REPORT of
RESEARCH ACTIVITIES

July 1, 1970 – June 30, 1973

COWLES FOUNDATION
FOR RESEARCH IN ECONOMICS
AT YALE UNIVERSITY
PURPOSE AND ORIGIN

The Cowles Foundation for Research in Economics at Yale University, established as an activity of the Department of Economics in 1935, has as its purpose the conduct of research in economics, finance, commerce, and industry, including problems of the organization of these activities. The Cowles Foundation seeks to foster the development of logical, mathematical, statistical, computational and other information processing methods of analysis for application in economics and related social sciences. The professional research staff are, as a rule, faculty members with appointments and teaching responsibilities in the Department of Economics or other departments.

The Cowles Foundation continues the work of the Cowles Commission for Research in Economics, founded in 1932 by Alfred Cowles at Colorado Springs, Colorado. The Commission moved to Chicago in 1939 and was affiliated with the University of Chicago until 1955. In 1955 the professional research staff of the Commission accepted appointments at Yale and, along with other members of the Yale Department of Economics, formed the research staff of the newly established Cowles Foundation.
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IN ECONOMICS AT YALE UNIVERSITY  
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**During a part of or the entire period July 1, 1970-June 30, 1973.  
***Active during the period of the report.

Note: For a list of GUESTS see page 37.

Administrative Assistant: ALTHEA G. STRAUSS.
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NOTE ON REFERENCES TO PUBLICATIONS

The following abbreviations are used throughout this report in referring to publications or working papers of the Cowles Foundation:

CFP: Cowles Foundation Papers (see p. 47)
CFDP: Cowles Foundation Discussion Papers (see p. 52)

Monographs (see p. 44) are referred to by number.

Other publications of each staff member are designated by letter in the list on pp. 55-58 and are referred to by author and title in the text.
RESEARCH ACTIVITIES
JULY 1, 1970—JUNE 30, 1973

INTRODUCTION

The Cowles Commission for Research in Economics was founded approximately forty years ago by Alfred Cowles, in collaboration with a group of economists and mathematicians concerned with the application of quantitative techniques to economics and the related social sciences. This methodological interest was continued with remarkable persistence during the early phase at Colorado Springs, then at the University of Chicago, and since 1955 at Yale.

One of the major interests at Colorado Springs was in the analysis of economic data by statistical methods of greater power and refinement than those previously used in economics. This was motivated largely by a desire to understand the chaotic behavior of certain aspects of the American economy—the stock market in particular—during the Depression years. The interest in statistical methodology was continued during the Chicago period with a growing appreciation of the unique character and difficulties of statistical problems arising in economics. An important use of this work was made in the description of the dynamic characteristics of the U. S. economy by a system of statistically estimated equations.

At the same time, the econometric work at Chicago was accompanied by the development of a second group of interests—also explicitly mathematical but more closely connected with economic theory. The activity analysis formulation of production and its relationship to the expanding body of techniques in linear programming became a major focus of research. The Walrasian model of competitive behavior was examined with a new generality and precision, in the midst of an increased concern with the study of interdependent economic units, and in the context of a modern reformulation of welfare theory.

The move to Yale in 1955 coincided with a renewed emphasis on empirical applications in a variety of fields. The problems of economic growth, the behavior of financial intermediaries, and the embedding of monetary theory in a general equilibrium formulation of asset markets were studied both theoretically and with a concern for the implications of the theory for economic policy. Earlier work on activity analysis and the general equilibrium model was extended with a view to eventual ap-
lications to the comparative study of economic systems and to economic planning at a national level. Along with the profession at large, we have engaged in the development of analytical methods oriented to contemporary social and economic problems, in particular the specifics of income distribution, the economics of exhaustible resources and other limitations on the growth of economic welfare.

For the purposes of this report it is convenient to categorize the research activities undertaken at Cowles during the last three years in the following way:

A. Descriptive and Optimal Growth Theory
B. Game Theory and Equilibrium Analysis
C. Rational Behavior under Risk and the Economics of Information
D. Macroeconomics and Monetary Theory
E. Econometrics
F. Public Sector

A. Descriptive and Optimal Growth Theory

During the last several years Koopmans has continued his work on the extension of the theory of optimal growth to models with many goods and in particular, the inclusion of both capital goods and natural resources. The last three-year report described his study of a stationary optimal growth path in such a model. Such a path starts from an initial capital stock so constituted that optimization into the future implies its reproduction with the same composition and level at the end of each future period. The condition to be met by such an "invariant" capital stock is that the shadow prices for all capital goods, taken at the beginning and the end of each period of one year, say, are proportional, with a factor of proportionality equal to the factor whereby the utility derived from future consumption is discounted annually. The validity of this condition was proven in the period of this report and compared with related work by Sutherland. *

An invariant optimal capital stock may be specified as a fixed point of a particular continuous mapping. This led Terje Hansen to develop, in the specific terms of this problem, an algorithm in the class of fixed-point algorithms originated by Scarf and developed by Scarf and Hansen. CFP 575 contains both Koopmans' theoretical constructions and results from Hansen's algorithm, with an application to a constructed example. To facilitate computation of an invariant capital stock, the technology was

assumed to be based on a finite set of processes with given ratios of inputs to outputs.

In related work, Stiglitz has investigated (CFDP 306) the transition between steady states in multi-sector linear models with a single primary factor. Stiglitz first establishes the circumstances under which the transition between the steady states of two different technologies can be made without unemployment of any resource along the way. Under these circumstances he shows that the rate of return for the transition is equal to the rate of interest at the point at which the two technologies are equally profitable.

In the past, much of growth theory has been concerned with models in which all, or all but one, of the factors of production are reproducible. In the last few years several members of the staff have become interested in the analysis of the role of non-reproducible factors, in particular, energy and natural resources, in the growth process. In "World Dynamics, Measurement without Data," Nordhaus analyzed the growth theoretic structure of World Dynamics. His paper noted that the theoretical assumptions of Forrester* and Meadows, et al.,** were quite arbitrary, even contrary to established empirical regularities, and further, that their predictions about the future are very sensitive to specification of the form of the functional relations.

Partly as an outgrowth of his service on a committee of the National Academies of Science and of Engineering to plan studies in the field of energy, Koopmans has been attempting to apply optimization models to the problems of exhaustible resources in general, and of energy resources in particular. In CFDP 356, he contrasts the effect of discounting of future utilities on the optimal path in the classical Ramsey model of aggregate capital with that in a model containing a single exhaustible resource.

Future research planned by Koopmans (partly in collaboration with Alan S. Manne of Stanford University) goes in the direction of construction and analysis of models of natural resource use, especially energy resources, and the introduction of uncertainty about future technologies in these models. A general problem also arising in this context is that of specifying terminal capital stock conditions in development programming. Finally a long-run aim of Koopmans' work is to find a theoretical and empirical basis for aggregative production functions (with labor, capital and

resources as inputs) in less aggregative process analysis models of the productive system.

In a paper currently in progress Nordhaus focuses on the problem of the pricing of appropriable natural resources. The question which is raised in this paper is whether the market mechanism can be relied on to generate the proper scarcity prices for resources. After an examination of the shortcomings of current resource markets—in particular the absence of futures and insurance markets—he concludes that we should well be skeptical about the correctness of market prices for such commodities. This work is still in progress and will require considerable further exploration. Two related problems which Nordhaus hopes to spend time on in the near future are the following: First, it appears that without futures markets the market for resources is dynamically unstable. If this is so, what are its implications? Second, it has often been proposed by non-economists that a separate discount rate be applied to unique, non-reproducible resources or to very important investments, particularly research. Nordhaus will be concerned with the appropriateness of separate discount rates in the absence of perfect futures markets. In addition, Nordhaus intends to consider some of the international aspects of resource scarcity, in particular the impact on our balance of payments and the interaction of this with political considerations.

It seems apparent that detailed specification of an energy or resource sector will require inputs from individuals with a variety of technical knowledge and skills. At the joint initiative of Koopmans and Alan S. Manne, the Cowles Foundation was host to a small informal conference (held on November 17-18, 1972) of economists and operations researchers interested in modeling and projecting the energy sector of the economy. A sequel to this conference was a seminar held at Stanford on July 23-26, 1973. Also, as a part of their teaching activities in the spring semester of 1973 Nordhaus and Koopmans established a workshop on economic models of the resource sector in the hope that this may stimulate the writing of dissertations in this area.

Given the amount of research in growth theory and the substantial empirical work attempting to find the determinants of growth, it is somewhat surprising that there has not been a greater concern with the conceptual relationship between economic growth and the growth of economic welfare. In "Is Growth Obsolete?" Tobin and Nordhaus address the question: Do existing measures of the economic progress of nations, namely GNP or NNP, provide adequate statistical tools for the measurement of economic welfare? The immediate answer is negative, for GNP is
an index of production rather than consumption, and economic welfare is concerned with consumption. Tobin and Nordhaus therefore made an exploratory attempt to determine whether an index better designed to measure economic welfare could be constructed. Such a measure was derived in the paper and designated a Measure of Economic Welfare (MEW). Of particular interest was how the MEW behaved compared to NNP: It was shown that the MEW grew more slowly than the NNP for the United States over the period 1929 to 1965. They also examined in this paper the role of natural resources in determining future growth patterns. And finally, they turned to the question of the effect of reduced, or even zero population growth on the level of per capita consumption. Using a life cycle model they estimated that the growth in per capita consumption stemming from a reduction in population growth to a zero level would be in the order of ten per cent.

The problem of optimal economic growth can be formulated as a non-linear programming problem. However, the present state of convex programming theory for infinite horizon models is not entirely satisfactory. Roughly speaking, classical duality principles can be shown to apply to finite subsections of an optimal trajectory and this avoids inefficiencies over any finite horizon. But it has never been completely clear how to avoid the kind of non-optimality which results from piling up too much "left over" capital forever. In Weitzman's paper (CFDP 317) a rigorous treatment of the subject is undertaken. Under a set of general axioms, a certain limiting "transversality condition," in conjunction with other duality conditions, is shown to be necessary and sufficient for optimality over an infinite horizon.

A technical point in the theory of optimization, also pertinent to intertemporal optimization, is whether the assumption of "quasi-concavity" of an objective function is really weaker than that of "concavity." While in general this is the case, it need not be so if the objective function falls in a special class frequently assumed in optimal growth theory. Koopmans clarified this question in a not yet distributed paper with the title, "If \( f(x) + g(y) \) is Quasi-convex at least One of \( f(x) \), \( g(y) \) is Convex." This paper was presented in July 1972 at a Symposium on Mathematical Methods of Economics, organized by the Mathematical Institute of the Academy of Sciences, Warsaw, Poland.

B. Game Theory and General Equilibrium Analysis

The study of the general Walrasian model of economic equilibrium has been a continuing interest at the Cowles Foundation for a number of
years. This report will describe four directions of research relating to this topic, which have recently been undertaken. They are, in order of presentation, the application of nonstandard analysis to the study of economic equilibria with an infinite number of agents; the development of techniques for the numerical solution of general equilibrium models; the relationship between game theory and the Walrasian model; and the incorporation of monetary considerations in a general equilibrium framework.

