Optimal Unilateral Carbon Policy*

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Abstract

Carbon policies differ by region, potentially generating inefficient shifts in location, an effect known as leakage. To understand the effects of differential carbon prices, we derive the optimal unilateral policy where one region of the world imposes a carbon tax and the rest of the world does not. The optimal policy can be implemented through (i) a nominal tax on extraction at a rate equal to the global marginal harm from emissions, (ii) a border adjustment on imports and exports of energy and on imports, but not exports, of goods, at a lower rate than the extraction tax rate, and (iii) an export subsidy designed to expand the export margin. By combining supply side and demand side taxes, the optimal policy controls leakage by controlling the price of energy. It exploits international trade to expand the reach of the climate policy. We calibrate and simulate the model to illustrate how the optimal policy compares to more traditional policies such as extraction, production and consumption taxes and combinations of those taxes.

Keywords: carbon taxes, border adjustments, leakage, climate change
JEL Codes: F18, H23, Q54

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1 Introduction

Global negotiations have given up trying to achieve a uniform approach to climate change, such as a harmonized global carbon tax. Instead, current negotiations focus on achieving uniform participation, with each country pursuing its own approach and its own level of emissions reductions. As a result, policies to control emissions of greenhouse gases vary widely by country, and are likely to continue to do so for the indefinite future.

Widely varying carbon policies potentially affect the location of activities such as extraction, production, and consumption, the effectiveness of the policies, and the welfare of people in various countries or regions. These effects are of critical importance to the design of carbon policy and to its political feasibility. For example, trade and location effects were central to the design of the European Union Emissions Trading System, the Regional Greenhouse Gas Initiative, and California's carbon pricing system. One of the reasons that the United States did not ratify the Kyoto Protocol was concern about the lack of emissions policies in developing countries and the resulting trade effects. Unless concerns about the effects of differential carbon prices are addressed, it may be difficult to achieve significant reductions in global emissions.

To address this problem, we develop an analytic general equilibrium model of international trade, where one region (Home) imposes a carbon policy and the rest of the world (Foreign) does not. The model stacks Markusen (1975) and Dornbusch, Fisher, and Samuelson (1977; henceforth DFS). We interpret Markusen as a model of extraction and trade in fossil fuels, with DFS bringing in production, trade, and consumption of goods produced with fossil fuels. Following Böhringer, Lange, and Rutherford (2014), we restrict policies adopted by Home to those that do not make Foreign worse off. Our solution strategy borrows from Costinot, Donaldson, Vogel, and Werning (2015; henceforth CDVW).

A planner seeking to optimize Home's welfare balances three wedges: (i) the wedge between the planner's marginal valuation of a unit of energy that is extracted and the Foreign energy price (the *extraction wedge*); (ii) the wedge between the planner's marginal valuation of energy and the Foreign energy price (the *consumption wedge*); and (iii) the wedge between the shadow cost of Home's exports of goods to Foreign and marginal utility to consumers in Foreign of those goods (the *export wedge*).

Each of these wedges corresponds to an activity in Foreign that is not directly under the planner's control. The extraction wedge reflects changes in Foreign extraction due to changes in the price of energy, an effect we think of as extraction leakage. The consumption wedge reflects an increase in the energy intensity of goods consumed in Foreign, an effect sometimes referred to as the fuel price effect. The export wedge reflects a reduction in Home's exports of goods, an effect similar to leakage in its standard usage. The planner sets the optimal policy to balance the marginal costs of these responses.

The taxes and subsidies that generate this policy in a decentralized equilibrium match these wedges: a tax on domestic extraction equal to the extraction wedge, a tax on all domestic production and domestic consumption equal to the consumption wedge, and an export subsidy equal to the export wedge. These taxes and subsidies can be implemented via nominal taxes and border adjustments as follows: (i) a domestic carbon tax on the extraction of fossil fuels at the global marginal harm from emissions, i.e., at the full Pigouvian rate; (ii) a border tax on imports and a tax rebate for exports of fossil fuels, both at a rate equal to the consumption wedge (which we will call a "partial border adjustment" because it is at a lower rate than the underlying nominal extraction tax); (iii) a border tax on the energy content of imports at that same partial rate; and (iv) an export subsidy designed to expand low-carbon exports from Home to the rest of the world, set at the export wedge. While the nominal extraction tax is equal to the Pigouvian rate, the partial border adjustment removes some of that tax, leaving the effective extraction tax equal to (minus) the extraction wedge.

To compare the optimal policy to more conventional policies, such as extraction taxes, production taxes, consumption taxes, and combinations of these taxes, we solve the model when the planner is limited in the outcomes it can control. The planner's solution in each case follows the same logic as the optimal policy. The planner accounts for the wedges between the prices Foreign actors actually see and what they would optimally see. For example, an extraction tax increases the price of energy seen by Foreign extractors relative to the after-tax price received by Home extractors. As a result, Foreign extractors increase their extraction generating extraction leakage. Policies that include extraction taxes take this wedge, and the resulting behavioral response by Foreign extractors, into account. Policies that have more levers to control the prices seen by Foreign actors are able to achieve better outcomes. Most centrally, policies that combine taxes on both the supply and demand for energy are able to moderate the net effect on the price of energy seen by Foreign actors, and, as a result, tend to generate good outcomes.

To understand the quantitative implications of our analysis, we calibrate the model and solve it numerically for both the optimal policy and the various constrained policies. In our core calibration, we assume that the OECD countries impose a carbon price and the rest of the world does not. Following the intuition just described, policies that combine taxes on the supply and demand for fossil fuels perform well in our simulations, considerably outperforming the more standard taxes on emissions from domestic production and those taxes combined with border adjustments. As a result, pairwise combinations, particularly, the combination of an extraction tax and a production tax, may be desirable approaches for implementing a unilateral carbon policy.[1]

[1] This tax can be imposed with a nominal tax on extraction combined with border adjustments (at a lower rate) on the imports and exports of energy, but not goods. As suggested by Metcalf and Weisbach (2009), an extraction tax would be easy to impose because there are a relatively small number of large extractors who would need to remit taxes. Border adjustments on energy would also be easy to impose because imports and exports of energy are already carefully tracked. As a

Our core model does not include renewable energy, and stimulating renewables is often seen as a central goal of carbon pricing. To examine this issue, we extend the analysis to show that including renewables only requires modest adjustments to the optimal policy. Not surprisingly, renewables are exempt from the tax on extraction. If they can be sold in the market at the same price as fossil fuels, this exemption stimulates the production of renewables.

The paper proceeds as follows. The remainder of this section provides additional motivation and reviews the relevant literature. Section 2 lays out the basic elements of the model. Section 3 solves the problem of a planner designing an optimal carbon policy for one region with the other region behaving as in the competitive equilibrium. In Section 4 we derive a set of taxes and subsidies that implement the optimal policy. Section 5 derives the taxes that Home would impose if it is constrained to using simpler policies. We explore the quantitative implications of the optimal policy in Section 6, using a calibrated version of the model. Section 7 extends the analysis to include a renewable energy sector. Section 8 concludes.

1.1 Prior Literature

Because of its prominence, there is a voluminous prior literature studying this problem. The overwhelming majority of studies use computable general equilibrium models to simulate carbon taxes and border adjustments. By our count, there are over 50 CGE studies of the general problem of differential carbon prices in the peer-reviewed literature (and many more in the gray literature) and each study considers multiple different scenarios, which means that there are hundreds of simulations of the problem.[2] For example, Branger and Quirion (2014) perform a meta-analysis of 25 studies of differential carbon taxes (20 of which were CGE studies while 5 were partial equilibrium studies). These 25 studies, which make up only a portion of the literature, had 310 different modeled scenarios.

CGE studies almost uniformly use leakage as their measure of the effects of differential carbon prices. Leakage is commonly defined as the increase in emissions in non-taxing regions as a percentage of the reduction in emissions in the taxing region (Hence, 100% leakage means the policy is totally ineffective in reducing global emissions). Leakage estimates fall within a relatively consistent range. The Branger and Quirion meta-study finds leakage rates between 5% and 25% with a mean of 14% without border adjustments. With border adjustments, leakage ranges from −5% to 15%, with a mean of 6%. Similarly, as summarized by Böhringer et al. (2012), the Energy Modeling Forum commissioned 12 modeling groups to study the effects of border adjustments on leakage using a common data set and common set of scenarios.

result, the simulations suggest that the combination of an extraction tax and a production tax is a promising policy to explore. It is also likely that the extraction/production hybrid raises fewer concerns about WTO compatibility than do the optimal tax or conventional border adjustments imposed on goods.

[2] For surveys of the leakage literature, see Droge et al. (2009), Zhang (2012) and Metz et al. (2007)

They considered emissions prices in the Kyoto Protocol Annex B countries (roughly the OECD) that reduce global emissions by about 9.5%. Without border adjustments, leakage rates were in the range of 5% to 19% with a mean value of 12%. These studies find that border adjustments reduce leakage by about a third, with a range between 2% and 12% and a mean value of 8%. Elliott et al. (2013) replicated 19 prior studies within their own CGE model, finding leakage rates between 15% and 30% for a tax on Annex B countries that reduced global emissions by about 13%.[3]

Rather than a large CGE model, we use an analytic general equilibrium model of trade to study the problem. This approach allows us to uncover the underlying economic logic for why some policies perform better than others, as well as solve for the optimal policy. It means, however, that our quantitative analysis is more illustrative than definitive.

There are a number of studies that precede us in this approach. The classic study, which we build on, is Markusen (1975). Markusen analyzes a two-country, two-good model in which production of one of the goods generates pollution that harms both countries. Writing before climate change was a widespread concern, he considers a simple pollutant, such as the release of chemicals into Lake Erie by polluters in the United States, which harms Canada (as well as the United States). One of the countries imposes policies to address the pollution; the other is passive. Markusen finds that the optimal tax is a Pigouvian tax on the dirty good combined with a tariff (if the good is imported) or a subsidy (if it is exported). The optimal tariff or subsidy combines terms of trade considerations and considerations related to leakage and is generally lower than the Pigouvian tax.[4] Other analytic models of the problem include Fowlie and Reguant (2020), Böringher, Lange and Rutherford (2014), Holladay et al (2018), Hemous (2016), Baylis et al. (2014), Jakob, Marschinski and Hubler (2013), Fischer and Fox (2012, 2011), Fowlie (2009), and Hoel (1994). There are also a number of studies, including Fowlie et al (2016) that consider leakage in particular industries, such as the Portland cement industry.

2 Basic Model

Two countries, Home and Foreign, are endowed with energy deposits and with labor, L and L^*. The $*$ distinguishes Foreign from Home, whose carbon policy we seek to

[3] A smaller number of studies focus on the effects of carbon taxes on particular energy-intensive and trade-exposed sectors. For example, Fowlie et al. (2016) consider the effects of a carbon price on the Portland cement industry. They find that a carbon price has the potential to increase distortions associated with market power in that industry. Leakage compounds these costs. They find that border adjustments induce negative leakage because of how industry actors respond, and can generate significant welfare gains at high carbon prices.

[4] Hoel (1996) generalizes Markusen's analysis and produces similar results in the context of climate change and carbon taxes. He also considers the case where the country may not impose tariffs. In this case, the optimal policy will involve carbon taxes that vary by sector (even though the harms from emissions do not vary by sector).

optimize.

There are three sectors: energy e, goods g, and services s. Energy is extracted from deposits using labor, goods are produced by combining labor and energy, and services are provided with labor only. As in DFS, goods come in a continuum, indexed by $j \in [0, 1]$. Labor is perfectly mobile across the continuum of goods and across the three sectors within a country.[5]

2.1 Preferences

Home preferences are represented as:

$$U = C_s + \int_0^1 u(c_j)dj - \varphi Q_e^W. \tag{1}$$

Here C_s is consumption of services, c_j is consumption of good j, φ is the marginal harm from global emissions, and Q_e^W is global energy extraction.[6] We impose the functional form:

$$u(c) = \eta^{1/\sigma} \frac{c^{1-1/\sigma} - 1}{1 - 1/\sigma},$$

where η governs demand for goods relative to services and σ is the elasticity of substitution between goods. We denote marginal utility as $u'(c) = (\eta/c)^{1/\sigma}$. Defining an index of goods consumption:

$$C_g = \left(\int_0^1 c_j^{(\sigma-1)/\sigma} dj \right)^{\sigma/(\sigma-1)},$$

we get $U = C_s + u(C_g) - \varphi Q_e^W$. Foreign preferences are the same except with η^* and σ^* (hence u^*) and φ^*.[7]

2.2 Technology

Energy is deposited in a continuum of fields, characterized by different costs of extraction. The quantity of energy that can be extracted at a unit labor requirement below a is given by $E(a)$ in Home ($a \geq \underline{a}$) and $E^*(a)$ in Foreign ($a \geq \underline{a}^*$).[8] The

[5] What we call "labor" can be interpreted as a combination of labor and capital used to extract energy, produce goods, and provide services.

[6] Prior to introducing multiple energy sources, including renewables, in Section 7, we equate energy with a homogeneous fossil fuel measured by its carbon content.

[7] We follow Grossman and Helpman (1994) in adopting quasi-linear preferences, which greatly simplifies the analysis. To ensure that the marginal utility of income is 1 we assume $C_s > 0$ and $C_s^* > 0$, a condition which is easily checked. If $\sigma = 1$ preferences simplify to:

$$U = C_s + \eta \ln C_g + \varphi^W Q_e^W = C_s + \eta \int_0^1 \ln c_j dj + \varphi^W Q_e^W$$

.

[8] We assume $E(\underline{a}) = E^*(\underline{a}^*) = 0$, with the derivatives strictly positive above these lower bounds: $E'(a) > 0$ and $E^{*\prime}(a) > 0$.

minimum labor L_e required by Home to extract a quantity of energy $Q_e = E(\bar{a})$ is:

$$L_e = \int_{\underline{a}}^{\bar{a}} a\, E'(a)da. \tag{2}$$

Foreign extraction, Q_e^*, is determined in the same way, with $Q_e + Q_e^* = Q_e^W$. The output of the energy sector is used as an intermediate input by the goods sector.

Goods $j \in [0, 1]$ are produced with input requirement a_j in Home using a Cobb-Douglas combination of labor and energy:

$$q_j = \frac{1}{\nu a_j} L_j^{\alpha} E_j^{1-\alpha}, \tag{3}$$

where L_j is the labor input, E_j is the energy input, $0 < \alpha < 1$ is the output elasticity of labor, and $\nu = \alpha^{\alpha} (1-\alpha)^{1-\alpha}$. The production function in Foreign is the same, but with a_j^* in place of a_j.[9]

Services, in quantitites Q_s and Q_s^*, are provided in both countries with a unit labor requirement. We take services to be the numéraire, with price 1.[10]

2.3 International Trade

We assume that energy and services are costlessly traded between Home and Foreign, with the relative price of energy denoted by p_e. This price will dictate outcomes in Foreign within the planning problem that we consider below. We will also consider an unfettered competitive equilibrium of the model, which we call the *business as usual* (BAU) baseline. We choose units of energy so that $p_e = 1$ clears the global energy market in the BAU baseline. Hence, we can interpret the parameter $\varphi^W = \varphi + \varphi^*$ as the marginal global harm from combusting a unit of fossil fuel relative to its value in a competitive-equilibrium.

Trade in the continuum of manufactured goods follows DFS. Goods are ordered by Home comparative advantage:

$$\frac{a_j^*}{a_j} = F(j), \tag{4}$$

with $F(j)$ assumed to be a strictly decreasing continuous function.[11] Goods trade incurs iceberg costs $\tau \geq 1$ on Home exports and $\tau^* \geq 1$ on Home imports . The total input requirement for Home to supply good j to Foreign is thus τa_j and for Foreign to supply good j to Home $\tau^* a_j^*$.

[9]In line with our Ricardian assumptions, we treat α as common across goods and countries. Including the constant ν in the production function simplifies expressions for costs that will appear later. This technology is nearly identical to the production and pollution technology in Shapiro and Walker (2018), although α here is $1-\alpha$ there. They use it to assess the reduction of air pollution in US manufacturing from 1990-2008.

[10]We will assume that $Q_s^* > 0$ so that, given the unit labor requirement for services, the wage in Foreign is $w^* = 1$. This outcome is guaranteed with a large enough labor endowment in Foreign.

[11]In order to have well defined integrals in what follows, we also assume that a_j and a_j^* can be treated as continuous functions of j.

2.4 Labor and Energy Requirements

We introduce notation for energy and labor input requirements that will be used throughout the rest of the paper. At energy intensity, $z_j = E_j/L_j$, we can invert Home's production function (3) to get the unit energy requirement for good j:

$$e_j(z_j) = \nu a_j z_j^{\alpha}, \tag{5}$$

with corresponding unit labor requirement $l_j(z_j) = e_j(z_j)/z_j$. Unit energy and labor requirements in Foreign, $e_j^*(z_j)$ and $l_j^*(z_j)$, are defined in the same way but with a_j^* in place of a_j.[12]

So as not to constrain the optimal policy, the energy intensity for good j may depend not only on where the good is produced but also on where it is shipped. For each good j we distinguish between Home exports, $x_j \geq 0$ and Home production for consumption there, $y_j = q_j - \tau x_j \geq 0$. We also distinguish between Home imports, $m_j \geq 0$ and Foreign production for consumption in Foreign, $y_j^* = q_j^* - \tau^* m_j \geq 0$. (Note that we define exports and imports in terms of the quantity that reaches the destination.) For each good j we allow for the possibility of four different energy intensities z_j^y, z_j^x,z_j^m, and z_j^*, one for each of the four lines of production y_j, x_j, m_j, and y_j^*.[13]

2.5 Carbon Accounting

We take a unit of energy to be a unit of carbon. Energy can be extracted in both countries and Home may either export or import energy from Foreign. Carbon is released when the energy is used to produce goods. These goods, embodying carbon emissions, may be traded before being consumed by households. We can therefore trace carbon from its extraction through its release into the atmosphere and finally to its implicit consumption.

We define G_e as total intermediate demand for energy by the goods sector in Home and G_e^* by the goods sector in Foreign. Home net exports of energy, positive or negative, are $Q_e - G_e$. These expressions account for the first level of trade in carbon.

The second level of trade in carbon is embodied in goods. Table 1 depicts the bilateral flows, with rows indicating the location of consumption and columns the location of production. For example, Home implicit consumption of carbon C_e (in the upper right) is the sum of carbon released by producers in Home serving the local market, C_e^{HH}, and carbon released by Foreign producers in supplying Home imports, C_e^{HF}.

[12]Our *unit energy requirement*, $e_j(z_j)$, is sometimes called *emissions intensity* in the environmental economics literature, e.g. Shapiro and Walker (2018). We instead use the term *energy intensity* for energy per worker, z_j (by analogy to the common use of *capital intensity* for capital per worker).

[13]Because Foreign can set z_j^* independently from how it sets z_j^m, we do not include a so-called Brussels effect, as suggested by Bradford (2020).

Table 1: Carbon Accounting Matrix

	Home	Foreign	Total
Home	$C_e^{HH} = \int_0^1 e_j(z_j^y) y_j dj$	$C_e^{HF} = \tau^* \int_0^1 e_j^*(z_j^m) m_j dj$	$C_e = C_e^{HH} + C_e^{HF}$
Foreign	$C_e^{FH} = \tau \int_0^1 e_j(z_j^x) x_j dj$	$C_e^{FF} = \int_0^1 e_j^*(z_j^*) y_j^* dj$	$C_e^* = C_e^{FH} + C_e^{FF}$
Total	$G_e = C_e^{HH} + C_e^{FH}$	$G_e^* = C_e^{HF} + C_e^{FF}$	$G_e^W = C_e^W = Q_e^W.$

3 The Planning Problem

A planner allocates the resources that it controls to maximize Home welfare (1), subject to three constraints: (i) its use of labor in the three sectors of the economy cannot exceed its supply of labor; (ii) the global use of energy in manufacturing cannot exceed global extraction of energy; and (iii) its policies cannot make Foreign worse off. To meet the Foreign welfare constraint, the planner can adjust transfers of services from Home to Foreign, subject to $C_s + C_s^* = Q_s + Q_s^*$. The planner is not constrained by trade balance. Consumption, production, and energy extraction in Foreign are dictated by market prices. We consider these outcomes in Foreign and set out the constraints below before stating the planning problem.

3.1 Foreign

Energy extractors in Foreign can sell energy at price p_e and can hire labor at wage $w^* = 1$. They tap all energy fields with a labor requirement below p_e:

$$Q_e^* = E^*(p_e), \tag{6}$$

or if $p_e \leq \underline{a}^*$ then $Q_e^* = 0$. Goods producers can purchase energy at price p_e and can hire labor at wage $w^* = 1$. Their cost-minimizing energy intensity is $z^* = (1-\alpha)/(\alpha p_e)$. They supply good j at price equal to unit cost:

$$p_j^* = l_j^*(z^*) + p_e e_j^*(z^*) = a_j^* p_e^{1-\alpha}. \tag{7}$$

Consumers in Foreign can purchase any good j from domestic producers at price p_j^*, creating an upper bound on their marginal utility, $u^{*\prime}(c_j^*) \leq p_j^*$.

3.2 Constraints

3.2.1 Home Labor Constraint

From (2), the labor L_e required to extract a quantity of energy Q_e is:

$$L_e = \int_{\underline{a}}^{E^{-1}(Q_e)} a\, E'(a) da. \tag{8}$$

The labor L_g required in goods production is:

$$L_g = \int_0^1 \left(l_j(z_j^y) y_j + \tau l_j(z_j^x) x_j \right) dj.$$

Accounting for labor to provide services, $L_s = Q_s$, Home's labor constraint is:

$$L_e + L_g + L_s = L. \tag{9}$$

3.2.2 Global Energy Constraint

The global constraint on use of energy is:

$$G_e + G_e^* \leq Q_e + Q_e^* = Q_e^W, \tag{10}$$

where Q_e is chosen by the planner and Q_e^* is given by (6). Expressions for G_e and G_e^*, the quantity of energy used in production, are in the last row of Table 1.

3.2.3 Foreign Welfare Constraint

We require that the planner's policy not reduce welfare in Foreign, yet Home has no obligation to raise Foreign welfare either. Hence:

$$C_s^* + u^*(C_g^*) - \varphi^* Q_e^W = U_{BAU}^*, \tag{11}$$

where U_{BAU}^* is Foreign welfare in the BAU baseline. In evaluating (11) below, we will employ the Foreign analog of (9).

3.3 The Planner's Lagrangian

The planner's objective is to maximize Home welfare, $U = C_s + u(C_g) - \varphi Q_e^W$, subject to the three constraints above: (9), (10), and (11). Substituting in the labor constraint (9) and the Foreign welfare constraint (11), in place of C_s, the objective becomes global welfare:[14]

$$U = u(C_g) + u^*(C_g^*) - \varphi^W Q_e^W + L + L^* - L_e - L_e^* - L_g - L_g^* - U_{BAU}^*,$$

[14] Accounting for labor constraints, the supply of global services is:

$$C_s + C_s^* = L + L^* - L_e - L_e^* - L_g - L_g^*.$$

Substituting in the Foreign welfare constraint in place of Foreign consumption of services yields an expression for Home consumption of services:

$$C_s = L + L^* - L_e - L_e^* - L_g - L_g^* + u^*(C_g^*) - \varphi^* Q_e^W - U_{BAU}^*.$$

Substituting into Home welfare yields the new expression for the planner's objective.

where $\varphi^W = \varphi + \varphi^*$ is the global marginal harm from emissions.

We apply a Lagrange multiplier λ_e to the energy constraint and drop constants such as L, L^*, and U^*_{BAU} to form the planner's Lagrangian:

$$\mathcal{L} = \int_0^1 u(y_j + m_j)dj + \int_0^1 u^*(y_j^* + x_j)dj - \varphi^W Q_e^W$$

$$-L_e^W - \int_0^1 \left(l_j(z_j^y)y_j + \tau l_j(z_j^x)x_j + l_j^*(z^*)y_j^* + \tau^* l_j^*(z_j^m)m_j\right) dj$$

$$- \lambda_e \left(\int_0^1 \left(e_j(z_j^y)y_j + \tau e_j(z_j^x)x_j + e_j^*(z^*)y_j^* + \tau^* e_j^*(z_j^m)m_j\right) dj - Q_e^W\right). \qquad (12)$$

The terms are, line-by-line: (i) global utility from goods consumption less harm from emissions, (ii) the opportunity cost (in terms of lost consumption of services) from labor employed in energy extraction and goods production, and (iii) the global energy constraint, weighted by the Lagrange multiplier.

Because the planner's objective is global welfare, the Lagrangian encompasses a number of different cases, which are determined by the resources that the planner is assumed to control. In our core planning problem, to derive the *unilateral* optimum, the planner can choose the quantities of each good that Home consumes and each good that it exports, $\{y_j\}$, $\{x_j\}$, $\{m_j\}$, their energy intensities, $\{z_j^y\}$, $\{z_j^x\}$, $\{z_j^m\}$, its energy extraction Q_e, and the price of energy, p_e. To derive the *global* optimum, the planner can also choose $\{y_j^*\}$, $\{z_j^*\}$, and Q_e^*.[15] Restricting the planner's choices to narrower sets of variables allows us to derive simpler or restricted policies to the unilateral optimum (which we explore in Section 5 and in our simulations).

We solve the maximization problem, starting with what CDVW call the *inner problem*, involving optimality conditions for an individual good given values for Q_e, λ_e, and p_e. We then evaluate the optimality conditions for Q_e and p_e in what they call the *outer problem.* The Lagrange multiplier λ_e clears the energy market.

The results that follow become more intuitive by anticipating that the solution satisfies $\lambda_e \geq p_e$, with a strict inequality in all but extreme cases. This inequality is derived in Appendix B.2. In the case of $\varphi^W = 0$ we get $\lambda_e = p_e$ and the planner's problem collapses to the BAU baseline.

3.4 Inner Problem

The inner problem is to maximize a Lagrangian for any arbitrary good j:

$$\begin{aligned}\mathcal{L}_j = {} & u\left(y_j + m_j\right) + u^*(y_j^* + x_j) \\ & - \left(l_j(z_j^y)y_j + \tau l_j(z_j^x)x_j + l_j^*(z^*)y_j^* + \tau l_j^*(z_j^m)m_j\right) \\ & - \lambda_e \left(e_j(z_j^y)y_j + \tau e_j(z_j^x)x_j + e_j^*(z^*)y_j^* + \tau^* e_j^*(z_j^m)m_j\right).\end{aligned}$$

[15] In this case p_e is redundant. Appendix A provides a step-by-step solution.

We consider, in turn: (i) optimal energy intensities, z_j^y, z_j^m, and z_j^x; (ii) optimal quantities for Home consumers, y_j and m_j; and (iii) optimal quantities for Foreign consumers, x_j.

