents, and parties do not receive any direct benefit from holding office. That is, when party \( i \in \{A, B\} \) is the incumbent, it chooses the experiment that maximizes the expected payoff of voter \( i \).

Intuitively, if the payoff functions of the two groups are sufficiently aligned, then a fully informative experiment is optimal, independently of the incumbent’s valence. This is in spite of the fact that a fully informative experiment might reduce the probability of re-electing the incumbent. That is, the policy-motivated IP is willing to sacrifice its re-election probability in order to generate more information to both candidates and guarantee a better policy.

However, the incumbent faces a more intricate problem when there is a large conflict of preferences (for example, see the preferences in Proposition 3). In this case, the IP prefers a more-informative experiment when the incumbent’s valence is large and she is almost sure to win. In this case, it is valuable to provide information to the likely winner. In contrast, when the incumbent is very incompetent and likely to lose, the IP prefers to implement a less-informative experiment. Again, the policy-motivated IP is willing to forgo the possibility of implementing a more-informative experiment — and, hence, increase its re-election probability — simply to avoid detrimental policies from the opposing party.

Interestingly, it seems that the purely policy-motivated case resembles the case of a purely office-motivated party, with a log-convex valence distribution. If, empirically, one finds that more competent incumbents tend to implement weakly more-informative experiments, then it is harder to say whether politicians are policy-motivated or office-motivated with a log-
convex distribution of valence. However, if, empirically, one finds that more competent incumbents choose less-informative experiments, then it is more likely that parties are purely office-motivated with a log-concave valence distribution.

It would be interesting to consider an alternative model in which parties are both policy- and office-motivated. We leave this promising agenda to future work.

5.4 Political Disagreement vs. Polarization

It is important to highlight that our notion of “political disagreement” is different from the notion of “polarization” seen in many papers in the literature. That is, an increase in political disagreement does not imply an increase in polarization, and vice versa. See the example in online Appendix B, Section B.5.

6 The Role of Belief Disagreement

Heterogeneous prior beliefs play an important role in politics — see Millner, Ollivier, and Simon (2014) for a recent review of the literature on heterogeneous priors in politics. We now extend our analysis to the case in which voters in the same group share a common prior belief, but voters in opposite groups openly disagree over the likelihood of state $\theta$.

Formally, voters in group $i$ have a common prior belief $p^i$ in the interior of the simplex $\Delta(\Theta)$, but prior beliefs differ across groups, $p^A \neq p^B$. To simplify presentation, assume that each party shares its affiliates’ beliefs. Preferences and prior beliefs are common knowledge — voters “agree to disagree.” If we interpret $\theta$ as describing the mapping between policy $x$ and outcomes, then different prior beliefs represent differences in voters’ views about the outcomes that different government policies produce.

Let $q^A$ and $q^B$ be the posterior beliefs of voters in groups $A$ and $B$ after observing the experiment’s results. Voters can correctly predict the policies $x^A^*(q^A)$ and $x^B^*(q^B)$ that each candidate would implement if elected. From the point of view of voters in group $A$, political

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19 Although there are different definitions of polarization in the literature, here, we define polarization as the Euclidean distance between the policies supported by the candidates, as in Dixit and Weibull (2007).
disagreement (3) becomes
\[ D(q^A, q^B) \equiv \sum_{\theta \in \Theta} q^A_\theta \left[ u^A(x^{*A}(q^A), \theta) - u^A(x^{*B}(q^B), \theta) \right]. \] (9)

As in the common priors case, voters in majority group \(A\) are decisive, and candidate \(A\) wins with probability \(F(D(q^A, q^B) + v^A)\). Again, candidate \(A\) wins the election with a probability that increases in the degree of political disagreement — candidate \(A\) has a “policy advantage” because a majority of voters believe that she has not only the “correct” preference, but also the “correct” belief, and, thus, she will implement the “correct” policy.

Let \(r_\theta \equiv \frac{p^B_\theta}{p^A_\theta}\) and \(r \equiv (r_\theta)_{\theta \in \Theta}\) capture the likelihood ratio of prior beliefs. We can then use the results from Alonso and Câmara (2015a) to express disagreement \(D(q^A, q^B)\) as a function \(D(q^A)\), which depends only on the beliefs of voters in group \(A\):
\[ D(q^A) \equiv D \left( q^A, q^A, \frac{r}{\langle q^A, r \rangle} \right). \]
Victory probability then becomes \(F(D(q^A) + v^A)\), and our results continue to hold for this new function — see online Appendix B for a more detailed analysis.

### 6.1 Increasing Belief Disagreement

To shed some light on the role of belief disagreement, we now focus on cases in which all voters share the same preferences, so that political disagreement is zero when voters hold a common belief. As Callander (2011, pg. 657) notes, “[o]n some policy issues it is conceivable that we all share common outcome preferences (or at least similar preferences), yet we disagree as to how best to go about achieving the desired outcome. […] Viewed this way, much political disagreement is over beliefs rather than outcomes.”

