REPORT of
RESEARCH ACTIVITIES

July 1, 1967 – June 30, 1970

COWLES FOUNDATION
FOR RESEARCH IN ECONOMICS
AT YALE UNIVERSITY
COWLES FOUNDATION FOR RESEARCH IN ECONOMICS AT YALE UNIVERSITY

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*As of June 30, 1970.
**During a part of or the entire period July 1, 1967–June 30, 1970.
***Active during the period of the report.

Note: For a list of GUESTS see page 39.

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PURPOSE AND ORIGIN

The Cowles Foundation for Research in Economics at Yale University, established as an activity of the Department of Economics in 1955, has as its purpose the conduct and encouragement of research in economics, finance, commerce, industry, and technology, including problems of the organization of these activities. The Cowles Foundation seeks to foster the development of logical, mathematical, and statistical methods of analysis for application in economics and related social sciences. The professional research staff are, as a rule, faculty members with appointments and teaching responsibilities in the Department of Economics and other departments.

The Cowles Foundation continues the work of the Cowles Commission for Research in Economics, founded in 1932 by Alfred Cowles at Colorado Springs, Colorado. The Commission moved to Chicago in 1939 and was affiliated with the University of Chicago until 1955. In 1955 the professional research staff of the Commission accepted appointments at Yale and, along with other members of the Yale Department of Economics, formed the research staff of the newly established Cowles Foundation.
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NOTE ON REFERENCES TO PUBLICATIONS

The following abbreviations are used throughout this report in referring to publications or working papers of the Cowles Foundation:

CFP: Cowles Foundation Papers (see p. 48)
CFDP: Cowles Foundation Discussion Papers (see p. 52)

Monographs (see p. 46) are referred to by number.

Other publications of each staff member are designated by letter in the list on pp. 55-59 and are referred to by author and title in the text.
The Cowles Commission for Research in Economics was founded approximately forty years ago by Alfred Cowles, in collaboration with a group of economists and mathematicians concerned with the application of quantitative techniques to economics and the related social sciences. This methodological interest was continued with remarkable persistence during the early phase at Colorado Springs, then at the University of Chicago, and since 1955 at Yale.

One of the major interests at Colorado Springs was in the analysis of economic data by statistical methods of greater power and refinement than those previously used in economics; this was motivated largely by a desire to understand the chaotic behavior of certain aspects of the American economy—the stock market in particular—during the Depression years. The interest in statistical methodology was continued during the Chicago period with a growing appreciation of the unique character and difficulties of statistical problems arising in economics. An important use of this work was made in the description of the dynamic characteristics of the U.S. economy by a system of statistically estimated equations.

At the same time, the econometric work at Chicago was accompanied by the development of a second group of interests—also explicitly mathematical but more closely connected with economic theory. The activity analysis formulation of production, and its relationship to the expanding body of techniques in linear programming, became a major focus of research. The Walrasian model of competitive behavior was examined with a new generality and precision, in the midst of an increased concern with the study of interdependent economic units, and in the context of a modern reformulation of welfare theory.

The move to Yale in 1955 coincided with a renewed emphasis on empirical applications in a variety of fields. The problems of economic growth, the behavior of financial intermediaries, and the embedding of monetary theory in a general equilibrium formulation of asset markets, were studied both theoretically and with a concern for the implications of the theory for economic policy. Earlier work on activity analysis and the general equi-
librium model was extended, with a view to eventual applications to the comparative study of economic systems and to economic planning at a national level. Along with the profession at large, we have also seen in recent years a greater interest in the specifics of income distribution, in the analysis of the effects of discrimination, and in the development of analytical methods oriented to contemporary social and economic problems.

During the three year period covered by this report, as in the past, the Cowles Foundation staff has exhibited a variety of individual research interests and methodological orientations. The composition of the staff has changed gradually over the years, and its intellectual activities have been stimulated by a substantial number of distinguished visitors from the United States and abroad. Yet, despite this diversity, the broad themes outlined above have continued to characterize current research interests. In the report that follows, an effort has been made to relate particular papers and studies of the last three years to these general topics, though no pattern of organization can do complete justice to all the work that is summarized.

I. General Equilibrium Models and Game Theory

The Walrasian model of economic equilibrium under perfect competition exemplifies one of the central themes of economic theory: the equilibrium arising from the interaction of a variety of agents (persons or organizations) with differing economic motivations. As distinct from the more general formulations of n person game theory, the Walrasian model is quite specific about the nature of the agents and the strategies which are available to them. The producing units of the economy are assumed to be fully aware of the technological possibilities of production, and of the sequence of factor and output prices which are expected to prevail at all moments of time in the future. Production and investment decisions are motivated exclusively by the desire to maximize the present value of the stream of profits flowing from these decisions.

The remaining agents in the economy are consuming units or households, each of which derives its income from the sale of privately owned productive factors—including labor—and allocates that income or wealth among the many categories of consumer goods and saving by maximizing some indicator of utility. The decentralized production and consumption decisions are linked together only by a knowledge of prevailing and expected prices, and are in equilibrium with each other if these prices produce an equality of supply and demand in all markets.

The Cowles Foundation has been associated for many years with the modern development of the general Walrasian model. One of the major research achievements, summarized in Debreu's fundamental monograph,
Theory of Value (Cowles Foundation Monograph No. 17), has been to establish the mathematical theorem that equilibrium prices do indeed exist for a model of this complexity and generality.

While there are special examples of Walrasian models in which the existence of equilibrium prices can be demonstrated by elementary arguments of a geometric or algebraic character, the difficulties of the general problem have required the introduction of complex and abstract mathematical techniques. An older generation of economists may have been satisfied by the observation that there are as many equations as there are unknown equilibrium prices, but this is by no means a convincing argument for the existence of appropriate solutions. Nor can an argument be given by formalizing the conventional intuition that the price of a commodity will rise if its demand exceeds supply and fall otherwise. There is no difficulty in constructing acceptable demand functions for which this sequence of prices, adjusted in response to the discrepancy between supply and demand, fails to converge to an equilibrium system of prices. The use of abstract mathematical tools, such as "fixed point" theorems, is fully justified by the intrinsic difficulty of the subject.