3.4.1 Energy Intensity

The optimal energy intensities z_j^y and z_j^x solve $\min_z \{l_j(z) + \lambda_e e_j(z)\}$ while z_j^m solves $\min_z \{l_j^*(z) + \lambda_e e_j^*(z)\}$. Using (5) it is apparent that in all three cases, the solution is:

$$z_j^y = z_j^m = z_j^x = \frac{1-\alpha}{\alpha \lambda_e} = z.$$

The planner chooses a common energy intensity z for the production of any good consumed in Home (whether produced in Home or Foreign) and for all production in Home (whether serving consumers in Home or Foreign).

For any good produced in Home for domestic consumption the energy requirement is:

$$e_j(z) = (1-\alpha) a_j \lambda_e^{-\alpha}$$

while the overall shadow cost is

$$l_j(z) + \lambda_e e_j(z) = a_j \lambda_e^{1-\alpha}.$$

If good j is exported from Home, the shadow cost is $\tau a_j \lambda_e^{1-\alpha}$, while if it is imported by Home, the shadow cost is

$$\tau^* \left(l_j^*(z) + \lambda_e e_j^*(z)\right) = \tau^* a_j^* \lambda_e^{1-\alpha}.$$

3.4.2 Goods for Home Consumers

The first order conditions for y_j and m_j, after substituting in results for shadow costs, can be written as:

$$u'(y_j + m_j) - a_j \lambda_e^{1-\alpha} \leq 0,$$

with equality if $y_j > 0$ and

$$u'(y_j + m_j) - \tau^* a_j^* \lambda_e^{1-\alpha} \leq 0,$$

with equality if $m_j > 0$. To distill the implications of these two FOC's we define the good $\bar{j}_m$ for which they both hold with equality. Applying (4), this threshold good satisfies:

$$F(\bar{j}_m) = \frac{1}{\tau^*}. \quad (13)$$

For goods $j < \bar{j}_m$, Home has a comparative advantage, the second FOC holds with a strict inequality so that $m_j = 0$, and the first holds with equality to determine y_j. For goods $j > \bar{j}_m$, Foreign has a comparative advantage, the first FOC holds with a strict inequality so that $y_j = 0$, and the second holds with equality to determine m_j. (We can ignore the outcome for the measure-zero threshold good $j = \bar{j}_m$.)

3.4.3 Goods for Foreign Consumers

As noted above, Foreign's marginal utility for good j is capped by p_j^*, the cost (7) at which it can supply the good to itself. Whether or not that upper bound binds makes two cases to consider in determining the optimal x_j. Case I pertains to goods j for which Foreign's marginal utility remains strictly below p_j^*. In this case we can set $y_j^* = 0$ so that Foreign consumption is $c_j^* = x_j$. Case II pertains to goods j for which Foreign's marginal utility equals p_j^*. In this case c_j^* is invariant to a decline in x_j as it will be exactly offset by a rise in y_j^* that keeps marginal utility equal to p_j^*.

Consider a good j in Case I. The first order condition for x_j, after substituting in the result for the shadow cost of serving the export market, can be written as:

$$u^{*\prime}(x_j) - \tau a_j \lambda_e^{1-\alpha} = 0.$$

Foreign's marginal utility is equated to the shadow cost, $\tau a_j \lambda_e^{1-\alpha}$. This cost is strictly below p_j^* for any good $j < j_0$, where j_0 satisfies:

$$F(j_0) = \tau \left(\frac{\lambda_e}{p_e} \right)^{1-\alpha}. \tag{14}$$

Case I applies to goods $j \in [0, j_0)$.

Now consider a good j in Case II, so that $j \geq j_0$. Foreign's marginal utility no longer depends on x_j, since c_j^* is fixed. But, resources used in Foreign are reduced when x_j increases, since $y_j^* = c_j^* - x_j$. After substituting in the relevant shadow values, the derivative of the Lagrangian is:

$$\frac{\partial \mathcal{L}_j}{\partial x_j} = -\tau a_j \lambda_e^{1-\alpha} + l_j^*(z^*) + \lambda_e e_j^*(z^*). \tag{15}$$

The right-hand side of (15) is the planner's value of the global resources saved by increasing x_j. The last two terms capture the labor (valued at 1) and energy (valued at λ_e by the planner) that Foreign would have used to produce an additional unit of good j for itself. This derivative is predicated on $y_j^* > 0$, but otherwise doesn't depend on x_j. The good $j = \bar{j}_x$ at which (15) is zero satisfies:

$$F(\bar{j}_x) = \tau \frac{\left(\frac{\lambda_e}{p_e} \right)^{1-\alpha}}{\alpha + (1-\alpha)\frac{\lambda_e}{p_e}}. \tag{16}$$

For $\lambda_e > p_e$ it follows that $\bar{j}_x > j_0$. For any good $j > \bar{j}_x$, Foreign has a strong comparative advantage and $x_j = 0$ since the value that the planner places on the resources saved in Foreign doesn't offset the shadow cost of Home producing the good for export. (We can ignore the outcome for the measure-zero threshold good $j = \bar{j}_x$.)

For any good $j \in (j_0, \bar{j}_x)$ Home comparative advantage is stronger so that (15) is strictly positive. Exports are pushed to the non-differentiable limit at which y_j^* is driven to zero. The quantity exported equates Foreign's marginal utility to p_j^*:

$$u^{*\prime}(x_j) - a_j^* p_e^{1-\alpha} = 0.$$

Table 2: Production and Distribution of a Good

	Home		Foreign	
Home	$y_j = \eta \left(a_j \lambda_e^{1-\alpha}\right)^{-\sigma}$	$j < \bar{j}_m$	$m_j = \eta \left(\tau^* a_j^* \lambda_e^{1-\alpha}\right)^{-\sigma}$	$j > \bar{j}_m$
Foreign	$x_j = \begin{cases} \eta^* \left(\tau a_j \lambda_e^{1-\alpha}\right)^{-\sigma^*} \\ \eta^* \left(a_j^* p_e^{1-\alpha}\right)^{-\sigma^*} \end{cases}$	$j < j_0$ $j_0 < j < \bar{j}_x$	$y_j^* = \eta^* \left(a_j^* p_e^{1-\alpha}\right)^{-\sigma^*}$	$j > \bar{j}_x$

Thresholds: $F(\bar{j}_m) = 1/\tau^*$, $F(j_0) = \tau(\lambda_e/p_e)^{1-\alpha}$, and $F(\bar{j}_x) = \frac{\tau(\lambda_e/p_e)^{1-\alpha}}{\alpha+(1-\alpha)\lambda_e/p_e}$

Table 2 displays the results of the inner problem. As in Table 1, the rows indicate the location of consumption while the columns indicate the location of production. These terms are as expected except for Home exports, x_j, for goods $j \in (j_0, \bar{j}_x)$: (i) exports of such goods reflect the price of energy p_e in Foreign rather than the planner's shadow price λ_e, (ii) although produced in Home, they reflect Foreign's input requirement a_j^* rather than Home's, and (iii) they do not reflect the iceberg costs of export τ. That is, $x_j \neq \eta^* \left(\tau a_j \lambda_e^{1-\alpha}\right)^{-\sigma}$ as is the case when $j \leq j_0$. The reason is that for goods $j \in (j_0, \bar{j}_x)$ Home crowds out Foreign production, in order to produce these goods with lower energy intensity, but its comparative advantage in these goods is not strong enough to justify exporting enough to push Foreign marginal utility below p_j^*. (BAU outcomes result from simply replacing λ_e with p_e throughout Table 2.)

3.5 Outer Problem

We now turn to the optimality conditions for Q_e and p_e, rewriting the Lagrangian in terms of aggregate magnitudes:

$$\begin{aligned} \mathcal{L} &= u(C_g) + u^*(C_g^*) - \varphi^W \left(Q_e + Q_e^*\right) \\ &\quad - L_e - L_e^* - L_g - L_g^* - \lambda_e \left(G_e + G_e^* - Q_e - Q_e^*\right). \end{aligned} \tag{17}$$

3.5.1 Energy Extraction

The first order condition with respect to Q_e is:

$$\frac{\partial \mathcal{L}}{\partial Q_e} = -\varphi^W - \frac{\partial L_e}{\partial Q_e} + \lambda_e \leq 0,$$

with equality if $Q_e > 0$. The extra labor to extract a bit more energy in Home is the labor requirement for the marginal energy field there, $E^{-1}\left(Q_e\right)$.[16] Applying this

[16]Integrating (8) by parts:

$$L_e = E^{-1}(Q_e)Q_e - \int_{\underline{a}}^{E^{-1}(Q_e)} E(a)da.$$

result, the first order condition simplifies to:

$$Q_e = E\left(\lambda_e - \varphi^W\right), \tag{18}$$

for $\lambda_e - \varphi^W \geq \underline{a}$ and $Q_e = 0$ otherwise. (To obtain the BAU outcome, replace $\lambda_e - \varphi^W$ with p_e, as with Foreign extraction (6).)

3.5.2 Energy Price

The first order condition with respect to p_e can be written as:

$$u^{*\prime}(C_g^*)\frac{\partial C_g^*}{\partial p_e} - \varphi^W \frac{\partial Q_e^*}{\partial p_e} - \frac{\partial L_e^*}{\partial p_e} - \frac{\partial L_g}{\partial p_e} - \frac{\partial L_g^*}{\partial p_e} = \lambda_e \left(\frac{\partial G_e}{\partial p_e} + \frac{\partial G_e^*}{\partial p_e} - \frac{\partial Q_e^*}{\partial p_e}\right).$$

To make sense of this condition requires computing the partial derivatives of C_g^*, Q_e^*, L_e^*, L_g, L_g^*, G_e, and G_e^*, each evaluated at the optimal unilateral policy itself.

Foreign energy extraction depends directly on the energy price, via (6), so that $\partial Q_e^*/\partial p_e = E^{*\prime}(p_e)$. The response of Foreign employment in the energy sector is $\partial L_e^*/\partial p_e = p_e E^{*\prime}(p_e)$. Dependence on the energy price is more subtle for the other aggregates as pieces of them have already been chosen by the planner in the inner problem.[17]

In Appendix B we compute all the partial derivatives and substitute them into the first order condition above to get:

$$\left((\lambda_e - \varphi^W) - p_e\right)\frac{\partial Q_e^*}{\partial p_e} = (\lambda_e - p_e)\frac{\partial C_e^{FF}}{\partial p_e} + \int_{j0}^{\bar{j}_x}\left(\tau a_j \lambda_e^{1-\alpha} - p_j^*\right)\frac{\partial x_j}{\partial p_e}dj. \tag{19}$$

This optimality condition balances Foreign supply and demand responses to a change in p_e with the deviation between the planner's valuation and the Foreign market

Differentiating it in this form:

$$\frac{\partial L_e}{\partial Q_e} = E^{-1}(Q_e) + Q_e\frac{\partial E^{-1}}{\partial Q_e} - E(E^{-1}(Q_e))\frac{\partial E^{-1}}{\partial Q_e} = E^{-1}(Q_e).$$

[17]For example, energy use by Foreign producers:

$$G_e^* = \int_{\bar{j}_x}^1 e_j^*(z^*)y_j^* dj + \tau^* \int_{\bar{j}_m}^1 e_j^*(z)m_j dj,$$

depends on the energy price only through the first integral, C_e^{FF}. The partial derivative we seek is therefore:

$$\frac{\partial G_e^*}{\partial p_e} = \frac{\partial C_e^{FF}}{\partial p_e} = -\epsilon_D^* \frac{C_e^{FF}}{p_e} < 0.$$

A change in the energy price affects Foreign's use of energy only through its domestic consumption C_e^{FF} and not through its exports of goods to Home C_e^{HF}. The planner has chosen and optimized $\bar{j}_m$, m_j, and $z^m = z$, which determine C_e^{HF}.

valuation of each response. We refer to these deviations as wedges: (i) the wedge between the planner's marginal valuation of a unit of energy extracted and the energy price (*extraction wedge*), (ii) the wedge between the planner's marginal valuation of energy used and the energy price (*consumption wedge*), and (iii) the wedges:

$$s_j = \tau a_j \lambda_e^{1-\alpha} - p_j^*,$$

for each good $j \in (j_0, \bar{j}_x)$, between the shadow cost of Home supplying exports of j and the marginal utility to consumers in Foreign (*export wedges*).

We get a compact expression for the energy-price condition by aggregating the export wedges into a single term:

$$S = \int_{j_0}^{\bar{j}_x} s_j x_j dj.$$

We can then rewrite (19) as:

$$\lambda_e - p_e = \frac{\varphi^W \epsilon_S^* Q_e^* - \sigma^*(1-\alpha)S}{\epsilon_S^* Q_e^* + \epsilon_D^* C_e^{FF}}, \tag{20}$$

where $\epsilon_S^* \geq 0$ and $\epsilon_D^* > \alpha$ are the Foreign elasticities of supply and demand for energy:[18]

$$\epsilon_S^* = E^{*\prime}(p_e)p_e/E^*(p_e); \qquad \epsilon_D^* = \alpha + (1-\alpha)\sigma^*. \tag{21}$$

3.6 Properties of the Solution

We can now compute the optimal policy, in principle: (i) the inner problem gives G_e and G_e^* in terms of p_e and λ_e, (ii) equations (6) and (18) give Q_e^* and Q_e as functions of p_e and λ_e, and (iii) equation (19) and the global energy constraint (10), which binds, jointly nail down p_e and λ_e.

We can also go further in characterizing the optimal wedges.

3.6.1 The Pigouvian Wedge

Adding the absolute value of the extraction wedge and the consumption wedge yields φ^W, the marginal global externality from carbon emissions. The wedge between extraction and use of energy in Home is Pigouvian. As shown in Appendix A, a global planner, that could also control outcomes in Foreign, would impose this Pigouvian wedge there as well. A unilaterally optimal policy cannot achieve that international uniformity, yet still imposes the Pigouvian wedge in Home.

[18]The supply elasticity will typically be a function of the energy price while the demand elasticity is a constant, depending on both the elasticity α of the unit energy requirement for producing a good and the elasticity $(1-\alpha)\sigma^*$ of the quantity demanded with respect to the energy price. With α close to 1, so too will be ϵ_D^* for moderate values of σ^*.

3.6.2 Balancing Extraction and Consumption Wedges

Appendix B.2 shows that the planner picks the consumption wedge, $\lambda_e - p_e$, from the interval $[0, \varphi^W)$, strictly positive if $\varphi^W \epsilon_S^* Q_e^* > 0$. The consumption wedge will approach the upper bound if $\epsilon_S^* Q_e^*$ is large relative to $\epsilon_D^* C_e^{FF}$. In this case the planner chooses a low energy price to keep Foreign extraction of energy in check. As the consumption wedge approaches φ^W the extraction wedge approaches 0.

The consumption wedge will approach the lower bound if $\epsilon_S^* Q_e^*$ is small relative to $\epsilon_D^* C_e^{FF}$. In this case the planner chooses a high price to keep Foreign demand for energy in check. With perfectly inelastic Foreign supply, the extraction wedge equals the Pigouvian wedge and the consumption wedge is 0. The unilateral policy then achieves the global optimum.[19]

3.6.3 Export Wedges and Crosshauling

The import threshold, $\bar{j}_m$, is the same under the optimal policy as under BAU. The export threshold is greater, $\bar{j}_x > \bar{j}_{x,BAU}$.[20] The planner implicitly subsidizes exports of goods $j \in (j_0, \bar{j}_x)$ as dictated by the export wedges, s_j. These properties of the solution create the possibility for crosshauling. Under the optimal policy there may be a set of goods that Home simultaneously imports and exports. Such a set of goods always exists in the absence of trade costs since then $F(\bar{j}_m) = 1$ while $F(\bar{j}_x) < F(\bar{j}_{x,BAU}) = 1$ implying $\bar{j}_x > \bar{j}_m$.[21]

Trade costs mute this effect. With high enough trade costs $F(\bar{j}_x) > F(\bar{j}_m)$ so that $\bar{j}_x < \bar{j}_m$. The inherent inefficiency of crosshauling overcomes its advantage in reducing the shadow value of resources used in production. Yet, even when there is no crosshauling the optimal policy broadens the range of goods that Home exports. The planner controls energy intensity not only for all production in Home but also for

[19] Following this logic, Harstad (2012) makes a case that the policy maker buy marginal energy fields from Foreign to create a locally vertical Foreign supply curve. We have ruled out such an international market in Foreign energy fields in our analysis here.

[20] Denote the right-hand side of (16) by $f(\lambda_e/p_e)$. Evaluating it at $\lambda_e/p_e = 1$ gives $f(1) = \tau$, hence $\bar{j}_x = \bar{j}_{x,BAU}$ if $\lambda_e = p_e$. Differentiating it:

$$\frac{\partial f(\lambda_e/p_e)}{\partial(\lambda_e/p_e)} = \alpha(1-\alpha)\frac{F(\bar{j}_x)^2}{F(j_0)}\left(\frac{1}{\lambda_e/p_e} - 1\right),$$

which is negative for $\lambda_e/p_e > 1$. Since $\lambda_e > p_e$ for $\varphi^W > 0$ it follows that $F(\bar{j}_x) < F(\bar{j}_{x,BAU})$ and hence $\bar{j}_x > \bar{j}_{x,BAU}$.

[21] The economic rationale for crosshauling is to save global resources, with labor valued at 1 and energy valued at λ_e. To make the argument precise, we exploit results from the inner problem. If $\varphi^W > 0$ then for any $j \in (j_0, \bar{j}_x)$ the planner saves resources if j is produced in Home and exported rather than being produced in Foreign (see Section 3.4.3). For any $j < \bar{j}_m$ the planner saves resources if j is produced in Foreign and imported rather than being produced in Home (see Section 3.4.2). In the absence of trade costs, $\tau = \tau^* = 1$, we have $\bar{j}_m \in (j_0, \bar{j}_x)$. In this case there is a range of goods $j \in (j_0, \bar{j}_m)$ for which resources are saved (in both directions) if Home produces them for Foreign consumers while Foreign produces them for Home consumers.

production in Foreign that Home imports. Goods produced in Foreign, for consumption there, escape the policy. The planner uses exports to reduce Foreign production for itself, with the export wedge inducing Foreign consumers to buy them.

4 Optimal Taxes and Subsidies

We now describe a set of taxes and subsidies that deliver the optimal outcomes in a competitive equilibrium. In shifting from a planning problem to a market economy, recall that services are the numéraire and the unit labor requirement for services pins the wage to 1 in both countries. We treat p_e as the *global* energy price, the base to which we apply carbon taxes. The taxes and subsidies we introduce into this competitive equilibrium must generate the wedges that appear in the optimal policy.

4.1 A Simple Implementation

We focus on a policy that is easy to describe, with three elements of intervention:

1. Impose a nominal tax on Home energy extraction, t_e^N, equal to the Pigouvian wedge:
$$t_e^N = \varphi^W.$$
2. Impose a border adjustment, t_b, on Home imports or exports of energy and on the energy content of Home imports of goods, equal to the consumption wedge:
$$t_b = \frac{\varphi^W \epsilon_S^* Q_e^* - \sigma^*(1-\alpha)S}{\epsilon_S^* Q_e^* + \epsilon_D^* C_e^{FF}}. \tag{22}$$
3. Provide an export subsidy s_j per unit exported of any good $j \in (j_0, \bar{j}_x)$ equal to the export wedge:
$$s_j = \tau a_j (p_e + t_b)^{1-\alpha} - p_j^*,$$
where $F(j_0) = \tau\,(1 + t_b/p_e)^{1-\alpha}$ and $F(\bar{j}_x) = \frac{F(j_0)}{\alpha + (1-\alpha)t_b/p_e}$.

The resulting *effective* extraction tax t_e equals the absolute value of the extraction wedge, $t_e = t_e^N - t_b$. In the special case of $\varphi^W = 0$ there are no wedges and the optimal policy sets $t_e^N = t_b = 0$ and $s_j = 0$ for all j, resulting in the BAU competitive equilibrium.

4.2 After-Tax Prices

To eliminate ambiguity about how this policy would work, we list the net prices faced by the different agents in the global economy:

1. The global price of energy, p_e, is paid by users of energy in Foreign and is received by energy extractors in Foreign.

2. If energy is imported by Home, it is subject to a border adjustment t_b, raising the price of energy for users in Home to $p_e + t_b$.

3. Energy extractors in Home sell energy domestically at price $p_e + t_b$, but after paying the extraction tax they net $p_e + t_b - t_e^N = p_e - t_e$.

4. Extractors export at price p_e, but also receive a partial rebate of t_b on the nominal extraction tax they paid, leaving their net price $p_e - t_e$ the same as if they sell domestically.

5. Goods $j < \bar{j}_m$ are produced in Home, using energy costing $p_e + t_b$, so that local consumers pay $p_j = a_j(p_e + t_b)^{1-\alpha}$.

6. Goods $j > \bar{j}_m$ are imported by Home. Foreign produces them with energy intensity z in anticipation of the border adjustment levied on their energy content. Their cost of production (including delivery to Home) is $\tau^* l_j^*(z) + p_e \tau^* e_j^*(z)$. (The energy they use is not taxed directly.) Adding in the border adjustment, $t_b \tau^* e_j^*(z)$, the price to consumers in Home becomes $p_j^m = \tau^* a_j^*(p_e + t_b)^{1-\alpha}$.

7. Goods $j < j_0$ are produced in Home and exported. The producers use energy costing $p_e + t_b$ with no adjustment when the goods are exported. The price in Foreign, including the trade cost, is $p_j^x = \tau a_j(p_e + t_b)^{1-\alpha}$.

8. Goods $j \in (j_0, \bar{j}_x)$ are also exported by Home. The producers use energy priced at $p_e + t_b$, with no relief from the energy tax when the goods are exported. They sell at price $p_j^x = p_j^* = a_j^* p_e^{1-\alpha}$ in Foreign, but get a subsidy from Home of s_j per unit to cover cost: $p_j^* + s_j = \tau a_j(p_e + t_b)^{1-\alpha}$.

9. Goods $j > \bar{j}_x$ are produced in Foreign, using energy at price p_e. They are sold to local consumers at price $p_j^* = a_j^* p_e^{1-\alpha}$.

4.3 Discussion

We can understand the optimal tax rates by considering how they are shaped by responses in Foreign. Extraction in Foreign and production of goods there for local consumption face no tax but respond to the equilibrium price of energy. The latter has two components: the energy intensity of this production (the intensive margin) and the set of goods produced (the extensive margin). These three margins—Foreign extraction, Foreign energy intensity, and the range of goods produced in Foreign for local consumers—can be thought of as three different sources of leakage. Home sets its combination of an extraction tax, a border adjustment, and an export subsidy to indirectly affect these margins, in effect controlling all these sources of leakage.

If Foreign's extraction elasticity is large, extraction leakage is potentially high, resulting in costs to Home that go up with φ^W. Border adjustments on energy moderate this effect. Increasing the border adjustment lowers the price of energy, thereby reducing extraction leakage. Lowering p_e, however, introduces distortions on the production and consumption side. As p_e goes down, the set of goods produced in Foreign increases, and Foreign's energy intensity in producing those goods goes up. The set of goods produced in Foreign roughly corresponds to traditional (production) leakage, while the energy intensity of those goods is sometimes called the "fuel price effect."[22] The principle of optimizing over the two tax instruments, t_e and t_b, given $t_e + t_b = t_e^N = \varphi^W$, is at the heart of the seminal paper of Markusen (1975).[23]

The optimal policy also controls production leakage through a combination of a border adjustment on imports and a goods-specific subsidy for exports. The border tax on imports means that imports face the same effective energy price as goods produced in Home. As a result, the border tax leaves the extensive margin for imports the same as without tax and causes the energy intensity of imports to be the same as that of goods produced in Home. The policy might have controlled the export margin in a parallel fashion, by rebating taxes on export, leaving the export margin the same as it would be without tax. Doing so, however, would remove the incentive for exporters to lower their energy intensity. Rather than removing the tax on export, therefore, the policy offers good specific subsidies. Because these subsidies do not depend on energy usage, they retain incentives for exporters to produce goods with low energy intensity.[24]

The subsidy goes beyond merely restoring Home's export margin: it applies to goods for which Home would not be competitive in the absence of any carbon policy. The reason follows the argument above for potential cross-hauling under the optimal policy. The policy is designed to crowd out some of Foreign's energy-intensive production for its domestic consumers. The same logic does not apply to the import margin because the border tax on imports ensures that all goods consumed in Home are produced with the same (low) energy intensity. The asymmetry between imports and exports arises because a unilateral policy can't directly control the energy intensity of goods produced in Foreign that are consumed in Foreign. The optimal export policy seeks to crowd out this activity.

[22] These terms, however, are not clearly distinguished in the literature, and our use of them is only suggestive. The fuel price effect appears to refer to any change in Foreign production or consumption due to a reduction in p_e. If true, then traditional production leakage is limited to shifts in import or export margins holding p_e fixed. Our usage does not precisely correspond to these definitions because our expressions all use the equilibrium value of p_e.

[23] This connection to Markusen (1975) is disguised by differences in terminology. Our extraction tax is what he refers to as a production tax. Our border adjustment is what he refers to as a trade tax. Furthermore, his taxes are ad valorem while ours are specific. More fundamentally, he imposes trade balance, so that his trade tax incorporates terms-of-trade considerations. Finally, in his model there is no analog of our production sector, which uses energy to produce tradable goods. Hence, his analysis doesn't speak to how the border adjustment applies to the energy embodied in these goods.

[24] This basic logic comes from Fischer and Fox (2012), who point out that rebating carbon tax revenue to producers, in proportion to their production (without regard to their tax payments), retains the incentive for them to use less carbon. An optimal subsidy to production in the context of carbon pricing emerges in Fowlie et al. (2016). There it is designed to offset the output-reducing effect of market power among cement producers.

5 Constrained Optimal Policies

To assess the optimal policy, we compare it to more conventional polices: an extraction tax, a consumption tax, and a production tax. We also consider hybrids of these taxes, which are optimal combinations of the three conventional policies.

We derive each policy as a variant of the planner's problem from Section 3. The Lagrangian (12) remains the same in each case, but is solved assuming that the planner can control only those variables subject to a given policy. For example, an extraction tax emerges when the planner controls Q_e and p_e, with all other variables determined in the competitive equilibrium. For a consumption tax, the planner chooses $\{z_j^y\}$,$\{z_j^m\}$, $\{y_j\}$, $\{m_j\}$, and p_e, with all other variables determined in the competitive equilibrium. Full solutions to the Lagrangian for each case are shown in Appendix C. Here we focus on the optimality condition for p_e, which conveys the essential intuition, and then show how the solution can be implemented through taxes.

5.1 The Planner's Solution

We can write the conditions for p_e for each of the constrained policies in terms of the first two wedges seen in the optimal policy, the extraction wedge, $(\lambda_e - \varphi^W) - p_e$, and the consumption wedge, $\lambda_e - p_e$. In each case, the planner uses the wedge to evaluate the cost of the response in Foreign of the corresponding item outside of its control. The planner sets the size of these marginal costs equal to each other.

