Spatial Linkages, Global Shocks, and Local Labor Markets: Theory and Evidence*

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Abstract
We analyze the impact of trade and productivity shocks in a generalized spatial model with aggregate elasticities controlling cross-market links in productivity, labor supply, and trade flows. We show how spatial linkages determine the effect on a local labor market of its direct shock exposure, as well as its indirect general equilibrium exposure to adjustments in other markets. We propose a class of consistent estimators for the model’s aggregate elasticities based on cross-market variation in exposure to observed trade shocks and actual changes in trade and labor outcomes. The “optimal” estimator in this class uses the local responses to the trade shock predicted by our general equilibrium model – the Model-implied Optimal IV (MOIV). Applying our methodology to US states, we find a limited role for indirect effects in the response to trade shocks, implying that the model’s general equilibrium predictions are well approximated by difference-in-difference designs based on simple measures of local shock exposure.

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1 Introduction

How do shocks to foreign and domestic economic fundamentals, such as trade costs and technology changes, affect the performance of local labor markets? The answer to this profound question occupies several mainstream economists, as a growing literature exploits differential local exposure to shocks from various sources to measure differential effects on various local economic outcomes.\(^1\) However, answering such question presents two main challenges. First, it is unclear how the fundamental elasticities that need to be measured are affected by general equilibrium forces that arise when local markets are interconnected. Second, given these general equilibrium effects, it is difficult to credibly identify the exposure of a market to economic shocks. We argue that these challenges arise because of the existence of cross-market links itself: it implies that any local shock percolates through the entire network, indirectly affecting other markets in equilibrium. We then develop a new methodology to theoretically assess and empirically measure these cross-market links and the fundamental elasticities necessary to evaluate the effects of economic shocks on labor market outcomes.

Our starting point is a Generalized Spatial Competitive Economy featuring countries constituted by multiple local labor markets, i.e. sector-region pairs. In each market, production of a composite good is subject to agglomeration and congestion forces – i.e., productivity is a function of employment in different markets. Countries have a representative household with preferences for consumption and labor supply across markets. The utility maximization problem determines, for each market, bilateral trade flows as a function of good prices, and labor supply as a function of real wages. Thus, our general equilibrium economy features three aggregate mappings that control cross-market links in terms of (i) productivity, (ii) labor supply, and (iii) bilateral trade flows.

In this environment, we theoretically assess the impact on local labor markets of changes in local shifters, “shocks” henceforth, of labor supply, productivity, or trade costs. Our first result establishes that, conditional on trade and labor outcomes in the initial equilibrium, knowledge of the three aggregate mappings of cross-market links in our model is sufficient to uniquely characterize the effect of trade and productivity shocks on local labor markets. This insight holds in a large class of spatial models with rich trade and labor mobility frictions as

\(^1\)There is a growing literature studying the local effects of international trade shocks – e.g., see Topalova (2010), Kovak (2013), Autor, Dorn, and Hanson (2013), Dix Carneiro and Kovak (2016) and, for a review, Muendler (2017). Several papers have also documented local effects of technological shocks – see Autor and Dorn (2013), Bustos, Caprettini, Ponticelli, et al. (2016), and Acemoglu and Restrepo (2017). Finally, an emerging literature evaluates the local effects of macroeconomic shocks – see Mian and Sufi (2014), Nakamura and Steinsson (2014) and Beraja, Hurst, and Ospina (2016). Muendler (2017) discusses the issues associated with extrapolating aggregate effects of a shock from the estimated regional effects.
well as in models where space plays less of a role such as macroeconomic and development setups with goods and labor mobility across markets. In fact, we show that, by properly specifying the shape of the aggregate mappings, our model generates labor market outcomes in equilibrium that are observationally equivalent to those implied by existing frameworks. This implies that the different microeconomic structure of these frameworks matters only insofar it affects the elasticity structure of the key aggregate mappings in our model.

Our second result characterizes the role of cross-market linkages in determining the general equilibrium impact of trade and productivity shocks on local labor markets. Following shocks to any particular market, we show how the shape of the aggregate mappings in our model regulates not only the direct response of employment and wages in that market, but also the indirect spillover effects on all other markets in the economy. The magnitude of these indirect effects is intrinsically related to the strength of the cross-market linkages in the world economy. These indirect spillover effects can only be ignored under the restrictive assumption of shocks to segmented labor markets in a small open economy. In the special case of constant elasticity aggregate mappings and frictionless trade, the indirect spillover effects arise in general equilibrium but they are common to all markets. In general, spatial heterogeneity in cross-market linkages creates spatial variation in the indirect exposure of markets to local shocks in any particular location.

These results have important implications for the empirical analysis of the effect of economic shocks on local economic outcomes. The common component of the indirect spillover effects is a part of the aggregate impact of the shock absorbed by the time fixed effect in empirical specifications. In case of strongly symmetric cross-market linkages, this common component can be large, which implies that the time fixed effect may contain a large fraction of the aggregate impact of the shock. Instead, any heterogeneous component of the spillover effects is part of the differential local exposure to the shock, whose importance depends on the spatial heterogeneity in cross-market linkages. Therefore, it is important to

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2 These frameworks include: (i) Neoclassical trade theories with external economies of scale (as in Ethier (1982a,b)) and without (as in Anderson (1979) and Eaton and Kortum (2002)); (ii) New trade theory models with homogeneous firms (as in Krugman (1980)); (iii) New economic geography models (as in Krugman (1991) and Allen and Arkolakis (2014)); (iv) Roy spatial assignment models (as in Galle, Rodríguez-Clare, and Yi (2015) and Adão (2015)). Finally, the model specified without trade frictions yields predictions equivalent to the Rosen (1979)-Roback (1982) setup.

3 We also show that the model can be extended to allow for: i) workers commuting across regions, as in Caliendo and Parro (2014); iii) multiple worker groups in production, as in Cravino and Sotelo (2017); and iv) firm heterogeneity as in Chaney (2008). In all these settings, besides the three aggregate mappings that arise in the Generalized Spatial Competitive Economy, there are other functions determining labor and trade outcomes in equilibrium.

4 This is the main intuition behind the multiple-sector model that motivates the empirical specifications in Autor, Dorn, and Hanson (2013) and Kovak (2013).
evaluate the magnitude and the dispersion of indirect general equilibrium effects to determine how important it is to account for cross-market linkages in any empirical investigation. If such indirect effects are quantitatively large, their omission can introduce a bias in the measurement of both the differential and aggregate effects of cost shocks. In contrast, if they are quantitatively small, empirical specifications based on cross-market variation in direct shock exposure recover the effect of economic shocks on local labor markets.

Given these results, we turn to the problem of empirically measuring the cross-market links embedded in the elasticity structure of the central aggregate mappings in our model. To estimate these key elasticities, we characterize a class of consistent estimators based on observed trade shocks that are orthogonal to unobservable local shocks in productivity and labor supply. We exploit the fact that, conditional on the shape of the aggregate mappings, these unobservable local shocks can be recovered from changes in trade and labor outcomes across local markets. Thus, the aggregate mappings can be estimated using a class of moment conditions that combine the recovered local shocks and arbitrary functions of observed trade shocks.5

We then use our general equilibrium model to characterize the “optimal” estimator within this class of consistent estimators. That is, we compute the function of observed trade shocks that minimizes the estimator’s asymptotic variance. We show that such a function is the impact of observed trade shocks on the endogenous variables predicted by our general equilibrium model – the Model-implied Optimal IV (MOIV). Intuitively, through the lens of the model, these predicted changes correspond to the total effect of the exogenous trade shock on the endogenous local outcomes and, therefore, contain all sources of variation available to estimate the unknown structural parameters. We further show that the optimal estimator can be easily implemented through a Two-Stage GMM estimator.

In the second tier of our analysis, we apply our methodology to empirically evaluate the role of cross-market linkages in determining the effect of trade shocks on local labor markets in the United States. In the application, we use changes in tariffs applied to agricultural goods produced in the United States by top trading partners between 1993 and 2012. Tariffs on agricultural products are typically higher and more widespread than in manufacturing, being agriculture a traditionally very protected sector in many countries (see Anderson et al. (2013) and Bagwell, Bown, and Staiger (2016)). Under the assumption that changes in foreign

5 The central orthogonality assumption of our methodology is typically maintained in empirical papers measuring the impact of trade shocks on local labor markets (see Topalova (2010), Kovak (2013), Autor, Dorn, and Hanson (2013), and Hakobyan and McLaren (2016)). However, as in Berry and Haile (2014) and Adao, Costinot, and Donaldson (2017), the recovery of unobservable local shocks requires the invertibility of the central functions in the model. In our setting, the mappings of labor supply and bilateral trade must be invertible, which holds in a variety of existing frameworks in the literature.
agricultural tariffs are orthogonal to local shocks in the US, we use our model’s predicted impact of these tariff changes on trade and labor outcomes across US states to consistently estimate cross-market linkages in terms of labor supply and productivity.

In our empirical application, we consider a parametrization of the Generalized Spatial Competitive Economy that captures the main forces embedded in existing quantitative papers. We obtain estimates of intensive and extensive margins elasticities of labor supply that are consistent with the estimates reviewed by Chetty et al. (2013). While we find evidence on mobility across sectors within regions, we find no evidence on between-state mobility in response to trade-induced changes in real wages, consistent with previous evidence for the US in Autor, Dorn, and Hanson (2013), and Autor, Dorn, Hanson, and Song (2014). In addition, we find strong local agglomeration forces, close to the forces implied by models with increasing returns to scale and firm entry as in Krugman (1980), and local congestion forces, as in Bartelme (2015) and Allen, Arkolakis, and Takahashi (2014). Finally, we show that changes in local labor market outcomes observed in the data are highly correlated with those predicted by our general equilibrium model in response to the observed tariff shocks. This suggests that the general equilibrium mechanisms of our model capture a meaningful and significant component of the response of local labor markets to trade shocks.

In the final part of the paper, we use our estimates to assess the effect of trade shocks on local labor markets in the United States. As a first step, we recover changes in bilateral trade costs between US states and foreign countries from observed changes in bilateral spending shares and real exchange rates between 1993 and 2007. We use these inverted trade costs to implement a counterfactual exercise where, starting from the observed equilibrium in 2007, we analyze the impact of reverting trade costs to their level of 1993. Interestingly, we find that the shock would increase the relative wage of workers in the tradable sector, with stronger impact on the states around the Rust Belt. We then use our framework to evaluate the quantitative importance of indirect spillover effects for the model’s predictions. We show that the direct effect of the shock captures almost entirely the predicted change in labor outcomes across US states, implying a very limited role for indirect spillover effects in general equilibrium. We further attest that the model’s predictions are well approximated by simpler measures of regional shock exposure computed using observable trade and revenue shares in the initial equilibrium. This suggests that these intuitive exposure measures can be readily used in difference-in-difference designs for the estimation of the effect of trade shocks.

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6We estimate the Marshallian labor elasticity of hours and participation, while Chetty et al. (2013) review mostly estimates of the Hicksian elasticity of labor supply along these margins. However, Chetty et al. (2013) and Chetty (2012) argue that the two are very similar, given that income effects are typically small.

7We extend the methodology proposed by Head and Ries (2001) to the case of asymmetric trade costs. This requires using information on aggregate price indices as well, similarly to Eaton et al. (2016).
on local labor markets.

Our theoretical approach is consistent with the spirit of the analysis suggested by Rosenzweig and Wolpin (2000) and Wolpin (2013), applied to models that allow for spatial linkages. In this sense, we characterize the “deep” elasticity mappings that have to be estimated to characterize the response of labor market outcomes to shocks on economic fundamentals. For that purpose, we generalize previous findings in gravity trade and new economic geography models where the bilateral trade, labor supply, and agglomeration spatial linkages take a constant elasticity functional form – see Arkolakis, Costinot, and Rodríguez-Clare (2012), Allen, Arkolakis, and Takahashi (2014), and Bartelme (2015) – or do not feature externalities and regional mobility – as in the work of Adao, Costinot, and Donaldson (2017), who have discussed general neoclassical models of trade with multiple sectors and factors. Thus, our approach provides a new robust way of estimating the various spatial linkages emphasized in the literature, and use them to credibly measure the general equilibrium effect of global shocks on local labor markets.

In the class of models covered by our environment, we explicitly show that the existence of cross-market links gives rise to indirect spillover effects that, if ignored, may result in an incomplete measurement of the differential and aggregate effect of shocks on economic outcomes. To address the issue of measurement of the differential and aggregate effects of economic shocks in general equilibrium, a growing body of literature proposes structural models that follow two distinct approaches. On one hand, by imposing symmetry assumptions regarding the effect of spatial linkages in general equilibrium, a group of papers have proposed general equilibrium models that yield a structural interpretation for the response of regional economies to local shocks – see Kovak (2013), Kline and Moretti (2014), Beraja, Hurst, and Ospina (2016) and Acemoglu and Restrepo (2017). On the other hand, several papers specify structural spatial models featuring rich patterns of spatial linkages in general equilibrium – see Allen and Arkolakis (2014), Caliendo et al. (2014), Galle, Rodriguez-Clare, and Yi (2015), Caliendo, Dvorkin, and Parro (2015), and, for a review, Redding and Rossi-Hansberg (2016). However, given the many primitive assumptions dictating the structure of aggregate linkages, it is often hard to evaluate the importance of cross-market linkages for the quantitative predictions of the model. In this sense, our methodology combines both approaches by proposing a model that, while featuring a rich structure of cross-market linkages, can be readily estimated using the impact of trade shocks on trade and labor outcomes across markets, in a way that is robust to any specific micro-foundation.

Finally, to identify these elasticity mappings we propose an instrumental variables procedure that consists of two steps: i) recovering from the data the structural residuals representing the unobserved productivity and labor supply shifters, and ii) using these
residuals together with plausibly exogenous variation in trade cost shocks to implement an efficient estimator of the key structural elasticities in the model. In the first step, we provide a distinct approach in that we can invert structural residuals without requiring the use of market clearing conditions, as proposed in Allen and Arkolakis (2014), Monte, Redding, and Rossi-Hansberg (2015), and Faber and Gaubert (2016). In the second step, we propose an estimator by applying the Optimal IV approach of Chamberlain (1987) in general equilibrium. Model-implied instruments have been also used in Monte, Redding, and Rossi-Hansberg (2015), Faber and Gaubert (2016), Eckert and Peters (2018), and Allen and Donaldson (2018), and most closely to our approach by Allen, Arkolakis, and Takahashi (2014). Our contribution is to formally establish a class of consistent estimators and conditions such that our MOIV estimator, effectively an aggregation function over a plausibly exogenous instrument, is the variance minimizing estimator in this class.

The rest of the paper is structured as follows. Section 2 describes our Generalized Spatial Competitive Economy and its equivalence to existing models in the literature. Section 3 characterizes the direct and indirect effects on local labor markets of shocks in labor supply, productivity, or trade costs. Section 4 describes our novel empirical methodology, which we then implement in Section 5 using state-level data in the United States. Using the theoretical model and the estimates, Section 6 conducts a number of counterfactual exercises. Section 7 concludes.

2 Generalized Spatial Competitive Economy

We begin by proposing a general environment to analyze international trade and labor markets. The framework is based on the premise that each location potentially produces one differentiated commodity, as in Armington (1969) and Anderson (1979), but departs from this setup in three fundamental ways. First, we consider a general utility function across goods, instead of CES preferences. Second, we allow for a flexible structure of labor supply choice across regions and sectors. Third, we incorporate agglomeration and congestion externalities into production with labor productivity depending on employment in different regions and sectors. These three departures imply that our model is flexible enough to capture the forces in three main strands of the geography and trade literature: i) generalizations of the gravity setup of trade; ii) new economic geography models with labor mobility; iii) trade

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8This has the advantage that only mis-specification in the model’s estimating equations constitutes a threat to the estimator consistency. In contrast, consistency of approaches using the entire general equilibrium structure for inversion requires the correct specification of every equation in the model.

9 The approach of Chamberlain (1987) has been used in partial equilibrium models by Berry, Levinsohn, and Pakes (1999) and Reynaert and Verboven (2014).
and geography models incorporating agglomeration and congestion forces in production.

2.1 Environment

We assume that the world economy is constituted of countries, \( c \), that are collections of regions, \( r \in R_c \), with multiple sectors, \( s \in K \). In the rest of the paper, we follow the convention of calling a region-sector pair a “local market”, and denote origin markets as \( i \), and destination markets as \( j \). We denote the vector of outcome \( x \) for all markets in the world as \( \{x_j\}_{j} \), and the vector for all markets in country \( c \) as \( \{x_j\}_{j \in J_c} \) with \( J_c \equiv \{(s,r) | r \in R_c, s \in K\} \).

Preferences. We assume that in each country there exists a representative agent with preferences for consumption and labor supply in different markets. In particular, the representative agent’s preferences are given by

\[
U_c \left( \{\nu_j C_j\}_{j \in J_c}, \{L_j\}_{j \in J_c} \right),
\]

where, in each market \( j \), \( L_j \) is the labor supply, \( C_j \) is a consumption index, and \( \nu_j \) is a market-specific preference shifter. The utility function \( U_c(\cdot) \) is increasing in consumption and quasi-concave in consumption and labor. Notice that we allow labor supply in different markets to be perfect substitutes in the utility of the representative agent.\(^{11}\)

The consumption index is an aggregator of the quantities consumed of the differentiated goods produced by different origin markets,

\[
C_j \equiv V_j \left( \{c_{ij}\}_{i} \right),
\]

where, in market \( j \), \( c_{ij} \) is the consumption of the good produced in market \( i \). We assume that the function \( V_j (\cdot) \) is increasing and quasi-concave in all arguments. Importantly, we also restrict \( V_j (\cdot) \) to be homogeneous of degree one, so that we can separate the problem of allocating spending shares across origin markets from the problem of determining labor supply across markets in the country.

We assume that consumption in each market is financed by labor income and transfers. Specifically, in each market \( j \in J_c \), the representative agent faces the following budget

\[10\]We treat regions and sectors in a similar way because there is a labor market clearing constraint that holds at that level for all models covered in our analysis. Of course, different assumptions on the degree of mobility of labor across sectors and regions imply different labor supply mappings, affecting wage differentials across markets in equilibrium. We get back to this point in Section 2.3.

\[11\]The utility function in (1) does not explicitly impose that the sum of labor supply across markets is constant, allowing our model to capture endogenous changes in aggregate labor supply in the country. The assumption of constant aggregate labor supply is common in the geography literature – see, for example, Allen, Arkolakis, and Takahashi (2014); Allen and Arkolakis (2014); Bartelme (2015). As discussed below, such a restriction can be imposed directly in the specification of the utility function in (1).
constraint:

\[ \sum_{i} c_{ij} p_{ij} = \rho_{j} (w_{j} L_{j}), \quad (3) \]

where \( w_{j} \) is the wage rate per unit of labor, \( \rho_{j} \) is the transfer rate per unit of labor income, and \( p_{ij} \) is the price of the good produced in market \( i \).

We consider a competitive environment where, in deciding consumption and labor supply, the representative agent takes as given the transfer rate \( \{\rho_{j}\}_{j} \), the wage rate \( \{w_{j}\}_{j} \), and the goods prices \( \{p_{ij}\}_{i,j} \).

**Production.** In each market, there exists a representative firm that operates under perfect competition. Production requires only labor and it is subject to external economies of scale. We assume that there are iceberg trade costs of shipping a good to a different market. The production function of market \( i \) to deliver goods to market \( j \) is

\[ Y_{ij} = \frac{\zeta_{i}}{\tau_{ij}} \Psi_{i} \left( \{L_{j}\}_{j} \right) L_{ij}, \quad (4) \]

where \( L_{ij} \) is the labor input and \( \tau_{ij} \) is the iceberg trade cost.

In equation (4), \( \zeta_{i} \Psi_{i} (\cdot) \) is the labor productivity of origin \( i \). It has two components: \( \zeta_{i} \) is an exogenous productivity shifter, and \( \Psi_{i} (\cdot) \) is an endogenous component that is allowed to depend on equilibrium employment levels in all markets. This endogenous productivity term governs the strength of agglomeration and congestion forces in our model.

**2.2 Competitive Equilibrium**

We now proceed to characterize the optimal choices of consumption, labor supply and production levels.

**Utility Maximization.** We start with the problem of the representative agent in country \( c \), who maximizes the utility in (1)–(2) subject to the budget constraint in (3) for every \( j \in \mathcal{J}_{c} \). We solve this problem in two stages. In the first-stage, the representative agent chooses the demand for products from various origins conditional on prices and total expenditure. Using the homotheticity of \( V_{j} (\cdot) \), we define the price index in \( j \) as

\[ P_{j} = P_{j} (\{p_{o_{j}}\}_{o}) \equiv \min_{\{c_{o_{j}}\}_{o}} \sum_{o} p_{o_{j}} c_{o_{j}} \quad \text{s.t.} \quad V_{j} (\{c_{o_{j}}\}_{o}) = 1, \quad (5) \]

with the associated spending share on goods from origin \( i \) given by

\[ x_{ij} \in X_{ij} (\{p_{o_{j}}\}_{o}). \quad (6) \]
The price index and spending shares inherit the usual properties of demand implied by utility maximization. Specifically, \( P_j (\{p_{oj}\}_o) \) is homogeneous of degree one, concave, and differentiable almost everywhere on the vector of prices, \( \{p_{oj}\}_o \). The assumption that \( V_j (\cdot) \) is quasi-concave implies that \( X_{ij} (\{p_{oj}\}_o) \) is a convex set. Notice also that any \( x_{ij} \in X_{ij} (\{p_{oj}\}_o) \) is a solution of the minimization problem in the definition of \( P_j (\{p_{oj}\}_o) \), so the envelope theorem in Milgrom and Segal (2002) implies that, for any selection \( x_{ij} \) satisfying (6), we can write

\[
x_{ij} = \frac{\partial \log P_j (\{p_{ij}\}_i)}{\partial \log p_{ij}}.
\]

This relation yields integrability: we can use any selection \( x_{ij} \) to recover \( P_j (\cdot) \) from \( X_{ij} (\cdot) \). Thus, if we know the functional form of \( X_{ij} (\cdot) \), we have immediate knowledge of the price index. We make use of this property below.

We now turn to the second-stage problem of choosing employment across markets in the country. We use the solution of the first-stage problem to write the budget constraint (3) as \( P_j C_j = \rho_j w_j L_j \), which we then substitute into the consumer’s problem to obtain

\[
\max_{\{L_j\}_{j \in J_c}} U_c \left( \{\nu_j \rho_j \omega_j L_j\}_{j \in J_c}, \{L_j\}_{j \in J_c} \right)
\]

where \( \omega_j \equiv w_j / P_j \) is the real wage in market \( j \).

The solution of this problem yields the labor supply in market \( j \):

\[
L_j \in \Phi_j (\{\nu_i \rho_i \omega_i\}_{i \in J_c}) ,
\]

where \( \Phi_j (\cdot) \) is a convex set because of the quasi-concavity of \( U_c (\cdot) \). Notice that the labor supply in market \( j \) depends on the entire vectors of real wages, productivity shifters and transfers in the country.

**Profit Maximization.** Assume that there are traders that can arbitrage any price differentials across markets. Thus, iceberg trade costs imply that

\[
p_{ij} = \frac{\tau_{ij}}{\zeta_i} p_i ,
\]

where \( p_i \) satisfies, from the profit maximization problem of the representative firm in \( i \),

\[
p_i \leq \frac{w_i}{\Psi_i (\{L_j\}_j)} \quad \text{with equality if } L_i > 0.
\]
By dividing the two sides of this expression by \( P_i \), we obtain the ratio between the cost of domestic producers, \( p_i \), and the domestic cost of the goods supplied by producers around the world, \( P_i \):

\[
\frac{p_i}{P_i} \leq \frac{\omega_i}{\Psi_i \left( \{L_j\}_j \right)} \quad \text{with equality if } L_i > 0.
\] (9)

By revealed preferences, \( p_i/P_i \) is directly related to consumption choices and import flows in market \( i \). We exploit this property to connect equation (9) to the data on bilateral trade flows in Section 4.

**Market Clearing.** To close the model, we specify the labor market clearing condition: in each market, total labor payments must be equal to total revenues,

\[
w_i L_i = \sum_j x_{ij} \rho_j w_j L_j.
\] (10)

We consider the world equilibrium where transfers \( \{\rho_j\}_j \) follow a set of rules that are determined outside of our model. However, we require that any rule guarantees trade balance at the world level, for any vector of wages and prices.\(^{12}\)

**Competitive Equilibrium.** The Competitive Equilibrium consists of \( \{p_i, P_i, L_i, \omega_i\}_i \) such that equations (5)–(10) hold, conditional on a set of transfers rules, \( \{\rho_i\}_i \), that satisfy world-level trade balance and a numeraire, \( p_{mm} \equiv 1 \), for an arbitrary reference market \( m \).

## 2.3 Discussion and Equivalences

We now discuss how our theoretical environment unifies a number of existing frameworks in spatial economics. We show that the shape of the mappings \( \{\{X_{ij}(\cdot)\}_i, \Phi_j(\cdot), \Psi_j(\cdot)\}_j \) encompasses the central forces in a wide range of spatial and trade models. We start by introducing a formal definition of the models for which the equilibrium outcomes of the Generalized Spatial Competitive Economy are observationally equivalent to.

**Definition 1.** The Generalized Spatial Competitive Economy is observationally equivalent to Economy \( N \) with respect to the shifters \( \{\{\tau_{ij}\}_i, \zeta_j, \nu_j\}_j \) if

\(^{12}\)An obvious special case of this specification is an environment without transfers, with \( \rho_j = 1 \). The transfer rule of constant transfers suggested by Dekle, Eaton, and Kortum (2007), which has become a staple of quantitative trade models, can be attained by specifying the transfer rule \( \rho_j = \frac{T_j}{w_i L_j} + 1 \), where \( T_j \) is a constant in terms of world GDP. We discuss further examples in conjunction with specific models in the Appendix of Section 2.3.
1. There exist unique mappings \( \left\{ \{ X_{ij}^N(\cdot) \}_i, \Phi_j^N(\cdot), \Psi_j^N(\cdot) \} \right\}_j \) such that the equilibrium of Economy \( N \) is characterized by conditions (5)–(10) for any levels of \( \left\{ \{ \tau_{ij}, \zeta_j, \nu_j \} \right\}_j \).

2. There exist preferences, (1)–(2), and technology, (4), that imply \( \left\{ \{ X_{ij}^N(\cdot) \}_i, \Phi_j^N(\cdot), \Psi_j^N(\cdot) \} \right\}_j \).

This definition requires that, independent of the levels of the exogenous shifters, Economy \( N \) must satisfy the equilibrium conditions (5)–(10) for unique mappings \( \left\{ \{ X_{ij}^N(\cdot) \}_i, \Phi_j^N(\cdot), \Psi_j^N(\cdot) \} \right\}_j \). This implies that any combination of shocks to the shifters \( \left\{ \{ \tau_{ij}, \zeta_j, \nu_j \} \right\}_j \) yields identical counterfactual outcomes in labor markets. We use Definition 1 to establish that our model is observationally equivalent to several existing frameworks under specific parametric restrictions on the shape of \( \left\{ \{ X_{ij}^N(\cdot) \}_i, \Phi_j^N(\cdot), \Psi_j^N(\cdot) \} \right\}_j \). We discuss the main equivalence results below by means of some parametric examples, and present the detailed proofs in Appendix D.1.

In the Generalized Competitive Spatial Economy, the mapping \( X_{ij}(\cdot) \) controls how bilateral trade costs affect bilateral trade flows. To see this, consider the case in which \( X_{ij}(\cdot) \) is a differentiable function, and define the cross-price elasticity \( \chi_{oij} \equiv \frac{\partial \log X_{ij}}{\partial \log p_{ij}} \) to represent the response of market \( j \)'s demand for goods from \( i \) to changes in the price of goods from \( o \).\(^{13}\) To explain how this cross-price elasticity matrix governs the dependence of the demand for labor in a market to changes in production costs in all other markets, we consider the following constant-elasticity gravity form.

**Assumption 1a.** The bilateral trade demand is CES with an elasticity structure given by:

\[
\chi_{oij} = -\chi \left( \mathbb{I}_{i=o} - X_{oj} \left\{ \{ p_{i'j} \}_{i'} \right\} \right)
\]

where \( \mathbb{I}_{i=o} \) is an indicator function that equals one if \( i = o \).

Assumption 1a is, by far, the most common assumption in trade models, oftentimes referred to as gravity. There is a variety of ways to micro-found this functional form, starting from simple Armington preferences, as in Anderson (1979), to heterogeneous technology in a Ricardian model, as in Eaton and Kortum (2002). This functional form is implied by a number of other forms of competition and demand structures discussed in Arkolakis, Costinot, and Rodríguez-Clare (2012), Costinot and Rodriguez-Clare (2013), and Arkolakis et al. (2017). In addition, the generalized mapping we consider is observationally equivalent to the one studied by Adao, Costinot, and Donaldson (2017) in the context of a one-factor neoclassical economy (with \( \Psi_i(\cdot) = \Phi_i(\cdot) = 1 \) for all \( i \)).

\(^{13}\)The assumption that the utility function of the representative agent in (1)–(2) is strictly quasi-concave and twice differentiable implies that \( X_{ij}(\cdot) \) and \( \Phi_j(\cdot) \) are differentiable functions.
The function $\Psi_i(\cdot)$ determines how productivity endogenously changes in response to changes in employment in the country. Whenever $\Psi_i(\cdot)$ varies with employment, our model is observationally equivalent to models with Marshallian externalities in production and “New Trade Theory” models with firm entry. The connection between our model and existing frameworks relies on the shape of $\Psi_i(\cdot)$, as summarized by the elasticity matrix, $\psi_{ij} \equiv \frac{\partial \log \Psi_i}{\partial \log L_j}$.

To see this, we consider the following parametric restriction.

