PURPOSE AND ORIGIN

THE COWLES FOUNDATION FOR RESEARCH IN ECONOMICS AT YALE UNIVERSITY, established as an activity of the Department of Economics in 1955, is intended to sponsor and encourage the development and application of quantitative methods in economics and related social sciences. The Cowles Foundation continues the work of the Cowles Commission for Research in Economics, founded in 1932 by Alfred Cowles at Colorado Springs, Colorado. The Commission moved to Chicago in 1939 and was affiliated with the University of Chicago until 1955. At that time, the professional research staff of the Commission accepted appointments at Yale and, along with other members of the Yale Department of Economics, formed the research staff of the newly established Cowles Foundation. The members of the professional staff typically have faculty appointments and teaching responsibilities in the Department of Economics or other departments at Yale University.
COWLES FOUNDATION FOR RESEARCH IN ECONOMICS AT YALE UNIVERSITY

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RESEARCH ACTIVITIES
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INTRODUCTION

The Cowles Commission for Research in Economics was founded approximately fifty years ago by Alfred Cowles, in collaboration with a group of economists and mathematicians concerned with the application of quantitative techniques to economics and the related social sciences. This methodological interest was continued with remarkable persistence during the early phase at Colorado Springs, then at the University of Chicago, and since 1955 at Yale.

One of the major interests at Colorado Springs was in the analysis of economic data by statistical methods of greater power and refinement than those previously used in economics. This was motivated largely by a desire to understand the chaotic behavior of certain aspects of the American economy—the stock market in particular—during the Depression years. The interest in statistical methodology was continued during the Chicago period and into the present with a growing appreciation of the unique character and difficulties of statistical problems arising in economics. An important use of this work was made in the description of the dynamic characteristics of the U.S. economy by a system of statistically estimated equations.

At the same time, the econometric work at Chicago was accompanied by the development of a second group of interests—also explicitly mathematical but more closely connected with economic theory. The activity analysis formulation of production and its relationship to the expanding body of techniques in linear programming became a major focus of research. The Walrasian model of competitive behavior was examined with a new generality and precision, in the midst of an increased concern with the study of interdependent economic units, and in the context of a modern reformulation of welfare theory.

The move to Yale in 1955 coincided with a renewed emphasis on empirical applications in a variety of fields. The problems of economic growth, the behavior of financial intermediaries, and the embedding of monetary theory in a general equilibrium formulation of asset markets were studied both theoretically and with a concern for the implications of the theory for economic policy. Earlier work on activity analysis and the general equilibrium model was extended with a view to eventual applications to the comparative study of economic systems and to economic planning at a national level. Algorithms for the numerical solution of general equilibrium models were developed, the study of non-convex production sets was pursued, and a variety of applica-
tions of game theory to economic problems were investigated. Along with the profession at large, we have engaged in the development of analytical methods oriented to contemporary social and economic problems, in particular the specifics of income distribution, the economics of exhaustible resources and other limitations on the growth of economic welfare.

For the purposes of this report it is convenient to categorize the research activities undertaken at Cowles during the last three years in the following way:

A. Macroeconomics
B. Mathematical Economics
C. Game Theory
D. Microeconomics
E. Econometrics

A. Macroeconomics

The effects of monetary and fiscal policies on economic activity—on production, employment, and prices—are a prominent practical concern of national and international politics. They are also a central topic of theoretical and empirical economics. They have been the major focus of James Tobin’s research for forty years. Most of it has taken place at the Cowles Foundation since its establishment at Yale in 1955, in association and collaboration with colleagues and students. Previous Reports have described the approach and the findings of much of this research.

Tobin has been particularly interested in the processes by which financial markets and institutions transmit the monetary and fiscal measures of government to the economy at large. These markets and institutions balance the changing demands for and supplies of assets of various types, from currency to capital goods. They also determine interest rates and asset prices, and through them affect expenditures on goods and services, for capital investment or consumption. Asset demands and supplies reflect the saving, portfolio, and balance sheet choices and adjustments of households, businesses, foreigners, and governments. Central government policies work by altering the amounts and terms of supply of important assets—especially currency and bank reserves, and public debt obligations of various maturities. In its emphasis on a broad spectrum of assets, generally imperfect substitutes for each other, and on the relative attractiveness of real assets, or claims to real assets, to savers and portfolio managers, the approach differs sharply from theories that concentrate on the volume of an arbitrarily defined monetary aggregate.

In the recent past Tobin has restated, revised and extended his theoretical framework in several publications. In the articles, “Deficit Spending and Crowding Out in Shorter and Longer Runs” and “Fiscal and Monetary Policies, Capital Formation and Economic Activity” (with Willem Buiter), Tobin presents models of the accumulation of savings in the forms of currency,
interest-bearing public debt, and real capital. The papers concern the short- and long-run effects of government deficits, and of their division between currency issue and debt issue, on real output and capital formation. They attempt, in particular, to delineate the conditions under which government demands for private saving ‘crowd out’ private investment. A discussion and summary of the basic framework also appears in Tobin’s Yrjo Jahnsson Lectures, delivered in Finland in 1979.

In December 1981 Tobin took the opportunity of his Nobel Lecture in Stockholm to present a full discussion, exposition and defense of his theoretical model. In a sense, this is a restatement and revision of the earlier exposition in ‘‘A General Equilibrium Approach to Monetary Theory’’ (Journal of Money, Credit and Banking, 1969).

The ‘‘flow-of-funds’’ statistics compiled by the Federal Reserve System are data on sectoral holdings of various assets and debts, which provide the possibility of empirical implementation of the theoretical framework just described. Research at the Cowles Foundation designed to estimate sectoral asset demands and supplies and to combine them in a complete system of asset market equations is described in previous Reports. Tobin contributed to ‘‘A Model of U.S. Financial and Nonfinancial Economic Behavior,’’ a recent report of the status of this empirical research.

The theory and practice of monetary and fiscal policy have recently been the scene of intense controversy, not just in the public and political arena but in the economics profession itself. A ‘‘new classical macroeconomics,’’ involving the principle and methodology of ‘‘rational expectations,’’ has questioned the effectiveness of systematic policies of countercyclical stabilization. The new school rejects Keynesian theories of economic fluctuations, from which Tobin’s framework is in important respects a descendant. It is not surprising, therefore, that Tobin has participated actively in the controversy. In the second of his Yrjo Jahnsson lectures, Tobin criticizes the new classical theories for their inability to explain the common features of observed business fluctuations and argues that this failure renders suspect their policy conclusions. In a major paper, ‘‘Stabilization Policy Ten Years After,’’ invited for the tenth anniversary of the Brookings Panel on Economic Activity of which he was a charter member, Tobin reviewed the events and policies of the 1970s in the light of their congruities or incongruities with macroeconomic theories and models.

Tobin has had a long-standing interest in saving behavior. During the period of this Report, his Paish Lecture in York, England in 1978, endeavored to refute the hypothesis of Robert Barro that government deficits financed by interest-bearing debt do not absorb saving because taxpayers save extra in anticipation of higher future taxes to service the debt. If the hypothesis were true, Tobin’s theoretical framework, described above, would not apply; Tobin
argues that most savers' horizons are much shorter than the infinite horizons Barro assumes. An empirically oriented discussion of the same issue is contained in the article "Debt Neutrality: A Brief Review of Doctrine and Evidence" with Willem Buiter. In "Mandatory Retirement, Saving and Capital Formation," Tobin and Walter Dolde presented simulations to describe the effects of social and private retirement systems on national saving and investment. This is a sequel to their earlier study "Wealth, Liquidity and Consumption" (in Consumer Spending and Monetary Policy: The Linkages, Federal Reserve Bank of Boston, 1971).

In 1982, a third volume of Tobin's professional papers was published.

It has long been recognized that the effects of monetary policy on the economy are influenced by the legal restrictions placed on the behavior of financial intermediaries, such as Regulation Q ceilings on commercial bank savings and time deposit interest rates. In CFDP 605, Christophe Chamley and Heraklis Polemarchakis used a general arbitrage argument of the Modigliani-Miller type to show that in a model with unrestricted trading, government trades in existing assets can have no effect on the real allocation of resources in equilibrium. In such a setting, monetary policies such as open-market operations would have no impact on the pattern of resource use. Thus real effects of such policies can only occur when restrictions are placed on private trading.

The recent theoretical debates on the effectiveness of macroeconomic policies are in part a reflection of the continued tension between economists' vision of market-clearance at the microeconomic level, and their macroeconomic prescriptions. During the period of this report, Katsuhiro Iwai published Cowles Foundation monograph No. 27, Disequilibrium Dynamics—A Theoretical Analysis of Inflation and Unemployment which was awarded the "Grand Prix of Nikkei Keizaitosho Bunka Sho" (grand prize for books in Economics) in 1982 in which his aim is to provide a microeconomic model of a market economy with no necessary tendency towards optimal employment of resources. To this end, Iwai dropped the conventional assumption of perfect competition and proposed instead a model of a monopolistically competitive economy in which the numerous interdependent firms set their own prices and fix their own wage offers without knowing what demands and supplies will be forthcoming. On this basis Iwai has tried to build a structure that explains the evolution of prices, wages, employment, and output for the economy as a whole, not as a smooth trajectory of equilibrium positions, but as a causal process that is moved by the complex pattern of dynamic interactions among firms.

The monograph consists of three parts. Part I reformulates Knut Wicksell's theory of cumulative processes. It shows that in a monetary economy, if prices and wages are flexible, a deviation from equilibrium, however small,
inevitably produces errors in firms' expectations and starts a dynamic process that tends to drive prices and wages cumulatively away from equilibrium. Such a process of inflation or deflation breeds, in the course of its own development, both accelerating and decelerating forces, and whether or not it will eventually return to equilibrium is decided only by the relative strength of these conflicting forces. With flexible prices and wages there is no a priori ground for a belief in the self-adjusting character of the economic system. On the contrary, it is argued in Part II, inflexibility rather than flexibility of money wages is what stabilizes a monetary economy. With sticky money wages, the system normally approaches a Keynesian equilibrium where employment is determined by effective demand. It is only in response to a macroeconomic disturbance large enough to break the inflexibility of money wages that the system abandons the Keynesian equilibrium and sets off on a cumulative process of inflation or deflation. A Keynesian principle of effective demand is thus integrated with a Wicksellian theory of cumulative processes. Part III then undertakes a long-run analysis of inflation and unemployment. It demonstrates that a monetary economy never outlives its monetary history. In particular, if money wages rise more readily than they fall, the Phillips curve is never vertical. Part III concludes with an analysis of the problem of wage-push stagflation, showing how this can be approached by the method developed in the monograph.

In other work studying macroeconomic variables out of equilibrium, Peter C. B. Phillips together with V. Hall and R. Bailey report the theoretical development of a small aggregative model of output, employment, capital formation and inflation (CFDP 552). The model is designed to explain medium term cyclical growth in a small open economy. It allows explicitly for disequilibrium in the markets for goods and factors of production and has a wage-price sector in which the movements in these variables are specified to allow for intended price setting behavior by firms while, in addition, responding to realizations which may differ from these intentions as well as responding to the effects of disequilibrium in the real sector. The model is formulated in continuous time as a system of non-linear differential equations and has a particular solution which corresponds to plausible steady state growth behavior for the variables of the model. The properties of this particular solution are analyzed directly, and solution trajectories for the variables corresponding to various intitial values which deviate from the steady state growth paths are computed numerically and compared with the steady state growth paths. The model has been developed with a view to subsequent empirical application to a small open economy and, as a foundation for this later work, some econometric methodology for the treatment of non-linear differential equations is developed in the paper.

Fair has recently completed a book (Specification, Estimation, and Analysis
of Macroeconometric Models, Harvard University Press, forthcoming 1983) that is a summary of much of his research in the last few years. The 'specification' part contains a discussion of both his theoretical and econometric macro models. The theoretical model is an attempt to integrate three main ideas. The first is that macroeconomics should be based on better microeconomic foundations. In particular, macroeconomics should be consistent with the view that decisions are made by maximizing objective functions. The second idea is that macroeconomic theory should allow for the possibility of disequilibrium in some markets. The third idea is that a model should account explicitly for balance sheet and flow of funds constraints. Contrary to previous disequilibrium work, including the work on fixed price equilibria, the model provides a choice-theoretic explanation of market failures. Firms set prices in a profit maximizing context, but because of possible expectations errors, these prices may not be market clearing. The original discussion of the theoretical model is in A Model of Macroeconomic Activity, Volume I: The Theoretical Model (Ballinger Publishing Co., 1974). The model is expanded to two countries in "A Model of the Balance of Payments." The original discussion of the United States econometric model is in A Model of Macroeconomic Activity, Volume II: The Empirical Model (Ballinger Publishing Co., 1976) and "The Sensitivity of the Fiscal Policy Effects to Assumptions about the Behavior of the Federal Reserve" (Econometrica, September 1978), and the original discussion of the multicountry econometric model is in CFDP 541R and in "Estimated Output, Price, Interest Rate, and Exchange Rate Linkages Among Countries" (Journal of Political Economy, June 1982).

The 'estimation' part contains a discussion of the estimation of large scale nonlinear models by various methods. The methods include two stage least squares, three stage least squares, full information maximum likelihood, and two stage least absolute deviations. Fair has worked on various aspects of these estimators during the past few years, particularly the computational aspects. The original discussions are in "The Estimation of Simultaneous Equation Models with Lagged Endogenous Variables and First Order Serially Correlated Errors" (Econometrica, May 1970), and "Full-Information Estimates of a Nonlinear Macroeconometric Model" with William Parke. The results in the book show that it is now computationally feasible to estimate large scale nonlinear models by full information methods and by robust methods like two stage least absolute deviations.

The main theme of the 'analysis' part of the book is the argument that more testing of models should be done once they are specified and estimated. It is only by testing models against each other that there is any hope of narrowing the current range of disagreements in macroeconomics regarding the structure of the economy. Fair has recently developed a method for estimating the pre-
dictive accuracy of models that takes into account the four main sources of uncertainty of a prediction: uncertainty due to 1) the error terms, 2) the coefficient estimates, 3) the exogenous variables, and 4) the possible misspecification of the model. Because the method accounts for all four sources, it can be used to make comparisons across models. The method has been used to compare a number of models in the book, including Fair’s U.S. model and two vector autoregressive models. The original discussions of this work are in “An Analysis of the Accuracy of Four Macroeconometric Models,” “Estimating the Expected Predictive Accuracy of Econometric Models,” and “The Effects of Misspecification of Predictive Accuracy.” The method relies heavily on the use of stochastic simulation, which, as seen in the book, can now be routinely done. The analysis part also contains a discussion of the estimation of the uncertainty of policy effects in models (originally in “Estimating the Uncertainty of Policy Effects in Nonlinear Models”) and a discussion of the solution and analysis of optimal control problems (originally in “On the Solution of Optimal Control Problems as Maximization Problems” (Annals of Economic and Social Measurement, January 1974) and “The Use of Optimal Control Techniques to Measure Economic Performance” (International Economic Review, June 1978)).