Returning to the problem that leads to an optimal extraction tax, the planner chooses Q_e according to (18) while Foreign extraction remains outside its control. The planner does not control the demand side of the market in either region, so the global consumption response is also outside its control. The condition for p_e balances the cost of the Foreign extraction response and the cost of the global demand response to changes in p_e:

$$\left((\lambda_e - \varphi^W) - p_e\right) \frac{\partial Q_e^*}{\partial p_e} = (\lambda_e - p_e) \frac{\partial C_e^W}{\partial p_e}.$$

The same logic holds for a consumption tax and a hybrid of an extraction and consumption taxes. In each case, the planner sets the marginal cost due to the extraction wedge equal to marginal cost due to the appropriate consumption wedge. Table 3 summarizes these cases.

Deriving optimal production taxes is more complex. Such policies change the cost of energy to producers in Home relative to the cost to producers in Foreign, which means that the trade margins $\bar{j}_x$ and $\bar{j}_m$ change, generating leakage, which the planner must take into account.

To compactly represent leakage, let r_e be the cost of energy in Home relative to

Table 3: Conditions for p_e for Extraction and Consumption Taxes

Extraction tax	$\left((\lambda_e - \varphi^W) - p_e\right)\frac{\partial Q_e^*}{\partial p_e} = (\lambda_e - p_e)\frac{\partial C_e^W}{\partial p_e}$
Consumption tax	$\left((\lambda_e - \varphi^W) - p_e\right)\frac{\partial Q_e^W}{\partial p_e} = (\lambda_e - p_e)\frac{\partial C_e^*}{\partial p_e}$
Extraction/Consumption	$\left((\lambda_e - \varphi^W) - p_e\right)\frac{\partial Q_e^*}{\partial p_e} = (\lambda_e - p_e)\frac{\partial C_e^*}{\partial p_e}$

its cost in Foreign. The planner chooses r_e so that:[25]

$$(r_e p_e - \lambda_e)\frac{\partial G_e}{\partial r_e} = (\lambda_e - p_e)\frac{\partial G_e^*}{\partial r_e}$$

We define leakage from a production tax in terms of the (negative) change in energy use in Foreign relative to the change in Home, both with respect to a change in r_e.

$$\Lambda = -\frac{\partial G_e^*/\partial r_e}{\partial G_e/\partial r_e}.$$

Using this expression, we can write the condition for p_e, leading to a production tax, in terms of the extraction and consumption wedges:

$$\left((\lambda_e - \varphi^W) - p_e\right)\frac{\partial Q_e^W}{\partial p_e} = (\lambda_e - p_e)\left(\frac{\partial G_e^*}{\partial p_e} + \Lambda\frac{\partial G_e}{\partial p_e}\right).$$

Here, the planner does not control either domestic or Foreign extraction, so the extraction wedge is multiplied by the change in Q_e^W. On the consumption side, the planner cares about both the direct effect of the energy price on Foreign use of energy and the indirect effect on Home use via production leakage. Faced with leakage, $\Lambda > 0$, the planner sets the cost of energy to Home producers below its shadow value, $r_e p_e < \lambda_e$. The greater the potential for leakage, the more the planner is concerned about the difference in the price of energy seen by Home producers relative to Foreign producers.

To derive an optimal combination of a production tax and consumption tax, the planner must control all goods produced domestically (no matter where consumed) and all goods consumed domestically (no matter where produced). Only C_e^{FF} and C_e^{FH} are outside the reach of the planner, on the demand side. The expression for leakage simplifies to:

$$\Lambda = -\frac{\partial C_e^{FF}/\partial r_e}{\partial C_e^{FH}/\partial r_e}.$$

[25] In terms of the choice variables in the Lagrangian (12), allowing the planner to choose r_e is equivalent to the planner choosing p_e and z, restricting $z_j^x = z_j^y = z$ while setting $x_j = \eta^*\left(\tau a_j\left(\frac{1-\alpha}{\alpha z}\right)^{1-\alpha}\right)^{-\sigma^*}$ (or $x_j = 0$ if $\tau a_j\left(\frac{1-\alpha}{\alpha z}\right)^{1-\alpha} \geq a_j^* p_e^{1-\alpha}$) and $y_j = \eta\left(a_j\left(\frac{1-\alpha}{\alpha z}\right)^{1-\alpha}\right)^{-\sigma}$ (or $y_j = 0$ if $a_j\left(\frac{1-\alpha}{\alpha z}\right)^{1-\alpha} \geq \tau^* a_j^* p_e^{1-\alpha}$). The formulation in terms of r_e is more convenient.

The expressions for p_e follow accordingly. For example, the optimality condition for this hybrid is:

$$\left((\lambda_e - \varphi^W) - p_e\right) \frac{\partial Q_e^W}{\partial p_e} = (\lambda_e - p_e) \left(\frac{\partial C_e^{FF}}{\partial p_e} + \Lambda \frac{\partial C_e^{FH}}{\partial p_e} \right).$$

5.2 Taxes and Implementation

Implementing these outcomes involves imposing extraction, production, and consumption taxes, as the case may be, for each policy. The extraction tax in each case is equal to the extraction wedge: $t_e = (\lambda_e - \varphi^W) - p_e$. The consumption tax is equal to the consumption wedge: $t_c = \lambda_e - p_e$. As we will discuss below, due to concerns about leakage, the production tax is equal to the consumption wedge reduced by the extent of leakage: $t_p = (1 - \Lambda)(\lambda_e - p_e)$. While the expressions for the various taxes are the same, the values of the wedges are different because of how the various policies affect energy prices and use in Home and Foreign.

Table 4 summarizes the effective taxes for each case, stated in terms of Foreign elasticities defined in (21), the global elasticity of supply, ϵ_S^W, and demand, ϵ_D^W, and the elasticities of energy use in Home, ϵ_G, and in Foreign, ϵ_G^*.[26]

If Home imposes one of the pure taxes–an extraction tax, a production tax, or a consumption tax–the overall tax rate does not reach the Pigouvian wedge, φ^W. In the case of an extraction tax, if Foreign is a major energy extractor with elastic supply, so that the value of $\epsilon_S^* Q_e^*$ is high, the planner will choose a lower extraction tax because of concerns that raising the extraction tax would stimulate Foreign extraction. Similarly, if Home imposes a consumption tax and the Foreign demand response, $\epsilon_D^* C_e^*$, is high, the planner chooses a lower tax rate because of concerns that a higher rate would stimulate Foreign demand.

The tax rate under a production tax is lower than the Pigouvian rate, φ^W, because of the Foreign response to the tax, $\epsilon_G^* G_e^*$, the domestic response due to leakage, $\Lambda \epsilon_G G_e$, and by a leakage parameter $(1 - \Lambda)$. The latter term, in particular, drives the production tax down because of concerns about leakage: As leakage approaches 1, the production tax becomes increasingly ineffective, and the optimal rate approaches 0.

[26] Defining ϵ_S and ϵ_D as Home's analogs of the Foreign elasticities in (21), the global elasticities are the weighted sums:

$$\epsilon_S^W = (Q_e/Q_e^W)\epsilon_S + (Q_e^*/Q_e^W)\epsilon_S^* \qquad \epsilon_D^W = (C_e/C_e^W)\epsilon_D + (C_e^*/C_e^W)\epsilon_D^*.$$

The energy use terms are

$$\epsilon_G = (C_e^{HH}/G_e)\epsilon_D + (C_e^{FH}/G_e)\epsilon_D^* \qquad \epsilon_G^* = (C_e^{HF}/G_e^*)\epsilon_D + (C_e^{FF}/G_e^*)\epsilon_D^*,$$

where the global analog is:

$$\epsilon_G^W = (G_e/G_e^W)\epsilon_G + (G_e^*/G_e^W)\epsilon_G^* = \epsilon_D^W.$$

Table 4: Effective Taxes

Policy	Effective Taxes	$(\lambda_e - p_e)$
Extraction tax	$t_e = \varphi^W - (\lambda_e - p_e)$	$\frac{\varphi^W \epsilon_S^* Q_e^*}{\epsilon_S^* Q_e^* + \epsilon_D^W C_e^W}$
Consumption tax	$t_c = \lambda_e - p_e$	$\frac{\varphi^W \epsilon_S^W Q_e^W}{\epsilon_S^W Q_e^W + \epsilon_D^* C_e^*}$
Production tax	$t_p = (1-\Lambda)(\lambda_e - p_e)$	$\frac{\varphi^W \epsilon_S^W Q_e^W}{\epsilon_S^W Q_e^W + \epsilon_G^* G_e^* + \Lambda \epsilon_G G_e}$
Extraction/Consumption	$\begin{cases} t_e = \varphi^W - (\lambda_e - p_e) \\ t_c = \lambda_e - p_e \end{cases}$	$\frac{\varphi^W \epsilon_S^* Q_e^*}{\epsilon_S^* Q_e^* + \epsilon_D^* C_e^*}$
Extraction/Production	$\begin{cases} t_e = \varphi^W - (\lambda_e - p_e) \\ t_p = (1-\Lambda)(\lambda_e - p_e) \end{cases}$	$\frac{\varphi^W \epsilon_S^* Q_e^*}{\epsilon_S^* Q_e^* + \epsilon_G^* G_e^* + \Lambda \epsilon_G G_e}$
Production/Consumption	$\begin{cases} t_p = (1-\Lambda)(\lambda_e - p_e) \\ t_c = \lambda_e - p_e \end{cases}$	$\frac{\varphi^W \epsilon_S^W Q_e^W}{\epsilon_S^W Q_e^W + \epsilon_D^* C_e^{FF} + \Lambda \epsilon_D^* C_e^{FH}}$
Extraction/Production/Cons	$\begin{cases} t_e = \varphi^W - (\lambda_e - p_e) \\ t_p = (1-\Lambda)(\lambda_e - p_e) \\ t_c = \lambda_e - p_e \end{cases}$	$\frac{\varphi^W \epsilon_S^* Q_e^*}{\epsilon_S^* Q_e^* + \epsilon_D^* C_e^{FF} + \Lambda \epsilon_D^* C_e^{FH}}$

Turning to the hybrid policies, when Home combines an extraction tax and a consumption tax, it can control both sides of the market. As a result, the overall tax rate is equal to the Pigouvian wedge, similar to what it is able to do with the optimal unilateral policy.

This is not the case, however, for combinations involving a production tax. For example, with the extraction/production hybrid, the combined taxes are lower than φ^W. The extraction component is set equal to the extraction wedge, but the production component is less than the consumption wedge by a factor of $(1-\Lambda)$. As leakage goes up, Home shifts away from the production tax. The extraction tax goes up somewhat (due to the $\Lambda\epsilon_G$ in the denominator). The combined taxes, however, are less than φ^W and go down with Λ: leakage reduces the power of this hybrid compared to the extraction/consumption hybrid.

While an extraction/production tax has to contend with leakage, it has an offsetting advantage over other policies: it can be implemented simply and accurately. To implement this tax, Home would impose a nominal extraction tax of $t_e = t_e^N + t_p = \varphi^W - \frac{\Lambda}{1-\Lambda} t_p$ and border adjustments on energy (but not on goods) at rate $t_b = t_p$. By avoiding border adjustments on goods, the tax avoids the need to estimate the marginal emissions from the production of goods in foreign countries, which is the key problem in imposing border adjustments. (Kortum and Weisbach (2016)).

Turning to the production/consumption tax hybrid, if leakage is zero, the planner sets the rates on the two taxes the same. This is similar to the optimal policy: all production and all consumption in Home is taxed the same way. If leakage is positive, the planner reduces its reliance on the production tax, shifting toward taxing consumption. The two taxes, however, add up to less than φ^W because leakage reduces the power of this hybrid.

To implement this hybrid, Home imposes a nominal extraction tax at rate $t_e^N = t_c$ together with a full border adjustment on energy as well as on goods imports, at rate $t_b = t_c$. The partial border adjustment on goods exports (which removes some of tax on these goods) would be at rate $t_b^x = t_b - t_p$, reducing the tax from t_c to t_p on energy used by Home to produce exports.

Finally, when Home can impose the combination of all three taxes, the sum of the extraction and consumption rates is equal to the Pigouvian rate, as with the hybrid of just those two taxes. The production tax rate, which applies only to exports, however, is lower due to a concern about leakage. As leakage goes up, the use of the production tax goes down (and the planner also shifts away from consumption taxes and toward extraction taxes).

6 Quantitative Illustration

We now turn to the quantitative implications of the optimal policy. We pursue a strategy, based on Dekle, Eaton, and Kortum (2007), calibrating the BAU competitive equilibrium to data on global carbon flows and then computing the optimal policy relative to this baseline. We also compare the BAU and optimal policies to the more conventional policies derived in the previous section.

6.1 Setup

We start by providing some of the key elements of our procedure (with a full treatment relegated to Appendix D), and then present our key results.

6.1.1 Functional Forms

To solve the model numerically we employ convenient functional forms for the comparative advantage curve $F(j)$ and the distributions of energy fields, $E(a)$ and $E^*(a)$.

Energy Supply We parameterize the distribution of energy fields by treating the supply elasticities, ϵ_S and ϵ_S^*, as parameters (setting $\underline{a} = \underline{a}^* = 0$) so that for $a \geq 0$:

$$E(a) = Ea^{\epsilon_S}; \qquad E^*(a) = E^*a^{\epsilon_S^*},$$

where $E = E(1)$ and $E^* = E^*(1)$ are shift parameters.[27]

Comparative Advantage We parameterize the efficiency of the goods sector in each country by:

$$a_j = \left(\frac{j}{A}\right)^{1/\theta}; \qquad a_j^* = \left(\frac{1-j}{A^*}\right)^{1/\theta},$$

where A and A^* determine absolute advantage in either country, and θ determines (inversely) the scope of comparative advantage. Taking the ratio of these two gives the comparative advantage curve:

$$F(j) = \frac{a_j^*}{a_j} = \left(\frac{A}{A^*}\frac{1-j}{j}\right)^{1/\theta},$$

continuous and strictly decreasing as specified in (4).

This functional form allows us to solve for the import and export thresholds in the BAU. In the BAU baseline a country's average spending per good doesn't depend on the source of the good. Since the share of energy in the cost of any good is the same, the baseline value of the import margin is:

$$\bar{j}_m = \frac{C_e^{HH}}{C_e} = \frac{A}{A + (\tau^*)^{-\theta} A^*},$$

while the baseline value of the export margin is:

$$\bar{j}_x = \frac{C_e^{FH}}{C_e^*} = \frac{\tau^{-\theta} A}{\tau^{-\theta} A + A^*}.$$

6.1.2 Calibration of BAU Scenario.

We calibrate the BAU baseline to carbon accounting data for 2015 from the Trade Embodied in CO_2 ($TECO_2$) database made available by the OECD.[28] Units are gigatonnes of CO_2. Energy extraction data for 2015 is from the International Energy Agency World Energy Statistics Database. We use emissions factors to convert units of energy to units of CO_2.

For most of our results, members of the OECD form the taxing region, Home, and the non-OECD countries are Foreign. Table 5 provides the data that we calibrate to. By this CO_2 metric the OECD represents about one-third of the world. It represents a smaller share of extraction and a larger share of implicit consumption, nearly twenty percent of which is imported.

[27] As noted in Section 2.3, we choose units of energy so that in the BAU baseline the global energy price is 1 and hence baseline extraction is $Q_e = E$ and $Q_e^* = E^*$.

[28] The values that we take from $TECO_2$ are broadly consistent with those available from the Global Carbon Project.

Table 5: Baseline Calibration for Home as the OECD

	Home	Foreign	Total
Home	$C_e^{HH} = 11.3$	$C_e^{HF} = 2.5$	$C_e = 13.8$
Foreign	$C_e^{FH} = 0.9$	$C_e^{FF} = 17.6$	$C_e^* = 18.5$
Total	$G_e = 12.2$	$G_e^* = 20.1$	$G_e^W = C_e^W = 32.3$
Extraction	$Q_e = 8.6$	$Q_e^* =23.7$	$Q_e^W = 32.3$

Table 6: Parameter Values

α	ϵ_S	ϵ_S^*	σ	σ^*	θ
0.85	0.5	0.5	1	1	4

In addition to the carbon accounting data, we need values for six parameters: θ, ϵ_S, ϵ_S^*, σ, σ^*, and α, the last three of which determine the demand elasticities, ϵ_D and ϵ_D^*.[29] Table 6 lists our central values for these parameters, which we have determined using a variety of sources.[30] Appendix E provides additional details on our calibration procedure.

Prior studies, such as Elliott et. al. (2009) show that the foreign elasticity of energy supply, ϵ_s^*, is the key parameter affecting leakage and the effectiveness of a production tax. We estimate that $\epsilon_S = \epsilon_S^* = 0.5$ using data in Asker, Collard-Wexler, and De Loecker (2018), by fitting the slope of $E(a)$ and $E^*(a)$ among oil fields with costs above the median. Based on a literature review, Kotchen (2021) uses much higher values for the United States, with a point estimate for coal of $\epsilon_s^{coal} = 1.9$, for natural gas of $\epsilon_s^{NG} = 1.6$, and for gasoline of $\epsilon_s^{gas} = 2.0$. To account for the uncertainty in these values, we show most of our results using both our baseline calibration and also setting $\epsilon_s^* = 2.0$.

[29]The eight other parameters: A, A^*, E, E^*, η, η^*, τ, and τ^* are all subsumed by calibrating to the carbon accounts.

[30]We choose $\alpha = 0.85$ based on the ratio of the value of energy used in production to value added. (In our model that ratio is $(1-\alpha)/\alpha$.) We take $\theta = 4$ based on the preferred estimate in Simonovska and Waugh (2014). The values for $\sigma = \sigma^* = 1$ are chosen as a compromise between a likely higher elasticity of substitution between individual goods and a lower elasticity of demand for the goods aggregate. Note that neither ϵ_D nor ϵ_D^* are very sensitive to this choice of σ and σ^* since α is close to 1.

6.1.3 From BAU to Optimal

For any endogenous variable x we denote the value under the optimal policy as $x(p_e, t_b)$, where $t_e^N = \varphi^W$ if $t_b > 0$. In the BAU baseline the value is $x(1, 0)$, denoted simply as x.[31]

Under the optimal policy, Home energy extraction is simply:

$$Q_e(p_e, t_b) = \left(p_e + t_b - \varphi^W\right)^{\epsilon_S} Q_e,$$

for $p_e + t_b - \varphi^W \geq 0$ and $Q_e(p_e, t_b) = 0$ otherwise. Foreign extraction is even simpler:

$$Q_e^*(p_e, t) = p_e^{\epsilon_S^*} Q_e^*,$$

for $p_e \geq 0$ and $Q_e^*(p_e, t) = 0$ otherwise.

The import margin for the optimal policy is unchanged from the BAU baseline, while the export margin changes to:

$$\bar{j}_x(p_e, t_b) = \frac{\left(p_e + (1-\alpha)\, t_b\right)^{\theta} C_e^{FH}}{\left(p_e + (1-\alpha)\, t_b\right)^{\theta} C_e^{FH} + \left(p_e^{\alpha} \left(p_e + t_b\right)^{1-\alpha}\right)^{\theta} C_e^{FF}}.$$

Consumption of energy in Foreign from Foreign production is:

$$C_e^{FF}(p_e, t_b) = p_e^{-\epsilon_D^*} \left(\frac{1 - \bar{j}_x(p_e, t_b)}{1 - \bar{j}_x}\right)^{1 + (1-\sigma^*)/\theta} C_e^{FF}.$$

Using similar derivations we can express the values under the optimal policy for each of the other components of energy demand and for the implicit subsidy to exporters S. See Appendix D for details.

To compute the optimal border adjustment t_b along with the equilibrium energy price p_e, we require that they clear the global energy market and satisfy (22). In particular, we require:

$$C_e^W(p_e, t_b) = Q_e^W(p_e, t_b),$$
$$t_b = \frac{\varphi^W \epsilon_S^* Q_e^*(p_e, t_b) - \sigma^*(1-\alpha) S(p_e, t_b)}{\epsilon_S^* Q_e^*(p_e, t_b) + \epsilon_D^* C_e^{FF}(p_e, t_b)}.$$

Our algorithm simply iterates between the first two equations until we find the vector (p_e, t_b) that satisfies them. We follow similar procedures for the optimal constrained policies.

We can evaluate any outcome of the model at the equilibrium (p_e, t_b) to explore the implications of the optimal policy. A key implication is the welfare benefit of the policy to Home. Our measure starts with the change in the planner's objective, $U(p_e, t_b) - U$. This term is equivalent to increased spending on services by Home,

[31] In the BAU baseline $t_e^N = 0$. While we model specific taxes, their magnitudes have an ad-valorem interpretation relative to the baseline energy price of 1.

since consumption of services enters preferences linearly with price 1. To interpret the magnitude, and to make it scale free, we normalize it by Home baseline spending on goods, $C_e/(1-\alpha)$. The measure we present is thus:

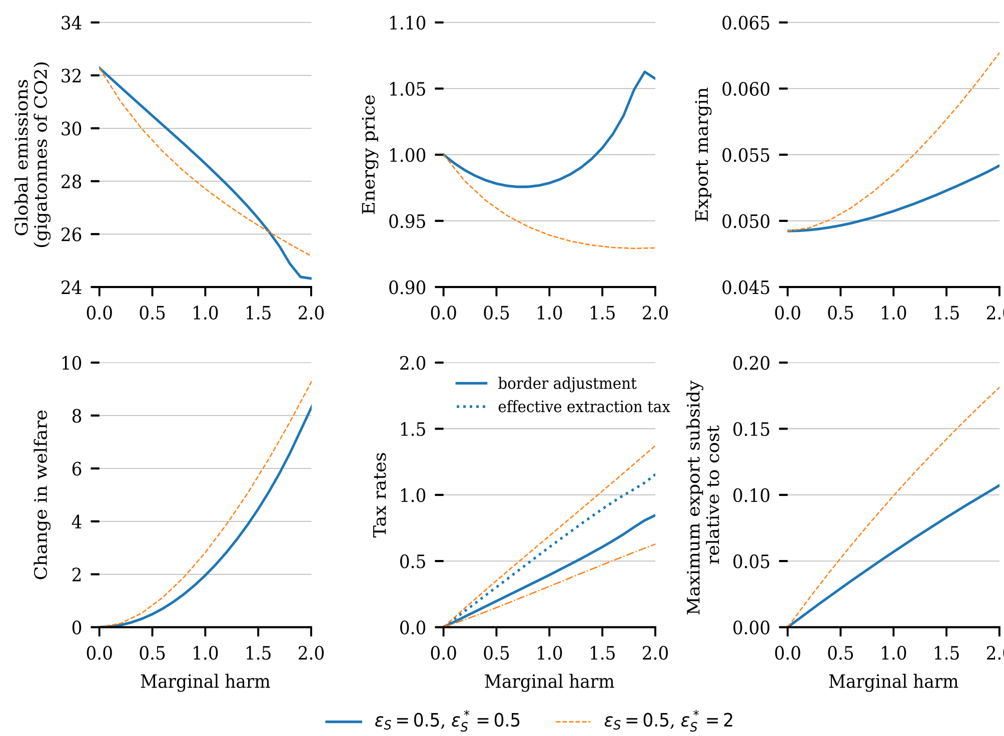
$$W = \frac{1-\alpha}{C_e}\left(U(p_e, t_b) - U\right).$$

Our script is in Matlab. We use the solving procedure described above rather than a built-in solver. Our code is available at `https://github.com/dweisbach/Optimal-Unilateral-Carbon-Policy`.

6.2 Results

6.2.1 Optimal Policy

Figure 1: Optimal Policy in the OECD



We begin with a simulation of the optimal policy in the OECD (Figure 1). We illustrate the policy for marginal harm ranging from $\varphi^W = 0$ to $\varphi^W = 2$, showing the result for our baseline calibration of $\epsilon_S^* = 0.5$ and for $\epsilon_S^* = 2.0$. We show (i) the emissions reductions, (ii) the change in welfare (W), (iii) the change in p_e, (iv) the tax rates under the optimal policy, (v) the change in Home's export margin, $\bar{j}_x$, and (vi) the export subsidy, S.

Focusing on our baseline calibration, global emissions go down by about ¼ when $\varphi^W = 2$, a substantial reduction given that emissions in the OECD are only about ⅓ of global emissions (as reflected in the value of G_e in Table 5). Note that the substantial reduction from the OECD policy does not mean that the OECD's emissions are near zero. Some of the reductions arise in other parts of the world because of how the optimal policy expands the carbon price to trading partners.

Notably, the OECD would choose to impose a significant carbon policy even when the rest of the world does not. For $\varphi^W = 2$, the optimal carbon policy reduces global emissions by 7.6 Gt CO_2. That the OECD would choose these policies on its own may have important implications for the design of climate negotiations: even if one or more countries hold out, it makes sense for the remaining countries to impose a substantial carbon price.

With $\epsilon_s^* = 0.5$, Home relies substantially on the extraction tax. The value of t_e is always higher than t_b, and increasingly so as φ^W goes up. Recalling that the tax rate can be interpreted in the ad-valorem sense, the optimal tax rates range from 0 to up to about 1.5 times the initial price of energy. The OECD's policy, however, still pushes the energy price (top middle) below 1 until φ^W approaches 1.5. For even higher values of φ^W, the net price received by energy extractors in the OECD, $p_e - t_e$, approaches zero. As a result, extraction in the OECD hits zero as φ^W approaches 2, which can be seen in the kink in the lines for high values of φ^W (reflecting a corner solution in the simulation).

Examining the two graphs on the right-hand column of Figure 1, we can see that Home expands its export margin as marginal damages increase. By expanding its export margin, Home is able to broaden the application of its carbon policy, which becomes more important as the marginal harm from emissions increases. This feature of the policy comes at a cost that rises with φ^W.

Our alternative calibration sets $\epsilon_s^* = 2.0$. With a higher foreign elasticity of energy supply, Home is less able to use an extraction tax, because the tax would induce a significant response in Foreign. Instead, Home shifts most of the tax to the demand side: in the bottom middle panel, t_b now exceeds t_e. The value of p_e, correspondingly, goes down. Because Home relies more on demand-side taxes, it adjusts the trade margins more aggressively, as seen in the two right hand panels. Notably, emissions reductions (top left panel) are similar in the two cases. By shifting the mix of taxes and subsidies, the optimal policy is able to achieve roughly the same outcome regardless of the value of ϵ_s^*.

To further examine the features of the optimal policy, we present four simulations that vary different elements of Home's policy.