For example, consider the spatial policy model from Section 2.4. Suppose that voters share the same payoff function \(u^A(x, \theta) = u^B(x, \theta) = -(x - \theta)^2\). Recall that the optimal policy is \(x^{*i}(q^i) = E[\theta|q^i]\). Political disagreement (9) translates naturally into the degree of belief disagreement over expectations,
\[ D(q^A, q^B) = \sum_{\theta \in \Theta} q^A_\theta \left[ u^A(x^{*A}(q^A), \theta) - u^A(x^{*B}(q^B), \theta) \right] \]
\[ = \sum_{\theta \in \Theta} q^A_\theta \left[ -(E[\theta|q^A] - \theta)^2 + (E[\theta|q^B] - \theta)^2 \right] \]
\[ = (E[\theta|q^A] - E[\theta|q^B])^2. \] (10)

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Similarly, consider the tax model from Section 4.1. Suppose that voters have the same income normalized to one, $\beta^A = \beta^B = 1$, and the production technology of the public good is $\psi = \frac{1}{2}$. The optimal tax rate becomes $x^{\star\star}(q^i) = \frac{E[\theta|q^i]^2}{4}$, and political disagreement takes the simple form $\mathcal{D}(q^A, q^B) = \frac{1}{4}(E[\theta|q^A] - E[\theta|q^B])^2$. In this case, disagreement over the optimal tax derives solely from the belief disagreement over the marginal value of the public good.

When voters have the same payoff function but different beliefs, can the IP increase political disagreement? As in these two applications, suppose that political disagreement is a strictly increasing function of the difference between voters’ expectation over the state. Although voters share a common payoff function, we show in the next proposition that if the state space is rich enough, then the IP can generically design an experiment that increases political disagreement with probability one.

**Proposition 4** Suppose that political disagreement strictly increases in the degree of belief disagreement over expectations, $\mathcal{D}(q^A, q^B) = R(|E[\theta|q^A] - E[\theta|q^B]|)$, where $R \geq 0$ and $R' > 0$. If $N \geq 4$, then the IP can generically design an experiment that increases political disagreement with probability one. Consequently, if $F$ has full support on the real numbers, the value of persuasion is positive for each finite incumbent’s valence $v^A$.

The following example illustrates Proposition 4 and shows how the IP can guarantee a higher disagreement.\textsuperscript{21,22}

**Example 4 — Increasing Belief Disagreement:** Let $\Theta = \{1, 2, 3, 4\}$. Consider priors $p^A = (.05, .45, .45, .05)$ and $p^B = (.45, .05, .05, .45)$, so that $E[\theta|p^A] = E[\theta|p^B] = 2.5$. Although prior beliefs are different, initial political disagreement is zero. The following binary experiment $S = \{s_1, s_2\}$ is optimal for the IP. States 1 and 3 induce signal $s_1$ with probability one, while states 2 and 4 induce signal $s_2$ with probability one. After observing

\textsuperscript{20}Genericity is interpreted over the space of pairs of prior beliefs.

\textsuperscript{21}In Section B.2.1 of online Appendix B, we present a budget allocation model in which the degree of political disagreement is endogenously given by the degree of belief disagreement, as measured by the relative entropy. In that case, any informative experiment decreases the expected political disagreement. Nevertheless, the IP still finds it optimal to implement a partially informative experiment when the expected valence of candidate $A$ is sufficiently low.

\textsuperscript{22}See Kartik, Lee and Suen (2015) for conditions such that a Blackwell more informative experiment, on average, brings posterior beliefs closer to each other.
signal $s_1$ beliefs become $E[\theta | q^A] = 2.8$ and $E[\theta | q^B] = 1.2$, while $s_2$ induces $E[\theta | q^A] = 2.2$ and $E[\theta | q^B] = 3.8$. That is, from the point of view of voters $A$, candidate $B$ not only “overreacts” to the information (updates her policy “too much”), but also moves the policy in the wrong direction. Therefore, the experiment induces a strictly higher belief disagreement for any of its realizations. This experiment is optimal independently of valence $v^A$ and distribution $F$, because each realization induces the maximum feasible disagreement, given the prior beliefs.

It is important to note that, although all voters share the same payoff function, this information strategically provided by the IP can decrease the expected payoff of a majority of voters. Without the IP’s experiment, both candidates support the same policy. Consequently, the candidate with the highest realized valence wins. From the point of view of majority voters $A$, the information provided by the IP’s optimal experiment has a small positive effect on candidate $A$’s policy, but a large negative effect on candidate $B$’s policy. From the point of view of voters $A$, information moves the policy of candidate $B$ by too much and in the wrong direction. Consequently, as in Proposition 3, this negative effect dominates if candidate $B$ is sufficiently more likely to win ($v^A$ is sufficiently small). □

7 Extensions

In this section, we extend our model to cases in which the IP supports the minority candidate and experiments are costly. In online Appendix B, we examine the role of post-election information and consider valence shocks that are independent across voters.