The final chapter of the book contains a discussion of the solution and estimation of nonlinear rational expectations models. The original discussion is in “Analysis of a Macro-Econometric Model with Rational Expectations in the Bond and Stock Markets” and “Solution and Maximum Likelihood Estimation of Dynamic Rational Expectations Models” with John Taylor. The methods discussed in this chapter have considerably expanded the range of rational expectations models that can be estimated and analyzed.

Fair and Parke have recently completed a computer program that allows all the techniques discussed in the book to be easily applied once the model has been set up in the program. The hope is that this program will allow more testing and analysis of models than has been true in the past.

Papers presenting early versions of Tobin’s theoretical framework “Pitfalls in Financial Model Building” (American Economic Review, May 1968) with William Brainard and “A General Equilibrium Approach to Monetary Theory,” did not take account of international financial markets and capital movements. Nevertheless the “Yale” portfolio approach proved to be the foundation for a vigorous and fruitful literature on international payments balances and exchange rates in a world of capital mobility across currencies. In “The Short-Run Macroeconomics of Floating Exchange Rates: An Exposition,” Tobin, in collaboration with Jorge de Macedo, essays his own extension of the basic model to open economies, concluding that the qualitative conclusions of the closed economy models survived, whether exchange rates were fixed or floating. Asset holdings in different currencies are also allowed
in "Fiscal and Monetary Policies, Capital Formation, and Economic Activity," and in Tobin’s Yrjo Jahnsson Lectures. Other papers on international economics are "A Proposal for International Monetary Reform," which argues for taxing inter-currency transfers of funds in order to give national monetary and fiscal policies more autonomy and "The State of Exchange Rate Theory: Some Skeptical Observations," which reviews critically current fashions in the theory of foreign exchange rates.

In another article on open economy macroeconomics, "Macroeconomic Tradeoffs in an International Economy with Rational Expectations," John Taylor, who visited the Cowles Foundation during 1980, considered alternative exchange rate rules in conjunction with alternative monetary policy rules. One of the main features of the monetary rules considered was the dependence of the rate of growth of the money supply on the recent rate of inflation—that is, the degree of monetary accommodation to inflation. There is a close relationship between monetary accommodation and exchange rate accommodation—the latter being defined as the degree of response of a managed exchange rate to a change in price competitiveness. A purchasing power parity rule in which the exchange rate adjusts to fully offset any change in the home country price level relative to the rest of the world is analogous to a fully accommodative monetary policy rule. Corresponding to zero monetary accommodation is a fixed exchange rate regime. The interaction between these two alternative types of accommodation has important implications for macroeconomic fluctuations of an open economy.

In order to provide a framework for examining some of the accommodation issues quantitatively, a general N-country international model based on wage contracts and rational expectations was developed. Econometric work described in the article indicated that the international connections are strong and that a small open-economy framework could be misleading for econometric examinations of alternative policies. A general procedure for evaluating policy in this international context was developed.

One outcome of this research is the finding that international price linkages call for more macroeconomic coordination. If such price linkages—modelled formally by including foreign prices evaluated in the domestic currency in the domestic price determination equation—are quantitatively significant, then it is difficult for individual countries to manage their internal economy without external effects. And perhaps more importantly, such linkages suggest that a policy mix of less accommodative exchange rate rules and more accommodative monetary policy would be preferred to a mixture which calls for exchange rate rules which are close to purchasing power parity rules.

Other research on the effects of economic policy undertaken at the Cowles Foundation used data from various countries as a source of information. Such international comparisons must be carefully performed, since no policy analy-
sis which effectively uses international evidence can ignore structural differences between countries. Structural differences—arising from behavioral, technological, or institutional factors—influence economic performance and may prevent the repeated success of an economic policy attempted at a different time or place.

In his article, "Policy Choice versus Economic Structure," Taylor adopted an econometric approach to sorting out policy choice from economic structure in a comparison of macroeconomic performance in several large OECD countries. In particular, the aim was to determine whether international differences in cyclical fluctuations in inflation and real GNP are due to policy differences in monetary and fiscal procedures, or to structural differences in wage and price setting arrangements and the susceptibility of each country to external shocks.

The criterion of economic performance used in this comparison was the magnitude of fluctuations in inflation and output around longer term secular trends. Economic performance is rated poor according to this criterion if the fluctuations are large and long-lasting; a good rating results if the fluctuations are small and temporary.

The macroeconomic policies which were examined have the objective of holding down the size of these cyclical fluctuations. Although every country would like to use these stabilization policies to minimize both inflation and output fluctuations, there is a macroeconomic tradeoff which forces a choice between the two. When a country is up against this tradeoff, smaller fluctuations in output can only be achieved through larger fluctuations in inflation. Since some countries have greater concern with inflation stability while others have greater concern with output stability, they will naturally choose different policies when faced with this tradeoff. Hence, policy choice will differ across countries.

The approach is illustrated in Figure 1 (taken from the article), where tradeoff curves for six OECD countries are presented. On each curve the darkened circle represents the actual economic performance of each of the countries. The time period for which the econometric parameters were derived is 1955-75.

The actual performances of the six countries in Figure 1 are substantially different from each other. But since their tradeoff curves are also quite different, we are safe in saying that these differences are not entirely due to policy choice. Only the U.S. and Canada have tradeoff curves which are nearly the same. For these two countries a large part of the difference between their economic performances can be attributed to policy choice, with Canada choosing a relatively accommodative policy and the U.S. choosing a relatively nonaccommodative policy. It is interesting that Canada is located on a very flat part of the tradeoff curve. According to these results Canadian economic
POLICY TRADEOFFS FOR SIX COUNTRIES

- ACTUAL POLICY, 1955-1975
- POLICY UNDER A COMPROMISED SET OF IDENTICAL PREFERENCES DETERMINED BY AVERAGING THE OBSERVED PREFERENCES OF EACH COUNTRY

FIGURE I
policy could have been made considerably less accommodative with only a negligible deterioration of output performance, but big gains in inflation performance.

In Figure 1 it is also shown how preferences or tastes could be taken into account, by extending a ray from the origin of the diagram with a slope determined by the average ratio of inflation fluctuations to output fluctuations in each country. Along this ray each country would have the same ratio of output to inflation stability as the average of all countries during the observation period: one definition of compromised preferences. If each country had average tastes (by this definition), there would still be significant differences in economic performance; but the differences in price stability would be considerably smaller. These results indicate that much of the difference in economic performance across countries is due to structural differences rather than to taste differences. In particular, it is interesting to observe the extent to which high price stability in Germany appears to be due to a favorable economic structure rather than solely to more "dislike" for price instability.

In related work Stanley Black, who visited the Cowles Foundation during 1980-81 studied the conduct of monetary policy in a paper "The Use of Monetary Policy for Internal and External Balance in Ten Industrial Countries" presented at the NBER Conference on Exchange Rates and International Macroeconomics in November 1981. Monetary policy reaction functions were estimated using advanced regression techniques. The results yielded some interesting cross-country comparisons, leading to the following conclusions: (a) There is an inverse correlation between the importance given to inflation objectives in formulating monetary policy in different countries and observed average rates of inflation in the 1970s. (b) The importance attached to inflation and unemployment objectives varies inversely across countries. (c) There appears to be little relationship across countries between the importance of unemployment objectives and observed average rates of unemployment. (d) There is an inverse correlation across countries between the importance of internal and external objectives for monetary policy. (e) There is an inverse correlation between the flexibility of the exchange rate and the relative importance of external compared to internal objectives, both over time and across countries. (f) Finally, conservative election victories have often led to tighter monetary policies.

In recent years, one of the major influences on the macroeconomic performance of the industrial economies has been the price of oil and other energy sources. Since 1979, William Nordhaus has been investigating the incorporation of models of energy systems into more general macroeconomic models. The model and results were presented in Brookings Papers on Economic Activity (1980: 2).

The model begins with a model of the supply and demand for oil. For a
given price, energy demand appears to be approximately proportional to output in the short run, due to the lack of substitutability with a given capital stock. Substitution away from energy takes place only as the capital stock embodying old technology is replaced with capital of newer vintages.

A crucial element in any model concerned with the interaction between world markets and economic activity is the specification of OPEC oil pricing behavior. The article argues that the behavior of key producers is best viewed as "noncooperative" rather than (either) as competitive or monopolistic. Formally, whenever world oil supply approaches short-run capacity—either because of strong demand or capacity disruptions—spot prices rise above list or contract prices. If such a situation is maintained for long, list prices are raised. Subsequently, if demand slackens, output of individual producing countries, particularly those that are financially unconstrained, will be restricted so as to maintain the higher list prices. Thus the mechanism generates an upward ratchet in the price of oil through time.

In the long run, oil supplies and prices depend on new discoveries. Although drilling activity has increased sharply in response to higher oil prices since 1973, it appears that additions to reserves have not.

Higher world oil prices affect industrial economies in a number of ways. They directly add to inflation by raising the average price level and contributing to the wage-price spiral; they transfer real income to oil producers; and they alter production techniques, reducing labor productivity and potential output. In addition, depending on the response of policy, they may substantially reduce actual output relative to potential and increase unemployment. To examine these effects, a simplified econometric model of the major OECD economies is presented, incorporating a world oil market and paying particular attention to the mechanisms by which oil affects the industrial countries. The model is used to investigate the effects of past OPEC price increases and to explore the policy alternatives available to the industrial nations. It is concluded that the first OPEC price increase of 1973-74 added only between 0.6 to 1.2 percentage points to the annual inflation rate for 1973-79 and subtracted less than 0.2 percentage point from the annual growth in labor productivity. Real incomes were reduced by 2.9 percent over this period, primarily through the transfer of real income abroad.

The inclusion of a world oil market with explicit behavioral equations for OPEC oil supply and pricing enables projection of future conditions in industrial economies under alternative assumptions for both oil supply and policy responses. In his projections the greater economic costs come from loss of real income, either through higher world oil prices or through slow economic growth that reduces domestic output and employment. To avoid narrowing the gap between output and capacity in world oil markets, and thus driving up the spot and then the list price of oil, policy must restrain the demand for oil.
But if this is done by slowing growth, the real income lost through lower production is even greater than the real income saved by avoiding higher oil import bills. According to Nordhaus’ simulations, the optimal policy is to raise domestic energy prices for users in the industrial countries, through either taxes or equivalent conservation measures. By restraining demand for oil, high domestic prices keep world crude oil prices low while permitting a normal expansion of output and real incomes.

B. Mathematical Economics

Researchers at the Cowles Foundation have continued to investigate the Walrasian model of general equilibrium. The classical study of existence of Walrasian equilibria is Gerard Debreu’s *Theory of Value*, published as Cowles Foundation monograph No. 16 in 1959. Since this demonstration of the consistency of the Walrasian framework, general equilibrium theorists have pursued a variety of directions. Major issues associated with the Walrasian model, such as uniqueness and stability of equilibria, and the generality of excess demand functions, have been explored. Other work has made the equilibrium framework available for policy analysis by developing and applying algorithms for the computation of equilibrium prices. A third major research goal has been to relax the assumptions of earlier work, and so extend the range of applicability of the theory. All three of these directions have been represented in recent research of Cowles staff members and visitors.

An early hope of general equilibrium theorists was that the assumption of rational behavior of consumers would place significant restrictions on the nature of aggregate excess demand functions. In 1974, Debreu completed the work initiated by Hugo Sonnenschein by demonstrating that any continuous function from the unit simplex in $\mathbb{R}^l$ into $\mathbb{R}^l$ satisfying the Walras’ Law could be decomposed into $l$ functions each representing the excess demand of a rational consumer (Debreu, “Excess Demand Functions,” *Journal of Mathematical Economics*, 1, 1974; Sonnenschein, “Market Excess Demand Functions,” *Econometrica*, 40, 1972). In CFDP 643 John Geanakoplos gave a geometric proof of Debreu’s theorem. In CFDP 642 with Polemarchakis, Geanakoplos showed that when there are fewer consumers than goods, then the standard neoclassical assumptions do put restrictions on the aggregate excess demand function: with one consumer there are the Slutsky conditions (negative definiteness and symmetry of its Jacobian on a space of dimension $l-1$) and conversely any function satisfying the Slutsky conditions is the excess demand of a rational consumer. Furthermore, for any $m < l$, a function $x(p)$ satisfies Slutsky conditions on a space of dimension $l-m$ if and only if it is the aggregate excess demand of $m$ rational consumers.

In a similar spirit Geanakoplos and Geoffrey Heal in CFDP 651 gave a geo-
metric proof of the transfer paradox analyzed by Leontief, Samuelson, and Chichilnisky: in a two agent, Walrasian stable economy the transfer of endowment from agent 1 to agent 2 cannot hurt the utility of 2, but if there is a third agent then in fact the transfer can hurt agent 2 and help agent 1.

All of the above results depend crucially on the income effects in a rational consumer's excess demand: indeed, it is the income effect matrix of rank 1 that gradually erodes the Slutsky restrictions on excess demands as consumers are added one by one to the model.

Tjalling Koopmans and Hirofumi Uzawa also completed a theoretical study of demand functions in the period of this report. This study originated in a discussion that took place in the summer of 1974 at the International Institute for Applied Systems Analysis in Laxenburg, Austria. The discussion, which involved Uzawa, T. N. Srinivasan, Nordhaus and Koopmans, was brought to its present state in CFDP 654, and was submitted for publication by Koopmans and Uzawa under the title "'Constancy and Constant Differences of Price Elasticities of Demand.'" The starting point was a concern that econometric practice in estimating such elasticities often proceeds from an assumption that own-price and cross-price elasticities of demand for several goods are constant over a wide area of the space.

The first half of the study shows that, under these assumptions of constancy, the constant values can only be \(-1\) for all own-price elasticities, and 0 for all cross-price elasticities. Thus, from mere logic, the assumption of constant elasticities is found to contradict the idea that the values of the price elasticities should be estimated. In particular, the demand functions for the \(n\) goods, say, must have the simple form

\[
q_i = \frac{\theta_i}{p_i}, \quad i = 1, \ldots, n, \text{ all } \theta_i > 0,
\]

which, in turn, can be derived by maximization of a utility function of the form

\[
U(q_1, \ldots, q_n) = \sum_{i=1}^{n} \theta_i \log q_i
\]

under the budget constraint

\[
\sum_{i=1}^{n} p_i q_i = y,
\]

where \(y\) is a given positive number. Conversely, postulating this form of the utility function implies the constant values of own- and cross-price elasticities of demand already indicated.