**Assumption 1b.** The agglomeration function has an elasticity structure given by:

$$
\psi_{ij} = \psi \mathbb{I}_{i=j} - \tilde{\psi} \Psi_j(\{L_o\}_o).
$$

As before, there are different micro foundations leading to this functional form. In Krugman (1980), firm entry and increasing returns to scale in production give rise to agglomeration forces that depend on the size of the employment in each market – in this case, $\psi = 1/(\sigma - 1)$ and $\tilde{\psi} = 0$ for $i \neq j$, where $\sigma$ is the elasticity of substitution across varieties. In the same spirit, the specification in Allen and Arkolakis (2014) corresponds to the case of $\psi = \alpha$ and $\tilde{\psi} = 0$. Marshallian external economies of scale, as introduced in Ethier (1982b), correspond to similar specifications, with $i$ denoting a sector in a region. As discussed in the Appendix, this specification also captures congestion forces arising from other factors in production, such as housing or capital, as in Allen and Arkolakis (2014) and Caliendo et al. (2014). In this case, production costs are increasing in other market’s employment, and thus $\tilde{\psi} > 0$. Furthermore, we show that the combination of the mappings $X_{ij}(\cdot)$ and $\Psi_i(\cdot)$ implies that our framework is observationally equivalent to a general one-factor neoclassical economy with a flexible form of market-level external economies of scale.\(^{14}\)

Finally, restrictions on $\Phi_j(\cdot)$ allow our model to accommodate different assumptions regarding worker mobility across regions and sectors. Our environment encompasses perfect mobility between two markets whenever any real wage differential leads to employment concentration in the market with the highest real wage. Our model also covers worker immobility between two markets whenever the labor supply in one market does not depend on the real wage of the other market. Between these two extreme cases, our Generalized Spatial Competitive Economy is observationally equivalent to most of the traditional models.

\(^{14}\)Technology levels that are proportional to population are also postulated in Kortum (1997); Eaton and Kortum (2001). Models with spatial diffusion of knowledge, as in Fujita, Krugman, and Venables (1999) and Lucas and Rossi-Hansberg (2003), specify cross-location spillovers of knowledge so that $\psi_{ij} > 0$ even if $i \neq j$. Our environment can also easily accommodate models with multiple sectors that differ in terms of market structure and strength of economies of scale – e.g., Krugman and Venables (1995), Balistreri, Hillberry, and Rutherford (2010), Kucheryavyy, Lyn, and Rodríguez-Clare (2016).
of trade and geography in the literature. This is the case for “New Economic Geography” models as in Krugman (1991) and Helpman (1998). Similarly, this is the case for traditional trade models, such as the neoclassical setup and more recent quantitative multiple-sector gravity setups surveyed by Costinot and Rodriguez-Clare (2013), where there is mobility across sectors but not across regions.

In addition, the flexibility of $\Phi_j(\cdot)$ allows our model to cover spatial quantitative models exhibiting partial worker mobility across markets. To illustrate the dependence on labor supply on real wages, we consider the case in which $\Phi_j(\cdot)$ is a differentiable function with the following elasticity structure, $\phi_{ij} \equiv \frac{\partial \log \Phi_i}{\partial \log \omega_j}$.

**Assumption 1c.** The labor supply mapping has an elasticity structure given by:

$$\phi_{ij} = \phi_{[i=j]} - \tilde{\phi}\Phi_j \left( \{\omega_o\}_o \right).$$

Several recent geography models impose this constant-elasticity gravity structure on labor supply across either sectors or regions. Under specific parametric assumptions, this labor supply structure arises in geography models from either externalities in preference for locations or worker heterogeneity in location-specific preferences or efficiency – e.g., Allen and Arkolakis (2014), Redding (2012), Allen, Arkolakis, and Takahashi (2014), Bartelme (2015) and Bryan and Morten (2015). In recent trade models, this gravity-like labor supply function is implied by the sorting decision of heterogeneous workers that draw sector-specific efficiency from an extreme value distribution – e.g., Galle, Rodriguez-Clare, and Yi (2015). More generally, the unrestricted function $\Phi_j(\cdot)$ implies that our models is observationally equivalent to a generalized Roy model without parametric restrictions on the distribution of market-specific preferences or efficiency units across workers.

Furthermore, combined restrictions on $(X_{ij}(\cdot), \Phi_j(\cdot), \Psi_j(\cdot))$ imply that the Generalized Spatial Competitive Economy is observationally equivalent to existing quantitative spatial models reviewed in Redding and Rossi-Hansberg (2016). The formal equivalence requires transfer rules that specify how income of non-labor factors are allocated across markets. For instance, Allen and Arkolakis (2014) impose a local transfer that is proportional to the income of the residents in a location, and Caliendo et al. (2014) allow for the possibility that rental income is concentrated in a national portfolio and then split equally to the residents of each location.

Finally, our Generalized Spatial Economy can be easily extended to incorporate additional

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15We can also extend our model to allow markets to be region-sector-occupation triples. With this more general definition of a market, the model is also observationally equivalent to environments with gravity-like labor supply function across occupations, as in Burstein, Morales, and Vogel (2016) and Lee (2015). See Appendix D.2.
features highlighted by the trade and geography literature. As we describe in the Online Appendix D.2, we extend the model to allow for: i) workers commuting across regions, as in Ahlfeldt, Redding, Sturm, and Wolf (2015), Monte, Redding, and Rossi-Hansberg (2015) and Allen, Arkolakis, and Li (2015); ii) input-output linkages in production, as in Caliendo and Parro (2014); iii) multiple worker groups in production, as in Cravino and Sotelo (2017); and iv) firm heterogeneity as in Chaney (2008). In all these settings, besides the three aggregate mappings that arise in the Generalized Spatial Competitive Economy, there are other functions determining labor and trade outcomes in equilibrium. Thus, counterfactual predictions in the models considered in the extensions require additional data and mappings compared to our baseline model.

3 Counterfactual Changes in Response to Shocks

We now turn to the counterfactual predictions of our model regarding changes in equilibrium variables following exogenous shocks in local shifters of trade costs, productivity and labor supply. Our objective is to trace down the implications of these shocks as they are propagated through cross-market linkages, and assess their effects on employment and wages in different local labor markets. Subsequently, we evaluate the implications of our theoretical results for the design of empirical specifications aimed at measuring the impact of these shocks on local labor markets.

3.1 Counterfactual Changes in Response to Shocks

Our starting point is a characterization of the counterfactual predictions of our model following changes in the exogenous shifters of productivity, labor supply and trade costs. In this section, we follow the “exact hat algebra” approach proposed by Dekle, Eaton, and Kortum (2007), and use $\hat{z}_j = z_j' / z_j$ to denote the ratio between the variables in the new and the initial equilibrium, and use a superscript 0 to denote variables in the initial equilibrium. To simplify the exposition, we focus on the particular case in which $\{X_{ij}(.), \Phi_i(.), \Psi_i(.)\}_i$ are differentiable functions, and we henceforth assume that there is positive employment in every market.16

We define the “local labor market module” as the equilibrium conditions (7)–(9) written in changes:

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16These assumptions imply that the equilibrium conditions (6)–(9) can be written with equality. We rely on those assumptions to prove uniqueness and to write a first-order approximation of the model’s predictions. Without such assumptions, our analysis remains otherwise unchanged, except for the fact that these expressions would be written in terms of sets.
\[ L_i^0 \hat{L}_i = \Phi_i \left( \left\{ \left( \nu_j^0 \rho_j^0 \omega_j^0 \right) \left( \hat{\nu}_j \hat{\rho}_j \hat{\omega}_j \right) \right\}_{j \in J_i} \right) \tag{11} \]

\[ \hat{w}_i = \hat{p}_i \frac{\Psi_i \left( \left\{ L_j^0 \hat{L}_j \right\}_{j} \right)}{\Psi_i \left( \left\{ L_j^0 \right\}_{j} \right)}. \tag{12} \]

We also need to consider the changes in the remaining equilibrium conditions, which we call the “trade module.” The market clearing condition in (10) yields

\[ w_i^0 L_i^0 (\hat{w}_i \hat{L}_i) = \sum_j x_{ij}^0 \hat{x}_{ij} \rho_j^0 w_j^0 L_j^0 \left( \hat{\rho}_j \hat{w}_j \hat{L}_j \right), \tag{13} \]

where the changes in spending shares and price indices in (5)–(6) are given by

\[ x_{ij}^0 \hat{x}_{ij} = X_{ij} \left( \left\{ \frac{\tau_{ij}^0 \rho_i^0}{\zeta_o} \right\}_{o} \right) \quad \text{and} \quad \hat{P}_j = P_j \left( \left\{ \frac{\tau_{ij}^0 \rho_i^0}{\zeta_o} \right\}_{o} \right). \tag{14} \]

The system (11)–(14) determines the changes in endogenous variables, \( \{ \hat{p}_i, \hat{\rho}_i, \hat{L}_i, \hat{\omega}_i \} \), implied by any combination of shocks, \( \{ \hat{\zeta}_i, \hat{\nu}_i, \hat{\tau}_{ij} \} \). It depends on the aggregate mappings \( \{ X_{ij}(.), \Phi_i(.), \Psi_i(.) \}_{i} \) as well as initial outcomes, \( \{ x_{ij}^0, \rho_i^0, w_i^0, L_i^0 \}_{i,j} \), and initial prices and shifters, \( \{ \tau_{ij}^0, p_i^0, P_i^0, \nu_i^0 \}_{i,j} \). Notice however that our model – and thus a large number of spatial models – is over-identified: there are multiple degrees of freedom to match observed labor and trade outcomes in the initial equilibrium. Specifically, in Appendix A.1, we show that it is always possible to choose the location of the preference and productivity shifters in \( \{ X_{ij}(.), \Phi_i(.), \Psi_i(.) \}_{i} \) to replicate the initial levels of trade flows and labor market outcomes across markets, while normalizing shifters of trade costs, productivity, and labor supply in the initial equilibrium. \(^{17}\) Thus, in equations (11)–(14), we choose initial shifters such that

\[ \frac{\tau_{ij}^0 \rho_i^0}{\zeta_o} = 1, \quad P_j^0 \equiv 1, \quad \nu_i^0 \equiv 1, \quad \Psi_i \left( \left\{ L_j^0 \right\}_{j} \right) \equiv 1 \quad \forall i, j. \tag{15} \]

Given the normalization in (15), we can use the system in (11)–(14) to characterize the counterfactual predictions in the Generalized Spatial Competitive Economy.

**Lemma 1.** Consider the Generalized Spatial Competitive Economy under the normalization in (15). Conditional on transfer rules \( \{ \hat{p}_j \}_{j} \) and initial levels of endogenous variables

\(^{17}\)The normalization of bilateral effective prices in the initial equilibrium is analogous to that imposed in neoclassical economies by Adao, Costinot, and Donaldson (2017).
\[
\left\{ x^0_{ij}, \rho^0_j, w^0_j, L^0_j \right\}, \text{ the counterfactual changes in endogenous outcomes, } \left\{ \hat{p}_j, \hat{L}_j \right\}, \text{ implied by any combination of shocks, } \left\{ \hat{\tau}_{ij}, \hat{\zeta}_j, \hat{\nu}_j \right\}, \text{ solve }
\]

\[
L^0_i \hat{L}_i = \Phi_i \left( \left\{ \left( \rho^0_j w^0_j \right) \left( \hat{v}_j \hat{p}_j \Psi_j \left( \left\{ L^0_o \hat{L}_o \right\}_o \right) \right) \right\}_j \right) \quad (16)
\]

\[
w^0_i \left( \hat{E}_i \hat{p}_i \right) = \sum_j X_{ij} \left( \left\{ \hat{\tau}_{oj} \hat{p}_o / \hat{\zeta}_o \right\}_o \right) \left( \rho^0_j w^0_j \right) \left( \hat{p}_j \hat{p}_j \hat{E}_i \right), \quad (17)
\]

where \( \hat{p}_j = P_j \left( \left\{ \hat{\tau}_{oj} \hat{p}_o / \hat{\zeta}_o \right\}_o \right), \) and \( \hat{E}_i \equiv L^0_i \hat{L}_i \Psi_j \left( \left\{ L^0_o \hat{L}_o \right\}_o \right). \)

Lemma 1 outlines the system that characterizes the response of the economy to changes in the shifters of trade costs, productivity, and labor supply. The local labor market module in (16) yields a relationship between changes in local competitiveness, \( \hat{p}_i / \hat{P}_i \), and changes in employment, \( \hat{L}_i \), driven by the cross-market linkages in labor mobility and productivity spillovers embedded in the mappings \( \Phi_i(\cdot) \) and \( \Psi_i(\cdot) \). In contrast, the trade module in (17) determines a relationship between changes in competitiveness, \( \hat{p}_i / \hat{P}_i \), and changes in market size, \( \hat{E}_i \), that depends on the properties of the bilateral trade network in \( X_{ij}(\cdot) \).\(^{18}\)

To investigate the predictions of our model and to simplify the exposition of our results, we introduce additional notation. We use bold variables to denote stacked vector, \( z \equiv [z_i]_i \), and bar bold variables to denote matrices, \( \bar{z} \equiv [z_{ij}]_{i,j} \). The solution of the system (16)–(17) depends on the shape of the cross-market linkages in the mappings \( \left\{ \{X_{ij}(\cdot)\}_i, \Phi_j(\cdot), \Psi_j(\cdot) \right\}_j \) locally summarized by the elasticity matrices,

\[
\bar{\chi} \equiv \left[ \sum_d y_{id} \chi_{ijd} \right]_{i,j}, \quad \bar{\phi} \equiv [\phi_{ij}]_{i,j}, \quad \bar{\psi} \equiv [\psi_{ij}]_{i,j}, \quad (18)
\]

as well as the following matrices of equilibrium trade linkages,

\[
\bar{y} \equiv [y_{ij}]_{i,j}, \quad \bar{x} \equiv [x_{ji}]_{i,j}, \quad (19)
\]

where \( y_{ij} \) denotes the share of market \( i \)'s revenue from sales to market \( j \), and \( x_{ji} \) denotes the

\(^{18}\)Equation (16) generalizes previous approaches to model endogenous labor supply that postulate a logit form (as in assumption 1b-1c) discussed in Allen, Arkolakis, and Takahashi (2014). Equation (14) generalizes previous approaches modeling trade demand, departing from the gravity assumption (as in assumption 1a), consistent with the approach of Adao, Costinot, and Donaldson (2017). Finally, equation (17), as a result of the incorporation of the transfer function, \( \rho_j \), is flexible enough to capture various forces that affect the redistribution of income across markets and sectors, stressed out in previous work. Examples of that, such as transfers, profits, land, capital rents and others are discussed in the Online Appendix.
share of market $i$’s expenditure on goods from market $j$.

The elasticity matrices defined in (18) capture responses along the three margins of cross-market linkages in our model: $\bar{\chi}$ is the elasticity of total sales to production costs, $\bar{\phi}$ is the elasticity of labor supply to real wages, and $\bar{\psi}$ is the elasticity of productivity to labor supply. Notice that these elasticity matrices are allowed to depend on the endogenous variables in the initial equilibrium – as in the parametric examples introduced in Section 2.3.

Since in our system of equations multiple equilibria may arise, we establish intuitive conditions under which the model delivers unique predictions.\(^{19}\) Specifically, we impose the following assumption.

**Assumption 2.** Assume that (i) the matrix $\bar{\Lambda} \equiv (\bar{I} - \bar{\psi}\bar{\phi})^{-1}\bar{\phi}$ exists, and (ii) the off-diagonal elements of $\bar{\Gamma} \equiv \bar{I} - \bar{y} - \bar{\chi} + (\bar{I} - \bar{y}) (\bar{I} + \bar{\psi}) \bar{\Lambda} (\bar{I} - \bar{x})$ are strictly negative for all $\bar{y}$ and $\bar{x}$.

The first part of Assumption 2 imposes that the matrix $\bar{I} - \bar{\psi}\bar{\phi}$ is non-singular. This guarantees that, locally, $\{\bar{L}_i\}_i$ is a function of $\{\hat{\tau}_o/\bar{P}_i\}_i$ implicitly characterized by the solution of the labor module in (16). The second part of Assumption 2 imposes restrictions on $\bar{\Gamma}$: the elasticity matrix of the excess trade demand system in (17) to production cost changes. This matrix combines two well known forces. As in gravity trade models, such changes trigger responses in market spending shares – i.e., the revenue effect of $X_{ij} (\{\hat{\tau}_o/\bar{P}_o\}_o)$ in equation (16), whose elasticity matrix is $\bar{\chi}$. As in spatial models, cost changes affect the spatial allocation of labor and, therefore, the relative size of destination markets – i.e., the revenue effect of $\hat{p}_j \hat{E}_j$ in equation (17), whose elasticity matrix is $(\bar{I} - \bar{y})(\bar{I} + \bar{\psi}) \bar{\Lambda} (\bar{I} - \bar{x})$. Assumption 2 is guaranteed whenever changes in production costs of competitors trigger gains in market shares that are larger than any potential loss in relative market size.

Assumption 2 is implied by restrictions on the shape of the mappings $\{X_{ij}(\cdot)\}_i$, $\Phi_j(\cdot)$, $\Psi_j(\cdot)$\(^{j}\). In the gravity model with Assumption 1a-1c, it is easy to show that Assumption 2 holds for a sufficiently large trade elasticity: $\chi > \phi/(1 - \phi\psi) > 0$.\(^{20}\) Similarly, in the general model, Assumption 2 requires that, for every $x_{od} \in (0, 1)$,

$$\chi_{oij} > (\mu_{io} - \mu_{jo}) + \sum_d (\mu_{jd} - \mu_{id}) x_{od} > 0 \quad \forall o \neq i \quad \forall j,$$

where $\bar{\mu} = (\bar{I} + \bar{\psi}) \bar{\Lambda}$ is the elasticity matrix of $\hat{E}_i$ to changes in $p_j/\hat{P}_j$ implicitly determined by the solution of (16).

\(^{19}\)Although we provide sufficient conditions for uniqueness that guarantee the solution of the model for large changes in parameters, for much of our theoretical analysis we consider small deviations around the observed equilibrium, so that the possibility of multiple equilibria does not affect our analysis.

\(^{20}\)Note that, in the case of inelastic labor supply, this condition becomes $\chi > 0$, which guarantees uniqueness in gravity trade models.
Under Assumption 2, the excess trade demand system satisfies the gross substitution property and, therefore, the equilibrium is unique (see Proposition 17.F.3 in Mas-Colell et al. (1995a)). To prove our first result, recall that, as discussed in Section 2, the integrability property of demand guarantees that knowledge of \{X_{ij}(\cdot)\}_i yields knowledge of \(P_j(\cdot)\). So, the price index is fully determined by the bilateral demand function. Thus, we state the following proposition.

**Proposition 1.** Consider the Generalized Spatial Competitive Economy satisfying Assumption 2. Conditional on transfer rules \(\{\hat{\rho}_j\}_j\) and initial levels of endogenous variables \(\{x_{ij}^0\}_i, \rho_j^0, w_j^0, L_j^0\)_j, the mappings \(\{X_{ij}(\cdot)\}_i, \Phi_j(\cdot), \Psi_j(\cdot)\)_j are sufficient to uniquely characterize counterfactual changes in endogenous outcomes, \(\{\hat{p}_j, \hat{P}_j, \hat{L}_j, \hat{\omega}_j\}_j\), implied by any combination of shocks, \(\{\hat{\tau}_{ij}, \hat{\zeta}_j, \hat{\nu}_j\}_j\), as a solution of (16)–(17).

The key insight of this result is that, in the class of models covered by the Generalized Spatial Competitive Economy, the researcher does not need to take a stance on the underlying microeconomic assumptions that generate the unobservable shocks. Instead, by conditioning on the initial equilibrium outcomes, \(\{x_{ij}^0\}_i, \rho_j^0, w_j^0, L_j^0\)_j, one can cleanse the model from any microeconomic assumptions. The microeconomic structure matters only insofar it affects the shape of the mappings \(\{X_{ij}(\cdot)\}_i, \Phi_j(\cdot), \Psi_j(\cdot)\)_j, which themselves can be represented with macroeconomic elasticity functions.

The counterfactual predictions of our model depend exclusively on the shape of the mappings \(\{X_{ij}(\cdot)\}_i, \Phi_j(\cdot), \Psi_j(\cdot)\)_j around the initial equilibrium. These mappings put discipline on the cross-market linkages and, therefore, determine the propagation of shocks between markets. We now turn to a careful investigation of such properties of model.

### 3.2 Cross-market Effects of Local Exposure to Shocks

To illustrate how the elasticity structure of \(\{X_{ij}(\cdot)\}_i, \Phi_j(\cdot), \Psi_j(\cdot)\)_j affects the counterfactual predictions of our model, we consider the first-order approximation of the log-change in endogenous variables following shocks in trade costs, \(\hat{\tau}_{ij}\), and productivity, \(\hat{\zeta}_j\). As discussed in Appendix A.3, all our insights also hold for local shocks in labor supply, \(\hat{\nu}_i\).

It is useful to define market \(i\)'s direct shock exposure to trade cost and productivity shocks, henceforth a "cost shock", as an importer,

\[
\log \hat{\eta}_i^T \equiv \sum_j a_{ji}^0 \left( \log \hat{\tau}_{ji} - \log \hat{\zeta}_j \right),
\]
and as an exporter,

$$\log \hat{\eta}_i^E \equiv \sum_j y^0_{ij} \sum_l \chi_{lij} \left( \log \hat{\tau}_{lj} - \log \hat{\zeta}_l \right).$$

(21)

These expressions capture the effect on market $i$ if wages and employment do not adjust to the cost shock. Specifically, $\hat{\eta}_i^I$ is the change in the cost of the consumption bundle triggered by the shock, while holding fixed wages and employment in all markets. Notice that, due to the optimality of the initial consumption bundle, market $i$’s direct import exposure depends solely on spending composition in the initial equilibrium, $x^0_{ji}$. In addition, $\hat{\eta}_i^E$ is the shift in total sales of market $i$ stemming from shocks affecting destination markets, while holding fixed employment and wages in the world economy. It is a function of $i$’s revenue exposure to a market, $y^0_{ij}$, interacted with the demand response of that market, $\chi_{lij}$, triggered by cost shocks to competitors, $\hat{\tau}_{lj}$ and $\hat{\zeta}_l$. Thus, $\hat{\eta}_i^E$ includes the effect on $i$’s revenue of changes in the competitive environment in the markets where $i$ sells to – the so-called third market effects.

Under Assumption 2, we obtain a precise characterization of the changes in endogenous outcomes caused by exogenous trade cost shocks. Proposition 2 describes the effect of such shocks on local labor market outcomes.

**Proposition 2.** Suppose that Assumption 2 holds. Then, the impact of cost shocks on local labor markets is, up to a first-order approximation, given by

$$\log \hat{L} = \tilde{\Lambda} \left( \log \hat{p} - \log \hat{P} \right)$$

(22)

$$\log \hat{\omega} = (I + \tilde{\psi} \tilde{\Lambda}) \left( \log \hat{p} - \log \hat{P} \right)$$

(23)

$$\log \hat{p} - \log \hat{P} = -\tilde{\Gamma}^I \log \hat{\eta}_i^I + \tilde{\Gamma}^E \log \hat{\eta}_i^E.$$  

(24)

where $\tilde{\Gamma}^I \equiv I - \tilde{\Gamma}^E \left( I - \tilde{\gamma} \right) \tilde{\mu}$, $\tilde{\Gamma}^E \equiv (I - \tilde{x}) \tilde{\Gamma}_m$, $\tilde{\Gamma}_m \equiv \tilde{M}' \left( \tilde{M} \tilde{\Gamma} \tilde{M}' \right)^{-1} \tilde{M}$, and $\tilde{M}$ is the matrix obtained from deleting the $m$-th row from the identity matrix.

Proposition 2 implies that, in general, understanding the impact of cost shocks on local labor markets requires not only measuring the impact of local shock exposure, but also knowledge of how shocks propagate to all other markets in the world economy. With small shocks, matrix inversions of the elasticity matrices $(\tilde{\chi}, \tilde{\phi}, \tilde{\psi})$ are thus sufficient to measure all these various linkages and associated propagation across markets.

In equations (22)–(23), the matrix $\tilde{\Lambda}$ controls the effect of endogenous changes in local competitiveness, $\hat{p}_i/\hat{P}_i$, on employment, $\hat{L}_i$, and real wages, $\hat{\omega}_i$. This matrix summarizes the spillovers effects of local competitiveness in its off-diagonal entries – that is, how much
competitiveness in one market affects employment and real wages in other markets. Under the assumption that spillovers are bounded, the matrix $\tilde{\Lambda}$ can be written as a series expansion of cross-market linkages in labor mobility and productivity, $\tilde{\Lambda} = \sum_{d=0}^{\infty} (\psi \tilde{\varphi})^d \tilde{\varphi}$. Intuitively, this series expansion captures the multiple adjustment rounds necessary to guarantee labor market clearing everywhere given changes in local competitiveness, $\hat{p}_{ii}/\hat{P}_i$.

An even more crucial role in our analysis is played by the transmission of shocks through $\hat{p}_i/\hat{P}_i$. The direct exposure to trade shocks, $(\hat{\eta}^E, \hat{\eta}^I)$, causes changes in competitiveness in both the own market and other markets as captured, respectively, by the diagonal and the off-diagonal entries of the matrices $\tilde{\Gamma}^E$ and $\tilde{\Gamma}^I$. Cross-market effects arise from the fact that, in response to direct shock exposure, good market clearing requires changes in local competitiveness, which by itself triggers further endogenous responses in trade flows due to substitution and market size effects in demand. The definition of $\tilde{\Gamma}$ indicates that the strength of cross-market links is controlled by the elasticity of bilateral trade flows, $\tilde{\chi}$, and the initial trade network in $\tilde{y}$ and $\tilde{x}$. In fact, under parameter restrictions, the matrix can be written as an infinite series sum of those elasticity and initial trade network capturing the spatial linkages of locations and the percolation of global shocks through the network of locations.

We now turn to an investigation of the consequences of cross-market spillovers to the empirical assessment of the effect of global economic shocks on local labor markets.

### 3.3 Measuring the Effect of Global Shocks on Labor Markets

Can variation in local exposure to global shocks identify the effect of these shocks on local labor markets? We answer this question using the predicted impact of economic shocks on local employment in the Generalized Spatial Competitive Economy. To illustrate the role played by cross-market links in the model, we compare predictions obtained under different particular cases of our general framework, and discuss their implications for design of empirical specifications.

We start by analyzing the model’s predicted impact of a generic cost shock on local employment, and compare it to the predicted impact of a shock to a segmented market in a Small Open Economy.

**Proposition 3.** Consider trade cost and productivity shocks in the world economy. The effect on the local employment is, up to a first-order approximation:

1) For a market in the Generalized Spatial Competitive Economy,

\[ 1 \) For a market in the Generalized Spatial Competitive Economy, $\sum_{j} | \sum_{o} \psi_{io} \phi_{oj} | < 1. \]

\[ 21 \) Formally, the series expansion holds if the spillovers are such that $\sum_{j} | \sum_{o} \psi_{io} \phi_{oj} | < 1. \]
\[
\log \hat{L}_i = \beta \log \hat{\eta}^E_i - \beta^I \log \hat{\eta}^I_i + \sum_{j \neq i} (\beta_{ij}^E \log \hat{\eta}^E_j - \beta_{ij}^I \log \hat{\eta}^I_j)
\]

where \(\beta_{ij}\) and \(\beta_{ij}^I\) are computed from the entries of \(\tilde{\Lambda}, \tilde{\Gamma}^E, \tilde{\Gamma}^I\).

2) For a segmented market in a Small Open Economy,

\[
\log \hat{L}_i = \alpha \log \hat{\eta}^E_i - \alpha^I \log \hat{\eta}^I_i,
\]

where \(\alpha \equiv (1 - x_{ii}) \lambda_{ii} \sigma_{ii} \) and \(\alpha^I \equiv \lambda_{ii} (1 - (1 - x_{ii}) \sigma_{ii} (1 + \psi_{ii}) (1 - y_{ii}) \lambda_{ii})\).

Equation (25) shows that, in the general case, employment in market \(i\) responds not only to local shocks, \((\hat{\eta}^E_i, \hat{\eta}^I_i)\), but also to the shocks hitting other markets, \((\hat{\eta}^E_j, \hat{\eta}^I_j)\) for \(j \neq i\). As discussed above, spatial links imply that the change in local employment induced by local shock exposure affect demand and supply conditions in other markets in the world economy, which trigger wage and employment responses elsewhere.

Such indirect spillover effects cannot be disregarded in an empirical analysis of the effect of economic shocks on local labor markets. They constitute an additional general equilibrium exposure channel of market \(i\) to economic shocks that, if omitted, has the potential to introduce bias in the measurement of both the differential and aggregate effects of cost shocks. Notice that, with a flexible structure of cross-market links, the indirect spillover effects are heterogeneous across markets and, therefore, cannot be accounted by fixed effects in empirical specifications.

In the second part of Proposition 3, we consider a cost shock affecting a segmented market in a Small Open Economy.\(^{22}\) In this case, there are no indirect spillover effects, since local outcomes can adjust to guarantee local market clearing without affecting the goods and labor markets in the rest of the country, in particular, and in rest of the world economy, in general. Equation (26) clearly illustrates how, in this environment, the relationship between local shock exposure and changes in local employment yields both the differential and the aggregate impact of cost shocks on labor market outcomes. This is the main intuition behind the multiple-sector models that motivates the empirical specification in Kovak (2013) and Autor, Dorn, and Hanson (2013).

To further understand the role of cross-market spillovers, we now consider the gravity economy under Assumptions 1a-1c.

\(^{22}\)We define formally a segmented market in a Small Open Economy in Appendix A. We say that market \(d\) is a small open economy if, for all other markets \(i \neq d\), (i) \(d\) is a trivial part of revenue and expenditure, \(y^0_{id} \approx 0\) and \(x^0_{di} \approx 0\); and (ii) wages and prices in \(d\) do not affect endogenous outcomes, \(\chi_{dij} = \phi_{id} = \psi_{id} = 0\).
Proposition 4. Consider trade cost and productivity shocks in the world economy. The
effect on the local employment in the parametric gravity model under Assumptions 1a-1c is,
up to a first-order approximation:

1) Without trade costs,

\[
\log \hat{L}_i = \beta^E \log \hat{\eta}^E_i + \beta^I \log \hat{\eta}^I_i + \sum_j \gamma_{ij} \left( \beta^E \log \hat{\eta}^E_j + \beta^I \log \hat{\eta}^I_j \right),
\] (27)

where \( \log \hat{\eta}^I_i = \log \hat{\eta}^I_i \) for all \( i \).