1. Let us use the term "standard economy" to refer to a model of exchange with a finite number of traders each of whom has an initial endowment and a preference relation. If one assumes that each trader’s preference relation is complete, continuous, transitive, monotonic, and convex and that each trader is positively endowed in each commodity, a competitive equilibrium can then be shown to exist. Having shown existence, we would next like to know if competitive equilibria are unique and if they are a continuous function of the defining data—the set of preferences and endowments. The best results to date pertaining to continuity and uniqueness of the competitive correspondence of a standard exchange economy can be found in Debreu.*

Standard exchange economies provide an admirable mathematical formulation of the economist’s notions of conflicting tastes and limited or constrained opportunities. In one respect, however, they are inadequate for describing a major feature of perfect competition. The assumption, which underlies the bulk of neoclassical economics, is that each economic agent has a negligible influence in determining the market clearing prices. Both the continuous and the nonstandard exchange economies are attempts at modeling not only conflicting tastes and limited opportunities, but also the economic negligibility of individual traders, by assuming an infinite number of agents. In the continuous model agents are identified with the points on the real line. In contrast the nonstandard model makes use of the mathematical technique introduced by Professor Abraham Robinson which permits the arithmetic manipulation of infinitesimals and infinitely large numbers.

Continuous exchange economies were first introduced by Aumann in his important paper of 1964.** In that paper he proved Edgeworth's con-

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jecture that in a perfectly competitive economy, every core allocation is a competitive equilibrium, a result which has come to be known as the equivalence theorem. In a subsequent paper he provided an independent demonstration of the existence of a competitive equilibrium in a continuous exchange economy.

Kannai and Hildenbrand have defined limiting processes which allow the infinite continuous exchange economies of Aumann to be interpreted as limits of generalized sequences of standard exchange economies. These limiting processes allow us to interpret properties of continuous economies as approximate properties of standard economies. For example, Edgeworth's conjecture becomes the statement that in very large economies every core allocation is almost a competitive equilibrium. Since the Aumann existence proof does not assume convexity of preferences, one can show that in very large economies, even without assuming convex preferences, there exist quasi-competitive equilibria. These are striking results and justify the amount of mathematical machinery needed to prove them.

Nonstandard exchange economies were first defined by Brown and Robinson in CFDP 308 in which they proved the equivalence of the core and set of competitive equilibria. Recently Brown (CFDP 342) has shown the existence of a nonstandard competitive equilibrium under assumptions analogous to those of Schmeidler, i.e., nonconvex, continuous monotonic partial orders as preferences.

The primary motivation for introducing nonstandard exchange economies as an alternative way of modeling perfectly competitive markets was to provide a more direct link between the idealized infinite economy and the large standard economies than exists in the continuum approach. The principal goal has been to obtain limit theorems and asymptotic properties of large standard exchange economies, as is done for example in CFDP 326.

Having demonstrated the equivalence and existence theorems for nonstandard exchange economies, the immediate research problem areas are:

(a) Continuity of the core correspondence of a nonstandard exchange economy.

(b) Differentiability of the core correspondence of a nonstandard exchange economy. The work of Debreu and Stephen Smale indicates that this is the way to approach the uniqueness question for nonstandard economies.

(c) Nonstandard representations of continuous economies.
Brown has obtained some results concerning (c), which are contained in a preliminary paper. The major conclusion is that the nonstandard equivalence and existence theorems imply the continuous equivalence and existence theorems, an indication of the richness of the nonstandard approach.

2. The second major area to be discussed in this section concerns the construction of numerical methods for the approximation of fixed points of a continuous mapping. Fixed-point theorems were first introduced into mathematical economics by von Neumann in 1937 in order to demonstrate the existence of equilibrium prices and capital proportions in his disaggregated model of an expanding economy. Since that time these techniques have become part of the standard equipment of the economist concerned with the simultaneous equations and inequalities of the Walrasian model of general equilibrium. Along with the activity analysis model of production and associated considerations of convexity, fixed-point methods have enlarged the mathematical tools available to the present generation of economists.

On the production side of the economy, linear programming has provided a superior alternative to the calculus in discussing the relationship between competitive prices and the choice of efficient productive techniques. The simplex method, the major tool for the solution of linear programming problems, has also suggested important analogies between numerical methods and the use of information concerning prices in guiding decentralized economic decisions. But important as these theoretical insights may be, linear programming would hardly have achieved its current prominence had it not been for its ability to provide a remarkably effective computational procedure for the explicit solution of a wide variety of problems with considerable practical importance.

Fixed-point methods, on the other hand, have typically been used by economists in order to demonstrate the existence of a solution to economic models describing the interaction between a variety of consumers and producing units. When applied to the Walrasian model, the goal has been to demonstrate the consistency of underlying behavioral assumptions, rather than using this type of formulation for the evaluation of economic policy.

Our ability to employ the general equilibrium model for the purpose of policy analysis has been enhanced by the development of a series of effective computational algorithms for the solution of fixed-point problems. The original work by Scarf and Hansen on this topic is alluded to in the previous three-year report, along with contributions by Harold Kuhn and Curtis Eaves. These methods are described in detail in the Cowles
Foundation monograph *The Computation of Economic Equilibria*, by Scarf with the collaboration of Terje Hansen.

During the period covered by this three-year report, work has continued at the Cowles Foundation and elsewhere in several directions: the further refinement of methodology itself, a search for potential applications beyond those previously discussed, and a series of specific numerical implementations. Perhaps the most significant development in the first of these directions is the recent work of Curtis Eaves* of Stanford University, who visited the Cowles Foundation in the summer of 1973. Instead of considering a single continuous mapping of the price simplex into itself—as is customary in studying the Walrasian model—Eaves works with a continuous family of mappings indexed by a parameter ranging, say, between zero and one. Such a family of mappings might naturally arise if a particular parameter of the general equilibrium model—a tax rate, for example—were varied continuously over an interval. One is then concerned with the way in which the entire family of price equilibria depend upon the parameter in question.

The most satisfying situation would be that in which a unique equilibrium exists for each value of the parameter, varying continuously with the parameter. Unfortunately the possibility of multiple equilibria may cause a more complex type of dependence to emerge, as the following figure illustrates.

An earlier theorem of Browder* tells us that it is possible to select a subset of equilibria, including at least one equilibrium for each parameter value, which forms a continuous path. The path may be forced to turn backwards on itself, but it is possible to connect the equilibria corresponding to any pair of parameter values in a continuous fashion. What Eaves has shown is that the numerical algorithms previously mentioned may be modified so as to approximate the entire path in a single computation. Since many applications of fixed-point methods involve a policy instrument which varies over some interval, Eaves' method may turn out to have substantial practical importance.

Earlier work on the approximation of fixed points of a continuous mapping typically required the degree of approximation to be specified in advance. If a higher degree of accuracy was required, it was necessary to initiate the algorithm again with no use made of the earlier, rough approximation. Eaves' work also contains a procedure which overcomes this numerical difficulty and permits a continual refinement of accuracy. This technique involves a number of ideas which were independently introduced by Orin Merrill in his doctoral thesis presented to the University of Michigan in 1971.** In another doctoral thesis, by Michael H. Wagner, similar considerations are applied to the solution of nonlinear programming problems.†

Fixed-point algorithms are not required for the solution of nonlinear programming problems, since a variety of techniques which exploit the conventional convexity assumptions have been available for a number of years. Nevertheless it is conceivable that fixed-point methods may compete successfully with standard approaches, such as Newton's method, particularly in a search for local maxima of nonconvex programming problems. Further exploration of this possibility is being undertaken.

The application of fixed-point methods to the determination of an optimal invariant capital stock is described earlier in this report. From a purely methodological point of view this work of Hansen and Koopmans (CFP 375) may be seen as a problem of optimal control theory. As such, there is an interesting parallel to the recent work of Allgower and Jepp-

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son who apply fixed-point methods to the numerical solution of nonlinear differential equations,* admittedly a topic far removed from the economic considerations which motivated the early development of these techniques.

A number of studies have recently been initiated in which the numerical calculations of equilibrium prices have been undertaken in order to evaluate the consequences of a specific policy change. In these instances the computer is used either to cope with a level of disaggregation which rules out the possibility of diagrammatic analysis, or because the contemplated policy change is sufficiently large so as to cast suspicion on a purely local analysis. For example, in 1971, Marcus Miller and John Spencer presented an initial attempt to analyze the economic consequences of the United Kingdom's joining the European Economic Community, based upon a general equilibrium model involving four countries (U.K., EEC, Australia-New Zealand, and the United States), eight final commodities (two per country) and two factors per country. Another application, to the Hungarian economy, was presented at the meeting of the Econometric Society held at Budapest in September 1972. In their paper entitled, "Experiences in the Application of Scarf's Method to General Equilibrium in the Hungarian National Economy," Kondor, Simon and Gabor construct a model based on an underlying Leontief technology, augmented by import and export sectors, and with the explicit introduction of demand functions for final goods. In order to determine equilibrium prices and activity levels, they made use of a computational variant based on the observation that the more nearly the technology is given by a pure input-output table, the more closely the equilibrium prices will be determined by the production side alone.

As discussed below in the Public Sector section, John Shoven and John Whalley have used the Scarf algorithm to examine the distortionary impact of the taxation of income from capital in the United States (CFDP 328). In another work (forthcoming in the Review of Economic Studies) Shoven and Whalley examine the general problem of incorporating ad valorem taxes in a general equilibrium model and discuss the modifications of the basic computational procedures caused by this extension.

3. The basic mathematical idea underlying this variety of numerical techniques and applications was originally introduced by Scarf in 1967 in

the proof of a general theorem providing a set of sufficient conditions that an n-person game have a nonempty core. As may be demonstrated quite easily, these conditions are satisfied by market models in which the customary convexity assumptions are placed on individual preferences and on the aggregate production set. In the paper "On Cores and Indivisibilities," by Scarf and Lloyd Shapley of the RAND Corporation, a simple example is given of an exchange economy with indivisible commodities which also satisfies this set of sufficient conditions for the core to be nonempty. The example may be described as follows: Let each of n consumers own a specific indivisible good, say, a house, and let each consumer have an arbitrary ordinal ranking of the collection of houses. Then there exists a permutation of the houses such that no coalition could have done better for all of its members by an alternative permutation of the houses which they initially owned.

Scarf also collaborated with Gerard Debreu on a paper which appeared in the volume Decision and Organization honoring Jacob Marschak (CFP 369). The paper places in a modern context Edgeworth's original argument for the convergence of the core to the set of competitive equilibria. Shubik was the first economist to recognize that Edgeworth's discussion of the contract curve in 1886 could be seen as a game theoretic result identifying the core of a replicated economy with its set of competitive equilibria. More recently, Shapley and Shubik have been able to establish the convergence of the value solution of an economy under replication to its competitive equilibria. In addition, they have been able to show, under somewhat more restrictive conditions, that a similar result obtains for the bargaining set.

The game theoretic solution concepts mentioned above are all cooperative in nature and can be interpreted in terms of the bargaining power of groups, equity or fair division procedures, and bilateral bargaining among individuals threatening to defect from coalitions. It is remarkable that such different behavioral assumptions seem to lead, in the limit, to identical conclusions. An interesting negative result was obtained by Shapley and Shubik when they established that there exists a market for which no von Neumann-Morgenstern stable set solution exists. This has raised questions as to the appropriateness of the stable set solution for economic problems.