The second half of the study starts by postulating maximization of the so-called CES (''constant elasticity of substitution'') utility function,
$$U(q_1, \ldots, q_n) = (\alpha_1 q_1^{-\rho} + \ldots + \alpha_n q_n^{-\rho})^{-\frac{1}{\rho}},$$

where $0 < \rho < 1$ and the $\alpha_i$ are positive numbers, derives the form of the corresponding demand function, and looks for any constancy properties that the price elasticities of demand may exhibit. It is found that in this case it is the pair-wise differences between the elasticities of demand for any one good $i$ with regard to the prices of any two goods, $j, k$ which are constant. For instance, in the case of $n = 3$ goods altogether, the nine price elasticities $\eta_{ij}$ of this kind depend on only four numbers, the three own-price elasticities $\eta_{11}, \eta_{22}, \eta_{33}$, and the parameter $\rho$, in the manner indicated in the following table:

<table>
<thead>
<tr>
<th>$\eta_{ij}$ =</th>
<th>w.r.t. the price of good</th>
</tr>
</thead>
<tbody>
<tr>
<td>elasticity of</td>
<td>$j = 1$</td>
</tr>
<tr>
<td>demand for</td>
<td>$\eta_{11}$</td>
</tr>
<tr>
<td>good $i = 1$</td>
<td>$\eta_{11} + \frac{1}{1+\rho}$</td>
</tr>
<tr>
<td>2</td>
<td>$\eta_{11} + \frac{1}{1+\rho}$</td>
</tr>
<tr>
<td>3</td>
<td>$\eta_{11} + \frac{1}{1+\rho}$</td>
</tr>
</tbody>
</table>

The elasticities $\eta_{ij}$ themselves need not be constant. The paper, to be circulated first as CFD 554, gives explicit formulae for the price elasticities $\eta_{ij}$ and the underlying demand functions.

Koopmans also completed a second study in which he had had a long-standing interest. In its simplest form, the problem concerns the properties of a function of the form $F(x, y) = f(x) + g(y)$, (x and y scalars) which, besides being additively separable, is assumed to be quasiconvex. That is, if $(x_0, y_0)$ and $(x_1, y_1)$ are two distinct points in a two-dimensional space, then the maximum of $F(\lambda x_0 + (1-\lambda)x_1, \lambda y_0 + (1-\lambda)y_1)$ on the interval $0 \leq \lambda \leq 1$ is attained in one, the other, or both endpoints, $\lambda = 0$ or $\lambda = 1$. The most frequent application of this concept in economics has been to quasiconcavity of utility functions, the subject of a much cited paper by Arrow and Enthoven. Since a function is quasiconcave if and only if its negative is quasiconvex, the above problem also bears on that of the form of utility functions that are both quasiconcave and additively separable.

One of the early findings with respect to the above quasiconvex function $F(x, y)$ was that both $f(x)$, $g(y)$ are continuous and right and left differentiable in all points, while at most one of them can fail to be convex.

A research visit by Gerard Debreu to the Cowles Foundation in 1976 led to a collaboration in which earlier findings and conjectures were reformulated and additional new ones developed and proved.
A finding of considerable interest is illustrated by the diagram shown below. It describes a property linking the graphs of the component functions $f(x)$ and $g(y)$ of $F(x,y)$ in a different way for each pair of arbitrarily selected points, $x_0$ in the domain of $f$, $y_0$ in that of $g$. The drawn curve represents a monotonic segment of the graph of $f$, containing the point $(x_0, f(x_0))$. The dotted curve represents a linear transform of a similar segment of the graph of $g$. The linear transform has the effect that the dotted curve has the point $(x_0, f(x_0))$ in common with the drawn curve. In addition, the fraction $\alpha/\beta$ in the definition of the transform can be so chosen that the dotted curve in no point exceeds the drawn curve. Moreover, the construction permits a separating function, represented by the dashed curve, where the number $\alpha$ has been so chosen that each point on the dashed curve is located, on its vertical, between the points of the other two curves on that same vertical. Finally, the separating function has the mathematical form defined by

$$
\text{if } \theta \neq 0, \quad h_\theta(z) = -\frac{1}{\theta} \log (1 - \theta z), \quad \text{where } 0 < z < 1
$$

$$
\text{if } \theta = 0, \quad h_0(z) = z.
$$

It is puzzling to see a nonlinear separating curve, defined in terms of a logarithmic function, arise as an implication of quasiconvexity combined with additive separability. Since a quasiconvex function is only the negative of a quasiconcave function, the question of the possible application of this finding to utility and its maximization may well be raised.
In CFDP 524, Roger Howe demonstrated that differentiability is a generic property of convex functions (defined on some fixed convex set), in the sense that the set of differentiable convex functions is a countable intersection of open dense subsets of the space of all convex functions. This is in contrast to the well-known fact that in the space of all continuous functions, the differentiable functions are non-generic.

In another paper, Howe studied the tendency to convexity of the vector sum of sets (CFDP 538). The paper focuses on the extent to which the vector sum $\Sigma V_i$ of subsets $V_i$ fills up its convex hull $\text{Co}(\Sigma V_i)$. It starts with a proof of the Shapley-Folkman theorem which bounds the distance between a general point of $\text{Co}(\Sigma V_i)$ and the closest point of $\Sigma V_i$. Then it shows that under various regularity assumptions on the $V_i$ (non-empty interior, smoothness of boundary, non-zero measure, arcwise connectivity), the set $\Sigma V_i$ tends to fill up the inside of $\text{Co}(\Sigma V_i)$.

One of the assumptions needed to guarantee continuity of demand functions, and thus existence of competitive equilibria, is that consumers have convex preferences. There have been many papers proving existence of approximate equilibria in exchange economies with the bounds on the excess demands depending on some measure of the non-convexity of preferences.

The recent joint work of Robert Anderson (who visited the Cowles Foundation during 1980-81), M. Ali Khan and Salim Rashid takes a different approach. Under minimal assumptions, it is shown that any exchange economy possesses a price so that market excess demand is bounded above by a constant which depends on the number of agents and the size of the endowments but is independent of the preferences. The constant is of the order of the square root of the number of agents times the largest endowment. The paper, entitled “Approximate Equilibria with Bounds Independent of Preferences,” will appear shortly in the Review of Economic Studies.

There is an intimate connection between the existence of Walrasian equilibria for an exchange economy and Brouwer’s fixed point theorem. All proofs of existence of equilibria rely critically on Brouwer’s theorem. In addition, in 1962, Uzawa demonstrated that existence of equilibrium prices for all economies whose excess demand functions are continuous and obey Walras’ Law implies Brouwer’s theorem (Uzawa, “‘Walras’ Existence Theorem and Brouwer’s Fixed Point Theorem,” Economic Studies Quarterly, 13, 1962). Debreu’s proof that Walras’ Law and continuity characterize market excess demand functions then shows the full equivalence of the Walrasian problem for an exchange economy and Brouwer’s fixed point theorem. When production is introduced into the general equilibrium model, Kakutani’s fixed point theorem is the tool used to demonstrate existence.

In Cowles Foundation monograph No. 24 published in 1973, Herbert Scarf in collaboration with T. Hansen developed techniques for computing equilib-
rium prices for general equilibrium models, both with and without production. These techniques involve the identification of approximate fixed points of continuous mappings of the simplex into itself. Since the publication of this monograph there have been several major refinements of Scarf's original algorithm.

A. Talman, who visited the Cowles Foundation in 1980-81, co-authored two monographs on simplicial fixed point algorithms with G. Van der Laan (Variable Dimension Fixed Point Algorithms and Triangulations, and Simplicial Fixed Point Algorithms, Mathematisch Centrum, Amsterdam, 1980). In these monographs they developed procedures which have the novel property that they can be initialized at an arbitrary point in the simplex. This allows efficient use of information as the grid size is refined and closer approximations to fixed points are sought. Talman and van der Laan's procedure involves working with subsets of the simplex whose dimension varies in the course of the algorithm. Computational experience suggests that this variable dimension algorithm significantly decreases the time necessary to achieve the desired degree of agreement between a point in the simplex and its image under the mapping.

In related work, Talman and Van der Heyden have used the ideas of the variable dimension algorithm to generalize Lemke's algorithm for the linear complementarity problem (CFDP 600). While Lemke's procedure always begins at a pre-assigned point, the new algorithm can start anywhere, and is thus better suited to performing sensitivity analyses and other forms of parametric analysis of programming problems.

In CFDP 542, Howe studied the linear complementarity problem from a different perspective. This paper observed that the formulation of the linear complementarity problem as the problem of inverting a piecewise linear, positive homogeneous map from \( \mathbb{R}^n \) to itself allows one, under non-degeneracy assumptions, to apply degree theory toward understanding linear complementarity. A number of well-known results on linear complementarity are explained in terms of degree theory. It is shown that all known algorithms, in particular Lemke's algorithm, apply only to problems whose associated map has degree 1, whereas there exist problems of arbitrarily large degree; in fact, the maximum degree for problems of a given dimension grows exponentially with the dimension.

Another application of fixed point algorithms is to the constructive proofs of a theorem stating that n-person games which obey a certain technical condition, known as balancedness, have non-empty cores. Interest in this result arises partly from the fact that market games involving consumers with convex preferences satisfy the balancedness conditions. In CFDP 575, Van der Heyden generalized Scarf's procedure for computing a point in the core of a balanced n-person game. Scarf's procedure allows each coalition only a finite
number of choices. Confronted with a game that is not finite, Scarf’s procedure must first approximate the characteristic set of each coalition by a finite union of translated non-negative orthants and then compute a point in the core of the approximating discrete game. Van der Heyden develops a procedure which works directly with the characteristic sets, bypassing the discrete approximation required by Scarf.

For the calculation to be easily implemented, the characteristic sets must be given by a union of polyhedral sets. This case is developed in “A Note on Scarf’s Theorem,” within a framework which is an abstraction of the core problem, and which also generalizes the main theorem [Theorem 4.2.3] in Scarf’s monograph, “The Computation of Economic Equilibria.”

The applicability of fixed point algorithms to models of urban land use was investigated by Donald Richter, who visited the Cowles Foundation during 1979-80. In his paper, Richter synthesized and generalized recent literature on the use of fixed point methods to compute approximate numerical solutions to general equilibrium models of urban land use (“A Computational Approach to Resource Allocation in Spatial Urban Models,” *Regional Science and Urban Economics*, 10, 1980). He showed that a broad array of spatial urban models, including ones involving endogenously generated externalities, can be studied within the context of a single unifying computational framework.

In the standard formulation of the Walrasian model the number of agents and the number of commodities are both taken to be finite. There are several situations of economic interest in which it is natural to allow infinite numbers of agents or commodities.

First, the assumption of an infinite number of agents is necessary to formalize the notion of a large but finite economy or an economy having “many” (as opposed to a “few”) agents. The many-agent assumption is at the heart of the perfect competition notion which posits that agents are price takers or, at best, have a negligible or infinitesimal influence on the equilibrium price.

Second, infinite dimensional commodity spaces arise naturally when the notion of commodity is extended to include the time of production and consumption or the state of the world in which production or consumption occurs.

In economies with infinite dimensional commodity spaces, the assumption that utility functions are continuous in the given topology can have significant behavioral implications. These behavioral implications were investigated by Donald Brown and Lucinda Lewis in CFP 525 as a first step in analyzing economies with infinite dimensional commodity spaces. Brown and Lewis continued their analysis of infinite economies in CFDP 581, where they established the existence of a competitive equilibrium in an economy having a continuum of agents and a countable number of commodities, using nonstandard analysis.

The commodity spaces arising in the study of financial markets are quite rich and require new techniques for their analysis. In fact, it appears that a
natural mathematical structure for the space of securities is a Riesz space or vector lattice; to express calls or puts as functions of the underlying security, the nonlinear operations of max and min are introduced and are required to be compatible with vector addition and scalar multiplication: hence a vector lattice. Stephen Ross and Brown are currently looking at the issues of spanning and arbitrage in the framework of Riesz spaces (Cowles Foundation Preliminary Paper 82682).

One of the difficulties arising in the equilibrium analysis of financial markets is demonstrating the existence of demand functions, given the standard assumptions on tastes—technically, budget sets need not be compact. C. D. Aliprantis and Brown have shown the existence of a competitive equilibrium in an economy with a Riesz space of commodities, assuming that demand functions exist and are continuous (CalTech Social Science Working Paper #427).

Geanakoplos studied the equilibria of the “consumption loan” model introduced by Samuelson in his seminal paper in 1958 (“An Exact Consumption Loan Model with or without the Social Contrivance of Money,” *Journal of Political Economy*, 66, 1958). This model, which has played an important role in many recent works in macroeconomics, involves an infinite number of finite-lived agents and an infinite number of commodities. Geanakoplos demonstrated that such models, despite incorporating the twin hypotheses of agent maximization and market-clearing are isomorphic to finite models in which not all markets clear and in which the social endowment exceeds the sum of the individual endowments. This explains the remarkable properties of these models, that typically possess a continuum of equilibria which are not Pareto optimal. Moreover, using nonstandard analysis Geanakoplos analyzed the dimension of the equilibrium manifold of such economies.

By adding production Geanakoplos managed to retain the above qualitative results and also to address some of the issues claiming the attention of the non-Walrasian schools, namely the Keynesian and Sraffian schools. Only in such a model with a continuum of equilibria can the twin Walrasian axioms referred to above be maintained while an analysis of government purchases of paper assets is undertaken. If in addition one supposes that the labor market need not clear then one can derive the familiar Keynesian-Hicksian apparatus of IS-LM curves, but now moving through time in a dynamic model.

One of the central assumptions employed by general equilibrium theorists is that production sets are convex, ruling out economies of scale in production. This assumption guarantees the existence of a price vector supporting any efficient production plan, from which the decentralization theorems of general equilibrium analysis follow. However, it is widely recognized that economies of scale are significant in many industries dominated by large cor-
porations, so that the assumption of convexity may be insufficient to account for the behavior of producers in these industries.

With non-convex production sets, alternatives to price-taking behavior are needed both in the selection of efficient production plans and in the specification of behavioral rules consistent with economy-wide equilibrium. Over the last three years, Brown has collaborated with Geoffrey Heal in undertaking a general equilibrium analysis of an economy in which certain producers have increasing returns to scale. Their notion of equilibrium in an economy with increasing returns is that suggested by Hotelling. A marginal cost pricing equilibrium is a set of consumption plans, production plans, prices and lump sum taxes such that households are maximizing utility subject to after-tax income; firms with decreasing returns to scale are maximizing profits; firms with increasing returns to scale are controlled and price at marginal cost; all markets clear and finally the lump-sum taxes cover the losses of the increasing returns to scale sector.

The major results are the existence of a marginal cost pricing equilibrium (CalTech Social Science Working Paper #415); a second welfare theorem for economies with increasing returns to scale (University of Essex Discussion Paper #179), and the demonstration that the first welfare theorem does not hold for economies with increasing returns to scale (CFP 519).

Richard Mclean, who visited the Cowles Foundation during 1979-80, studied the effects of indivisibilities (which are an economically important type of non-convexity) on the optimal assignment of activities among locations (CFDP 540).

Another study of the impact of indivisibilities is that of Mamuro Kaneko, who was at the Cowles Foundation from 1980-82. In CFDP 571, Kaneko presents a model of a rental housing market in which houses are treated as indivisible commodities. He presents several comparative static propositions demonstrating how competitive rents change with certain parameters of the model.