**IP Supports the Minority:** So far, we have assumed that the IP supports the majority candidate — that is, candidate $A$ is the incumbent. Now suppose that the minority party $B$ is in power (hence controls the experiment) and supports the incumbent candidate $B$. Since the political advantage of the majority candidate is due solely to political disagreement, the IP now benefits from *decreasing* political disagreement. The results from Section 3 now apply to the valence $v^B$ of the incumbent: the IP uses less-informative experiments when the minority incumbent is more competent ($v^B$ is high) and more-informative experiments when she is less competent. Interestingly, the optimal experiment in Proposition 2 becomes *lower-censoring*: the minority party pools low disagreement states and fully reveals high
disagreement states.

Moreover, consider the models of Section 6.1, in which citizens share the same payoff function but hold different prior beliefs. In these cases, regardless of prior beliefs, full information disclosure is always optimal for the minority candidate. Complete transparency eliminates political disagreement and the policy advantage of the majority candidate, thus increasing the chances of the minority candidate. Therefore, for policy issues in which political disagreement derives solely from belief disagreement, we should empirically observe that policy experiments by minority incumbents are more informative than those of majority incumbents.

Costly Policy Experiments: Our basic model assumes that experiments are costless. If every experiment is equally costly, then, whenever the IP decides to implement an experiment, it implements the optimal experiment we describe. The only change is that the IP implements an experiment only if the value of persuasion is higher than the fixed cost of implementation.

What if different experiments have different costs? Following Gentzkow and Kamenica (2014), suppose that the cost of an experiment is given by the expected relative entropy of the beliefs that it induces. Consequently, more-informative experiments are more costly. In this case, one can show that our results from Section 3 continue to hold. However, perfectly revealing a state is infinitely costly. Therefore, full information disclosure and upper-censoring experiments are never optimal.

8 Conclusion

In this paper, we study strategic policy experimentation by office-motivated parties. That is, we study strategic learning about how different policies map into the payoffs of different voters. In our environment, information changes the degree of political disagreement and sways future elections — experimental outcomes that increase disagreement increase the victory probability of the candidate who has similar preferences and beliefs as a majority of voters. Therefore, office-motivated party bureaucrats supporting this majority candidate benefit from policy experiments that create more dissent between the majority and the minority.
We derive several results regarding the informational content of optimal policy experiments. First, we define conditions such that the informativeness of optimal experiments has a monotonicity property with respect to the competence of the incumbent politician. Optimal experiments under a more competent politician are never more informative than optimal experiments under a less competent politician. Second, when disagreement is an increasing function of the expected state, there exists an optimal experiment that is upper censoring — it fully reveals low disagreement states and pools high disagreement states. Under these conditions, we establish the sharper result that the informativeness of the optimal experiment, in fact, decreases with the incumbent’s competence. Finally, we consider cases in which all voters share the same payoff function, so that political disagreement is due solely to belief disagreement. We show that, even in these cases, policy experiments can be used to increase disagreement and benefit the majority candidate.

A Appendix

Before we present the proof of Lemma 1, we provide the following lemma.

Lemma A.1 Fix any \( a, b, c \in \mathbb{R} \). Define

\[
G(a, b, v^A) \equiv \frac{F(b + v^A) - F(a + v^A)}{f(a + v^A)}.
\]

(11)

If \( F \) satisfies (A1), then \( G(a, b, v^A) \) is non-increasing in \( v^A \).

Proof of Lemma A.1: We first rewrite the function \( G \) as

\[
G(a, b, v^A) = \int_0^{b-a} \frac{f(a + v^A + z)}{f(a + v^A)} \, dz.
\]

Since \( f \) is log-concave, it exhibits decreasing ratios in the sense that for every \( z > 0 \) and \( v^A \geq v'^A \) we have

\[
\frac{f(a + v'^A + z)}{f(a + v^A)} \geq \frac{f(a + v^A + z)}{f(a + v^A)}.
\]

(12)

Suppose first that \( b > a \). Then integrating both sides of (12) between 0 and \( b - a \) shows that \( G(a, b, v'^A) \geq G(a, b, v^A) \). Now suppose that \( a > b \). Then for any \( z \in [0, a - b] \) we can
rewrite (12) as
\[ \frac{f(a + v^A - z)}{f(a + v^A)} \geq \frac{f(a + v^{A'} - z)}{f(a + v^{A'})}. \]

Integrating between 0 and \(b - a\) we conclude that
\[ -G(a, b, v^A) = \int_0^{a-b} \frac{f(a + v^A - z)}{f(a + v^A)}dz \geq \int_0^{a-b} \frac{f(a + v^{A'} - z)}{f(a + v^{A'})}dz = -G(a, b, v^{A'}), \]
or, in other words, \(G(a, b, v^A) \geq G(a, b, v^{A'})\).