But however satisfying it may be from an intellectual point of view to demonstrate the internal consistency of the Walrasian model by means of an existence theorem, such results are of little use in applying the competitive formulation to specific policy questions. In order to discuss the influence of tariff changes in a multi-sector model of international trade—to take only one example—it is necessary to be able to solve explicitly for equilibrium price and production decisions, rather than merely asserting the existence of prices which equilibrate supply and demand.

In a series of papers (CFP's 262, 271, 277, 308, and CFDP 272), Scarf and Terje Hansen have developed a new class of numerical algorithms which have, as one of their applications, the determination of approximate equilibrium prices and production decisions for a general Walrasian model. The algorithm has been tried on a large number of specific though artificial examples in order to estimate the number of elementary iterations and total computation time required to solve problems of a given size. The time seems to increase roughly as the square of the number of commodity sectors; problems involving twenty or fewer sectors can be done with considerable accuracy in less than 10 minutes of computing time on an IBM 7094. This is considerably less efficient than the simplex method for a linear programming problem of comparable size—but it must be stressed that the determination of equilibrium prices is a problem to which the conventional methods of linear and non-linear programming cannot be applied.

At present several experimental applications of these computational
techniques have either been initiated or are being contemplated. These include a study of the implications of various tariff proposals in an international trade model; the determination of equilibrium prices for a centrally planned economy; and the redistributive aspects of certain welfare proposals such as a negative income tax plan.

The details of the algorithm may be found in the series of papers previously referred to. It should be remarked that this class of algorithms can be applied to a number of problems other than that of calculating equilibrium price and production levels. An appropriate variant of the algorithm can be used to approximate a "fixed point" of an arbitrary continuous mapping of a closed, bounded, convex set into itself, and another variant can be used—under certain circumstances—to determine a vector in the core of an n person game. As Hansen has shown in CFDP 277, the methods can also be used to solve non-linear programming problems, under customary convexity assumptions, and seem to compare quite favorably—in terms of speed of computation—with alternative numerical procedures such as the gradient method. Hansen and Scarf are engaged in writing a monograph in which this methodology is summarized, along with parallel developments by others including Harold Kuhn, Lloyd Shapley and Curtis Eaves.

For a number of years various authors have been concerned with the specific introduction of monetary phenomena into the Walrasian model. In CFDP 295 Starr has investigated the role of money in avoiding the necessity of coincidence in the wants of pairs of consumers who are engaged in trade, thus making explicit and rigorous Jevon's analysis in Money and the Mechanism of Exchange. In CFDP 300 Starr extends the Walrasian analysis to markets where exchange is a resource using activity and takes place between pairs of traders rather than in the single market for all traders of general equilibrium analysis. This results in a well defined demand for media of exchange. A subsequent paper studies the existence and optimality of equilibrium in a monetary economy, and the role of government policy—especially taxation—in maintaining the value of currency. The argument is made that the government's willingness to accept tax receipts in monetary rather than real form offers some constraints on the value of money which might otherwise be absent from the general equilibrium model.

Demand functions, and the underlying individual preferences from which they are derived, form one of the basic ingredients of the Walrasian model. Traditionally economists, and more recently mathematical political scientists, have assumed that individuals are able to order the alternatives confronting them in a complete and transitive fashion. The completeness
assumption, which requires that an individual be capable of explicit preference—or indifference—between any pair of alternatives, regardless of their dissimilarity, has been criticized as being unnecessarily strong. In a recent paper, "Acyclicity and Choice," Brown is concerned with a relaxation of customary assumptions on individual preferences by requiring merely that every finite set of alternatives contains at least one "maximal" element, i.e., one which is not inferior to any other element of the set. The extent to which traditional demand theory can be modified in the light of this more general approach, is still an open question.

The basic motivation of n person game theory is to provide an analytical setting for economic and political problems, which is free of the specific behavioristic assumptions of the Walrasian model, and therefore capable of addressing some important aspects of economic reality—such as increasing returns to scale, and the public sector—to which the neo-classical model is not applicable. Game theory works with a series of concepts which are logically prior to those of the Walrasian model: rather than assuming that producers and consumers respond passively to competitively determined prices, this analytic framework permits the typical agent to have at his disposal an essentially arbitrary collection of strategies. These may range from the selection of a sequence of votes in a political model to the choice of price and output plans in a study of oligopolistic behavior. All that is required, for a game to be specified in "normal" form, is that each player have a fully delineated set of strategies—not necessarily finite in number—and a systematic procedure for ranking all the outcomes that they may arise from independent choices of strategies by all of the participants.

Of the several procedures which have been suggested for the solution of an n person game, two may be emphasized because of their similarity to traditional methods of analysis in economic theory. The concept of a non-cooperative equilibrium point proposed by Nash, is a direct generalization of Cournot's solution of the oligopoly problem, and is also a basic notion in theories of imperfect competition. According to this method of solution, a selection of strategies, one by each player in the game, is in equilibrium if no player can improve his utility by a unilateral action, assuming no change on the part of the remaining players.

In Cournot's original work each of several firms are engaged in producing the same commodity, under possibly different cost conditions, and each has as its only strategic choice the selection of a level of output. Shubik, in collaboration with Richard Levitan of IBM, has explored (CFDP's 270, 287, 289) a number of variations of this basic model, including the pos-
sibility of the simultaneous selection of price and output levels, capacity constraints on production, and the introduction of uncertainty in demand.

In addition to these theoretical studies, Shubik and Stern (CFDP's 236, 240, 274) have reported on a series of experimental games whose object is to provide statistical evidence about the type of solution procedure actually adopted in specific situations. In these papers the Nash equilibrium point is compared with alternative solutions, for example that selection of strategies which maximizes the sum of the payoffs to the two players. A related contribution by Friedman (CFDP 246) surveys the area of experimental research in oligopoly.

A second major proposal for the solution of n person games is that of the core, a notion which has its roots in Edgeworth’s treatise, "Mathematical Psychics," published in 1886. In order for a cooperative concept, such as the core, to be applicable, the game must be described by specifying for each coalition of players the collection of utility vectors which it can achieve. A utility vector designates a level of utility for each player. It is then in the core if, first, it is achievable by all of the players acting collectively and secondly, no coalition can by itself achieve a higher utility for each of its members.