6.2.2 Coalition Size

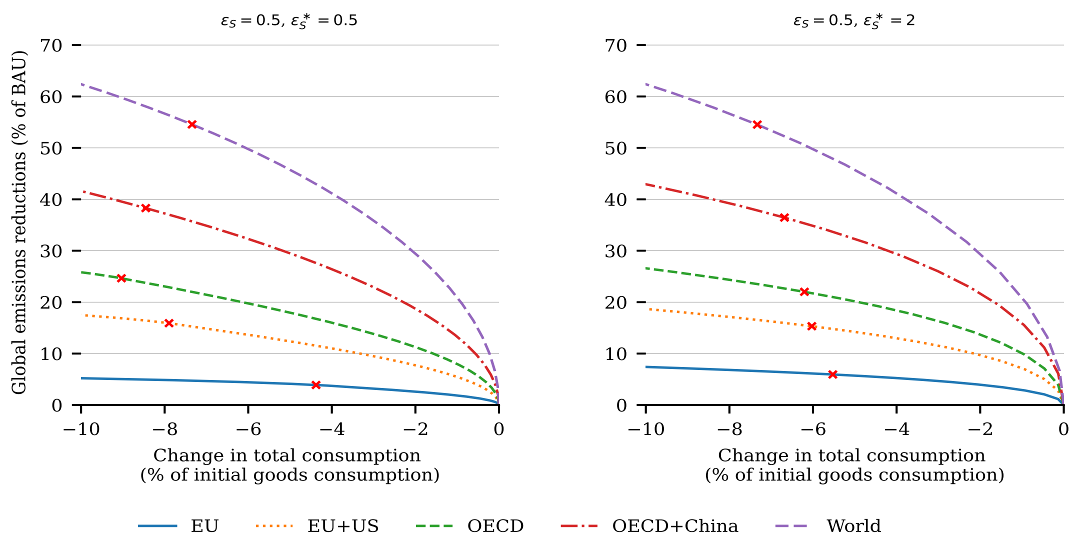
A key factor in global climate negotiations is the set of countries that will agree to emissions reductions. As noted, one of the major criticisms of the Kyoto Protocol was that it left some large emitters out of the emissions reduction coalition. The

Paris Agreement was, in part, designed to address this criticism by trying to achieve universal participation.

To examine the effects of coalition size, Figure 2 shows the effects on global emissions of optimal policies with five increasingly large coalitions, starting with just the EU and eventually going up to a globally harmonized tax.[32] Tables 7, 8 and 9 provide the calibrations for the three new scenarios. We show effects for our baseline calibration of ϵ_s^* (left panel) and our alternative calibration (right panel). All other parameters remain the same across each case.

Figure 2 can be thought of as a production possibility frontier showing the trade-offs between emissions reductions and cost for a given pricing coalition. Cost is measured as the reduced consumption needed to achieve a given percentage reduction in emissions from the 2015 level (32.3 Gt CO_2).[33] The x's in each line show the optimal emissions reduction when $\varphi^W = 2$.

Figure 2: Choice of Pricing Coalition



Both panels show a consistent story, which is that there are substantial gains from expanding the taxing coalition. The EU alone has almost no power to reduce emissions. Adding the United States or the rest of the OECD countries helps significantly. Adding

[32]We treat the global case as the limit of our two-region model as Foreign becomes infinitesimally small. For the EU-only case, we treat the EU as having 28 members as it had, prior to Brexit, in 2015.

[33]Our measure of economic cost of the policy to Home starts with the welfare measure W given above, but adds $\varphi^W \left(Q_e^W(p_e, t_b, t_e) - Q_e^W\right)$ (which is negative) to the numerator. The result is necessarily a negative number, becoming more negative as a larger φ^W leads to greater emissions reductions. This measure is convenient to compute, but implicitly assumes $\varphi^* = 0$. If $\varphi^* > 0$ then we overstate the economic cost to Home by ignoring transfers from Foreign to Home that offset gains to Foreign from reduced global emissions. Given a non-zero value for φ^* it is straightforward to make the necessary adjustment, which would push our measure of economic cost toward zero.

Table 7: Calibration for the European Union

	Home	Foreign	Total
Home	$C_e^{HH} = 3.0$	$C_e^{HF} = 1.0$	$C_e = 4.0$
Foreign	$C_e^{FH} = 0.5$	$C_e^{FF} = 27.8$	$C_e^* = 28.3$
Total	$G_e = 3.5$	$G_e^* = 28.8$	$G_e^W = C_e^W = 32.3$
Extraction	$Q_e = 1.0$	$Q_e^* =31.3$	$Q_e^W = 32.3$

Table 8: Calibration for the EU and the United States

	Home	Foreign	Total
Home	$C_e^{HH} = 7.7$	$C_e^{HF} = 2.0$	$C_e = 9.8$
Foreign	$C_e^{FH} = 0.7$	$C_e^{FF} = 21.8$	$C_e^* = 22.5$
Total	$G_e = 8.5$	$G_e^* = 23.8$	$G_e^W = C_e^W = 32.3$
Extraction	$Q_e = 5.4$	$Q_e^* =26.9$	$Q_e^W = 32.3$

China to the taxing coalition further reduces the costs of any given level of reductions and increases the willingness of the coalition to reduce emissions.

Looking at the calibration tables, we can see that the size of the extraction base is the key difference between the EU and the coalition of the EU and the United States. Production and consumption roughly double, reflecting the relative size of the two economies, but extraction goes up by a factor of more than 5. With almost no extraction, the EU on its own is unable to take advantage of the extraction tax portion of the optimal policy, which means that acting alone, it is ineffective at reducing emissions. Adding the United States expands the extraction base and makes the policy more effective.

Comparing the left and right panels, we can see that regardless of the value of ϵ_s^*, the taxing coalition is able to achieve about the same emissions reductions for a given cost. With the exception of the EU-only tax, however, the taxing coalition is willing to incur a higher cost when ϵ_s^* is low than when it is high. For example, the OECD would choose to reduce emissions by 24.6% at a cost of 9% when $\epsilon_s^* = 0.5$, but would only be willing to spend 6.2% to reduce emissions by 22% when $\epsilon_s^* = 2.0$.

Table 9: Calibration for the OECD plus China

	Home	Foreign	Total
Home	$C_e^{HH} = 20.1$	$C_e^{HF} = 1.7$	$C_e = 21.8$
Foreign	$C_e^{FH} = 1.4$	$C_e^{FF} = 9.1$	$C_e^* = 10.5$
Total	$G_e = 21.5$	$G_e^* = 10.8$	$G_e^W = C_e^W = 32.3$
Extraction	$Q_e = 16.24$	$Q_e^* =16.1$	$Q_e^W = 32.3$

6.2.3 Choice of tax

The top panels of Figure 3 compares the optimal tax to the six constrained optimal taxes, under our baseline calibration and for $\epsilon_S^* = 2.0$.[34] The bottom panels show the effects on p_e for each tax.

With a low value of ϵ_S^*, extraction taxes perform much better than the demand-side taxes (production or consumption taxes, or a hybrid of the two). The bottom panel illustrates why: the extraction tax raises p_e while the demand-side taxes lower it. Increasing p_e in this case induces a demand-side response in Foreign without generating a large supply-side response. In fact, when ϵ_s^* is low, both hybrids involving an extraction tax perform almost as well as the optimal tax.

When $\epsilon_S^* = 2.0$ (the right hand panel) extraction taxes are no longer desirable. Increasing p_e would cause a substantial increase in Foreign extraction, offsetting the effectiveness of the tax. Demand-side taxes are correspondingly more effective because lowering p_e causes a significant reduction in Foreign extraction. For example, the pure production tax goes from an optimal emissions reduction of 4.8% when $\epsilon_s^* = 0.5$ to reductions of 10.6% when $\epsilon_s^* = 2.0$. Looking at the bottom right panel, we can see that Home is less willing to allow p_e to change when ϵ_s^* is high.

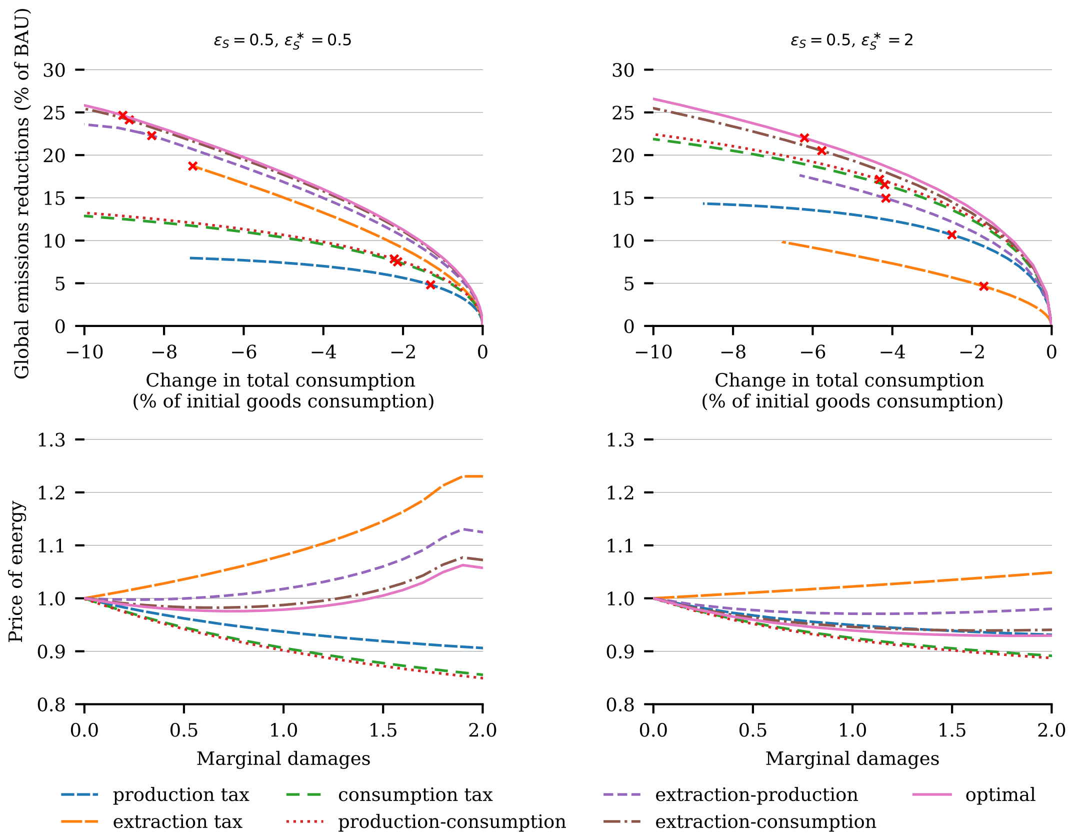
6.2.4 Location

Figure 4 explores the effects of taxes on leakage and other shifts in location, focusing on how activities in Foreign change in response to Home's taxes. It illustrates the optimal tax and four of the constrained taxes (dropping the extraction/consumption hybrid to reduce clutter). It shows the percent changes in Q_e^*, G_e^*, and C_e^* relative to their values with no tax. The bottom right panel shows the change in global emissions that the OECD would choose if it were constrained to using each of these taxes.

Changes to extraction (top left) are consistent with the changes to p_e seen in Figure 3. Extraction taxes drive up p_e and as a result, cause Foreign to increase its

[34]Some of the lines in Figure 3 stop short of a cost of 10%. This is for two reasons. First, we only ran our simulation up to values of $\varphi^W = 20$. Second, Home extraction goes to zero for sufficiently high values of φ^W, so an extraction tax become ineffective beyond that point.

Figure 3: Effects of different taxes on emissions and p_e



extraction. Production and consumption taxes drive p_e down, causing Foreign to reduce its extraction. The optimal tax and the extraction/production hybrid moderate the effects on Foreign extraction.

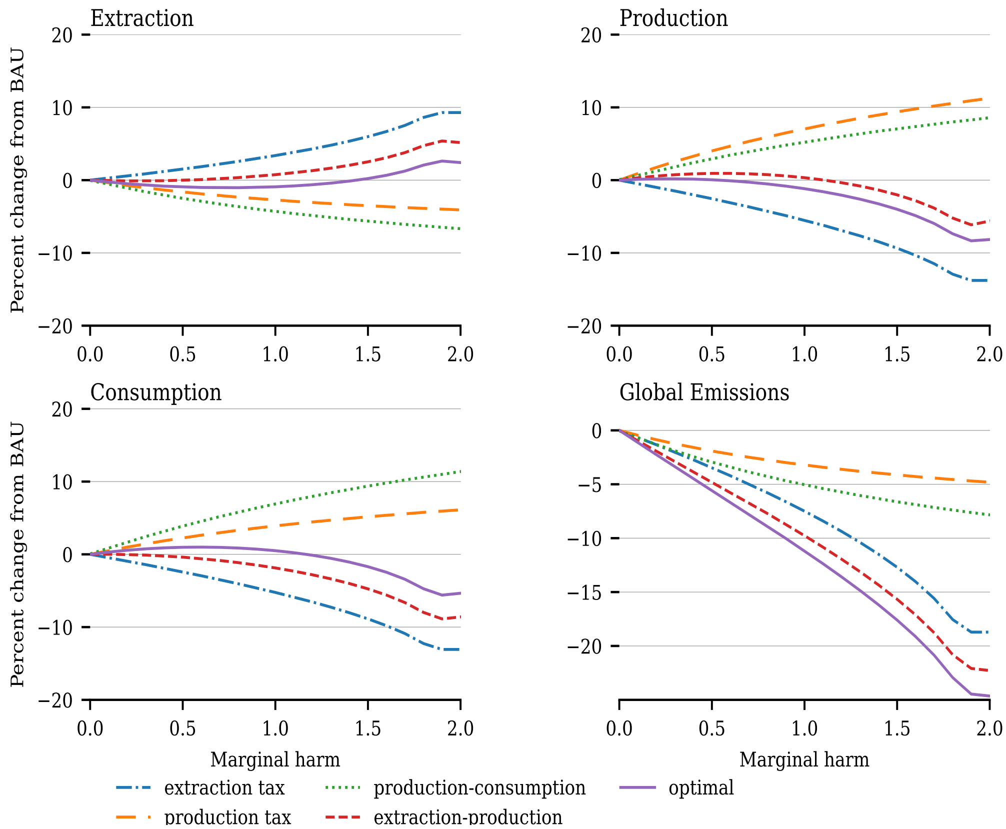
The opposite occurs for G_e^* and C_e^* (top right, bottom left). Because production and consumption taxes drive p_e down, G_e^* and C_e^* both go up when Home imposes those taxes. Correspondingly, Foreign production and consumption both go down when Home imposes an extraction tax. And once again, the optimal and the extraction/production hybrid operate in the middle.

7 Multiple Energy Sources

Up to this point we have assumed that all energy is from fossil fuel with a fixed carbon content. We could therefore normalize a unit of CO_2 to be a unit of energy, treating energy and CO_2 interchangeably. We also assumed that energy is costlessly traded, crude oil being the closest example. Here we briefly explore how our analysis can accommodate a variety of energy sources.

We introduce $K \geq 1$ sources, indexed by k, such as coal, natural gas, and solar.

Figure 4: Effects on Foreign Activities



We assume that these sources are perfect substitutes in providing energy, but may differ in their CO_2 content, h_k. We take $k = 1$ to be crude oil, and normalize $h_1 = 1$. If k is a renewable source, $h_k = 0$. Each source has a corresponding distribution of energy fields, $E_k(a)$ in Home and $E^*_k(a)$ in Foreign.[35] This formulation, in terms of energy fields, fits renewable sources as well since costs of generating solar, wind, and water power are also dictated by scarce geographic factors.

Our basic simplifying assumption is that the world energy market is integrated through trade in oil. Furthermore, we assume that other sources of energy are not tradable. This second assumption rules out potential policy interventions by Home to shift Foreign supply toward sources with lower CO_2 content.[36] While clearly abstracting from critical features of the energy market, these assumptions lead to a simple and intuitive generalization of our analysis above.

[35]In parallel to our assumptions above on $E(a)$ and $E^*(a)$, for any k we take $E_k(a)$ to be a continuous and strictly increasing function on $a \geq \underline{a}_k \geq 0$, with $E(a) = 0$ for $a < \underline{a}_k$. The same applies to $E^*(a)$, with $\underline{a}^*_k \geq 0$ replacing $\underline{a}_k$.

[36]For example, if renewables were tradable, Home might want to import them to stimulate their production in Foreign. If Home imported all that Foreign supplied, it could raise their relative price in Foreign, thus stimulating supply.

The quantity of energy from Home becomes:

$$Q_e = \sum_{k=1}^{K} Q_{e,k},$$

with total CO_2 content per unit of energy of:

$$h = \sum_{k=1}^{K} h_k \frac{Q_{e,k}}{Q_e}.$$

At an energy price p_e Foreign extraction is:

$$Q_e^* = \sum_{k=1}^{K} E_k^*(p_e).$$

The analysis above is the special case of $K = 1$.

7.1 Amendments to the Planner's Problem

This extension requires only a slight amendment to the planner problem. First, the planner now chooses the quantity of extraction of each type, $\{Q_{e,k}\}_{k=1}^{K}$. The inner problem is unchanged but the outer problem must be extended.

The first order condition for energy extraction from source k is:

$$\frac{\partial \mathcal{L}}{\partial Q_{e,k}} = -h_k \varphi^W - \frac{\partial L_e}{\partial Q_{e,k}} + \lambda_e \leq 0,$$

with equality if $Q_e > 0$. The extra labor in Home to extract a bit more energy from source k is the labor requirement on the marginal energy field for that source, $E_k^{-1}(Q_{e,k})$. If $Q_{e,k} > 0$ the first order condition therefore simplifies to:

$$Q_{e,k} = E\left(\lambda_e - h_k \varphi^W\right).$$

If $\lambda_e - h_k \varphi^W \leq \underline{a}_k$ then $Q_{e,k} = 0$, a more likely outcome if the CO_2 content h_k is high.

The first order condition for the energy price becomes:

$$u^{*\prime}(C_g^*) \frac{\partial C_g^*}{\partial p_e} - \varphi^W \sum_{k=1}^{K} h_k \frac{\partial Q_e^*}{\partial p_e} - \sum_{k=1}^{K} \frac{\partial L_{e,k}^*}{\partial p_e} - \frac{\partial L_g}{\partial p_e} - \frac{\partial L_g^*}{\partial p_e}$$
$$= \lambda_e \left(\frac{\partial G_e}{\partial p_e} + \frac{\partial G_e^*}{\partial p_e} - \sum_{k=1}^{K} \frac{\partial Q_{e,k}^*}{\partial p_e} \right),$$

where:

$$\frac{\partial L_{e,k}^*}{\partial p_e} = p_e \frac{\partial E_k^*(p_e)}{\partial p_e}.$$

In its final simplified form, the condition is nearly the same as (20):

$$\lambda_e - p_e = \frac{\varphi^W \tilde{\epsilon}_S^* Q_e^* - \sigma^*(1-\alpha)S}{\tilde{\epsilon}_S^* Q_e^* + \epsilon_D^* C_e^{FF}}. \tag{23}$$

The only new ingredient is that the elasticity of Foreign CO_2 "extraction" has become:

$$\tilde{\epsilon}_S^* = \sum_{k=1}^{K} h_k \epsilon_{S,k}^* \frac{Q_{e,k}^*}{Q_e^*},$$

where:

$$\epsilon_{S,k}^* = \frac{dE_k^*}{dp_e} \frac{p_e}{E_k^*}.$$

The key insight is that a single elasticity $\tilde{\epsilon}_S^*$, which is endogenous to the policy, nonetheless captures all that is relevant about Foreign energy supply in formulating the optimal unilateral policy.

7.2 Amendments to Optimal Taxes

The optimal policy can still be implemented with an extraction tax, a border adjustment, and a subsidy to Home's marginal exporters. The last feature is unchanged by the addition of multiple energy sources.

At first blush it appears that the extraction tax must be specific to each source of energy, at a rate of $t_{e,k} = h_k \varphi^W$ per unit of energy (with $Q_{e,k}$ units in total). The policy is simpler to implement, however, with a common extraction tax of $t_e = \varphi^W$ per unit of CO_2 (with the tax applied to the $h_k Q_{e,k}$ units of CO_2 in energy from source k).

The level of the border adjustment is $t_b = \lambda_e - p_e$, given by (23). It is now applied per unit of energy, not per unit of carbon. The marginal source of energy supply in Foreign is taken into account in determining the level of the border adjustment, but not in its application. Similarly, after applying the extraction tax in Home, in which the rate depends on the CO_2 content, the price of energy to users is $p_e + t_b$ without regard to the source.

Putting the extraction tax and border adjustment together yields further insight. Extraction from source k by Home is:

$$Q_{e,k} = \max\left\{E\left(p_e + t_b - h_k t_e\right), 0\right\}.$$

Extraction from a low-carbon source k will be stimulated under the optimal policy if $h_k < t_b/t_e$. This inequality is satisfied for renewables.

In considering the application of carbon border adjustments, a seemingly intractable issue is whether to take account of the energy source used in producing the imported good. Kortum and Weisbach (2017) discuss this issue at length and argue that what matters is the carbon content of the marginal energy source of the country exporting the good, not the actual source of energy used to produce the good. Equation (23) formalizes that argument. The optimal unilateral policy ignores the energy source for each exporting firm, applies the border adjustment to energy content only, and sets the level of the border adjustment based on the marginal sources of Foreign's energy extraction, captured in $\tilde{\epsilon}_S^*$. What seemed like an intractable issue has a simple solution.

8 Conclusion

While the model in this paper is highly stylized, its simplicity yields analytical insights into the features of an optimal unilateral carbon policy.

To see which features are of first-order importance, it is critical to push the analysis in a more quantitative direction, extending it to multiple countries and perhaps to multiple periods of time as well. For the first extension, the multi-country model of Eaton and Kortum (2002) retains the Ricardian structure of trade in goods used here, while the model of Larch and Wanner (2019) contains a natural multi-country extension of the energy sector. On the second extension, the dynamic analysis in Golosov, Hassler, Krusell, and Tsyvinski (2014) appears amenable to nesting within a multi-country world.

Another important extension, in a multi-country world, is to consider endogenizing the region we call Home. Our current approach follows Markusen (1975) and CDVW in assuming that Foreign is totally passive. Home's optimal policy will likely be different in a setting where it seeks to entice (or coerce) Foreign countries to join its coalition. Such policies are studied by Nordhaus (2015), while Farrokhi and Lashkaripour (2020) have made advances in solving them in more realistic settings.

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A Global Planner's Problem

Suppose the planner controls all decisions in Foreign as well as Home. We pointed out in the paper that we can solve this problem with the same Lagrangian (12), simply enlarging the set of choice variables. (We remove the choice of p_e since it no longer appears in the problem.) For convenience we repeat the Lagrangian here:

$$\begin{aligned}\mathcal{L} = & \int_0^1 u(y_j + m_j)dj + \int_0^1 u^*(y_j^* + x_j)dj - \varphi^W Q_e^W \\ & - L_e^W - \int_0^1 \left(l_j(z_j^y)y_j + \tau l_j(z_j^x)x_j + l_j^*(z_j^*)y_j + \tau^* l_j^*(z_j^m)m_j\right) dj \\ & - \lambda_e \left(\int_0^1 \left(e_j(z_j^y)y_j + \tau e_j(z_j^x)x_j + e_j^*(z_j^*)y_j^* + \tau^* e_j^*(z_j^m)m_j\right) dj - Q_e^W \right).\end{aligned}$$

The global planner chooses Q_e, Q_e^*, $\{y_j\}$, $\{y_j^*\}$, $\{x_j\}$, $\{m_j\}$, $\{z_j^y\}$, $\{z_j^*\}$, $\{z_j^x\}$, and $\{z_j^m\}$ to maximize $\mathcal{L}$.

A.1 Solution

Following CDVW, we first solve the inner problem, involving conditions for an individual good j, given λ_e. We then turn to the outer problem, optimizing over Q_e and Q_e^* while solving for λ_e.

A.1.1 Inner Problem

The inner problem amounts to a Lagrangian for good j (as in the paper):

$$\begin{aligned}\mathcal{L}_j = & u(y_j + m_j) + u^*(y_j^* + x_j) \\ & - \left(l_j(z_j^y)y_j + \tau l_j(z_j^x)x_j + l_j^*(z^*)y_j^* + \tau l_j^*(z_j^m)m_j\right) \\ & - \lambda_e \left(e_j(z_j^y)y_j + \tau e_j(z_j^x)x_j + e_j^*(z^*)y_j^* + \tau^* e_j^*(z_j^m)m_j\right).\end{aligned}$$

We maximize it by choosing: y_j, y_j^*, x_j, m_j, z_j^y, z_j^*, z_j^x, and z_j^m.

The first order conditions for energy intensities of production imply:

$$z_j^y = z_j^x = z_j^* = z_j^m = z = \frac{1-\alpha}{\alpha\lambda_e}.$$

The unit energy requirement in Home is thus:

$$e_j(z) = (1-\alpha)a_j\lambda_e^{-\alpha},$$

while in Foreign:

$$e_j^*(z) = (1-\alpha)a_j^*\lambda_e^{-\alpha}.$$

The FOC for y_j implies:

$$u'(y_j + m_j) \leq a_j \lambda_e^{1-\alpha},$$

with equality if $y_j > 0$. The FOC for m_j implies:

$$u'(y_j + m_j) \leq a_j^* \tau^* \lambda_e^{1-\alpha},$$

with equality if $m_j > 0$. The good $\bar{j}_m$ at which the FOC's for y_j and m_j both hold with equality satisfies:

$$F(\bar{j}_m) = \frac{1}{\tau^*}.$$

Thus, for $j < \bar{j}_m$:

$$y_j = \eta \left(a_j \lambda_e^{1-\alpha}\right)^{-\sigma}$$

and $m_j = 0$ while for $j > \bar{j}_m$:

$$m_j = \eta \left(a_j^* \tau^* \lambda_e^{1-\alpha}\right)^{-\sigma}$$

and $y_j = 0$.

The FOC for y_j^* implies:

$$u^{*\prime}(y_j^* + x_j) \leq a_j^* \lambda_e^{1-\alpha},$$

with equality if $y_j^* > 0$. The FOC for x_j implies:

$$u^{*\prime}(y_j^* + x_j) \leq a_j \tau \lambda_e^{1-\alpha},$$

with equality if $x_j > 0$. The good $\bar{j}_x$ at which the FOC's for y_j^* and x_j both hold satisfies:

$$F(\bar{j}_x) = \tau.$$

Since F is monotonically decreasing, it follows that $\bar{j}_x < \bar{j}_m$. For $j < \bar{j}_x$:

$$x_j = \eta^* \left(a_j \tau \lambda_e^{1-\alpha}\right)^{-\sigma^*}$$

and $y_j^* = 0$ while for $j > \bar{j}_x$:

$$y_j^* = \eta^* \left(a_j^* \lambda_e^{1-\alpha}\right)^{-\sigma^*}$$

and $x_j = 0$.