The most important example of non-convex production sets are those arising from an activity analysis model in which the activity levels are required to assume integral values. During the period of this report, Scarf continued his study of these production sets. Let the columns of the matrix

$$ A = \begin{bmatrix} a_{01} & \cdots & a_{0n} \\ a_{11} & \cdots & a_{1n} \\ \vdots \\ a_{m1} & \cdots & a_{mn} \end{bmatrix} $$
be the list of available production plans with inputs represented by negative numbers and outputs by positive entries. The production set \( Y \) consists of all vectors \( x \) in \( \mathbb{R}^{m+1} \) with

\[
x \leq Ah,
\]

as \( h \) ranges over the integral vectors in \( n \)-dimensional Euclidean space. An example of such a production set (with \( m \) and \( n \) both equal to 2) is given in the following figure, which is taken from the paper "Production Sets with Indivisibilities, Part I: Generalities," published in *Econometrica* in January, 1981.

The economic content of the duality theorem for linear programming asserts that a vector of activity levels satisfying the resource constraints will be optimal if there is a vector of prices which yields zero profit for the activities in use, and non-positive profits for the remaining activities. The simplex method, which is the most frequently used algorithm for solving linear programming problems, can be viewed as a price adjustment mechanism in which prices, and activity levels, are systematically revised until the appropriate profitability conditions are satisfied. In the context of the activity analysis model with continuous activity levels, the role of prices provides an important interpretation of the functioning of markets in the optimal allocation of resources.
When activity levels are restricted to being integers, and more generally when the production set is not convex, prices are no longer available to test whether a feasible set of activities is optimal. A systematic search for a vector of prices which satisfy appropriate profitability conditions can no longer be the basic goal of an algorithm for solving integer programming problems.

The approach adopted by Scarf is to replace the use of prices in solving integer programs by the concept of a neighborhood system which limits the search required to verify that a given vector of activity levels is the optimal solution.

Let us assume that every vector \( h \) of integral activity levels in \( \mathbb{R}^n \) has associated with it a neighborhood \( N(h) \). The association is arbitrary, aside from the following two conditions:

1. \( N(h) = N(0) + h \), so that the neighborhood associated with two lattice points are translates of each other, and

2. If \( k \in N(h) \) then \( h \in N(k) \), which states that the property of being neighbors is a symmetric relation.

A vector of activity levels, satisfying the constraints of the programming problem is then defined to be a local maximum (with respect to the neighborhood system) if every neighbor either violates some of the inequalities or yields a smaller value of the objective function. A major result of the paper "Production Sets with Indivisibilities, Part I: Generalities," is to demonstrate that under mild conditions an activity analysis matrix \( A \) will have associated with it a unique, minimal neighborhood system for which a local maximum is global—for every specification of factor endowments.

This particular neighborhood system which depends on the technology matrix, but not on the factor endowment—is given by the following construction. By selecting the vector \( b = (b_0, b_1, \ldots, b_m)' \), place the inequalities \( Ah \geq b \) so that they define a lattice free region in \( n \) dimensional space. Then relax the inequalities, systematically, so that no further relaxation is possible without introducing a lattice point. In this process some of the inequalities will be relaxed forever; the remaining constraint planes will each contain a lattice point which satisfies all of the other inequalities. The following figure illustrates this process when \( m = 5 \) and \( n = 2 \).

Each such application of this process will yield a finite set of lattice points whose convex hull is a polyhedron with integral vertices and which contains no lattice points other than its vertices; such a polyhedron is termed an integral polyhedron. A specific collection of integral polyhedra associated with the matrix \( A \) is obtained as the relaxations are carried out in different ways starting from arbitrary lattice free regions. The unique, minimal neighborhood system for which a local maximum is global is then obtained by defining two lattice points to be neighbors if they are vertices of a common integral polyhedron associated with \( A \).
If the collection of integral polyhedra associated with a given technology is known, then the related integer programming problems can be solved by path following techniques which are virtually identical to simplicial algorithms for approximating fixed points of a continuous mapping. Depending on the factor endowment, a specific sequence of integral polyhedra is calculated which terminates with a polyhedron one of whose vertices is the optimal solution to the integer programming problem.

In CFDP 649, Philip White, Andrew Caplin, and Van der Heyden demonstrate an intimate relationship between this path of simplices and the path followed by a version of the dual simplex method for the linear programming relaxation of the integer programming problem. They show that this particular dual simplex path is always contained in the polyhedral path generated by Scarf's procedure, and that any polyhedron encountered in the integer programming algorithm intersects this simplex path. Thus as the grid of lattice points is successively refined, Scarf's procedure converges to an efficient algorithm for the linear program. These results also show that the integer programming algorithm can be initialized at the solution of the corresponding linear programming relaxation.

In the paper "Production Sets with Indivisibilities, Part II: The Case of Two Variables," Scarf has given a complete description of the integral polyhedra associated with a matrix A in which the number of columns is equal to two. The specific information is then used to develop an algorithm terminating in a number of steps which is polynomial in the data of the problem. In this paper Scarf conjectured that the general integer program with a fixed number of variables has a polynomial algorithm; a result which was demonstrated in a remarkable paper by H. W. Lenstra, Jr., "Integer Programming
with a Fixed Number of Variables,' making use of sophisticated techniques from that branch of mathematics known as the Geometry of Numbers.

Aside from special cases, a complete description of the collection of integral polyhedra is difficult to obtain when the number of variables is greater than or equal to three. As a consequence Scarf’s research has concentrated on an analysis of properties possessed by such a collection with a view to improving numerical techniques and finding economically significant implications for the theory of the firm. The following two examples are illustrative.

First of all, we note that it is easy to demonstrate that an integral polyhedron in \( n \) dimensions can have no more than \( 2^n \) vertices. This observation leads immediately to the following conclusion, which was independently obtained by Bell and Doignon: Let the integer program

\[
\begin{align*}
\text{max } & a_0 h_1 + \ldots + a_m h_n \\
& a_1 h_1 + \ldots + a_m h_n \geq b_1 \\
& \quad \vdots \\
& a_m b_1 + \ldots + a_m h_n \geq b_m \\
& h \quad \text{integral}
\end{align*}
\]

have a finite maximum. If \( m > 2^n - 1 \) then at least one of the inequalities can be discarded and the resulting integer program will have the same solution. Moreover the bound is sharp in the sense that there are integer programs with \( m = 2^n - 1 \) whose optimal solution is changed by discarding any of the inequalities.

This conclusion should be contrasted with the corresponding situation in linear programming, in which an inequality can be discarded as long as \( m > n \). The simplex method can, in fact, be viewed as a systematic search for that subset of \( n \) inequalities whose optimal solution satisfies the remaining constraints. Part of the difficulty associated with solving integer programs can be attributed to the fact that a large number of inequalities are required to characterize the solution.

As a second example, Scarf has given a detailed study of integer programs in which the technology matrix \( A \) has four rows and three columns, in the paper ‘‘Integral Polyhedra in Three Space.’’ The major conclusion, drawing on previously unpublished work by Roger Howe, states that each matrix has associated with it a family of parallel planes

\[ l_1 h_1 + l_2 h_2 + l_3 h_3 = c, \]

where \( (l_1, l_2, l_3) \) are relatively prime integers, and \( c \) ranges over all integral values. For any \( b = (b_0, b_1, b_2, b_3) \) if the system \( Ah \geq b \) has integral solu-
tions on the two planes \( l_1h_1 + l_2h_2 + l_3h_3 = c \) and \( l_1h_1 + l_2h_2 + l_3h_3 = c' \) it will also have integral solutions on each intermediary plane \( l_1h_1 + l_2h_2 + l_3h_3 = c'' \), with \( c'' \) an arbitrary integer between \( c \) and \( c' \). This conclusion permits us to solve the integer programs associated with \( A \), by solving the two variable problems on each such plane. Specifically the solution on such a plane will permit us to say on which side of the plane the optimal solution to the original three variable problem lies. An economic interpretation might permit us to say that the three variable problem can be decentralized: one agent selects the plane \( l_1h_1 + l_2h_2 + l_3h_3 = c \), and a second agent solves the simpler problem on this plane. The argument which yields an optimal solution is transferred back to the first agent, who can then determine the direction in which \( c \) should be changed in order to move to the optimal solution. A suitable analogue of this type of decentralization for higher dimensions would be of great significance for integer programming.

C. Game Theory

Another major topic of research at the Cowles Foundation has been the development of n-person game theory in relation to economics. Game theory involves a richer specification of the decision-making environment than does general equilibrium theory. In the Walrasian framework the price system serves as the sole means by which individuals are guided to mutually consistent decisions. In contrast, game theory explicitly recognizes the interdependence between agents' choices, so that both the strategies open to decision-makers and the extent of their information of each others choices must be specified. In addition the degree of co-operation between agents becomes a major consideration.

The originators of game theory felt that analysis of these more intricate decision-making environments would serve both to clarify the range of issues for which the simple Walrasian assumptions suffice, and also to provide alternative analytic tools where the Walrasian framework would not be appropriate. The subsequent development of the field has seen the introduction of many solution concepts for n-person games, such as the bargaining set and the core for cooperative games, and the Nash equilibrium for non-cooperative games. Much work has gone into clarifying the connections between game-theoretic concepts and the Walrasian equilibrium for market games. More recently, the economics profession as a whole, and game theory in particular, has witnessed a resurgence of interest in the role of agents' information in shaping their decisions. Several Cowles staff members and visitors are recent contributors to these areas of research.

One of the most important equilibrium concepts for cooperative games is the core of the game. This was first introduced by Edgeworth in 1881 in his
book *Mathematical Psychics* (Kegan Paul, London), and was then reintroduced and placed in an explicitly game-theoretic setting by Martin Shubik in 1959 ("Edgeworth Market Games," in A. W. Tucker and R. D. Luce, eds., *Contributions to the Theory of Games IV*, Princeton University Press, Princeton). Edgeworth's original analysis demonstrated that in an economy with two types of agents, both with convex preferences, the core shrinks to the set of competitive equilibria as the economy is replicated. This core convergence theorem was generalized to economies with an arbitrary number of types of agent by Debreu and Scarf in 1963 ("A Limit Theorem on the Core of an Economy," *International Economic Review*, Vol. 4). Anderson's results provide the most general convergence statements currently available when agents have convex preferences ("An Elementary Core Equivalence Theorem," *Econometrica*, Vol. 46, 1978).

Anderson's recent work on core theory extended convergence results for cores of exchange economies with convex preferences to "most" exchange economies with nonconvex preferences. Several different formulations for "most" are given. Perhaps the most appealing is the following: suppose that a sequence of economies is produced by a sequence of independent samples from a fixed distribution of agents' characteristics. Under appropriate assumptions, convergence will hold with probability one. The resulting paper, "Strong Core Theorems with Nonconvex Preferences," has been tentatively accepted, subject to revision, by *Econometrica*.

One of the original motivations for studying the core of market economies was the belief that the core would be non-empty in many economies for which no Walrasian equilibrium exists. In particular it was hoped that the core might provide an analytic apparatus suited to the study of indivisibilities and other forms of nonconvexity.

Martine Quinzii, who visited the Cowles Foundation during 1982, studied the existence of the core in an economy with indivisibilities. More precisely, the model studied was the following: there are n agents in the economy and two goods. The first one is a perfectly divisible good called money, the second is a good which exists in the form of indivisible items (houses, for example). It is assumed that no agent has initially more than one indivisible item, and that he has no use for more than one item. The core of this economy is a distribution of money and houses among the agents such that no coalition of agents can find a redistribution of its own resources which is preferred by all its members.

This model is a generalization of the two models of exchange of indivisible goods previously studied in the literature from the point of view of the core. These are the model of Shapley-Scarf of exchange of houses among n agents with compensation in money and the model of Shapley-Shubik of a market between buyers and sellers of houses. It was proved for these two models that
the core was non-empty. Quinzii generalized this result to the general exchange model by proving that the economy is balanced (a property introduced by Shapley and sufficient to imply the existence of the core). Another interesting property of this model is that, under assumptions ensuring the presence of money in the economy, the core and the competitive equilibria coincide. This means that all core allocations can be obtained by a competitive market for the indivisible items.

In CFDP 563, Kaneko considers an assignment game without side payments and proves the nonemptiness of the core. He then analyzes a market model with indivisible goods but without the transferable utility assumption; the nonemptiness of the core and the existence of a competitive equilibrium of the market model are shown, using the first result.

In CFDP 620, co-authored with Myrna Wooders, Kaneko considers a generalization of assignment games, called partitioning games. Given a finite set N of players, there is an a priori given subset π of coalitions of N which are available for blocking. Necessary and sufficient conditions for the nonemptiness of the cores of all games with essential coalitions π are developed.

Convergence results for finite economies generally have even stronger analogues for games in which there is a non-atomic continuum of players. For instance, Aumann demonstrated that in the setting of a continuum the core and the competitive equilibria actually coincide (“Markets with a Continuum of Traders,” Econometrica, Vol. 32, 1964). More recently, Geanakoplos showed that in a continuum of traders economy with transferable utility any allocation in the bargaining set must give to each agent his marginal contribution, and hence be in the core.

Another solution concept for cooperative games is the Shapley value, which is a measure of the contribution made by each participant to a cooperative game. The Shapley value is always efficient, in the sense that it specifies allocations such that no alternative allocation would improve everyone’s welfare. Recently, attention has been focused on generalizations and analogues of the Shapley value that do not enjoy the efficiency property.

In their recent paper, Pradeep Dubey together with Abraham Neyman and Robert Weber consider this topic from an axiomatic viewpoint (“Value theory without Efficiency,” Mathematics of Operations Research, Vol. 6, 1981). They characterize the class of operators that is obtained by omitting the efficiency axiom from the axioms defining the Shapley value.

In CFDP 610, Dubey and Neyman provide an axiomatic analysis of the equivalence of various equilibrium concepts for non-atomic games with transferable utility. When thus restricted to games with transferable utility and smooth preferences, the equivalence phenomenon is even more striking in that many solutions coincide at a unique outcome. Dubey and Neyman derive a “meta-equivalence” theorem: any solution coincides with this payoff if and only if it satisfies their axioms.
The best-known solution concept for games without cooperation among players is the Nash equilibrium. This is a set of choices such that each player's chosen strategy is optimal in the face of others' choices. Such equilibria are frequently inefficient, as illustrated by well-known games such as the prisoners' dilemma, in which both prisoners could be made better off if they acted in cooperation.

The issue of whether Nash equilibria of market games are efficient was studied by Dubey and J. Rogawski in CFDP's 622 and 631. A general theorem is presented and applications are made to non-cooperative market games.