4. In the work noted above, one solution concept noticeably missing from the study of markets under replication is the noncooperative equilibrium. In distinction to cooperative game theory, this particular concept leads quite naturally to a dynamic formulation. Furthermore, it does not
make use of the assumption concerning Pareto optimality that is typically employed in all cooperative solutions.

Shubik has recently succeeded in formulating a model of a closed trading economy in terms of a strategic game which can be played noncooperatively (CFDP 324 and a subsequent series of papers on "Theory of Money and Financial Institutions"). In order to do so, it was natural to treat one of the commodities in a nonsymmetric manner; this special commodity could be regarded as a commodity money used to pay for purchases. Upon replicating this noncooperative market game Shubik observed that the limiting noncooperative equilibrium points were not necessarily the same as the set of competitive equilibria. A cash flow constraint had been introduced by the requirement that a certain commodity be used in payment. If the supply of this commodity were not appropriate, the competitive solution would not be obtained. In order to relax this constraint, it is necessary to model a loan market with an explicit bankruptcy law.

In terms of the replicated noncooperative game it then becomes possible to define what is meant by an optimal bankruptcy law. It is a rule which has the property that when the game is played noncooperatively, the rule is sufficiently severe that strategic bankruptcy is never profitable when compared with strategies not involving bankruptcy. The law must be sufficiently lenient that in the limit the cash flow constraint for every individual is completely relaxed; thus, an individual will be able to borrow without fear up to an amount which in the limit will be his ex post budget constraint rather than his cash constraint.

There are many extensions of this basic model of the noncooperative economy which Shubik plans to pursue over the next few years. For example, the models studied so far typically have no exogenous uncertainty and stipulate a finite time horizon. Both the introduction of uncertainty and the accommodation of an infinite horizon require modifications of the bankruptcy law. Shubik and Whitt have recently given a solution to a dynamic infinite horizon model with fiat money and a single commodity using an extension of the concept of a perfect equilibrium point (CFDP 389). Another area in which work will be done is in the modeling of the loan market, distinguishing among various types of banking functions.

Much of the work described above is closely related to the economics of information. In particular, an implicit assumption is made concerning the nature of the information structure as the number of traders in a market is increased. It is not unreasonable to consider different limiting states of information as numbers grow. For example, one might consider a mar-
ket in which individuals always move simultaneously and forget all previous history. This can be regarded as an extremely low information state and can be contrasted with a game in which all moves are sequential with perfect information. Thus it may be of interest to examine the sensitivity of noncooperative equilibria to variations in the information structure. An example of such a sensitivity analysis has been given in "Information Duopoly and Competitive Markets" (CFDP 347). Shubik expects to do further work on the economics of information in conjunction with the work on money and financial institutions.

Ross Starr has also been concerned with the integration of monetary theory and the theory of value. Starr's approach is the inclusion of monetary variables in a mathematical general equilibrium model. In CFP 365 a family of classical contentions on the relation of money and barter exchange is investigated. It is shown there that when the restrictions of "double coincidence of wants" is placed on bilateral trade, in general a barter economy will not be able to achieve a competitive equilibrium allocation. On the other hand, a monetary economy, with the double coincidence condition suitably reinterpreted will be able to do so. The classical view of the superiority of monetary over barter exchange is verified in this simple model.

CFDP 300 investigates the existence of equilibrium and the demand for media of exchange as it may vary with transactions costs and initial endowments. CFDP 310 proposes a solution to a long troublesome technical problem in this field. A monetary economy may be demonetized, in a fashion that is difficult to treat mathematically, if the value in exchange (price) of money becomes zero. This can be a serious theoretical problem if the currency is unbacked. It is occasionally noted that a government can guarantee that this currency will be accepted in payment of taxes. It is shown in CFDP 310 that this guarantee can be made sufficient to keep the value of the currency above zero.

In "Money and the Decentralization of Exchange" (CFDP 349) Starr and Joseph Ostroy of UCLA investigate the role of money in facilitating the process of exchange. It is shown that a barter economy can achieve an equilibrium allocation by use of a great deal of coordination and information transmission. In a monetary economy achievement of an equilibrium allocation requires dramatically less information and coordination. This study succeeds in formalizing and analyzing the often recurring and seldom analyzed concept of "difficulties of barter" and advantages of monetary exchange. Thus, CFDP 349 generates an analytic basis for the transactions demand for money.
C. Rational Behavior under Risk and the Economics of Information

During the most recent period, research on behavior under uncertainty has continued in the areas discussed in the last three-year report: the development of basic concepts, extensions of portfolio models, and the development of general equilibrium models incorporating considerations of risk. Work on the economics of information has also been initiated during this period.

1. Behavior under Risk. In work reported earlier (CFP 341), Michael Rothschild of Princeton and Stiglitz examined several possible interpretations of the intuitive idea that one distribution is riskier than another. They found that a number of apparently different definitions were equivalent to one based on the notion of a "mean-preserving spread," which may be stated in terms of the indefinite integrals of differences between cumulative distribution functions. Further, they demonstrated that in applications where one is willing to assume that individuals and firms are risk averse, nothing is gained by a more restrictive definition, e.g., using variance as measure of risk. In current research, Peter Diamond of M.I.T. and Stiglitz have extended this concept to that of "expected utility-preserving spread." They have established a general theorem specifying those situations in which it is possible to determine the qualitative responses of households and firms when mean-utility-preserving increases in risk occur.

In other work on the basic concepts used in analyzing behavior under uncertainty, "A Note on 'The Ordering of Portfolios in Terms of Mean and Variance,'" Klevorick investigated how wide the class of expected utility functions is that can serve simultaneously as expected utility functions for both the normal family of distribution functions and the two-point even-chance family of distribution functions. (The latter family consists of all lotteries offering outcomes a and b, each with probability 1/2, when a and b are allowed to take on any real value.) He demonstrated that the class is quite narrow, consisting only of the family of expected utility functions derived by taking the expectation of cubic utility-of-income functions. The result is of interest because in an earlier paper, John Chipman had demonstrated that a great variety of expected utility functions could serve for the normal family alone and for the two-point even-chance family alone.

Most of the early portfolio models took a narrow view of the allocation problem facing an individual. Typically, the consumption-saving decision was taken as given, and transactions costs and the uncertainty in
labor income and consumption needs were ignored. The last three-year report summarized some of the continuing efforts at Cowles and elsewhere to understand the importance of these limitations and to build models which avoid them. For example, Stiglitz (CFP 330) has analyzed the term structure of interest rates in a utility-maximizing model involving consumption during each of several periods. In the face of an uncertain future short-term interest rate, the consumer is given the choice of buying short or long-term bonds in order to finance future consumption. In a related Ph.D. thesis (supervised by Tobin and Stiglitz), J. Schaeffer is investigating optimal portfolio behavior in the presence of transactions costs in the purchase and sale of assets and uncertainty about future labor income. In order to focus on the implication of these conditions for liquidity preference, Schaeffer assumes perfect knowledge about future asset prices.

In most portfolio analyses the joint distribution of returns on physical assets is taken as given. A direction in which future work is planned is in the analysis of the economic determination of the structure of risk. It seems desirable to incorporate in this analysis the assumption that the investor simultaneously takes into account the risk in the return to investments and the related uncertainty about final product prices. Stiglitz has begun by exploring a simple two commodity model in which the output of one commodity is stochastically related to the input of the primary factor. The consequences of this technological uncertainty on the riskiness of investment in the two sectors is not obvious. The uncertainty about the output of one commodity creates uncertainty in the relative prices, and hence in the return to investment in both sectors. In a simple example, Stiglitz has established that when investors take into account the implications of the variation in relative prices of their consumption possibilities, which investment appears riskier depends on the elasticity of substitution of demand for the two commodities. Indeed, for an elasticity less than unity the investor acts as if the sector without technological uncertainty is in fact the riskier investment.

One of the most important analytical devices available to economists for the analysis of uncertainty is the concept of a conditional commodity introduced by Arrow and Debreu almost twenty years ago. According to this point of view a commodity may be characterized not only by its physical attributes and the date and location of its availability but by one or another in a series of uncertain events. Thus what is ordinarily called an umbrella may be viewed as a vector of commodities contingent upon the state of the weather and other relevant states of nature. Using this device
one may immediately apply economic arguments developed for the treatment of a certain world to the analysis of uncertainty.

This point of view has proved to be very fruitful in clarifying the conceptual problems involved in a variety of applications. Examples are the determination of the appropriate social discount for risk, the evaluation of firms with different financial structures and the analysis of insurance markets. Indeed, the framework has been useful in understanding the consequences of limiting the markets for contingent commodities themselves.

In the Arrow-Debreu framework, social risk may be defined as variation in the social endowment across states of nature—uncertainty about the aggregate consumption opportunities of society. The consequence of a given amount of social risk for the welfare of the individuals comprising society depends crucially on the financial arrangements society makes for its distribution. If financial markets are perfect, the "distribution of risk" would be Pareto optimal. In that case, prices, or "risk premia," would be associated with each competitive equilibrium allocation indicating the terms on which individuals would be willing to trade certain income for risky outcomes. In the absence of perfect markets, in particular in the absence of a complete set of contingent commodity markets, risk premia do not provide a direct indication of the presence and magnitude of social risk. In CFP 350, Brainard and Dolbear discuss the conceptual problems of measuring social risk in such a situation and attempt to make a rough estimate of the extent to which, for particular sectors of the economy, private and social risk differ. They tentatively conclude that there is a substantial difference between private and social risk and that even in the case of the returns to capital traded in the stock market there would be a substantial reduction in private risk if there existed a way to diversify the returns to capital with the returns to other factors. Brainard and Dolbear then discuss the extent to which the markets for composite commodities—stocks, bonds, labor, etc.—are able to provide opportunities for the efficient distribution of risk. Such an analysis is useful both in providing an insight into the reasons particular types of financial claims exist in the real world and in suggesting areas in which particular types of risk-spreading assets are missing.

Another example in which the absence of a complete set of contingent commodity markets plays a central role is Stiglitz's analysis of a primitive economy which depends on sharecropping to diversify agricultural risk (CFDP 353). In a model of such an economy Stiglitz investigates the extent to which restricting the type of allowable contracts interferes with the efficient allocation of risk and affects the distribution of income.
In order to obtain specific results about the effects of risk on resource allocation it is sometimes useful to place restrictions either on preferences or on the distribution of returns. Assuming that a typical investor's expected utility depends only on the mean and variance of returns, Sharpe and Lintner have provided a simple expression for the value of a firm whose assets yield risky returns. In CFP 362, Stiglitz uses the Sharpe-Lintner formula to analyze the investment decisions of firms. If a firm purchases assets so as to maximize its stock market value as given by the Sharpe-Lintner formula, the allocation of investment will not be optimal. Since the equities of the various firms provide differentiated types of risk to the investor, firms have some monopoly power. This leads to a systematic underinvestment in the more risky activities.