The successful application of this concept to economics has been largely to those models of production and exchange in which the collection of achievable utility vectors can readily be defined for each coalition. In a model in which each consumer begins the trading period with a stock of commodities and has a utility function for final consumption, the utility vectors achievable by a coalition are most naturally taken to be those arising from an arbitrary redistribution of that coalition’s assets among its members.

But this simplicity disappears if the game is expressed in normal form—in terms of the strategies open to each player—since no coalition of less than all the players will then be able to dictate the outcome of the game, independently of the strategic choices made by the complementary coalition. (This point is illustrated by Shubik [CFDP 288] in a discussion of externalities in production.) An appropriate generalization of the concept of the core to games in normal form was first suggested by Aumann: for him a joint strategy choice is in the core if no coalition can select alternative strategies which guarantee higher utility levels for all of its members, regardless of what the complementary coalition chooses to do.

The distinctions between the Nash equilibrium point, and the core defined in this way are illustrated by Shubik (CFDP 274) using the well-known example of the Prisoner’s Dilemma, a two person non-zero sum
game, with two strategies for each player and with the pair of payoff matrices:

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<td>1 10 0</td>
<td>1 10 11</td>
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<td>2 11 1</td>
<td>2 0 1</td>
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representing the utilities of each player. The Nash equilibrium for this game requires each player to select his second strategy, and thereby results in a pair of utilities distinctly inferior to that which can be achieved by cooperation. On the other hand the utility vector (10, 10) is in the core, according to our definition—no coalition can guarantee a higher utility vector for its members, independently of the actions of the complementary coalition. For example if player one were to attempt a higher utility he would have to move to his second strategy, and would be vulnerable to a corresponding change in player two's strategy.

In CFDP 293, Scarf has given a set of conditions on a general n person game which imply that the core is not empty. Aside from some technical conditions, it is sufficient to require that each player has a convex set of strategies, and that each player's utility function be a quasi-concave function of all of the strategies jointly.

One interesting application of this theorem is to an exchange model with external effects in consumption, so that each individual's utility depends not only on his own vector of consumption, but is also a function of the bundles consumed by some or all of the remaining consumers. If this generalized utility function has indifference surfaces which are convex from above (see figure on page 8), then the theorem of this paper is applicable. We conclude that there is an allocation of society's initial holdings, which cannot be improved upon by any specific redistribution of the assets of any coalition, if the complementary coalition is subsequently permitted to redistribute its own initial assets in an arbitrary fashion.

There is an intimate connection between the core of a market game and the competitive equilibria for the same model as the number of con-
An Indifference Curve for Player 1

sumers tends to infinity. If the appropriate neo-classical assumptions are made, these two modes of analysis yield in the limit precisely the same set of production and distribution plans. This observation has stimulated a considerable body of work in which the number of agents are assumed to be infinite—an idealization which is designed to avoid the technicalities of passing to the limit. In CFDP 258, Kannai provides a set of sufficient conditions for the existence of a non-empty core in such a game.

In a game theoretic formulation, alternative social outcomes are ordered independently by each of the participants, and partially controlled by the selection of appropriate strategies. This may be contrasted with those theories of voting in which strategic choices are downgraded, and replaced by rules such as majority voting which amalgamate individual preferences into a social choice.

Voting mechanisms are often unstable because of the phenomenon of "cyclical" majorities pointed out by Kenneth J. Arrow in Cowles Foundation Monograph 12, Social Choice and Individual Values. As Arrow's theorem indicates, any specific mechanism—such as majority voting—which satisfies a few intuitively plausible conditions will inevitably produce an intransitive social ordering (i.e. one containing cycles) for some assortment of individual preferences. One possible resolution of this difficulty is to
relax the requirement that alternative social states be completely ordered; perhaps it is sufficient to assume merely that the social ordering be "acyclic," in the sense used by Brown for individual orderings.

There are a number of conditions—requiring for example, some degree of similarity in preferences—which have been shown sufficient to eliminate intransitivities and to restore stability to majority rule. As Kramer has demonstrated, however (CFDP 284), these conditions turn out to be extraordinarily restrictive when applied to problems involving the distribution of economic goods, or in models of the public sector; for many public sector decisions pure majority rule appears to be inherently unstable.

Kramer's subsequent research has moved in the direction of employing game theoretic concepts and taking account of the institutional structure in which voting occurs. Some preliminary results were reported in a paper, "Theory of Electoral Systems," presented at the Eighth World Congress of the International Political Science Association in September 1970.

In collaboration with Lepper, Kramer has also extended some earlier empirical work on Congressional elections. As in the case of the previous work, their paper is based on the assumption that constituents vote for the incumbent if they are satisfied with the results of the policies attributed to the incumbent's party; the policy consequences specifically incorporated in the regression analysis are measures of economic prosperity. The paper, "Congressional Elections," which was completed and will appear in a forthcoming volume concerned with applications of quantitative methods in political and economic history (edited by R. W. Fogel), extends the earlier analysis to tentative experiments with data at the county level.

II. Planning and Comparative Economic Systems

One of the major questions in the theory of economic planning is the extent to which the multiplicity of economic decisions confronting a central planning board can be effectively decentralized. Must the planners themselves be involved in the choice of elementary techniques of production, and the determination of highly disaggregated levels of output, or can decisions of this sort be delegated to smaller economic units, constrained only by broad guidelines issued by the central planning board?

This question has been examined by many authors since the initial proposal by Oscar Lange that certain aspects of the Walrasian model could be directly translated into a program for the efficient decentralization of a planned economy. According to this analysis the choices of consumers—sensitive to relative prices—would constitute the ultimate source of demand for all goods and services, and producers would be constrained only to maxi-
mize profit at the equilibrium price system. All information about preferences and scarcity would be transmitted to producers exclusively in terms of relative prices.

Not only is the actual practice of planning considerably more subtle than this bare scheme—taking into consideration information about quantities both as indicators of scarcity and as goals—but there may also be sound theoretical reasons for departing from the Walrasian model. The inadequacies of the competitive model for planning purposes form the central theme of Janos Kornai's profound study, "Anti-Equilibrium," which will be published shortly by Springer. The book, which was completed during Kornai's visit to the Cowles Foundation in the spring of 1970, is a comprehensive discussion of virtues and deficiencies of various analytical approaches to planning, and should be a focal point of discussions for years to come.