Aggregating over goods, the implicit consumption of energy in Home is:

$$C_e(\lambda_e) = (1-\alpha)\,\eta \left(\int_0^{\bar{j}_m} a_j^{1-\sigma} dj + (\tau^*)^{1-\sigma} \int_{\bar{j}_m}^1 \left(a_j^*\right)^{1-\sigma} dj \right) \lambda_e^{-\epsilon_D},$$

which is a function of the Lagrange multiplier λ_e. Similarly, in Foreign:

$$C_e^*(\lambda_e) = (1-\alpha)\eta^* \left(\tau^{1-\sigma^*} \int_0^{\bar{j}_x} a_j^{1-\sigma^*} dj + \int_{\bar{j}_x}^1 \left(a_j^*\right)^{1-\sigma^*} dj \right) \lambda_e^{-\epsilon_D^*}.$$

A.1.2 Outer Problem

In the outer problem we choose Q_e and Q_e^* while solving for the value of λ_e that clears the global energy market. We can rewrite the outer Lagrangian in terms of aggregate magnitudes as:

$$\begin{aligned}\mathcal{L} =& u(C_g) + u^*(C_g^*) - \varphi^W (Q_e + Q_e^*) \\ & - \left(L_e + L_e^* + L_g + L_g^*\right) - \lambda_e (C_e + C_e^* - Q_e - Q_e^*) .\end{aligned}$$

The first order condition with respect to Home energy extraction implies:

$$Q_e = E(\lambda_e - \varphi^W),$$

for $\lambda_e - \varphi^W \geq \underline{a}$, or else $Q_e = 0$. Likewise for Foreign energy extraction:

$$Q_e^* = E^*(\lambda_e - \varphi^W),$$

for $\lambda_e - \varphi^W \geq \underline{a}^*$, or else $Q_e^* = 0$. The Lagrange multiplier solves:

$$C_e(\lambda_e) + C_e^*(\lambda_e) = E\left(\lambda_e - \varphi^W\right) + E^*\left(\lambda_e - \varphi^W\right).$$

A.2 Decentralized Global Optimum

We can interpret the solution in terms of a decentralized economy with a price of energy:

$$p_e = \lambda_e.$$

An extraction tax in both countries, equal to global marginal damages from emissions, solves the global externality:

$$t_e = t_e^* = \varphi^W.$$

In this case the nominal and effective tax are the same. Energy extractors in both countries receive an after-tax price of $p_e - \varphi^W$. With a globally harmonized policy, a consumption tax at rate φ^W results in the same outcomes.[37]

A.3 Competitive Equilibrium

In a competitive equilibrium agents ignore the global externality. All outcomes other than global welfare are the same as if we simply set $\varphi^W = 0$ in the decentralized global optimum above, and hence $\lambda_e = p_e$. For later reference, we list the outcomes for any good j in Table 10. We treat this case as our business-as-usual (BAU) baseline.

[37] Inspection of the global market clearing condition for energy shows that extraction and consumption of energy remain the same if we instead set $p_e = \lambda_e + \varphi^W$. This change corresponds to adding full border adjustments, $t_b = t_b^* = \varphi^W$, to a nominal extraction tax, $t_e^N = t_e^{N*} = \varphi^W$, turning it into a consumption tax. Any differences in the distribution of services consumption between these two policies (a global extraction tax versus a global consumption tax) can be addressed with transfers.

Table 10: BAU Competitive Equilibrium (Good-j Outcomes)

	Home		Foreign	
Home	$y_j = \eta \left(a_j p_e^{1-\alpha}\right)^{-\sigma}$	$j < \bar{j}_m$	$m_j = \eta \left(\tau^* a_j^* p_e^{1-\alpha}\right)^{-\sigma}$	$j > \bar{j}_m$
Foreign	$x_j = \eta^* \left(\tau a_j p_e^{1-\alpha}\right)^{-\sigma^*}$	$j < \bar{j}_x$	$y_j^* = \eta^* \left(a_j^* p_e^{1-\alpha}\right)^{-\sigma^*}$	$j > \bar{j}_x$

Trade thresholds: $F(\bar{j}_m) = 1/\tau^*$ and $F(\bar{j}_x) = \tau$

B Home Planner's Problem: Additional Details

Here we provide missing steps from Section 3 of the text, which derives the optimal unilateral policy.

B.1 The Energy-Price Condition

The first order condition with respect to p_e can be written as:

$$\frac{\sigma^*}{\sigma^* - 1} \frac{\partial V_g^*}{\partial p_e} - \varphi^W \frac{\partial Q_e^*}{\partial p_e} - \frac{\partial L_e^*}{\partial p_e} - \frac{\partial L_g}{\partial p_e} - \frac{\partial L_g^*}{\partial p_e} = \lambda_e \left(\frac{\partial G_e}{\partial p_e} + \frac{\partial G_e^*}{\partial p_e} - \frac{\partial Q_e^*}{\partial p_e} \right), \tag{24}$$

where we have introduced:

$$V_g^* = u^{*\prime}(C_g^*) C_g^* = (\eta^*)^{1/\sigma^*} (C_g^*)^{1-1/\sigma^*}.$$

We now turn to the partial derivatives (with respect to the energy price) of the seven aggregate variables that appear in (24): Q_e^*, L_e^*, G_e^*, G_e, L_g, L_g^*, and V_g^*. While we don't make it explicit in our notation that follows, all of these partial derivatives are evaluated at the optimal unilateral policy itself.

B.1.1 Dependence on the Energy Price

Foreign energy extraction depends directly on the energy price via (6), with elasticity ϵ_S^*. The response of Foreign labor employed in the energy sector is:

$$\frac{\partial L_e^*}{\partial p_e} = \frac{\partial L_e^*}{\partial Q_e^*} \frac{\partial Q_e^*}{\partial p_e} = p_e \frac{\partial Q_e^*}{\partial p_e} > 0. \tag{25}$$

Dependence on the energy price is more subtle for the other five aggregates. Since Home directly chooses z, $\bar{j}_m$, $\bar{j}_x$, $\{m_j\}$, and $\{y_j\}$, the envelope theorem allows us treat them as fixed when differentiating the Lagrangian with respect to p_e. From the inner problem, each satisfies its own first-order condition with equality.[38] Furthermore, we

[38]Thus, C_g in (17) does not appear in (24) since it depends only on terms that were optimized in the inner problem.

can take as fixed the unit energy requirement for Home producers, whether supplying the domestic or export market. On the other hand $\{y_j^*\}$ and z^* are not chosen by the planner while for $j \in (j_0, \bar{j}_x]$ the export levels $\{x_j\}$ are optimized at a corner solution. They must be considered in the first order condition. We apply (7) and results in the bottom half of Table 2 to compute the partial derivatives of the five aggregates.

Energy use by Foreign producers:

$$G_e^* = \int_{\bar{j}_x}^{1} e_j^*(z^*) y_j^* dj + \tau^* \int_{\bar{j}_m}^{1} e_j^*(z) m_j dj,$$

depends on the energy price only through the first integral, C_e^{FF} . The partial derivative we seek is therefore:

$$\frac{\partial G_e^*}{\partial p_e} = \frac{\partial C_e^{FF}}{\partial p_e} = -\epsilon_D^* \frac{C_e^{FF}}{p_e} < 0. \tag{26}$$

That is, a change in the energy price affects Foreign's use of energy only through its domestic consumption C_e^{FF} and not through its exports of goods to Home C_e^{HF}. Home has chosen and optimized the determinants of C_e^{HF} ($\bar{j}_m$, m_j, and $z^m = z$).

Energy use by Home producers:

$$G_e = \int_{0}^{\bar{j}_m} e_j(z) y_j dj + \int_{0}^{j_0} \tau e_j(z) x_j dj + \int_{j_0}^{\bar{j}_x} \tau e_j(z) \eta^* \left(a_j^* p_e^{1-\alpha}\right)^{-\sigma^*} dj,$$

depends on the energy price only through the third term (while j_0 also depends on the energy price, its derivative adds to the second term exactly what it subtracts from the third term). The partial derivative we seek is therefore:

$$\frac{\partial G_e}{\partial p_e} = -(1-\alpha)\sigma^* \frac{1}{p_e} \int_{j_0}^{\bar{j}_x} \tau e_j(z) x_j dj. \tag{27}$$

Goods-sector employment is closely related to energy use. In Home:

$$L_g = \frac{G_e}{z} = \frac{\alpha}{1-\alpha} \lambda_e G_e,$$

so that:

$$\frac{\partial L_g}{\partial p_e} = \frac{\alpha}{1-\alpha} \lambda_e \frac{\partial G_e}{\partial p_e}. \tag{28}$$

In Foreign:

$$L_g^* = \frac{C_e^{FF}}{z^*} + \frac{C_e^{HF}}{z} = \alpha \int_{\bar{j}_x}^{1} p_j^* y_j^* dj + \frac{C_e^{HF}}{z}.$$

Since only the first term depends on the price of energy:

$$\frac{\partial L_g^*}{\partial p_e} = \alpha(1-\sigma^*) C_e^{FF}. \tag{29}$$

The new term, V_g^*, can be written as:

$$\begin{aligned} V_g^* = (\eta^*)^{1/\sigma^*}(C_g^*)^{1-1/\sigma^*} &= (\eta^*)^{1/\sigma^*}\int_0^1 (c_j^*)^{(\sigma^*-1)/\sigma^*}dj \\ &= \int_0^{j_0} a_j\tau\lambda_e^{1-\alpha}x_j dj + \int_{j_0}^{\bar{j}_x} p_j^* x_j dj + \int_{\bar{j}_x}^1 p_j^* y_j^* dj. \end{aligned}$$

Since only the last two integrals depend on the energy price, the derivative is:

$$\frac{\partial V_g^*}{\partial p_e} = (1-\alpha)(1-\sigma^*)\frac{1}{p_e}\int_{j_0}^{\bar{j}_x} p_j^* x_j dj + (1-\sigma^*)C_e^{FF}. \tag{30}$$

B.1.2 Restatement of the Optimality Condition

Using the partial derivatives from above, we can simplify the first order condition (24). We start by rewriting it as

$$\lambda_e\left(\frac{\partial Q_e^*}{\partial p_e} - \frac{\partial G_e^*}{\partial p_e}\right) = \frac{\sigma^*}{1-\sigma^*}\frac{\partial V_g^*}{\partial p_e} + \varphi^W\frac{\partial Q_e^*}{\partial p_e} + \frac{\partial L_e^*}{\partial p_e} + \frac{\partial L_g}{\partial p_e} + \frac{\partial L_g^*}{\partial p_e} + \lambda_e\frac{\partial G_e}{\partial p_e}.$$

Applying (25), adding $p_e \partial G_e^*/\partial p_e$ to both sides, and substituting in (28) we get:

$$(\lambda_e - p_e)\left(\frac{\partial Q_e^*}{\partial p_e} - \frac{\partial G_e^*}{\partial p_e}\right) = \frac{\sigma^*}{1-\sigma^*}\frac{\partial V_g^*}{\partial p_e} + \varphi^W\frac{\partial Q_e^*}{\partial p_e} + \frac{\partial L_g^*}{\partial p_e} + p_e\frac{\partial G_e^*}{\partial p_e} + \frac{1}{1-\alpha}\lambda_e\frac{\partial G_e}{\partial p_e}.$$

Substituting in (30), (29), (27), and (26):

$$\begin{aligned} (\lambda_e - p_e)\left(\frac{\partial Q_e^*}{\partial p_e} - \frac{\partial G_e^*}{\partial p_e}\right) =& \varphi^W\frac{\partial Q_e^*}{\partial p_e} + (1-\alpha)\sigma^*\frac{1}{p_e}\int_{j_0}^{\bar{j}_x} p_j^* x_j dj + \sigma^* C_e^{FF} \\ & + \alpha(1-\sigma^*)C_e^{FF} - \epsilon_D^* C_e^{FF} - \lambda_e\sigma^*\frac{1}{p_e}\int_{j_0}^{\bar{j}_x} \tau e_j(z^x)x_j dj. \end{aligned}$$

Combining and cancelling terms:

$$(\lambda_e - p_e)\left(\frac{\partial Q_e^*}{\partial p_e} - \frac{\partial G_e^*}{\partial p_e}\right) = \varphi^W\frac{\partial Q_e^*}{\partial p_e} - \frac{\sigma^*(1-\alpha)}{p_e}\int_{j_0}^{\bar{j}_x}\left(\tau a_j\lambda_e^{1-\alpha} - a_j^* p_e^{1-\alpha}\right)x_j dj.$$

Applying equation (26) and rearranging, we obtain:

$$\left((\lambda_e - \varphi^W) - p_e\right)\frac{\partial Q_e^*}{\partial p_e} = (\lambda_e - p_e)\frac{\partial C_e^{FF}}{\partial p_e} + \int_{j_0}^{\bar{j}_x}\left(\tau a_j\lambda_e^{1-\alpha} - p_j^*\right)\frac{\partial x_j}{\partial p_e}dj,$$

which is (19) in the text.

B.2 Bounds on the Consumption Wedge

We establish a lower bound on $\lambda_e - p_e$ by decomposing the wedges (or subsidies) s_j, for $j \in (j_0, \bar{j}_x)$, which enter (20) through S. Adding and subtracting $\lambda_e e_j^*(z^*)$ from each wedge:

$$s_j = (\lambda_e - p_e)e_j^*(z^*) - \pi_j,$$

where $\pi_j = l_j^*(z^*) + \lambda_e e_j^*(z^*) - \tau a_j \lambda_e^{1-\alpha} > 0$ is the planner's value of global resources saved when a unit of good j is produced in Home and exported rather than being produced in Foreign. Equation (15) shows that π_j is also the derivative of the inner problem with respect to x_j, so is strictly positive for $j < \bar{j}_x$ and zero at $j = \bar{j}_x$.

Substituting this expression for s_j into the overall implicit subsidy S, we can rewrite (20) as:

$$\lambda_e - p_e = \frac{\varphi^W \epsilon_S^* Q_e^* + \sigma^*(1-\alpha)\int_{j_0}^{\bar{j}_x} \pi_j x_j dj}{\epsilon_S^* Q_e^* + \epsilon_D^* C_e^{FF} + \int_{j_0}^{\bar{j}_x} e_j^*(z^*) x_j dj}.$$

The denominator is strictly positive while the numerator is weakly positive, establishing the result that $\lambda_e - p_e \geq 0$. If $\varphi^W \epsilon_S^* Q_e^* = 0$ then $\lambda_e - p_e = 0$, with $j_0 = \bar{j}_x$.

Having shown that $\lambda_e \geq p_e$, it follows that $j_0 \leq \bar{j}_x$ and hence $S > 0$. We get an upper bound on λ_e by using (20) to write:

$$\varphi^W - (\lambda_e - p_e) = \frac{\epsilon_D^* C_e^{FF}}{\epsilon_S^* Q_e^* + \epsilon_D^* C_e^{FF}} \varphi^W + \frac{\sigma^*(1-\alpha)}{\epsilon_S^* Q_e^* + \epsilon_D^* C_e^{FF}} S.$$

The right-hand side is positive, which implies $\lambda_e - p_e \leq \varphi^W$, with a strict inequality if $\varphi^W > 0$.

C Constrained-Optimal Policies

Here we derive the formulas for the constrained-optimal policies that appear in Tables 3-4 of the paper. As in the paper, we first consider extraction and consumption taxes before turning to the more intricate derivations for policies that involve a production tax.

C.1 Pure-Extraction Policy

We constrain the planner to choose only Q_e and p_e. Energy intensities, quantities produced, and quantities consumed of each good j are as in the BAU competitive equilibrium, given p_e. Hence we can skip the inner problem and go directly to the outer problem:

$$\begin{aligned}\mathcal{L} = &\frac{\sigma}{\sigma-1} V_g + \frac{\sigma^*}{\sigma^*-1} V_g^* - \varphi^W (Q_e + Q_e^*) \\ &- L_e - L_e^* - L_g^W - \lambda_e \left(C_e^W - Q_e - Q_e^*\right),\end{aligned}$$

where $V_g = u'(C_g) \cdot C_g$ and $V_g^* = u^{*\prime}(C_g^*) \cdot C_g^*$. All terms but the first are the same as in (17). This first term V_g depends on C_g, which is no longer optimized within the inner problem.

C.1.1 Solution

The first order condition for Q_e is identical to that for the unilaterally optimal policy. For $\lambda_e - \varphi^W \geq \underline{a}$:

$$Q_e = E(\lambda_e - \varphi^W),$$

otherwise $Q_e = 0$.

The first order condition for p_e is:

$$\lambda_e \left(\frac{\partial Q_e^*}{\partial p_e} - \frac{\partial C_e^W}{\partial p_e} \right) = \frac{\sigma}{1-\sigma} \frac{\partial V_g}{\partial p_e} + \frac{\sigma^*}{1-\sigma^*} \frac{\partial V_g^*}{\partial p_e} + \varphi^W \frac{\partial Q_e^*}{\partial p_e} + \frac{\partial L_e^*}{\partial p_e} + \frac{\partial L_g^W}{\partial p_e}.$$

As in Appendix B, we can simplify this condition using results on each partial derivative. Since:

$$V_g = \eta p_e^{(1-\alpha)(1-\sigma)} \left(\int_0^{\bar{j}_m} a_j^{1-\sigma} dj + \int_{\bar{j}_m}^1 \left(\tau^* a_j^* \right)^{1-\sigma} dj \right) = \frac{p_e C_e}{1-\alpha},$$

we have:

$$\frac{\partial V_g}{\partial p_e} = \frac{(1-\alpha)(1-\sigma)}{p_e} V_g = (1-\sigma) C_e.$$

Likewise, we have: $\partial V_g^* / \partial p_e = (1-\sigma^*) C_e^*$. Since:

$$L_g^W = \frac{\alpha}{1-\alpha}(p_e G_e + p_e G_e^*) = \frac{\alpha}{1-\alpha} p_e (C_e + C_e^*) = \alpha \left(V_g + V_g^* \right),$$

we have:

$$\frac{\partial L_g^W}{\partial p_e} = \alpha \left(\frac{\partial V_g}{\partial p_e} + \frac{\partial V_g^*}{\partial p_e} \right) = \alpha(1-\sigma) C_e + \alpha(1-\sigma^*) C_e^*.$$

Finally, we still have $\partial L_e^* / \partial p_e = p_e \partial Q_e^* / \partial p_e$.

Plugging in these partial derivatives, and using $\epsilon_D = \sigma + (1-\sigma)\alpha$ (similarly for ϵ_D^*), the first order condition becomes:

$$\lambda_e \left(\frac{\partial Q_e^*}{\partial p_e} - \frac{\partial C_e^W}{\partial p_e} \right) = \varphi^W \frac{\partial Q_e^*}{\partial p_e} + \epsilon_D C_e + \epsilon_D^* C_e^* + p_e \frac{\partial Q_e^*}{\partial p_e}.$$

We can rewrite this expression as:

$$(\lambda_e - p_e) \left(\frac{\partial Q_e^*}{\partial p_e} - \frac{\partial C_e^W}{\partial p_e} \right) = \varphi^W \frac{\partial Q_e^*}{\partial p_e},$$

hence:

$$\lambda_e - p_e = \varphi^W \frac{\epsilon_S^* Q_e^*}{\epsilon_S^* Q_e^* + \epsilon_D^W C_e^W}.$$

Table 11: Pure-Consumption Policy (Good-j Outcomes)

	Home		Foreign	
Home	$y_j = \eta \left(a_j \lambda_e^{1-\alpha}\right)^{-\sigma}$	$j < \bar{j}_m$	$m_j = \eta \left(\tau^* a_j^* \lambda_e^{1-\alpha}\right)^{-\sigma}$	$j > \bar{j}_m$
Foreign	$x_j = \eta^* \left(\tau a_j p_e^{1-\alpha}\right)^{-\sigma^*}$	$j < \bar{j}_x$	$y_j^* = \eta^* \left(a_j^* p_e^{1-\alpha}\right)^{-\sigma^*}$	$j > \bar{j}_x$

Trade thresholds: $\tau^* F(\bar{j}_m) = 1$ and $F(\bar{j}_x) = \tau$

C.1.2 Decentralization

In a market economy we can impose an extraction tax of $t_e = \varphi^W - (\lambda_e - p_e)$ so that the after-tax price, $p_e - t_e = \lambda_e - \varphi^W$, induces the optimal level of extraction in Home. The extraction tax rate is thus:

$$t_e = \varphi^W \frac{\epsilon_D^W C_e^W}{\epsilon_S^* Q_e^* + \epsilon_D^W C_e^W}.$$

C.2 Pure-Consumption Policy

We now constrain the planner to choose only: $\left\{z_j^y\right\}$, $\left\{z_j^m\right\}$, $\{y_j\}$, $\{m_j\}$, and p_e.

C.2.1 Solution

We first consider the inner problem (conditions for an individual good j given values for p_e and λ_e) then return to the outer problem (optimizing p_e and solving for λ_e).

Inner Problem The terms involving Foreign consumption drop out of the inner problem, as they are determined by p_e, leaving:

$$\mathcal{L}_j = u(y_j + m_j) - l_j(z_j^y) y_j - \tau l_j^*(z_j^m) m_j - \lambda_e \left(e_j(z_j^y) y_j + \tau^* e_j^*(z_j^m) m_j\right).$$

The first order conditions for z_j^y, z_j^m, y_j, and y_j^m will clearly be identical to those for the unilaterally optimal policy. The results, together with the market-determined outcomes, are summarized in Table 11.

All producers serving consumers in Home, whether domestic or foreign, use the same energy intensity, but Home uses a different energy intensity for serving consumers in Home and Foreign (unlike in the unilaterally optimal case). The import and export thresholds are the same as in the BAU competitive equilibrium.

Outer Problem As in the unilaterally optimal problem, we can treat C_g and C_e as constants in the outer problem since they are fully determined by the inner problem. Unlike the unilaterally optimal problem, we can also treat the energy sector as being globally integrated, since in the pure-consumption policy Q_e is determined by the global energy price as in the BAU competitive equilibrium. Taking these properties into account, the outer problem becomes:

$$\mathcal{L} = \frac{\sigma^*}{\sigma^* - 1} V_g^* - \varphi^W Q_e^W - L_e^W - L_g^W - \lambda_e \left(C_e + C_e^* - Q_e^W \right),$$

We maximize this expression with respect to p_e.

The first order condition is:

$$\lambda_e \left(\frac{\partial Q_e^W}{\partial p_e} - \frac{\partial C_e^*}{\partial p_e} \right) = \frac{\sigma^*}{1 - \sigma^*} \frac{\partial V_g^*}{\partial p_e} + \varphi^W \frac{\partial Q_e^W}{\partial p_e} + \frac{\partial L_e^W}{\partial p_e} + \frac{\partial L_g^W}{\partial p_e}.$$

Like the pure extraction case, consumption in Foreign is determined as:

$$V_g^* = \eta^* p_e^{(1-\alpha)(1-\sigma^*)} \left(\int_0^{\bar{j}_x} (\tau a_j)^{1-\sigma^*} dj + \int_{\bar{j}_x}^1 (a_j^*)^{1-\sigma^*} dj \right) = \frac{p_e C_e^*}{1 - \alpha},$$

so that:

$$\frac{\partial V_g^*}{\partial p_e} = \frac{(1-\alpha)(1-\sigma^*)}{p_e} V_g^* = (1 - \sigma^*) C_e^*.$$

Labor used in goods production is affected by the energy price through Foreign demand for goods. Hence:

$$\frac{\partial L_g^W}{\partial p_e} = \alpha \frac{\partial V_g^*}{\partial p_e} = \alpha (1 - \sigma^*) C_e^*.$$

Finally, we have $\partial L_e^W / \partial p_e = p_e \partial Q_e^W / \partial p_e$.

Substituting these results into the first order condition while adding $p_e \partial C_e^* / \partial p_e = -\epsilon_D^* C_e^*$ to both sides:

$$(\lambda_e - p_e) \left(\frac{\partial Q_e^W}{\partial p_e} - \frac{\partial C_e^*}{\partial p_e} \right) = \sigma^* C_e^* + \varphi^W \frac{\partial Q_e^W}{\partial p_e} - \epsilon_D^* C_e^* + \alpha (1 - \sigma^*) C_e^*.$$

All the terms on the right-hand side involving C_e^* cancel out so that we can rewrite this condition as:

$$\lambda_e - p_e = \varphi^W \frac{\epsilon_S^W Q_e^W}{\epsilon_S^W Q_e^W + \epsilon_D^* C_e^*}.$$

C.2.2 Decentralization

In a market economy we can impose a consumption tax of $t_c = \lambda_e - p_e$ so that the after-tax price of energy embodied in goods consumed in Home, $p_e + t_c = \lambda_e$, induces the optimal level of demand. The consumption tax rate is thus:

$$t_c = \varphi^W \frac{\epsilon_S^W Q_e^W}{\epsilon_S^W Q_e^W + \epsilon_D^* C_e^*}.$$

C.3 Extraction-Consumption Hybrid Policy

We now augment the pure-consumption policy by allowing the planner to choose the amount of energy extraction in Home. To solve this problem we need only tweak the pure consumption case by replacing the competitively determined Q_e with the optimally chosen value.

C.3.1 Solution

The inner problem is identical to the optimal pure-consumption policy. The outer problem is only slightly changed (to distinguish between Q_e and Q_e^* as well as L_e and L_e^*):

$$\mathcal{L} = \frac{\sigma^*}{\sigma^* - 1} V_g^* - \varphi^W (Q_e + Q_e^*) - L_e - L_e^* - L_g^W - \lambda_e \left(C_e + C_e^* - Q_e - Q_e^*\right).$$

We solve the Lagrangian by maximizing this expression with respect to Q_e and p_e.

The first order condition for Q_e delivers the same result as for the optimal pure-extraction (and unilaterally optimal) policy. For $\lambda_e - \varphi^W \geq \underline{a}$:

$$Q_e = E(\lambda_e - \varphi^W),$$

otherwise $Q_e = 0$.

The first order condition with respect to p_e becomes:

$$\lambda_e \left(\frac{\partial Q_e^*}{\partial p_e} - \frac{\partial C_e^*}{\partial p_e}\right) = \frac{\sigma^*}{1 - \sigma^*} \frac{\partial V_g^*}{\partial p_e} + \varphi^W \frac{\partial Q_e^*}{\partial p_e} + \frac{\partial L_e^*}{\partial p_e} + \frac{\partial L_g}{\partial p_e} + \frac{\partial L_g^*}{\partial p_e}.$$

We can simplify it just like for the pure-consumption case, but now using $\partial L_e^* / \partial p_e = p_e \partial Q_e^* / \partial p_e$ as in the pure-extraction case. The result is:

$$(\lambda_e - p_e) \left(\frac{\partial Q_e^*}{\partial p_e} - \frac{\partial C_e^*}{\partial p_e}\right) = \sigma^* C_e^* + \varphi^W \frac{\partial Q_e^*}{\partial p_e} - \epsilon_D^* C_e^* + \alpha(1 - \sigma^*) C_e^*.$$

Canceling out the C_e^* terms and rearranging yields:

$$\lambda_e - p_e = \varphi^W \frac{\epsilon_S^* Q_e^*}{\epsilon_S^* Q_e^* + \epsilon_D^* C_e^*}.$$

C.3.2 Decentralization

In a market economy, the optimal consumption tax is:

$$t_c = \lambda_e - p_e = \varphi^W \frac{\epsilon_S^* Q_e^*}{\epsilon_S^* Q_e^* + \epsilon_D^* C_e^*}.$$

Since the optimal nominal extraction tax is $t_e^N = \varphi^W$ (as in the unilaterally optimal policy) the corresponding effective extraction tax is $t_e = \varphi^W - (\lambda_e - p_e)$.