Dubey's paper, “Price-Quantity Strategic Market Games” published in *Econometrica*, Vol. 50, No. 1, January 1982, is in the rapidly growing series of articles on strategic approaches to economic equilibrium. He begins with a standard Walrasian exchange economy with a finite number of traders and commodities. This is recast as a game in strategic form in essentially two different ways. There is a trading-post for each commodity to which traders send contingent statements about how much they wish to buy and sell, and at what prices. In Model 1, the trading point is determined by the intersection of the aggregate supply and demand curves. In Model 2, trade takes place so as to meet as many contingent statements as possible. Each buyer whose orders are filled pays the price he quoted, using a fiat money which can be borrowed costlessly and limitlessly. But after trade is over there is a settlement of accounts, and a penalty is levied on those who are bankrupt. The Nash equilibria of each of these games are then examined.

In CFDP 601, Kaneko introduces a new solution concept which he calls the conventionally stable set for an n-person noncooperative game. This new solution concept is based on the von Neumann-Morgenstern stable set, particularly on their interpretation of it as "standards of behavior." This first paper provides the definition of conventionally stable sets and applies the new solution to zero-sum 2 person games, the prisoner’s dilemma, the battle of the sexes, and games with a continuum of players.

In CFDP 614, Kaneko applies his theory to monopolistic and oligopolistic markets. A market model with a finite number of producers and a continuum of buyers is presented and is then formulated as a strategic game in which the producers' strategies are prices and the buyers' strategies are demands for commodities. It is shown that a conventionally stable set in this game corresponds to a conventionally stable one in a game where the producers are the only players but the buyers are replaced by demand functions. Furthermore, it is shown that the theory of the conventionally stable set is compatible with the classical monopoly solution, the kinked-demand-curve solution and the leader-follower solution.

When repeated plays of a game are being considered, it becomes important to specify each player's information about others' moves at each stage of the game. In two recent papers (CFDPs 625 and 629) Dubey and Kaneko explore
the relation between information patterns and Nash equilibria in such extensive games.

Within the economic literature, much attention has been focused on the revelation of information through the market prices of commodities in rational expectations equilibria. In recent work with Hugo Sonnenschein of Princeton University, Anderson proposed a new idealization of rational expectations equilibrium, and proved several general existence theorems. Their central point is that agents must form models of the economy based on a finite number of observations, and the resulting approximations made by agents rule out the examples of non-existence of equilibria previously exhibited.

An alternative method of modelling the connections between prices and information is to imbed the issue in an explicitly game-theoretic setting. In CFDP 634, Geanakoplos together with Dubey and Shubik formulated a critique of the theory of rational expectations. That theory is designed to show how the diverse pieces of information held in various hands can be utilized by the economy as if it were all known by one agent (or all agents). What it in fact shows is how prices can reveal information. It does not begin to explain how information is put into the prices to be revealed. The theory also suffers from a grave paradox: if information is costly to acquire, no agent will bother getting any, since he can costlessly wait for the prices to reveal them anyway. In this paper, it was demonstrated that if one realistically models the economic process, paying close attention to how information is disseminated in a well-defined game, then indeed prices eventually reveal information, but not until agents with superior information have been able to profit from that information.

In an earlier paper on the revelation of information in strategic market games, Geanakoplos and Polemarchakis showed that given two agents 1 and 2 with the same priors but different information and given any event A, the simple communication of the agents’ opinions back and forth must lead them to the same opinion (CFDP 639). This opinion, however, need not be the same common opinion they would hold if they communicated all their information rather than just their opinions. Moreover, Geanakoplos showed that this communication process could contain an arbitrarily long repetition of the same message: to the outside observer it could appear that 1 and 2 were simply repeating themselves, although in fact such repetition is still informative. Later Geanakoplos extended this framework to allow the agents to accept or refuse a generalized bet: eventually one refused, even though for an arbitrarily long time period they might both be willing to take opposite sides of the bet. This process is reminiscent of labor strikes, where both parties repeat their demands, until one suddenly compromises.

Carolyn Pitchik, who visited the Cowles Foundation during 1980-81 studied the theory and economic applications of games of timing. In CFDP 579,
Pitchik extends the classical existence and uniqueness results for games of timing, relaxing the zero-sum and differentiability conditions on the payoff kernels. The classical games of timing were such that equilibria always existed; in contrast, necessary and sufficient conditions for existence of equilibria in a wider class of games are exhibited along with a characterization of a subset of any existing set of equilibria.

The applicability of games of timing to economics is exemplified in the two papers by Pitchik (joint with Martin J. Osborne of Columbia University): 

"Equilibria for a Three-Person Location Problem" (CFDP 628) and "Are Large Firms More Powerful than Small Ones?" (Columbia discussion paper no. 124, revised version to appear as a CFDP).

In the former paper, they find all equilibria, within a wide class, of the following simplified three-firm location model of Hotelling (in which the price variable is ignored), thus filling a gap in the location literature. Three firms produce the same good with constant unit costs. Each firm selects a point on some line segment at which to locate. The potential consumers of the good are uniformly distributed on the line segment. Each consumer buys one unit of the good from the least costly source (where the constant unit cost of travelling is included). In the latter paper, they study a model of duopoly in which the firms, which have different capacities, share the profits from collusion according to the relative potency of their threats of price-cutting. Up to its capacity, each firm can produce with constant unit costs. The solution concept used (the Nash variable-threat bargaining solution) predicts that the large firm always earns lower profits per unit of capacity than the small one. Thus, the model provides an explanation for the higher profit rate earned by smaller firms in industries where there is implicit or explicit collusion.

During the last three years, Shubik's research has been on five closely intertwined topics with major intellectual concerns being the theory of money and financial institutions and the development of game theory.

The five topics have been 1) the theory of money and financial institutions, 2) the development of game theory and its applications in general, 3) the applications of economic analysis and game theory to defense problems, 4) the applications of economic analysis and game theory to problems in bidding, auctions and in oligopolistic competition, and 5) the methods and uses of experimental, teaching and operational gaming.

The forging of a basic link between micro and macro economics involves recasting the Arrow-Debreu general equilibrium model as a game in strategic form. Since 1970, when Shubik first succeeded in constructing a closed exchange economy as an intrinsically symmetric game in strategic form, he, together with several colleagues, has elaborated models of strategic market games demonstrating the need for the invention of various financial institutions and instruments. In particular, models where commodity money, fiat
money, other types of credit, bankruptcy laws, stocks, bonds, notes, and future markets appear have been studied. The models themselves are neither equilibrium nor disequilibrium models. They are merely well defined and playable games. This means that the rules of the game have to be sufficiently well described that some method for price formation, for the functioning of markets, for the trade in bonds, etc., will be operationally specified.

Shubik has been pursuing this approach to money and financial institutions for the last 12 years and expects to continue this work in collaboration with Dubey and other scholars. Some 25 years ago, Shapley and Shubik commenced work on what originally was to be a collection of volumes on the theory of games and its applications to political economy. Shapley has disassociated himself from these books although research collaboration continues.


Volume III in the series is planned to cover money and financial institutions. Shubik expects to complete this work partially in collaboration with Dubey and others in the course of the next few years.

Shubik has always maintained an interest in defense problems and in the uses and limitations of mathematics in the study of defense. In particular, he has collaborated with Robert Weber on the extension of game theoretic analysis to network defense, and they have also succeeded in linking these results up with some theoretical problems in game theory concerning the relationship between the value of a cooperative game and the noncooperative equilibria of a related game in strategic form. Shubik has also been working with Paul Bracken on problems of nuclear warfare and command and control.

Shubik has continued his interest in the investigation of oligopoly theory and together with Matthew Sobel, is taking some steps toward the construction of multi-stage models of the firm with an explicit capital and financial asset structure.

Allied to much of the work noted above, is an ongoing interest in the techniques of gaming. Some years ago Garry Brewer and Shubik did a major study on the uses of war gaming. Shubik has recently been concerned with the value of war gaming as a strategic planning device and possibly even more importantly as an educational device to draw attention to the extreme dangers of nuclear war.

**D. Microeconomics**

Nordhaus' recent work has centered on the economics of energy and natural resources. In Cowles Foundation Monograph No. 26, *The Efficient Use of Energy Resources*, Nordhaus addressed the issue of how fast low-cost energy resources such as oil and gas should be exploited.
Economic theory suggests that efficient use of energy resources entails using cheap before expensive resources. In addition, each resource will, in a competitive market, have a "royalty" attached to it. The royalty will be zero for resources that are not scarce, positive for those that are. For all resources, the royalty will be rising at the market interest rate.

By working backward from exhaustion, we can determine what an efficient price for oil or other resources would be. The basic result can easily be seen where there are no extraction costs—roughly accurate for Mideast oil today. In this case, at the point when substitution of the next resource (higher-cost oil, coal, etc.) occurs, the price of Mideast oil and its substitute must be equal. For concreteness, call that year 2020. In 2020, then, the royalty on Mideast oil must equal the cost of the substitute. Since, in an efficient market, the royalty must rise at the interest rate, the royalty today must be the discounted value of the royalty in 2020. If the discount rate is 6 percent, and the substitute costs $40.00, then in 1975 the royalty on Mideast oil, and its efficiency price, must equal 

$$\frac{40.00}{1.06^{45}} = \$2.90.$$  

The monograph describes the construction of a model designed to determine the efficient path for using energy resources under far more realistic assumptions than in the simple example above. The model has two components. The first component is the "demand" side of the energy market. It reports the results of a detailed econometric model of energy demand and then shows how these results can be used in the energy model. The second component is the technology: estimates of the extent of energy resources, as well as the costs of extraction and conversion. Alternative models of cost of extraction are briefly described.

One major spillover from the model construction are the estimates of energy demand functions. These rely on a combination of techniques for estimating the price-responsiveness of energy demanded in the United States and Europe. The basic result is that energy demand is shown to be moderately elastic with respect to price, with elasticities in the range of −0.5 to −1.0 depending on the sector, country, and specification.

The most important investigation relates to the estimate of the efficiency price of oil, given in Chapter 5. Relying on the model, and the (clearly unrealistic) assumption that the energy market is competitive, Nordhaus estimates that the efficient price of oil (for the mid-1980s in 1982 prices) is $3.70 per barrel. This compares with a price of approximately $32 per barrel in 1981 (again in 1982 prices). The reason the calculated efficiency price is so surprisingly low is that the cost of the next substitute resource is relatively modest, and the time at which substitution occurs is distant. Extensive sensitivity analysis in Chapter 5 gives a range of $3.35 to $6.15 per barrel—still well below the present market price.

To see whether the Organization of Petroleum Exporting Countries (OPEC) is responsible for the enormous discrepancy between actual and calculated
efficiency price, Chapter 1 investigates the theory of monopoly in resource markets. Under limited but plausible assumptions it is shown that the monopoly price will be set at approximately the substitute price. In the example above, then, if a monopolist had control of the oil market, he would set the price at slightly below the substitute (say $39.00), rather than at the competitive price of $2.90.

The temptation to attribute the rise of the world oil price from 1972 to 1982 to the effective monopolization of the world oil market by OPEC is reinforced by the result in Chapter 5 that the market price in the late 1960s and early 1970s was virtually equal to the calculated efficiency price. Chapter 6 looks more carefully at the empirical support for this hypothesis, both in the current study and in other studies. Most studies make a motivational hypothesis that OPEC is interested in maximizing its discounted profits (the “wealth maximizing monopolist”). The basic result of this and other economic studies indicates that the wealth-maximizing price for OPEC oil today lies at the top end of the $17 to $33 per barrel range (in 1982 prices). These studies confirm that the price rise after 1972 can be traced basically to the virtual monopolization of the international oil market.

In other work with applications to the pattern of energy exploitation, Geanakoplos considered the optimal behavior of an oligopolistic corporation with significant positions in several markets. Geanakoplos showed that an oligopolistic oil producer with access to two sources of oil, cheap and expensive, should under some circumstances not extract all the cheap oil before beginning to produce the expensive oil. On the contrary, the production of cheap oil has a potentially large strategic cost stemming from the increased aggression of the rival firms in subsequent time periods once the first producers “trump cards” have been played. Similarly a subsidy of a firm’s output in a given market will necessarily hurt the firm in another market, if the firm has increasing marginal costs. Thus a firm that suddenly finds itself able to produce alone in another market will always lose total profits if that new opportunity is not extremely favorable.

In CFDP 640, Geanakoplos and Takatoshi Ito applied general equilibrium methods to examine the consequences of optimal contracting between a risk averse worker and a risk neutral employer whose labor requirements are random. If severance pay is allowed and employers know workers outside prospects, then the optimal contract will set severance pay so that workers will be indifferent to whether they are laid off or not, and there will thus be no involuntary unemployment. However, if the firm is unable to observe the actual outside pay of laid off workers, then this indifference need not hold. In particular, it is shown that if workers have decreasing absolute risk aversion, the optimal contract will make the average worker prefer the uncertainty associated with a lay-off to the certain wage with the firm.
In two recent studies of the evolution of industrial structure, Iwai has developed models in which firms constantly strive for survival and growth through their innovative and imitative activities. The principal objective of this research is to illustrate the Schumpeterian process of "creative destruction."

In CFDP 602, Iwai proposed a simple stochastic model of industrial structure, which explains how the dynamic processes of firms' technological innovations and imitations interact with each other and shape the evolutionary pattern of the industry's state of technology both in the middle- and long-run. In the middle-run, it was shown that the process of technological imitations works essentially as an equilibrating force that continually moves the industry towards a static equilibrium of perfect technological knowledge, whereas the function of technological innovation lies precisely in upsetting this tendency by forcing the state of technology to be more diverse. It was also demonstrated that in the long-run a certain statistical regularity emerges out of this seemingly random pattern of the dynamic interactions between imitations and innovations, in the sense that the cross-sectional distribution of technological efficiencies among firms asymptotically approaches a non-degenerate long-run average distribution. While new technological knowledge constantly flows into the industry, actual production methods of a majority of firms always lag behind it, and a multitude of diverse production methods with a wide range of efficiencies will forever coexist in the industry. Indeed, it is the macroscopic equilibrium of microscopic (technological) disequilibria that characterizes the "long-run" of the industry.

In CFDP 603, Iwai extended the model of the first paper to take account of the differential impact of diverse technological developments among firms on their growth capabilities and the consequent repercussions on the evolutionary pattern of the state of technology. It was first demonstrated that if neither innovation nor imitation were possible, only the most efficient firms would survive in the long-run competitive struggle for limited resources for capacity expansion. This is exactly what the doctrine of "economic selection" has been telling us. Once, however, the possibility of technological imitations was allowed in the model, this doctrine was shown to lose much of its relevance. In this case the most efficient firms will again monopolize the whole productive capacity of the industry, but the technological imitation of the efficient firms by the less efficient ones will eventually allow most of the existing firms to join the rank of the most efficient. This is more akin to the Larmarkian mechanism than the rigid natural selection process. Finally, if the possibility of occasional technological innovations was also allowed in the model, it was possible to prove that the processes of capacity growth, imitation and innovation will interact with each other and work to maintain the relative shape of the industry's state of technology in a statistically balanced form in the long-run. The blind force of economic selection working through
the differential growth rates among firms with diverse efficiencies is constantly outwitted by the firms' imitation activities and intermittently disrupted by their innovative activities.