2) With trade costs,

\[
\log \hat{L}_i = \beta^1 \sum_j \gamma^E_{ij} \log \hat{\eta}^E_j + \beta^2 \sum_j \gamma^I_{ij} \log \hat{\eta}^I_j + \sum_j \left( \tilde{\beta}^1_j \sum_i \gamma^E_{ij} \log \hat{\eta}^E_j + \tilde{\beta}^2_j \sum_i \gamma^I_{ij} \log \hat{\eta}^I_j \right),
\] (28)

where the constants \( \beta^i \) depend on \( \epsilon, \phi, \psi, \tilde{\phi}, \tilde{\psi} \), and the constants \( \tilde{\beta}^i_j \) depend on \( L_j, w_j \) as well.

When \( \tilde{\phi} = \tilde{\psi} = 0 \), we have \( \tilde{\beta}^1_j = \tilde{\beta}^2_j = 0 \).

Proposition 4 outlines the direct and indirect effects of local exposure to cost shocks in
the gravity economy under Assumptions 1a-1c. The first part of the proposition considers
the case without trade costs, where expenditure and revenue shares are symmetric (\( y_{ij} = y_j \nand \( x_{ji} = x_j \) for all \( i \)). Under these restrictions, the matrices of cross-market links, \( (\hat{\chi}, \hat{\phi}, \hat{\psi}) \),
entail identical effects of changes in the outcomes of market \( j \) on outcomes of every other
market \( i \) in the economy.\(^23\) This symmetry translates into a symmetric indirect spillover
effect in general equilibrium which constitutes an “endogenous” fixed effect for all markets.

This particular case has important implications for empirical analysis. Conditional on
the fixed effect, differential variation in direct shock exposure is associated with differential
changes in employment across markets. Thus, when symmetry is strong, a standard difference-
in-difference specification is able to precisely measure the differential effects of local shock
exposure. However, these differential effects do not correspond to the complete effect of
the shock on local markets: the fixed effect contains the component of the response that is
identical to all markets. Thus, even in this case, the magnitude of the effect of economic
shocks on employment depends on the strength of cross-market links.\(^24\)

In the presence of trade costs under Assumptions 1a-1c, revenue and spending shares are
asymmetric, implying that cross-market links in trade flows are heterogeneous. As shown in
equation (28), this introduces heterogeneity in indirect spillover effects through the weights

\(^{23}\)Specifically, the matrices \( (\hat{\chi}, \hat{\phi}, \hat{\psi}) \) have a form: \( A = [a_{ij}] \) such that \( a_{ij} = \alpha 1_{i=j} - a_j \) for all \( i \).

\(^{24}\)These symmetry assumptions are routinely maintained in empirical papers in macroeconomics, develop-
ment, and urban economics – for example, see Kovak (2013), Kline and Moretti (2014), Nakamura and
\( \gamma_{ij}^E \) and \( \gamma_{ij}^I \). Only the part of indirect effects associated with cross-market links in labor supply and productivity is common to all markets and, therefore, is absorbed by the “endogenous” fixed effect — in fact, this term disappears if \( \tilde{\phi} = \tilde{\psi} = 0 \).

Taken together, our theoretical results indicate that any investigation of the complete effect of cost shocks on local labor markets requires a careful assessment of the magnitude of indirect spillover effects in general equilibrium. Only when such indirect effects are quantitatively small, empirical specifications based on cross-market variation in direct shock exposure recover the differential and the aggregate effect of economic shocks on local labor markets. Thus, in the rest of the paper, we turn to the problem of estimating cross-market links within countries, and quantifying the magnitude of indirect spillover effects in general equilibrium.

4 Econometric Methodology

We now turn to the problem of estimating the central aggregate mappings in our model. In particular, we focus on the estimation of the elasticity structure of labor supply, \( \Phi_i(\cdot) \), and agglomeration, \( \Psi_i(\cdot) \). Throughout our analysis, we assume that the elasticity for bilateral trade flows, \( X_{ij}(\cdot) \), is known, since its estimation has been the goal of an extensive literature in international trade.

We develop our methodology in three steps. First, we propose an econometric version of our model where \( \Phi_i(\cdot) \) and \( \Psi_i(\cdot) \) depend on a vector of unknown parameters. In this parametric model, we establish that, conditional on \( \Phi_i(\cdot) \) and \( \Psi_i(\cdot) \), local unobserved shocks in productivity and labor supply are identified from observable data on trade and labor outcomes. Second, we construct a class of moment conditions using observed trade cost shifters that are orthogonal to local shocks in productivity and labor supply. Finally, we use our general equilibrium model to show that the “optimal” instrument in this class is the impact of observed trade shocks on the endogenous variables predicted by our general equilibrium model – the Model-implied Optimal IV (MOIV).

\[25\] In single-sector gravity models, the demand for bilateral trade flows only depends on the trade elasticity that has been studied by an extensive empirical literature — for a review, see Head and Mayer (2013). In addition, Caliendo and Parro (2014) and Costinot, Donaldson, and Komunjer (2011) consider multiple-sector gravity models where these functions only depend on the sector-level trade elasticity that is estimated using sector-level bilateral trade flows. More recently, Adao, Costinot, and Donaldson (2017) consider the problem of non-parametrically identifying the functions controlling bilateral trade flows in a competitive environment. It is possible to show that a similar argument holds in our environment, leading to the non-parametric identification of \( X_{ij}(\cdot) \).
4.1 Parametric Econometric Model

In every period \( t \), we assume that the world economy is generated by the Generalized Spatial Competitive Economy described in Section 2. Specifically, we impose that the aggregate mappings \( \{ \{ X_{ij}(\cdot) \}_i, \Phi_j(\cdot), \Psi_j(\cdot) \}_j \) are the same in all periods, but markets are subject to changes in the exogenous shifters, \( \{ \{ \tau_{ij} \}_i, \zeta_j, \nu_j \}_j \). In equilibrium, endogenous trade and labor outcomes are determined by the solution of (6)–(10).

Let \( y_t \) denote the value of variable \( y \) in period \( t \), and \( \hat{y}_t = y_t/y_0 \) denote its change between a base period 0 and period \( t \), with \( \Delta \log y_t \equiv \log \hat{y}_t \). Assume that we observe bilateral trade flows, \( \{ X_{ij}^t \} \), and labor market outcomes, \( \{ \rho^t_i, \omega^t_i, L^t_i \} \), but we do not observe the shifters, \( \{ \nu^t_i, \zeta^t_i, \tau^t_{ij} \} \).

We start by introducing restrictions on the functions controlling labor supply, \( \Phi_j(\cdot) \), and agglomeration forces, \( \Psi_j(\cdot) \). We assume that these functions are known, except for a vector of unknown parameters, \( \theta \). Formally, we impose the following assumption.

**Assumption 3a.** \( \Phi_j(\cdot) = \Phi_j(\cdot | \theta) \) and \( \Psi_j(\cdot) = \Psi_j(\cdot | \theta) \) are differentiable functions on the vector of unknown parameters \( \theta \in \mathbb{R}^s \).

In the rest of this section, we use our general equilibrium model to propose a methodology to consistently estimate \( \theta \). To this end, we impose additional restrictions in our model that allow the recovery of the unobserved local shocks, \( \{ \nu^t_i, \zeta^t_i \} \), from the observed changes in trade and labor market outcomes, \( \{ x_{ij}^t, \omega^t_i, L^t_i, \rho^t_i \} \), and the observed equilibrium variables in the base period, \( W^0 = \{ X^0_{ij}, L^0_j \} \).

**Assumption 3b.** \( \Phi_j(\cdot) \) and \( X_{ij}(\cdot) \) are functions such that

1. \( \{ L_j \}_{j \in \mathcal{J}_c} = \{ \Phi_j(\omega | \theta) \}_{j \in \mathcal{J}_c} \) is invertible,
2. \( \{ x_{ij} \}_i = \{ X_{ij}(\{ p_{oj} \}_o) \}_i \) is invertible (up to a scalar) for all \( j \in \mathcal{J}_c \).

The first part of Assumption 3b imposes that the system \( \{ L_j \}_{j \in \mathcal{J}_c} = \{ \Phi_j(\omega | \theta) \}_{j \in \mathcal{J}_c} \) is invertible, so we can write \( \omega_j = \Phi_j^{-1}(L_j | \theta) \). The combination of this restriction with the invertibility of the structural residuals is a crucial step in many empirical structural frameworks – see Berry (1994); Berry and Haile (2014). In the Generalized Spatial Competitive Economy, invertibility is guaranteed if the utility function has following separable form:

\[
U_c = \sum_j \nu_j C_j + U(\{ L_j \})
\]

with \( U(\cdot) \) strictly quasi-concave. In this case, the labor supply function is the unique solution of the system of first-order conditions: \( \nu_j \rho_j \omega_j = \frac{\partial U}{\partial L_j} \). We can also consider the weaker condition of invertibility up to a
labor supply equation in (7) yields the change in labor supply shifters:

\[ \Delta \log \nu^t_j = -\Delta \log \rho^t_j - \Delta \log \omega^t_j + \Delta \log \Phi^{-1}_j (L^t|\theta). \]  

(29)

Moreover, our methodology requires a measure of the local competitiveness, \( \hat{p}_i/\hat{P}_i \). This implies either taking an explicit stance on the price data to measure \( p_i \), or using data on trade shares and invert the function \( X_{ij}(\cdot) \) to obtain prices. Since our model yields equivalent counterfactual outcomes to a wide class of existing frameworks, one should be cautious in using lead to different disaggregated prices, as argued by Arkolakis, Costinot, and Rodríguez-Clare (2012) and Simonovska and Waugh (2014b), and thus different measured aggregate prices. Thus, in the second part of Assumption 3b, we effectively impose that it is possible to recover relative prices from observed trade flows: \( p^t_{ij}/p^t_{jj} = X^{-1}_{ij}(x^t_j) \) where \( x^t_j \equiv \{x^t_{ij}\}_i \).

27 The demand function \( X_{ij}(\cdot) \) is invertible (up to scalar) if it satisfies the connected substitutes property – see Berry, Gandhi, and Haile (2013). Adao, Costinot, and Donaldson (2017) show that the invertibility of \( X_{ij}(\cdot) \) is a central property for its non-parametric identification, and that it is guaranteed in a generalized class of Ricardian economies.

Recalling that, as in Section 2, knowledge of \( X_{ij}(\cdot) \) yields knowledge of \( P_j(\cdot) \), we can write local competitiveness, \( Q_j(x^t_j) = P_j(\{p^t_{ij}/p^t_{jj}\}_j) \), as a function of the vector of local spending shares:

\[ Q_j(x^t_j) = P_j(\{X^{-1}_{ij}(x^t_j)\}_i). \]

Since \( p^t_{ii} = p^t_{i}/\zeta^t_i \), we can apply this expression into (9) to recover productivity shifters from the following expression:

\[ \Delta \log \zeta^t_j = \Delta \log \omega^t_j - \Delta \log Q_j(x^t_j) - \Delta \log \Psi_j(L^t|\theta). \]

(30)

Given the parameter vector \( \theta \), expressions (29)–(30) relate changes in the unobserved shocks, \( \{\nu^t_j, \zeta^t_j\} \), to changes in observed variables, \( \{x^t_j, \omega^t_j, L^t_j, \rho^t_j\} \). We summarize this relationship in the following expression:

\[
\begin{bmatrix}
\Delta \log \nu^t_j \\
\Delta \log \zeta^t_j
\end{bmatrix}
= -
\begin{bmatrix}
\Delta \log \rho^t_j + \Delta \log \omega^t_j \\
\Delta \log Q_j(x^t_j) - \Delta \log \omega^t_j
\end{bmatrix}
+ \begin{bmatrix}
\Delta \log \Phi^{-1}_j (L^t|\theta) \\
-\Delta \log \Psi_j (L^t|\theta)
\end{bmatrix}.
\]

(31)

Expression (31) is our main estimating equation: it yields the underlying local shocks from observable data on trade and labor outcomes. This, of course, relies on two premises. First, because our approach is parametric, it requires the correct functional forms of the scalar when the labor supply function is also homogeneous of degree zero, as in spatial assignment models. In this case, the labor supply function is invertible (up to scalar) if it satisfies the connected substitutes property – see Adão (2015).

25
mappings \( \Phi_j(\cdot | \theta) \) and \( \Psi_j(\cdot | \theta) \). Second, since expression (31) follows directly from the equilibrium conditions (7) and (9), these conditions must specify correctly the labor supply and production decisions in the economy. Violations of these premises introduce additional endogenous variables in (31) that give rise to usual concerns regarding omitted variable bias in the estimation of \( \theta \). 28 Importantly, expression (31) is robust to mis-specification in other equilibrium conditions of the model and, therefore, relies on weaker assumptions than those necessary to recover local shocks using the economy’s full general equilibrium structure – as in Bartelme (2015); Allen, Arkolakis, and Takahashi (2014).

To estimate \( \theta \) using expression (31), notice that, in general equilibrium, local trade and local labor outcomes are correlated with the unobserved local shocks. In fact, \( \{x^t_j, \omega^t_j, L^t_i, Q^t_i\} \) are endogenous variables that depend on exogenous shocks, \( \{\hat{\tau}^t_{ij}, \hat{\nu}^t_{ij}, \hat{\zeta}^t_j\} \). Thus, we design an instrumental variables methodology that exploits the structure of our model to construct moment conditions for the consistent estimation of \( \theta \).

4.2 Model-implied Optimal IV

We now derive moment conditions for the estimation of \( \theta \) using the recovered error terms in (31) and observed shifters of bilateral trade costs. To this end, assume that we have an observable variable, \( \hat{z}^t \equiv \{\hat{z}^t_{ij}\} \), that satisfies the following conditions.

**Assumption 3c.** There exists a trade cost shifter, \( \hat{z}^t \equiv \{\hat{z}^t_{ij}\} \), such that

1. \( E [\Delta \log \tau^t_{ij}|\hat{z}^t, W^0] = \kappa \Delta \log z^t_{ij} \),
2. \( E [\Delta \log \nu^t_j|\hat{z}^t, W^0] = E [\Delta \log \zeta^t_j|\hat{z}^t, W^0] = 0 \).

Assumption 3c imposes two conditions on the observable shifter \( \hat{z}^t \). First, changes in trade costs in the model, \( \tau^t_{ij} \), are log-linearly related to changes in the observable shifter, \( \hat{z}^t_{ij} \). This log-linearity restriction significantly simplifies the conditions for optimality of our methodology. However, it is not necessary to compute our instrument.

In addition, the second part of the Assumption 3c states that, conditional on the vector of endogenous variables in the initial world equilibrium \( W^0 \), the trade cost shifter \( \hat{z}^t \) is

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28 Any parametric empirical approach is subject to similar concerns. In our case, the separability of local unobserved shocks in equation (31) allows the non-parametric identification of the functions \( \Psi_j(\cdot), \Phi_j(\cdot) \) as long as instrumental variables satisfy the completeness condition proposed by Newey and Powell (2003) – for similar strategies, see Berry and Haile (2014), and Aado, Costinot, and Donaldson (2017). Expression (31) is subject to misspecification of the channels determining labor supply and production costs in conditions (7) and (9). For instance, as discussed in Appendix D.2, extensions of our model introduce extra variables into these conditions and, therefore, imply modified versions of (31).
mean-independent from local shocks to productivity and labor supply. This exogeneity condition is similar to the assumption imposed in empirical works in international trade, such as Topalova (2010), Kovak (2013), and Pierce and Schott (2016). In the empirical application below, we will use worldwide tariff changes as observable shifters of bilateral trade costs that are mean-independent from local shocks.\(^{29}\)

To construct moment conditions, we introduce a function capturing the exposure of each market to the observable trade cost shock: \(H_i(\hat{z}^t, W^0)\). By the law of iterated expectations, Assumption 3c immediately implies that, for any function \(H_i(\cdot)\),

\[
E[H_i(\hat{z}^t, W^0) e_i^t] = 0.
\]

\[(32)\]

where \(e_i^t \equiv (\Delta \log \nu^t_j, \Delta \log \zeta^t_j)^\prime\).

The moment condition in (32) yields the following class of GMM estimators.

**Definition 2.** Let \(H_i(\hat{z}^t, W^0)\) be a \(S \times 2\) matrix of functions. The GMM estimator is

\[
\hat{\theta}_H \equiv \underset{\theta}{\text{arg min}} \left[ \sum_{i,t} H_i(\hat{z}^t, W^0) e_i^t(\theta) \right]' \left[ \sum_{i,t} H_i(\hat{z}^t, W^0) e_i^t(\theta) \right]^{-1} \left[ \sum_{i,t} H_i(\hat{z}^t, W^0) e_i^t(\theta) \right]
\]

\[(33)\]

where \(e_i^t(\theta) \equiv (\Delta \log \nu^t_j, \Delta \log \zeta^t_j)^\prime\) defined in equation (31).

Definition 2 outlines a standard GMM estimator based on the moment condition in (32). Notice that the dimension of \(H_i(\cdot)\) is such that the number of moments is equal to the number of parameter in \(\theta\). This does not imply any loss of generality, since it is always possible to define \(H_i(\cdot)\) to include an optimal weighting matrix if there are multiple shifters of trade costs. Under standard regularity conditions, \(\hat{\theta}_H\) converges in probability to the true parameter vector, \(\theta\), and has an asymptotically normal distribution.\(^{30}\)

\(^{29}\)Our choice of instrument differs from other work in that we do not use trade and labor outcomes to invert exogenous shifters for estimation. Therefore we avoid imposing that the entire vector of endogenous trade flows is mean independent from unobserved local shocks, as in Autor, Dorn, and Hanson (2013). Similarly, we avoid using the full general equilibrium structure of the model alongside with trade and labor outcomes to invert instrumental variables, as in Monte, Redding, and Rossi-Hansberg (2015); Faber and Gaubert (2016); Allen and Arkolakis (2013); Bartelme (2015); Allen, Arkolakis, and Takahashi (2014). By relying on the entire general equilibrium structure, misspecification in any part of the model is a threat to identification. In contrast, under Assumption 3c, our approach is only subject to the misspecification of equation (31).

\(^{30}\)Newey and McFadden (1994) provide regularity conditions for consistency and normality of GMM estimators of the form in Definition 2 – for consistency, see Theorems 2.6–2.7 and, for normality, see Theorem 3.4. Such regularity conditions require \(\theta\) to be the unique solution of the moment condition in (32). This is implied by usual rank conditions establishing identification in GMM estimators. For instance, if \(\Delta \log \nu^t_j(\theta)\) and \(\Delta \log \zeta^t_j(\theta)\) are linear in \(\theta\), then uniqueness requires \(H_i(\hat{z}^t, W^0)\) to be correlated with the endogenous variables multiplying \(\theta\).
Although any exposure function $H_i(\cdot)$ yields a consistent estimator of $\theta$, different exposure functions $H_i(\cdot)$ vary in terms of asymptotic variance — that is, the estimators differ in terms of precision. To choose the exposure function $H_i(\cdot)$, we follow the approach in Chamberlain (1987) and select the one minimizing the asymptotic variance of the estimator. Applying the result in Chamberlain (1987), we show in Appendix B.1 that the most efficient estimator in the class of estimators in Definition 2 is

$$H_i^*(\hat{z}_t^i, W_0) \equiv E\left[\nabla_\theta c_i^j(\theta)|\hat{z}_t^i, W_0\right] (\Omega_i^{t})^{-1}$$

(34)

where $\Omega_i^{t} \equiv E\left[c_i^j(\theta)c_i^j(\theta)'|\hat{z}_t^i, W_0\right]$.

The optimal exposure function in (34) has two components. The matrix $\Omega_i^{t}$ attributes higher weight to observations with a lower variance of local unobserved shocks – if local shocks are homoskedastic, $\Omega_i^{t} = \Omega$. The optimal exposure function also depends on $E\left[\nabla_\theta c_i^j(\theta)|\hat{z}_t^i, W_0\right]$: the expected response of the endogenous variables associated with $\theta$ induced by the exogenous trade cost shifter $\hat{z}_t^i$. The definition of the structural residuals in (31) implies that, up to a first-order approximation,

$$E\left[\nabla_\theta c_i^j(\theta)|\hat{z}_t^i, W_0\right] \approx \nabla_\theta B(\theta, W_0) E\left[\log \hat{L}_t^i|\hat{z}_t^i, W_0\right],$$

(35)

where $B(\theta, W_0) \equiv \left(\bar{\phi}^{-1}(\theta, W_0), \bar{\psi}(\theta, W_0)\right)$. Note that, in our model, $E\left[\nabla_\theta c_i^j(\theta)|\hat{z}_t^i, W_0\right]$ depends only on the expected change in the employment but, in another settings, it could be a function of other endogenous variables.

We use our general equilibrium model to approximate the expected change in employment conditional on the change in the trade cost shifter. The combination of Assumption 3c and the log-linear version of our model in Proposition 2 implies that

$$E\left[\log \hat{L}_t^i|\hat{z}_t^i, W_0\right] \approx \kappa \log \hat{L}_t^P(\hat{z}_t^i, W_0|\theta)$$

(36)

where $\hat{L}_t^P(\hat{z}_t^i, W_0|\theta)$ is the ‘‘exact hat-algebra’’ solution of the system (11)–(14) given the observed trade cost shock, $\tau' = \hat{z}_t^i$, while holding constant all other shocks, $\log \hat{\nu}_j^t = \log \hat{\zeta}_j^t = 0$.

The expression in equation (36) is the first-stage implied by our general equilibrium model: $\hat{L}_t^P(\hat{z}_t^i, W_0|\theta)$ is the predicted impact of the trade cost shifter, $\hat{z}_t^i$, on the endogenous changes in employment, $\hat{L}_t^i$. Through the lens of our model, $\hat{L}_t^P(\hat{z}_t^i, W_0|\theta)$ is the Optimal IV since it is the best predictor of changes in employment induced by $\hat{z}_t^i$. Intuitively, $\hat{L}_t^P(\hat{z}_t^i, W_0|\theta)$ captures all the channels through which $\hat{z}_t^i$ affects endogenous labor market outcomes across markets, and thus leads to the most precise estimates.

This discussion is summarized by the following proposition:
Proposition 5. Suppose the world economy is generated by the model in Section 2, satisfying Assumptions 3a–3c. The function \( H^*_i(\cdot) \) in (34) determines the estimator with the minimum asymptotic variance in the class of estimators in Definition 2, and is approximately given by

\[
E \left[ \nabla_\theta e^t_i(\theta) | \hat{z}^t, W^0 \right] \approx \kappa \nabla_\theta B \left( \theta, W^0 \right) \log \hat{L}^p \left( \hat{z}^t, W^0 | \theta \right).
\] (37)

We now characterize a simple two-stage procedure that is asymptotically equivalent to the MOIV estimator. Using a guess of the structural parameters, we compute the predicted changes in endogenous variables, \( \log \hat{L}^p \left( \hat{z}^t, W^0 | \theta^0 \right) \), and the the instrumental variable, \( H^*_i \left( \hat{z}^t, W^0 | \theta^0 \right) \). In the first-stage, we use this instrument to estimate the model, leading to estimates \( \hat{\theta}^1 \). Since the instrument is a function of \( \left( \hat{z}^t, W^0 \right) \), the first-stage estimator is a consistent estimator of \( \theta \), but it is not optimal instrument since it was computed using an arbitrary guess of the model’s parameters. To improve on efficiency, in the second-stage, we use \( \hat{\theta}^1 \) to compute the instrument, \( H^*_i \left( \hat{z}^t, W^0 | \hat{\theta}^1 \right) \), and estimate the model again, leading to estimates \( \hat{\theta}^2_{MOIV} \).

The following proposition establishes that the asymptotic properties of this two-stage estimator are identical to those of the estimator based on (37).

Proposition 6. The Model-implied Optimal IV estimator is asymptotically equivalent to the estimator obtained from the following two-stage procedure.

Stage 1. For an initial guess for the parameter vector \( \theta^0 \),

\[
\hat{\theta}^1 = \arg \min_{\theta} \left[ \sum_{i,t} H^*_i \left( \hat{z}^t, W^0 | \theta^0 \right) e^t_i(\hat{\theta}) \right] \left[ \sum_{i,t} H^*_i \left( \hat{z}^t, W^0 | \theta^0 \right) e^t_i(\hat{\theta}) \right]^{'}.
\] (38)

Stage 2. Using the first-stage estimates \( \hat{\theta}^1 \),

\[
\hat{\theta}^2_{MOIV} = \arg \min_{\theta} \left[ \sum_{i,t} H^*_i \left( \hat{z}^t, W^0 | \hat{\theta}^1 \right) e^t_i(\hat{\theta}) \right] \left[ \sum_{i,t} H^*_i \left( \hat{z}^t, W^0 | \hat{\theta}^1 \right) e^t_i(\hat{\theta}) \right]^{'}.
\] (39)

Proof. Appendix B.2.

Having characterized the optimal estimator for our Generalized Economy, we now use such estimator and our framework to empirically assess the impact of trade shocks on local labor markets.
5 Estimation of Cross-Market Linkages in the US

Our theoretical results describe the importance of cross-market linkages in measuring the effect of shocks on local labor markets. We have developed a methodology to estimate such linkages from cross-market variation in trade and labor outcomes induced by exogenous shocks to observable trade cost shocks. We now use our theoretical results to evaluate the importance of cross-market effects of local shocks among states in the United States.

5.1 Multiple-Sector Spatial Model

The first step of our methodology is the parametrization of the Generalized Spatial Competitive Economy. Overall, while we focus on the parametric modeling of certain spatial links, we try to strike a balance between the tractability of our empirical application and the generality of our theoretical environment. To achieve such a balance, we rely on existing frameworks in the literature to guide our parametric choices of the functional forms governing cross-market links in labor supply, productivity, and trade flows.

We introduce a multiple-sector version of the spatial gravity model discussed in Section 2.3. Our parametric model is flexible enough to capture the main forces in the quantitative spatial models reviewed by Redding and Rossi-Hansberg (2016). Yet, it remains tractable: the strength of cross-market linkages depend on a parsimonious number of parameters, which allows us to estimate the model’s key elasticities of labor supply and agglomeration. In this section, we use $k$, $s$ to denote sectors, and $o$, $d$ to denote regions.

**Bilateral Trade Flows.** We follow the extensive literature on recent quantitative trade models, and in particular Costinot and Rodriguez-Clare (2013), to incorporate multiple sectors in our framework, and impose nested CES preferences for goods from different sectors and regions. Thus, conditional on bilateral good prices $\{p_{ko, sd}\}_{ko}$, the share of expenditure on goods from sector $k$ of region $o$ by workers in sector $s$ of region $d$ is

$$x_{ko, sd}^t = \left( \frac{p_{ko, sd}^t}{P_{k, sd}^t} \right)^{1-\chi_k} \left( \frac{P_{k, sd}^t}{\sum_{k'} (P_{k', sd}^t)^{1-\chi}} \right)^{1-\chi}$$  \hspace{1cm} (40)

where the sector-level price index is $P_{k, sd}^t = \left[ \sum_o (p_{ko, sd}^t)^{1-\chi_k} \right]^{1/\chi_k}$. In equation (40), $\chi_k$ is the elasticity of substitution across origins in sector $k$, and $\chi$ is the elasticity of substitution across sectors.

An important step of our empirical strategy is the inversion of local competitiveness from observed trade outcomes. The expenditure shares in (40) implies that
\[ Q_{ko}^t = (x_{kr,kr}^t)^{1-\chi_k} \left( \sum_o x_{ko,kr}^t \right)^{\frac{1}{1-\chi_k}}. \]  

(41)

A necessary and sufficient condition for the invertibility of the trade demand in (40) is \( \chi_k \neq 1 \) and \( \chi \neq 1 \). This rules out an unitary elasticity of substitution across good from different sectors and region, a knife-edge case where spending shares are not sensitive to changes in bilateral prices. Thus, our approach follows a strand of recent papers that feature a non-unitary elasticity of substitution across sectors – e.g., Costinot and Rodriguez-Clare (2013), Herrendorf, Rogerson, and Valentinyi (2013), Comin, Lashkari, and Mestieri (2015), and Cravino and Sotelo (2017).

**Labor Supply.** We assume that, as described in Appendix C.1, the representative household maximizes a nested utility function over the number of worked hours in a sector-region pair, \( h_{sd}^t \), and the allocation of labor across sector-region pairs, \( l_{sd}^t \). We incorporate non-employment in the model by introducing a home sector, \( s = 0 \), that yields an identical level of outside utility in all regions. Such a preference structure implies three margins of employment responses:

\[ \Delta \log h_{sd}^t = \phi_h \Delta \log (\rho_{sd}^t \omega_{sd}^t) + \nu_{sd}^t, \]  

(42)

\[ \Delta \log \left( \frac{l_{sd}^t}{l_{0d}^t} \right) = \phi_e \Delta \log (\rho_{sd}^t \omega_{sd}^t) + \tilde{\nu}_{sd}^t, \]  

(43)

\[ \Delta \log \left( \frac{n_{sd}^t}{n_{0d}^t} \right) = \phi_m \Delta \log \left( \frac{l_{0d}^t}{l_{0o}^t} \right) + \tilde{\nu}_d^t, \]  

(44)

where \( n_{sd}^t = \sum_s l_{sd}^t \) is the share of national population in region \( d \), and \( l_{0d}^t \) is the share of non-employed individuals residing in region \( d \).

Equations (42) and (43) specify the choices of hours and relative employment in the different sectors of region \( d \). Following changes in the real wage, the parameters \( \phi_h \) and \( \phi_e \) regulate labor supply responses in hours worked and sectoral employment shares. These parameters capture two main margins of labor supply adjustment: \( \phi_h \) is the intensive labor supply elasticity and \( \phi_e \) is the extensive labor supply elasticity. In addition, equation (44) characterizes the allocation of labor across regions. The parameter \( \phi_m \) regulates the elasticity of relative labor supply to changes in relative non-employment rate in any two regions of the country.