A related set of concerns has been the determination of the optimal financial structure of the firm under uncertainty. Some of this work, which relaxed the assumptions necessary to demonstrate the validity of the Modigliani-Miller theorem, was described in the last three-year report. In this work, as well as in the multiperiod extension due to Stiglitz, it appeared necessary to maintain the unattractive assumption of a universal belief in the impossibility of bankruptcy. Stiglitz has begun an investigation in which individuals have different expectations about the returns to investments, including bankruptcy. Early results suggest that it will be optimal for firms to borrow to such an extent that, in the judgment of the lender, there will be a non-zero probability of bankruptcy. Stiglitz also proposes to investigate the implications of divergent expectations and bankruptcy on the determination of equilibrium prices and the allocation of investment.

The after tax returns of shareholders are extremely sensitive to the form in which corporate profits are disbursed. The structure of taxes may therefore have a substantial impact on corporate financial policy. In the absence of uncertainty, the optimal debt equity ratio for individuals in a high personal income tax bracket requires financing of investment first through retained earnings, and subsequently through debt. The cost of capital is merely the market rate of interest. Stiglitz has also analyzed the cost of capital in the presence of uncertainty, assuming firms are constrained to avoid bankruptcy as well as a number of other simplifying assumptions. The impact of taxes on the financial structure of the firm, taking into account the choice of corporate form, the possibility of bankruptcy, and the related questions of control is a fundamental and difficult problem, deserving continued interest.

2. Risk and the Economics of Information. The inadequacy of the
assumption of perfect information commonly made in competitive models has long been recognized. Information, like economic goods in general, is scarce, and its production requires the input of scarce resources. Recent and continuing research at Cowles has been concerned with the extent to which private markets provide incentives for the production of the Pareto optimal amount of information, and the implications of imperfect information for the behavior of markets themselves.

Some types of information—for example, information about new processes or commodities—are in principle appropriable. In work reported in the last three-year report Nordhaus analyzed the optimal life of patents. Since typically there is a trade off between the life of the patent and the social benefit derived from an invention, optimal patent systems only allow partial appropriation of the benefits by the inventor. This would seem to give a presumption that in the absence of subsidy there will be an undersupply of inventive activity. In recent preliminary work, Stiglitz has found that this presumption may not be correct. A distortion exists since the patent system rewards an inventor for being first in making a discovery rather than for the amount by which he has advanced the time of its arrival. The implications of this distortion are investigated in a simple model in which a firm can influence the probability of making a discovery in a given time interval by varying the factor inputs. Equilibrium in the model has too many firms doing research, each doing it at too fast a rate. One may view this result intuitively as firms or individuals drawing upon an unpriced “pool” of potential inventions; an externality exists which is analogous to the resource stock externalities which occur in fisheries and some extractive industries.

Many kinds of information, such as that concerning the distribution of prices in a market, are not patentable. Indeed, some types of economic information are not produced by a single agent in a visible fashion, but rather are revealed by the functioning of the market itself. One such example is the job market where potential employees have different productive capabilities not perfectly known to employers. The employer is concerned with the productivities of those he is about to hire, but must rely upon characteristics such as education which are imperfect predictors of ability and may be subject to the control of the potential employee. Another example occurs in insurance markets. Individuals may know their own probabilities of accident but it may be difficult for insurance companies to determine them ex ante. As a consequence insurers are led to offer complex contracts, involving deductability and coinsurance clauses, designed to take advantage of the self-selection process, and minimize the
problem of adverse selection. Stiglitz and Rothschild are considering in a forthcoming paper the existence and properties of equilibrium in a market for insurance with these characteristics. An equilibrium is specified by a set of insurance policies, such that no other insurance policy can be offered, which will be chosen in preference to those presently selected by individuals and which will at the same time be profitable for the insurer.

D. Macroeconomics and Monetary Theory

In recent years, developments in macroeconomic theory and policy have continued to broaden and refine the insights of the Keynesian revolution. In both theoretical and empirical dimensions, the thrust of research at the Cowles Foundation has been to extend simple aggregative specifications of economic relationships by an analysis of underlying microeconomic behavior. On a theoretical level, this involves examination of the properties of optimizing behavior by individual economic agents and explicit recognition of the problems of aggregation. On the empirical side, there has been an attempt to disaggregate to greater sectoral detail, even to individual households or firms.

In his Presidential Address to the American Economic Association in 1971, "Inflation and Unemployment," Tobin presented an overview of post-Keynesian theoretical and empirical research bearing on macroeconomic employment policy: Is unemployment an equilibrium or a disequilibrium phenomenon; is there a single "natural rate" of unemployment associated with a stable rate of inflation; and is such a rate optimal? Tobin criticizes the view that satisfactory answers to these questions may be obtained by treating labor as a single homogeneous commodity traded in a single market. His approach views aggregate wage and unemployment statistics as reflections of conditions in numerous loosely-connected labor and product markets subject to shocks which are only partially correlated. According to this view the system never arrives at a long-run equilibrium in all markets. The responses of money wages and prices to changes in aggregate demand reflect differences among markets in the speeds of adjustment of wages to disequilibria, institutional constraints (such as downward rigidity in money wages) and relative wage patterns which lead workers to emulate the wage demands of those in markets that are contiguous in geography, industry, or skill.

Such a general approach seems more consistent with the observed behavior of money wages and unemployment than do the results of a search model which focuses only on the movement of labor from sector to sector. While search models have provided a valuable contribution to the under-
standing of frictional unemployment, their implications for the cyclical behavior of quit rates and hence for the shape of the short-run Phillips curve are ambiguous.

Tobin's research plans call for continued work in the field of unemployment and inflation. He is continuing to develop a more completely articulated model of wage and employment behavior which extends earlier work by Lipsey, Archibald, and Holt.

Price and wage dynamics have also been given intensive examination by Nordhaus in a number of papers. In CFDP 296, he reviewed the theoretical foundations of work on price dynamics and inflation theory and surveyed U. S. econometric studies of price determination. Nordhaus concluded that the theoretical basis of price dynamics had been largely ignored in empirical work, leading to misspecifications that could bias the results. This paper was presented at a conference sponsored by the Board of Governors of the Federal Reserve System. At the same conference Tobin provided the summary comment (CFDP 315). He presented the basic specifications underlying the empirical work that had been discussed at the conference and commented upon its relevance.

In "Pricing in the Trade Cycle" (CFP 371) Nordhaus and W. A. H. Godley of Cambridge University attempt to determine whether cost-push or demand-pull theories of price inflation more accurately describe the process of price formation in industrial economies. The hypothesis tested is whether price is simply a mark-up over normal historical cost or whether there are cyclical variables which determine price movements. The study employed a new method of testing such a hypothesis, using a constructed or predicted price series and then comparing the predicted with the actual. The test was applied to British data in non-food manufacturing over the period 1954 to 1969. The results confirmed the mark-up hypothesis.

Nordhaus intends to continue his collaboration with Godley in an examination of the problem of the shifting of direct company taxation. Nordhaus and Godley have been able to formulate measures of tax shifting and are now in the process of testing these on data for British manufacturing. A monograph reporting on this work, with estimates covering five or six manufacturing industries, is scheduled for completion in the next eighteen months.

A third recent paper by Nordhaus (CFP 374) in the area of short-run price and wage dynamics considered whether any unifying explanation of the surprising worldwide acceleration of wages after 1968 could be found. It appeared that low unemployment rates in the U. S. late in the 1960s, the subsequent growth in inflationary expectations, and the inter-
national spread of this inflation offered the best though not fully satisfactory explanation.

Concern about inflation stems of course from its impact on the welfare of households and institutions. Although empirical studies of the distributional effects of inflation have been conducted in the past, few of these studies rest on a theoretical foundation which clearly defines an appropriate concept of welfare. In CFDP 329, Nordhaus attempted to remedy the difficulties of using only current income as a measure of welfare. A concept of “annuity income” is developed which measures the ability to sustain a consumption path over the expected life of the household. A simple macroeconomic model is then presented which determines both the cyclical and long-run response of capital, wealth, and employment to macroeconomic policies. Drawing on a data set provided by the Federal Reserve System, this model is able to simulate the impact of different countercyclical policies on the distribution of income. The results indicate that if there is a long-run trade-off between inflation and unemployment, expansionary monetary and fiscal policies lead to significantly greater equality in the distribution of economic welfare.

In a related paper (CFDP 321), Nichols investigates the definition of income that should be used by households or perpetual institutions in arranging their consumption decisions. It is shown that neither dividends alone nor dividends plus capital gains is appropriate.

Work on the macroeconomics of consumption has been concerned with the aggregation of life cycle models which incorporate liquidity constraints faced by households. These models permit an estimate of the influence of monetary and fiscal policy on aggregate consumption assuming a population with specified demographic characteristics. The life cycle theory originally set forth in Tobin’s CFP 272 is expanded with explicit attention to liquidity constraints in “Wealth, Liquidity and the Propensity to Consume” (CFDP 314, published in *Human Behavior in Economic Affairs*, Essays in Honor of George Katona). The theory was the basis of a simulation designed to evaluate a series of monetary and fiscal policy measures carried out by Tobin and Walter Dolde (CFP 360). In this model consumers are assumed to maximize the discounted sum of utilities from consumption in each period over a lifetime of certain length. Consumers are also assumed to borrow in order to invest in consumer durables early in their life and are then subject to contractual saving requirements in order to repay this loan. The amount of additional borrowing in which they engage may be limited by quantity restrictions and is subject to a penalty rate of interest higher than the lending rate. In the policy simulations this
model exhibited small responses to temporary changes in tax rates and relatively large responses to long-run changes in interest rates. The liquidity constraints, which were binding on the young and the poor, had the effect of raising the marginal propensity to consume from current wealth and from current disposable income. The results of the simulations appeared plausible both in magnitude and time path.

Other work in macroeconomic models builds on areas discussed in the report of research for the 1967-70 period. In particular, Bischoff has continued his investigations of the determinants of business investment decisions. His econometric work has focused on (1) choosing a satisfactory theoretical formulation, (2) developing investment relations that make accurate forecasts in the presence of changes in tax and monetary policies, and (3) giving particular attention to lag distributions. In CFP 372, for example, Bischoff investigated the effect of alternative tax incentives on capital spending. He started with a model allowing substitution ex ante and fixed proportions ex post. He then derived the optimal path of investment under the assumption that firms choose factor proportions in such a way as to minimize cost. The major advantage of Bischoff's work over earlier works is the separation of plant and equipment into two different categories, which allows for different lags on price and output terms, and for non-unitary elasticities of substitution between capital and labor. The principal conclusions of the work substantially confirm the value of the neoclassical approach to investment. Relative prices appear to have a significant effect on equipment expenditures; and the investment tax credit is shown to have an additional, independent, and statistically significant effect.

Bischoff intends to extend his research on investment behavior to a complete model of demand for factors and outputs. The major work, on which he is now proceeding, is a five-equation structural model which would simultaneously determine factor demand (equipment, structures, and labor), output, and prices. He hopes to estimate this model, using nonlinear methods, for a number of major industries and sectors using post-World War II data and for the total private sector using yearly data from 1929 to 1968. A further study in which Bischoff is currently engaged is the study of relative wages, capital and industrial growth in the new South for the period 1865 to 1900. It is hoped that this foray into econometric history will add evidence on the question of determination of investment and factor price behavior in competitive market economies.