In any theory of corporate growth, research and development expenditures play a central role. An examination of the empirical evidence led John Beggs to propose tentatively that an important component of research and development, and subsequent patenting, occurs in response to adverse external circumstances in a firm's product market. This so-called "defensive R&D hypothesis" is supported by two of Beggs' studies, CFDP 588 and NBER Working Paper 952. The first is an analysis of industry data at the two-digit SIC level considering the joint time series interaction between R&D and industry profits measured as rate of return on stockholder equity. The data are for 14 industries for the period 1959 to 1979. The empirical results indicated that a "shock" to industry profits has an inverse effect on R&D effort. Increases in the rate of growth of profits slow the rate of growth of R&D and, conversely. The second study involved data collected for twenty more narrowly defined industries during the period 1850 through to 1954. Patent data were used as the indicator of technological change. The explanatory variable of concern was industry value added. For each industry the rate of change of patenting and the rate of change of value added were measured relative to the national aggregate rates of change of patenting and gross domestic product respectively. This was done to remove trade cycle effects. Again the results showed a negative correlation between the two variables, supporting the defensive R&D hypothesis.

In CFDP 580, Christophe Chamley investigated the role of capital markets and institutions such as limited liability in determining the allocation of funds among would-be entrepreneurs. He analyzed these issues in a simple theoretical model of occupational choice (with entrepreneurs, employees, and financial institutions). Entrepreneurs may use their liability form as a signal of their ability. Equilibria (which may be multiple) are inefficient. Moreover, the social value of the institution of limited liability is ambiguous. In particular a tax on limited liability firms (corporations are an example) may be desirable when the possibilities of substitution between occupations are relatively large, and the diseconomies of scale in each firm are relatively small.

Chamley has also investigated the effects of taxation on the intertemporal allocation of resources, using a series of aggregate stylized models incorporaring rational expectations of the future price changes induced by tax reform. In CFP 535, Chamley analyzed the efficiency cost of capital income taxation in this framework. This cost is induced by the price distortion in the intertemporal allocation of resources. It increases with the elasticity of substitution between capital and labor in the production function and with the elasticity of substitution between consumption levels at different dates. The first
effect seems significantly more important than the second. The welfare gain obtained by the abolition of the capital income tax is smaller when expectations are not rational (for example, it can be cut in half when expectations are myopic). Also the allocation efficiency cost of the corporate tax is larger than the intertemporal welfare cost.

Recently, Chamley extended this framework to analyze the optimal combination of taxes on incomes of capital and labor (or on consumption). One of the results is that the differential efficiency cost of the capital income tax depends only on the production technology when the growth rate and the discount rate are identical.

In CFDP 554, the tax rates are optimized over time together with the government deficit. The optimal formulae for the second best taxation of different goods consumed in the same period are similar but not identical to those obtained in standard atemporal models. The difference arises because of the incidence of tax reform on accumulated assets which are in fixed supply in the short-run. This effect is the reason for the time inconsistency of optimal policies.

Alvin Klevorick's research on economic theory and antitrust policy has focused on the pricing behavior of dominant firms and has involved collaborative work with Paul Joskow of the Massachusetts Institute of Technology and Richard Levin of Yale. In their article, "A Framework for Analyzing Predatory Pricing Policy," 89 Yale Law Journal, 213 (1979), Joskow and Klevorick develop a decision-theoretic analytical framework that explains and clarifies how various factors and judgments enter into the formulation of a policy toward "predatory pricing"—a dominant firm's use of price to restrict competition by driving out existing rivals or excluding potential ones. Consideration of the links between certain firm and market characteristics, on the one hand, and the probabilities of error, error costs, and rule-implementation costs, on the other, lead Joskow and Klevorick to propose that a two-tiered "structuralist" rule-of-reason approach be applied in cases of alleged predatory pricing. They argue that such an approach most appropriately accounts for the uncertainty and the costs of making incorrect decisions that inhere in the formulation of a policy toward pricing behavior. In the first stage, both the structural characteristics of the market in question and the market power of the alleged predator firm would be examined to determine if they generate a reasonable expectation that predatory pricing could occur and would impose significant economic losses on society. A claim that predatory pricing had taken place could be pursued only if a reasonable case could be made that there was a serious monopoly problem in the industry. Only in such instances would one go on to the second stage: a rule-of-reason inquiry into the behavior of the dominant firm. The substantive content of the second-tier examination of pricing behavior draws, in an eclectic way, on the insights suggested
by predatory pricing "rules" that had been previously proposed by other scholars—including Areeda and Turner, Williamson, and Baumol—but it finds no one rule adequate to the task.

These relatively simple rules that have been proposed to set the boundaries for legal pricing behavior of dominant firms are the principal focus of the theoretical work by Kleverick and Levin in "A Welfare Analysis of Pricing Constraints on Dominant Firms." The paper analyzes the welfare consequences of each of these rules using a particular parameterization of the now-standard Gaskin's model of a dominant firm that faces a competitive fringe of existing firms and potential entrants. The aim is not to find the "best" rule for, as Joskow and Kleverick argued, no one rule will produce optimal results for all market situations. Instead, the objective is to characterize the different sets of market conditions (demand elasticity, speed of entry and exit, cost advantage of the dominant firm, social discount rate, etc.) under which each of the different rules is likely to lead to welfare improvements over the unconstrained situation.

Kleverick and Levin find that if an industry's initial conditions are such as to induce an unconstrained dominant firm to permit entry (in the sense of Gaskin), none of the rules offered by Areeda and Turner, Williamson, and Baumol will affect behavior. Furthermore, under these conditions, a laissez-faire approach dominates a rule that proscribes the sacrifice of short-term profits. A specific implication of this result is that the discounted total surplus generated by Gaskin's style dynamic limit pricing is larger than that generated by short-run profit-maximizing behavior ("umbrella pricing"). It is shown that when the initial conditions in an industry are such as to induce an intertemporally profit-maximizing dominant firm to drive competitors from the market, the Baumol rule will always impose a binding constraint, and the Areeda-Turner rule will be binding under some conditions. It is interesting, though, that neither the Baumol nor Areeda-Turner rule unambiguously produces increased welfare. Moreover, neither of the two rules dominates the other. The paper characterizes the specific conditions that favor application of the Areeda-Turner rule, the Baumol rule, or a policy of non-intervention.

Kleverick has also continued his research on mathematical models of jury decision making. His most recent work has focused on one of the virtues claimed for a jury decision—namely, that such a decision is based on more complete and better processing of the information available than the verdict of any one juror deciding alone would be. During the deliberation process, the argument goes, jurors exchange points of view and assemble the evidence into a coherent picture that is more likely to be correct than is the view of any one juror. Given the central and valuable role attributed to the information processing that jury deliberation is supposed to achieve, it is striking that previous models of the jury decision process—including both abstract mathematical
formulations and simulation models—are inattentive to this aspect of the jury’s work. Although these models depict how a jury might move to a verdict from the initial views of its members, they do not provide any description or specification of how the jurors’ views are combined or how their various observations and insights are assimilated.

In a paper entitled “Information Processing and Jury Decisionmaking,” Kleverick, together with Michael Rothschild of the University of Wisconsin-Madison and Christopher Winship of Northwestern University, use a formal model to explore the information processing function that jury deliberation performs. In particular, they investigate when a jury that deliberates to a unanimous verdict can reach better decisions—ones with lower probabilities of error—than a jury that bases its decision on the view of the majority of its members immediately after the trial is concluded. What is the gap between the quality of decisions reached using a first-ballot, majority-rule procedure and the quality of those that would be generated by a jury making optimal use of the information provided at the trial? The authors’ strategy is to consider a simple model of juror observations, though one that is richer than characterizations in the literature, for which they can define precisely what an optimal jury decision rule would be. It is assumed that jurors’ observations are correlated normal random variables or, equivalently, it is supposed that each juror receives the same information from the trial but different jurors make independent errors of observation. With this specification, the jury faces a problem in discriminant analysis, and Kleverick, Rothschild, and Winship use that statistical theory to discuss the optimal jury decision rule. The paper shows that deliberation has the potential for generating substantial improvement in the quality of decisions, and demonstrates how that potential arises, especially the central role that heterogeneity among jurors—in terms of what they see and hear, what they believe about the costs of erroneous decisions, and what differences there are in their information processing capacities—plays in determining how much improvement is possible. Of particular interest is the fact that if jurors differ in what they see and hear at the trial but share the same view of the relative cost of erroneous convictions and erroneous acquittals and have the same individual abilities to process information, then for large juries both the first-ballot majority rule procedure and optimal deliberation yield results on the efficiency frontier but the former fails to produce the socially optimal mix of false acquittals and false convictions.

Kleverick has been engaged in research on the economics of mental health, and he has focused, in particular, on the effects of regulation on the delivery of mental health services. He and Thomas McGuire of Boston University are formulating and analyzing theoretical models of the demand for and supply of mental health services that take explicit account of the extent of insurance coverage for such services, the multiplicity of types of providers, and the im-
pact of direct government regulation on the delivery system. This theoretical work will form the basis for an empirical analysis using a short term series of cross-section observations.

In his paper, "Regulation and Cost Containment in the Delivery of Mental Health Services," Klevorick considers the various forms in which regulations appear in the mental health services area. After discussing a cost-benefit specification of the regulatory goal, he focuses on the importance and complexity of substitution relationships in the mental health area and the impact of these relationships on regulation.

Gerald Kramer's research on the theory of electoral systems continued in the period of this report. His recent paper on existence of electoral equilibrium extended the theory of electoral competition by establishing the existence of candidate equilibria under quite general assumptions, in particular concerning the dimensionality of the underlying policy or issue space over which the candidates compete. In the classical Hotelling-Downs case this space is one-dimensional, and under the usual assumptions on voter and candidate preferences, there exists a pure strategy equilibrium in which both candidates adopt policies at the median of the voter policy-preference distribution. It is well known, however, that when the policy space is of greater dimensionality, pure strategy equilibria generally will not exist. Thus analysis of electoral competition in the general and substantively important multidimensional case must be based either on explicit hypotheses about disequilibrium behavior, or else on a more general equilibrium concept, such as that involving the use of mixed strategies. In the article, "Existence of Electoral Equilibrium," Kramer focuses on the second of these possibilities, characterizing candidate behavior in terms of an equilibrium in the domain of mixed strategies.

In a paper on electoral stability and the dynamics of electoral competition, Kramer is concerned with stability properties of a class of competitive electoral processes. The basic structure is one in which two political parties compete for votes over a multidimensional space \( \mathbb{R}^k \) of issues or policies, by adopting specific electoral platforms, or positions in the issue space; each is interested in maximizing its electoral prospects by finding a platform which is preferred by as many voters as possible. If this electoral contest is modeled as a symmetric two-player game, a pure strategy equilibrium of the classical Hotelling-Downs variety almost never exists (if \( k > 1 \)). Moreover, this formulation ignores an obvious asymmetry in the roles of the incumbent party and its opponent, since an incumbent's choice of platform is heavily constrained by the need to defend its record in office.

This asymmetry suggests a natural dynamic reformulation of the electoral process: the two parties are assumed to compete repeatedly, over an infinite sequence of elections. In each election the party whose platform is preferred by a majority is elected; it is assumed to then enact the policy package it advo-
cated, and to defend this same policy in the next election. The "out" party may adopt any policy it wishes, to maximize its prospects. In general, the incumbent's policy will always be defeated, and the two parties will alternate in office. As the process is repeated over time, a sequence of successively-enacted "winning" policies is thus generated. The analysis focuses on the long-run behavior of these sequences, or trajectories, of policies.

Voter political preferences are assumed to be representable by satiated, additive utility functions with parameters $a_1, \ldots, a_n$ of the form $u(x) = f_0(x) + a_1f_1(x) + \ldots + a_nf_n(x)$ (satisfying standard concavity and smoothness conditions). Alternatively, if we think of $x$ as a vector of arbitrary characteristics or attributes of parties or candidates, voter political attitudes toward these objects are assumed to have a common underlying structure, which reflects a shared set $f_0, \ldots, f_l$ of underlying criteria or factors (specific factors may be weighted very differently, or not at all, by different sectors of the electorate). With this structure, a society or electorate is then characterized as a probability distribution $l$ on $\mathbb{R}^l$, the parameter space.

These assumptions imply the existence of a set $S \subset \mathbb{R}^k$ of policies, typically a small, proper subset of the Pareto Optimal set, which is a dynamic equilibrium for the system in any of several senses. $S$ contains all the equilibrium points, or possible steady states, of the system, and is also a region of recurrence for any trajectory. $S$ is also asymptotically stable, in the following sense: there exists a neighborhood $N_\delta(S)$ of $S$ such that any trajectory must eventually enter and thereafter remain inside $N_\delta(S)$; moreover, $\delta$ tends to zero as the electorate increases in size.

Thus, in large electorates, any vote-maximizing trajectory will eventually be found in or near the set $S$.

The size of $S$ in some sense reflects the degree or extent of possible instabilities in the system. In general this will depend on the distribution of voter preferences; if $l < k$, however, sharper and essentially distribution-free results can be obtained. In this case $S$ coincides with a particular set which has been studied in the voting social choice literature, the minmax set. The minmax set is typically quite small in large electorates. Moreover, as the size of the electorate increases in the manner described above the minmax set collapses to a point. Thus in large electorates satisfying these conditions, competitive vote maximization will eventually draw both parties into the immediate vicinity of the unique minmax point, and two-party electoral competition will be very stable.

The minmax set is also a natural extension of the Condorcet criterion for social choice under majoritarian democratic rule. Hence from a normative point of view, political competition under these conditions leads not simply to stability alone, but also to the attainment of a democratic social optimum.

The work on election theory outlined above shares with most recent work
the assumption that political parties compete over a multidimensional space of issues or policy variables. In his paper, "Electoral Politics in the Zero-Sum Society," Kramer develops a theory of competition under an alternative structure, in which candidates compete by directly offering particular benefits and services to voters. The analysis presumes a symmetry in the roles of incumbent and challenger, in that the former necessarily commits himself to an allocation first, by his actions in office, thereby presenting the challenger with a fixed target to optimize against. Voters tend to discount the challenger's promises to some degree in comparing them to the benefits currently being received under the incumbent, and cast their votes so as to maximize the level of benefits received. Under these circumstances, Kramer establishes that challengers optimally pursue a "divide and conquer" strategy of bidding for a minimum winning coalition of voters. Incumbents, by contrast, pursue a more even-handed strategy, attempting to appeal to all their constituents. The model thus predicts distinctive differences in the behavior of challengers and incumbents, with no tendency for the candidates to converge on a common strategy or position, as in the classical Downsian case.