The theory of planning has been illuminated by the introduction of numerical techniques for the solution of programming problems, which are simultaneously capable of interpretation as decentralized decision making processes. For example the "decomposition" method for linear programming—first introduced by Dantzig and Wolfe—can be interpreted as an iterative procedure in which the prices of some factors of production are quoted by the planners; the sectors independently maximize profits based on these prices, which are then systematically revised in terms of the discrepancy between supply and demand.

In CFP 332, Weitzman investigates the interaction between the center and various sectors of an economy in which price guidelines are replaced by quantity targets, which are revised if the sectors are able to convince their superiors that technological considerations preclude the fulfillment of their assigned quota. In the process the sectors attempt to impart a notion of the constraints that are binding and the direction in which a new quota must move if it is to be feasible. The paper then demonstrates how efficient planning can take place even though each sector—and the center—has incomplete information about the economy as a whole.

The function of the method of material balances in Soviet type economies is to insure coordination between anticipated supplies and projected demands of factors of production. In "Material Balances under Uncertainty" (CFDP 286), Weitzman studies a simplified aggregate model of this procedure, which is used to determine the optimal balance between the production of intermediate and final goods.

Koopmans, in collaboration with J. M. Montias, has been working on the development of formal methods for the description and comparison of
economic systems. The research was originally undertaken as an assignment for a Conference on the Comparison of Economic Systems, held at the University of Michigan in November, 1968, at the initiative of Professor Alexander Eckstein of that University. They have approached the assignment in the spirit of a search for ways in which formal theorizing and model construction can contribute ideas and footholds for analysis in the description and comparison of economic systems.

Ideas that were emphasized in the description include the notion of custody as a relation between persons (or organizations) and goods, that is found to be important in all modern economic systems; the notion of transfers of custody in specific stages of processing of the good in question; the use of linear graphs in representing organizational relations between persons; the relation between information available to the members of an organization and the actions they take; and the use of the concept of a hierarchy, for instance, in the discussion of the international corporation.

The evaluative comparison discusses alternative criteria that may be adopted or rejected, or given different weights, by different comparos. The effects of the attrition of information under vertical transmission in a hierarchical organization is then examined as an application of the criterion of efficiency.

Finally, on the empirical side of comparative economic studies undertaken at the Cowles Foundation, Weitzman has attempted to measure production relationships for the Soviet economy. In "Soviet Postwar Economic Growth and Capital Labor Substitution," CFP 333, a constant elasticity of substitution production function is directly estimated on the basis of Soviet data, the results are interpreted and comparisons are made with results obtained from U.S. data.

III. Micro-Economic Theory and Policy Applications

One of the major virtues that economic theorists have long attributed to a market economy is that the resulting production and distribution decisions are optimal in the sense that no alternative suggestions for production and redistribution will improve the utility of every member of society. The range of application of this theorem may be somewhat diminished by an awareness of the restrictive assumptions required for its validity—for example, non-increasing returns to scale, perfect futures and insurance markets, and the costless flow of information, to name only a few. There is also substantial difficulty in incorporating a variety of considerations which lie between economics and political theory, such as external effects in production and consumption, and the taxing, spending and regulatory activities
of the government sector. But the major objection undoubtedly resides in the weakness of the optimality criterion itself, and its complete indifference to considerations of equity.

An older tradition typified by Bentham saw in the social welfare function a major technique for describing society's ethical judgments on economic matters. But in the late 19th and early 20th centuries economists from Pareto to Samuelson tended to view utility as an ordinal and subjective rather than cardinal and tangible concept. On the other hand, everyday discussions which attempt to judge the fairness of a specific distribution of society's resources are rarely couched in terms of utility. Fairness and equity can be seen in terms of principles—such as the approximate equality of income, and the provision of essential goods and services at some agreed upon level—which can be expressed in real terms and debated with no direct reference to utility.

Proposals for a negative income tax—in which households receive a graduated supplement if their income falls below a minimum level—are specific examples of redistribution plans designed to alleviate the inequalities of privately generated income. In the previous three year report reference was made to the joint work of Mieszkowski, Tobin and Joseph Pechman of the Brookings Institution on the mechanics of alternative negative income tax proposals. A summary of their findings appear in the paper "Is a Negative Income Tax Practical?", published in the Yale Law Review (November 1967). During the period of this report both of these staff members continued their interest in the economics of poverty and inequality. Tobin's debate with W. Allan Wallis on Welfare Programs: An Economic Appraisal was published in 1968. He also contributed to the 1968 Brookings volume Agenda for the Nation, an essay "Raising the Incomes of the Poor," which gave a general assessment of the sources and possible remedies for poverty in the United States. In April 1970, Tobin gave the Henry Simons Memorial Lecture at the University of Chicago, "On Limiting the Domain of Inequality" (subsequently published in the Journal of Law and Economics, October 1970). In this lecture he considered, with illustrative examples, the appropriate role of "specific egalitarianism," i.e., efforts to distribute particular goods less unequally than the market would distribute them—given the extent of the prevailing income inequality.

Tobin was also associated, as a consultant on the experimental design, with the experimental test of "graduated income incentives," i.e., negative income tax, conducted in New Jersey by the Institute for Research on Poverty of the University of Wisconsin, and by Mathematica, Inc. This series of experiments is aimed, in part, at exploring the labor supply re-
spose to various forms of a negative income tax—an important ingredient in discussing the implications for efficiency, and the costs, of income maintenance plans.

Taxation is one form of governmental economic activity that has raised classical problems in regard both to efficiency and to the distribution of real income. Mieszkowski has continued to work on the theory of tax incidence; his survey article on this topic, "Tax Incidence Analysis: The Effects of Taxes on the Distribution of Income," appeared in the Journal of Economic Literature, in December 1969. He has also revised and extended the work on the distributional implications of the property tax, referred to in the last three year Report. Basically the paper (CFDP 304) argues (1) that the property tax system decreases the after tax return on capital by an amount equal to the mean rate of tax for the nation and (2) raises the cost of capital to cities with high rates of tax and lowers it for those jurisdictions whose tax rates are low relative to the mean rate.