Research in monetary economics has proceeded in two directions which were discussed in the previous report. Tobin continued to be involved in
current debates about the channels of influence and the magnitudes of the
effect of changes in the money supply on other macroeconomic variables.
In the 1970-72 period, this debate revolved around articles by Tobin
(CFP 370) and Friedman in the Journal of Political Economy concerned
with the consistent specifications of models determining real income and
prices and the ability of monetary authorities to control the real rather
than the nominal money supply.

Work at the Cowles Foundation has also continued toward estimation
of a complete disaggregated model of financial markets and flows of
funds. The theoretical foundations for this project were developed in
erlier work by Brainard and Tobin (see pages 29-30 of Report of Re-
search Activities 1967-70). A starting point for the empirical estimation
was the 1971 doctoral dissertation of Gary Smith (who subsequently
joined the staff of the Cowles Foundation). Smith estimated a financial
model, using postwar quarterly flow of funds data, which accounted for
the supply of and demand for 12 types of financial instruments and the
government sector. In each sector's demand equations, the proportion of
the sector's wealth which it desires to hold in an asset is a function of
interest rates on all assets and a set of additional explanatory variables.
Actual holdings of assets respond with lags to deviations of last period's
holdings from desired holdings. Whether in equilibrium or disequi-
librium, each sector's holdings of assets and liabilities are constrained to
sum to net worth.

Smith's demand equations forecast quantities of sectoral asset holdings
moderately well over the first two years (1966-I through 1967-IV) of a
four-year period but then drifted off the historical path and performed
worse than a naive autoregressive model. The most dramatic characteristic
of this particular out-of-sample period was the extraordinary level of
nominal interest rates, and the primary forecasting error was a predicted
but unrealized massive shift out of money holdings into interest-bearing
assets. This suggests that the primary problem may have been that the
variation in rates was too large to maintain the assumption of linearity in
the demand equations.

Subsequent work building on these foundations is now underway in-
volving Brainard, Nordhaus, Smith and Tobin in collaboration with in-
vestigators at Massachusetts Institute of Technology, the University of
Pennsylvania and the American University. The model being developed is
designed to be linked to the real sectors of the FMP* econometric model

*The FMP—Federal Reserve, MIT, Penn—model is one of a family of models
developed initially by a group from the Board of Governors of the Federal Reserve
of the United States economy. The specifications of variables and equations will permit the "Yale model" to receive inputs of relevant variables from the real FMP sectors and to transmit outputs to them. The performance of the full FMP model can then be examined with the Yale financial sector substituted for the regular FMP monetary and financial equations. The same approach can hopefully be extended to other econometric models, so that there will be a detachable and interchangeable Yale "module" which can be used as an alternative to the regular financial sectors of large scale models.

The Yale model will recognize explicitly the interrelatedness of financial markets. Many financial assets are imperfect, though close substitutes for each other in the portfolios of financial institutions and other economic agents. Moreover, certain balance sheet identities must be respected. For individual agents and sectors, a decision to hold funds in one asset is, simultaneously, a decision not to hold them in other assets. Likewise, assets acquired or debts issued by one sector must be balanced by assets sold or debt absorbed elsewhere. In the Yale model these identities are satisfied in the asset demand or supply equations of a sector by including the same list of explanatory variables—always including interest rates, exogenous constraints on disposable funds, and initial positions in all assets—in every equation.

E. Econometrics

Applied econometric work by staff members and visitors at the Cowles Foundation has been described under the appropriate substantive subheadings of this report. Research on problems of econometric methodology has also been undertaken in a number of different areas.

Grether and Maddala, in CFDP 301, studied the properties of several commonly used two-stage procedures for estimating distributed lag models. In the presence of serially correlated errors, these procedures, though still consistent, are known to be asymptotically less efficient than the method of maximum likelihood. Grether and Maddala explored the large-sample performance of these estimators under various conditions to determine how large the loss of efficiency is, and to develop guidelines for choosing among alternative estimators. They found that with high posi-
tive serial correlation, which is characteristic of many economic time series, the loss in efficiency from all of the two-stage procedures is considerable, and that maximum likelihood estimators, despite their greater computational complexity, should generally be used instead. They also found that in some cases the two-stage procedures are even less efficient than the instrumental variable estimators used in the first step of the procedures.

In CFDP 302, Maddala and Rao used Monte Carlo sampling experiments to investigate the performance of two tests proposed by Durbin for testing serial correlation in distributed lag models. They found that both of Durbin's tests generally do about equally well, and that in most cases they compare favorably with the computationally more difficult likelihood ratio test. Their results also provide some clues as to when the tests are likely to lead to wrong conclusions.

Peck's CFDP 325 is concerned with the properties of several procedures for using a cross section of time series to estimate a regression containing a lagged endogenous variable. The estimators he considers were developed and their properties investigated via Monte Carlo methods by Nerlove in an earlier paper (CFP 348). Peck's study was based on a different analytical approach, developed and applied by Kadane to simultaneous equation problems (CFDP 269, 326), which used a new type of approximation of bias and mean-square error as the variance of the disturbance tends to zero. Peck applies this asymptotic theory to the time series of cross sections problems and derives analytic approximations of the bias and mean-square error of several estimators. His results indicated that, even among large-sample-equivalent estimators, which of several estimators is best depends upon the true coefficient of the lagged variable, the relative variances of the time-series and cross-sectional error components, and the autocorrelation of the exogenous variables.

Another continuing interest of Peck's is the problem of data merging. This problem, which is of substantial practical interest, is to develop methods of combining samples which are drawn from the same population but do not contain the same observations, and which cover different but not disjoint sets of variables. The objective is to be able to estimate relationships among variables in the "joint" sample. An early empirical attempt to deal with these difficulties is criticized in Peck's "Comment" on Benjamin Okner, "Constructing a New Data Base from Existing Microdata Sets: The 1966 Merge File." Peck's current research in this area is directed at analytical evaluation of the cases where these techniques may succeed.

Peck is also engaged in research on models involving limited dependent variables. He and Kleverick are exploring the possibility of extending the
standard multiple probit model, as presented in Tobin's CFDP 1, to the case where there are several critical indices and each index is a linear combination of a set of independent variables. They intend to derive estimating equations for this "multi-index probit model," examine the properties of the estimators, and compare the results obtained using the multi-index model with those obtained in empirical studies which have used single-index probit models. Peck is also currently using the standard probit and limited-dependent variable models, and extensions due to Amemiya and Boskin, to investigate the problem of bias due to attrition in the New Jersey graduated work incentive experiment. His paper on this subject will be published as part of the forthcoming final report on the experiment.

F. The Public Sector

During the last several years research at Cowles on the economics of the public sector has proceeded along the following three lines: social choice and voting, legal policy and regulation, and taxation and public expenditure.

1. Social Choice and Voting. The study of collective choice by voting or other non-market socio-political mechanisms has a long tradition at the Cowles Foundation, dating back to Kenneth J. Arrow's Cowles Foundation Monograph 12, Social Choice and Individual Values. Current and prospective research by staff members continues this tradition on a variety of fronts, ranging from abstract mathematical theory to more applied theoretical and empirical work.

In Arrow's original formulation of the problem of social choice, i.e., the aggregation of individual preference relations into a social preference relation, he assumed that the individual and social preference relations were complete and transitive. The Arrow Possibility Theorem is that any aggregation procedure which is Pareto efficient, which exhibits positive responsiveness to individual and social values, and which satisfies the condition of independence of irrelevant alternatives is dictatorial. Brown, in "Aggregation of Preferences," forthcoming in the Quarterly Journal of Economics, has shown that if the social preference relation is only required to be acyclic, then there exist nondictatorial aggregation procedures satisfying the remainder of Arrow's conditions. These aggregation procedures are characterized by a distinguished family of coalitions of individuals in society, which may be termed a polity. The social preference relation defined by a polity is that society prefers x to y if and only if the set of in-
dividuals who prefer $x$ to $y$ belongs to the polity. A typical polity over a society of $k$ individuals is given by choosing $m$ individuals and some number $n$ such that $m + n \leq k$. Society will then prefer the state $x$ to the state $y$ if the chosen $m$ individuals and at least $n$ other members of society prefer $x$ to $y$.

The most common social choice mechanisms, based on majority rule or other voting mechanisms, are often unstable because of the phenomena of "cyclical" majorities. Typical public sector problems which might arise for resolution by some voting mechanism might include determining the levels of each element in a set of governmental services or public goods, or choosing from a set of alternative social states which can be partially characterized by quantitative indices or social indicators. In such problems the feasible alternatives constitute a point set in some appropriately defined multidimensional choice space, and citizen preferences can often be represented by utility functions with the usual properties. A variety of preference restrictions—which require, in effect, some degree of similarity in individual preferences—have been shown in the literature to be sufficient to eliminate intransitivities and to restore stability to majority rule. In an earlier discussion ("On a Class of Equilibrium Conditions for Majority Rule") Kramer showed that these conditions, when applied to multidimensional choice problems, turn out to be extraordinarily restrictive and are tantamount to requiring unanimity of individual preferences. In a subsequent paper ("Sophisticated Voting over Multidimensional Choice Spaces") the problem was re-examined under somewhat different institutional and strategic assumptions about individual voters. The actual course of voting is sequential, with each of the underlying decision variables being considered and voted on separately. Voters are "sophisticated" in Farquharson's sense, thus permitting the possibility of strategic voting (but not explicit collusion). Kramer showed that under these assumptions a voting equilibrium will exist if the preferences of the individual voters satisfy a well-known additive separability condition. This result holds for a wide variety of voting rules, and a corollary of one of the results is that the well-known single peakedness condition is sufficient for a voting equilibrium under any voting rule which defines a simple game (a much broader class than voting games based on the simple majority voting rule, which have received most of the attention in the literature).

In a paper "A Voting Model for the Allocation of Public Goods: Existence of an Equilibrium" (a part of a forthcoming Ph.D. dissertation supervised by Brainard, Brown, and Kramer), Steven Slutsky has ex-
amined a related problem in the context of a general equilibrium model which includes public goods provided by an endogenous public sector. The public sector is financed by taxes on individual income, and the amounts to be provided of each of the public goods are determined by majority votes of the consumers who are assumed to vote non-strategically on each of the goods separately. Slutsky establishes the existence of a public competitive equilibrium, defined as a vector of prices for both public and private goods, a vector of tax rates on individuals, a vector of commonly consumed public goods and a distribution of private goods to the members of the economy. In addition to the conventional properties of a competitive equilibrium the voting equilibrium requires that no majority coalition of consumers will vote to alter the level of any of the public goods. Investigation of the efficiency and distributional properties of this equilibrium is now under way.

Voting equilibria in the social-choice-theory sense are closely related to the game-theoretic solution concept of the core. In CFDP 343, Kramer and Kleverick find a condition (which turns out to be a rather natural generalization of the well-known single peakedness condition) for the existence of a "local" core—that is, an outcome which will not be blocked by any "nearby" alternative—in the class of voting processes that can be represented as simple games. In CFDP 351, Shubik surveys the application of game-theoretic concepts to a variety of political science problems, and he is currently working on a game theoretic treatment of logrolling in voting games.