This parametric specification encompasses recent models with endogenous choices of employment across sectors and regions. Whenever \( \phi_m = 0 \), our labor supply structure is isomorphic to that implied by Roy models with Frechet distribution of sector-specific efficiency and preferences – such as Galle, Rodriguez-Clare, and Yi (2015). In addition, if \( \phi_m \to \infty \), our labor supply structure is equivalent to a static version of the model in Caliendo,
Dvorkin, and Parro (2015), which impose the same elasticity of relative employment across sectors and regions.\footnote{We can show that our environment is isomorphic to a model where heterogeneous individuals maximize a separable quasi-linear utility function in consumption goods and labor, and draw idiosyncratic preferences for sectors and regions from a Generalized Extreme Value distribution, as in McFadden (1980).}

**Technology.** We impose two main restrictions on the production structure of the Generalized Spatial Competitive Economy. The first assumption is that the cost of shipping goods from sector $k$ of region $o$ to any sector of region $d$ is the same. That is, we assume that

$$\tau_{ko,sd}^t = \tau_{ko,d}^t \quad \forall s.$$

When combined with the bilateral trade demand in (40), this restriction implies that, for any sector $s$ of region $d$, spending shares are identical: $x_{ko,sd}^t = x_{ko,d}^t$ for all $s$. This is an important assumption for our empirical application given the lack of expenditure data for individuals employed in different sectors of regional labor markets.

We also need to specify the function form governing agglomeration and congestion forces in the model. Specifically, we assume that productivity in sector $s$ of region $d$ is

$$\Psi_{sd} = (h_{sd}^t l_{sd}^t n_{d}^t)^{\psi_e} \left( \sum_k h_{kd}^t l_{kd}^t n_{d}^t \right)^{\psi_m - \psi_e}.$$

The combination of this expression with equation (9) yields

$$\Delta \log \left( \omega_{sd}^t / Q_{sd}^t \right) = \psi_e \Delta \log (h_{sd}^t l_{sd}^t) + \psi_m \Delta \log (n_{d}^t) + (\psi_m - \psi_e) \Delta \log \left( \sum_k h_{kd}^t l_{kd}^t \right) + \zeta_{sd} \quad (45)$$

where $Q_{sd}^t$ is given by (41).

Expression (45) entails two parameters to separately control the strength of agglomeration/congestion forces across sectors and regions. While the parameter $\psi_e$ regulates the effect arising from sectoral employment concentration in the region, $\psi_m$ regulates the effect of spatial population concentration. As discussed in Section 2.3, our specification captures local productivity gains arising from knowledge spillovers, increasing economies of scale, and firm entry balanced by congestion costs arising from the effect of employment concentration on the price of local factors in limited supply, as highlighted in Allen and Arkolakis (2014) and Redding and Rossi-Hansberg (2016).

### 5.2 Data

To apply our methodology, we construct a dataset of trade and labor outcomes for two sectors, tradeable and non-tradeable, in US states between 1993 and 2012. We now describe the main
variables in our analysis, and discuss the details of the data construction in Appendix C.2.

**Trade Cost Shock.** Our methodology requires trade cost shifters that are orthogonal to local shocks in productivity and labor supply affecting regional markets in the United States. In our empirical application, we use changes in tariffs on agricultural goods applied by foreign countries. We use this specific trade shock for several reasons.

First, tariffs on agricultural products are typically higher than in manufacturing, agriculture being a traditionally very protected sector in many countries – see Anderson, Rausser, and Swinnen (2013). This provides substantial variation in trade costs that can be used to identify the structural parameters of our model. Second, we use the tariffs applied by foreign countries to US products, which are less likely to be affected by changes in productivity in particular regions of the United States, and thus are arguably an exogenous source of variation for estimation. Lastly, as highlighted by equations (21), the exposure of a market to a trade shock is given by the interaction between the trade shock itself and the relative size of the trading partner, as summarized by the revenue shares. Therefore, we focus on tariff shocks in important foreign destinations for US products, and use tariffs applied by the US top 10 trading partners.

We obtain tariff schedules at the country-product level from the TRAINS database in the World Bank’s WITS website, for the years 1997-2012. For each country pair, we create ad-valorem import tariffs at the 2-digit HS level by taking the simple average of tariff lines within each HS code. We then construct the bilateral tariff between each US state and each foreign country by computing the trade-weighted average of HS2 tariff rates.

**Labor Market Data.** We extract employment and wage information from the CPS-MORG. We compute state-level outcomes using reported weekly earnings and weekly hours. We classify individuals into three aggregate sectors: (i) the home sector that includes non-employed individuals; (ii) the tradeable sector that includes all manufacturing and agriculture industries; and (iii) the non-tradeable sector that includes all remaining industries.

We construct state-level price indices using data from the Cost of Living Index, published by the Council for Community and Economic Research (C2ER). This is a well-known source of living cost differentials among cities in the U.S – see Moretti (2013). Our state-level price

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32 As documented in Bown (2004), Bown (2012), and Bagwell, Bown, and Staiger (2016), the implementation of contingent protection measures like anti-dumping and countervailing duties remains frequent and widespread for agricultural products. See Table 3 in Bagwell, Bown, and Staiger (2016), which shows sharp differences between average applied tariffs in agricultural products relative to overall rates of protection.

33 The countries are Brazil, Canada, China, Germany, India, Japan, Mexico, Netherlands, Singapore and UK. They account for 70% of total US exports in our sample period. Appendix C.3 reports results obtained with tariff shocks to different sets of countries and sectors.

34 There is large number of missing tariff information for the years prior to 1995. Thus, we focus on tariff changes observed after 1997, when the dataset coverage is high for the set of countries analyzed.
index is the average of the price index changes of urban areas in the state weighted by the area’s population.35

**Trade Data.** Our main source of international trade data is the Eora Multi Region Input Output Tables—see Lenzen et al. (2013). We construct bilateral trade flows for 59 countries in two sectors, tradeable and non-tradeable. We merge this matrix of international trade flows with a matrix of bilateral flows between US states. To construct this regional trade matrix, we use between-states shipments data from the Commodity Flow Survey, which is conducted every five years by the US Census Bureau. Finally, we use Census data at the port of entry/exit level to obtain trade flows between each of the US states and each of the 59 countries in the MRIO. By merging these data sources, we construct a matrix of bilateral trade flows between the 50 US states and 59 countries. Appendix C describes the details of the trade data construction.

### 5.3 Results

We now turn to the estimation of the structural parameters $\theta \equiv (\phi_h, \phi_e, \phi_m, \psi_h, \psi_m)$ using the methodology derived in Section 4. For each period, we obtain the structural residual $e_t^i(\theta) = (\nu_{sd}^t, \nu_{sd}', \nu_d^t, \zeta_d^t)$ by combining equations (42)–(45) with data on trade and labor outcomes for US states. Following Proposition 6, we implement the Two-Step GMM where, in each step, the moment conditions use the model’s predicted values of $H_i^* (\hat{z}^t, W^0|\theta)$ given the observed changes in import tariffs, $\hat{z}^t$.

#### 5.3.1 Structural Parameters

Table 1 reports the parameter estimates obtained with the methodology described in Section 4, along with the standard errors clustered at the state-level. In all specifications, we include period and sector dummies in the stacked sample of state-sector pairs for the periods of 1993-1997, 1997-2002, 2002-2007 and 2007-2012. We calibrate the elasticity of substitution across origins to $\chi_k = 5$ for both sectors, as in Simonovska and Waugh (2014a), and the elasticity of substitution across sectors to $\chi = 0.3$, as in Buera, Kaboski, and Rogerson (2015).

Columns (1)-(3) report our estimates of the structural parameters governing regional labor supply. First, our point estimate for the intensive margin is 0.217—i.e. the elasticity of hours worked with respect to the real wage. Second, the extensive margin elasticity is

35 As highlighted by Feenstra (1994), our price index does not fully capture the effect of new varieties in the cost of living. One possible solution is to use the local prices in the Nielsen Homescan Dataset to construct state-level price indices, as in Handbury and Weinstein (2014). However, this would significantly reduce our sample since bar-code price data is only available for a subset of states after 2004. A deeper investigation of these issues is a fruitful agenda of future research.
\( \phi_e = 0.914 \), implying that a 1% increase in the sector relative wage triggers a 0.91% increase in the sector relative employment share. In terms of employment shares, if real wages in sector \( k \) increase by 1%, the employment share in sector \( k \) increases by \( \phi_e l_{ko}(1 - l_{ko}) \). Using the national employment composition in 2007, this implies an extensive margin elasticity of 0.11% for the manufacturing sector and 0.25% for the non-manufacturing. Therefore, our estimates of the extensive and intensive margins of labor supply elasticity are in the lower end of the estimates in the literature reviewed by Chetty et al. (2013).\(^{36}\) Finally, we obtain an imprecise estimate of the migration elasticity. This result is somewhat consistent with recent empirical literature finding no evidence of responses of migration flows to international trade shocks in the United States – e.g., see Autor, Dorn, and Hanson (2013) and Autor et al. (2014).

Column (4) reports our estimates of the elasticity of agglomeration and congestion forces under the restriction that \( \psi_e = -\psi_m = \psi \). Our point estimate of \( \psi \) is 0.245, which multiplied by the trade elasticity implies an agglomeration elasticity close to the local agglomeration forces implied by models with increasing returns to scale and firm entry as in Krugman (1980). These sectoral agglomeration forces are balanced by the regional congestion forces stemming from lower cost advantage as population becomes more concentrated in space.\(^{37}\)

### 5.3.2 Fit of the Model

We now turn to an investigation of the model’s ability to generate a cross-market pattern of labor outcomes response to observed tariff shocks that is consistent with the actual pattern of cross-market changes in labor outcomes in the United States. This comparison is important for two main reasons. First, it clearly outlines the source of variation in the data behind the estimates of the structural parameters reported in Table 1. Second, under the assumption that observed tariff shocks are orthogonal to local productivity and supply shocks, this comparison highlights our model’s ability to replicate the causal impact of trade shocks on local labor markets and, therefore, provides support for the model’s counterfactual predictions.

\(^{36}\)It is important to notice that we are estimating the uncompensated labor elasticity of hours and participation. In contrast, Chetty et al. (2013) provide estimates of the compensated elasticity, but they argue that, due to small income effects on labor supply, estimates of uncompensated and compensated labor supply elasticities are typically very similar.

\(^{37}\)As shown in Table 5 in Appendix C.3, we are unable to jointly estimate these parameters, but their separate estimation indicates that this is a good approximation for the forces in the data. The estimates of agglomeration forces in the literature present large variation. Kline and Moretti (2014) estimate an elasticity of county productivity with respect to manufacturing density of 0.4. Peters (2017) also finds a large agglomeration elasticity. In a recent paper, Bartelme et al. (2017) find that the productivity elasticity to employment across countries and sectors is positive, but below our point estimates.
To this end, we estimate the following linear model:

\[ \Delta \log Y_{kr}^t = \beta \Delta \log Y_{kr}^p \left( \tilde{z}_t | \hat{\theta} \right) + X_{kr}^t \gamma + \epsilon_{kr}^t, \]

where \( \Delta \log Y_{kr}^t \) is the change in outcome \( Y \) in sector \( k \) of region \( r \) at time \( t \), \( \Delta \log Y_{kr}^p \left( \tilde{z}_t | \hat{\theta} \right) \) is the impact of shock \( \tilde{z}_t \) on outcome \( Y \) predicted by our general equilibrium model, and \( X_{kr}^t \) is a set of sector-region controls. In this exercise, we consider the same data used in the estimation of structural parameters, and compute predicted effects with the estimates reported in Table 1.

Panel A of Table 2 reports the relationship between the actual changes in endogenous variables and the predicted changes in the same endogenous variables implied by our general equilibrium model. For all variables, there is a positive and statistically significant relationship between actual and predicted changes across US states. This indicates that, following trade costs shocks, the general equilibrium responses implied by our model are consistent with the observed pattern of responses in labor market outcomes across US states. Notice that the estimated coefficients for all variables are around 15. As discussed in Section 4, up to a first-order approximation, this estimated coefficient captures the pass-through of tariff shocks to trade cost shocks, which seems to be high in our empirical application.

The remaining panels of Table 2 investigate the fit of alternative exposure measures to the same tariff shock. Panel B reports estimates using the “direct effect” of the tariff shock, which we defined as the sector-region exposure to shocks in the region.\(^{38}\) The direct shock exposure yields similar point estimates for the model fit, but they are more imprecise for hours and non-employment rate. Similarly, Panel C shows that even the simpler sector-region revenue exposure, \( \hat{\eta}_E \), has a statistically significant impact on real wages and relative sector employment. In our model, these two measures are components of the total effect of trade shocks on local labor markets and, therefore, partially drive the fit of our model reported in Panel A.

Finally, Panel D considers the differential effect on labor market outcomes of changes in trade-weighted agricultural tariffs faced by producers across US states. This Bartik-like measure of shock exposure combines aggregate changes in tariffs applied to US products with region-sector variation in the initial share of exports across destination markets. By definition, this naive exposure measure contains part of the direct revenue effect of tariff shocks – see equation (21). Our results indicate that exposure to higher tariffs trigger declines in real wages, hours, and employment. Intuitively, states faced with higher tariffs experience

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\(^{38}\) We compute direct and indirect effects using the first-order approximation of the model’s predicted response in Proposition (2). The direct effect corresponds to the terms of the matrix associated with the response implied by shocks to all sectors of the region.
a relative decline in labor demand and, therefore, a relative decline in wages and employment.

Together, the results in Table 2 indicate that the adjustment margins in our model provide a good approximation for those controlling the effect of trade shocks on labor market outcomes across US states. Moreover, Panels B–D suggest that simpler measures of direct shock exposure encompass an important part of the cross-market variation in the predicted effect of trade shocks in our model.\textsuperscript{39} The next section returns to this point in detail.

5.3.3 Robustness

Our results are robust to a number of specification choices. In particular, Tables 7 and 8 in Appendix C.3 show that we obtain similar results when (i) controlling for period dummies interacted with the state’s manufacturing employment share and the state’s demographic characteristics (Panel B–C), (ii) restricting the sample to the tradeable sector (Panel E) and to the period prior to the great recession (Panel F), and (iii) weighting sector-state pairs by the state’s national population in 1993.\textsuperscript{40} Moreover, Tables 9 and 10 in Appendix C.3 show that we obtain similar results as we change the set of foreign trade partners (Panels B–D), but the model’s fit is significantly worse as we consider also changes in manufacturing tariffs of top trade partners.

6 Application: Effect of Trade Shocks on US Local Labor Markets

We now apply our framework to assess the quantitative importance of indirect general equilibrium effects arising from observed trade cost shocks across local labor markets in the United States. To this end, we analyze a counterfactual exercise in which, starting from the initial equilibrium in 2007, we revert the trade costs between all countries and US states to their level of 1993. We evaluate the importance of indirect spillover effects by comparing the predicted effect implied by different measures of market exposure to the trade shock.

\textsuperscript{39}Table 6 in Appendix C.3 report the estimation of the structural parameters using these alternative measures of shock exposure. As we go from the more sophisticated Model-implied IV to these simpler exposure measures, the estimates become substantially more imprecise – especially for the elasticity of hours and agglomeration. This follows the fact that, as shown in Table 2, the effect of the alternative exposure measures on hours and non-employment is statistically weaker. This is consistent with our theoretical results of Section 4, which establish that MOIV is a more efficient estimator and, therefore, yields lower standard errors.

\textsuperscript{40}In this case, our estimates are more imprecise owning to the fact that shocks in agricultural tariffs are more important for small states specialized in the agricultural production.
6.1 Impact of Trade Shocks on US States

The first step of the analysis is to use our general equilibrium model to recover trade shocks from the data. This allows us to exploit not only tariff changes but all other sources of bilateral shocks affecting trade flows. Specifically, the spending shares in equation (40) imply that changes in trade costs are given by

\[
\hat{\tau}_{ko,d} = \left( \frac{\hat{x}_{k,d}}{\hat{x}_{k,o}} \right)^{1 - \chi_{k}} \left( \frac{\hat{x}_{ko,d}}{\hat{x}_{ko,o}} \right)^{1 - \chi_{k}} \frac{\hat{P}_d}{\hat{P}_o},
\]

(46)

where \(\bar{x}_{k,d} \equiv \sum_{o} x_{ko,d}\) is the total expenditure share on sector \(k\) by individuals residing in \(d\).

Note that equation (46) allows for asymmetric changes in trade costs between trade partners, and thus it generalizes the approach proposed by Head and Ries (2001).\(^{41}\)

We then use equation (46) to compute the change in bilateral trade costs between 1993 and 2007 for each pair of foreign country and US state. Our main counterfactual exercise considers the impact of this trade cost shock in the observed equilibrium in 2007. In this exercise, we compute predicted changes in labor market outcomes using our parametric model of Section (5) using the estimated parameters reported in Table 1. Given our imprecise estimates of the migration elasticity, we report baseline results without migration (\(\phi_m = 0\)), and evaluate their sensitivity to the strength of migration responses.

Figure 1 plots, for each US state, the associated change in import and export exposures, given by equations (20) and (21), in the tradeable sector. The shock is quite heterogeneous across states and is large: the average change in the cost of exporting from US states to other countries is -4%, while the average trade costs of importing from other countries increase by 28%.

Figures 2 and 3 illustrate the impact of the trade shock on labor market outcomes in both the tradeable and non-tradeable sectors. The effects are very heterogeneous, owing to the different exposure of the regions to foreign markets, and to their different degrees of specialization on tradeable and non-tradeable. Interestingly, in almost all states, the tradeable sector gains both in terms of real wages and employment, as a result of the lower exposure to foreign competition. The gain of workers in the tradable sector is especially large in the states around the rust belt between the mid-west and the northeast. Notice that a few states also experience an increase in non-tradeable real wages due to demand spillover.

\(^{41}\)The inversion of bilateral trade costs in equation (46) follows a similar approach as that proposed by Eaton, Kortum, Neiman, and Romalis (2016). The main difference is that, by assuming CES preferences across sectors, equation (46) does not require the change in the price index for each sector-region pair, since these can be inverted from changes in sectoral spending shares, \(\bar{x}_{k,d}\). Our procedure still requires the price index in each region, which we obtain from the Penn World Tables as in Eaton, Kortum, Neiman, and Romalis (2016).
triggered by the gains in the local tradeable industry. However, for most states, real wages and employment in the non-tradeable sector present small declines. On average, the change in the real wage is 1.2 log-points and -0.7 log-points in the tradable and non-tradable sectors.

6.2 Importance of Indirect Spillover Effects in General Equilibrium

We now evaluate the quantitative importance of indirect spillover effects to the changes in real wages predicted by our general equilibrium model. To this end, we take advantage of the expressions in Proposition 3 that yields a decomposition of predicted effects into direct and indirect components. In particular, we compute the direct shock exposure of a sector-region pair \( i \) by summing the terms in equation (25) related to all sectors in that region.

Figure 4 plots, for each US states, the direct effect of the shock against the total predicted effect by our model. We can see that the two exposure measures are highly correlated – the correlation is around 0.9 for both variables – and, for almost all states, the direct exposure to the shock is sizable relative to the total effect. Interestingly, the two measure are also very similar on average, with the average direct effect being slightly lower for the tradable sectors and slightly higher for the non-tradable sector. This tight correlation indicates that indirect spillover effects across US states are very weak in our baseline specification without migration.

To summarize, cross-market links in our parametric model do not seem to generate sizeable indirect effects in the general equilibrium response to trade shocks. Our results are consistent with the findings in Caliendo et al. (2014) in context of a quantitative spatial model featuring free mobility of workers across regions and input-output linkages. Specifically, they show that, following a state-specific productivity shock, the indirect effect on other states is one order of magnitude smaller than the direct effect of the shock.

This result suggests that the use of simple measures of direct shock exposure should capture a sizeable fraction of the effect of trade shocks on local labor markets. To evaluate this claim, we now implement a formal comparison of the ability of different exposure measures to capture the predictions of our general equilibrium model. In this exercise, we also consider the other exposure measures introduced in Section 5.3.2: the direct revenue exposure to the trade shock in the region as defined in equation (21), and the Bartik-like trade-weighted exposure to changes in trade costs.

We denote each exposure measure by \( m \), with \( m = GE \) representing our baseline general equilibrium model. For each \( m \), we use the estimated coefficients in column (1) of Table 2 to compute the predicted shock effect on real wages, \( \Delta \omega^m_{kr} \equiv \beta_m \Delta \log \omega^m_{kr} (\hat{\tau}) \), where
\( \Delta \log \omega_{kr}^m (\hat{r}) \) is the shock exposure according to measure \( m \). We then compare predictions using the following linear regressions:

\[
\frac{\Delta \omega_{kr}^{GE}}{\Delta \omega_{kr}^{GE}} = \alpha^m + \rho^m \left( \frac{\Delta \omega_{kr}^m}{\Delta \omega_{kr}^{GE}} \right) + e_{kr}^m, \tag{47}
\]

where the bar denotes the national average of the corresponding variable.

The linear regression in (47) captures different margins of measure \( m \)’s ability to replicate the predicted responses of our general equilibrium model. First, the constant \( \alpha^m \) corresponds to the percentage difference in the average effect predicted by measure \( m \) and the baseline model. Second, the slope \( \rho^m \) is the cross-market correlation between the predictions of measure \( m \) and the baseline model. Finally, \( e_{kr}^m \) is the difference between the predictions with the two measures, with the \( R^2_m \) capturing the overall predictive power of measure \( m \).

Table 3 reports the estimates of equation (47), along with 95% confidence intervals computed using the asymptotic distribution of estimated parameters. In line with Figure 4, Panel A indicates the direct effect is a good predictor of the overall effect of trade shock on real wages, implying that indirect spillover effects play a limited role in our general equilibrium model. Interesting enough, Panel B shows that the direct revenue exposure is tightly related to the model’s predictions across regions, with the \( R^2_m \) above 85% for both sectors. This is a powerful result: in order to capture almost entirely the effect of the shock across regions, one can use a simple measure of shock exposure computed using observable trade and revenue shares in the initial equilibrium. However, the large value of the constant indicates that the direct revenue exposure under-predicts the effect of the shock on real wages. In our particular application, given the small magnitude of the aggregate effect of the shock, this difference corresponds to a difference of less than 1 log-point of real wage growth.

Lastly, we investigate how the relationship between the direct and the total effect of the trade shock is affected by different degrees of migration of workers across regions. Figure 5 plots the constant and the slope of a regression of the total linear effect predicted by the general equilibrium model on the corresponding direct effect, as a function of different values for the migration elasticity. The figure highlights that the magnitude of the migration

\[\text{To see this formally, denote deviations from the national average as } \ddot{Y} \equiv Y - \bar{Y} \text{ and notice that, by definition, } \]

\[\alpha^m = \frac{\Delta \omega_{kr}^{GE} - \rho^m \Delta \omega_{kr}^m}{\Delta \omega_{kr}^{GE}} \text{ and } \rho^m = \frac{\sum_{kr} \Delta \omega_{kr}^{GE} \Delta \omega_{kr}^m}{\sum_{kr} \Delta \omega_{kr}^{GE} \Delta \omega_{kr}^m}.\]

\[\text{Specifically, we take 100 draws of the structural parameters vector, } \hat{\theta}, \text{ from the asymptotic normal distribution of estimates reported in Table 1. For each draw, we compute the predicted effect of the trade shock on real wages for the different exposure measures, and estimate equation (47). The confidence intervals correspond to the percentiles 2.5 and 97.5 of the empirical distribution of estimates of } (\alpha^m, \rho^m) \text{ across different draws of the parameter vector.}\]
elasticity does not significantly affect the relationship between the direct and the total effect.\(^{44}\)

7 Conclusions

In this paper, we bring endogenous labor supply and agglomeration forces to the forefront of the analysis regarding the relationship between trade and labor markets. Our analysis stresses the need to directly estimate the relationship between trade and labor market outcomes at regional level, using it to discipline the structural parameters of the model. To this end, we propose a new empirical methodology that uses as an instrument the impact of exogenous trade shocks on changes of the endogenous variables predicted by our general equilibrium model. This yields the most efficient estimator of the structural elasticities, i.e. a Model-implied Optimal IV. We then apply our methodology to evaluate the aggregate impacts of trade shocks affecting regional labor markets in the U.S. Interesting avenues for future research emerge from our study. We hope that our novel methodology, easily implementable with a simple GMM procedure, can be used for the estimation of structural parameters in a wide class of general equilibrium models.

\(^{44}\)Figure 6 in the Appendix shows the response of employment. Although the response is more sensitive to migration than the real wages, the overall positive relationship between total and direct effects is similar to the baseline.
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8 Tables and Figures
Table 1: Estimation of Structural Parameters

<table>
<thead>
<tr>
<th></th>
<th>Labor Supply Elasticity</th>
<th>Agglomeration Elasticity</th>
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<tbody>
<tr>
<td></td>
<td>Hours</td>
<td>Between sectors</td>
</tr>
<tr>
<td>$\phi_h$</td>
<td>0.219**</td>
<td>0.914***</td>
</tr>
<tr>
<td>(0.090)</td>
<td>(0.201)</td>
<td>(0.553)</td>
</tr>
<tr>
<td>F-stat</td>
<td>16.9</td>
<td>16.9</td>
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Note. Stacked sample of 400 state-sector pairs in columns (1), (2) and (4), and 200 states in column (3). All regressions include period and sector dummies. Standard errors in parenthesis are clustered at the state-level. *** p < .01, ** p < .05, * p < .10
Table 2: Estimation of the Model Fit

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<thead>
<tr>
<th></th>
<th>Real Wage</th>
<th>Hours</th>
<th>Sectoral relative employment</th>
<th>Non-Participation Share</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\Delta \log w^t_{kr}$</td>
<td>$\Delta \log h^t_{kr}$</td>
<td>$\Delta \log l^t_{kr}/l^t_{0r}$</td>
<td>$\Delta \log l^t_{0r}$</td>
</tr>
<tr>
<td>Panel A: Baseline, MOIV</td>
<td>13.648***</td>
<td>15.352***</td>
<td>15.428***</td>
<td>17.944**</td>
</tr>
<tr>
<td></td>
<td>(3.314)</td>
<td>(3.671)</td>
<td>(4.694)</td>
<td>(9.060)</td>
</tr>
<tr>
<td>F-stat</td>
<td>16.964</td>
<td>17.492</td>
<td>10.802</td>
<td>3.923</td>
</tr>
<tr>
<td></td>
<td>(1.793)</td>
<td>(8.273)</td>
<td>(4.698)</td>
<td>(10.958)</td>
</tr>
<tr>
<td>F-stat</td>
<td>60.985</td>
<td>2.470</td>
<td>8.674</td>
<td>2.067</td>
</tr>
<tr>
<td>Panel C: Tariff Shock Exposure</td>
<td>2.133***</td>
<td>0.596</td>
<td>2.942***</td>
<td>-0.585</td>
</tr>
<tr>
<td></td>
<td>(0.311)</td>
<td>(0.384)</td>
<td>(0.831)</td>
<td>(0.378)</td>
</tr>
<tr>
<td>F-stat</td>
<td>47.091</td>
<td>2.411</td>
<td>12.519</td>
<td>2.393</td>
</tr>
<tr>
<td>Panel D: Naïve Tariff Shock Exposure</td>
<td>-0.769***</td>
<td>-0.256*</td>
<td>-1**</td>
<td>-0.088</td>
</tr>
<tr>
<td></td>
<td>(0.215)</td>
<td>(0.155)</td>
<td>(0.402)</td>
<td>(0.254)</td>
</tr>
<tr>
<td>F-stat</td>
<td>12.826</td>
<td>2.713</td>
<td>6.171</td>
<td>0.120</td>
</tr>
</tbody>
</table>

Note. Sample of state-sector pairs in columns (1)-(3) and states in column (4). All regressions include period and sector dummies. Standard errors in parenthesis are clustered at the state-level. *** p < .01, ** p < .05, * p < .10
Table 3: Comparison across Models

<table>
<thead>
<tr>
<th></th>
<th>Real Wages</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Tradeable</td>
</tr>
<tr>
<td>Panel A: Direct Effect</td>
<td>(1)</td>
</tr>
<tr>
<td>Constant</td>
<td>0.486</td>
</tr>
<tr>
<td></td>
<td>[0.465 0.514]</td>
</tr>
<tr>
<td>Slope</td>
<td>0.922</td>
</tr>
<tr>
<td></td>
<td>[0.892 0.945]</td>
</tr>
<tr>
<td>R2</td>
<td>0.564</td>
</tr>
<tr>
<td></td>
<td>[0.544 0.578]</td>
</tr>
<tr>
<td>Panel B: Trade Shock Exposure</td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>1.005</td>
</tr>
<tr>
<td></td>
<td>[0.954 1.066]</td>
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<tr>
<td>Slope</td>
<td>0.410</td>
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<td>[0.393 0.424]</td>
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<td>R2</td>
<td>0.864</td>
</tr>
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<td></td>
<td>[0.858 0.870]</td>
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<tr>
<td>Panel C: Naïve Trade Shock Exposure</td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>1.413</td>
</tr>
<tr>
<td></td>
<td>[1.351 1.481]</td>
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<tr>
<td>Slope</td>
<td>1.328</td>
</tr>
<tr>
<td></td>
<td>[1.274 1.373]</td>
</tr>
<tr>
<td>R2</td>
<td>0.668</td>
</tr>
<tr>
<td></td>
<td>[0.663 0.673]</td>
</tr>
</tbody>
</table>

Note. 95% Confidence intervals in brackets computed by bootstrap from the asymptotic distribution of parameter estimates.
Figure 1: Trade costs shock

The map on the left shows, for each US state, the log-change from 2007 to 1993 in the import exposure, shown in equation (20). Values not reported for Alaska and Hawaii are 0.04 for both. The map on the right instead shows log-change in the export exposure, shown in equation (21). Values not reported for Alaska and Hawaii are -0.3 for both.
Figure 2: Impact of trade shock, real wage

The map on the left shows, for each US state, the log-change in the real wage in the tradeable sector after the counterfactual trade shock shown in Figure 1. Values not reported for Alaska and Hawaii are 0.01 and 0.02, respectively. The map on the right instead shows the corresponding change in the non-tradeable sector. Values not reported for Alaska and Hawaii are -0.01 and 0.002, respectively.

Figure 3: Impact of trade shock, employment

The map on the left shows, for each US state, the log-change in total employment in the tradeable sector after the counterfactual trade shock shown in Figure 1. Values not reported for Alaska and Hawaii are 0.007 and 0.018, respectively. The map on the right instead shows the corresponding change in the non-tradeable sector. Values not reported for Alaska and Hawaii are -0.006 and 0.001, respectively.
Figure 4: Predicted Change in Real Wages: Total Effect $\times$ Direct Effect

Figure 5: Predicted Change in Real Wages with Migration
A Proofs for Section 3

A.1 Lemma 1

We obtain (11)–(14) immediately by writing equations (6)–(10) in changes. To obtain equations (16)–(17), we substitute equations (12), (14) and (15) into (11) and (13).