**Proof of Lemma 1:** Consider an experiment \(\pi\) that generates a distribution \(\sigma \in \Delta(\Delta(\Theta))\) over posterior beliefs. Note that this distribution is independent of valences. For any \(q\) in the support of \(\sigma\), the change in the victory probability of the majority candidate is
\[ W(q; v^A) - W(p; v^A) = F(D(q_A) + v^A) - F(D(p) + v^A) = f(D(p) + v^A)G(D(p), D(q), v^A), \]
where \(G\) is defined by (11). Therefore, the expected change in victory probability from experiment \(\pi\) can be written as
\[ E_\pi[W(q; v^A) - W(p; v^A)] = f(D(p) + v^A) \int_{q \in \text{supp}(\sigma)} G(D(p), D(q), v^A)d\sigma. \]
Because \(f > 0\) rewrite
\[ \frac{E_\pi[W(q; v^A) - W(p; v^A)]}{f(D(p) + v^A)} = \int_{q \in \text{supp}(\sigma)} G(D(p), D(q), v^A)d\sigma. \tag{13} \]
From Lemma A.1, we know that \(G\) is non-increasing in \(v^A\), hence the LHS of (13) is non-increasing in \(v^A\). This implies that if \(E_\pi[W(q; v^A) - W(p; v^A)] \leq 0\) then \(E_\pi[W(q; v^{A'}) - W(p; v^{A'})] \leq 0\) for any \(v^{A'} > v^A\), concluding the proof.

**Proof of Proposition 1:** Suppose \(\pi^*\) is an optimal experiment given valence \(v^A\). Take any \(v^{A'} > v^A\) and any \(\pi'\) that is Blackwell more informative than \(\pi^*\). The proof has two steps. In the first step we construct a grand experiment \(\{\pi^*, \{\pi_s\}_{s \in S}\}\) that is payoff equivalent to \(\pi'\). In the second step we show that since \(\pi^*\) is weakly better than \(\{\pi^*, \{\pi_s\}_{s \in S}\}\) when valence is \(v^A\), then \(\pi^*\) is weakly better than \(\{\pi^*, \{\pi_s\}_{s \in S}\}\) when the valence is higher. Consequently, \(\pi^*\) is weakly better than \(\pi'\) for any \(v^{A'} > v^A\).

**Step 1:** Let \(Q^*\) be the set of posterior beliefs \(q\) in the support of \(\pi^*\), and \(Q'\) be the set of posterior beliefs \(q\) in the support of \(\pi'\). The fact that \(\pi'\) is Blackwell more informative than
implies that $Q^*$ is contained in the convex hull of $Q'$. Consequently, we can decompose the more informative experiment into two parts. Players first observe the realization $s \in S$ of experiment $\pi^*$. Then they observe the realization of an additional experiment $\pi_s$. Note that experiment $\pi_s$ can be different for different signal realizations. This grand experiment \{$(\pi^*, \{\pi_s\}_{s \in S})$\} induces the same probability distribution over posterior beliefs as $\pi'$, hence they are payoff equivalent.

**Step 2:** When the incumbent’s valence is $v^A$, optimality of $\pi^*$ implies that after each signal realization $s$ of $\pi^*$ the IP does not benefit from further disclosing information. That is, for every posterior belief $q^*$ in the support of $\pi^*$ and every experiment $\pi_s$, we have

$$E_{\pi_s}[W(q;v^A)|q^*] \leq W(q^*;v^A).$$

Apply Lemma 1-(i) to (14): for each posterior belief $q^*$ in the support of $\pi^*$, for every $v^{A'} > v^A$, and every experiment $\pi_s$, we have $E_{\pi_s}[W(q;v^{A'})|q^*] \leq W(q^*;v^{A'})$. Consequently, after each signal realization of $\pi^*$ the IP does not benefit from further disclosing information. Taking expectations over the realizations of $\pi^*$ yields $E_{(\pi^*,\{\pi_s\}_{s \in S})}[W(q;v^{A'})] \leq E_{\pi^*}[W(q;v^{A'})]$.

**Proof of Corollary 1:** Suppose that for some $v^A_2$ a completely uninformative experiment is optimal, and note that every possible experiment is Blackwell more informative than no information. Then Proposition 1 immediately implies that a completely uninformative experiment is weakly better than every other experiment for any $v^A > v^A_2$.