At a more applied level, Kleverick and Kramer in CFP 387 have undertaken an examination of the Genossenschaften, a collection of agencies responsible for managing water quality in the Ruhr area of Germany. These agencies are widely cited in the environmental literature as successful models of basin-wide management of water resources. The typical Genossenschaft has the authority to set water quality standards, raise revenue by imposing effluent charges on industries and towns within its jurisdiction, and use the revenues so raised to construct treatment facilities. Ultimate authority within each agency rests with a voting body composed of representatives of communities and industries in the area, and typically voting strength in the assembly is approximately proportional to each member's financial contribution to the agency. Since contributions depend primarily upon effluent charges, the largest polluters have the most votes in determining water quality standards, an arrangement which seems strange. From a theoretical point of view, the existence of an equilibrium or self-sustaining water quality standard under such a representation
scheme is not obvious. Different agents will respond differently (depending on their concern for water quality and their ability to treat their own waste) when the level of effluent charges is changed. Thus, if the assembly were to vote a change in the charges, under the new charges a different distribution of votes results, and the newly-reapportioned assembly might then vote for a further change in the effluent charges, leading to the possibility of succession of changes in effluent charges. Using theoretical voting models, Kleverick and Kramer establish conditions for the existence of a self-sustaining voting equilibrium under this representation scheme, and they compare it with the water quality standard that would prevail under alternative political mechanisms. They also find that introduction of a more efficient technology for pollution treatment may, under certain circumstances, actually result in a lowering of the prevailing water quality standard when the Genossenschaften representation scheme is used to decide on water quality.

In general, a majority voting rule may result in either an over or undersupply of public goods, depending upon the nature of the good, the distribution of individual preferences, the tax schedule, and so forth. The situation is even more complicated when a number of communities are involved. F. Westoff, in a Yale Ph.D. dissertation in progress (supervised by Brainard, Brown, and Stiglitz), has been exploring the application of voting theory to the “ Tiebout” local public goods problem. Consider a society composed of a number (possibly variable) of small communities, with individuals free to move from one community to another, and with the level of public goods provided in each community determined by majority vote of the residents of that community. The existence of an equilibrium allocation of individuals to communities and set of levels of public goods is not obvious in such a system, even if individual preferences satisfy the “ single peakedness” assumption. Westoff has shown the existence of an equilibrium under reasonably general conditions, and is currently exploring the question of efficiency in this context.

In a joint research project, Kleverick and Professor A. B. Atkinson of the University of Essex use voting theory to study the social choice of rules for achieving distributive justice. They envision a representative constitutional assembly whose function is to select the principle that the society’s government shall follow in designing its income redistribution policy. In one case the government’s instruments are limited to a linear income tax and in another case to lump-sum taxation. They also restrict the menu of possible principles before the assembly to three: average utilitarianism, a Rawlsian maximin policy, and laissez faire nonintervention. Individuals
in the society are all assumed to have the same utility as a function of consumption and leisure but to differ in their ability levels, which in turn determine their respective wage rates. Finally, Atkinson and Kleverick consider several alternatives with respect to the information each individual has about his own ability level—perfect knowledge, knowledge only about the distribution of ability levels in the society as a whole, and an intermediate case of imperfect knowledge. Individuals are assumed to act to further their own interests, as represented by maximizing utility or (in the case of limited knowledge) expected utility. Given the knowledge people have about their position, conflicts of interest arise about which principle the government should use in designing the tax structure, and these conflicts are assumed to be resolved by a voting rule (for example, simple majority rule) in the assembly. The research in progress examines what principle is adopted by the assembly under alternative conditions concerning, for example, the distribution of ability levels in society, the degree of information people have about their own ability levels and the degree of risk aversion people display.

In CFDP 333, Nordhaus has examined the behavior of a political voting mechanism in dealing with economic stabilization. Starting with a model of political behavior developed earlier by Kramer in CFP 344, Nordhaus added an economic model postulating a dynamic relationship between unemployment and inflation. Two basic theoretical results were demonstrated in the paper. First, it is shown that as a result of the dynamics of the economic and political system the long-run equilibrium in such a model will have a lower unemployment rate and higher inflation rate than would be determined by a conventional social welfare function. Second, it was shown that economic policy within the period of incumbency will display a marked cyclical effect. It is optimal from a political point of view to deflate the economy in the early stages of incumbency and to expand in the latter stages.

In an earlier paper (CFP 344) Kramer investigated econometrically the relationship between aggregate economic fluctuations and election outcomes in the United States. Kramer and Lepper subsequently ("Congressional Elections") have pursued these questions at a more disaggregated level. Most recently, in CFDP 341, Lepper has extended this line of research both theoretically, clarifying the micro-behavioral assumptions underlying the specification of the earlier econometric models, and empirically, using a variety of econometric techniques.

2. Legal Policy and Regulation. Kleverick has continued his research on the behavior of regulated public utility firms. In "The Behavior of a
Firm Subject to Stochastic Regulatory Review” (CFP 393) he proceeded with his efforts to provide a more realistic model of the regulatory process than the standard Averch-Johnson model. In particular, the model Klevorick proposes in this paper considers the firm's operations in a dynamic context (with the firm looking to the future in making today's decisions), and it incorporates some of the interplay between the regulatory agency and the firm. The model captures the price-setting role of the regulators, and it encompasses the phenomenon of regulatory lag. Rate reviews are assumed to occur stochastically through time, and the model also incorporates technical change generated by the firm's program of research and development. The regulated firm's optimal policy is characterized, and the implications of this policy for two traditional issues in regulatory economics—the input efficiency of firms and the effect of regulatory lag on research and development—are examined.

Klevorick's research suggests the need for a re-examination of the economic and legal rationales offered for regulation and a more precise statement of our societal goals in controlling public utilities. Klevorick has also begun some interdisciplinary research bridging the fields of economics and law. He is concerned with the possible contribution of economics to our understanding of several issues in constitutional law, specifically, the "new" equal protection doctrine and freedom of speech. In the case of the former, he is focusing on the economic basis, if any, for the central concept of a "fundamental interest" (analogous to the concept of a "merit want" suggested by Musgrave). With respect to the freedom of speech issue, he is attempting to use economic theory to explore what legal and policy implications flow from the justification of freedom of speech which rests on the economic metaphor of "the market place of ideas."

3. Taxation and Public Expenditure. The primary emphasis of work discussed in Section 1 above is on the mechanism of social choice rather than the economic consequences of the choices themselves. Various members of the staff have also been active in examining, both theoretically and empirically, the distributional and efficiency consequences of government taxation and expenditure policies.

The investigation of efficient taxation is a continuing problem of general research interest. Recently, Samuelson, Diamond, and Mirrlees have returned to the classic work of Ramsey (1928) and Boiteux (1943) and placed it in a modern setting. Ramsey argued that in the case of independent demand curves, efficiency requires that commodity taxes be proportional to the sum of the reciprocal of the demand and supply elasticities; Diamond and Mirrlees, on the other hand, derive tax formulae depending
only on demand elasticities. In CFP 352, Stiglitz and Dasgupta show both to be special cases of a more general formulation: Ramsey implicitly assuming that rents cannot be taxed, and Diamond and Mirrlees implicitly assuming either no rents or 100% rent taxes. More generally, the optimal tax formulae depend on the set of restrictions which are imposed on the class of admissible taxes, and whether there is an overall budget constraint on the government.

Further questions of optimal tax structure in the case of additive utility functions are discussed by Atkinson and Stiglitz in CFP 367. In this paper it is noted that the Ramsey rule alluded to above assumes implicitly a constant marginal utility of leisure. On the other hand, if labor is completely inelastically supplied, it is well known that a uniform tax on all commodities (i.e., a tax on labor income alone) is optimal. This suggests that more generally, the optimal tax formulae might be written as a weighted “average” of the Ramsey rates and a set of uniform rates, with the weights related to the elasticity of the marginal utility of leisure. In the special case of additive utility functions this in fact turns out to be the case.

Although the “optimal tax structure” obviously involves less deadweight loss than a simple income tax, it is also likely to be administratively more complex, and the desirability of adopting it depends at least in part on the magnitude of the resource savings. Using estimated demand functions, Stiglitz and Atkinson are attempting to derive optimal tax rates and to estimate the welfare gains involved in switching to the optimal tax structure.

Stiglitz has also applied the analysis of efficient taxation to the specific problem of the taxation of risky assets in a general equilibrium context in CFPD 305. The question posed is whether there should be differential treatment of risky and safe securities. The answer is shown to depend upon the source of uncertainty, the objectives of government policy, and the effects of taxation on the financial structure of firms. In CFP 383, the effects of taxation on financial structure and on the cost of capital are explored further. It is shown that under certain circumstances (for example, when the corporate tax rate is less than the personal income tax rate) the combined effects of the corporate profits tax, the special treatment of capital gains and the provisions for interest deductibility need not create a divergence between the marginal cost of capital in the incorporated and unincorporated sectors.

Work has also continued on the distributive as well as the efficiency aspects of taxation. The negative income tax is perhaps the most widely discussed of redistribution schemes. As indicated in the last two research
reports, there is continuing interest in the design and analysis of such plans. Most recently, Peck and Harold W. Watts (Irving Fisher Professor of Economics, 1972-73) have evaluated a variety of linear negative income tax plans in a paper, "On the Comparison of Income Redistribution Plans," written for the Institute for Research on Poverty. A variety of plans differing in their marginal tax rates and intercepts were contrasted with the current (1970) tax structure. It was found that substantial amounts of poverty could be alleviated without increased taxation of middle income taxpayers.

Studies analyzing the distribution effects of government taxation or expenditure policy frequently need to make use of summary measures of inequality. A difficulty arises in that different measures often give contrasting estimates of the degree of inequality. Atkinson, in a recent paper, has utilized the conceptual framework developed by Rothschild and Stiglitz (CFP 341) in their analysis of risk to provide measures of income inequality. In CFDP 344, Rothschild and Stiglitz present an axiomatic formulation of the ordering under which one distribution can be said to be more unequal than another and explore the implications of this information for summary measures of inequality.

Theoretical and empirical studies at Cowles have pursued a "descriptive" analysis of several questions related to taxation. An example of a theoretical study is Mieszkowski's work (CFDP 304), which investigated the conditions under which the property tax is essentially an excise tax and those under which it is essentially a profits tax. At the empirical level, Shoven and Whalley have used Scarf's algorithm to consider the distributional and the efficiency implications of the distortionary taxation of income from capital in the U. S. using a two-sector general equilibrium model. Their estimate of the efficiency loss differs substantially from Harberger's earlier results.*

A second application of Scarf's algorithm is John Whalley's examination of proposals for tax changes in the United Kingdom. In April 1973 the U.K. abolished the purchase tax and selective employment tax and replaced these by a value-added tax; in addition, changes in the systems and rates of corporation tax and personal income tax were introduced. A competitive model of the U.K. economy has been developed with industrial divisions chosen so as to capture the major discriminatory features of these taxation arrangements. This model is used to examine equilibrium solu-

tions for the economy under these alternative tax changes. It is found that
gains to the economy from these changes are small if not negative in spite
of official expectations to the contrary. This is in contrast to large potential
benefits from the replacement of all discriminatory aspects of the tax sys-
tem with a broadly based value-added tax of perhaps 2-3%. Impacts on
the personal and functional distributions of income, and on the balance
of payments are also considered.