In the paper, "Tax Incentives for Low Income Housing," Nordhaus examined, using a simple model of a two-sector housing market, the effects on rents and on housing supply of various subsidy plans and tax incentives for low income housing. This review identified one problem—external diseconomies, such as fire hazards associated with slum housing—which may require subsidies to correct the associated market inefficiency. On the other hand, poverty would call either for general income supplements or—if a "specific egalitarian" approach is taken—for particular types of rent supplements or rebates.

The economics of housing markets and real estate values was also examined in two studies by Mieszkowski. One project with Thomas King, a Yale graduate student, is based on a sample of rental units in New Haven containing information on the quality of the units and the characteristics of the household that occupies them. The basic motivation for this survey was to determine whether Negroes pay more for housing of comparable quality and hence have their real incomes reduced by the rent differential. Previous studies on this topic relied on aggregate census data and suffered from imperfect controls on the quality of the units. Mieszkowski's study, which is virtually complete, reaches the following conclusions:

(1) Controlling for the size, quality and other characteristics of the unit, the sample of black households pay about 11% more for rental housing than do whites.

(2) Households headed by a male in all black areas pay about 11% more than in integrated areas.
(3) Whites who live in mixed areas pay slightly less for housing than whites who live in exclusively white areas.

(4) The discrimination against households headed by black women is substantially greater than discrimination against households headed by black men. Households headed by white women do not appear to be discriminated against relative to households headed by white men, and

(5) An education variable, which is taken as a proxy for socio-economic status decreases rents paid by blacks—but has no effect on the rents paid by white households.

Mieszewski and Grether are also involved in a study of the determinants of real estate values in New Haven, sponsored by the Institute of Social Science at Yale. Several years ago the New Haven Board of Realtors made available to members of the Yale Department of Economics their master cards containing detailed information on original asking and final selling prices, and physical characteristics of properties sold in approximately 10 thousand transactions. Based on these data, the project will estimate the effect on housing value of variables such as property tax burdens, the distance between the unit and central work locations, and "neighborhood effects," e.g., the volume of traffic, crime rates and the quality of public services. The results may provide indirect evidence on the demand for public services (and the willingness to be taxed for them), information related to alternative land use patterns and consequences of zoning, and some further evidence on the existence and degree of market discrimination.

Higher rental costs based on discrimination may tend to increase disparities in nominal earnings; in a similar fashion, the alleged phenomenon that the poor pay more for goods and services than their affluent counterparts would—if true—worsen the problems of poverty otherwise identified on the basis of money income. A study conducted by Kleverick, in conjunction with Roger Alcaly of Columbia University, investigated the relationship between food prices and income levels in different neighborhoods of New York City. The basic data bank used for the analysis was a comprehensive survey of food prices, conducted in New York City during the summer of 1967 by the New York City Council of Consumer Affairs. Using the price data from the New York City survey and data on the median family income for forty-six neighborhoods in New York, the simple relationship between price and neighborhood income was investigated employing a single-equation linear regression model. The regression results suggest that, after separating chain stores from small independent stores, the commodity-
by-commodity prices of food items on retail merchant's shelves do not, with few exceptions, rise with decreases in neighborhood income.

Care must be taken in interpreting these results. For example, while the direction of commodity price variations with income and the significance of such variations are similar for chain stores and other stores, for most commodities the mean price of the item is higher in other stores than in chain stores. And, the evidence suggests that low-income neighborhoods have a higher proportion of small independent stores than do higher-income areas. This point and other qualifications of the regression results are presented in the paper "Do the Poor Pay More for Food," CFDP 290. These qualifications, taken in conjunction with the results themselves, make it clear that a complete investigation of the important question of whether the cost of food is greater to the poor will require data of even higher quality than those in the New York City survey and discussion that goes beyond the level of comparing relative prices in stores serving areas with different incomes.

Klevorick also contributed to a study Higher Education in the Boston Metropolitan Area: A Study of the Potential and Realized Demand for Higher Education in the Boston SMSA undertaken for the Boston Metropolitan Area Planning Council and the Board of Higher Education of the Commonwealth of Massachusetts. The objective of the study was to provide the basic analysis necessary for planning the future of higher education in the Boston SMSA (Standard Metropolitan Statistical Area). Klevorick's participation in the study centered on the preparation of policy proposals for financial aid to students, taking into account the egalitarian objective of equal educational opportunity, the role of this individual state in the nation's effort to attain this goal, and the incentive effects of various forms of aid on the student's educational decisions.

During the academic year 1969-70, Guy Orcutt—then on leave from the Urban Institute in Washington, D.C.—held the Irving Fisher visiting professorship in the Department of Economics, and was also a member of the staff of the Cowles Foundation. One of Orcutt's major research interests is in the construction of detailed microeconomic models—applicable to the evaluation of alternative policies concerned with such issues as poverty, congestion, and waste disposal—and in the use of simulation techniques for their solution. He continued, during his visit, to collaborate with the staff of the Urban Institute on the development of a micro analytical model of the United States population, oriented toward assessing the effects of a range of government policies that directly affect the distribution of income and assets.
Regulation of the prices charged by particular industries—for example, public utilities—is another way in which governments intervene in market economies. Beginning with the contribution by H. Averch and L. L. Johnson in 1962, a number of economists have discussed the behavior of a profit-maximizing monopolist whose output price is constrained so as to yield a preassigned rate of return on capital. The paper "Input Choices and Rate-of-Return Regulation: An Overview of the Discussion," by Kleverick and William J. Baumol of Princeton University, focuses particularly on the consequences of rate regulation for efficiency and the implications for public policy.

Kleverick has continued his interest in this topic by asking whether the Averch-Johnson formulation can offer any guidance about what the "fair rate of return" ought to be. He uses the Averch-Johnson model to predict how the firm will behave when the fair rate of return is set at various levels. With this firm reaction function in hand, the regulators then choose the value of the allowed rate of return that will lead the firm to maximize social welfare. The results of this research raise questions about the conventional view, as expressed through the history of judicial and regulatory commission proceedings, that the fair rate of return allowed to a regulated firm always should be equal to the market cost of capital. Because of the large number of specific assumptions that are needed to obtain concrete results, the specific conclusions are primarily suggestive. Nevertheless, the research does make the important point that the choice of values for regulatory control instruments should be viewed in a broader context than that suggested by current practices.