Finally, we show that, by specifying preferences and technology, we obtain an equilibrium with identical trade and labor outcomes as the initial equilibrium under the normalization in (15).

**Initial Equilibrium.** Consider an initial equilibrium such that

\[
L_j^0 = \Phi_j \left( \left\{ \nu_i^0 \rho_i^0 \frac{w_i^0}{P_i^0} \right\} \right)_{i \in \mathcal{I}_c}
\]

\[
p_i^0 = \frac{w_i^0}{P_i^0} \Psi_i \left( \left\{ \frac{L_j^0}{j} \right\} \right)
\]

\[
w_i^0 L_i^0 = \sum_j x_{ij}^0 \rho_i^0 w_j^0 L_j^0
\]

\[
x_{ij} = X_{ij} \left( \left\{ \frac{\tau_{ij}^0 P_o^0 / \zeta_0^0}{o} \right\} \right) \quad \text{and} \quad P_j^0 = P_j \left( \left\{ \frac{\tau_{ij}^0 P_o^0 / \zeta_0^0}{o} \right\} \right).
\]

**Alternative Economy.** Let us construct an alternative economy without trade costs (\(\tilde{\tau}_{ij} \equiv 1\)), where technology is given by

\[
\tilde{\zeta}_i^0 = w_i^0
\]

\[
\tilde{\Psi}_i \left( \{L_j\} \right) \equiv \frac{\Psi_i \left( \{L_j\} \right)}{\Psi_i \left( \{\frac{L_j^0}{j} \} \right)},
\]

and preferences are given by

\[
\tilde{U}_c \left( \{C_j\}_{j \in \mathcal{J}_c}, \{L_j\}_{j \in \mathcal{J}_c} \right) \equiv U_c \left( \left\{ \frac{\nu_i^0 C_j}{P_j^0} \right\}_{j \in \mathcal{J}_c}, \{L_j\}_{j \in \mathcal{J}_c} \right)
\]

\[
\tilde{V}_j \left( \{c_{ij}\} \right) \equiv V_j \left( \left\{ \frac{\zeta_0^0 P_j^0}{\tau_{ij}^0 P_i^0} \right\}_{i} \right).
\]

In this case, we immediately get that

\[
\tilde{\Phi}_j \left( \{\rho_i^0 \omega_i^0 \} \right) = \Phi_j \left( \left\{ \frac{\nu_i^0}{P_i^0} \rho_i^0 \omega_i^0 \right\} \right)
\]

and

\[
X_{ij} \left( \{p_{oj}\} \right) = X_{ij} \left( \left\{ \frac{\tau_{ij}^0 P_o^0}{\zeta_0^0 P_j^0 p_{oj}} \right\} \right) \quad \text{and} \quad \tilde{P}_j \left( \{p_{oj}\} \right) = P_j \left( \left\{ \frac{\tau_{ij}^0 P_o^0}{\zeta_0^0 P_j^0 p_{oj}} \right\} \right).
\]
Equilibrium of Alternative Economy. In this economy, the equilibrium entails \( \hat{w}_i = w_i^0 \), \( \hat{L}_i = L_i^0 \), \( \hat{x}_{ij} = x_{ij}^0 \), and \( \hat{p}_{ii} = \hat{P}_i = 1 \). To see this, notice that

\[
\hat{p}_{ii} = \frac{\hat{p}_i}{\zeta_i} = \frac{\hat{w}_i}{\zeta_i^0 \hat{\Psi}_i \left( \left\{ L_j^0 \right\}_j \right)} = 1,
\]

\[
\hat{\Phi}_j \left( \left\{ \rho_j^0 \hat{w}_i \right\}_i \in \mathcal{J}_c \right) = \Phi_j \left( \left\{ \frac{\nu_j^0}{\bar{P}_i} \rho_j^0 w_i^0 \right\}_i \in \mathcal{J}_c \right) = L_j^0,
\]

\[
X_{ij} \left( \left\{ \tau_{oij} \hat{p}_{oo} \right\}_o \right) = X_{ij} \left( \left\{ \frac{\tau_{oij} \rho_j^0}{\bar{P}_i} \right\} \right) = x_{ij}^0 \quad \text{and} \quad \hat{P}_j \left( \{ \tau_{oij} \hat{p}_{oo} \}_o \right) = P_j \left( \left\{ \frac{\tau_{oij} \rho_j^0}{\bar{P}_i} \right\} \right) = \hat{P}_j = 1.
\]

Finally, the labor market clearing condition holds

\[
w_i^0 L_i^0 = \sum_j x_{ij}^0 \rho_i^0 (\hat{w}_j^0) \hat{P}_j.
\]

\[\Box\]

A.2 Proposition 1

Part 1. We start by showing that if the matrix \( \Lambda \equiv (I - \Phi \bar{\Psi})^{-1} \Phi \) exists, then the system (16) implicitly defines \( \left\{ L_j^1 \hat{L}_j \right\}_j \) as a function of \( \left\{ \left( \rho_j^1 \hat{w}_i^0 \right) \left( \nu_j \hat{\rho}_j \hat{P}_j \hat{P}_j \right) \right\} \). To see this, notice that the system (16) can be written as

\[
F_i \left( \{ u_j, v_j \}_j \right) \equiv \log u_i - \log \Phi_i \left( \{ \Psi_j \left( \{ \bar{u}_o \}_o \right) v_j \}_j \right) = 0
\]

with \( u_i \equiv L_j^0 \hat{L}_j \) and \( v_j \equiv \left( \rho_j^0 w_i^0 \right) \left( \hat{v}_j \hat{\rho}_j \hat{P}_j \right) \).

By the Implicit Function Theorem, it is possible to locally write \( \{ u_j \}_j \) as a function of \( \{ v_j \}_j \) if the Jacobian of the system is non-singular. In the system above, the Jacobian is \( \nabla_u F = I - \Phi \bar{\Psi} \). Thus, under Assumption 2,

\[
\nabla_v L_j^1 \hat{L}_j = \left( \nabla_u F \right)^{-1} \nabla_u F = \Lambda.
\]

Part 2. Now we show that \( \hat{\Gamma} \) is the Jacobian of the excess trade demand system in (17). We can write the system as

\[
G_i \left( \{ \hat{p}_j \}_j \right) = \log w_i^0 \left( \hat{E}_i \hat{p}_i \right) - \log \sum_j X_{ij} \left( \left\{ \tau_{oij} \hat{p}_{oo} / \bar{c}_o \right\}_o \right) \left( \rho_j^0 w_i^0 \right) \left( \hat{\rho}_j \hat{p}_j \hat{E}_j \right) = 0,
\]

where \( \left\{ \hat{E}_j \right\}_j = \left\{ L_j^1 \hat{L}_j \Psi_j \left( \{ \bar{L}_o \} \right)_o \right\}_j \) with \( L_j^1 \hat{L}_j \) implicitly determined by the solution of the system in (16).

Notice that the excess demand system is homogeneous of degree zero: \( G_i \left( \{ \hat{t}_j \}_j \right) = G_i \left( \{ \hat{p}_j \}_j \right) \).

In addition, taking the derivative of the expression above, we get that

\[
\frac{\partial G_i}{\partial \log \bar{p}_o} = \frac{\partial}{\partial \log \bar{p}_o} \left( x_{oij} \log \hat{E}_j \right) \frac{\partial \log \hat{E}_j}{\partial \log \bar{p}_o} - \frac{\partial \log \hat{E}_i}{\partial \log \bar{p}_i} \]

(48)
where \(y_{ij}\) are the revenue shares.

This implies that
\[
\nabla_{\log \hat{p}} G = \bar{I} - \bar{y} - \bar{\chi} + (\bar{I} - \bar{y}) \nabla_{\log \hat{c}} \log \bar{E}.
\]

Notice that, by Shepard’s lemma, \(\nabla_{\log \hat{p}} \log P_j \left( \left\{ \hat{\tau}_{oj} \hat{p}_o / \hat{\zeta}_o \right\}_o \right) = \bar{x} \). Thus, the implicit function theorem implies that
\[
\nabla_{\log \hat{p}} \log \bar{E} = (\bar{I} + \bar{\psi}) \bar{\Lambda} (\bar{I} - \bar{x}).
\]

Combining the expressions above, we obtain the Jacobian of the excess demand system:
\[
\nabla_{\log \hat{p}} G = \bar{I} - \bar{y} - \bar{\chi} + (\bar{I} - \bar{y}) (\bar{I} + \bar{\psi}) \bar{\Lambda} (\bar{I} - \bar{x}).
\]

Therefore, \(\nabla_{\log \hat{p}} G = \bar{\Gamma}\). Under Assumption 2, \(\nabla_{\log \hat{p}} G\) has off-diagonals with the same sign. By Proposition 17.F.3 in Mas-Colell, Whinston, and Green (1995b), this property implies that there is at most one \(\{\hat{p}_i\}\) such that \(G_j (\{\hat{p}_i\}) = 0\) for all \(j\).

Defining \(\bar{\mu} \equiv (\bar{I} + \bar{\psi}) \bar{\Lambda}\), the system in (48) implies that the off-diagonals of \(\bar{\Gamma}\) are given by
\[
\gamma_{io} = -y_{io} - \sum_j y_{ij} \left( \chi_{oij} + \mu_{jo} - \mu_{io} + \sum_d (\mu_{jd} - \mu_{id}) x_{od} \right).
\]

Thus, if \(\chi_{oij} + \mu_{jo} - \mu_{io} + \sum_d (\mu_{jd} - \mu_{id}) x_{od} > 0\) for all \(x_{od} \in (0, 1)\), then \(\gamma_{oi} < 0\).

**A.3 Proposition 2**

**Part 1: Equations (22)–(23).** We totally differentiate equations (7) and (9):
\[
\log L_i = \sum_j \phi_{ij} (\log \hat{\omega}_j + \log \hat{\rho}_j + \log \hat{\nu}_j).
\]
\[
\log L_i = \phi_i (\log \hat{\omega} + \log \hat{\rho} + \log \hat{\nu}).
\]
\[
\log \hat{\omega}_i = \log \bar{q}_i + \sum_j \psi_{ij} \log \hat{L}_j,
\]

where \(q_i \equiv p_i / P_i\).

In matrix notation, these expressions can be written as
\[
\log \hat{L} = \bar{\phi} (\log \hat{\omega} + \log \hat{\rho} + \log \hat{\nu})
\]
\[
\log \hat{\omega} = \log \bar{q} + \bar{\psi} \log \hat{L}.
\]

Plugging the second expression into the first expression,
\[
(\bar{I} - \bar{\phi} \bar{\psi}) \log \hat{L} = \bar{\phi} (\log \bar{q} + \log \bar{\rho} + \log \bar{\nu}).
\]

Under Assumption 2, the matrix \(\bar{\Lambda} \equiv (\bar{I} - \bar{\phi} \bar{\psi})^{-1} \bar{\phi}\) is well defined. Thus,
\[
\log \hat{L} = \bar{\Lambda} (\log \bar{q} + \log \bar{\rho} + \log \bar{\nu})
\]

Substituting it into the expression for real wages,
\[
\log \hat{\omega} = (\bar{I} + \bar{\psi} \bar{\Lambda}) \log \bar{q} + \bar{\psi} \bar{\Lambda} (\log \bar{\rho} + \log \bar{\nu}).
\]
Equations (49)–(50) immediately yield equations (22)–(23) by setting \( \log \hat{\rho} = \log \hat{\nu} = 0 \).

**Part 2: Equation (24).** We totally differentiate the definition of \( q_j \equiv p_j / p_j \). By Shepard’s lemma,

\[
\log \hat{q}_j = \log \hat{p}_j - \sum_i x^0_{ij} \left( \log \hat{\tau}_{ij} - \log \hat{\zeta}_i + \log \hat{p}_i \right).
\]

In matrix notation, the expression above becomes

\[
\log \hat{q} = -\log \hat{\eta}^E + (\bar{I} - \bar{x}) \log \hat{\rho}.
\]

By substituting (9) into (10), we get

\[
\bar{p} \Psi_i (L) \bar{L}_i = \sum_j x_{ij} \rho_j \bar{p}_j \Psi_j (L) \bar{L}_j,
\]

with \( x_{ij} = X_{ij} \left( \{ \tau_{oj}p_o / \zeta_o \}_o \right) \).

The total differentiation of this expression implies that

\[
\sum_j \left( I_{ij=1} - y_i^0 - \sum_d y^0_{id} x_{jid} \right) \log \hat{p}_j = \log \hat{\eta}^E + \sum_j y^0_{ij} \log \hat{p}_j - \sum_j \left( I_{ij=1} - y^0_{ij} \right) \log \hat{e}_j,
\]

where \( I_{[o=i]} \) is an indicator function that equals 1 whenever \( o = i \), and

\[
\log \hat{e}_j = \log \hat{L}_j + \sum_o \psi_{jo} \log \hat{L}_o.
\]

In matrix form,

\[
(\bar{I} - \bar{y} - \bar{\chi}) \log \hat{\rho} = \log \hat{\eta}^E - (\bar{I} - \bar{y}) \log \hat{\rho} + \bar{y} \log \hat{\rho} \\
\log \hat{\rho} = (\bar{I} + \bar{\psi}) \log \hat{L}.
\]

Substituting (49) and (51),

\[
\bar{\Gamma} \log \hat{\rho} = \log \hat{\eta}^E + (\bar{I} - \bar{y}) \bar{\mu} \log \hat{z}^T + (\bar{y} - (I - \bar{y}) \bar{\mu}) \log \hat{\rho} - (I - \bar{y}) \bar{\mu} \log \hat{\nu}.
\]

Notice that the matrix \( \bar{\Gamma} \) is singular because, by definition, the transfer rule guarantees trade balance at the world level. So, in the system above, we use the matrix \( \bar{M} \) to set \( \hat{p}_{mm} = 0 \) and eliminate the \( m \)-th equation. Under Assumption 2,

\[
\log \hat{p} = \bar{\Gamma}_m \left( \log \hat{\eta}^E + (\bar{I} - \bar{y}) \bar{\mu} \log \hat{y}^T + (\bar{y} - (I - \bar{y}) \bar{\mu}) \log \hat{\rho} - (I - \bar{y}) \bar{\mu} \log \hat{\nu} \right).
\]

We obtain (24) by substituting (53) into (51), while setting \( \log \hat{\rho} = \log \hat{\nu} = 0 \).
A.4 Proposition 3

A.4.1 General Model

Inserting expression (22) into (24),
\[
\log \hat{L} = \Lambda \left( \Gamma^E \log \hat{\eta}^E - \Gamma^I \log \hat{\eta}^I \right)
\]

Thus,
\[
\hat{L}_j = \left( \sum_d \lambda_{id} \gamma_{jd}^E \right) \log \hat{\eta}^E_j - \left( \sum_d \lambda_{id} \gamma_{jd}^I \right) \log \hat{\eta}^I_j,
\]

which immediately implies expression (25). ■

A.4.2 Small Open Economy

We say that a market is a small open economy if its outcomes do not impact other markets in the world economy. Formally, we define a small open economy as follows.

Definition 3. Market \( d \) is a segmented market in a small open economy if, for all other markets \( i \neq d \),
1) The share of revenue and spending associated with \( d \) is approximately zero: \( y_{id}^0 \approx 0 \) and \( x_{di}^0 \approx 0 \) for all \( i \neq d \).
2) Spending shares, employment, and productivity do not respond to outcomes in market \( d \):
\[
\chi_{dij} = \phi_{id} = \psi_{id} = 0 \quad \text{for all} \quad i, j \neq d.
\]

We use this definition to derive the impact of trade cost shocks for a small open economy. The equilibrium satisfies the following system of equations:
\[
\begin{align*}
\log \hat{L}_i &= \sum_j \phi_{ij} \left( -\log \hat{\eta}_j^E + \log \hat{p}_j - \sum_o x_{oj}^0 \log \hat{p}_o + \sum_l \psi_{jl} \log \hat{L}_l \right), \\
\sum_o \left( I_{o=i} - y_{io}^0 - \chi_{io} \right) \log \hat{p}_o &= \log \hat{\eta}_i^E - \sum_j \left( I_{j=i} - y_{ij}^0 \right) \left( \log \hat{L}_j + \sum_o \psi_{jo} \log \hat{L}_o \right).
\end{align*}
\]

Consider a shock only to the small open economy \( d \): \( \log \hat{\tau}_{ij} = 0 \) and \( \log \hat{\zeta}_i \) if \( i \neq d \) and \( j \neq d \). By the definition of a Small Open Economy, we have that \( x_{di}^0 = 0 \) and \( \chi_{dij} = 0 \) all \( i, j \neq d \), which implies that
\[
\begin{align*}
\log \hat{\eta}_i^E &= x_{di}^0 \log \hat{\tau}_{di} - x_{di}^0 \log \hat{\zeta}_d = 0, \quad \text{and} \quad \log \hat{\eta}_i^I = \sum_j y_{ij}^0 \left( \chi_{dij} \left( \log \hat{\tau}_{dj} - \log \hat{\zeta}_d \right) \right) = 0.
\end{align*}
\]

Applying \( \log \hat{\eta}_i^E = \log \hat{\eta}_i^I = 0 \) for all \( i \neq d \) to the system above, we obtain that
\[
\begin{align*}
\log \hat{L}_i &= \sum_{j \neq d} \phi_{ji} \left( \log \hat{p}_j - \sum_{i \neq d} x_{ij}^0 \log \hat{p}_i + \sum_{l \neq d} \psi_{il} \log \hat{L}_l \right), \\
\sum_{o \neq d} \left( I_{o=i} - y_{io}^0 - \sum_{j \neq d} y_{ij}^0 \chi_{oj} \right) \log \hat{p}_o &= - \sum_{j \neq d} \left( I_{j=i} - y_{ij}^0 \right) \left( \log \hat{L}_j + \sum_{o \neq d} \psi_{jo} \log \hat{L}_o \right).
\end{align*}
\]
where the first equation uses \( x_{id}^0 = \phi_{id} = \psi_{id} = 0 \) and the second equation uses \( \psi_{id} = y_{id} = \chi_{dj} = 0 \).

Thus, we have \( \log p_i = \log \hat{L}_i = 0 \) for all \( i \neq d \), and we only have to solve the following system for market \( d \):

\[
\log \hat{L}_d = \phi_{dd} \left( -\log \hat{\eta}_d^I + (1 - x_{dd}^0) \log \hat{\rho}_d + \psi_{dd} \log \hat{L}_d \right)
\]

\[
(1 - y_{dd}^0 - \chi_{dd}) \log \hat{\rho}_d = \log \hat{\eta}_d^E - (1 - y_{dd}^0) (1 + \psi_{dd}) \log \hat{L}_d.
\]

Define \( \lambda_{dd} \equiv (1 - \phi_{dd} \psi_{dd})^{-1} \phi_{dd} \). Rearranging the first expression in this system, we get

\[
\log \hat{L}_d = \lambda_{dd} \left( -\log \hat{\eta}_d^I + (1 - x_{dd}^0) \log \hat{\rho}_d \right).
\] (54)

Substituting (54) into the second equation in the system above, we get that

\[
(1 - y_{dd}^0 - \chi_{dd}) \log \hat{\rho}_d = \log \hat{\eta}_d^E - (1 - y_{dd}^0) \lambda_{dd} \left( -\log \hat{\eta}_d^I + (1 - x_{dd}^0) \log \hat{\rho}_d \right)
\]

Define \( \gamma_{dd} \equiv (1 - y_{dd}^0 - \chi_{dd} + \lambda_{dd} (1 + \psi_{dd}) (1 - y_{dd}^0) (1 - x_{dd}^0))^{-1} \). Thus

\[
\log \hat{\rho}_d = \gamma_{dd} \left( \log \hat{\eta}_d^E + \lambda_{dd} (1 - y_{dd}^0) (1 + \psi_{dd}) \log \hat{\eta}_d^I \right).
\] (55)

The combination of (55) and (54) yields expression (26).

\[\Box\]

### A.5 Proposition 4

#### A.5.1 Gravity Economy without Trade Costs

Consider an economy without trade costs, \( \tau_{ij} = 1 \) for all \( i \) and \( j \), such that \( y_{id} = y_d \) and \( x_{od} = x_o \). We analyze the effect of a productivity shock. Thus, Assumption 1a implies that

\[
\log \hat{\eta}_d^E \equiv \chi \sum_d y_d \sum_o (1_{|o=i|} - x_o) \log \hat{\zeta}_o =
\]

\[
\log \hat{\eta}_d^E = \chi \log \hat{\zeta}_i - \chi \sum_o x_o \log \hat{\zeta}_o
\] (56)

Notice that the symmetry of trade costs implies that the price index is the same everywhere such that

\[
\log \hat{\rho}_i = \log \hat{\eta}_i^I + \sum_o x_o \log \hat{w}_o,
\]

with \( \hat{\eta}_i^I = \sum_o x_o \hat{\zeta}_o \).

Log-linearizing the market clearing constraint, given Assumption 1a and symmetric trade costs, we have

\[
\log \hat{w}_i + \log \hat{L}_i = \chi \left( -\log \hat{w}_i + \log \hat{\zeta}_i + \log \hat{\Psi}_i + \sum_o x_o \hat{w}_o - \sum_o x_o \log \hat{\zeta}_o - \sum_o x_o \log \hat{\Psi}_o \right),
\] (57)
where we normalize the world GDP to one, \( \sum_j w_j L_j \equiv 1 \), so that \( \sum_j x_{ij} \left( \log \hat{w}_j + \log \hat{L}_j \right) = 0 \).

Given Assumption 1b, labor supply is
\[
\log \hat{L}_i = \phi \left( \log \hat{w}_i - \log \hat{P}_j \right) - \tilde{\phi} \sum_j L_j \left( \log \hat{w}_j - \log \hat{P}_j \right) \tag{58}
\]
and with Assumption 1c, we have
\[
\log \hat{\Psi}_i = \psi \log \hat{L}_i - \tilde{\psi} \sum_j \Psi_j \log \hat{L}_j. \tag{59}
\]
Combining equations (56)-(59), we have
\[
(1 + \phi + \chi - \chi \psi \phi) \log \hat{w}_i = \log \hat{\eta}_E + \left( \phi - \tilde{\phi} \sum_j L_j \right) \log \hat{P} + \chi (1 - \psi \phi) \sum_j x_{ij} \log \hat{w}_j + \tilde{\phi} \sum_j L_j \log \hat{w}_j
\]
\[
\log \hat{w}_i = \tilde{\beta}^E \log \hat{\eta}_E + \tilde{\beta}^I \log \hat{\eta}_I + \sum_j \tilde{\gamma}_j \log \hat{w}_j
\]
where \( \tilde{\beta}^E \equiv (1 + \phi + \chi - \chi \psi \phi)^{-1} \), \( \tilde{\beta}^I \equiv \tilde{\beta}^E \left( \phi - \tilde{\phi} \sum_j L_j \right) \), and \( \tilde{\gamma}_j \equiv \tilde{\beta}^E \tilde{\phi} L_j + \tilde{\beta}^E x_{ij} \left( \phi - \tilde{\phi} \sum_o L_o \right) + \chi (1 - \psi \phi) x_j \).

Summing across \( i \), we get that
\[
\sum_j \tilde{\gamma}_j \log \hat{w}_j = \sum_j \gamma_j \left( \tilde{\beta}^E \log \hat{\eta}_E + \tilde{\beta}^I \log \hat{\eta}_I \right)
\]
with \( \gamma_j \equiv \tilde{\gamma}_j \left( 1 - \tilde{\beta}^E (\phi + \chi (1 - \psi \phi)) \right)^{-1} \).

Thus,
\[
\log \hat{w}_i = \tilde{\beta}^E \log \hat{\eta}_E + \tilde{\beta}^I \log \hat{\eta}_I + \sum_j \tilde{\gamma}_j \left( \tilde{\beta}^E \log \hat{\eta}_j^E + \tilde{\beta}^I \log \hat{\eta}_j^I \right). \tag{60}
\]
and
\[
\log \hat{L}_i = \phi \log \hat{w}_i - \left( \phi - \tilde{\phi} \sum_j L_j \right) \log \hat{\eta}_I + \sum_j \left( \left( \phi - \tilde{\phi} \sum_j L_j \right) x_j - \tilde{\phi} L_j \right) \log \hat{w}_j,
\]
which admits the representation in equation (27).
A.5.2 Gravity Economy with Trade Costs

For the second part, we start with the Local Labor Market Module. The total differentiation of equations (7) and (9) under Assumption 1b-1c implies that

$$\log \hat{L}_i = \phi \left( \log \hat{Q}_i + \log \hat{\zeta}_i + \psi \log \hat{L}_i - \psi \sum_j \psi_j \log \hat{L}_j \right)$$

$$- \tilde{\phi} \sum_j L_j \left( \log \hat{Q}_j + \log \hat{\zeta}_j + \psi \log \hat{L}_j - \psi \sum_o \psi_o \log \hat{L}_o \right)$$

Defining $\alpha_j^1 \equiv \phi \tilde{\phi} \psi_j + \phi \psi L_j - \phi \tilde{\phi} \psi \sum_o L_o$, this expression becomes

$$(1 - \psi \phi) \log \hat{L}_i + \sum_j \alpha_j^1 \log \hat{L}_j = \phi \left( \log \hat{Q}_i + \log \hat{\zeta}_i \right) - \phi \sum_j L_j \left( \log \hat{Q}_j + \log \hat{\zeta}_j \right)$$

Let us multiply both sides by $\alpha_i^1$ and sum across all $i$:

$$\sum_i \alpha_i^1 \log \hat{L}_i = \frac{1}{(1 - \psi \phi + \sum_o \alpha_o^1)} \sum_i \alpha_i^1 \left( \phi - \tilde{\phi} L_i \sum_o \alpha_o^1 \right) \left( \log \hat{Q}_i + \log \hat{\zeta}_i \right).$$

Substituting this expression back into the expression above,

$$(1 - \psi \phi) \log \hat{L}_i = \phi \left( \log \hat{Q}_i + \log \hat{\zeta}_i \right)$$

Thus,

$$\log \hat{L}_i = \alpha^2 \left( \log \hat{Q}_i + \log \hat{\zeta}_i \right) - \alpha^2 \sum_j \alpha_j^3 \left( \log \hat{Q}_j + \log \hat{\zeta}_j \right)$$

where

$$\alpha^2 \equiv \frac{\phi}{1 - \psi \phi}$$

$$\alpha_j^3 \equiv \frac{1}{\phi} \frac{(1 - \psi \phi) \tilde{\phi} L_j + \phi \alpha_j^1 + (1 - \alpha_j^1) \tilde{\phi} L_j \sum_o \alpha_o^1}{1 - \psi \phi + \sum_o \alpha_o^1}.$$

which are functions only of parameters and initial conditions.