Suppose that for some $v^A_1$ the fully informative experiment $\pi^{FD}$ is optimal. Alonso and Câmara (2015a, Corollary 2) show that a fully informative experiment is optimal if and only if $E_{\pi^{FD}}[W(q';v^A_1)|q] \geq W(q;v^A_1)$ for all $q \in \Delta(\Theta)$. Lemma 1 implies that for every $v^A < v^A_1$, we have $E_{\pi^{FD}}[W(q';v^A)|q] \geq W(q;v^A)$ for all $q \in \Delta(\Theta)$. Hence, $\pi^{FD}$ is optimal for all $v^A < v^A_1$.

In Proposition A.1 below, we show that upper-censoring is an optimal experiment for a large class of Bayesian persuasion games. Then, in the proof of Proposition 2, we show that our model satisfies the conditions of Proposition A.1.

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23 Lemma 1 implies that if $E_{\pi}[W(q;v^A) - W(p;v^A)] \geq 0$, then $E_{\pi}[W(q;v^{A'}) - W(p;v^{A'})] \geq 0$ for any $v^{A'} < v^A$. 

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Proposition A.1 Consider a Bayesian persuasion game between a sender and a receiver, as in KG. Suppose that the sender’s payoff \( u_S(a, \theta) \) and receiver’s optimal action \( a^*(q) \) satisfy

\[
\sum_{\theta \in \Theta} q_\theta u_S(a^*(q), \theta) = K(E[\theta|q]),
\]

where \( K(\cdot) \) is twice differentiable and strictly increasing, and \( E[\theta|q] \) denotes the expected state given posterior belief \( q \). If \( K' \) is single-peaked (single-dipped), then there exists an optimal signal that is upper-censoring (lower-censoring).

Proof of Proposition A.1:

Note that \( U_S(q) = \sum_{\theta \in \Theta} q_\theta u_S(a^*(q), \theta) \) is the sender’s expected payoff, as a function of posterior belief \( q \). Therefore, (15) implies that the sender’s expected utility depends on posterior beliefs only through the posterior expectation of the state. Furthermore, function \( K \) is assumed to be twice differentiable, with a strictly positive derivative \( K'(E) \equiv \frac{dK(e)}{de}\bigg|_{e=E} > 0 \).

First, suppose that \( K' \) is single-peaked — that is, there exists an \( \bar{E} \) in the extended real line such that \( K''(E) \geq 0 \) for all \( E < \bar{E} \), and \( K''(E) \leq 0 \) for all \( E > \bar{E} \). Consequently, \( K \) is locally convex in the range \( E < \bar{E} \), and locally concave in the range \( E > \bar{E} \). Since \( K' \) might be “flat” at its peak, we define \( \bar{E} \) as the lowest expectation at the peak. That is, \( \bar{E} \) is defined such that \( K'(E) < K'(\bar{E}) \) for all \( E < \bar{E} \), and \( K'(\bar{E}) \geq K'(E) \) for all \( E \geq \bar{E} \).

Since \( \theta_1 < \ldots < \theta_N \), players’ posterior expectation of the state must be in \([\theta_1, \theta_N]\). If \( \bar{E} \geq \theta_N \), then the sender’s payoff is everywhere convex and a fully informative experiment is optimal; if \( \bar{E} \leq \theta_1 \), then the sender’s payoff is everywhere concave and a completely uninformative experiment is optimal (see KG for details). Note that full disclosure and no disclosure are the extreme cases of upper-censoring, with cutoff states \( \theta_N \) and \( \theta_1 \), respectively.

Now consider the remaining case: \( \theta_1 < \bar{E} < \theta_N \). We next construct an optimal experiment that is upper-censoring. The proof has two steps.

Step 1) We first show that, among the class of optimal experiments, there is always one that induces at most one non-degenerate posterior belief. To see this, take any optimal experiment \( \pi^* \) and let \( \sigma^* \) be the the distribution of posterior beliefs induced by this experiment. All beliefs \( q^- \) in the support of \( \sigma^* \) such that \( E[\theta|q^-] < \bar{E} \) are in the locally convex region of \( K \). Hence, the sender weakly benefits from further disclosing some information. All beliefs \( q^+ \) in the support of \( \sigma^* \) such that \( E[\theta|q^+] \geq \bar{E} \) are in the locally concave region of \( K \). Hence, the
sender weakly benefits from combining all these beliefs into a single belief. Repeated use of this argument implies the following. There exists an experiment \( \pi' \) that (i) is weakly better than \( \pi^* \), hence \( \pi' \) is also optimal, and (ii) letting \( \sigma' \) be the distribution of posterior beliefs induced by \( \pi' \), there is at most one non degenerate belief in the support of \( \sigma' \). This single non degenerate belief is in the concave region of \( K \), while every belief in the convex region is degenerate.