Kleverick has also attempted to provide more realistic models of the regulatory process, with particular emphasis on its dynamic character and on the interplay between the regulatory agency and the firm it is regulating. One of the models, for example, assumes that the occurrence of regulatory reviews can be described by a stochastic process. The firm believes there is a certain probability that a review will occur in a particular year and it acts on this belief. Kleverick has investigated what the optimal policy would be for a regulated firm maximizing the discounted present value of profit in such an environment. Models of this type enable one to predict how the "rightness" of regulatory policy—for example, the mean time between reviews—affects the operating efficiency, in both static and dynamic terms, of the regulated firm.

Technological change has been viewed by many economists as a highly significant component of economic growth, yet it is incorporated only in rather simple and stylized form in aggregated growth models.
Nordhaus has been concerned with a study of technological change and invention—primarily at a microeconomic level—which might give more insight into the ways in which a variety of government policies could influence these components of economic growth. He has, first, proposed certain theoretical models of the inventive process which can explain the rate and direction of inventive activity. These include models of microeconomic determination of the level of research, models of the economics of patents, and aggregate movements in the rate and direction of technological change. Secondly, he has considered, within the context of these models, the influence of such policies on technological change as the duration of patents, the rate and direction of government-produced research, and subsidy systems for private research and development. Finally, he has made some empirical estimates of these models to determine their predictive ability.

During the past two years Brainard, together with several graduate students, has been conducting a study of the economics of pollution of the Connecticut River. The study is an attempt to determine the optimal pattern of sewage treatment for obtaining any desired level of water quality and to compare the costs of such treatment with uniform code enforcement of the type embodied in most legislation. In order to investigate this question a programming model of the river has been constructed which builds on technological information provided by the Army Corp of Engineers, and which incorporates estimates of the cost of various levels of sewage treatment. The model breaks the river into approximately 100 sections and includes a similar number of "pollution activities" which can be varied by changing the level of treatment. In addition to providing a means of computing the "least cost" method of pollution, subject to a water quality constraint, the model provides shadow prices on the water quality at various places on the river. These "prices" in turn are valuable in determining how quality standards should be changed and are useful in the selection of river sites to be developed for recreational and other purposes.

IV. Rational Behavior Under Risk

One of the continuing interests of the Cowles Foundation over the last two decades has been the development of a theory of economic behavior in the presence of uncertainty. During the period of this report, research has focused on three areas: the development of basic concepts, extensions of portfolio models, and the development of general equilibrium models incorporating considerations of risk.

The modern theory of consumer behavior works with two distinct approaches: the "ordinal" utility theory of consumers' choice among many
commodities, and the Von Neumann-Morgenstern "cardinal" utility theory of consumer behavior under risk—generally with one commodity or with many commodities whose price ratios are fixed. Stiglitz (CFDP 262) showed that these two theories are in fact intimately related; he investigated the mutual relationships between alternative restrictions on the indifference map and assumptions about consumer behavior under risk. For instance, it is shown that if an individual is risk neutral at all price ratios in a given region, then the income-consumption curves must all be straight lines (in that region).

In another paper dealing with basic concepts, Stiglitz and Michael Rothschild (Massachusetts Institute of Technology) explore the problem of characterizing degrees of uncertainty. In CFDP 275, they consider four possible criteria by which riskiness might be judged: (1) If uncorrelated noise is added to a random variable, the new random variable is riskier than the original. (2) If $X$ and $Y$ have the same mean but every risk averter (a person with a concave utility function is a risk averter) prefers $X$ to $Y$, then $X$ is less risky than $Y$. (3) If the probability distribution of $Y$ has more weight in the tails than the distribution of $X$, then $Y$ is riskier than $X$. (4) If $Y$ has a greater variance than $X$, then $Y$ is riskier than $X$. Rothschild and Stiglitz show that the first three criteria lead to single definition of greater riskiness different from that of the fourth. This definition may be phrased in terms of the indefinite integrals of differences of cumulative distribution functions. They also show how their definition can be applied to a variety of economic and statistical problems; for example, the effects of increased risk on portfolio allocation, savings behavior, and firm investment policy are investigated.

One of the earliest and most fruitful areas of application of the theory of behavior under uncertainty has been to problems of portfolio allocation among alternative risky assets. Until recently, the models examined in detail have usually been based either on the simplifying assumptions of only one period and two assets, one of which is safe (money) and the other risky, or on the assumption that a mean-variance analysis is applicable. Recent research in this area has been primarily concerned with extending the portfolio model to eliminate these limitations.

In an earlier Cowles Foundation paper, Tobin had showed that, with mean-variance analysis, the portfolio decision could be divided into two parts: what proportion of the portfolio to invest in the risky assets (as a whole) and, independently, the proportion to be invested in each particular risky asset. Cass and Stiglitz (CFP 329) investigated the necessary and sufficient conditions for such a portfolio separation theorem to obtain. They also examined the more general question of when a set of mutual funds
could provide all the market opportunities desired by a class of individuals. The general class of utility functions for which mutual funds are acceptable include as special cases the quadratic utility function, those with constant relative risk aversion, and those with constant absolute risk aversion.

Cass and Stiglitz have continued their research into the structure of portfolios with many assets, by investigating the effects of a change in wealth on portfolio allocation. When there are only two assets, it can be shown that (a) the wealth elasticity of demand for the safe asset is greater or less than unity as relative risk aversion is increasing or decreasing; (b) the variance of the rate of return increases or decreases with wealth as relative risk aversion is decreasing or increasing; (c) the certainty equivalent rate of return to the optimal portfolio increases or decreases with wealth as relative risk aversion is increasing or decreasing. In a multi-asset world, if there are as many states of nature as securities, only the second and third properties hold, and if there are more states of nature than securities, only the third remains valid.