Next we continue with the Trade Module. Consider $\log \hat{e}_i = -\psi \log \hat{L}_i + \psi \hat{\psi} \log \hat{L}$, which under Assumption 1b is

$$\log \hat{e}_i = \log \hat{\zeta}_i + (1 + \psi) \log \hat{L}_i - \psi \sum_j \psi_j \log \hat{L}_j$$

Substituting expression (61),

$$\log \hat{e}_i = \log \hat{\zeta}_i + (1 + \psi) \alpha^2 \left( \log \hat{\zeta}_i + \log \hat{Q}_i - \sum_j \alpha_j^3 \left( \log \hat{Q}_j + \log \hat{\zeta}_j \right) \right)$$

$$- \psi \alpha^2 \sum_j \psi_j \log \hat{L}_j \left( \log \hat{\zeta}_j + \log \hat{Q}_j - \sum_o \alpha_o^3 \left( \log \hat{Q}_o + \log \zeta_o \right) \right).$$
Thus, \[
\log \hat{e}_i = (1 + (1 + \psi)\alpha^2) \log \hat{\xi}_i + (1 + \psi)\alpha^2 \log \hat{Q}_i - \hat{\xi}
\] (62)
where
\[
\hat{\xi} \equiv \alpha^2 \sum_j \left( (1 + \psi)\alpha_j^3 + \bar{\psi}\Psi_j - \bar{\psi}\alpha_j^2 \sum_o \Psi_o \right) \left( \log \hat{Q}_j + \log \xi_j \right).
\]

To solve for the change in relative competitiveness \(\hat{Q}_i\), we totally differentiate the market clearing condition under Assumption 1a:

\[
\log \hat{p}_{ii} + \log \hat{e}_i = \log \hat{\eta}_i^E - \chi \sum_j y_{ij} \left( \log \hat{p}_{ii} - \sum_o x_{oj} \log \hat{p}_{oo} \right) + \sum_j y_{ij} \left( \log \hat{p}_{jj} + \log \hat{e}_{jj} \right)
\]

Substituting \(\log \hat{e}_i\) with expression (62),

\[
(1 + \chi) \log \hat{p}_{ii} - \sum_j (y_{ij} + \chi \sum_l y_{il} x_{jl}) \log \hat{p}_{jj} = \log \hat{\eta}_i^E - (1 + (1 + \psi)\alpha^2) \left( \log \hat{\xi}_i - \sum_j y_{ij} \log \hat{\xi}_j \right) - (1 + \psi)\alpha^2 \left( \log \hat{Q}_i - \sum_j y_{ij} \log \hat{Q}_j \right)
\]

Define \(\alpha^4 \equiv (1 + \chi + (1 + \psi)\alpha^2)^{-1}\). Thus,

\[
\log \hat{p}_{ii} - \sum_j b_{ij} \log \hat{p}_{jj} = \alpha^4 \left[ \log \hat{\eta}_i^E + (1 + \psi)\alpha^2 \left( \log \hat{\eta}_i^I - \sum_j y_{ij} \log \hat{\eta}_j^I \right) \right] - \alpha^4 (1 + (1 + \psi)\alpha^2) \left( \log \hat{\xi}_i - \sum_j y_{ij} \log \hat{\xi}_j \right)
\] (63)

where
\[
b_{ij} \equiv \alpha^4 \alpha^2(1 + \psi)x_{ji} + \alpha^4 (1 + (1 + \psi)\alpha^2) y_{ij} + \alpha^4 \alpha^2 (\chi - (1 + \psi)\alpha^2) \sum_l y_{il} x_{jl}
\]

Define the matrix
\[
\mathbf{B} = \mathbf{M} \left( \alpha^4\alpha^2(1 + \psi)\mathbf{x} + \alpha^4 (1 + (1 + \psi)\alpha^2) \mathbf{y} + \alpha^4 \alpha^2 (\epsilon - (1 + \psi)\alpha^2) \mathbf{y} \mathbf{x} \right) \mathbf{M}'
\] (64)

Expression (63) can be written as

\[
(\mathbf{I} - \mathbf{B}) \log \hat{p} = \alpha^4 \log \hat{\eta}^E - \alpha^4 (1 + \psi)\alpha^2 (\mathbf{I} - \mathbf{y}) \log \hat{\eta}^I - \alpha^4 (1 + (1 + \psi)\alpha^2) (\mathbf{I} - \mathbf{y}) \log \hat{\xi}.
\]

If \(\chi > (1 + \psi)\alpha^2\) and \(\phi \psi < 1\), then \((\mathbf{I} - \mathbf{B})\) is non-singular and \((\mathbf{I} - \mathbf{B})^{-1} = \sum_{d=0}^{\infty} \mathbf{B}^d\). To see this, notice that, in this case, \(b_{ij} > 0\), which implies that, for all \(i\),

\[
\sum_{j \neq m} |b_{ij}| = \alpha^4 \alpha^2(1 + \psi)(1 - x_{mi}) + \alpha^4 (1 + (1 + \psi)\alpha^2) (1 - y_{im}) + \alpha^4 (\chi - (1 + \psi)\alpha^2) (1 - x_{mi}),
\]
where the inequality follows from the fact that gravity structure implies $x_{ji} > 0$ for all $i$ and $j$. We define

$$\Gamma \equiv (\mathbf{I} - B)^{-1}.$$ 

This implies that

$$\log \hat{p} = \Gamma \left( \alpha^4 \log \hat{\eta}^E + \alpha^4 (1 + \psi) \lambda (\mathbf{I} - \mathbf{y}) \log \hat{\eta}' - \alpha^4 (1 + (1 + \psi) \alpha^2) (\mathbf{I} - \mathbf{y}) \log \hat{\zeta} \right).$$

Let us set $\log \hat{\zeta} = 0$. Thus,

$$\log \hat{Q} = \alpha^4 \hat{\Gamma}^E \log \hat{\eta}^E - \hat{\Gamma}' \log \hat{\eta}'. \tag{65}$$

where

$$\hat{\Gamma}^E \equiv (\mathbf{I} - \mathbf{x}) \hat{\Gamma} \quad \text{and} \quad \hat{\Gamma}' \equiv \mathbf{I} - \alpha^4 (1 + \psi) \alpha^2 (\mathbf{I} - \mathbf{x}) \hat{\Gamma} (\mathbf{I} - \mathbf{y}).$$

Applying (65) into (61),

$$\log \hat{L}_i = \alpha^2 \alpha^4 \sum_j \gamma_{ij}^E \log \hat{\eta}_{ij}^E + \alpha^2 \sum_j \gamma_{ij}^I \log \hat{\eta}_{ij}^I + \xi (\hat{\eta}', \hat{\eta}^E)$$

where

$$\xi (\hat{\tau}', \hat{\tau}^E) \equiv -\alpha^2 \sum_j \alpha_j^3 \left( \alpha^4 \sum_{ij} \gamma_{ij}^E \log \hat{\eta}_{ij}^E + \sum_j \gamma_{ij}^I \log \hat{\eta}_{ij}^I \right).$$

Thus, to get expression (28), we just have to define $\beta^1 \equiv \alpha^2 \alpha^4$, $\beta^2 \equiv \alpha^2$, $\tilde{\beta}^1 \equiv \alpha^2 \alpha^3 \alpha^4$, $\tilde{\beta}^2 \equiv \alpha^2 \alpha^3$, $\beta^3 \equiv \alpha^4 \alpha^2 (1 + \psi)$, $\beta^4 \equiv \alpha^4 (1 + (1 + \psi) \alpha^2)$, and $\beta^5 \equiv \alpha^4 \alpha^2 (\chi - (1 + \psi) \alpha^2)$. Finally, notice that $\alpha^3_x = 0 \implies \xi (\hat{\eta}', \hat{\eta}^E) = 0$, if $\tilde{\phi}, \tilde{\psi} = 0$.■

B Proofs for Section 4

B.1 Proof of the Optimal IV in Expression (34)

The asymptotic variance of the GMM estimator for any function $H_i(\cdot)$ is

$$V (H) = \left( E \left[ H_i (\hat{z}^t, \mathbf{W}^0) G_i \right] \right)^{-1} \left( E \left[ H_i (\hat{z}^t, \mathbf{W}^0) e_i e_i' H_i (\hat{z}^t, \mathbf{W}^0)' \right] \right) \left( E \left[ H_i (\hat{z}^t, \mathbf{W}^0) G_i \right] \right)^{-1} \tag{66}$$

where $G_i \equiv E \left[ \nabla_{\theta} e_i (\theta) | \hat{z}^t, \mathbf{W}^t \right]$, $e_i (\theta') = e_i (\theta)$.

The asymptotic variance of the Optimal IV estimator in (34) is

$$V (H^*) = E \left[ G_i' \left( \Omega (\hat{z}^t, \mathbf{W}^0) \right)^{-1} G_i \right] \tag{67}$$

with $\Omega (\hat{z}^t, \mathbf{W}^t) = E \left[ e_i (\theta) e_i (\theta)' | \hat{z}^t, \mathbf{W}^t \right]$. 65
We now show that \( V(H) - V(H^*) \) is positive semi-definite for any \( H_i(\cdot) \):

\[
V(H) - V(H^*) = (E[H_iG_i^{it}])^{-1} \left( E \left[ \left( H_i e_i^t \right)^2 \right] \right) (E[H_iG_i^{it}])^{-1} \left( E \left[ G_i^{it} \Omega^{-1} G_i^{it} \right] \right) - (E[H_iG_i^{it}])^{-1} \left( E \left[ G_i^{it} \Omega^{-1} G_i^{it} \right] \right) \left( E[H_iG_i^{it}] \right)^{-1}.
\]

Let us define

\[
U_i^t = H_i e_i^t - E \left[ \left( H_i e_i^t \right) \left( G_i^{it} \Omega^{-1} e_i^t \right) \right] (E[G_i^{it} \Omega^{-1} G_i^{it}])^{-1} G_i^{it} \Omega^{-1} e_i^t,
\]

which implies that

\[
E[U_i^t U_i^{it}'] = E \left[ \left( H_i e_i^t \right) \left( H_i e_i^t \right) \right] - E[H_iG_i^{it}] (E[G_i^{it} \Omega^{-1} G_i^{it}])^{-1} E[H_iG_i^{it}]'.
\]

Therefore,

\[
V(H) - V(H^*) = (E[H_iG_i^{it}])^{-1} \left( E[U_i^t U_i^{it}] \right) (E[H_iG_i^{it}])^{-1}.
\]

Since \( E[U_i^t U_i^{it}] \) is positive semi-definite, \( V(H) - V(H^*) \) is also positive semi-definite. Therefore, the asymptotic variance is minimized at \( H^* \).

B.2 Proof of the Two-Stage Procedure in Proposition 6

We use the strategy in Section 6.1 of Newey and McFadden (1994) to establish asymptotic properties of two-step estimators. To this end, we define the joint moment equation for the two estimating steps:

\[
\left( \hat{\theta}_2, \hat{\theta}_1 \right) \equiv \arg \min_{\theta_2, \theta_1} \left( \sum_{i,t} v_i^t(\theta_2, \theta_1) \right)' \left( \sum_{i,t} v_i^t(\theta_2, \theta_1) \right) \quad (68)
\]

where

\[
v_i^t(\theta_2, \theta_1) \equiv \left[ H_i^*(\hat{z}^t, W^0|\theta_0) e_i^t(\theta_2) \quad H_i^*(\hat{z}^t, W^0|\theta_0) e_i^t(\theta_1) \right]
\]

\[
H_i^*(\hat{z}^t, W^0|\theta) \equiv E \left[ \nabla \theta e_i^t(\hat{\theta}) | \hat{z}^t, W^0 \right] (\Omega_i^t)^{-1}.
\]

We have that \( \left( \hat{\theta}_2, \hat{\theta}_1 \right) \overset{p}{\rightarrow} (\theta, \theta) \), with an asymptotic variance given by

\[
\text{Var} \left( \hat{\theta}_2, \hat{\theta}_1 \right) = \left( \hat{G}' \hat{\Omega}^{-1} \hat{G} \right)^{-1}
\]

where \( \hat{G} \equiv \left[ \nabla \left( \theta_2, \theta_1 \right) \right] v_i^t(\theta_2, \theta_1) \) and \( \hat{\Omega} \equiv E \left( v_i^t(\theta_2, \theta_1) \right)^t \left( v_i^t(\theta_2, \theta_1) \right)' \).

Define \( h_i^t \equiv H_i^*(\hat{z}^t, W^0|\theta) e_i^t(\theta) \) and \( \hat{h}_i^t \equiv H_i^*(\hat{z}^t, W^0|\theta_0) e_i^t(\theta) \). Thus, \( \hat{G} \) and \( \hat{\Omega} \) are given by

\[
\hat{\Omega} = E \left[ \frac{h_i^t}{h_i^t} \frac{h_i^t}{h_i^t} \frac{h_i^t}{h_i^t} \frac{h_i^t}{h_i^t} \right] \quad \text{and} \quad \hat{G} = \left[ \begin{array}{cc} G & G_1 \\ 0 & G_2 \end{array} \right]
\]

where
\[
G \equiv E \left[ H_i^* (\hat{z}^t, W^0 | \theta) \nabla_{\theta} e_t^i (\theta) \right]
\]

\[
G_1 \equiv E \left[ \nabla_{\theta} H_i^* (\hat{z}^t, W^0 | \theta) e_t^i (\theta) \right]
\]

\[
G_2 \equiv E \left[ H_i^* (\hat{z}^t, W^0 | \theta_0) \nabla_{\theta} e_t^i (\theta) \right].
\]

By Assumption 3c, any function of \((\hat{z}^t, W^t)\) is orthogonal to \(e_t^i (\theta)\), which implies that \(G_1 = 0\). Thus, \((\hat{G}' \Omega^{-1} \hat{G})^{-1}\) is block diagonal and the marginal distribution of \(\theta_2\) is asymptotically normal with variance

\[
\text{Var} \left( \hat{\theta}_2 \right) = (G' \Omega^{-1} G)^{-1},
\]

which is equivalent to the asymptotic distribution of the Optimal IV in (67).

\[
\boxed{
C \text{ Online Appendix: Empirical Application}
\]

\[
C.1 \text{ Derivations in Section 5.1}
\]

**Bilateral Trade Demand.** In sector \(s\) of region \(d\), we impose that preferences have the following nested CES form:

\[
C_{sd} = \left[ \sum_k (C_{k, sd})^{\chi_k^{-1} \chi^{-1}} \right]^{\chi} \chi^{-1} \chi
\]

where \(\chi_k, \chi > 0\).

The first-stage cost minimization problem is

\[
\min_{C_{ko, sd}} \sum p_{ko, sd} C_{ko, sd} \quad \text{s.t.} \quad \left[ \sum_k \left( \sum_o (C_{ko, sd})^{\chi_k^{-1} \chi} \right)^{\chi_k^{-1} \chi} \chi^{-1} \chi \right] = 1.
\]

The first-order condition implies that

\[
\left[ \sum_k \left( \sum_o (C_{ko, sd})^{\chi_k^{-1} \chi} \right)^{\chi_k^{-1} \chi} \chi^{-1} \chi \right]^{-1} \left( \sum_o (C_{ko, sd})^{\chi_k^{-1} \chi} \chi^{-1} \chi \right) (C_{ko, sd})^{-\frac{1}{\chi_k}} = \lambda p_{ko, sd}
\]

Thus,

\[
\left( \frac{C_{ko, sd}}{C_{ko', sd}} \right) = \left( \frac{p_{ko, sd}}{p_{ko', sd}} \right)^{-\chi_k}
\]

and
Thus,

\[
x_{k'o',sd} = \frac{p_{k'o',sd} C_{k'o',sd}}{\left[ \sum_k \left( \frac{\sum_o \left( p_{ko,md} \right)^{1-\chi_k}}{\chi_k} \right)^{\frac{1-\chi_k}{1-\chi}} \right]^{\frac{1}{1-\chi}}} = \left( \frac{C_{k'o',sd}}{C_{k'o',md}} \right)^{-1}.
\]

Defining \( P_{k,md} = \left[ \sum_o \left( p_{ko,md} \right)^{1-\chi_k} \right]^{\frac{1}{1-\chi}} \), we get that

\[
x_{k'o',sd} = \left( \frac{p_{k'o',sd}}{P_{k',md}} \right)^{1-\chi_k} \left( \frac{P_{k',md}}{P_{k,md}} \right)^{1-\chi}.
\]

**Labor Supply.** Assume that the representative household preferences have the following nested structure, if \( \sum_{ko} l_{ko} = 1 \),

\[
U_c = \left[ \sum_d \nu_d \left( \sum_s V_{sd} \left( l_{sd} \frac{\phi_e - 1}{\phi_e} \right) \right) \right]
\]

where \( V_{sd} = \left( \nu_{sd} C_{sd} \frac{1+\phi_h}{\phi_h} \right)^{\frac{1}{1+\phi_h}} \)

and \( U_c = -\infty \) whenever \( \sum_{ko} l_{ko} \neq 1 \).

In sector \( s \) of region \( d \), the total labor supply is \( L_{sd} = h_{sd} l_{sd} \) and, using the spending shares in (40), the real consumption is \( C_{sd} = \omega_{sd} h_{sd} l_{sd} \). Thus, the representative household solves the following second-stage problem:

\[
\max_{\{l_{kd},h_{sd}\}} \left[ \sum_d \nu_d \left( \left( l_{0d} \frac{\phi_e - 1}{\phi_e} \right) + \sum_s V_{sd} \left( l_{sd} \frac{\phi_e - 1}{\phi_e} \right) \right) \right]
\]

subject to

\[
V_{sd} = \nu_{sd} \rho_{sd} \omega_{sd} h_{sd} - \left( h_{sd} \right)^{\frac{1+\phi_h}{\phi_h}}
\]

\[
\sum_{ko} l_{ko} = 1.
\]

The first-order condition for \( h_{sd} \) implies

\[
h_{sd} = (\tilde{\nu}_{sd} \rho_{sd} \omega_{sd})^{\phi_h}, \tag{69}
\]

so that

\[
V_{sd} = \tilde{\nu}_{sd} \rho_{sd} \omega_{sd}.
\]

let \( \mu \) be Lagrange multiplier of the additivity constraint. The first-order condition for \( l_{sd} \)
\[ \kappa_c \tilde{w}_d (l_{0d})^{-\frac{1}{\phi_e}} = \mu \]
\[ \kappa_c \tilde{v}_d \omega_{sd} (l_{sd})^{-\frac{1}{\phi_e}} = \mu \]

where

\[ \kappa_c \equiv \left[ \sum_d \nu_d \left( (l_{0d})^{\frac{\phi_e - 1}{\phi_e}} + \sum_s \left( \nu_{sd} \rho_{sd} \omega_{sd} h_{sd} - (h_{sd})^{\frac{1+\phi_h}{\phi_h}} \right) \frac{1}{\frac{1+\phi_h}{\phi_h}} \left( l_{sd} \right)^{\frac{\phi_e - 1}{\phi_e}} \right) \right] \]

\[ \tilde{w}_d \equiv \nu_d \left( (l_{0d})^{\frac{\phi_e - 1}{\phi_e}} + \sum_s \left( \nu_{sd} \rho_{sd} \omega_{sd} h_{sd} - (h_{sd})^{\frac{1+\phi_h}{\phi_h}} \right) \frac{1}{\frac{1+\phi_h}{\phi_h}} \left( l_{sd} \right)^{\frac{\phi_e - 1}{\phi_e}} \right) \frac{\phi_m \phi_e}{\phi_m \phi_e - 1} \]

Thus,

\[ \frac{l_{sd}}{l_{0d}} = (\tilde{v}_{sd} \omega_{sd})^{\phi_e} \]

and

\[ \frac{l_{0d}}{l_{0d} + \sum_s l_{sd}} = \frac{1}{1 + (\tilde{v}_{sd} \omega_{sd})^{\phi_e}}. \]

The first-order condition for \( l_{sd} \) also implies that

\[ \frac{\tilde{w}_d}{\tilde{w}_0} = \left( \frac{l_{0d}}{l_{00}} \right)^{\frac{1}{\phi_e}} \]

such that

\[ \tilde{w}_d = \nu_d \left( (l_{0d})^{-\frac{1}{\phi_e}} \left( l_{0d} + \sum_s l_{sd} \right) \right) \frac{1}{\phi_m \phi_e - 1}. \]

Thus,

\[ \frac{\nu_d}{\nu_0} \left( \left( \frac{l_{0d}}{l_{00}} \right)^{-\frac{1}{\phi_e}} \left( \frac{l_{0d}}{l_{00}} + \sum_s l_{sd} \right) \right) \frac{1}{\phi_m \phi_e - 1} = \left( \frac{l_{0d}}{l_{00}} \right)^{\frac{1}{\phi_e}} \]

and finally

\[ \frac{n_{0d}}{n_{00}} = \frac{l_{0d} + \sum_s l_{sd}}{l_{00} + \sum_s l_{s0}} = \left( \frac{\nu_d}{\nu_0} \right)^{\phi_m \phi_e - 1} \left( \frac{l_{0d}}{l_{00}} \right)^{\phi_m}. \]  

\[ (70) \]

\textbf{C.2 Data Construction}

\textbf{C.2.1 World Trade Matrix}

We construct a matrix of bilateral sector-level trade flows among 50 US states and 59 foreign countries for 1993, 1997, 2002, 2007 and 2012. To this end, we merge information on bilateral trade flows of 59 countries extracted from the Multi Region Input Output database (MRIO) and information on domestic and foreign trade flows of 50 US states extracted from the
Commodity Flow Survey (CFS) and US Census Foreign Trade Database (FTD). We then aggregate the data at the manufacturing and non-manufacturing level. In order to avoid zeros in the trade matrix, we merge DC and Maryland into a single state, and consider the set of countries in Table 4. We proceed in four steps.

1) First, we construct foreign trade flows of US states for each sector, year and foreign country. Let \((Z_{dj}^{kt}, Z_{jd}^{kt})\) denote the trade flows between each of the 40 US custom districts, \(d\), and foreign country, \(j\), by sector \(k\) and year \(t\). We obtain \((Z_{dj}^{kt}, Z_{jd}^{kt})\) from the US Merchandise Trade Files released annually by the US Census between 1990 and 2016. The exports and imports of state \(i\) to foreign country \(j\) are

\[
X_{ij}^{kt} = \sum_d a_{dij,kt} \cdot Z_{dj}^{kt}
\]

\[
X_{ji}^{kt} = \sum_d b_{dij,kt} \cdot Z_{jd}^{kt}
\]

where \(a_{dij,kt}\) and \(b_{dij,kt}\) correspond to the share of total exports and imports in district \(d\) whose respective origin and destination are state \(i\). We normalize the size of international trade flows so that the total value of export shipments in the CFS and total value of exports in the FTD are equal in 2012.

2) Second, we construct bilateral trade flows between US states for each sector and year. Let \(\tilde{\lambda}_{ir}^{kt}\) denote the value of shipments from state \(i\) to state \(r\) of goods in sector \(k\) at year \(t\). We obtain \(\tilde{\lambda}_{ir}^{kt}\) from the Commodity Flow Survey released by the US Census in 1993, 1997, 2002, 2007 and 2012. The trade flows between state \(i\) to state \(r\) are

\[
X_{ir}^{kt} = \tilde{\lambda}_{ir}^{kt} - \sum_{d,j} \left( \tilde{a}_{dij,kt}^{ir} \cdot Z_{dj}^{kt} + \tilde{b}_{dij,kt}^{ir} \cdot Z_{jd}^{kt} \right)
\]

where \(\tilde{a}_{dij,kt}^{ir}\) and \(\tilde{b}_{dij,kt}^{ir}\) correspond respectively to the share of total exports and imports in district \(d\) transiting between states \(i\) and \(r\). To compute the variables above, we assume that the transit route is the same for all export and import of all sectors with identical state of origin/destination, port of exit/entry, and foreign country of origin/destination. Using the US Census data on state of origin exports by port and destination, we compute the following variables:

\[
a_{dij,kt} = \frac{\text{exports}_{dij,t}}{\sum_l \text{exports}_{dij,t}} \quad \text{and} \quad \tilde{a}_{dij,kt}^{ir} = \frac{\text{exports}_{dij,t}^{ir}}{\sum_{r,l} \text{exports}_{rdj,t}^{rl}}
\]

3) Third, we adjust domestic sales of the residual sector to include local spending in services:
\[ X_{ii}^{NT,t} = \left( \sum_{k \neq NT} \sum_r X_{rt}^{kt} \right) e^t_i \]

where \( e^t_i \) is the expenditure ratio between non-tradeable and tradeable goods of state \( i \) at year \( t \) obtained from the BEA state-level accounts. This adjustment is equivalent to ignoring trade between states in goods and services excluded from the CFS, \( X_{ir}^{NT,t} = 0 \) for \( i \neq r \).

4) Fourth, we merge the trade bilateral trade flows of US states with the bilateral trade flows of the US and other countries in the MRIO database. To this end, we use US domestic sales in the MRIO to normalize total expenditures of US states on goods produced from other US states. We also distribute the bilateral trade flows of the US in the MRIO among US states using each state share in total trade flows to/from other foreign countries obtained in step 1.

**Methodology to replace suppressed values.** Using observed data on bilateral shipments between US states in the tradeable sector, we estimate the following gravity equation, for every year \( t \):

\[
\log \hat{X}_{ij} = \beta_0 + \beta_1 \ln d_{ij} + \beta_2 \ln Y_i + \beta_2 \ln E_j + e_{ij}
\]

where \( d_{ij} \) is the bilateral distance between state \( i \) and \( j \), \( Y_i \) is the total production in state \( i \), \( E_j \) is the total expenditure in state \( j \), and \( e_{ij} \) is the econometric error. Then we obtain the predicted values

\[
\log \hat{X}_{ij} = \hat{\beta}_0 + \hat{\beta}_1 \ln d_{ij} + \hat{\beta}_2 \ln X_i + \hat{\beta}_2 \ln X_j.
\]

We compute the residual outflows for each state as:

\[
\bar{Y}_i = Y_i - \sum_j \hat{X}_{ij}
\]

and the residual inflows:

\[
\bar{E}_j = E_j - \sum_i \hat{X}_{ij}
\]

For suppressed values, we assume that the true trade flow equals:

\[
\hat{X}_{ij} = \hat{X}_{ij} \xi_i \gamma_j.
\]

We must have that the summation of predicted flows across destinations for each origin has to be equal to total production:

\[
\sum_j \hat{X}_{ij} = \bar{Y}_i
\]

and the summation of predicted flows across origins for each destination has to be equal to total expenditures

\[
\sum_i \hat{X}_{ij} = \bar{E}_j
\]

To compute \( \xi_i \) and \( \gamma_j \), we use the following algorithm. For state \( k \), consider the vector of exports to all states \( \hat{X}_{kj} \) and the imports \( \hat{X}_{jk} \). Then compute the following ratios
\[ \xi_k = \frac{\sum_j \tilde{X}_{kj}}{\bar{Y}_k} \]
\[ \gamma_k = \frac{\sum_j \tilde{X}_{jk}}{\bar{E}_k} \]

and adjust the vectors to match the aggregate flows:

\[ \hat{X}_{kj} = \tilde{X}_{kj} / \xi_k \]
\[ \hat{X}_{jk} = \tilde{X}_{jk} / \gamma_k \]

For state \( k + 1 \), repeat the same procedure, but keeping constant the exports and imports of the previous adjusted states \( 1 \) to \( k \), and adjusting the total expenditures and production. Thus first compute:

\[ \bar{Y}_{k+1}^{adj} = \bar{Y}_{k+1} - \sum_{s=1}^{k} \bar{Y}_s \]
\[ \bar{E}_{k+1}^{adj} = \bar{E}_{k+1} - \sum_{s=1}^{k} \bar{E}_s \]

and then:

\[ \xi_{k+1} = \frac{\sum_j \tilde{X}_{k+1,j} - \sum_{s=1}^{k} \tilde{X}_{k+1,s}}{\bar{Y}_{k+1}^{adj}} \]
\[ \gamma_{k+1} = \frac{\sum_j \tilde{X}_{j,k+1} - \sum_{s=1}^{k} \tilde{X}_{s,k+1}}{\bar{E}_{k+1}^{adj}} \]

and adjust the vectors to match the aggregate flows:

\[ \hat{X}_{k+1,s} = \tilde{X}_{k+1,s} / \xi_{k+1} \]
\[ \hat{X}_{s,k+1} = \tilde{X}_{sk+1} / \gamma_{k+1} \]

for all \( s = 1, \ldots, k \). Finally, we use these predicted (and consistent with the aggregates) values to fill the suppressed shipments.

C.2.2 Labor Market Data

We use the Current Population Surveys - Merged Outgoing Rotation Groups (CPS-MORG) to construct labor market outcomes in 50 US states and 2 sectors. We consider the sample all individuals aged between 16 and 64 years in the survey, and we follow the cleaning procedure of Autor, Katz, and Kearney (2008) to adjust for top censoring, outliers, and time consistency of variables. For each state and sector, we compute the nominal hourly wage as the weighted average of the weekly earnings divided by the number of weekly hours across employed
individuals, where individual weights correspond to the number of worked hours times the sampling weights. For each state and sector, we also compute the average hours worked as the weighted average number of weekly hours of employed individuals where individual weights correspond to sampling weights. Finally, we use individual sampling weights to compute total number of employed individuals in each sector and state. The total sector-level employment is the average number of hours worked times the total number of employed individuals, and the total home-sector employment is the total number of individuals either unemployed or out of the labor force.
C.2.3 Price data

To construct the price index by region, that we use to compute the real wage, we use data from the Cost of Living Index, published by the Council for Community and Economic Research (C2ER). Our state-level price index is the average of the price index changes of urban areas in the state, weighted by the area’s population. To construct, instead, the aggregate price index for the 50 US states and 59 foreign countries, we use data from the Penn World Tables 8.1 on the price of consumption goods (see Table 1 in Feenstra et al. (2015) for a description).\textsuperscript{45} We construct the state level aggregate price indices as follows. From the Cost of Living Index price indices, we compute the US average, and take the ratio of each state price over the national average. Then, we multiply this ratio by the aggregate price level of US.