**Step 2** We now solve for the optimal experiment in the class of experiments that induce at most one non degenerate belief. Given Step 1, this experiment is then optimal for the sender when she is unconstrained in her choice of experiment.

Consider any experiment that induces at most one non degenerate belief. Without loss of generality, define the signal space as \( S \equiv \{s_{\theta_1}, \ldots, s_{\theta_N}, s_{\text{pooling}}\} \). Each state \( \theta \in \Theta \) induces the pooling signal \( s_{\text{pooling}} \) with probability \( \alpha_\theta \in [0, 1] \), and induces the fully revealing signal \( s_\theta \) with probability \( 1 - \alpha_\theta \). Given \( \alpha = (\alpha_{\theta_1}, \ldots, \alpha_{\theta_N}) \), let \( q^+(\alpha) \equiv \left( \frac{\alpha_\theta p_\theta}{\sum_{\theta' \in \Theta} \alpha_{\theta'} p_{\theta'}} \right)_{\theta \in \Theta} \) be the updated posterior belief after observing \( s_{\text{pooling}} \), and \( E^+(\alpha) \equiv \langle q^+(\alpha), \theta \rangle = \sum_{\theta \in \Theta} \alpha_{\theta} p_{\theta} \theta \) be the updated expectation of \( \theta \). The sender’s problem then simplifies to choosing \( \alpha \) the maximizes her expected payoff:

\[
\max_{\alpha \in [0,1], \theta \in \Theta} \Pi(\alpha) \equiv \left( \sum_{\theta \in \Theta} \alpha_\theta p_\theta \right) K(E^+(\alpha)) + \sum_{\theta \in \Theta} (1 - \alpha_\theta) p_\theta K(\theta).
\]

We now solve for an optimal \( \alpha^* \) and show that the optimal experiment is upper-censoring, that is, there exists a cutoff state \( \theta_k \) such that \( \alpha^*_\theta = 0 \) if \( \theta < \theta_k \) and \( \alpha^*_\theta = 1 \) if \( \theta > \theta_k \).

From Step 1, the pooling posterior belief \( q^+(\alpha^*) \) is in the concave region, \( E^+[\alpha^*] \geq \bar{E} \). Moreover, since posterior beliefs in the convex region are degenerate, we have \( \alpha^*_\theta = 1 \) for all \( \theta \geq \bar{E} \). Now consider the convex region \( \theta < \bar{E} \). Taking the derivative of the objective function with respect to \( \alpha_{\theta'} \) for each state \( \theta' < \bar{E} \), and noting that \( \frac{\partial E^+(\alpha)}{\partial \alpha_{\theta'}} = \frac{p_{\theta'}}{\sum_{\theta' \in \Theta} \alpha_{\theta'} p_{\theta'}} \left[ \theta' - E^+(\alpha) \right] \), we have

\[
\frac{\partial \Pi(\alpha)}{\partial \alpha_{\theta'}} = p_{\theta'} K(E^+(\alpha)) - p_{\theta'} K(\theta') + \left( \sum_{\theta \in \Theta} \alpha_\theta p_\theta \right) K'(E^+(\alpha)) \frac{\partial E^+(\alpha)}{\partial \alpha_{\theta'}} = p_{\theta'} \left( K(E^+(\alpha)) - K(\theta') + K'(E^+(\alpha)) \left[ \theta' - E^+(\alpha) \right] \right)
\]

\[
= p_{\theta'} \int_{\theta'}^{E^+(\alpha)} \left[ K'(E) - K'(E^+(\alpha)) \right] dE.
\]
Since \( p_{\theta} > 0 \), the derivative \( \frac{\partial \Pi(\theta)}{\partial \alpha_{\theta'}} \) has the same sign as

\[
A(\theta') \equiv \int_{\theta'}^{E(\alpha)} [K'(E) - K'(E^{+}(\alpha))] \, dE.
\]

Suppose that \( \alpha_{\theta'} < 1 \) for some \( \theta' < \bar{E} \), which implies that \( A(\theta') \leq 0 \). Single-peakedness of \( K' \) implies that \( K' \) is increasing for \( \theta < \bar{E} \). Therefore, \( K'(\theta) \leq K'(\theta') \) for all \( \theta < \theta' \) which leads to \( A(\theta) < A(\theta') \leq 0 \) for all \( \theta < \theta' \). This establishes that, for all \( \theta < \theta' \), we must have \( \alpha_{\theta} = 0 \).

The same steps of the proof show the opposite (lower-censoring) result for the case of a single-dipped \( K' \).

**Proof of Proposition 2:** Suppose that (A.1) and (A.2') hold. Then we can write the IP’s payoff as a strictly increasing, twice differentiable function of the expected state, \( K(E) = F(H(E) + vA) \). Moreover, the derivative \( K'(E) = f(H(E) + vA)H'(E) \) is log-concave, therefore it is single-peaked. Consequently, the conditions of Proposition A.1 hold and there is an optimal experiment that is upper-censoring.