Table 12: Pure-Production Policy (Good-j Outcomes)

	Home		Foreign	
Home	$y_j = \eta \left(a_j(r_e p_e)^{1-\alpha}\right)^{-\sigma}$	$j < \bar{j}_m$	$m_j = \eta \left(\tau^* a_j^* p_e^{1-\alpha}\right)^{-\sigma}$	$j > \bar{j}_m$
Foreign	$x_j = \eta^* \left(\tau a_j(r_e p_e)^{1-\alpha}\right)^{-\sigma^*}$	$j < \bar{j}_x$	$y_j^* = \eta^* \left(a_j^* p_e^{1-\alpha}\right)^{-\sigma^*}$	$j > \bar{j}_x$

Trade thresholds: $F(\bar{j}_m) = (1/\tau^*) r_e^{1-\alpha}$ and $F(\bar{j}_x) = \tau r_e^{1-\alpha}$

C.4 Pure-Production Policy

We constrain the planner to choose p_e and the relative price of energy for Home producers, r_e. Goods prices are thus: $p_j^* = a_j^* p_e^{1-\alpha}$ and $p_j = a_j(r_e p_e)^{1-\alpha}$ with $p_j^m = \tau^* p_j^*$ and $p_j^x = \tau p_j$.

We get the export margin by equating p_j^x with p_j^* at $j = \bar{j}_x$:

$$F(\bar{j}_x) = \tau r_e^{1-\alpha}.$$

For any good $j < \bar{j}_x$ the quantity of Home exports demanded by Foreign is:

$$x_j = \eta^*(\tau a_j(r_e p_e)^{1-\alpha})^{-\sigma^*}$$

while $y_j^* = 0$. For any good $j > \bar{j}_x$ the quantity demanded by Foreign from its local producers is:

$$y_j^* = \eta^*(a_j^* p_e^{1-\alpha})^{-\sigma^*}$$

while $x_j = 0$.

We get the import margin by equating p_j^m with p_j at $j = \bar{j}_m$:

$$F(\bar{j}_m) = \frac{1}{\tau^*} r_e^{1-\alpha}.$$

For any good $j > \bar{j}_m$, Home imports:

$$m_j = \eta(\tau^* a_j^* p_e^{1-\alpha})^{-\sigma}$$

while $y_j = 0$. For any good $j < \bar{j}_m$ Home purchases:

$$y_j = \eta(a_j(r_e p_e)^{1-\alpha})^{-\sigma}$$

from local producers, while $m_j = 0$.

Table 12 summarizes these results. The intensive margin of demand for goods produced in Home depends on $r_e p_e$, the intensive margin for goods produced in Foreign depends only on p_e, and the extensive margins of trade depend only on r_e.

C.4.1 Aggregates and Derivatives

We can compute aggregates that depend on r_e and p_e. A term that arises repeatedly in differentiating these aggregates with respect to r_e is the energy Home uses to produce a good at the import threshold:

$$e_{\bar{j}_m} y_{\bar{j}_m} = \left.(e_j(z) y_j)\right|_{j=\bar{j}_m} = (1-\alpha)\eta (p_e r_e)^{-\epsilon_D} \left(a_{\bar{j}_m}\right)^{1-\sigma}$$

and at the export threshold:

$$\tau e_{\bar{j}_x} x_{\bar{j}_x} = \tau \left.(e_j(z) x_j)\right|_{j=\bar{j}_x} = (1-\alpha)\eta^* (p_e r_e)^{-\epsilon_D^*} \left(\tau a_{\bar{j}_x}\right)^{1-\sigma^*}.$$

The following list reports the key aggregates and their derivatives:

1. Energy embodied in goods that Home produces for itself:

$$C_e^{HH} = \int_0^1 e_j(z) y_j dj = (1-\alpha)\eta (p_e r_e)^{-\epsilon_D} \int_0^{\bar{j}_m} a_j^{1-\sigma} dj$$

$$\frac{\partial C_e^{HH}}{\partial p_e} = -\epsilon_D \frac{C_e^{HH}}{p_e}$$

$$\frac{\partial C_e^{HH}}{\partial r_e} = -\epsilon_D \frac{C_e^{HH}}{r_e} + e_{\bar{j}_m} y_{\bar{j}_m} \frac{\partial \bar{j}_m}{\partial r_e}.$$

2. Energy embodied in goods that Home imports:

$$C_e^{HF} = \int_0^1 \tau^* e_j^*(z^*) m_j dj = (1-\alpha)\eta p_e^{-\epsilon_D} \int_{\bar{j}_m}^1 \left(\tau^* a_j^*\right)^{1-\sigma} dj$$

$$\frac{\partial C_e^{HF}}{\partial p_e} = -\epsilon_D \frac{C_e^{HF}}{p_e}$$

$$\frac{\partial C_e^{HF}}{\partial r_e} = -(1-\alpha)\eta p_e^{-\epsilon_D} \left(\tau^* a_{\bar{j}_m}^*\right)^{1-\sigma} \frac{\partial \bar{j}_m}{\partial r_e} = -r_e e_{\bar{j}_m} y_{\bar{j}_m} \frac{\partial \bar{j}_m}{\partial r_e}.$$

(The last equality applies the expression for the import threshold.)

3. The cost of the energy embodied in Home consumption:

$$V_g = \frac{p_e}{1-\alpha}(r_e C_e^{HH} + C_e^{HF})$$

$$\frac{\partial V_g}{\partial p_e} = \frac{V_g}{p_e} - \frac{p_e}{1-\alpha}\left(r_e \epsilon_D \frac{C_e^{HH}}{p_e} + \epsilon_D \frac{C_e^{HF}}{p_e}\right) = (1-\sigma)(1-\alpha)\frac{V_g}{p_e}$$

$$\frac{\partial V_g}{\partial r_e} = \frac{p_e}{1-\alpha}\left(C_e^{HH} + r_e \frac{\partial C_e^{HH}}{\partial r_e} + \frac{\partial C_e^{HF}}{\partial r_e}\right) = (1-\sigma) p_e C_e^{HH}.$$

4. Energy embodied in the goods that Home exports:

$$C_e^{FH} = \int_0^1 \tau e_j(z) x_j dj = (1-\alpha)\eta^* (p_e r_e)^{-\epsilon_D^*} \int_0^{\bar{j}_x} (\tau a_j)^{1-\sigma^*} dj$$

$$\frac{\partial C_e^{FH}}{\partial p_e} = -\epsilon_D^* \frac{C_e^{FH}}{p_e}$$

$$\frac{\partial C_e^{FH}}{\partial r_e} = -\epsilon_D^* \frac{C_e^{FH}}{r_e} + \tau e_{\bar{j}_x} x_{\bar{j}_x} \frac{\partial \bar{j}_x}{\partial r_e}.$$

5. Energy embodied in goods that Foreign produces for itself:

$$C_e^{FF} = \int_0^1 e_j^*(z^*) y_j^* dj (1-\alpha)\eta^* p_e^{-\epsilon_D^*} \int_{\bar{j}_x}^1 \left(a_j^*\right)^{1-\sigma^*} dj$$

$$\frac{\partial C_e^{FF}}{\partial p_e} = -\epsilon_D^* \frac{C_e^{FF}}{p_e}$$

$$\frac{\partial C_e^{FF}}{\partial r_e} = -(1-\alpha)\eta^* p_e^{-\epsilon_D^*} \left(a_{\bar{j}_x}^*\right)^{1-\sigma^*} \frac{\partial \bar{j}_x}{\partial r_e} = -r_e \tau e_{\bar{j}_x} x_{\bar{j}_x} \frac{\partial \bar{j}_x}{\partial r_e}.$$

(The last equality applies the expression for the export threshold.)

6. The cost of the energy embodied in Foreign consumption:

$$V_g^* = \frac{p_e}{1-\alpha}(r_e C_e^{FH} + C_e^{FF})$$

$$\frac{\partial V_g^*}{\partial p_e} = (1-\epsilon_D^*)\frac{V_g^*}{p_e} = (1-\sigma^*)(1-\alpha)\frac{V_g^*}{p_e}$$

$$\frac{\partial V_g^*}{\partial r_e} = \frac{p_e}{1-\alpha}\left(C_e^{FH} + r_e \frac{\partial C_e^{FH}}{\partial r_e} + \frac{\partial C_e^{FF}}{\partial r_e}\right) = (1-\sigma^*) p_e C_e^{FH}.$$

7. Labor employed globally in goods production:

$$L_g^W = L_g + L_g^* = \alpha\left(V_g + V_g^*\right)$$

$$\frac{\partial L_g^W}{\partial p_e} = \alpha(1-\epsilon_D)\frac{V_g}{p_e} + \alpha(1-\epsilon_D^*)\frac{V_g^*}{p_e}$$

$$\frac{\partial L_g^W}{\partial r_e} = \alpha(1-\sigma) p_e C_e^{HH} + \alpha(1-\sigma^*) p_e C_e^{FH}.$$

8. Energy used in Home to produce goods:

$$G_e = C_e^{HH} + C_e^{FH}$$

$$\frac{\partial G_e}{\partial p_e} = -\epsilon_D \frac{C_e^{HH}}{p_e} - \epsilon_D^* \frac{C_e^{FH}}{p_e}$$

$$\frac{\partial G_e}{\partial r_e} = -\epsilon_D \frac{C_e^{HH}}{r_e} - \epsilon_D^* \frac{C_e^{FH}}{r_e} + \left(e_{\bar{j}_m} y_{\bar{j}_m} \frac{\partial \bar{j}_m}{\partial r_e} + \tau e_{\bar{j}_x} x_{\bar{j}_x} \frac{\partial \bar{j}_x}{\partial r_e}\right).$$

9. Energy used in Foreign to produce goods:

$$G_e^* = C_e^{HF} + C_e^{FF}$$

$$\frac{\partial G_e^*}{\partial p_e} = -\epsilon_D \frac{C_e^{HF}}{p_e} - \epsilon_D^* \frac{C_e^{FF}}{p_e}$$

$$\frac{\partial G_e^*}{\partial r_e} = -r_e \left(e_{\bar{j}_m} y_{\bar{j}_m} \frac{\partial \bar{j}_m}{\partial r_e} + \tau e_{\bar{j}_x} x_{\bar{j}_x} \frac{\partial \bar{j}_x}{\partial r_e} \right).$$

10. Energy used globally to produce goods:

$$C_e^W = G_e + G_e^* = C_e + C_e^* = \left(C_e^{HH} + C_e^{HF}\right) + \left(C_e^{FH} + C_e^{FF}\right)$$

$$\frac{\partial C_e^W}{\partial p_e} = -\epsilon_D \frac{C_e}{p_e} - \epsilon_D^* \frac{C_e^*}{p_e}$$

$$\frac{\partial C_e^W}{\partial r_e} = -\epsilon_D \frac{C_e^{HH}}{r_e} - \epsilon_D^* \frac{C_e^{FH}}{r_e} - (r_e - 1) \left(e_{\bar{j}_m} y_{\bar{j}_m} \frac{\partial \bar{j}_m}{\partial r_e} + \tau e_{\bar{j}_x} x_{\bar{j}_x} \frac{\partial \bar{j}_x}{\partial r_e} \right).$$

11. Combining results above:

$$\frac{\partial C_e^W}{\partial r_e} = \frac{\partial G_e}{\partial r_e} + \frac{\partial G_e^*}{\partial r_e}$$

$$p_e \frac{\partial G_e}{\partial p_e} = r_e \frac{\partial G_e}{\partial r_e} + \frac{\partial G_e^*}{\partial r_e}.$$

The last set of results can be combined into a production leakage term, Λ. Due to a rise in r_e, the ratio of the increase in Foreign use of energy relative to the decline in Home use of energy is:

$$\Lambda = \frac{-\partial G_e^*/\partial r_e}{\partial G_e/\partial r_e} = \frac{r_e \left(e_{\bar{j}_m} y_{\bar{j}_m} \frac{\partial \bar{j}_m}{\partial r_e} + \tau e_{\bar{j}_x} x_{\bar{j}_x} \frac{\partial \bar{j}_x}{\partial r_e} \right)}{e_{\bar{j}_m} y_{\bar{j}_m} \frac{\partial \bar{j}_m}{\partial r_e} + \tau e_{\bar{j}_x} x_{\bar{j}_x} \frac{\partial \bar{j}_x}{\partial r_e} - \epsilon_D \frac{C_e^{HH}}{r_e} - \epsilon_D^* \frac{C_e^{FH}}{r_e}}. \tag{31}$$

C.4.2 Solution

The outer problem is:

$$\mathcal{L} = \frac{\sigma}{\sigma - 1} V_g + \frac{\sigma^*}{\sigma^* - 1} V_g^* - \varphi^W Q_e^W - L_e^W - L_g^W - \lambda_e \left(C_e^W - Q_e^W \right).$$

We maximize it over r_e and p_e.

The first order condition for p_e is:

$$\lambda_e \left(\frac{\partial Q_e^W}{\partial p_e} - \frac{\partial C_e^W}{\partial p_e} \right) = \frac{\sigma}{1 - \sigma} \frac{\partial V_g}{\partial p_e} + \frac{\sigma^*}{1 - \sigma^*} \frac{\partial V_g^*}{\partial p_e} + \varphi^W \frac{\partial Q_e^W}{\partial p_e} + \frac{\partial L_e^W}{\partial p_e} + \frac{\partial L_g^W}{\partial p_e}.$$

Using the derivatives enumerated above it simplifies to:

$$\lambda_e\left(\frac{\partial Q_e^W}{\partial p_e}-\frac{\partial C_e^W}{\partial p_e}\right)=\epsilon_D\left(r_eC_e^{HH}+C_e^{HF}\right)+\epsilon_D^*\left(r_eC_e^{FH}+C_e^{FF}\right)+\varphi^W\frac{\partial Q_e^W}{\partial p_e}+\frac{\partial L_e^W}{\partial p_e}.$$

Substituting in $\partial L_e^W/\partial p_e = p_e \partial Q_e^W/\partial p_e$ and rearranging:

$$(\lambda_e-p_e)\left(\frac{\partial Q_e^W}{\partial p_e}-\frac{\partial C_e^W}{\partial p_e}\right)=\varphi^W\frac{\partial Q_e^W}{\partial p_e}+\epsilon_D(r_e-1)C_e^{HH}+\epsilon_D^*(r_e-1)C_e^{FH},$$

from which we can derive the expression in the paper:

$$\left((\lambda_e-\varphi^W)-p_e\right)\frac{\partial Q_e^W}{\partial p_e}=(\lambda_e-r_ep_e)\frac{\partial G_e}{\partial p_e}+(\lambda_e-p_e)\frac{\partial G_e^*}{\partial p_e}. \tag{32}$$

The first order condition for r_e is:

$$-\lambda_e\frac{\partial C_e^W}{\partial r_e}=\frac{\sigma}{1-\sigma}\frac{\partial V_g}{\partial r_e}+\frac{\sigma^*}{1-\sigma^*}\frac{\partial V_g^*}{\partial r_e}+\frac{\partial L_g^W}{\partial r_e}.$$

Using the derivatives enumerated above it simplifies to:

$$-\lambda_e\frac{\partial C_e^W}{\partial r_e}=\sigma p_eC_e^{HH}+\sigma^*p_eC_e^{FH}+\alpha(1-\sigma)p_eC_e^{HH}+\alpha(1-\sigma^*)p_eC_e^{FH}.$$

Combining terms we get:

$$-\lambda_e\frac{\partial C_e^W}{\partial r_e}=\epsilon_Dp_eC_e^{HH}+\epsilon_D^*p_eC_e^{FH}$$

and hence:

$$\frac{\lambda_e}{p_e}\frac{\partial C_e^W}{\partial r_e}=p_e\frac{\partial G_e}{\partial p_e}.$$

We substitute the last two derivatives enumerated above into each side of this equation to get:

$$\frac{\lambda_e}{p_e}\left(\frac{\partial G_e}{\partial r_e}+\frac{\partial G_e^*}{\partial r_e}\right)=r_e\frac{\partial G_e}{\partial r_e}+\frac{\partial G_e^*}{\partial r_e}.$$

Rearranging delivers:

$$(\lambda_e-r_ep_e)\frac{\partial G_e}{\partial r_e}=(p_e-\lambda_e)\frac{\partial G_e^*}{\partial r_e}.$$

The optimal r_e balances the two wedges on the right hand side of (32).

From (31) we have:

$$\frac{\lambda_e-r_ep_e}{\lambda_e-p_e}=\Lambda.$$

Substituting into (32) yields:

$$\begin{aligned}\lambda_e-p_e &= \varphi^W\frac{\partial Q_e^W/\partial p_e}{\partial Q_e^W/\partial p_e-\partial G_e^*/\partial p_e-\Lambda\partial G_e/\partial p_e}\\ &= \varphi^W\frac{\epsilon_S^WQ_e^W}{\epsilon_S^WQ_e^W+\epsilon_G^*G_e^*+\Lambda\epsilon_GG_e}.\end{aligned}$$

Since $\Lambda \geq 0$ it's clear that $\lambda_e \geq p_e$ and hence $\lambda_e \geq r_ep_e$ as well.

C.4.3 Decentralization

In a market economy we can impose a production tax of $t_p = r_e p_e - p_e$ so that the after-tax price of energy used to produce goods in Home, $p_e + t_p = r_e p_e$, induces the optimal energy intensity. The production tax rate is thus:

$$t_p \;=\; r_e p_e - p_e = (\lambda_e - p_e) - (\lambda_e - r_e p_e) = \varphi^W \frac{(1-\Lambda)\epsilon_S^W Q_e^W}{\epsilon_S^W Q_e^W + \epsilon_G^* G_e^* + \Lambda \epsilon_G G_e}.$$

In the case of no trade in goods there is no leakage and the pure-production tax becomes the same as the pure-consumption tax.

C.5 Extraction-Production Hybrid Policy

Suppose we augment the pure-production policy by allowing the planner to choose Q_e. We now need to distinguish between Q_e and Q_e^* as well as between L_e and L_e^*.

C.5.1 Solution

The outer problem looks the same as for the pure extraction policy:

$$\begin{aligned}\mathcal{L} = & \frac{\sigma}{\sigma-1}V_g + \frac{\sigma^*}{\sigma^*-1}V_g^* - \varphi^W\left(Q_e + Q_e^*\right) \\ & - L_e - L_e^* - L_g^W - \lambda_e\left(C_e^W - Q_e - Q_e^*\right).\end{aligned}$$

We want to maximize this Lagrangian over Q_e, r_e, and p_e.

The first order condition for Q_e is identical to that for the unilaterally optimal policy. For $\lambda_e - \varphi^W \geq \underline{a}$:

$$Q_e = E(\lambda_e - \varphi^W),$$

otherwise $Q_e = 0$.

The first order condition for p_e is:

$$\lambda_e\left(\frac{\partial Q_e^*}{\partial p_e} - \frac{\partial C_e^W}{\partial p_e}\right) = \frac{\sigma}{1-\sigma}\frac{\partial V_g}{\partial p_e} + \frac{\sigma^*}{1-\sigma^*}\frac{\partial V_g^*}{\partial p_e} + \varphi^W\frac{\partial Q_e^*}{\partial p_e} + \frac{\partial L_e^*}{\partial p_e} + \frac{\partial L_g^W}{\partial p_e},$$

which is the same as the pure-production policy except that $\partial Q_e^*/\partial p_e$ is in place of $\partial Q_e^W/\partial p_e$ and $\partial L_e^*/\partial p_e$ is in place of $\partial L_e^W/\partial p_e$. We can thus jump to the expression in the paper:

$$\left((\lambda_e - \varphi^W) - p_e\right)\frac{\partial Q_e^*}{\partial p_e} = (\lambda_e - r_e p_e)\frac{\partial G_e}{\partial p_e} + (\lambda_e - p_e)\frac{\partial G_e^*}{\partial p_e}.$$

The first order condition for r_e is unchanged from the pure-production policy:

$$\frac{\lambda_e - r_e p_e}{\lambda_e - p_e} = \Lambda.$$

Table 13: Production-Consumption Hybrid Policy (Good-j Outcomes)

	Home		Foreign	
Home	$y_j = \eta\left(a_j\lambda_e^{1-\alpha}\right)^{-\sigma}$	$j < \bar{j}_m$	$m_j = \eta\left(\tau^* a_j^* \lambda_e^{1-\alpha}\right)^{-\sigma}$	$j > \bar{j}_m$
Foreign	$x_j = \eta^*\left(\tau a_j (r_e p_e)^{1-\alpha}\right)^{-\sigma^*}$	$j < \bar{j}_x$	$y_j^* = \eta^*\left(a_j^* p_e^{1-\alpha}\right)^{-\sigma^*}$	$j > \bar{j}_x$

Trade thresholds: $F(\bar{j}_m) = 1/\tau^*$ and $F(\bar{j}_x) = \tau r_e^{1-\alpha}$

Combining the two:

$$\begin{aligned}\lambda_e - p_e &= \varphi^W \frac{\partial Q_e^*/\partial p_e}{\partial Q_e^*/\partial p_e - \partial G_e^*/\partial p_e - \Lambda \partial G_e/\partial p_e} \\ &= \varphi^W \frac{\epsilon_S^* Q_e^*}{\epsilon_S^* Q_e^* + \epsilon_G^* G_e^* + \Lambda \epsilon_G G_e}.\end{aligned}$$

C.5.2 Decentralization

In a market economy, the optimal production tax rate is:

$$t_p = r_e p_e - p_e = (\lambda_e - p_e) - (\lambda_e - r_e p_e) = \varphi^W \frac{(1-\Lambda)\epsilon_S^* Q_e^*}{\epsilon_S^* Q_e^* + \epsilon_G^* G_e^* + \Lambda \epsilon_G G_e}.$$

From the first order condition for Q_e we know that the after-tax price received by extractors must satisfy:

$$r_e p_e - t_e^N = p_e - t_e = \lambda_e - \varphi^W.$$

The optimal nominal extraction tax is thus:

$$t_e^N = \varphi^W - (\lambda_e - r_e p_e) = \varphi^W - \Lambda(\lambda_e - p_e) = \varphi^W - \frac{\Lambda}{1-\Lambda} t_p,$$

while the corresponding effective extraction tax is:

$$t_e = t_e^N - t_p = \varphi^W - \frac{t_p}{1-\Lambda}.$$

C.6 Production-Consumption Hybrid Policy

We now augment the pure-consumption policy by allowing the planner to choose r_e, where $r_e p_e$ is the cost of energy for producing Home's exports. Since the choice of r_e doesn't interact with any of the good-specific choices under the pure-consumption policy, we simply summarize the results for a particular good j in Table 13.

C.6.1 Aggregates and Derivatives

Using these results we compute aggregates that depend on r_e and p_e. A term that arises in differentiating these aggregates with respect to r_e is the energy Home uses to produce a good at the export threshold:

$$\tau e_{\bar{j}_x} x_{\bar{j}_x} = \tau \left. \left(e_j(z) x_j\right)\right|_{j=\bar{j}_x} = (1-\alpha)\eta^*(p_e r_e)^{-\epsilon_D^*} \left(\tau a_{\bar{j}_x}\right)^{1-\sigma^*}.$$

The following list of aggregates and derivatives is shorter than that for the pure-production case, since some of the aggregates are fully determined by the planner's control of Home consumption:

1. Energy embodied in the goods that Home exports:

$$C_e^{FH} = \int_0^1 \tau e_j(z) x_j dj = (1-\alpha)\eta^*(p_e r_e)^{-\epsilon_D^*} \int_0^{\bar{j}_x} (\tau a_j)^{1-\sigma^*} dj$$

$$\frac{\partial C_e^{FH}}{\partial p_e} = -\epsilon_D^* \frac{C_e^{FH}}{p_e}$$

$$\frac{\partial C_e^{FH}}{\partial r_e} = -\epsilon_D^* \frac{C_e^{FH}}{r_e} + \tau e_{\bar{j}_x} x_{\bar{j}_x} \frac{\partial \bar{j}_x}{\partial r_e}.$$

2. Energy embodied in goods that Foreign produces for itself:

$$C_e^{FF} = \int_0^1 e_j^*(z^*) y_j^* dj (1-\alpha)\eta^* p_e^{-\epsilon_D^*} \int_{\bar{j}_x}^1 \left(a_j^*\right)^{1-\sigma^*} dj$$

$$\frac{\partial C_e^{FF}}{\partial p_e} = -\epsilon_D^* \frac{C_e^{FF}}{p_e}$$

$$\frac{\partial C_e^{FF}}{\partial r_e} = -(1-\alpha)\eta^* p_e^{-\epsilon_D^*} \left(a_{\bar{j}_x}^*\right)^{1-\sigma^*} \frac{\partial \bar{j}_x}{\partial r_e} = -r_e \tau e_{\bar{j}_x} x_{\bar{j}_x} \frac{\partial \bar{j}_x}{\partial r_e}.$$

(The last equality applies the expression for the export threshold.)

3. The cost of the energy embodied in Foreign consumption:

$$V_g^* = \frac{p_e}{1-\alpha}(r_e C_e^{FH} + C_e^{FF})$$

$$\frac{\partial V_g^*}{\partial p_e} = (1-\epsilon_D^*)\frac{V_g^*}{p_e} = (1-\sigma^*)(1-\alpha)\frac{V_g^*}{p_e}$$

$$\frac{\partial V_g^*}{\partial r_e} = \frac{p_e}{1-\alpha}\left(C_e^{FH} + r_e \frac{\partial C_e^{FH}}{\partial r_e} + \frac{\partial C_e^{FF}}{\partial r_e}\right) = (1-\sigma^*) p_e C_e^{FH}.$$

4. Labor employed globally in goods production:

$$L_g^W = L_g + L_g^* = \alpha \left(V_g + V_g^* \right)$$

$$\frac{\partial L_g^W}{\partial p_e} = \alpha(1 - \epsilon_D^*) \frac{V_g^*}{p_e}$$

$$\frac{\partial L_g^W}{\partial r_e} = \alpha(1 - \sigma^*) p_e C_e^{FH}.$$

(Note that V_g is invariant to p_e or r_e.)