The extensions of the portfolio model to a simultaneous consideration of portfolio allocation and saving behavior has been the concern of four papers (CFP's 288 and 330 and CFDP's 268 and 260) in the last three years. Tobin, in CFP 288 discussed below, considers a two period model which incorporates uncertainty about rates of return and future income. Stiglitz was primarily concerned with analyzing the allocation between short, and long term bonds in a two period model. Following earlier work of Tobin, Stiglitz points out that the conventional view whereby long term bonds are treated as risky and short term bonds as safe is incorrect; which asset is risky depends on the investor's consumption patterns. Accordingly, it is shown that even when the long term interest factor is equal to the product of the expected short term factors, all individuals who consume in both periods purchase some long term bonds, and they may in fact specialize in long term bonds (if they are not very risk averse). Properties of the demand functions for short and long term bonds are analyzed; the pattern of allocation is shown, for instance, to be sensitive to relative rates of return, but not necessarily to the level of the interest rate.

The capital-budgeting problem faced by a firm is analogous to the multi-period portfolio-selection problem faced by an individual investor. A prototype of the firm's capital-budgeting problem would picture an existing firm planning an investment program for the next T periods. Taking account of the anticipated cash throw-offs from the resources owned at the beginning of the investment program and the capital-market options available to it, the firm then would be depicted as choosing the optimal subset of invest-
ments to make from among the opportunities available over the planning horizon. Since the returns from (and, perhaps, the cash outlays on) the various projects are not known with certainty and can be characterized as random variables, the firm's capital-budgeting problem is an example of decision-making under risk in which a decision taken now has a stream of future effects.

Two aspects of the capital-budgeting problem just described have provided the focal points for Kleverick's research in this area during the last three years. First, while the criterion function of the capital-budgeting problem for a firm must necessarily evaluate risks that occur at different points in time, almost all previous characterizations of attitudes towards risk have been in terms of a single-period utility-of-wealth function (as, for example, in the traditional discussion of risk aversion and the more recent developments due to K. J. Arrow and J. W. Pratt). The question naturally arises, then, whether the discussion of attitudes toward risk and statements about the behavioral implications of these attitudes can be extended to a multi-period context.

One part of Kleverick's work on the capital-budgeting decision has addressed itself to just this question. The relationship between concavity of the decision-maker's utility function and risk aversion that is familiar in the case of a single-stage utility function can readily be shown to obtain in the case of a multi-period utility function. The emphasis of further research has been on extending the Arrow-Pratt concept of decreasing absolute risk aversion to the case of multi-period decisions. This extension has been completed successfully in terms of insurance-policy purchases in perfect capital markets. A set of sufficient conditions has been derived for the existence of multi-period decreasing absolute risk aversion and it has been shown that this set of conditions is a generalization of the Pratt condition for the single-period case. It has also been proved that, in the case of an additive multi-period utility function, the necessary and sufficient condition for decreasing absolute risk aversion is that the Pratt condition obtain for each individual period's utility function.

The second aspect of the capital-budgeting problem upon which Kleverick's research has concentrated concerns the appropriate form that the objective function in that problem should assume. There is general agreement on the appropriate criterion function for an investment-planning decision made in a world of perfect capital markets and perfect certainty. On the other hand, the question of what the appropriate function is when capital markets are imperfect and projects' returns and costs are uncertain remains open for debate. An examination of the alternative candidates for
the maximand has led to the development of a case in favor of the maximization of expected utility by a corporation budgeting capital in the presence of risk and imperfect capital markets.

The research on both issues just discussed will form part of the material to be included in a monograph on capital budgeting under risk which Kleverick has been preparing for publication.

Hakansson has also investigated multi-period models of capital budgeting. His use of a sequential decision-making model provides an extensive generalization to the model originally formulated by Phelps (CFP 192).

The effect of taxation on risk taking was one of the first problems investigated by means of portfolio analysis. It was shown (papers by Tobin and Lepper in Cowles Monograph 19) that proportional taxes with full loss-offsets increase risk taking if the investors' utility of income functions are quadratic (i.e. in one of the two cases where mean-variance analysis applies). Stiglitz has shown that this result does not hold generally; the effects of alternative taxes (with and without loss offset provisions) are shown to depend on the level of relative risk aversion as well as on how relative and absolute risk aversion change with wealth. In addition, a diagrammatic interpretation of increasing and decreasing relative (and absolute) risk aversion is provided in CFP 293.

Development of general equilibrium models for economies with uncertainty has been focused primarily on the formulation of models in which insurance markets are not complete and perfect. In "A Re-Examination of the Modigliani-Miller Theorem" (CFP 314), Stiglitz explicitly considered the problem of bankruptcy. He showed that the financial policy (debt-equity ratio) of firms had no effect on any of the real variables of the economy—the Modigliani-Miller result—provided firms did not go bankrupt. Furthermore, bankruptcy made no difference if individuals had identical expectations and their preferences could be described in terms of means and variances. However, when these conditions are not satisfied, there does exist an optimal financial policy for the firm. In more recent work, Stiglitz has been concerned with the analysis of investment policy and choice of technique in an economy with a stock market.

V. Descriptive and Optimal Growth Theory

Just as the static general equilibrium model under certainty can be extended to include uncertainty, so there is also an immediate way in which the model can be extended to accommodate economic problems of a dynamic character. If commodities are distinguished not only by their physical aspects but also by the calendar date of their availability, the entire apparatus
of the Walrasian model can be used to analyze the problems of allocation over time.

This procedure, attractive as it may be, does have at least one substantial drawback: it requires the independent decision making units to be in full possession of the relevant prices which will indeed prevail in the future. Production units must be aware of future labor and material costs, interest rates, and the competitive prices which will be obtained for their outputs. Consuming units are required to be informed, not only of the various components of their own future income streams, but also of future prices and interest rates, since these may influence current consumption and savings decisions.

Much of the research in that area known as "descriptive" growth theory has been concerned with the replacement of the complex maximizing behavior required in the Walrasian model by a variety of simpler and possibly more realistic assumptions about individual behavior. For example, savings behavior, which might otherwise be obtained by the maximization of utility subject to a budget constraint, may be replaced by an assumption that saving is—for each individual or in the aggregate—directly proportional to income. And production decisions may be described as motivated largely by short run profit maximization, independently of the course of future prices and interest rates.

These simplifications of descriptive growth theory are one attempt to cut through the complexities of a more elaborate model in order to focus on the dynamic aspects of a problem. A similar concentration on inter-temporal choices has been obtained by a number of other simplifications, within the context of maximizing behavior. For example a single utility function—or social welfare function—may replace the variety of individual utility functions typically assumed in a more disaggregated model; stocks of machinery may be replaced—as inputs into production—by a fictitious homogeneous capital good; and production possibilities may be described by a highly simplified relation between current output and aggregate inputs of capital and labor.