In the paper, an issue is defined as a measure which, if taken, would generate a fixed distribution of benefits and costs, and on which each candidate must take a position. Kramer obtains a simple classification of issues according to their electoral consequences, and shows that one important category of issues—which he labels the "controversial" issues—is strategically important. The existence of a controversial issue invariably works to the disadvantage of the incumbent; hence he always has an incentive to suppress or remove it from the electoral arena altogether, if he can. If he cannot, it will then be optimal for the incumbent to favor the issue if and only if it is one which produces a (positive) net social benefit. Even with this optimal position, however, under general conditions the incumbent will nevertheless be defeated, by a challenger who opposed the issue and who will therefore not enact it, even though it would be socially optimal to do so. These results thus support the doubts expressed by Thoow and others, concerning the inability of a competitive democratic system to deal effectively with major issues when distributional considerations become politically important. They also imply, however, that Thoow's proposed reforms, to strengthen party responsibility, would not help, since the problem lies in the nature of the competitive process itself.

In his paper, "The Ecological Fallacy Revisited: Aggregate-versus Individual-Level Findings on Economics and Elections," Kramer seeks to explain a puzzling feature of voting behavior. Several aggregate-level studies have found a relationship between macroeconomic conditions and election outcomes, operating in intuitively plausible directions. More recent survey-based studies, however, have been unable to detect any comparable relation-
ship operating at the individual-voter level. One recently proposed explanation for this persistent discrepancy is that voters actually behave in an altruistic or "sociotropic" fashion, responding to economic events only as they affect the general welfare, rather than in terms of self-interested "pocketbook" considerations.

Kramer argues that the discrepancies between the macro- and micro-level studies are a statistical artifact, arising from the fact that observable changes in individual welfare actually consist of two unobservable components, a government-induced (and politically relevant) component, and an exogenous component caused by life-cycle and other politically irrelevant factors. He shows that because of this, individual-level cross-sectional estimates of the effects of welfare changes on voting are badly biased and are essentially unrelated to the true values of the behavioral parameters of interest: they will generally be considerable underestimates, and may even be of the wrong sign. An aggregate-level time-series analysis, on the other hand, will often yield reasonably good (if somewhat attenuated) estimates of the underlying individual-level effects of interest. Thus, in this case, individual behavior is best investigated with aggregate- rather than individual-level data.

It is also shown that the evidence for sociotropic voting is artifactual, in the sense that the various findings and evidence which ostensibly show sociotropic behavior are all perfectly compatible with the null hypothesis of self-interested "pocketbook" voting.

In other works combining economic and political considerations, Donald Richter studied existence of equilibrium in the context of a Walrasian economy which includes a finite number of local governmental jurisdictions and a continuum of perfectly mobile consumers whose jurisdictions of residence are endogenously determined. In equilibrium, each jurisdiction finances its own provisions of public goods using a proportional endowment income tax, no other affordable tax-expenditure package for the jurisdiction exists which all its residents prefer, and no consumer wants to move.

John Roemer, who was a visitor at the Cowles Foundation for the 1979-80 academic year continued his construction of a general theory of exploitation. This general theory is intended to clarify the differences between Marxian and non-Marxian thinkers in the kinds of relationships they view as exploitative. In constructing this theory, it was necessary to remove the obsolete labor theory of value from its traditionally central role in models of exploitation.

The general theory of exploitation which Roemer has developed uses concepts of property relations, not labor value, to characterize exploitation. Some simple cooperative game theory was used to specify exploitation. Exploitation is defined at a given allocation with respect to some conception of an alternative, specified by the characteristic function of some game. How one chooses the characteristic function determines one's ethical preferences.
Roemer then showed how characteristic functions could be constructed which characterized notions of feudal, capitalist, socialist, status and neoclassical exploitation. Capitalist and Marxian exploitation are equivalent in simple models where both can be defined. In each case, the characteristic function was generated by imagining a change in underlying property relations. As an immediate consequence of the definition, and if certain simple assumptions hold (super-additivity of the characteristic function and Pareto-optimality of the initial allocation), the non-exploitative allocations with respect to a given theory of exploitation consist precisely of the core of the game in question. As a corollary to this approach, Roger Howe and Roemer developed as well an application of the method to Rawlsian justice, constructing the game whose core consists precisely of the Rawlsian-just allocations.

In addition to the theory of exploitation, an endogenous theory of classes was developed. Both class and exploitation status of agents in a general equilibrium model emerge endogenously, as a consequence of agents optimizing against constraints which specify their wealth. People put themselves into certain classes as an optimal procedure. They also end up being exploited or exploiting (or neither), and the important theorem relates these two characteristics of agents, and is called the Class Exploitation Correspondence Theorem.

The theorem states that an agent who optimizes by selling labor power is necessarily capitalistically exploited, and an agent who optimizes by hiring labor power is necessarily an exploiter. The CECP provides microfoundations for ideas of class which have heretofore been only macro concepts. One need not define an agent as being of a certain class, but that emerges from economic behavior derived from the initial asset positions of agents.

Roemer’s research at Cowles was presented in CFDP’s 543 to 545, while a more general summary of his research is contained in his recently published book, A General Theory of Exploitation and Class, published in 1982 by Harvard University Press.

E. Econometrics

A major and growing concern of econometric research in recent years has been the effective pooling of cross section and time series economic data. This topic has been the subject of theoretical and applied research by John J. Beggs. In CFDP 633 Beggs extends modern methods of time series analysis to the efficient aggregation of contemporaneous time series processes which have the same autoregressive-moving average (ARMA) form. Important applications include the analysis of stock prices of corporations trading in the same or similar markets, overlapping commodities futures contracts, time series on major economics aggregates, such as unemployment, drawn from
each of the 50 States individually, etc. Using an error components structure, an efficient method of pooling such contemporaneous time series can be developed which makes use of a spectral decomposition of each series. As the time series sample becomes large the periodogram ordinates are asymptotically uncorrelated across frequencies, so that the only remaining correlation is among the cross-sectional replications at each frequency. Optimally weighted, the periodogram ordinates at each frequency are chi-squared distributed with degrees of freedom equal to twice the number of available cross-sectional replications. This effectively removes the need for smoothing the periodogram through averaging adjacent periodogram ordinates, and hence removes the bias which this practice introduces in the formulation of the underlying ARMA model. Simulation, Monte Carlo, and actual empirical application all indicate that the proposed procedure performs exceedingly well, even in situations with time series samples as short as twenty-five periods. CFDP 646 successfully applies these techniques to the study of commodities futures prices.

The theory of nonlinear regression has been another topic of substantial interest in recent years in econometrics. In CFDP 573 and CFDP 549 P. C. B. Phillips takes issue with recent theoretical work on the nonlinear simultaneous equations model. Much of the latter work has emphasized the importance of the normality assumption and, more generally, correct distributional assumptions about the equation errors in establishing the consistency of the nonlinear full information maximum likelihood (FIML) estimator. In this respect, the general non-linear model appears to be very different from the linear simultaneous equations model, where it is known that the consistency of FIML based on the hypothesis of normally distributed errors is maintained for a wide class of alternative error distributions. One aim of CFDP 573 is to provide examples which show that normality is not necessary for the consistency of nonlinear FIML even when there are major non-linearities in the structural functions; and the analysis suggests a general procedure for constructing non-normal error distributions for which the consistency of non-linear FIML is maintained. The analysis also demonstrates the intimate relationship that exists between the form of the non-linear functions admitted into the structural specification of the model and the tail behavior of the error distribution which is permissible if an asymptotic theory is to be developed. An additional aim of the paper is to prove a possibility theorem which demonstrates that when non-linear FIML is consistent under normality, it is always possible to find non-normal error distributions for which the consistency of non-linear FIML is maintained. The procedure that is developed for finding a class of error distributions which preserve the consistency of non-linear FIML can be applied more generally and will be useful in other contexts. Many additional problems associated with the asymptotic properties of non-linear FIML under alterna-
tive, plausible error distributions arise in this discussion and some of these are currently the subject of a continuing investigation.

Another topic of theoretical research by Phillips concerns the exact finite sample distributions of econometric statistics. In "The Exact Finite Sample Density of Instrumental Variables Estimators in an Equation with \( n + 1 \) Endogenous Variables" the exact finite sample density function of the instrumental variable estimator was found for the most general single equation case. CFDP 609 extends some of this work to extract the marginal densities of individual coefficients and presents graphical analyses which illustrate the effect of various parameter changes on the sampling distributions. These computations indicate, amongst other things, how the sampling distributions concentrate more slowly from sample size increases as the number of endogenous variables in the equation grow.

CFDP 621 surveys the literature in the field of exact finite sample theory for simultaneous systems. This review covers methods of derivation, derives useful generic formulae for estimator forms and considers results that relate to both structural and reduced forms. A new line of approach to the distribution of the limited information maximum likelihood (LIML) estimator is suggested in CFDP 621 and is systematically explored in the general single equation case in CFDP 626. The latter paper shows that for a leading overidentified case, LIML has a multivariate Cauchy distribution.

Methods of approximating the complex analytic forms of many small sample distributions in econometrics have formed the basis of several additional papers by Phillips ("A Saddlepoint Approximation to the Distribution of the k-Class Estimator of a Coefficient in a Simultaneous System," with A. Holly, "Finite Sample Theory and the Distributions of Alternative Estimators of the Marginal Propensity to Consume," CFDPs 562, 608, 609). Certain of these (the first two papers in particular) have concentrated on traditional methods based on asymptotic expansions and evaluations of their adequacy, in these and other papers, have shown the need for improvements. The purpose of CFDPs 562 and 608 is to suggest a new approach to small sample theory that allows for a convenient integration of analytical, experimental and purely numerical directions of research. The approach centers on a flexible technique of approximating distributions which can accommodate information from sources as diverse as the following: (i) exact analytical knowledge concerning the distribution, its moments or its tail behavior; (ii) alternative approximations based on crude asymptotic theory or more refined asymptotic series; (iii) purely numerical data arising perhaps from numerical integrations of moments or at least isolated points in the distribution; or even (iv) soft quantitative information of the Monte Carlo variety. The first part of CFDP 608 by Phillips explores the properties of rational function approximants which are best according to the uniform norm for a general class of probability density
functions. Characterization, uniqueness and convergence theorems for these approximants are given. In the second part of the article, an operational procedure for extracting rational approximants with good global behavior is devised. It involves modifications to multiple-point Padé approximants which will typically utilize purely local information about the behavior of the body and the tails of the distribution. The new procedure is applied to a simple simultaneous equation estimator and gives exceptionally accurate results even for tiny values of the concentration parameter. Extensions to this work are currently under way.

In CFP 560 Phillips shows that the formula for the characteristic function of the F distribution in the literature was incorrect and implied the existence of all moments. Correct formulae for the central and non-central cases were derived. CFP 546 illustrates a simple method of finding the latent root sensitivity formulae for a matrix. CFD 567 proves a general theorem on tail expansions for densities given the asymptotic behavior of the characteristic function in the locality of the origin. The resulting formula is applied to the stable densities. In CFD 568 (written jointly with E. Maasoumi), some errors in the literature on the asymptotic theory of instrumental variable estimators in dynamic models are corrected. The paper also considers the adequacy of presently used design methods in experimental work of the Monte Carlo variety in econometrics and makes certain recommendations about the way in which such work is conducted and reported.

During 1980, Taylor and Fair initiated research on the estimation and solution of nonlinear rational expectations models. Most econometric applications of rational expectations techniques have concentrated on linear models. The reason for this concentration is largely pragmatic: econometric analysis of nonlinear models has been cumbersome and frequently intractable. However, nonlinearity arises naturally in many macroeconometric applications. Models incorporating the government budget constraint in a rational expectations framework, exchange rate models which emphasize the constraint that the current account surplus is equal to the rate of increase in claims on the rest of the world, and adding-up constraints or portfolio risk factors in financial models are some examples. Risk modelling brings in conditional second order moments of the underlying distribution of the endogenous variables, thereby giving rise to nonlinearities. Research on this topic is reported in CFP 561 by Fair and Taylor. The paper develops tractable procedures for extending the rational expectations approach to nonlinear systems, thereby broadening the range of problems which can be handled using this approach. A general class of nonlinear models, which included nonlinearities in variables, parameters, and expectations but with additive disturbances, was considered in this research. The basic approach is an extension from the linear case. In the linear case a reduced form version of a rational expectations model can be calculated.
explicitly, and this form can be used to calculate the likelihood function directly. In the nonlinear case it is not feasible to calculate the reduced form explicitly, but one can use numerical simulation rather than direct computation. Experimentation with the simulation technique in nonlinear models indicates that the approach is feasible even though it is likely to be dominated by more direct approaches in linear models. The experiments focused either on simulation of large nonlinear systems or full estimation of small linear systems.
COWLES FOUNDATION SEMINARS
AND CONFERENCES

Seminars

In addition to periodic Cowles Foundation staff meetings, at which members of the staff discuss research in progress or nearing completion, the Foundation also sponsors a series of Cowles Foundation Seminars conducted occasionally by staff but most frequently by colleagues from other universities or elsewhere in Yale. These speakers usually discuss recent results of their research on quantitative subjects and methods. All interested members of the Yale community are invited to these Cowles Foundation Seminars, which are frequently addressed to the general economist including interested graduate students. The following seminars occurred during the past three years.

1979


October 19. JOHN ROEMER, Yale and University of California, Davis, "Origins of Exploitation and Class: Value Theory of Pre-Capitalist Economies."

November 9. GEOFFREY HEAL, Columbia, "Necessary and Sufficient Conditions for a Resolution of the Social Choice Paradox."


November 16. WILLIAM SAMUELSON, Boston University, "The Simple Economics of Bargaining."

November 28. GYORGY SZAKOLCZAI, University of Texas, "Hungarian Price Reform of 1980."

November 30. JOHN BEGGS, Yale, "Pooling Cross Sections in Time Series Analysis."

December 7. ANDREW POSTELWAITE, Princeton, "Strategic Behavior and a Notion of Ex Ante Efficiency in a Voting Model."

1980

March 7. GREGORY CHOW, Princeton, "Estimation of Econometric Models with Rational Expectations."

March 14. VOLKER BOHM, University of Mannheim, "A Simple Macroeconomic Disequilibrium Model."

April 11. JOHN B. TAYLOR, Yale, "Measuring the Real Effects of Disinflation with Rational Expectations and Labor Contracts."
April 18. SCOTT BOORMAN, Yale, "Estimating Unreported Income."

April 22. PAUL MILGROM, Northwestern, "The Equilibrium Limit Pricing Doesn't Limit Entry."

April 23. GLENN LOURY, University of Michigan, "A Theory of 'Oil'igopoly."

April 25. DONALD BROWN, Cowles, "The Rate of Interest in a Perfect Loan Market."

May 14. ALBERTO HOLLY, Ecole Polytechnique and University of Lausanne, "The LR Test, the Wald Test and Kuhn Tucker Test in Non-linear Models with Inequality Constraints."

May 16. ANGUS DEATON, Princeton, "Labor Supply, Commodity Demand and Rationing."

May 19. HIROFUMI UZAWA, University of Tokyo, "Disequilibrium Analysis and Keynes' General Theory."