5. Energy consumed globally:

$$C_e^W = C_e^{HH} + C_e^{HF} + C_e^{FH} + C_e^{FF}$$

$$\frac{\partial C_e^W}{\partial p_e} = \frac{\partial C_e^*}{\partial p_e} = -\epsilon_D^* \frac{C_e^*}{p_e}$$

$$\frac{\partial C_e^W}{\partial r_e} = \frac{\partial C_e^{FH}}{\partial r_e} + \frac{\partial C_e^{FF}}{\partial r_e} = -\epsilon_D^* \frac{C_e^{FH}}{r_e} - (r_e - 1) \tau e_{\bar{j}_x} x_{\bar{j}_x} \frac{\partial \bar{j}_x}{\partial r_e}.$$

6. Combining results above:

$$p_e \frac{\partial C_e^{FH}}{\partial p_e} = r_e \frac{\partial C_e^{FH}}{\partial r_e} + \frac{\partial C_e^{FF}}{\partial r_e}.$$

The production leakage term becomes:

$$\Lambda = \frac{-\partial C_e^{FF} / \partial r_e}{\partial C_e^{FH} / \partial r_e} = \frac{r_e \tau e_{\bar{j}_x} x_{\bar{j}_x} \frac{\partial \bar{j}_x}{\partial r_e}}{\tau e_{\bar{j}_x} x_{\bar{j}_x} \frac{\partial \bar{j}_x}{\partial r_e} - \epsilon_D^* \frac{C_e^{FH}}{r_e}}, \tag{33}$$

which now depends only on terms related to Home exports.

C.6.2 Solution

The outer problem is:

$$\mathcal{L} = \frac{\sigma^*}{\sigma^* - 1} V_g^* - \varphi^W Q_e^W - L_e^W - L_g^W - \lambda_e \left(C_e^W - Q_e^W \right).$$

We want to maximize this Lagrangian over r_e, and p_e.

The first order condition for p_e is:

$$\lambda_e \left(\frac{\partial Q_e^W}{\partial p_e} - \frac{\partial C_e^*}{\partial p_e} \right) = \frac{\sigma^*}{1 - \sigma^*} \frac{\partial V_g^*}{\partial p_e} + \varphi^W \frac{\partial Q_e^W}{\partial p_e} + \frac{\partial L_e^W}{\partial p_e} + \frac{\partial L_g^W}{\partial p_e},$$

which simplifies, using the derivatives enumerated above, to:

$$\lambda_e \left(\frac{\partial Q_e^W}{\partial p_e} - \frac{\partial C_e^*}{\partial p_e} \right) = \epsilon_D^* \left(r_e C_e^{FH} + C_e^{FF} \right) + \varphi^W \frac{\partial Q_e^W}{\partial p_e} + \frac{\partial L_e^W}{\partial p_e}.$$

Substituting in $\partial L_e^W / \partial p_e = p_e \partial Q_e^W / \partial p_e$ and rearranging:

$$(\lambda_e - p_e) \left(\frac{\partial Q_e^W}{\partial p_e} - \frac{\partial C_e^*}{\partial p_e} \right) = \varphi^W \frac{\partial Q_e^W}{\partial p_e} + \epsilon_D^* (r_e - 1) C_e^{FH},$$

from which we can derive the expression in the paper:

$$\left((\lambda_e - \varphi^W) - p_e \right) \frac{\partial Q_e^W}{\partial p_e} = (\lambda_e - r_e p_e) \frac{\partial C_e^{FH}}{\partial p_e} + (\lambda_e - p_e) \frac{\partial C_e^{FF}}{\partial p_e}.$$

The first order condition for r_e is:

$$-\lambda_e \frac{\partial C_e^W}{\partial r_e} = \frac{\sigma^*}{1 - \sigma^*} \frac{\partial V_g^*}{\partial r_e} + \frac{\partial L_g^W}{\partial r_e},$$

which simplifies, by using the derivatives enumerated above, to:

$$-\lambda_e \frac{\partial C_e^W}{\partial r_e} = \sigma^* p_e C_e^{FH} + \alpha(1 - \sigma^*) p_e C_e^{FH} = \epsilon_D^* p_e C_e^{FH}.$$

Dividing through by p_e and employing the last two identities enumerated above:

$$\frac{\lambda_e}{p_e} \left(\frac{\partial C_e^{FH}}{\partial r_e} + \frac{\partial C_e^{FF}}{\partial r_e} \right) = p_e \frac{\partial C_e^{FH}}{\partial p_e} = r_e \frac{\partial C_e^{FH}}{\partial r_e} + \frac{\partial C_e^{FF}}{\partial r_e}.$$

We can simplify this expression to get the one in the paper:

$$(\lambda_e - r_e p_e) \frac{\partial C_e^{FH}}{\partial r_e} = (p_e - \lambda_e) \frac{\partial C_e^{FF}}{\partial r_e}.$$

The optimal r_e balances the two wedges on the right hand side of (32).

From (33) we have:

$$\frac{\lambda_e - r_e p_e}{\lambda_e - p_e} = \Lambda.$$

Substituting into (32) yields:

$$\begin{aligned} \lambda_e - p_e &= \varphi^W \frac{\partial Q_e^W / \partial p_e}{\partial Q_e^W / \partial p_e - \partial C_e^{FF} / \partial p_e - \Lambda \partial C_e^{FH} / \partial p_e} \\ &= \varphi^W \frac{\epsilon_S^W Q_e^W}{\epsilon_S^W Q_e^W + \epsilon_D^* C_e^{FF} + \Lambda \epsilon_D^* C_e^{FH}}. \end{aligned}$$

Since $\Lambda \geq 0$ it's clear that $\lambda_e \geq p_e$ and hence $\lambda_e \geq r_e p_e$ as well.

C.6.3 Decentralization

In a market economy the optimal consumption tax is:

$$t_c = \lambda_e - p_e = \varphi^W \frac{\epsilon_S^W Q_e^W}{\epsilon_S^W Q_e^W + \epsilon_D^* C_e^{FF} + \Lambda \epsilon_D^* C_e^{FH}}.$$

The optimal production tax on Home's exports is:

$$t_p = r_e p_e - p_e = (\lambda_e - p_e) - (\lambda_e - r_e p_e) = (1 - \Lambda) t_c.$$

C.7 Extraction-Production-Consumption Hybrid Policy

The final case augments the production-consumption policy to allow the planner to choose Q_e. Many of the results for the production-consumption case carry over, including those for individual goods shown in Table 13 as well as aggregates and derivatives derived in Section C.6.1.

C.7.1 Solution

The outer problem looks like the production consumption policy:

$$\mathcal{L} = \frac{\sigma^*}{\sigma^* - 1} V_g^* - \varphi^W Q_e^W - L_e^W - L_g^W - \lambda_e \left(C_e^W - Q_e^W \right),$$

except we now we maximize this Lagrangian over Q_e, r_e, and p_e.

The first order condition for Q_e is identical to that for the unilaterally optimal policy. For $\lambda_e - \varphi^W \geq \underline{a}$:

$$Q_e = E(\lambda_e - \varphi^W),$$

otherwise $Q_e = 0$. The first order condition for p_e is only slightly revised from the production-consumption case, with Q_e^* and L_e^* appearing in place of Q_e^W and Q_e^W:

$$\lambda_e \left(\frac{\partial Q_e^*}{\partial p_e} - \frac{\partial C_e^*}{\partial p_e} \right) = \frac{\sigma^*}{1 - \sigma^*} \frac{\partial V_g^*}{\partial p_e} + \varphi^W \frac{\partial Q_e^*}{\partial p_e} + \frac{\partial L_e^*}{\partial p_e} + \frac{\partial L_g^W}{\partial p_e}.$$

We can thus jump to the expression in the paper:

$$\left((\lambda_e - \varphi^W) - p_e \right) \frac{\partial Q_e^*}{\partial p_e} = (\lambda_e - r_e p_e) \frac{\partial C_e^{FH}}{\partial p_e} + (\lambda_e - p_e) \frac{\partial C_e^{FF}}{\partial p_e}.$$

The first order condition for r_e is unchanged from the production-consumption case:

$$-\lambda_e \frac{\partial C_e^W}{\partial r_e} = \frac{\sigma^*}{1 - \sigma^*} \frac{\partial V_g^*}{\partial r_e} + \frac{\partial L_g^W}{\partial r_e},$$

which simplifies to:

$$(\lambda_e - r_e p_e) \frac{\partial C_e^{FH}}{\partial r_e} = (p_e - \lambda_e) \frac{\partial C_e^{FF}}{\partial r_e}.$$

Combining terms, we have:

$$\begin{aligned}\lambda_e - p_e &= \varphi^W \frac{\partial Q_e^*/\partial p_e}{\partial Q_e^*/\partial p_e - \partial C_e^{FF}/\partial p_e - \Lambda \partial C_e^{FH}/\partial p_e} \\ &= \varphi^W \frac{\epsilon_S^W Q_e^*}{\epsilon_S^* Q_e^* + \epsilon_D^* C_e^{FF} + \Lambda \epsilon_D^* C_e^{FH}}.\end{aligned}$$

C.7.2 Decentralization

In a market economy the optimal nominal extraction tax is $t_e^N = \varphi^W$, while the effective rate is:

$$t_e = \varphi^W - (\lambda_e - p_e) = \varphi^W \frac{\epsilon_D^* C_e^{FF} + \Lambda \epsilon_D^* C_e^{FH}}{\epsilon_S^* Q_e^* + \epsilon_D^* C_e^{FF} + \Lambda \epsilon_D^* C_e^{FH}}.$$

The optimal consumption tax, applying to Home consumption of both domestically produced and imported goods is:

$$t_c = \lambda_e - p_e = \varphi^W \frac{\epsilon_S^W Q_e^*}{\epsilon_S^* Q_e^* + \epsilon_D^* C_e^{FF} + \Lambda \epsilon_D^* C_e^{FH}}.$$

The optimal production tax on Home exports of goods is:

$$t_p = r_e p_e - p_e = (\lambda_e - p_e) - (\lambda_e - r_e p_e) = (1 - \Lambda) t_c.$$

D Solutions for Quantitative Illustration

Here we provide a list of equations for the parameterized version of the model that we use for the quantitative results in Section 5 of the paper. For each outcome, we start with the BAU competitive equilibrium value that we calibrate the model to. We then show how to express the optimal outcomes in terms of these BAU outcomes. To distinguished the two, we express outcomes under the optimal policy as functions of p_e and t_b (we leave off t_e since under the policy we always have $t = \varphi^W$). We elminate these arguments to represent BAU outcomes. Thus for an outcome x we denote the optimal outcome as $x(p_e, t_b)$ (sometimes x' for short) and the BAU outcome as simply x. We impose the restrictions from Section 6.1.1. .

D.1 Expressions to Compute the Optimal Policy

Most of the expressions that follow are based on integrating energy use across the continuum of goods. We start with the three expressions for unit energy requirements per good under the optimal policy:

1. For production in Home to serve consumers in Home or Foreign

$$e_j(z) = (1 - \alpha) a_j (p_e + t_b)^{-\alpha}$$

2. For production in Foreign to serve consumers in Home

$$e_j^*(z) = (1-\alpha)a_j^*(p_e+t_b)^{-\alpha}$$

3. For production in Foreign to serve consumers in Foreign

$$e_j^*(z^*) = (1-\alpha)a_j^* p_e^{-\alpha}$$

These three expressions apply to BAU as well by setting $p_e = 1$ and $t_b = 0$.

What follows is a list of all the unilaterally optimal outcomes expressed in terms of p_e, t_b, and the corresponding outcomes under the BAU competitive equilibrium.

1. The import margin is invariant to the optimal policy:

$$\bar{j}_m' = \bar{j}_m(p_e, t_b) = \bar{j}_m = \frac{A}{A + (\tau^*)^{-\theta} A^*} = \frac{C^{HH}}{C_e}$$

2. Export margin:

   (a) Under unilateral optimal:

$$\bar{j}_x(p_e, t_b) = \frac{\tau^{-\theta} A p_e^{-\alpha\theta} (p_e + t_b)^{-(1-\alpha)\theta}}{\tau^{-\theta} A p_e^{-\alpha\theta} (p_e + t_b)^{-(1-\alpha)\theta} + A^* (p_e + (1-\alpha) t_b)^{-\theta}}$$

   (b) Under BAU:

$$\bar{j}_x = \frac{\tau^{-\theta} A}{\tau^{-\theta} A + A^*} = \frac{C_e^{FH}}{C_e^*}$$

   (c) Expressed in terms of BAU:

$$\bar{j}_x(p_e, t_b) = \frac{\bar{j}_x p_e^{-\alpha\theta} (p_e + t_b)^{-(1-\alpha)\theta}}{\bar{j}_x p_e^{-\alpha\theta} (p_e + t_b)^{-(1-\alpha)\theta} + (1 - \bar{j}_x) (p_e + (1-\alpha) t_b)^{-\theta}}$$

   (d) Shorthand:

$$\bar{j}_x' = \bar{j}_x(p_e, t_b).$$

3. Intermediate export margin:

   (a) Under unilateral optimal:

$$j_0(p_e, t_b) = \frac{\tau^{-\theta} A (p_e + t_b)^{-(1-\alpha)\theta}}{\tau^{-\theta} A (p_e + t_b)^{-(1-\alpha)\theta} + A^* p_e^{-(1-\alpha)\theta}}$$

(b) Under BAU:

$$j_0 = \bar{j}_x = \frac{\tau^{-\theta} A}{\tau^{-\theta} A + A^*} = \frac{C_e^{FH}}{C_e^*}$$

(c) Expressed in terms of BAU:

$$j_0(p_e, t_b) = \frac{j_0(p_e + t_b)^{-(1-\alpha)\theta}}{j_0(p_e + t_b)^{-(1-\alpha)\theta} + (1 - j_0) p_e^{-(1-\alpha)\theta}}$$

(d) Shorthand:

$$j_0' = j_0(p_e, t_b).$$

4. Energy used by producers in Home to supply Home consumers:

(a) Under unilateral optimal:

$$\begin{aligned} C_e^{HH}(p_e, t_b) &= \int_0^{\bar{j}_m'} e_j(z) y_j dj \\ &= \eta \left(1 - \alpha\right) \left(p_e + t_b\right)^{-\epsilon_D} \int_0^{\bar{j}_m'} a_j^{1-\sigma} dj \\ &= \eta \left(1 - \alpha\right) \left(p_e + t_b\right)^{-\epsilon_D} \frac{A^{(\sigma-1)/\theta}}{1 + \left(1 - \sigma\right)/\theta} \left(\bar{j}_m'\right)^{1+(1-\sigma)/\theta} \end{aligned}$$

(b) Under BAU:

$$C_e^{HH} = \eta \left(1 - \alpha\right) \frac{A^{(\sigma-1)/\theta}}{1 + \left(1 - \sigma\right)/\theta} \left(\bar{j}_m\right)^{1+(1-\sigma)/\theta}$$

(c) Expressed in terms of BAU:

$$C_e^{HH}(p_e, t_b) = (p_e + t_b)^{-\epsilon_D} C_e^{HH}$$

5. Energy used by producers in Home to supply exports of Home:

(a) Under unilateral optimal:

$$
\begin{aligned}
C_e^{FH}(p_e,t_b) &= C_e^{FH,1}(p_e,t_b)+C_e^{FH,2}(p_e,t_b)\\
C_e^{FH,1}(p_e,t_b) &= \tau\int_0^{j_0'} e_j(z)x_j dj\\
&= \tau^{1-\sigma^*}\eta^*(1-\alpha)(p_e+t_b)^{-\epsilon_D^*}\int_0^{j_0'} a_j^{1-\sigma^*}dj\\
&= \tau^{1-\sigma^*}\eta^*(1-\alpha)(p_e+t_b)^{-\epsilon_D^*}\\
&\quad \frac{A^{-(1-\sigma^*)/\theta}}{1+(1-\sigma^*)/\theta}\left(j_0'\right)^{1+(1-\sigma^*)/\theta}\\
C_e^{FH,2}(p_e,t_b) &= \tau\int_{j_0'}^{\bar{j}_x'} e_j(z)x_j dj\\
&= \tau\eta^*(1-\alpha)p_e^{-(1-\alpha)\sigma^*}(p_e+t_b)^{-\alpha}\int_{j_0'}^{\bar{j}_x'} a_j\left(a_j^*\right)^{-\sigma^*}dj\\
&= \tau\eta^*(1-\alpha)p_e^{-(1-\alpha)\sigma^*}(p_e+t_b)^{-\alpha}\frac{\left(A^*\right)^{\sigma^*/\theta}}{A^{1/\theta}}\\
&\quad \left(B\left(\bar{j}_x',\frac{1+\theta}{\theta},\frac{\theta-\sigma^*}{\theta}\right)-B\left(j_0',\frac{1+\theta}{\theta},\frac{\theta-\sigma^*}{\theta}\right)\right)
\end{aligned}
$$

where and $B(x,a,b)$ is the incomplete beta function[39]

(b) Under BAU:

$$
C_e^{FH}=\tau^{1-\sigma^*}\eta^*(1-\alpha)\frac{A^{(\sigma^*-1)/\theta}}{1+(1-\sigma^*)/\theta}\left(\bar{j}_x\right)^{1+(1-\sigma^*)/\theta}
$$

[39]The incomplete beta function is:

$$
B(x,a,b)=\int_0^x i^{a-1}(1-i)^{b-1}di,
$$

for $0\le x\le 1$, $a>0$, and $b>0$. Setting $x=1$ gives the beta function itself, $B(a,b)$.

(c) Expressed in terms of BAU:

$$C_e^{FH,1}(p_e, t_b) = (p_e + t_b)^{-\epsilon_D^*} \left(\frac{j_0'}{j_0}\right)^{1+(1-\sigma^\star)/\theta} C_e^{FH}$$

$$C_e^{FH,2}(p_e, t_b) = \tau^{\sigma^*} \frac{\theta + 1 - \sigma^*}{\theta} p_e^{-\epsilon_D^*} \left(\frac{p_e + t_b}{p_e}\right)^{-\alpha} \left(\frac{A^*}{A}\right)^{\sigma^*/\theta} \frac{\left(B\left(\bar{j}_x', \frac{1+\theta}{\theta}, \frac{\theta-\sigma^*}{\theta}\right) - B\left(j_0', \frac{1+\theta}{\theta}, \frac{\theta-\sigma^*}{\theta}\right)\right)}{\bar{j}_x^{1+(1-\sigma^*)/\theta}} C_e^{FH}$$

$$= \frac{\theta + 1 - \sigma^*}{\theta} \left(\frac{1 - \bar{j}_x}{\bar{j}_x}\right)^{\sigma^*/\theta} p_e^{-\epsilon_D^*} \left(\frac{p_e + t_b}{p_e}\right)^{-\alpha} \frac{\left(B\left(\bar{j}_x', \frac{1+\theta}{\theta}, \frac{\theta-\sigma^*}{\theta}\right) - B\left(j_0', \frac{1+\theta}{\theta}, \frac{\theta-\sigma^*}{\theta}\right)\right)}{\bar{j}_x^{1+(1-\sigma^*)/\theta}} C_e^{FH}$$

6. Energy used by producers in Foreign to supply Foreign consumers:

   (a) Under unilateral optimal:

$$C_e^{FF}(p_e, t_b) = \int_{\bar{j}_x'}^{1} e_j^*(z^*) y_j^* dj$$

$$= \eta^*(1-\alpha) p_e^{-\epsilon_D^*} \int_{\bar{j}_x'}^{1} \left(a_j^*\right)^{1-\sigma^*} dj$$

$$= \eta^*(1-\alpha) p_e^{-\epsilon_D^*} \frac{\left(A^*\right)^{(\sigma^*-1)/\theta}}{1 + (1-\sigma^*)/\theta} (1 - \bar{j}_x')^{1+(1-\sigma^*)/\theta}$$

   (b) Under BAU:

$$C_e^{FF} = \eta^*(1-\alpha) \frac{\left(A^*\right)^{(\sigma^*-1)/\theta}}{1 + (1-\sigma^*)/\theta} (1 - \bar{j}_x)^{1+(1-\sigma^*)/\theta}$$

   (c) Expressed in terms of BAU:

$$C_e^{FF}(p_e, t_b) = p_e^{-\epsilon_D^*} \left(\frac{1 - \bar{j}_x'}{1 - \bar{j}_x}\right)^{1+(1-\sigma^*)/\theta} C_e^{FF}$$

7. Energy used by producers in Foreign to supply imports of Home:

(a) Under unilateral optimal:

$$
\begin{aligned}
C_e^{HF}(p_e, t_b) &= \tau^* \int_{\bar{j}'_m}^{1} e_j^*(z) m_j dj \\
&= (\tau^*)^{1-\sigma} \eta(1-\alpha) (p_e + t_b)^{-\epsilon_D} \int_{\bar{j}'_m}^{1} \left(a_j^*\right)^{1-\sigma} dj \\
&= (\tau^*)^{1-\sigma} \eta(1-\alpha) (p_e + t_b)^{-\epsilon_D} \\
&\quad \frac{(A^*)^{(\sigma-1)/\theta}}{1 + (1-\sigma)/\theta} (1 - \bar{j}'_m)^{1+(1-\sigma)/\theta}
\end{aligned}
$$

(b) Under BAU:

$$
C_e^{HF} = (\tau^*)^{1-\sigma} \eta(1-\alpha) \frac{(A^*)^{(\sigma-1)/\theta}}{1 + (1-\sigma)/\theta} (1 - \bar{j}_m)^{1+(1-\sigma)/\theta}
$$

(c) Expressed in terms of BAU:

$$
C_e^{HF}(p_e, t_b) = (p_e + t_b)^{-\epsilon_D} C_e^{HF}
$$

8. Energy implicitly consumed by Foreign:

(a) Under BAU:

$$
\begin{aligned}
C_e^* &= C_e^{FH} + C_e^{FF} \\
&= \tau^{1-\sigma^*} \eta^*(1-\alpha) \frac{A^{(\sigma^*-1)/\theta}}{1 + (1-\sigma^*)/\theta} (\bar{j}_x)^{1+(1-\sigma^*)/\theta} \\
&\quad + \eta^*(1-\alpha) \frac{(A^*)^{(\sigma^*-1)/\theta}}{1 + (1-\sigma^*)/\theta} (1 - \bar{j}_x)^{1+(1-\sigma^*)/\theta} \\
&= \eta^*(1-\alpha) \frac{\left(\tau^{-\theta} A / \bar{j}_x\right)^{(\sigma^*-1)/\theta} \bar{j}_x + (A^*/(1-\bar{j}_x))^{(\sigma^*-1)/\theta} (1 - \bar{j}_x)}{1 + (1-\sigma^*)/\theta} \\
&= \eta^*(1-\alpha) \frac{\left(\tau^{-\theta} A + A^*\right)^{(\sigma^*-1)/\theta}}{1 + (1-\sigma^*)/\theta}
\end{aligned}
$$

9. Value of Home exports of goods:

(a) Under unilateral optimal:

$$
\begin{aligned}
V_g^{FH}(p_e,t_b) &= V_g^{FH,1}(p_e,t_b) + V_g^{FH,2}(p_e,t_b) \\
V_g^{FH,1}(p_e,t_b) &= \int_0^{j_0'} \tau a_j (p_e+t_b)^{1-\alpha} x_j dj \\
&= \tau^{1-\sigma^\star}\eta^* \left(p_e+t_b\right)^{1-\epsilon_D^*} \int_0^{j_0'} a_j^{1-\sigma^*} dj \\
&= \tau^{1-\sigma^\star}\eta^\star \left(p_e+t_b\right)^{1-\epsilon_D^\star} \frac{A^{-(1-\sigma^\star)/\theta}}{1+\left(1-\sigma^\star\right)/\theta} \left(j_0'\right)^{1+(1-\sigma^\star)/\theta} \\
V_g^{FH,2}(p_e,t_b) &= \int_{j_0'}^{\bar{j}_x'} a_j^* p_e^{1-\alpha} x_j dj \\
&= \eta^* p_e^{1-\epsilon_D^*} \int_{j_0'}^{\bar{j}_x'} \left(a_j^*\right)^{1-\sigma^*} dj \\
&= \eta^* p_e^{1-\epsilon_D^*} \frac{\left(A^*\right)^{(\sigma^*-1)/\theta}}{1+\left(1-\sigma^*\right)/\theta} \\
&\qquad \left((1-j_0')^{(\theta+1-\sigma^*)/\theta} - (1-\bar{j}_x')^{(\theta+1-\sigma^*)/\theta}\right)
\end{aligned}
$$

(b) Under BAU:

$$
V_g^{FH} = \tau^{1-\sigma^*}\eta^* \frac{A^{(\sigma^*-1)/\theta}}{1+\left(1-\sigma^*\right)/\theta} \left(\bar{j}_x\right)^{1+(1-\sigma^*)/\theta}
$$

(c) Expressed in terms of BAU:

$$
\begin{aligned}
V_g^{FH,1}(p_e,t_b) &= (p_e+t_b)^{1-\epsilon_D^*} \left(\frac{j_0'}{j_0}\right)^{1+(1-\sigma^\star)/\theta} V_g^{FH} \\
V_g^{FH,2}(p_e,t_b) &= p_e^{1-\epsilon_D^*} \frac{\left((1-j_0')^{(\theta+1-\sigma^*)/\theta} - (1-\bar{j}_x')^{(\theta+1-\sigma^*)/\theta}\right)}{\bar{j}_x \left(1-\bar{j}_x\right)^{(1-\sigma^\star)/\theta}} V_g^{FH}
\end{aligned}
$$

Substitute in $V_g^{FH} = \frac{1}{1-\alpha} C_e^{FH}$:

$$
\begin{aligned}
V_g^{FH,1}(p_e,t_b) &= (p_e+t_b)^{1-\epsilon_D^*} \left(\frac{j_0'}{j_0}\right)^{1+(1-\sigma^\star)/\theta} \frac{1}{1-\alpha} C_e^{FH} \\
V_g^{FH,2}(p_e,t_b) &= p_e^{1-\epsilon_D^*} \frac{\left((1-j_0')^{(\theta+1-\sigma^*)/\theta} - (1-\bar{j}_x')^{(\theta+1-\sigma^*)/\theta}\right)}{\bar{j}_x \left(1-\bar{j}_x\right)^{(1-\sigma^\star)/\theta}} \frac{1}{1-\alpha} C_e^{FH}
\end{aligned}
$$

10. Value of Home's imports of goods:

(a) Under unilateral optimal:

$$\begin{aligned}
V_g^{HF}(p_e, t_b) &= \int_{\bar{j}'_m}^{1} p_j^m m_j dj = \int_{\bar{j}_m}^{1} p_j^m m_j dj \\
&= (\tau^*)^{1-\sigma} \eta (p_e + t_b)^{1-\epsilon_D} \left(\frac{p_e + \alpha t_b}{p_e + t_b} \right) \int_{\bar{j}_m}^{1} \left(a_j^*\right)^{1-\sigma} dj \\
&= (\tau^*)^{1-\sigma} \eta (p_e + t_b)^{1-\epsilon_D} \left(\frac{p_e + \alpha t_b}{p_e + t_b} \right) \\
&\quad \frac{(A^*)^{(\sigma-1)/\theta}}{1 + (1-\sigma)/\theta} (1 - \bar{j}_m)^{1+(1-\sigma)/\theta}
\end{aligned}$$

(b) Under BAU:

$$V_g^{HF} = (\tau^*)^{1-\sigma} \eta \frac{(A^*)^{(\sigma-1)/\theta}}{1 + (1-\sigma)/\theta} (1 - \bar{j}_m)^{1+(1-\sigma)/\theta}$$

(c) Expressed in terms of BAU:

$$V_g^{HF}(p_e, t_b) = (p_e + t_b)^{-\epsilon_D} (p_e + \alpha t_b) V_g^{HF}$$

Substitute in $V_g^{HF} = \frac{1}{1-\alpha} C_e^{HF}$:

$$V_g^{HF}(p_e, t_b) = (p_e + t_b)^{-\epsilon_D} (p_e + \alpha t_b) \frac{1}{1-\alpha} C_e^{HF}$$

11. Energy extraction by Home:

(a) Under unilateral optimal (for $p_e + t_b - \varphi^W \geq \underline{a}$):

$$Q_e(p_e, t_b) = \left(p_e + t_b - \varphi^W\right)^{\epsilon_S} E$$

(b) Under BAU:

$$Q_e = E$$

(c) Expressed in terms of BAU:

$$Q_e(p_e, t_b) = \left(p_e + t_b - \varphi^W\right)^{\epsilon_S} Q_e$$

(d) If $p_e + t_b - \varphi^W < \underline{a}$ then $Q_e(p_e, t_b) = 0$

12. Energy extraction by Foreign:

(a) Under unilateral optimal (for $p_e \geq \underline{a}$):

$$Q_e^*(p_e, t_b) = (p_e)^{\epsilon_S^*} E^*$$

(b) Under BAU:

$$Q_e^* = E^*$$

(c) Expressed in terms of BAU:

$$Q_e^*(p_e, t_b) = (p_e)^{\epsilon_S^*} Q_e^*$$

(d) If $p_e < \underline{a}$ then $Q_e^*(p_e, t_b) = 0$

13. Total export subsidy:

(a) Under unilateral optimal:

$$\begin{aligned} S(p_e, t_b) &= \int_{j_0'}^{\bar{j}_x'} s_j x_j dj \\ &= \tau\eta^\star \int_{j_0'}^{\bar{j}_x{}'} a_j \left(p_e + t_b\right)^{1-\alpha} (p_j^\star)^{-\sigma^\star} dj - \eta^\star \int_{j_0'}^{\bar{j}_x{}'} (p_j^\star)^{1-\sigma^\star} dj \\ &= (p_e + t_b) \frac{C_e^{FH,2}(p_e, t_b)}{1-\alpha} - V_g^{FH,2}(p_e, t_b) \end{aligned}$$

(b) Under BAU: $S = 0$

14. Maximum export subsidy relative to production cost:

(a) Under unilateral optimal:

$$\begin{aligned} \frac{s_{\bar{j}_x'}}{\tau a_{\bar{j}_x'} \left(p_e + t_b\right)^{1-\alpha}} &= 1 - \frac{a^\star_{\bar{j}_x'} p_e^{1-\alpha}}{\tau a_{\bar{j}_x'} \left(p_e + t_b\right)^{1-\alpha}} \\ &= 1 - \frac{F(\bar{j}_x')}{F(j_0')} = 1 - \left(\frac{(1-\bar{j}_x')/\bar{j}_x'}{(1-j_0')/j_0'}\right)^{1/\theta} \end{aligned}$$

(b) Under BAU: $s_{\bar{j}_x'} = 0$

D.2 Expressions to Compute Welfare

Having solved for the optimal border adjustment and the corresponding change in the global energy price we can compute all other outcomes as well. A key outcome is Home's welfare in moving to the optimal unilateral policy from the BAU competitive equilibrium.