The fact that the optimal censoring cutoff weakly decreases in \( vA \) follows immediately from Proposition 1: strictly increasing the censoring cutoff increases the informativeness of the experiment, but the IP does not benefit from a more informative experiment if \( vA \) increases.

**Proof of Proposition 3:** Suppose \( \beta^{A}\beta^{B} < 0 \) or \( |\beta^{B}| > 2|\beta^{A}| \). This implies that \( \beta^{A} \neq \beta^{B} \) and \( \beta^{B}(2\beta^{A} - \beta^{B}) < 0 \).

We first show that, from the point of view of voter \( A \), the expected policy payoff if candidate \( B \) is elected is strictly lower if the candidate observes a fully informative experiment, compared to no information. That is, full information makes candidate \( B \) choose a strictly worse policy on average.

Without further information, candidate \( B \) chooses policy \( \beta^{B}E[\theta|p] \), which yields expected policy payoff \( E[-(\beta^{B}E[\theta|p] - \beta^{A}\theta)^2|p] \) to voter \( A \). With a fully informative signal, candidate \( B \) chooses policy \( \beta^{B}\theta \) after learning that the state is \( \theta \). This yields an expected policy payoff \( E[-(\beta^{B}\theta - \beta^{A}\theta)^2|p] \) to voter \( A \). No information yields a strictly higher payoff than full
information if and only if

\[ E[-(\beta^B E[\theta|p] - \beta^A \theta)^2|p] > E[-(\beta^B \theta - \beta^A \theta)^2|p] \]
\[ -(\beta^B)^2 E[\theta|p]^2 + 2\beta^A \beta^B E[\theta|p]^2 - (\beta^A)^2 E[\theta^2|p] > -(\beta^B)^2 E[\theta^2|p] + 2\beta^A \beta^B E[\theta^2|p] - (\beta^A)^2 E[\theta^2|p] \]
\[ (2\beta^A \beta^B - (\beta^B)^2) E[\theta|p]^2 > (2\beta^A \beta^B - (\beta^B)^2) E[\theta^2|p] \]
\[ 0 > \beta^B (2\beta^A - \beta^B) (E[\theta^2|p] - E[\theta|p]^2). \]

Since the variance \((E[\theta^2|p] - E[\theta|p]^2)\) is strictly positive given any interior prior belief, the inequality holds if and only if \(0 > \beta^B(2\beta^A - \beta^B)\), concluding this step of the proof.

Disagreement is a convex function of the posterior belief, \(D(q) = (\beta^B - \beta^A)^2 (E[\theta|q])^2\).

Consequently, if \(v^A\) is sufficiently low, then the IP’s optimal experiment is fully informative. From the point of view of voter \(A\), compared to no information, full information leads candidate \(B\) to choose a worse policy on average, while it leads candidate \(A\) to choose a better policy when elected. Moreover, if \(v^A\) is sufficiently low, then candidate \(B\) is sufficiently likely to win the election, and the strictly negative effect of a worse policy from candidate \(B\) dominates the positive effect from the better policy from candidate \(A\).

**Proof of Proposition 4:** Suppose political disagreement takes the form \(\mathcal{D}(q^A, q^B) = R(|E[\theta|q^A] - E[\theta|q^B]|),\) where \(R \geq 0\) and \(R' > 0\). The proof has two steps.

**Step 1** Define the vector \(v = r(\theta - E[\theta|q^B])\), the linear subspaces \(W_1 = \{x \in \mathbb{R}^{\text{card}(\theta)} : \langle x, 1 \rangle = 0\}\) and \(W_{\theta-v} = \{x \in \mathbb{R}^{\text{card}(\theta)} : \langle x, \theta - v \rangle = 0\}\). In this first step, we prove that if the projections of \(\theta\) and \(r\) are not negatively collinear with respect to \(W_1 \cap W_{\theta-v}\), then there exists an experiment \(\pi\) where all signal realizations increase political disagreement.

Use (24) to rewrite

\[ \mathcal{D}(q^A, q^B) = R(|E[\theta|q^A] - E[\theta|q^B]|) = R\left(\left|\frac{\langle q^A, \theta \rangle - \langle q^A r, \theta \rangle}{\langle q^A, r \rangle}\right|\right) \equiv D(q^A). \]

Define \(q^A = \varepsilon \lambda + p^A\), with \(\lambda \in W_1 = \{x : \langle x, 1 \rangle = 0\}\) and \(\varepsilon \in \mathbb{R}\), and let

\[ L(\varepsilon; \lambda) = \langle q^A, \theta \rangle - \frac{\langle q^A r, \theta \rangle}{\langle q^A, r \rangle} = \varepsilon \langle \lambda, \theta \rangle + E[\theta|q^A] - \frac{\varepsilon \langle \lambda, r \theta \rangle + E[\theta|q^B]}{\varepsilon \langle \lambda, r \rangle + 1}. \]