June 11. LAURENCE WEISS, Cowles, "Missing Information and the Cycle."


September 12. PRAKASH CHANDER, Indian Statistical Institute, "Dimensional Requirements for Efficient Decentralized Mechanisms."

October 10. STANLEY BLACK, Yale and Vanderbilt University, "On the Political Economy of Inflation in Open Economies."

October 17. ROBERTO MARIANO, University of Pennsylvania, "The Asymptotic Behavior of Predictors in a Non-Linear System."

October 24. WILLIAM TAYLOR, Bell Labs, "Panel Data and Unobservable Individual Effects: Estimating the Returns to Schooling."

October 31. EUGENE FAMA, University of Chicago, "Inflation, Output, Real Return and Capital Investment."

November 7. ALAN AUERBACH, Harvard University, "Taxation, Portfolio Choice and Debt-Equity Ratios: A General Equilibrium Model."

November 12. GERALD JAYNES, Yale, "Unemployment and Inflation in Macro Equilibrium."

November 14. DAVID WISE, Harvard University, "Test Scores, Educational Opportunities and Individual Choice."

November 21. OLIVIER BLANCHARD, Harvard University, "Production and Inventory Behavior of the U.S. Automobile Industry."

December 3. KATSUHITO IWAI, Cowles, "Schumpeterian Dynamics."

December 5. GEORGE RHODES, Jr., Colorado State University, "Interpretations and Extensions of Identifiability Testing in Linear Models."


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February 6.  STANLEY BLACK, Yale and Vanderbilt University, "Consistent Estimation of the Limited Dependent Variable Threshold Regression Model by Ordinary Least Squares."

February 13.  AVINASH DIXIT, Princeton, "Trade in Natural Resources and Capital Goods."


February 27.  PETER DIAMOND, M.I.T., "Aggregate Demand Management in Search Equilibrium."

March 6.  ROBERT SHILLER, University of Pennsylvania and NBER, "The Determinants of the Variability of Stock Prices."

April 3.  JEFFREY SACHS, Harvard University and NBER, "Current Account Movements and the Real Exchange Rate in the 1970s: A Comparative Study."

April 16.  DAVID KREPS, Stanford University, "Sequential Equilibria."

April 23.  LARRY WEISS, Cowles, informal workshop on current research

April 24.  ALVIN ROTH, University of Illinois, "The Economics of Matching: Stability and Incentives."

May 1.  DONALD BROWN, Cowles, "Existence of a Market Equilibrium Subject to a Budget Constraint."

May 8.  LARRY SUMMERS, M.I.T., "Inflation and the Valuation of Corporate Equities."

May 15.  MAMORU KANEKO, Yale and University of Tsukuba, "The Nash Social Welfare Function and the Nash Bargaining Solution."

June 5.  ROGER GORDON, Bell Labs, "Taxation of Corporate Capital Income: Tax Revenues vs. Tax Distortions."


September 24.  ROBERT HALL, Hoover Institution, Stanford University, "The Excessive Sensitivity of Employment to Demand."


October 9.  MARTIN WEITZMAN, M.I.T., "Increasing Returns to Scale and the Foundations of Unemployment Equilibrium Theory."


October 23.  GUILLERMO CALVO, Columbia University, "Staggered Contracts and Exchange Rate Policy."
November 6. PAUL R. MILGROM, Northwestern University, "A Theory of Auctions and Competitive Bidding."

November 13. THOMAS SARGENT, Univ. of Minnesota & Visiting Professor at Harvard, "The Real Bills Doctrine vs. The Quantity Theory: A Reconsideration."

November 20. JOHN HARTWICK, Queens University, "Learning About and Exploiting Exhaustive Resource Deposits of Uncertain Size."

December 11. JOHN WHALLEY, Univ. of Western Ontario, "General Equilibrium Calculations of Distributional and Efficiency Effects of Taxes."

1982

February 5. CHIRSTOPHE CHAMLEY, Yale, "Efficient Taxation in a Stylized Model of Intertemporal General Equilibrium."

February 19. BENGT HOLMSTROM, Northwestern University, "A Theory of Western Wage Dynamics."


April 2. DAVID KREPS, Stanford and Yale, "Rational Learning and Rational Expectation."

April 16. WILLIAM NORDHAUS, Yale, "Are Real Interest Rates Really High?"

April 23. RUDIGER DORNBUSCH, M.I.T., "Intertemporal Trade Theory."


May 3. Professor TAKATOSHI ITO, University of Minnesota, "A Comparison of Japanese and United States Macroeconomic Behavior by a Vector Autoregressive Model."

Conferences

The Cowles Foundation hosted and financed a Summer Workshop in Econometrics over the period June 20-21, 1982. P. C. B. Phillips initiated the project, organized the program and invited participants to Yale from Europe, Australia, Canada and the U.S.A. Five papers were presented during the day. Topics ranged from extensions of the theory of the linear model to encompass fully dependent variable regressors and errors to a new method of improving periodogram estimates by smoothing of economic unit cross sections and to developmental work on nonlinear stochastic prediction and Kalman filtering. Two of the papers involved empirical applications as well as methodological developments.

The Center for Competitive and Conflict Systems Research at the Cowles Foundation sponsored three conferences which were initiated and organized by Professor Martin Shubik. Two of these conferences were held at the Seven Springs Conference Center in Mt. Kisco, N.Y. In November, 1979 the conference topic was “Mathematical Models of Conflict and Combat: Uses and Theory”; in December, 1979, the conference topic was “Auctions and Competitive Bidding: Uses, Theory and Development”. The most recent conference was held in New Haven in May, 1982 and the topic was “Game Theory”. 
MONOGRAPHS
1934 – 1982*

The monographs of the Cowles Commission (Nos. 1-15) and Cowles Foundation (Nos. 16-25) are listed below.


No. 2. NRA Economic Planning, by CHARLES F. ROOS. 1937, Evanston, Ill.: Principia Press. 596 pages. (Out of print.)


No. 5. The Variate Difference Method, by GERHARD TINTNER. 1940. Evanston, Ill.: Principia Press. 175 pages. (Out of print.)


*Orders for Monographs 13, 16, 17, 22, 23, 24, 25, 26, and 27 should be sent to Yale University Press, 92A Yale Station, New Haven, Connecticut 06520. Order for, or inquiries concerning, all other Monographs should be sent to the Cowles Foundation for Research in Economics at Yale University, Box 2125 Yale Station, New Haven, Connecticut 06520.


No. 22. Economic Theory of Teams, by JACOB MARSHAK and ROY RADNER. 1971. New Haven: Yale University Press. This monograph emphasizes the informational aspect of the problem of designing efficient organizations. After an introduction to decision-making under uncertainty and to the economics of information, a wide variety of models is treated within a unifying conceptual framework.

mation in the presence of prior information is developed. How valuable is prior information in increasing the precision of parameter estimation and what are efficient methods of incorporating this information into estimation procedures are the two basic questions investigated.

No. 24. *The Computation of Economic Equilibria*, by HERBERT E. SCARF. 1973. New Haven: Yale University Press. The first general method for the explicit numerical solution of the price system and economic equilibrium is presented. An important connection between computational methods and economic theory is made which promises to be of use as a practical tool for the evaluation of economic policy.

No. 25. *Bank Management and Portfolio Behavior*, by DONALD D. HESTER and JAMES L. PIERCE. 1975. New Haven: Yale University Press. This monograph provides a microeconomic analysis of portfolio behavior and earnings by commercial and mutual savings banks using time series of cross-section data for individual banks. The results are shown to be of value in constructing an aggregate model of a system of banks.

No. 26. *The Efficient Use of Energy Resources*, by WILLIAM D. NORDHAUS. 1979. New Haven: Yale University Press. A theoretical and empirical model of the efficient allocation of scarce energy resources over time. The book uses linear programming to describe quantities and prices for energy resources under a number of different scenarios, such as under competitive and monopolistic conditions. Chapters include an empirical investigation of the energy demand, resources and control of the carbon dioxide buildup.

No. 27. *Disequilibrium Dynamics: A Theoretical Analysis of Inflation and Unemployment*, by KATSUHITO IWAI. 1981. New Haven: Yale University Press. This book focuses on the central issue in contemporary macroeconomics of whether prices and wages can be described as determined in perfectly competitive markets. After arguing that this classical framework is inadequate, the book goes on to establish a method of dynamic analysis in which monopolistic competition leads to wage and price inertia—a result that provides a theoretical foundation for modern Keynesian theory.
COWLES FOUNDATION PAPERS

July 1, 1979 – June 1, 1982


No. 484. MARTIN SHUBIK, "Opinions on How One Should Play a Three-Person Nonconstant Sum Game," Stimulation and Games, 1978.


No. 533. TJALLING C. KOOPMANS, "The Transition from Exhaustible to Renewable or Inexhaustible Resources," reprinted with permission from ECONOMIC GROWTH AND RESOURCES, St. Martin's Press. 1979.


No. 528. L. WEISS, Information Aggregation, and Policy.


530. R. ENGELBRECHT-WIGGANS and D. R. STRIP, On the Relation of Various Reliability Measures to Each Other and to Game Theoretic Values.

531. R. ENGELBRECHT-WIGGANS and R. J. WEBER, Competitive Bidding and Proprietary Information.

532. W. D. NORDHAUS, How Should We Revise Our Beliefs about Nuclear Power Safety After Three Mile Island?

533. W. D. NORDHAUS, Tax-Based Incomes Policies: a Better Mousetrap?


539. O. FUJIWARA, Morse Programs.


541. R. C. FAIR, A Multicountry Econometric Model.

542. R. HOWE, Linear Complementarity and the Degree of Mappings.

543. J. ROEMER, Exploitation and Class: Part II, Capitalist Economy.

544. J. ROEMER, Rawlsian Justice as the Core of a Game.


549. I. ADLER, R. P. McLEAN and J. S. PROVAN, An Application of the Khachian-Shor Algorithm to a Class of Linear Complementarity Problems.


554. C. CHAMLEY, Optimal Intertemporal Taxation and the Public Debt.


556. G. H. KRAMER, Extension of a Dynamical Model of Political Equilibrium.


559. M. SHUBIK, Perfect or Robust Non-cooperative Equilibrium: A Search for the Philosopher's Stone?


566. M. KANEKO, A Necessary and Sufficient Condition for the Nonemptiness of the Cores of Partitioning Games.


571. M. KANEKO, Housing Market with Individuality.


573. P. C. B. PHILLIPS, On the Consistency of Non-Linear FIML.

574. T. C. KOOPMANS and G. DEBREU, Additively Decomposed Quasi-convex Functions.

575. L. VAN DER HEYDEN, A Path Following Procedure for Finding a Point in the Core of a Balanced n-Person Game.

576. K. NTI and M. SHUBIK, Duopoly with Differentiated and Entry Barriers.


578. R. ANDERSON, Core Theory with Strongly Convex Preferences.

579. C. PITCHIK, Equilibria of a Two-Person Non-Zerosm Noisy Game of Timing.

580. C. CHAMLEY, Entrepreneurial Abilities and Liabilities in a Model of Self-Selection.


589. L. WEISS, Interest Rate Policies and Informational Efficiency.

590. R. ANDERSON, Strong Core Theorems with Nonconvex Preferences.

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591. C. CHAMLEY and D. DOWNING, Efficient Stationary Taxation and Intertemporal General Equilibrium.

592. R. C. FAIR, Estimated Output, Price Interest Rate and Exchange Rate Linkages Among Countries.

593. J. J. BEGGS, Market Structure on the "Research and Development" Phase.


595. R. ANDERSON, M. A. KHAN and S. RAS, Approximate Equilibria with Bounds Independent of Preferences.

596. J. J. BEGGS, Arbitrage Pricing Theory in a Finite Economy.

597. R. C. FAIR, Estimated Effects of Relative Prices on Trade Shares.

598. C. CHAMLEY, Optimal Inflation, Taxation & Short-Run Policy in an Infinite Horizon Model.


600. D. TAIMAN and L. VAN DER HEYDEN, Algorithms for the Linear Complementarity Problem which Allow an Arbitrary Starting Point.


603. K. IWAI, Schumpeterian Dynamics: II: Technological Progress, Firm Growth and "Economic Selection".


606. M. SHUBIK, Game Theory: The Language of Theory?

607. M. KANEKO, Some Remarks on the Folk Theorem in Game Theory.


611. W. D. NORDHAUS, Macroconfusion: The Dilemmas of Economic Policy.
612. M. WOODERS, The Epsilon Core of a Large Game.
615. W. D. NORDHAUS, How Fast Should We Graze the Global Commons?
620. M. KANEKO and M. H. WOODERS, Cores of Partitioning Games.
621. P. C. B. PHILLIPS, Exact Small Sample Theory in the Simultaneous Equations Model.
622. P. DUBEY and J. D. ROGAWSKI, Inefficiency of Nash Equilibria: Part I.
624. C. A. MONASH, Stochastic Games II: The Minmax Theorem.
625. P. DUBEY and M. KANEKO, Information about Moves in Extensive Games.
626. P. C. B. PHILLIPS, On the Exact Distribution of LIML.
627. Z. LIVNE and M. SHUBIK, Navy Procurement Problems: Theory & Practice.
628. C. PITCHIK and M. J. OSBORNE, Equilibria for a Three-Person Location Problem.
629. P. DUBEY and M. KANEKO, Information about Moves in Extensive Games.
PUBLICATIONS AND PAPERS BY STAFF MEMBERS

July 1, 1979 – June 30, 1982

This contains papers which were published during the period and resulted from work at the Cowles Foundation, papers published while the author was a staff member, and a few other papers referred to in the text in the Report.

ANDERSON, ROBERT
Discussion Papers: CFDP 572, 578, 590, 595

BEGGS, JOHN J.
Discussion Papers: CFDP 582, 583, 584, 585, 586, 588, 593, 596, 633, 636
Other Publications:

BRAINARD, WILLIAM C.
Papers: CFP 500, 513
Discussion Papers: CFDP 535, 548
Other Publications:

BROWN, DONALD J.
Papers: CFP 494, 497, 519, 525
Discussion Papers: CFDP 581

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HOWE, ROGER E.
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KOOPMANS, TJALLING C.
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   Papers: CFP 517
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   B. "The Interaction between Oil and the Economy in Industrial Countries,”

PHILLIPS, PETER C. B.
   Papers: CFP 546, 560
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   Other Publications:
   A. "A Saddlepoint Approximation to the Distribution of the k-Class Estimator of a Coefficient in a Simultaneous System” (with A. Holly), Econometrica, Vol. 47, No. 6, November 1979, pp. 1527-1548.


PITCHIK, CAROLYN
Discussion Papers: CFDP 579, 628

RICHTER, DONALD
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SCARF, HERBERT E.
Papers: CFP 516, 522, 541
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SHUBIK, MARTIN
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TALMAN, A. J.
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TOBIN, JAMES

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WEBER, ROBERT J.
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WEISS, LAURENCE
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