The foregoing studies are primarily concerned with the structure of
the tax system. Two ongoing studies focus on the interaction between the
level of government expenditures and the system of taxation used to raise
revenue for it.

Lepper has recently been analyzing variations in local expenditures for
education across communities. Preliminary analysis of data for 130 of the
169 towns in Connecticut was reported in "Fiscal Capacity and the De-
mand for Public Expenditures—With Special Reference to Education,”
presented at the Winter 1972 Meetings of the Econometric Society. In
the towns considered, local financing of public primary and secondary ed-
ucation is almost exclusively dependent on the local property tax, and per-
pupil expenditures vary directly with the size of the local property tax
base per pupil. It is found, however, that a higher property tax base
which is associated with higher family income makes a distinctly larger
positive contribution to per pupil expenditures for education than a higher
property tax base resulting from greater commercial and industrial activity.
Lepper intends to extend this analysis in a number of directions: (a)
further refining in statistical technique, (b) focusing more attention on
the role of preferences for private versus public education, and (c) con-
sidering the relative importance of private affluence and business wealth
on local expenditures other than education. Lepper is also considering the
usefulness of the notion of "equity-constrained” Pareto-optimal alloca-
tions for private goods and public services. One type of equity constraint
is that each consumer receive the same amount of benefits from the public
service. Another is that consumers who differ in ability to derive benefits
from the public service receive equal allocations of public resources.

On the theoretical side Stiglitz and Dasgupta have analyzed the conse-
quences of distortionary taxation on the optimal level of expenditure on
public goods. In CFDP 352 they established that the Samuelson rule for
the optimal supply of public goods (the sum of the marginal rates of sub-
stitution equals the marginal rate of transformation) is not valid with
distortionary taxation and have developed an alternative rule. An analysis
of the implications of this alternative rule has begun, but at this juncture,
it is not clear whether the Samuelson rule usually leads to an under or oversupply of public goods.

A related "second best" problem which arises when market prices do not correctly reflect social costs is the specification of a criterion to be used for the selection of government projects. In the absence of restrictions on the type of taxation Diamond and Mirrlees argued that the government's prices should be the same as the producer's prices in the private sector, even when there are strong redistributive objectives. In contrast, Stiglitz and Dasgupta (CFP 352) have established that with plausible restrictions, e.g., that pure rents are taxed at less than 100% or that the government obeys a budget constraint, the two sets of prices should systematically differ.

A quite distinct set of problems in the public sector relate to the management of common resources such as land, fishing grounds, or highways. For such resources a free access equilibrium is inefficient; average products rather than marginal products of the variable factor tend to be equated. Weitzman (CFDP 323) has proposed a formal model to characterize and compare the allocations of resources which occur under conditions of free access and of private property ownership.
GUESTS

The Cowles Foundation is pleased to have as guests scholars and advanced students from other research centers in this country and abroad. Their presence contributes stimulation and criticism to the work of the staff and aids in spreading the results of its research. The Foundation has accorded office, library, and other research facilities to the following guests who were in residence for various periods of time during the past three years.

RONALD G. BODKIN, The University of Western Ontario.
KåRE HAGEN, Norwegian School of Economics and Business Administration.
HARRY JOHNSON, University of Chicago and London School of Economics.
HAYNE E. LELAND, Stanford University.
TAMAS NAGY, Hungarian Academy of Sciences.
J. KIRKER STEPHENS, University of Oklahoma.
KNUT SYDSAETER, University of Oslo.
EDUARDAS VILKAS, Institute of Physics and Mathematics of Lithuanian Academy of Sciences.
COWLES FOUNDATION SEMINARS
AND CONFERENCES

Seminars

In addition to periodic Cowles Foundation staff meetings, at which
members of the staff discuss research in progress or nearing completion,
the Foundation also sponsors a series of Cowles Foundation Seminars con-
ducted by colleagues from other universities or elsewhere in Yale. These
speakers usually discuss recent results of their research on quantitative sub-
jects and methods. All interested members of the Yale community are
invited to these Cowles Foundation Seminars, which are frequently ad-
dressed to the general economist including interested graduate students.
The following seminars occurred during the past three years.

July 1, 1970–June 30, 1973

1970

October 23. DANIEL McFADDEN, M.I.T., "Revealed Stochastic Technology."

November 6. EYTAN SHESHINSKI, Harvard University, "Optimal Government
Production and Inflation."

November 13. TERRENCE GORMAN, University of North Carolina and London
School of Economics, "Aggregates for Variable Goods: An Application of
Duality."

December 4. HUGO SONNENSCHEIN, University of Massachusetts, "Three Prob-
lems in General Equilibrium and Welfare Economics."

1971

February 5. ROBERT SOLow, M.I.T., "Land Use in a Long Narrow City."

February 26. ROBERT E. LUCAS, Jr., Carnegie-Mellon University, "Cross Sec-
tion Tests of the Natural Rate Hypothesis."

March 5. MICHAEL BRUNo, M.I.T., "Disequilibrium Growth in an Open Eco-

nomy."

April 2. RICHARD D. PORTES, Princeton University, "A Quantity-Guided Decen-
tralized Planning Procedure."

April 16. CHRISTOPHER SIMS, National Bureau of Economic Research, "Money,
Income and Causality."

April 23. WITOLD TRZECIAKOWSKI, Warsaw, Poland, "The Application of
Short-Run Optimization Models in Foreign Trade Planning and Management
in Poland."

May 14. RICHARD ROSETT, University of Rochester, "The Effect of Health In-
surance on the Demand for Medical Care."

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October 27. Stephen Goldfield, Princeton University, "Econometric Model Selection in the Presence of Repeated Structural Change."

November 12. Peter Diamond, M.I.T., "Aggregate Production with Consumption Externalities."


1972


March 10. Sidney Winter, University of Michigan, "Simulation of Technical Change in an Evolutionary Model."

March 17. Wladyslaw Wlefe, University of Pennsylvania, "Medium-Term Econometric Models of Poland, Czechoslovakia and Hungary: Goals, Specification and Empirical Results."

April 14. Albert Madansky, City College of New York, "Improved Instrumental Variable Estimators."

April 21. Arthur Goldberg, University of Wisconsin, "Unobservable Variables."


May 12. Mordecai Kurz, Stanford University, "Equilibrium with Transaction Cost and Money."


October 27. Janos Kornai, Institute of Economics, Hungarian Academy of Sciences, "Interemporal Aspects of Hungarian Long-Term Planning."


November 17. Alan Manne, Stanford University, "Electricity Investments Under Uncertainty About the Date of Breeder Availability."

December 15. Truman Bewley, Harvard University, "Preliminary Work on a Dynamic Central Market Model."

1973

January 5. Michael Spence, Harvard University, "Market Signaling."

February 2. George Brown, Center of Naval Operations, "Some Problems Involving Disturbance-Variance Systems."


April 6. Jerzy Los, Computer Center of the Polish Academy of Sciences, "Recent Polish Work in the Field of von Neumann Models."


May 11. David Hoaglin, Harvard University, "Exploring Some Unemployment Data."

Conferences

The Cowles Foundation was also the host for a conference on Energy Sector Modeling, November 17 and 18, 1972.
FINANCING AND OPERATION OF THE COWLES FOUNDATION

The Cowles Foundation relies largely on gifts, grants and contracts to finance its research activities. Yale University contributes to the Cowles Foundation the use of a building at 30 Hillhouse Avenue which provides office space, a seminar room, and related facilities. The University also supports the Foundation's research and administration through paying or guaranteeing part or all of the non-teaching fractions of the salaries of three permanent staff members.

The annual gifts of the Cowles family are the cornerstone of the financial support of the Cowles Foundation. Two endowment funds were established at Yale during the past three years which also provide funds for the Foundation. The Cowles family initiated an endowment in 1970, the income from which provides additional general support. In 1972, the Marcus Goodbody Foundation made a gift to Yale from which another endowment fund was established. Income from this fund contributes to the research salary and expenses of a member of the Cowles Foundation staff designated as the Marcus Goodbody Fellow. These three sources provide dependable untied funds permitting a degree of intellectual and administrative flexibility which is extremely useful for any organization engaged in basic research.

The scale of activity at Cowles is dependent on the availability of a substantial amount of financial support from grants and contracts. During the period covered by this report, the Cowles Foundation has been fortunate in having sizeable grants from the National Science and Ford Foundations which are not tied to specific research projects, and has continued to receive support from the Office of Naval Research which has financed work at Cowles on operations research and game theory since the late 1940's. The National Science Foundation grant was a 'continuing' grant providing annual funding for the period July 1968 through June 1973. Additional funds for support of the general program of the Cowles Foundation and for a program of visiting staff members were generously provided by the Ford Foundation for the period 1968-1976. This Ford visitors program is intended specially to facilitate visits by Eastern European economists and also by scholars in disciplines other than economics but related to interests of Cowles Foundation staff. These guests are regular members of the Cowles Foundation staff for the period of their stay—generally four months or longer.

The major part of Cowles Foundation expenditures is accounted for by
research salaries (and associated fringe benefits). The rest of the budget consists of office and library salaries, overhead expenses such as the costs of preparing and distributing manuscripts, and the costs of computing services.

The pattern of Cowles Foundation income and expenditures in recent years is outlined in the table below:

### ANNUAL INCOME AND EXPENDITURES OF THE COWLES FOUNDATION

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<tr>
<th>Average for</th>
<th>INCOME</th>
<th>EXPENDITURES</th>
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<td></td>
<td>Permanent</td>
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<td>Cowles Family</td>
<td>including project support</td>
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<td>Gifts, and</td>
<td>Total</td>
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<tr>
<td></td>
<td>Endowments</td>
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<td>1961-64</td>
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<td>58</td>
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<td></td>
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</tbody>
</table>

During the period of this report, the research staff of the Cowles Foundation included 18 or 19 members in faculty ranks. This size was determined by an interplay of considerations including financial constraints, limitations of space at 30 Hillhouse Avenue, and opportunities to bring to the Foundation colleagues who would complement or supplement current research activities. The balance among ranks of the staff in residence varied from year to year depending largely upon leaves of absence and the opportunities to compensate for such absences by visiting appointments. Excluding staff members on such visiting appointments, the staff included seven tenured faculty of the Departments of Economics, Administrative Science and Political Science and 8 to 10 faculty on term appointments. On average, both the permanent and the younger members of the staff devoted about half of their professional effort in the academic year and up to two full months in the summer to their research and to seminars and discussions with their colleagues.
These activities were supported by the services of five secretaries and manuscript typists under the direction of Miss Althea Strauss. In addition to the office staff, a varying number of student research assistants and part-time computer programmers assisted directly in the research studies.