C.3 Additional Results

\textsuperscript{45}Using instead the price of expenditures gives virtually the same results.
Table 5: Agglomeration Functional Forms - Robustness

<table>
<thead>
<tr>
<th></th>
<th>$\psi_h$</th>
<th>$\psi_m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panel A: $\psi_h = -\psi_m$</td>
<td>0.248*</td>
<td>-0.248*</td>
</tr>
<tr>
<td></td>
<td>(0.143)</td>
<td>(0.143)</td>
</tr>
<tr>
<td>Panel B: $\psi_m = 0$</td>
<td>1.734*</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.962)</td>
<td></td>
</tr>
<tr>
<td>Panel C: $\psi_h = 0$</td>
<td>-0.649**</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.267)</td>
<td></td>
</tr>
</tbody>
</table>

### Table 6: Estimation of Structural Parameters - Alternative IVs

<table>
<thead>
<tr>
<th></th>
<th>Labor Supply Elasticity</th>
<th>Agglomeration Elasticity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Hours</td>
<td>Between sectors</td>
</tr>
<tr>
<td><strong>Panel A: Baseline, MOIV</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \phi_h )</td>
<td>0.219**</td>
<td>0.914***</td>
</tr>
<tr>
<td>(0.090)</td>
<td></td>
<td>(0.201)</td>
</tr>
<tr>
<td><strong>Panel B: Direct effect</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \phi_e )</td>
<td>0.201*</td>
<td>0.87***</td>
</tr>
<tr>
<td>(0.120)</td>
<td></td>
<td>(0.269)</td>
</tr>
<tr>
<td>F-stat</td>
<td>60.652</td>
<td>60.652</td>
</tr>
<tr>
<td><strong>Panel C: Tariff Shock Exposure</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \phi_m )</td>
<td>0.282</td>
<td>1.323***</td>
</tr>
<tr>
<td>(0.183)</td>
<td></td>
<td>(0.387)</td>
</tr>
<tr>
<td>F-stat</td>
<td>47.091</td>
<td>47.091</td>
</tr>
<tr>
<td><strong>Panel D: Naïve Tariff Shock Exposure</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \psi_e = -\psi_m )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \phi_h )</td>
<td>0.329</td>
<td>1.292**</td>
</tr>
<tr>
<td>(0.279)</td>
<td></td>
<td>(0.520)</td>
</tr>
<tr>
<td>F-stat</td>
<td>12.826</td>
<td>12.826</td>
</tr>
</tbody>
</table>

Table 7: Estimation of Structural Parameters - Alternative Specifications

<table>
<thead>
<tr>
<th></th>
<th>Labor Supply Elasticity</th>
<th>Agglomeration Elasticity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\phi_h$</td>
<td>$\phi_e$</td>
</tr>
<tr>
<td>Panel A: Time and sector dummies</td>
<td>0.219**</td>
<td>0.914***</td>
</tr>
<tr>
<td></td>
<td>(0.090)</td>
<td>(0.201)</td>
</tr>
<tr>
<td>Panel B: Include initial state demographic controls</td>
<td>0.189**</td>
<td>0.832***</td>
</tr>
<tr>
<td></td>
<td>(0.078)</td>
<td>(0.202)</td>
</tr>
<tr>
<td>Panel C: Include initial state demographic controls interacted with period dummies</td>
<td>0.203**</td>
<td>0.775**</td>
</tr>
<tr>
<td></td>
<td>(0.091)</td>
<td>(0.364)</td>
</tr>
<tr>
<td>Panel D: Weight states by population in 1992</td>
<td>0.164**</td>
<td>0.462</td>
</tr>
<tr>
<td></td>
<td>(0.081)</td>
<td>(0.391)</td>
</tr>
<tr>
<td>Panel E: Include only tradeable sector</td>
<td>0.233*</td>
<td>0.892*</td>
</tr>
<tr>
<td></td>
<td>(0.123)</td>
<td>(0.536)</td>
</tr>
<tr>
<td>Panel F: Drop great recession period of 2007-2012</td>
<td>0.211***</td>
<td>0.88***</td>
</tr>
<tr>
<td></td>
<td>(0.068)</td>
<td>(0.263)</td>
</tr>
</tbody>
</table>

Table 8: Model Fit - Alternative Specifications

<table>
<thead>
<tr>
<th>Real Wage</th>
<th>Hours</th>
<th>Sectoral relative employment</th>
<th>Non-Participation Share</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
</tbody>
</table>

Panel A: Time and sector dummies

|           |       | 13.648***                     | 15.352***               | 15.428***               | 17.944**               |
|           |       | (3.314)                       | (3.671)                 | (4.694)                 | (9.060)                |

Panel B: Include initial state demographic controls

|           |       | 13.874***                     | 13.85***                | 14.03***                | 13.976                 |
|           |       | (3.043)                       | (3.617)                 | (4.904)                 | (9.178)                |

Panel C: Include initial state demographic controls interacted with period dummies

|           |       | 11.848***                     | 12.056***               | 9.035**                 | 18.337**               |
|           |       | (2.971)                       | (3.714)                 | (3.695)                 | (8.752)                |

Panel D: Weight states by population in 1992

|           |       | 17.176***                     | 13.883***               | 8.021                   | 13.541                 |
|           |       | (5.568)                       | (4.268)                 | (5.235)                 | (11.653)               |

Panel E: Include only tradeable sector

|           |       | 9.738***                      | 10.294***               | 8.099*                  | 18.337**               |
|           |       | (2.924)                       | (4.209)                 | (4.243)                 | (8.752)                |

Panel F: Drop great recession period of 2007-2012

|           |       | 14.729***                     | 15.557***               | 12.652***               | 35.093**               |
|           |       | (2.472)                       | (3.858)                 | (4.391)                 | (14.471)               |

Table 9: Estimation of Structural Parameters - Alternative Tariff Shocks

<table>
<thead>
<tr>
<th>Labor Supply Elasticity</th>
<th>Agglomeration Elasticity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hours</td>
<td>Between sectors</td>
</tr>
<tr>
<td>$\phi_h$</td>
<td>$\phi_e$</td>
</tr>
</tbody>
</table>

Panel A: Tariffs in agriculture and manufacturing

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0.152</td>
<td>-0.009</td>
<td>1.883</td>
<td>0.841</td>
</tr>
<tr>
<td>(0.130)</td>
<td>(0.662)</td>
<td>(4.843)</td>
<td>(0.593)</td>
</tr>
</tbody>
</table>

Panel B: Tariffs in agriculture by top 5 partners

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0.219**</td>
<td>0.917***</td>
<td>0.698</td>
<td>0.245*</td>
</tr>
<tr>
<td>(0.090)</td>
<td>(0.201)</td>
<td>(0.551)</td>
<td>(0.143)</td>
</tr>
</tbody>
</table>

Panel C: Tariffs in agriculture by top 15 partners

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0.171</td>
<td>0.341</td>
<td>1.558</td>
<td>0.997</td>
</tr>
<tr>
<td>(0.128)</td>
<td>(0.525)</td>
<td>(3.093)</td>
<td>(1.188)</td>
</tr>
</tbody>
</table>

Panel D: Tariffs in agriculture by top 10 partners applied to all countries

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0.177</td>
<td>1.045***</td>
<td>0.451</td>
<td>0.539***</td>
</tr>
<tr>
<td>(0.112)</td>
<td>(0.239)</td>
<td>(0.437)</td>
<td>(0.192)</td>
</tr>
</tbody>
</table>

Table 10: Model Fit - Alternative Tariff Shocks

<table>
<thead>
<tr>
<th></th>
<th>Real Wage</th>
<th>Hours</th>
<th>Sectoral relative employment</th>
<th>Non-Participation Share</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
</tr>
<tr>
<td>Panel A: Tariffs in agriculture and manufacturing</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10.259***</td>
<td>4.562</td>
<td>-0.433</td>
<td>2.458</td>
<td></td>
</tr>
<tr>
<td>(2.000)</td>
<td>(3.444)</td>
<td>(6.959)</td>
<td>(8.831)</td>
<td></td>
</tr>
<tr>
<td>Panel B: Tariffs in agriculture by top 5 partners</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11.833***</td>
<td>7.758***</td>
<td>10.268***</td>
<td>10.534**</td>
<td></td>
</tr>
<tr>
<td>(2.905)</td>
<td>(1.944)</td>
<td>(2.846)</td>
<td>(5.287)</td>
<td></td>
</tr>
<tr>
<td>Panel C: Tariffs in agriculture by top 15 partners</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10.781***</td>
<td>5.466*</td>
<td>3.533</td>
<td>3.703</td>
<td></td>
</tr>
<tr>
<td>(3.353)</td>
<td>(3.259)</td>
<td>(5.398)</td>
<td>(9.408)</td>
<td></td>
</tr>
<tr>
<td>Panel D: Tariffs in agriculture by top 10 partners applied to all countries</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11.942***</td>
<td>6.278**</td>
<td>12.107***</td>
<td>15.143***</td>
<td></td>
</tr>
<tr>
<td>(2.464)</td>
<td>(3.059)</td>
<td>(3.658)</td>
<td>(5.583)</td>
<td></td>
</tr>
</tbody>
</table>

Figure 6: Predicted Change in Employment with Migration
D Online Appendix: Proofs of Equivalences and Extensions

In the first sub-section, we provide the proofs of the equivalence of existing models with our Generalized Spatial Competitive Economy. We show the equivalence with, respectively: (i) Neoclassical models with economies of scale, (ii) New trade theory models, (iii) New economic geography models, (iv) Spatial assignment models, and (v) Spatial assignment models with other factors of production. In the second sub-section, we extend our model to allow for: i) workers commuting across regions. ii) input-output linkages in production, iii) multiple worker groups in production; and iv) firm heterogeneity.

D.1 Equivalences

D.1.1 Neoclassical Economy

Environment. Consider a neoclassical economy with a single factor of production. We denote all the variables of this economy that are potentially different from the Generalized Spatial Economy with a superscript $N$. The proofs follows the logic of the proof of Adao, Costinot, and Donaldson (2017) but extending to the case of labor mobility and agglomeration spillovers. We assume that the agglomeration function, the labor supply function, the transfer function, the exogenous shifters are the same as for the Generalized Economy, so that we do not use superscripts for those objects.

As in the Generalized Spatial Competitive Economy, each country has a representative agent with preferences for consumption and labor supply in different markets, with utility function given by

$$U_c \left( \{ \nu_j C_j^N \} \right) \left( \{ L_j^N \} \right).$$

The main difference is that we explicitly allow for preferences over goods, $z$:

$$C_j^N = V^N \left( \{ c_{z,ij}^N \} \right),$$

where $V^N(.)$ is twice differentiable, quasi-concave, homothetic, and increasing in all arguments. Notice that $V^N(.)$ allows for the possibility that goods from different origins are imperfect substitutes.

The representative household’s budget constraint is

$$\sum_i \sum_z p_{z,ij}^N c_{z,ij}^N = \rho_j w_j^N L_j^N.$$

There are many perfectly competitive firms supplying each good in any market. The production technology uses only labor and entails external economies of scale at the market level. In particular, the technology of producing good $z$ in $i$ and delivering to $j$ is given by

$$Y_{z,ij}^N = \Psi_i \left( \{ L_j^N \} \right) \zeta_i \frac{L_{z,ij}^N}{\tau_{ij} \alpha_{z,ir}^N},$$

where $\alpha_{z,ij}^N$ is good-specific productivity shifter of producing in $i$ and delivering in $j$. 

82
**Equilibrium.** We use the fact that $V^N(\cdot)$ is homothetic to derive the price index in market $j$:

$$P_N^j = P_N^j \left( \{ p_{k,oj} \}_{k,o} \right) \equiv \min_{\{ c_{k,oj} \}_{k,o}} \sum_{k,o} p_{k,oj}^N c_{k,oj}^N \quad \text{s.t.} \quad V^N \left( \{ c_{k,oj} \}_{k,o} \right) \geq 1 \quad (71)$$

where the associated spending share on good $z$ from $i$ is

$$x_{ij,z}^N \in X_{ij,z}^N \left( \{ p_{k,oj}^N \}_{k,o} \right). \quad (72)$$

Conditional on prices, the representative household solves the utility maximization problem that yields the labor supply in market $j$:

$$L^N_j = \Phi_j \left( \{ \nu_i \rho_i \omega_i^N \} \right). \quad (73)$$

Profit maximization implies that

$$p_{z,ij}^N = \frac{\tau_{ij}}{\zeta_i} p_{z,ij}^N \quad (74)$$

where

$$p_i^N = \frac{\bar{w}_i^N}{\Psi_i \left( \{ L_j^N \} \right)} \quad (75)$$

Finally, the labor market clearing condition is

$$w_i^N L_i^N = \sum_j \sum_z x_{ij,z}^N \cdot \rho_j w_j^N L_j^N. \quad (76)$$

The competitive equilibrium corresponds to $\left\{ \{ p_{z,ij}^N \}_{z,i}^j, w_j^N, L_j^N, P_j^N \right\}$ such that equations (71)–(76) hold. Thus, the equilibrium can be written as $\left\{ p_i^N, \omega_i^N, L_i^N, P_i^N \right\}_i$ solving (5)–(10) with $\Phi_j(\cdot), \Psi_j(\cdot)$, and

$$X_{ij}^N \left( \left\{ \frac{\tau_{oj} p_o^N}{\zeta_o} \right\}_o \right) \equiv \left\{ x_{ij}^N : x_{ij,z}^N \in X_{ij,z}^N \left( \left\{ \frac{\tau_{oj} p_o^N \alpha_{k,oj}}{\zeta_o} \right\}_{o,k} \right) \right\} \quad (77)$$

such that

$$P_i^N \left( \left\{ \frac{\tau_{oj} p_o^N}{\zeta_o} \right\}_o \right) = P_i^N \left( \left\{ \frac{\tau_{oj} p_o^N \alpha_{k,oj}}{\zeta_o} \right\}_{o,k} \right) \quad (78)$$

**Equivalence.** We now construct an equivalent Generalized Spatial Competitive Economy. We only need to show that there exist preferences and technology in the Generalized Spatial Competitive Economy that are consistent with $\left\{ \{ X_{ij}^N \}_{i}^j, \Phi_j(\cdot), \Psi_j(\cdot) \right\}$. We also assume that the production function of the market-specific composite good is in the Generalized Economy is

$$Y_{ij} = \Psi_i \left( \{ L_j \} \right) \zeta_i \frac{L_{ij}}{\tau_{ij}}.$$
In addition, consider the preferences in Section 2 with

\[
V_j \left( \{c_{ij}\}_i \right) \equiv \max_{\{c_{z,ij}\}_{z,i}} V^N \left( \{c_{z,ij}\}_{z,i} \right) \quad \text{s.t.} \quad \sum_z \alpha_{z,ij}^N c_{z,ij} = c_{ij}.
\]

(77)

Intuitively, the preference structure in (77) implies that, if the representative household acquires \(c_{ij}\) units of \(i\)'s composite good for \(j\)'s consumption, then it optimally allocates the composite good into the production of different goods, given the exogenous weights \(\alpha_{z,ij}^N\) that are now embedded into the representative agent's preferences. Since the relative price of goods in market \(i\) only depends on \(\alpha_{z,ij}^N\), this decision yields allocations that are identical to those in the competitive equilibrium of the decentralized economy.

To see this, denote the spending shares associated with the cost minimization problem with \(V_j(.)\) by \(x_{ij} \in X_{ij} \left( \left\{ \frac{\tau_{ij}p_i}{\zeta_o} \right\}_o \right)\). Thus, the equivalence follows from showing that

\[
X_{ij} \left( \left\{ \frac{\tau_{ij}p_i}{\zeta_o} \right\}_o \right) = X_{ij}^N \left( \left\{ \frac{\tau_{ij}p_i}{\zeta_o} \right\}_o \right) \forall \left\{ \frac{\tau_{ij}p_i}{\zeta_o} \right\}_o.
\]

(87)

First, we show that \(x_{ij} \in X_{ij} \left( \left\{ \frac{\tau_{ij}p_i}{\zeta_o} \right\}_o \right) \implies \exists x_{ij}^N \in X_{ij}^N \left( \left\{ \frac{\tau_{ij}p_i}{\zeta_o} \alpha_{z,ij}^N \right\}_{k,o} \right)\) with \(x_{ij} = \sum_z x_{ij}^N\). Let \(\{c_{z,ij}\}_{z,i}\) be the solution of the good allocation problem in the definition of \(V_j \left( \{c_{ij}\}_i \right)\) in (77). We proceed by contradiction to show that \(\{c_{z,ij}\}_{z,i}\) implies spending shares, \(x_{z,ij} = \left\{ \frac{\tau_{ij}p_i}{\zeta_o} \alpha_{z,ij}^N \right\}_{z,i}\) such that \(x_{z,ij} \in X_{ij}^N \left( \left\{ \frac{\tau_{ij}p_i}{\zeta_o} \alpha_{z,ij}^N \right\}_{k,o} \right)\). Suppose there exists a feasible allocation \(\{c_{z,ij}^N\}_{z,i}\) such that

\[
V^N \left( \{c_{z,ij}^N\}_{z,i} \right) > V^N \left( \{c_{z,ij}\}_{z,i} \right) \quad \text{and} \quad \sum_z \sum_{ij} \frac{\tau_{ij}p_i}{\zeta_i} \alpha_{z,ij}^N c_{z,ij}^N \leq 1.
\]

(89)

Notice that \(\sum_z \sum_{ij} \frac{\tau_{ij}p_i}{\zeta_i} \alpha_{z,ij}^N c_{z,ij}^N \leq 1\), which implies that the allocation \(c_{ij}^N \equiv \sum_z \alpha_{z,ij}^N c_{z,ij}^N\) is feasible in the Generalized Spatial Competitive Economy. Thus,

\[
V^N \left( \{c_{z,ij}\}_{z,i} \right) = V_j \left( \{c_{ij}\}_i \right) \geq V_j \left( \{c_{ij}^N\}_i \right) \geq V^N \left( \{c_{z,ij}^N\}_{z,i} \right),
\]

which is a contradiction of inequality (89).

Second, we show that \(x_{ij} = \sum_z x_{ij}^N\) with \(x_{ij}^N \in X_{ij}^N \left( \left\{ \frac{\tau_{ij}p_i}{\zeta_o} \alpha_{z,ij}^N \right\}_{k,o} \right)\), and \(c_{ij} = \sum_z \alpha_{z,ij}^N c_{z,ij}^N \implies x_{ij} \in X_{ij} \left( \left\{ \frac{\tau_{ij}p_i}{\zeta_o} \right\}_o \right)\). We start with \(c_{ij}^N = \sum_z \alpha_{z,ij}^N c_{z,ij}^N\) implied by the solution of the consumer's problem in the Neoclassical Economy. We proceed by contradiction to show that \(\{c_{ij}^N\}_i\) is optimal in the Generalized Spatial Competitive Economy given prices \(\{\tau_{ij}p_i\}_i\). Suppose there exists a feasible allocation \(\{c_{ij}\}_i\) in the Generalized Spatial Competitive Economy such that

\[
V_j \left( \{c_{ij}\}_i \right) > V_j \left( \{c_{ij}^N\}_i \right) \quad \text{and} \quad \sum_i p_{ij} c_{ij} \leq \sum_i p_{ij} c_{ij}^N = 1.
\]

Let \(\{c_{z,ij}\}_{z,i}\) be the solution of the good allocation problem in the definition of \(V_j \left( \{c_{ij}\}_i \right)\) in (77). Thus,

\[
\sum_i \frac{\tau_{ij}p_i}{\zeta_i} \sum_z \alpha_{z,ij}^N c_{z,ij} = \sum_i \frac{\tau_{ij}p_i}{\zeta_i} c_{ij} \leq 1
\]

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and, by revealed preference,

\[ V_j \left( \{ c_{ij}^N \} \right) \geq V_j^N \left( \{ c_{z,ij}^N \} \right) \geq V_j \left( \{ c_{ij} \} \right) . \]

This establishes the contradiction. Since we have found preferences and technology that imply the mappings \( \left( \Phi_j(\cdot), \Psi_j(\cdot), X_{ij}^N(\cdot) \right) \), we have proven the equivalence.

**D.1.2 New Trade Theory**

**Environment.** The utility function is as in the Generalized Spatial Competitive Economy. We assume that \( C_j \) has a nested preference structure across sectors, \( C_j^N = V_j^N \left( \{ C_{k,j} \} \right) \) with \( V_j^N(\cdot) \) strictly quasi-concave and homogeneous of degree one. Sectors are divided into two groups: competitive sectors, \( k \in K^{NC} \), and monopolistic competitive sectors, \( k \in K^{NM} \).

In any competitive sector \( k \in K^{NC} \), firms in each country produce one homogeneous good with the production technology in (4). In particular, assume that technology is subject to external economies of scale with the marginal production cost given by \( \zeta_{kr} \Psi_{kr}^{NC} \left( \{ L_j \} \right) \). Let \( C_{k,j}^{NC} \) be an aggregator of goods from different origins \( r \), \( C_{k,j}^{NC} \equiv V_{k,j}^{NC} \left( \{ c_{kr,j} \} \right) \), where \( V_{k,j}^{NC}(\cdot) \) is twice differentiable, increasing, quasi-concave, and homogeneous of degree one. Notice that the utility function allows the goods produced in different regions to be perfect substitutes and, therefore, it covers homogeneous goods.

In any sector \( k \in K^{NM} \), there is a large mass of potential entrants in each region that produce a differentiated good, indexed by \( z \), and operate in monopolistic competition. We assume that all potential entrants in sector-region \( (k,r) \) have access to the same increasing returns technology where, in terms of labor, the fixed entry cost is \( \mu_{kr} \Psi_{kr}^{NE} \left( \{ L_j \} \right) \) and the marginal production cost is \( \zeta_{kr} \cdot \Psi_{kr}^{NP} \left( \{ L_j \} \right) \). We explicitly allow \( \Psi_{kr}^{NP}(\cdot) \) and \( \Psi_{kr}^{NE}(\cdot) \) to depend on employment, but we assume that firms perceive them as given. So, these functions incorporate external agglomeration and congestion forces at the market level.

We also assume that, for \( k \in K^{NM} \), preferences are CES across the available differentiated goods with elasticity \( \sigma > 1 \):

\[
C_{k,j}^N = \left[ \int_{z \in Z_{k,j}} \left( c(z) \right)^{\sigma/\sigma - 1} d z \right]^{\sigma - 1/\sigma},
\]

where \( Z_{k,j} \) is the set of goods in sector \( k \in K^{NM} \) available in market \( j \).

**Equilibrium.** As in the Generalized Spatial Competitive Economy, the representative household’s problem yields the labor supply in region-sector \( j \),

\[ L_j^N \in \Phi_j \left( \{ \nu_i \rho_i \omega_i^N \} \right) . \]  

Consider now a competitive sector \( k \in K^{NC} \). Cost minimization implies that

\[
p_{kr,j}^N = \tau_{kr,j} \frac{w_{kr}^N}{\zeta_{kr} \Psi_{kr}^{NC} \left( \{ L_j \} \right)}.
\]
For the monopolistic competitive sector $k \in K_{NM}$, all firms in region $r$ choose the same price:

$$p_{kr,j}^N = \frac{\tau_{kr,j}}{\zeta_{kr}} \frac{\Psi_{kr}^{NP} \left( \{ L_j^N \} \right)}{w_{kr}^N}.$$  \hspace{1cm} (82)

We now characterize the mass of operating firms, $M_{kr}$. The labor market clearing and the free entry conditions in $(k, r)$ imply

$$M_{kr} = \frac{1}{\sigma \mu_{kr}} \cdot \frac{L_{kr}^N}{\Psi_{kr}^{NE} \left( \{ L_j^N \} \right)}.$$

Thus, in the monopolistic competitive sector $k \in K_{NM}$, we can express prices as

$$p_{kr,j}^N = \frac{\tau_{kr,j}}{\zeta_{kr}} \frac{w_{kr}^N}{\Psi_{kr}^{NM} \left( \{ L_j^N \} \right)}$$  \hspace{1cm} (83)

with

$$\Psi_{kr}^{NM} \left( \{ L_j^N \} \right) \equiv \frac{\sigma - 1}{\sigma} \Psi_{kr}^{NP} \left( \{ L_j^N \} \right) \left( \frac{1}{\sigma \mu_{kr}} \cdot \frac{L_{kr}^N}{\Psi_{kr}^{NE} \left( \{ L_j^N \} \right)} \right)^{\frac{1}{1-\sigma}}.$$  \hspace{1cm} (84)

Using these expressions, it is straightforward to show that the labor market clearing condition in sector $k$ of region $r$, $i = (k, r)$, is

$$w_{kr}^N L_{kr}^N = \sum_j x_{kr,j}^N \rho_j w_j^N L_j^N.$$  \hspace{1cm} (85)

**Equivalence.** We now construct an equivalent Generalized Spatial Competitive Economy To establish the equivalence, we need to set $\Psi_{kr}(\cdot) = \Psi_{kr}^{NM}(\cdot)$ for $k \in K_{NM}$ and $\Psi_{kr}(\cdot) = \Psi_{kr}^{NC}(\cdot)$ for $k \in K_{NC}$. We also need to specify sector-level preferences such that $V_{k,j} \left( \{ c_{kr,j} \} \right) = V_{k,j}^{NC} \left( \{ c_{kr,j} \} \right)$ for $k \in K_{NC}$ and $V_{k,j} \left( \{ c_{kr,j} \} \right) = \left[ \sum_r (c_{kr,j})^{\frac{\sigma}{\sigma-1}} \right]^{\frac{\sigma-1}{\sigma}}$ for $k \in K_{NM}$. In addition, we must specify the same upper-level consumption aggregator across sectors: $V_j \left( \{ c_{kr,j} \} \right) = V_j^N \left( \{ V_{k,j} \} \right)$.

**D.1.3 New Economic Geography**

**Environment.** For the next equivalence result we consider an economy with production structure and preference for goods identical to those in the New Trade Theory Economy of Section D.1.2. We assume that each country $c$ is populated by a continuum of individuals with identical preferences for goods. These individuals differ in terms of mobility across markets. As in Krugman (1991), there are two groups of markets in each country, $J_c^{NI}$ and $J_c^{NM}$. Market $j \in J_c^{NI}$ is populated by a subset of completely immobile individuals such that

$$L_j^N = \bar{L}_j \forall j \in J_c^{NI},$$  \hspace{1cm} (86)
In addition, there is a mass \( \bar{L}_c \) of individuals that is completely mobile across markets \( j \in J_{cN}^M \) such that

\[
\sum_{j \in J_{cN}^M} L_j^N = \bar{L}_c. \tag{87}
\]

Mobile individuals have identical preferences for being employed in any \( j \in J_{cN}^M \):

\[
U_j (\omega_j^N, L_j^N) = \nu_j \omega_j^N (L_j^N)^\beta.
\]

where \( \omega_j^N \) is the real wage in market \( j \).

**Equilibrium.** We restrict attention to equilibria with positive employment in every \( j \in J_{cN}^M \), and analyze separately the cases of \( \beta = 0 \) and \( \beta = 0 \).

If \( \beta = 0 \), any employment allocation is feasible as long as \( \nu_i \omega_i^N = \bar{u} \). Thus, the labor supply is

\[
\{L_i^N\}_i = \Phi^N_c (\{\nu_i \omega_i^N\}_i) = \begin{cases} L_i^N = \bar{L}_c, & L_i^N = 0 \quad \text{if } \nu_i \omega_i^N > \nu_j \omega_j^N \forall i \in J_{cN}^M \\ \sum_j L_j^N = \bar{L}_c & \text{if } \nu_i \omega_i^N = \bar{u} \forall i \in J_{cN}^M \end{cases} \tag{88}
\]

If \( \beta \neq 0 \), in this case, any \( j \in J_{cN}^M \) with positive employment must have

\[
\nu_j \omega_j^N (L_j^N)^\beta = \bar{u} \quad \Rightarrow L_j^N = \left( \frac{\bar{u}}{\nu_j \omega_j^N} \right)^{\frac{1}{\beta}}
\]

From equation (87),

\[
L_j^N = \Phi^N_c (\{\nu_i \omega_i^N\}_i) = \bar{L}_c \left( \frac{\nu_j \omega_j^N}{\sum_{i \in J_{cN}^M} (\nu_i \omega_i^N)^{\beta}} \right)^{\frac{1}{\beta}} \tag{89}
\]

The equilibrium of this economy is \( \{p_i^N, p_i^N, L_i^N, \omega_i^N\} \) solving (5)–(10) with \( \Phi_j (\{\nu_i \omega_i^N\}_i) = \bar{L}_j \) if \( j \in J_{cN}^M \) and \( \Phi_j (\{\nu_i \omega_i^N\}_i) = \Phi^N_j (\{\nu_i \omega_i^N\}_i) \) if \( j \in J_{cN}^M \).

**Equivalence.** To establish equivalence, we construct preferences for the the representative household in the Generalized Spatial Competitive Economy that yield the labor supply function \( \Phi_j (\cdot) = \Phi^N_j (\cdot) \). Specifically, consider the following preferences:

\[
U_c (\{\nu_j C_j, L_j\}_j) = \begin{cases} \sum_{j \in J_{cN}^M} (\nu_j C_j) (L_j)^\beta \right)^{\frac{1}{1+\beta}} & \text{if } \sum_{j \in J_{cN}^M} L_j = \bar{L}_c \text{ and } L_i = \bar{L}_i \forall i \in J^I \\ \infty & \text{otherwise} \end{cases}
\]

Since the budget constraint implies that \( C_j = \rho_j \omega_j L_j \), the labor supply function is the solution of

\[
\{\Phi_j (\{\nu_i \omega_i\}_i)\}_j = \arg \max_{\{L_j\}} \left[ \sum_{j \in J_{cN}^M} (\nu_j \rho_j \omega_j) (L_j)^{1+\beta} \right]^{\frac{1}{1+\beta}} \text{ s.t. } \sum_{j \in J_{cN}^M} L_j = \bar{L}_c.
\]

If \( \beta = 0 \), it is straightforward to see that the solution of the utility maximization problem yields equation (88). If \( \beta \neq 0 \), the solution of the maximization problem is the same as equation (89). Since we have assumed a production structure and preferences for goods identical to those in the New Trade Theory Economy, the assumptions on technology and consumption aggregator imposed in the previous section imply that the functions \( X_{ij}(\cdot) \) and \( \Psi_i(\cdot) \) deliver the equivalence.
D.1.4 Spatial Assignment Models

Environment. Suppose that countries are populated by a continuum of individuals, \( \iota \in I_c \), that are heterogeneous in terms of preferences and efficiency across markets (i.e., sector-region pairs). We assume individual \( \iota \) has market specific preferences, \( a_j(\iota) \), and market specific efficiency, \( e_j(\iota) \). In particular, if employed in market \( j \), we assume that individual \( \iota \) has homothetic preferences given by

\[
U_j(\iota) = a_j(\iota) + \nu_j \cdot V_j \left( \{ c_{ij}^N \} \right),
\]

with a budget constraint given by

\[
\sum_i p_{ij} \cdot c_{ij}^N = p_j \cdot w_j^N \cdot e_j(\iota).
\]

We further assume that individuals take independent draws of \( (a_j(\iota), e_j(\iota)) \) from a common distribution:

\[
\{ a_j(\iota), e_j(\iota) \}_j \sim F^N(\alpha, \xi).
\]

On the production side, we maintain the same structure of the Generalized Spatial Economy. That is, there is a representative competitive firm in each market with the production technology in (4).

Equilibrium. We start by characterizing spending shares across markets. Conditional on choosing \( j \), individuals choose spending shares that minimize total cost:

\[
P_j^N \left( \{ p_{oij}^N \}_j \right) \equiv \sum_o p_{oij}^N \cdot c_{oij}^N \text{ s.t. } V_j \left( \{ c_{oij}^N \}_o \right) = 1,
\]

with associated spending shares given by

\[
x_{ij} \in X_{ij}^N \left( \{ p_{oij}^N \}_o \right).
\]

The solution of this problem implies that, for individual \( \iota \), the utility of being employed in \( j \) is \( U_j(\iota) = a_j(\iota) + \nu_j \cdot \omega_j^N \cdot e_j(\iota) \). Thus, the set of individuals choosing \( j \) is

\[
I_j \left( \{ \rho_i \cdot \nu_i \cdot \omega_i^N \}_i \right) \equiv \{ (\alpha, \xi) : a_j + e_j \cdot \nu_j \cdot \omega_j \geq a_i + e_i \cdot \rho_i \cdot \nu_i \cdot \omega_i \ \forall i \},
\]

with the associated labor supply given by

\[
L_j = \Phi_j^N \left( \{ \rho_i \cdot \nu_i \cdot \omega_i^N \}_i \right) \equiv \int_{I_j \left( \{ \rho_i \cdot \nu_i \cdot \omega_i^N \}_i \right) \} e_j \ dF_e(\alpha, \xi).
\]

Notice that the function \( \Phi_j^N(\cdot) \) is homogeneous of degree zero with \( \frac{\partial \Phi_j^N}{\partial \omega_j} \geq 0 \) and \( \frac{\partial \Phi_j^N}{\partial \omega_i} \leq 0 \).

Profit maximization and labor market clearing are still given by (8)–(10). Thus, the equilibrium can be written as \( \{ p_{oij}^N, \omega_i^N, L_i, Q_i \} \) solving (5)–(10) with \( \Psi_j(\cdot), X_{ij}(\cdot) = X_{ij}^N(\cdot) \), and \( \Phi_j(\cdot) = \Phi_j^N(\cdot) \).