Disagreement is a strictly increasing function of the absolute value of \(L(\varepsilon; \lambda)\). First suppose that \(L(\varepsilon; \lambda) \geq 0\). We will show that under the conditions of the proposition one can always
find a vector of “marginal beliefs” \( \lambda' \) such that \( L \) achieves a local minimum with respect to \( \varepsilon \) at \( \varepsilon = 0 \). This means that along the line \( \lambda' \) and in a neighborhood of 0, any belief \( q^A = \varepsilon \lambda' + p^A \) with \( \varepsilon > 0 \) increases \( L \), and thus \( D(q^A) > D(p^A) \), while any belief \( q^A = \varepsilon \lambda' + p^A \) with \( \varepsilon < 0 \) also increases \( L \), yielding \( D(q^A) > D(p^A) \). That is, we have found collinear beliefs that can average to the prior and that increase \( D \).

First, we have

\[
\frac{dL}{d\varepsilon} = \langle \lambda, \theta \rangle - \langle \lambda, r \theta \rangle - \langle \lambda, r \rangle \left( \frac{\theta | q^B \rangle}{(\varepsilon \langle \lambda, r \rangle + 1)^2} \right),
\]

\[
\frac{d^2L}{d\varepsilon^2} = \frac{2 \langle \lambda, r \rangle}{(\varepsilon \langle \lambda, r \rangle + 1)^3} \left( \langle \lambda, r \theta \rangle - \langle \lambda, r \rangle \left( \frac{\theta | q^B \rangle}{(\varepsilon \langle \lambda, r \rangle + 1)^2} \right) \right).
\]

For \( L(\varepsilon; \lambda) \) to achieve a local minimum at \( \varepsilon = 0 \), it is sufficient that there exists \( \lambda \in W \) such that

\[
\frac{dL}{d\varepsilon} \bigg|_{\varepsilon=0} = 0 \Rightarrow \langle \lambda, \theta \rangle = \langle \lambda, r \left( \theta - E(\theta | q^B \rangle) \right) \rangle,
\]

(16)

\[
\frac{d^2L}{d\varepsilon^2} \bigg|_{\varepsilon=0} > 0 \Rightarrow \langle \lambda, r \rangle \langle \lambda, r \left( \theta - E(\theta | q^B \rangle) \right) \rangle > 0.
\]

(17)

Since \( \theta \) and \( r \) are not negatively collinear with respect to \( W_1 \cap W_{\theta-v} \), then there exists \( \lambda' \in W_1 \cap W_{\theta-v} \) with \( \langle \lambda', \theta \rangle \langle \lambda', r \rangle > 0 \) — see Alonso and Cámara (2015a). Since \( \lambda' \in W_{\theta-v} \), then \( \lambda' \) satisfies (16). Then, given (16), the fact that \( \langle \lambda', \theta \rangle \langle \lambda', r \rangle > 0 \) implies that \( \lambda' \) also satisfies (17). Therefore, \( L(\varepsilon; \lambda) \) achieves a local minimum at \( \varepsilon = 0 \).

Now consider the remaining case, \( L(\varepsilon; \lambda) < 0 \). Since disagreement strictly increases in the absolute value of \( L \), we now can increase disagreement by decreasing \( L \). The same steps of the proof above can be used to show that under the conditions of the proposition one can always find a vector of “marginal beliefs” \( \lambda'' \) such that \( L \) achieves a local maximum with respect to \( \varepsilon \) at \( \varepsilon = 0 \). This follows as the fact that \( \theta \) and \( r \) are not negatively collinear with respect to \( W_1 \cap W_{\theta-v} \) implies the existence of \( \lambda'' \in W_1 \cap W_{\theta-v} \) with \( \langle \lambda'', \theta \rangle \langle \lambda'', r \rangle < 0 \) (see Alonso and Cámara 2015a), so that \( L(\varepsilon; \lambda'') \) is locally concave at \( \varepsilon = 0 \). This concludes the first step of the proof.

**Step 2)** The previous step shows that if the projection of \( \theta \) and \( r \) are not negatively collinear with respect to \( W_1 \cap W_{\theta-v} \) then persuasion is valuable. We now show that negative collinearity of \( \theta \) and \( r \) with respect to \( W_1 \cap W_{\theta-v} \) is a non-generic property if \( N \geq 4 \). First note that \( W_1 \cap W_{\theta-v} \) has at least dimension \( N - 2 \), and thus the projections of \( \theta \) and \( r \)
also have dimension $N - 2 \geq 2$. As collinearity is a non-generic property with vectors of dimension at least 2, this concludes the proof. ■

References