A small library, under the supervision of Patricia Graczyk, is maintained in the building of the Cowles Foundation. It makes research materials readily available to the staff and supplements the technical economics and statistics collections of other libraries on the Yale campus. The library includes a permanent collection of some 5,400 books and 158 journals primarily in the fields of general economics, mathematical economics, econometric studies and methods, statistical methods and data; numerous pamphlets from Government sources and international organizations; series of reprints from 22 research organizations at other universities in the United States and abroad; and a rotating collection of recent unpublished working papers. Although the library is oriented primarily to the needs of the staff, it is also used by other members of the Yale faculty and by students of the University.
MONOGRAPHS
1934–1973*

The monographs of the Cowles Commission (Nos. 1-15) and Cowles Foundation (Nos. 16-24) are listed below:


No. 4. Silver Money, by DICKSON H. LEAVENS. 1939. Evanston, Ill.: Principia Press. 459 pages. (Out of print.)

No. 5. The Variate Difference Method, by GERHARD TINTNER. 1940. Evanston, Ill.: Principia Press. 175 pages. (Out of print.)


No. 8. Price Flexibility and Employment, by OSCAR LANGE. 1944. Evanston, Ill.: Principia Press. 114 pages. A clarification of important concepts that have had much currency in the practical discussion of depression and of economic stabilization generally but had yet to be formalized in a complete theoretical structure.


*Orders for Monographs 12, 13, 14, 16, 17, 21, 22, 23, and 24 should be sent to Yale University Press, 92A Yale Station, New Haven, Connecticut 06520. Orders for, or inquiries concerning, all other Monographs should be sent to the Cowles Foundation for Research in Economics at Yale University, Box 2125 Yale Station, New Haven, Connecticut 06520.


No. 20. *Studies in Portfolio Behavior*, edited by DONALD D. HESTER and JAMES TOBIN. 1967. New York: John Wiley and Sons. 258 pages. Six essays applying the theoretical development of Monograph 19 to the circumstances and objectives of various economic units such as households, nonfinancial corporations, banks and life insurance companies.

No. 21. *Financial Markets and Economic Activity*, edited by DONALD D. HESTER and JAMES TOBIN. 1967. New Haven: Yale University Press. 256 pages. These essays are concerned with the conditions of general equilibrium in economy-wide financial markets. The micro-economic principles discussed in Monographs 19 and 20 are assumed to guide the behavior of economic units in supplying assets and debts in these markets.

No. 22. *Economic Theory of Teams*, by JACOB MARSHAK and ROY RADNER. 1971. New Haven: Yale University Press. This monograph emphasizes the informational aspect of the problem of designing efficient organizations. After an introduction to decision-making under uncertainty and to the economics of information, a wide variety of models is treated within a unifying conceptual framework.

No. 23. *Efficient Estimation with A Priori Information*, by THOMAS J. ROTHEMBAUGH. 1973. New Haven: Yale University Press. A unified theory of estimation in the presence of prior information is developed. How valuable is prior information in increasing the precision of parameter estimation and what are effi-
cient methods of incorporating this information into estimation procedures are the two basic questions investigated.

No. 24. The Computation of Economic Equilibria, by Herbert Scarf. 1973. New Haven: Yale University Press. The first general method for the explicit numerical solution of the price system and economic equilibrium is presented. An important connection between computational methods and economic theory is made which promises to be of use as a practical tool for the evaluation of economic policy.

Forthcoming (now at press):

Bank Management and Portfolio Behavior, by Donald D. Hester and James L. Pierce.
COWLES FOUNDATION PAPERS

July 1, 1970–June 30, 1973


No. 337. WILLIAM J. BAUMOL and ALVIN K. KLEVORICK, “Input Choices and Rate-of-Return Regulation: An Overview of the Discussion,” The Bell Journal


No. 296  WM. D. NORDHAUS, Recent Developments in Price Dynamics.
297  FUMINASA HAMADA, An Econometric Model of Postwar Japan.
298  E. J. HANNAF, Non-Linear Time Series Regression.
300  ROSS M. STARR, Equilibrium and Demand for Media of Exchange in a Pure Exchange Economy with Transactions Costs.
303  J. E. STIGLITZ, The Badly-Behaved Economy with the Well-Behaved Production Function.
304  P. M. MIESZKOWSKI, The Property Tax: An Excise Tax or a Profit Tax?
305  J. E. STIGLITZ, Taxation, Risk-Taking, and the Allocation of Investment in a Competitive Economy.
306  J. E. STIGLITZ, On Transitions Between Steady States.
307  THOMAS KING and P. MIESZKOWSKI, An Estimate of Racial Discrimination in Rental Housing.
308  D. J. BROWN and A. ROBINSON, Nonstandard Exchange Economics.
309  J. TOBIN, Friedman's Theoretical Framework.
310  ROSS M. STARR, The Price of Money in a Pure Exchange Monetary Economy.
311  WALTER DOLDE and JAMES TOBIN, Wealth, Liquidity, and Consumption.
312  WITOLD TRZECIAKOWSKI, Systems of Indirect Management in a Planned Economy: Effectiveness Models and Their Applications in Poland.
313  TERJE HANSEN and T. C. KOOPMANS, Definition and Computation of a Capital Stock Invariant Under Optimization.
314  JAMES TOBIN, Wealth Liquidity and the Propensity to Consume.
315  JAMES TOBIN, Summary Comment. (From FRB Price Econometrics Conference)
316  M. SHUBIK, Competitive Equilibrium and Game Theory Solutions, Part I: The Core and Value.
318  M. SHUBIK, On the Scope of Gaming.
319  WM. D. NORDHAUS and JAMES TOBIN, Is Growth Obsolete?
321  D. A. NICHOLS, A New Measure of Income from Wealth.

J. PECK, A Comparison of Alternative Estimators for a Dynamic Relationship Estimated From a Time Series of Cross-Sections When the Disturbances are Small.

D. J. BROWN and A. ROBINSON, The Cores of Large Standard Exchange Economies.

R. G. BODDIN, The Marginal Revolution After One Hundred Years.

J. E. SHOVEN and J. WHALLEY, A General Equilibrium Calculation of the Effects of Differential Taxation of Income From Capital in the U. S.


R. STARR, Notes on Transactions Costs and the Analysis of Microeconomic Monetary Theory.


M. SHUBIK and L. SHAPLEY, A Theory of Money and Financial Institutions, Part VI: The Rate of Interest, Noncooperative Equilibrium and Bankruptcy.

J. E. STIGLITZ, Alternative Theories of Wage Determination and Unemployment in L.D.C.'s: I. The Labor Turnover Model.


M. SHUBIK, A Theory of Money and Financial Institutions, Part V: The Rate of Interest on Fiat Money in a Closed Economy.

J. E. STIGLITZ, On the Irrelevance of Corporate Financial Policy.


S. J. LEPPER, Voting Behavior and Aggregate Policy Targets.


G. KRAMER and A. KLEVORICK, Existence of a 'Local' Cooperative Equilibrium in a Class of Voting Games.

J. STIGLITZ and M. ROTHSCCHILD, Some Further Results on the Measurement of Inequality.


J. STEPHENS, Input Quality, Obsolescence and Unemployment.

M. SHUBIK, Information, Duopoly and Competitive Markets; a Sensitivity Analysis.

J. TOBIN, Technological Development and Employment.


G. BRADLEY and M. SHUBIK, A Note on the Shape of the Pareto Optimal Surface.

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351 M. SHUBIK, Game Theory and Political Science.
352 M. SHUBIK, Voting, Preference Orderings and Cooperative Games.
353 J. STIGLITZ, Incentives and Risk-Sharing in Sharecropping.
354 J. STIGLITZ, The Theory of 'Screening,' Education and the Distribution of Income.
355 M. SHUBIK and W. WHITT, Fiat Money in an Economy with one Non-durable Good and No Credit (A Noncooperative Sequential Game).
356 T. KOOPMANS, Some Observations on 'Optimal' Economic Growth and Exhaustible Resources.
357 J. STIGLITZ, Alternative Theories of Wage Determination and Unemployment in L.D.C.'s: II. The Efficiency Wage Model.
358 N. MOISEEV and A. SCHMIDT, Some Problems of Centralized Economy.
359 M. SHUBIK, The General Equilibrium Model with Joint Ownership of the Corporation (Voting Stock and the Core).
360 D. BROWN, Acyclic Choice.
361 H. LELAND, Production Theory and The Stock Market.
PUBLICATIONS AND PAPERS BY STAFF MEMBERS
July 1, 1970–June 30, 1973*

*Contains papers which were published during the period and resulted from work at the Cowles Foundation, papers published while the author was a staff member, and a few other papers referred to in the text of the Report.

BISCHOFF, CHARLES W.
Papers: CFP 345, 372
Other Publications:

BRAINARD, WILLIAM
Papers: CFP 350
Other Publications:

BROWN, DONALD J.
Discussion Papers: CFDP 308, 326, 342, 360
Other Publications:

GRETHE, DAVID M.
Papers: CFP 339, 373

HAKANSSON, NILS H.
Papers: CFP 338

HANNAN, E. J.
Papers: CFP 380

KADANE, J.
Papers: CFP 364

KLEWORICK, ALVIN K.
Papers: CFP 345, 356, 387, 393
Discussion Papers: CFDP 343, 345
Other Publications:

KOOPMANS, TJALLING C.
Papers: CFP 353, 357, 366, 375, 396
Discussion Papers: CFDP 313, 356
Other Publications:
A. "If $f(x) + g(y)$ is Quasi-convex at least One of $f(x), g(y)$ is Con-
vex", presented at the Symposium on Mathematical Methods of Economics organized by the Mathematical Institute of the Academy of Sciences, Warsaw, Poland, July 1972.

**KRAMER, GERALD**

Papers: CFP 344, 387, 397
Discussion Papers: 343, 345

Other Publications:


**LEPPER, SUSAN J.**

Discussion Papers: CFDP 341

Other Publications:


**MADDALA, G. S.**

Papers: CFP 373

**MIESZKOWSKI, P. M.**

Discussion Papers: CFDP 307

**MOISEEV, N. N. and A. G. SCHMIDT**

Discussion Papers: CFDP 358

**NICHOLS, D. F.**

Papers: CFP 380

Discussion Papers: CFDP 321

**NORDHAUS, WILLIAM**

Papers: CFP 368, 371, 374, 377, 382, 386, 388, 398, 399

Discussion Papers: CFDP 319, 329, 333

**PECK, JON K.**

Discussion Papers: CFDP 325

**SCARF, HERBERT E.**

Papers: CFP 358, 369

Other Publications:

Cowles Foundation Monograph 24, *The Computation of Economic Equilibria* (with the collaboration of Terje Hansen).

**SHUBIK, MARTIN**


Other Publications:


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**STARR, ROSS M.**
Papers: CFP 365, 381
Discussion Papers: CFDP 310, 332, 349

Other Publications:

**STIGLITZ, JOSEPH**
Papers: CFP 341, 352, 362, 367, 376, 383, 384, 390, 394, 395
Other Publications:

TORIN, JAMES
Papers: CFP 360, 361, 370, 378, 398
Discussion Papers: CFDP 309, 311, 314, 315, 319, 348
Other Publications:

TRZECIAKOWSKI, WITOLD
Discussion Papers: CFDP 312

WEITZMAN, M. L.
Papers: CFP 347, 349, 385
Discussion Papers: CFDP 317, 323