Home's Utility (dropping constants) can be expressed as:

1. Under BAU:

$$\begin{aligned} U &= \frac{\sigma}{\sigma-1}\eta^{1/\sigma} C_g^{1-1/\sigma} + \frac{\sigma^*}{\sigma^*-1}(\eta^*)^{1/\sigma^*}(C_g^*)^{1-1/\sigma} \\ &\quad - \varphi^W(Q_e + Q_e^*) - L_g - L_g^* - L_e - L_e^* \\ &= \frac{\sigma}{\sigma-1} V_g + \frac{\sigma^*}{\sigma^*-1} V_g^* - \varphi^W Q_e^W - L_g^W - L_e^W \end{aligned}$$

2. Under unilateral optimal:

$$U(p_e,t_b) = \frac{\sigma}{\sigma-1}V_g(p_e,t_b)+\frac{\sigma^*}{\sigma^*-1}V_g^*(p_e,t_b)-\varphi^W Q_e^W(p_e,t_b)-L_g^W(p_e,t_b)-L_e^W(p_e,t_b)$$

3. The change in moving to the optimal unilateral policy from the BAU competitive equilibrium:

$$\begin{aligned} U(p_e,t_b) - U = & \frac{\sigma}{\sigma-1}(V_g(p_e,t_b) - V_g) + \frac{\sigma^*}{\sigma^*-1}(V_g^*(p_e,t_b) - V_g^*) \\ & - \varphi^W(Q_e^W(p_e,t_b) - Q_e^W) - (L_g^W(p_e,t_b) - L_g^W) - (L_e^W(p_e,t_b) - L_e^W) \end{aligned}$$

Our preferred measure of welfare is normalized by BAU spending on goods:

$$W = \frac{U(p_e,t_b) - U}{V_g}$$

For the terms in the welfare function above, we show:

1. Home's employment in energy extraction:
   (a) Change from BAU to unilateral optimal (for $p_e + t_b - \varphi^W \geq \underline{a}$)::

$$\begin{aligned} L_e(p_e,t_b) - L_e &= \int_1^{p_e+t_b-\varphi^W} a\,dE(a) \\ &= Q_e \int_1^{p_e+t_b-\varphi^W} \epsilon_S a^{\epsilon_S} da \\ &= \frac{\epsilon_S}{\epsilon_S+1}((p_e+t_b-\varphi^W)^{\epsilon_S+1}-1)Q_e \end{aligned}$$

   (b) If $p_e + t_b - \varphi^W < \underline{a}$ then $L_e(p_e,t_b) = 0$

2. Foreign's employment in energy extraction:
   (a) Change from BAU to unilateral optimal (for $p_e \geq \underline{a}$):

$$\begin{aligned} L_e^*(p_e,t_b) - L_e^* &= \int_1^{p_e} a^*\,dE^*(a^*) \\ &= Q_e^* \int_1^{p_e} \epsilon_S^*\,(a^*)^{\epsilon_S^*}\,da^* \\ &= \frac{\epsilon_S^*}{\epsilon_S^*+1}(p_e^{\epsilon_S^*+1}-1)Q_e^* \end{aligned}$$

   (b) If $p_e < \underline{a}$ then $L_e^*(p_e,t_b) = 0$

3. Labor employed in production in Home:

(a) Under unilateral optimal:

$$L_g(p_e, t_b) = \frac{\alpha}{1-\alpha} (p_e + t_b) \left(C_e^{HH}(p_e, t_b) + C_e^{FH}(p_e, t_b)\right)$$

(b) Under BAU:

$$L_g = \frac{\alpha}{1-\alpha} \left(C_e^{HH} + C_e^{FH}\right)$$

(c) Change from BAU to unilateral optimal:

$$L_g(p_e, t_b) - L_g = \frac{\alpha}{1-\alpha} \left((p_e + t_b) \left(C_e^{HH}(p_e, t_b) + C_e^{FH}(p_e, t_b)\right) - C_e^{HH} - C_e^{FH}\right)$$

4. Labor employed in production in Foreign:

(a) Under unilateral optimal:

$$L_g^*(p_e, t_b) = \frac{\alpha}{1-\alpha} \left((p_e + t_b) C_e^{HF}(p_e, t_b) + p_e C_e^{FF}(p_e, t_b)\right)$$

(b) Under BAU:

$$L_g^* = \frac{\alpha}{1-\alpha} \left(C_e^{HF} + C_e^{FF}\right)$$

(c) Change from BAU to unilateral optimal:

$$L_g^*(p_e, t_b) - L_g^* = \frac{\alpha}{1-\alpha} \left((p_e + t_b) C_e^{HF}(p_e, t_b) - C_e^{HF} + p_e C_e^{FF}(p_e, t_b) - C_e^{HF}\right)$$

5. The value of Home's spending on goods:

(a) Under unilateral optimal:

$$\begin{aligned} V_g(p_e, t_b) = \eta^{1/\sigma} C_g(p_e, t_b)^{1-1/\sigma} &= \frac{1}{1-\alpha} (p_e + t_b) \left(C_e^{HH}(p_e, t_b) + C_e^{HF}(p_e, t_b)\right) \\ = \eta \left(p_e + t_b\right)^{1-\epsilon_D} &\frac{\left(A + (\tau^*)^{-\theta} A^*\right)^{(\sigma-1)/\theta}}{1 + (1-\sigma)/\theta} \end{aligned}$$

(b) Under BAU:

$$V_g = \eta \frac{\left(A + (\tau^*)^{-\theta} A^*\right)^{(\sigma-1)/\theta}}{1 + (1-\sigma)/\theta} = \frac{1}{1-\alpha} C_e$$

(c) Expressed in terms of BAU:

$$V_g(p_e, t_b) = (p_e + t_b)^{1-\epsilon_D} V_g = (p_e + t_b)^{1-\epsilon_D} \frac{1}{1-\alpha} C_e$$

6. The term that enters the change in Home's welfare is:

$$\frac{\sigma}{\sigma-1}(V_g(p_e,t_b)-V_g)=\frac{\left((p_e+t_b)^{(1-\alpha)(1-\sigma)}-1\right)}{(\sigma-1)/\sigma}V_g$$

For the case of $\sigma=1$ this term reduces to:

$$\lim_{\sigma\to 1}\frac{(p_e+t_b)^{(1-\alpha)(1-\sigma)}}{(\sigma-1)/\sigma}V_g=-(1-\alpha)\ln(p_e+t_b)V_g=-\ln(p_e+t_b)C_e$$

7. The value of Foreign's spending on goods:

   (a) Under unilateral optimal:

$$\begin{aligned}V_g^*(p_e,t_b)&=(\eta^*)^{1/\sigma^*}(C_g^*(p_e,t_b))^{1-1/\sigma^*}\\&=V_g^{FH}(p_e,t_b)+V_g^{FF}(p_e,t_b)\\&=\eta^*\frac{\left(\tau^{-\theta}A(p_e+t_b)^{-(1-\alpha)\theta}+A^*p_e^{-(1-\alpha)\theta}\right)^{-(1-\sigma^*)/\theta}}{1+(1-\sigma^*)/\theta}\end{aligned}$$

   (b) Under BAU:

$$V_g^*=\eta^*\frac{\left(A^*+\tau^{-\theta}A\right)^{(\sigma^*-1)/\theta}}{1+(1-\sigma^*)/\theta}=\frac{1}{1-\alpha}C_e^*$$

   (c) Expressed in terms of BAU:

$$\begin{aligned}V_g^*(p_e,t_b)&=\left(\frac{j_0}{j_0'}(p_e+t_b)^{-(1-\alpha)\theta}\right)^{-(1-\sigma^*)/\theta}V_g^*\\&=\frac{\left(j_0(p_e+t_b)^{-(1-\alpha)\theta}+(1-j_0)p_e^{-(1-\alpha)\theta}\right)^{-(1-\sigma^*)/\theta}}{1-\alpha}C_e^*\end{aligned}$$

8. The term that enters the change in Foreign's welfare is:

$$\begin{aligned}&\frac{\sigma^*}{\sigma^*-1}(V_g^*(p_e,t_b)-V_g^*)\\&=\frac{\left(j_0(p_e+t_b)^{-(1-\alpha)\theta}+(1-j_0)p_e^{-(1-\alpha)\theta}\right)^{-(1-\sigma^*)/\theta}-1}{(\sigma^*-1)/\sigma^*}V_g^*\end{aligned}$$

For the case of $\sigma^*=1$ this term reduces to:

$$\begin{aligned}&\lim_{\sigma^*\to 1}\frac{\left(j_0(p_e+t_b)^{-(1-\alpha)\theta}+(1-j_0)p_e^{-(1-\alpha)\theta}\right)^{-(1-\sigma^*)/\theta}-1}{(\sigma^*-1)/\sigma^*}V_g^*\\&=\frac{\ln\left(j_0(p_e+t_b)^{-(1-\alpha)\theta}+(1-j_0)p_e^{-(1-\alpha)\theta}\right)}{(1-\alpha)\theta}C_e^*\end{aligned}$$

9. Global emissions:

   (a) Under unilateral optimal:

$$Q_e^W(p_e, t_b) = Q_e(p_e, t_b) + Q_e^*(p_e, t_b)$$

   (b) Under BAU:

$$Q_e^W = Q_e + Q_e^*$$

D.3 Expressions for Constrained-Optimal Policies

1. Export margin:

   (a) Under optimal production policies (production, extraction-production hybrid and production-consumption hybrid):

$$\bar{j}_x(r_e) = \frac{\tau^{-\theta} r_e^{-\theta(1-\alpha)} A}{\tau^{-\theta} r_e^{-\theta(1-\alpha)} A + A^*}$$

   (b) Under BAU:

$$\bar{j}_x = \frac{\tau^{-\theta} A}{\tau^{-\theta} A + A^*} = \frac{C_e^{FH}}{C_e^*}$$

   (c) Expressed in terms of BAU:

$$\bar{j}_x(r_e) = \frac{r_e^{-\theta(1-\alpha)}}{\bar{j}_x r_e^{-\theta(1-\alpha)} + 1 - \bar{j}_x}$$

   (d) Shorthand:

$$\bar{j}_x{}' = \bar{j}_x(r_e).$$

   (e) Under optimal non-production policies (extraction, consumption and extraction-consumption hybrid policy) the export threshold doesn't change from BAU:

$$\bar{j}_x{}' = \bar{j}_x$$

2. Import margin:

   (a) Under optimal production and extraction-production policies:

$$\bar{j}_m(r_e) = \frac{(\tau^*)^{\theta} r_e^{-\theta(1-\alpha)} A}{(\tau^*)^{\theta} r_e^{-\theta(1-\alpha)} A + A^*}$$

(b) Under BAU:

$$\bar{j}_m = \frac{(\tau^*)^\theta A}{(\tau^*)^\theta A + A^*} = \frac{C^{HH}}{C_e}$$

(c) Expressed in terms of BAU:

$$\bar{j}_m(r_e) = \frac{r_e^{-\theta(1-\alpha)}}{\bar{j}_m r_e^{-\theta(1-\alpha)} + 1 - \bar{j}_m}$$

(d) Shorthand:

$$\bar{j}_m{}' = \bar{j}_m(r_e).$$

(e) Under all other constrained optimal policies the import threshold doesn't change from BAU:

$$\bar{j}_m{}' = \bar{j}_m$$

3. Energy used by producers in Home to supply exports of Home:

(a) Under constrained optimal production policies:

$$\begin{aligned} C_e^{FH}(p_e, r_e) &= \tau \int_0^{j_x'} e_j(z) x_j dj \\ &= \tau^{1-\sigma^*} \eta^* (1-\alpha)(r_e p_e)^{-\epsilon_D^*} \int_0^{j_x'} a_j^{1-\sigma^*} dj \\ &= \tau^{1-\sigma^*} \eta^* (1-\alpha)(r_e p_e)^{-\epsilon_D^*} \\ &\quad \frac{A^{-(1-\sigma^*)/\theta}}{1 + (1-\sigma^*)/\theta} (j_x')^{1+(1-\sigma^*)/\theta} \end{aligned}$$

(b) Under BAU:

$$C_e^{FH} = \tau^{1-\sigma^*} \eta^* (1-\alpha) \frac{A^{(\sigma^*-1)/\theta}}{1 + (1-\sigma^*)/\theta} (\bar{j}_x)^{1+(1-\sigma^*)/\theta}$$

(c) Expressed in terms of BAU:

$$C_e^{FH}(p_e, r_e) = (r_e p_e)^{-\epsilon_D^*} \left(\frac{j_x'}{\bar{j}_x} \right)^{1+(1-\sigma^\star)/\theta} C_e^{FH}$$

4. Value of Home exports of goods:

(a) Under constrained optimal production policies:

$$\begin{aligned} V_g^{FH}(p_e, t_b) &= \int_0^{j_x'} \tau a_j (r_e p_e)^{1-\alpha} x_j dj \\ &= \tau^{1-\sigma^\star} \eta^* (r_e p_e)^{1-\epsilon_D^*} \int_0^{j_0'} a_j^{1-\sigma^*} dj \\ &= \tau^{1-\sigma^\star} \eta^\star (r_e p_e)^{1-\epsilon_D^\star} \frac{A^{-(1-\sigma^\star)/\theta}}{1 + (1-\sigma^\star)/\theta} (j_x')^{1+(1-\sigma^\star)/\theta} \end{aligned}$$

(b) Under BAU:

$$V_g^{FH} = \tau^{1-\sigma^*}\eta^* \frac{A^{(\sigma^*-1)/\theta}}{1+(1-\sigma^*)/\theta} \left(\bar{j}_x\right)^{1+(1-\sigma^*)/\theta}$$

(c) Expressed in terms of BAU:

$$\begin{aligned} V_g^{FH}(p_e, t_b) &= (r_e p_e)^{1-\epsilon_D^*} \left(\frac{\bar{j}_x'}{\bar{j}_x}\right)^{1+(1-\sigma^\star)/\theta} V_g^{FH} \\ &= \frac{1}{1-\alpha}(r_e p_e)^{1-\epsilon_D^*} \left(\frac{\bar{j}_x'}{\bar{j}_x}\right)^{1+(1-\sigma^\star)/\theta} C_e^{FH} \end{aligned}$$

5. Production leakage term under the production policies:

(a) The energy Home uses to produce a good at the export threshold:

$$\tau e_{\bar{j}_x'} x_{\bar{j}_x'} = (1-\alpha)\eta^* (p_e r_e)^{-\epsilon_D^*} \left(\tau a_{\bar{j}_x'}\right)^{1-\sigma^*} = \left(1 + \frac{1-\sigma^*}{\theta}\right) C_e^{FH} \frac{\hat{\bar{j}}_x^{\frac{1-\sigma^*}{\theta}}}{\bar{j}_x} (p_e r_e)^{-\epsilon_D^*}$$

(b) The energy Home uses to produce a good at the import threshold:

$$e_{\bar{j}_m'} y_{\bar{j}_m'} = (1-\alpha)\eta (r_e p_e)^{-\epsilon_D} \left(a_{\bar{j}_m'}\right)^{1-\sigma} = \left(1 + \frac{1-\sigma^*}{\theta}\right) C_e^{HH} \frac{\hat{\bar{j}}_m^{\frac{1-\sigma}{\theta}}}{\bar{j}_m} (r_e p_e)^{-\epsilon_D}$$

(c) Derivative of export threshold:

$$\frac{\partial \bar{j}_x'}{\partial r_e} = -\frac{\theta(1-\alpha)}{r_e} \bar{j}_x' \left(1 - \bar{j}_x'\right)$$

(d) Derivative of import threshold:

$$\frac{\partial \bar{j}_m'}{\partial r_e} = -\frac{\theta(1-\alpha)}{r_e} \bar{j}_m' \left(1 - \bar{j}_m'\right)$$

E Data and Calibration

E.1 Calibration

For our quantitative analysis we calibrate the model to fossil fuel extraction and the energy embodied in trade between the region that, in our model, will enact a carbon policy (Home) and the region that will remain with business as usual (Foreign). Our common unit for energy is gigatonnes of CO_2, based on the quantity released by its combustion.

We consider three scenarios for the regions representing Home and Foreign. In the first, the United States is Home and all other countries are Foreign. The alternative scenarios, respectively, are the European Union prior to Brexit (EU28) as Home (and all other countries as Foreign) and the members of the Organization for Economic Cooperation and Development (OECD37) as Home (and all others as Foreign).

Our data source for energy consumption is The Trade in Embodied CO_2 (TECO2) database from OECD. We use their measure of consumption-based CO_2 emissions embodied in domestic final demand and the country of origin of emissions. This database covers 83 countries and regional groups over the period 2005-2015. Carbon dioxide embodied in world consumption in 2015 is 32.78 gigatonnes. We cross-checked the results with a dataset from the Global Carbon Project. The overall difference is less than ten percent.

Extraction data are from the International Energy Agency (IEA), which provides the World Energy Statistics Database on energy supply from all energy sources, including fossil fuels, biofuels, hydro, geothermal, renewables and waste. This dataset covers 143 countries as well as regional and world totals. The data are provided in units of kilotonnes of oil equivalent (ktoe). In order to keep the units consistent with the energy consumption data (gigatonnes of carbon dioxide), we first convert to terajoules (TJ) (1 ktoe = 41.868 TJ) and then apply emission factors to the five fossil fuel types to calculate CO_2 emissions. The five fossil fuel types considered are coal and coal products, natural gas, peat and peat products, oil products, as well as crude, NGL and feedstocks. The emission factors are default emission factors for stationary combustion from the 2006 IPCC Guidelines for National Greenhouse Gas Inventories. To be specific, we convert 1 TJ of crude, NGL and feedstocks to 73,300 kg CO_2, 1 TJ of natural gas to $56,100$ kg CO_2, and 1 TJ of coal, peat and oil products to $94,600$ kg CO_2. Using this calculation, world extraction is 35.96 gigatonnes of carbon dioxide.

To explain the discrepancy between world consumption and world extraction, note that the OECD data for embodied carbon does not include non-energy use of fossil fuels. In other words, some fossil fuels extracted are not combusted to produce energy. Instead, they are consumed directly or as intermediate goods. For example, petroleum can be used as asphalt and road oil and as petrochemical feedstocks for agricultural land. However, given that combusted energy is the source of CO_2 emissions, non-energy use of fossil fuel extraction is excluded in our analysis.

To make this adjustment, we note that, according to EIA (2018), approximately 8 percent of fossil fuels are not combusted in the United States. Applying this rate to the world extraction, we get a number close to world consumption ($35.96 * 0.92 = 33.08$, vs. 32.78). Thus, we can simply re-scale the world extraction data so that world extraction is equal to world consumption. To be specific, the original extraction data is divided by 1.097 (the ratio of world extraction to world consumption). Tables 5, 7, 8, and 9 display the resulting data we use for our calibration.

E.2 Parameter Values

For the key parameter in the goods production function α, the output elasticity of labor, we calibrate $(1-\alpha)/\alpha$ to the value of energy used in production $p_e G_e$ relative to the value added.[40] The data from TECO2 records the carbon emissions embodied by sector and country. We can convert to barrels of oil based on 0.43 metric tons of CO_2 per barrel of crude oil (from EPA, 2019). The price per barrel of oil is taken from the average closing price of West Texas Intermediate (WTI) crude oil in 2015, which is \$48.66 per barrel. Value added data comes from OECD Input-Output Tables (2018). We consider three definitions of the goods sector, with both the numerator (value of energy) and the denominator (value added) computed for the same sector definition, either: (i) the manufacturing sector, (ii) manufacturing plus agriculture and construction, and (iii) manufacturing, agriculture, construction, wholesale, retail, and transportation. The values of α that we obtain are, respectively, 0.85, 0.79, and 0.84. Our preferred value is 0.85, very close to two of these three.

For the energy supply elasticities, ϵ_S and ϵ_S^*, we use data from Asker, Collard-Wexler, and De Loecker (2018) on the distribution across oil fields of extraction costs. The data come in the form of quantiles (q = 0.05, 0.10, ..., 0.95), separately for the EU, the US, OPEC, and ROW (q% of oil in the US is extracted at a cost below \$$a$ per barrel, for example). We approximate OECD countries by aggregating the EU and US while for the non-OECD region we aggregate OPEC and ROW. To aggregate the quantiles for two regions, we combine them, sort the combination by the cost level, and reassemble after taking account of total oil extraction for each region (available from the IEA). The data are plotted on log scales in Figures 5 and 6, to reveal the supply elasticity as the slope.

The most costly oil fields in either region would be the first to be abandoned under a carbon policy. Thus, the upper end of the cost distribution is the most relevant for calibrating the supply elasticities. Our baseline values of $\epsilon_S = 0.5$ and $\epsilon_S^* = 0.5$ are close to the slope shown in the figures when we consider only costs above the median. Our alternative value of $\epsilon_S^* = 1$ is closer to the slope if we were to use the upper 75% of costs or even all the data.

Lacking this distributional data for coal and natural gas fields, we assume that the distribution for oil extraction is representative of all fossil fuels.

[40] We think of value added as the closest proxy to labor cost in the model, since we interpret labor in the model as labor equipped with capital.

Figure 5: Calibration of the Extraction Supply Elasticity in Home

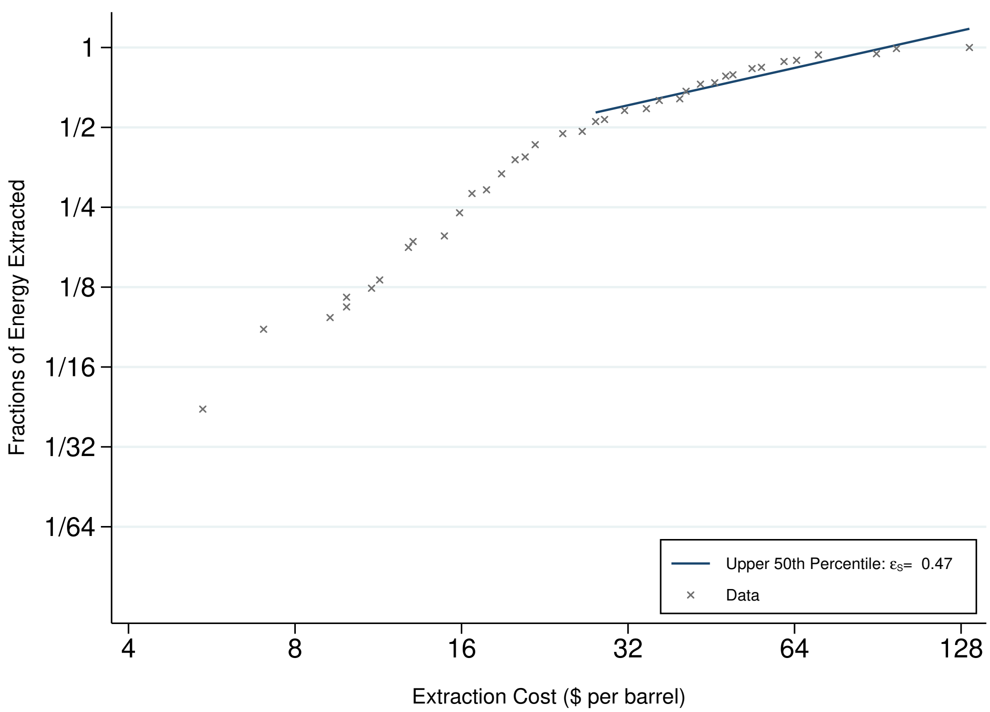

Figure 6: Calibration of the Extraction Supply Elasticity in Foreign