Equivalence. To establish the equivalence, it is sufficient to show that there are preferences

\[\text{The homogeneity of } \Phi_j^N(\cdot) \text{ follows immediately from the definition of } I_j. \text{ To see that } \frac{\partial \Phi_j^N}{\partial \omega_j} \geq 0 \text{ and } \frac{\partial \Phi_j^N}{\partial \omega_i} \leq 0, \text{ notice that } I_i(\tilde{\omega}_c) \subset I_i(\omega_c) \text{ and } I_j(\omega_c) \subset I_j(\tilde{\omega}_c) \text{ whenever } \tilde{\omega}_j > \omega_j \text{ and } \tilde{\omega}_i = \omega_i.\]
for the representative household of the Generalized Spatial Competitive Economy that yield 
\( \Phi_j(\cdot) = \Phi_j^N(\cdot) \) and \( X_{ij}(\cdot) = X_{ij}^N(\cdot) \). Specifically, consider the following preferences:

\[
C_j = V_j^N \left( \{c_{ij}\}_j \right),
\]

and

\[
U \left( \{\nu_j C_j\}_j \{L_j\}_j \right) = \max_{\{I_j(a,e)\}_j(a,e)} \sum_j \nu_j C_j + \int \sum_j a_j I_j(a,e) \, dF_N(a,e)
\]

subject to

\[
L_j = \int e_j I_j(a,e) \, dF_N(a,e) \quad \forall j
\]

\[
\sum_j I_j(a,e) = 1 \quad \forall (a,e),
\]

\[
I_j(a,e) \geq 0 \quad \forall j, \forall (a,e).
\]

It is straightforward to see that the first-stage problem in the Generalized Spatial Competitive Economy yields

\[
X_{ij}(\{p_{oj}\}_o) = X_{ij}^N(\{p_{oj}\}_o) \quad \text{and} \quad P_j(\{p_{oj}\}_o) = P_j^N(\{p_{oj}\}_o).
\]

Also, the second-stage problem in the Generalized Spatial Competitive Economy yields a labor supply function that solves

\[
\{\Phi_j(\{\rho_j \nu_j \omega_j\}_j)\}_j = \arg \max_{\{I_j(a,e)\}_j(a,e)} \int \sum_j (a_j + \rho_j \nu_j \omega_j e_j) I_j(a,e) \, dF_N(a,e)
\]

subject to

\[
\forall (a,e) : \sum_j I_j(a,e) = 1, \quad \text{and} \quad I_j(a,e) \geq 0.
\]

To solve this problem, we substitute the first constraint into the objective function to eliminate \( I_o(a,e) \) for an arbitrary \( o \). Then, we consider the problem's Lagrangian:

\[
\max_{\{I_j(a,e) \geq 0\}_j \neq o} \int (a_o + \rho_o \nu_o \omega_o e_o) \, dF(a,e) + \int \sum_j (a_j + \rho_j \nu_j \omega_j e_j - a_o - \rho_o \nu_o \omega_o e_o) I_j(a,e) \, dF_N(a,e).
\]
The first-order condition of this problem implies that, for all \( j \neq o \), \( I_j(a, e) = 0 \) if \( a_o + \rho_o \nu_j \omega_o e_o > a_j + \rho_j \nu_j \omega_j e_j \). Thus, \( I_o(a, e) = 1 \) if, and only if, \( a_o + \rho_o \nu_j \omega_o e_o \geq a_j + \rho_j \nu_j \omega_j e_j \). Since \( o \) was arbitrarily chosen, we can write

\[
\forall i : I_i^*(a, e) = 1 \Leftrightarrow (a, e) \in I_j (\{ \rho_i \nu_i \omega_i \}) \equiv \{(a, e) : a_j + \rho_j \nu_j \omega_j e_j \geq a_o + \rho_o \nu_o \omega_o e_o \ \forall o \}.
\]

Thus, the system of labor supply constraints implies that

\[
\Phi_j (\{ \rho_i \nu_i \omega_i \}) = \int_{I_j (\{ \rho_i \nu_i \omega_i \})} e_j dF^N (a, e),
\]

and, therefore,

\[
\Phi_j (\{ \rho_i \nu_i \omega_i \}) = \Phi_j^N (\{ \rho_i \nu_i \omega_i \}).
\]

### D.1.5 Spatial Assignment Models with Other Factors in Production

**Environment.** Consider an economy with a representative household with the preferences in (1)–(2) subject to the budget constraint in (3). We denote an origin sector-region pair as \( i \equiv (k, r) \) and a destination sector-region pair as \( j \equiv (s, d) \). We impose additional restrictions on preferences to obtain the equivalence result. First, assume that individuals employed in all sectors of region \( r \) have identical preferences, \( V_{sd}(\cdot) = V_d(\cdot) \), and face identical prices, \( p_{kr,sd} = p_{kr,d} \). Second, assume that preferences are such that the labor supply function is invertible (up to a scalar). Third, we restrict attention to counterfactual exercises in which \( \nu_j \) remains constant and, therefore, \( \nu_j \) can be normalized to one.

In sector-region pair, there is a representative competitive firm that uses labor, \( L_{kr}^N \), and another factor, \( T_{kr}^N \), in production, with the following Cobb-Douglas production function:

\[
Y_{kr}^N = \zeta_{kr} \tilde{\Psi}_{kr}^N \left( \{ L_{sd}^N \}_{sd} \right) \left( L_{kr}^N \right)^{\alpha_{kr}^N} \left( T_{kr}^N \right)^{1-\alpha_{kr}^N}.
\]  (93)

Each region \( r \) has an endowment of the other factor, \( \tilde{T}_r^N \). We assume that the other factor is mobile across sectors within a region, but that it is immobile across regions – like land in spatial models. Similar to Caliendo et al. (2014), there is a national mutual fund that owns the other factor in all regions. We assume that the local government in region \( r \) owns a share \( \kappa_r \) of the national fund, and it transfers all dividends to local residents. In particular, we impose that the transfer rate to individuals employed in sector \( k \) of region \( r \), \( \rho_{kr}^N \), is inversely proportional to the share of labor in the total cost of the sector,

\[
\rho_{kr}^N = \rho_r^N / \alpha_{kr}^N.
\]  (49)

**Equilibrium.** To characterize the equilibrium, it is useful to work with the adjusted wage rate, \( \tilde{w}_{kr}^N \equiv w_{kr}^N / \alpha_{kr}^N \). The representative household’s cost minimization problem yields spending share and price indices that are given by, for all \( s \),

\[
x_{kr, sd}^N \in X_{kr, sd} \left( \{ p_{kr, sd}^N \} \right) = X_{kr, d} \left( \{ p_{kr, d}^N \} \right) \quad \text{and} \quad P_{sd}^N = p_{sd} \left( \{ p_{kr, sd}^N \} \right) = p_d \left( \{ p_{kr, dd}^N \} \right).
\]  (95)

As in Section (2), the utility maximization problem of the representative household yields the labor supply function. Using the transfer rule in (49), the labor supply in \( j \) is

\[
L_{sd} \in \Phi_{sd} \left( \{ \rho_r^N \tilde{w}_{kr}^N \} \right).
\]  (66)
Thus, the optimization of consumption and labor choice is corresponds directly to the one of the Generalized Economy.

In addition, the profit maximization problem of firms implies that

\[ p^N_{kr, sd} = \frac{T^N_{kr, sd}}{\zeta_{kr}} p^N_{kr} \]

where

\[ p^N_{kr} = \frac{\tilde{w}^N_{kr}}{\Psi^N_{kr} (\{ L^N_{sd} \}_{sd})} \cdot \left( \frac{R^N_{kr}}{\tilde{w}^N_{kr}} \right)^{1-\alpha^N_{kr}} \]

where \( R^N_{kr} \) is the price of other factor faced by the producer in sector \( k \) of region \( r \), and, abusing notation, \( \zeta_{kr} \equiv \zeta_{kr} (1-\alpha^N_{kr})^{(1-\alpha^N_{kr})} \).

To obtain the equilibrium level of \( R^N_{kr} \), consider the market clearing condition for the other factor in region \( r \):

\[ \bar{T}^N_r = \sum_k T^N_{kr} = \sum_k (1-\alpha^N_{kr}) \tilde{w}^N_{kr} L^N_{kr} / R^N_{kr}. \]

Since the other factor is perfectly mobile across sectors, \( R^N_{kr} = R^N_r \) for all \( k \) and, therefore,

\[ R^N_r = \frac{\sum_k (1-\alpha^N_{kr}) \tilde{w}^N_{kr} L^N_{kr}}{\bar{T}^N_r} \]

We use this expression to eliminate \( R^N_{kr} \) in the expression of \( p^N_{kr, kr} \) for sector \( k \) in region \( r \). After some manipulation, we obtain

\[ p^N_{kr, kr} = \frac{\tilde{w}^N_{kr}}{\zeta_{kr} \Psi^N_{kr} (\{ L^N_{sd} \}_{sd})} \left( \frac{1}{\bar{T}^N_r} \sum_s (1-\alpha^N_{sr}) \frac{\rho^N_{sr} \omega^N_{sr}}{\rho^N_{kr} \omega^N_{kr}} L^N_{sr} \right)^{1-\alpha^N_{kr}} \]

Thus, the invertibility of the labor supply function yields

\[ p^N_{kr, kr} = \frac{\tilde{w}^N_{kr}}{\zeta_{kr} \Psi^N_{kr} (\{ L^N_{sd} \}_{sd})} \]

\[ p^N_{kr} = \frac{\tilde{w}^N_{kr}}{\zeta_{kr} \Psi^N_{kr} (\{ L^N_{sd} \}_{sd})} \]

(97)

with

\[ \Psi^N_{kr} (\{ L^N_{sd} \}_{sd}) \equiv \tilde{\Psi}^N_{kr} (\{ L^N_{sd} \}_{sd}) \left( \frac{1}{\bar{T}^N_r} \sum_s (1-\alpha^N_{sr}) \Phi^{-1}_{kr, sr} (\{ L^N_{sd} \}_{sd}) L^N_{sr} \right)^{\alpha^N_{kr}-1}. \]

(98)

where we used invertibility of labor supply up to a scalar to write

\[ \frac{\rho^N_{sr} \omega^N_{sr}}{\rho^N_{kr} \omega^N_{kr}} = \Phi^{-1}_{kr, sr} (\{ L^N_{sd} \}_{sd}). \]

To close the equilibrium, we consider the labor market clearing condition that can be written in terms of the revenue share accruing to labor in every sector-region pair:

\[ \tilde{w}^N_{kr} L^N_{kr} = \sum_{sd} x^N_{kr, sd} \rho^N_{sd} \bar{w}^N_{sd} L^N_{sd}. \]

(99)

Finally, the transfer rate in region \( r \) is determined by its share in the dividend paid by the
mutual fund:

\[ \kappa_r \sum_{sd} (1 - \alpha_{sd}^N) \tilde{w}_{sd}^N L_{sd}^N = \sum_k (\rho_{kr}^N - 1) \alpha_{kr} \tilde{w}_{kr}^N L_{kr}^N = \sum_k (\rho_{kr}^N - \alpha_{kr}) \tilde{w}_{kr}^N L_{kr}^N \]

\[ \rho_r^N = \frac{\kappa_r \sum_{sd} (1 - \alpha_{sd}^N) \tilde{w}_{sd}^N L_{sd}^N + \sum_k \alpha_{kr} \tilde{w}_{kr}^N L_{kr}^N}{\sum_k \tilde{w}_{kr}^N L_{kr}^N} \quad (100) \]

where the left hand side is region \( r \)'s total transfer payments, and the right hand side is region \( r \)'s share in the total land revenue in the country.

The equilibrium of this economy is characterized by \( \{ p_i^N, w_i^N, L_i^N, P_i^N \} \) that solve equations (95)–(99), with \( \{ \Phi_j(\cdot), \Psi_{kr}(\cdot), X_{ij}(\cdot) \} \), conditional on the transfer rule \( \{ \rho_r \} \) in (100).

**Equivalence.** To establish the equivalence, we consider the Generalized Spatial Competitive Economy of Section 2, with \( \Psi_{kr}(\cdot) = \Psi_{kr}^N(\cdot) \) in (98) and the transfer rule in (100). This establishes that the Generalized Spatial Competitive Economy is equivalent to spatial assignment models with other factors of production that are mobile across sectors but not across regions – e.g., land and other natural resources. A similar argument yields the equivalence with models with other factors of production that are mobile across both regions and sectors. The only restriction is that the invertibility step to obtain (98) requires the same transfer rate across markets in the country, as in Caliendo et al. (2014).

### D.2 Extensions

#### D.2.1 Generalized Spatial Competitive Economy with Multiple Labor Types

**Multiple Worker Types.** Consider the Generalized Spatial Competitive Economy with multiple worker groups – groups are indexed by \( g \) and \( g' \). We write the equilibrium in terms of factor-content of trade as in Adao, Costinot, and Donaldson (2017). Each market now is defined as a triple of sector-region-group. We denote origin markets as \( i \equiv (k, r, g) \), and destination markets as \( j \equiv (s, d, g') \). As before, the representative consumer has preferences over consumption and labor across markets (i.e., sector-region-group markers):

\[ U_c \left( \{ \nu_j C_j \}, \{ L_j \} \right). \]

We assume that the consumption index depends on the factor content of trade from different sectors and regions. That is, the consumption index depends directly on a composite good produced by each sector-region-group triple:

\[ C_j = V_j \left( \{ c_{ij} \}_i \right) \]

Finally, assume that there is a competitive firm producing the market-level composite good with production function given by

\[ Y_{ij} = \frac{\zeta_i \Psi_i \{ L_o \}_o}{\tau_{ij}} L_{ij}. \]

All our results remain valid in this environment with spending shares in terms of factor content of trade. That is, \( x_{ij} \) is the spending share on the composite good produced in the sector-region-group triple.

**Equivalent Armington Economy Multiple Worker Types.** To gain intuition for this
economy, we now derive preferences in terms of factor content of trade in the case of an Armington
economy with multiple labor types. Assume that the representative household has preferences over
the allocation of the multiple worker groups across sector-region pairs \( j \),

\[
U_c \left( \{ \nu_j^g C_j^g \}_{j,g}, \{ L_j^g \}_{j,g} \right).
\]

The consumption index is a function of the quantities consumed of goods produced in different
origin sector-region pairs \( i \):

\[
C_j^g = \tilde{V}_j \left( \{ \gamma_i^g \}_{i} \right).
\]

Assume that each sector-region pair \( i \) has a representative firm that combines labor from different
worker types with a constant returns to scale technology:

\[
Y_{ij} = F_{ij} \left[ \left\{ \frac{\zeta_j^g}{\tau_{ij}} \Psi_i^g \left( \{ L_j^g \}_{j,g} \right) \right\} \right] L_{ij}^g,
\]

where \( F_{ij}(\cdot) \) is homogeneous of degree one.

Thus, as in the equivalence with the Ricardian economy above, we must define preferences of
the representative agent that incorporate the technology to produce final goods,

\[
C_j = V_j \left( \{ L_{ij}^g \}_{i,g} \right) \equiv V_j^N \left( \{ F_{ij} \left( \{ L_j^g \}_{j,g} \right) \}_{i} \right)
\]

and define the production technology of the composite good of a sector-region-group to be

\[
Y_{ij}^g = \frac{\zeta_i \Psi_i \left( \{ L_j^g \}_{j,g} \right)}{\tau_{ij}} L_{ij}^g.
\]

In equilibrium, the price of the sector-region-group is

\[
p_{ij}^g = \tau_{ij}^g \frac{w_i^g}{\zeta_i^g \Psi_i^g \left( \{ L_j^g \}_{j,g} \right)}
\]

In this case, the spending share on factor \( g \) in sector-region pair \( i \) is simply

\[
x_{ij}^g = \alpha_i^g \left( \{ p_{ij}^g \}_{j,g} \right) x_{ij}
\]

where \( \alpha_i^g \left( \{ p_{ij}^g \}_{j,g} \right) \) is the share of factor \( g \) in the production cost of sector-region pair \( i \), and \( x_{ij} \) is
the spending share on goods from sector-region pair \( i \).
D.2.2 Generalized Spatial Competitive Economy with Intermediate Goods in Production

Assume that production uses labor and other intermediate goods according to the following production function:

\[ Y_{ij} = \frac{\zeta_i}{\tau_{ij}} \Psi_i \left( \{L_j\}_j \right) F_i \left( L_{ij}, \tilde{F}_i \left( \{M_{ij}\}_i \right) \right) \]

where \( F_i(.) \) and \( \tilde{F}_i(.) \) are homogeneous of degree one, and \( M_{ij} \) is the quantity of good \( l \) used in production of goods shipped from \( i \) to \( j \).

This production function covers multiple sector gravity models reviewed in Costinot and Rodriguez-Clare (2013). In this class of models, final preferences and production imply identical spending shares across origins within a sector. However, sector-level spending shares differ across industries and final consumption. In quantitative models, it is typically assumed that sector-level spending shares are constant, and identical to those in Input-Output Tables – for example, see Caliendo and Parro (2014).

Let \( \tilde{P}_i \) be the producer price index in region-sector pair \( i \):

\[ \tilde{P}_i \left( \{p_{li}\}_l \right) \equiv \min_{M_{li}} \sum p_{li} M_{li} \quad \text{s.t.} \quad \tilde{F}_i \left( \{M_{li}\}_i \right) = 1 \]

and the input shares are given by

\[ \pi_{li} = \pi_{li} \left( \{p_{li}\}_l \right) \equiv \frac{\partial \ln \tilde{P}_i \left( \{p_{li}\}_l \right)}{\partial \ln p_{li}}. \]

The cost minimization problem in market \( i \) is

\[ C_i \left( w_i, \tilde{P}_i \right) \equiv \min_{L, F} w_i L + \tilde{P}_i \tilde{F} \quad \text{s.t.} \quad F_i \left( L, \tilde{F} \right) = 1, \]

implying that

\[ p_i = \frac{C_i \left( w_i, \tilde{P}_i \right)}{\Psi_i \left( \{L_j\}_j \right)} \]

Notice that \( C_i(.) \) homogeneous of degree one, with associated labor spending shares given by

\[ \alpha_i \left( w_i, \tilde{P}_i \right) \equiv \frac{\partial \ln C_i \left( w_i, \tilde{P}_i \right)}{\partial \ln w_i} \]

Thus, we can write the profit maximization condition as

\[ p_{ij} = \frac{\tau_{ij}}{\zeta_i} p_i \]

\[ \frac{p_i}{\tilde{P}_i} \left[ C_i \left( 1, \tilde{P}_i \left( \{p_{li}\}_l \right) \right) \right]^{-1} = \frac{\omega_i}{\zeta_i \Psi_i \left( \{L_j\}_j \right)} \quad (101) \]

To close the model, consider the market clearing condition for labor in each market. The total revenue of market \( i \) from sales in market \( j \) is
\[ X_{ij} = x_{ij} \rho_j w_j L_j + \pi_{ij} (1 - \alpha_j) \sum_d X_{jd} \]

\[ X_{ij} = x_{ij} \rho_j w_j L_j + \pi_{ij} \frac{1 - \alpha_j}{\alpha_j} w_j L_j \]

\[ X_{ij} = \left( x_{ij} \rho_j + \pi_{ij} \frac{1 - \alpha_j}{\alpha_j} \right) w_j L_j \]

Thus,

\[ w_i L_i = \sum_j \alpha_i \left( x_{ij} \rho_j + \pi_{ii} \{p_{ii}\} \left( \frac{1}{\alpha_i \left( w_i, \hat{P}_t \{p_{ii}\} \right)} - 1 \right) \right) w_j L_j. \quad (102) \]

Notice that the equilibrium requires knowledge of the cost function, \( C_i(\cdot) \), the labor share function, \( \alpha_i(\cdot) \), the PPI function \( \hat{P}_t(\cdot) \), and the associated input share functions, \( \pi_{ii}(\cdot) \). The equilibrium entails \( \{w_i, P_j, L_i, p_i\} \) that satisfy (5)–(8) and (101)–(102).

### D.2.3 Generalized Spatial Competitive Economy with Commuting

We now define a Generalized Spatial Competitive Economy with Commuting between markets. We assume that the representative household has preferences over consumption and labor for individuals residing in market \( j \) and commuting to market \( d \):

\[ U \left( \{v_{jd}C_{jd}\}_{j,d}, \{L_{jd}\}_{j,d} \right) \]

where \( L_{jd} \) is the mass of workers residing in market \( j \) and working on market \( d \), and \( C_{jd} \) denoting the associated consumption index of these workers.

We assume that individuals consume in their market of residence. For labor in market \( j \) commuting to \( d \), the homothetic consumption index is

\[ C_{jd} = V_j \{c_{jd}\}_i \]

and the budget constraint is

\[ \sum_i p_{ij} c_{ijd} = w_d L_{jd}. \]

As in the baseline model, the first-stage problem yields the price index and the spending shares, \( P_j \{p_{ij}\}_i \) and \( X_{ij} \{p_{oj}\}_o \). \quad (103)

Notice that, because \( V_j(\cdot) \) does not vary with the commuting destination, the price index and the spending shares do not vary with the commuting destination. This implies that

\[ \sum_i p_{ij} c_{ijd} = P_j C_{jd} \Rightarrow C_{jd} = \frac{w_d}{P_j} L_{jd} = \omega_{jd} L_{jd} \]

where \( \omega_{jd} = w_d/P_j \) is the real wage of working in market \( d \) and residing in market \( j \).

Thus, the second-stage problem is

\[ \max_{\{L_{jd}\}_{j,d}} U \left( \{(v_{jd}\omega_{jd}) L_{jd}\}_{j,d}, \{L_{jd}\}_{j,d} \right) \]

95
which yields the labor supply mapping,

\[ L_{jd} \in \Phi_{jd}(\{\nu_{oi}\omega_{oi}\}_{oi}). \] (104)

As in the baseline model, we consider the profit maximization problem of firms in market \( i \) yields the same equilibrium conditions

\[ p_{ij} = \frac{\tau_{ij}p_i}{\zeta_i}, \] (105)

\[ p_i = \frac{w_i}{\Psi_i(\{L_{jd}\}_{j,d})}. \] (106)

To close the model, we consider the labor market clearing condition: total labor payments to labor in market \( i \) equals total revenue of market \( i \) from selling to all other markets in the world economy. That is,

\[ \sum_o w_i L_{oi} = \sum_j x_{ij} \left( \sum_d w_d L_{jd} \right). \] (107)

The competitive equilibrium in this economy corresponds to \( \{p_i, w_i, P_j, L_{ij}\} \) such that conditions (103)–(107) hold. In this case, we need to extend the notion of labor supply to capture commuting flows across markets. In other words, counterfactual predictions require knowledge of the extended labor supply mapping with between-market worker commuting flows, \( L_{jd} \in \Phi_{jd}(\{\nu_{oi}\omega_{oi}\}_{oi}) \).

**D.2.4 Spatial Generalized Competitive Economy with Selection in Production**

We next consider a version of the Melitz (2003) monopolistic competition model with firm heterogeneity and endogenous entry, augmented with Pareto distributed productivities, as in Helpman et al. (2004), Chaney (2008) and Arkolakis et al. (2008).

**Preferences.** We assume that the utility is given by

\[ U_c(\{\nu_j C_j\}_{j \in J_c}, \{L_j\}_{j \in J_c}) \]

where \( C_j \) is given by a Constant Elasticity of Substitution (CES) aggregator over a continuum of goods \( z \):

\[ C_j \equiv \left( \sum_i \int_{z \in Z_{ij}} (c(z))^{\sigma-1} \, dz \right)^{\frac{1}{\sigma-1}}, \] (108)

\( c_{ij}(z) \) is the consumption of good \( \omega \) produced in country \( i \) and sold in country \( j \), and \( \sigma \) is the elasticity of substitution between goods. Using the necessary first order conditions associated with the maximization problem, it is straightforward to show that the optimal choices for consumption and labor are, respectively,

\[ c_{ij}(z) = \frac{p_{ij}(z)^{-\sigma}}{P_j^{1-\sigma} \nu_j \rho_j w_j L_j}, \] (109)
\[ L_j = \Phi_j \left( \{ \nu, \rho, \omega \}_{i \in J} \right), \]  

where \( \omega_j \equiv w_j/P_j \) is the real wage in market \( j \), and \( P_j \) is the CES price index:

\[ P_j = \left( \sum_i \int_{z \in Z_{ij}} p_{ij}(z)^{1-\sigma} dz \right)^{\frac{1}{1-\sigma}}. \]

**Production.** Each firm can choose to pay an entry cost \( f_e \), and upon entry learns its productivity \( z \) from a pre-specified distribution. We denote the measure of equilibrium entrants as \( J_i \). In order to export the firm has to incur iceberg costs, as before, but also an additional fixed exporting cost \( f_{ij} \), in terms of foreign labor. We assume the same agglomeration function as in baseline, and thus the optimal price is a constant markup over the marginal cost:

\[ p_{ij}(z) = \frac{\sigma}{\sigma - 1} \frac{\tau_{ij} w_i}{z \zeta_i \Psi_i^M(L)}. \]

This result implies that bilateral sales of a firm \( z \) from country \( i \) to \( j \) are

\[ y_{ij}(z) = \left( \frac{\sigma}{\sigma - 1} \frac{\tau_{ij} w_i}{z \zeta_i \Psi_i^M(L)} \right)^{1-\sigma} \frac{w_j L_j}{P_j^{1-\sigma}}, \]

and profits are a share \( 1/\sigma \) of sales. If we equate marginal profit to fixed costs and substitute for the optimal labor choice, we can solve for the threshold productivity cutoff of entry for firms from country \( i \) selling to country \( j \):

\[ \left( \frac{z_{ij}^*}{z_{ij}} \right)^{\sigma - 1} = \frac{f_{ij} \sigma (P_j)^{(1-\sigma)}}{\left( \frac{\sigma}{\sigma - 1} \frac{\tau_{ij} w_i}{z \zeta_i \Psi_i^M(L)} \right)^{1-\sigma} L_j}. \]

**Trade equilibrium.** We specify the distribution of firm productivities following Helpman et al. (2004), Chaney (2008) and Arkolakis et al. (2008). In particular, we assume that the productivity of firms in country \( i \) is drawn from a Pareto distribution with density:

\[ g(z) = \theta z^{-\theta - 1}, \quad z \geq 0 \]

with shape parameter \( \theta > 0 \). In this market setting the price index is:

\[ P_j^{1-\sigma} = \sum_i J_i \int_{z_{ij}}^\infty \left( \frac{\sigma}{\sigma - 1} \frac{\tau_{ij} w_i}{z \zeta_i \Psi_i^M(L)} \right)^{1-\sigma} \theta z^{-\theta - 1} dz. \]

\[ P_j^{1-\sigma} = \sum_i J_i \gamma \left( \frac{\tau_{ij} w_i}{z \zeta_i \Psi_i^M(L)} \right)^{1-\sigma} (z_{ij}^*)^{-\theta - 1 + \sigma}, \]

97
where $\gamma \equiv \left( \frac{\sigma}{\sigma - 1} \right)^{1 - \sigma} \frac{\theta}{\theta - \sigma + 1}$. Expected profits in equilibrium are zero so that

$$
\sum_j \int_{z_{ij}^*}^{\infty} \left( \frac{\sigma}{\sigma - 1} \frac{\tau_{ij} w_i}{\zeta_i \Psi_i^M(L)} \right)^{1 - \sigma} \frac{w_j L_j}{\sigma P_j^{1 - \sigma}} \theta z_{ij}^{\sigma - \theta - 2} dz - \sum_j \int_{z_{ij}^*}^{\infty} w_{ij} \theta z_{ij}^{\sigma - \theta - 1} dz = w_i f_e.
$$

which can be simplified as:

$$
\frac{\sigma - 1}{\theta - \sigma + 1} \sum_j w_{ij} \left( z_{ij}^* \right)^{-\theta} = w_i f_e. \quad (117)
$$

The labor market clearing condition is:

$$
J_i \sum_j \int_{z_{ij}^*}^{\infty} \tau_{ij} \left( \frac{\sigma}{\sigma - 1} \frac{\tau_{ij} w_i}{\zeta_i \Psi_i^M(L)} \right)^{-\sigma} \frac{w_j L_j}{P_j^{1 - \sigma}} \theta z_{ij}^{\sigma - \theta - 2} dz + J_i f_e + \sum_j J_j \int_{z_{ji}^*}^{\infty} f_{ji} \theta z_{ji}^{\sigma - \theta - 1} dz = L_i,
$$

which we can simplify as:

$$
J_i \frac{\theta (\sigma - 1)}{\theta - \sigma + 1} \zeta_i \Psi_i^M(L) \sum_j w_{ij} \left( z_{ij}^* \right)^{-\theta} + J_i f_e + \sum_j J_j \sum_j f_{ji} \left( z_{ji}^* \right)^{-\theta} = L_i. \quad (118)
$$

Using the equations for free-entry, labor market clearing, and price index, it is easy to show that the number of entrants equals:

$$
J_i = \Psi_i^E(L) \equiv \frac{\left( \theta + 1 \right) (\sigma - 1)}{\theta \sigma f_e} \frac{L_i}{1 + \theta \zeta_i \Psi_i^M(L)}.
$$

(119)

Combining expressions (114), (116) and (119), we get that

$$
x_{ij} = \left( \frac{P_{ij}}{P_j} \right)^{-\theta} = \frac{P_{ij}^{-\theta}}{\sum_i P_{ij}^{-\theta}}
$$

such that

$$
P_{ij} = \tilde{\tau}_{ij} \frac{w_i}{\zeta_i \Psi_i^E(L)} \left( L_j \right)^{-\rho} \Psi_i(L) \equiv \left( \sigma^\sigma (\sigma - 1) \right)^{\frac{\sigma - 1}{\sigma + 1}} \left( \Psi_i^E(L) \gamma \right)^{\frac{1}{\sigma}} \Psi_i^M(L)
$$

with $\tilde{\tau}_{ij} \equiv \tau_{ij} \left( f_{ij} \right)^{\frac{\sigma - 1}{\sigma}} \left( \theta + 1 - \sigma \right)$, and $\rho \equiv \frac{1}{\theta} \frac{\theta + 1 - \sigma}{\sigma - 1}$.

In this model, the bilateral price index depends on employment in the destination market due to the selection forces embedded in the endogenous entry decision of heterogeneous firms. Thus, counterfactual analysis requires knowledge of the parameter $\rho$ that controls the effect
of destination market size on bilateral trade flows. This parameter can be calibrated using estimates of the trade elasticity, $\theta$, and the elasticity of substitution, $\sigma$. 