In a series of papers extending over a number of years, Koopmans has been concerned with the study of growth paths in an economy in which the social welfare function for consumption over time is assumed to have the property that the relative ranking of future consumption streams that coincide in the first period is independent of the levels of consumption achieved in that first period. In work described in an earlier three-year report, Koopmans has characterized the utility functions which possess this and related properties. The class of utility functions thus obtained is a sub-
stantial generalization of the discounted sum of single period utility functions so frequently encountered in optimal growth theory. In a joint publication with Richard Beals (CFP 309) this class of utility functions is applied to an aggregate growth model involving a single good which serves interchangeably as a capital or consumption good. The work differs from most of the published work on optimal growth in that the optimality criterion allows the rate of discounting of future utility flows to depend on the future consumption path, rather than being assumed constant.

In subsequent work Koopmans has reverted to the simple optimality criterion in which the discount rate is a constant, in order to study a different complication: that of introducing into the model many goods, classified as consumption goods, resources, and capital goods. The technology is of the von Neumann type with resources and consumption goods added, and is constant over time. The first object of study is a stationary state resulting from maximization of a discounted sum of utilities derived from future consumption flows—provided the initial stock is such as to be self-preserving under the maximization. It is studied how the stationary capital stock depends on the constant discount rate.

The mathematical essence of this problem appears to be a concave nonlinear programming problem in which additional constraints are placed on the dual variables (specifically, on the shadow prices of capital goods). It is hoped that analysis of this problem will help to obtain a better understanding of several difficulties in capital theory that have been discussed in the recent literature. It may also help in discussing problems of optimal taxation where capital goods may be among those taxed.

The problem of aggregation in a growth model involving many sectors was also examined by Weitzman in CFP 292. The paper attempts to define constant "stationary equivalents" for the principal variables in a nonstationary optimal growth path. For instance, the path of consumption is represented by the utility value of that constant consumption vector whose discounted sum of utilities over the infinite future equals that of the optimal path itself. Similar definitions are introduced for the capital stock and the total product. It is then found that the derivative of this aggregated total product with respect to the aggregated capital stock equals the discount rate.

Weitzman has also studied an extension of the classical optimal growth problem originally discussed by Frank Ramsey in 1928. In this paper (CFDP 273) output is limited by the smaller of two capacities: (1) that of a directly productive capital stock whose contribution is subject to decreasing returns, and (2) that of overhead capital, itself produced under increasing returns to scale. In a model in which the overhead capacity limitation is ignored, the op-
timal path is that derived by Ramsey. If the direct capital limitation is ignored, the problem is one of optimal stepwise capacity expansions similar to those studied by Alan S. Manne and other authors. When both limitations are recognized, the optimal path travels along Ramsey's curve in those phases when overhead capacity is ample, but holds consumption constant in alternate periods in order to accumulate goods to be embodied in the next increase in overhead capacity.

One of the major limitations of the simple neoclassical models of growth—developed for example by Ramsey and by Solow—was the requirement of a single, homogeneous, malleable capital good, i.e., a good whose complementary labor input was not fixed at the time of construction, and whose output could be increased by increasing the supply of such labor. Not only was the assumption unrealistic, but it also made it impossible to ask several of the central questions concerning economic growth: What determines the choice of technique; how are different capital goods allocated to different sectors, and what are the effects of capital gains on growth? Moreover, in the descriptive models, because of these assumptions about the nature of capital (as well as the requirement that savings be a simple function of income) future wage rates and interest rates play no role in the pattern of growth. Attempts to alleviate the limitations of the simple neoclassical model have been the subject of a number of papers by Cowles staff members and others during the past three years.

These limitations on the malleable capital model were, at least partially, realized by the early 1960's; both Solow and Johansen formulated models in which machinery of varying capital-labor ratios could be constructed, even though no modification of the labor requirement was permitted subsequent to production. Investigation of the dynamics of these models had, however, been limited because, unlike the simpler growth models, the growth processes were now described by a seemingly complicated mixed differential-difference equation system. In CFP 299 and CFP 287, Cass and Stiglitz developed methods for analyzing both descriptive and optimal versions of these models.

For the descriptive model, both of the polar cases of static expectations and perfect foresight about future factor prices were investigated. In the case of static expectations, convergence of the growth path was assured under weak conditions. Paths which are consistent with fully perfect foresight converge to a balanced growth path, but there exist many paths consistent with limited short-run perfect foresight which do not. These results confirmed those of earlier studies by Hahn, Shell and Stiglitz showing that with heterogeneous capital goods stability depended crucially
on the expectations hypothesis used. The optimal growth path with non-malleable capital is shown to be quite different from that of the malleable case; consumption need not be monotonic—indeed wages, and the capital intensity of the machine constructed, may fluctuate. There may also be discontinuities in the choice of technique, in which two types of machines are manufactured without constructing machines of intervening capital intensities.

The assumption of malleable capital has also been made in econometric estimates of production functions. If capital is not in fact malleable, a specification error is therefore being made. The nature of this error is investigated by Cass and Stiglitz in CFP 299 for three estimation problems, the estimation of the rate of technical change, the estimation of investment functions, and the estimation of production functions with constant elasticity of substitution.

The implications of a similar mis-specification in the context of an optimal growth model were examined by Newbery (CFDP 281). Assuming that the economy is accurately described by a vintage model with fixed coefficients, a series of numerical simulations are obtained, which describe the loss in utility occasioned by following an "optimal" path—the path being calculated under the assumption of malleable capital.

The property of malleability may be contrasted with another type of substitution possibility: capital is said to be "shiftable" if, after its production, it can be used in any one of several sectors, should economic conditions warrant such a choice. In CFDP 266, Weitzman introduced a generalization of a two sector model of optimal economic growth first considered by the Soviet economist G. A. Fel'dman in 1928. In addition to consumption and investment sectors, Weitzman's model includes a raw material sector which can be used to provide inputs into either of the other two sectors—thereby providing a degree of shiftability which is not captured by the previous formulation. Under fairly general conditions, however, the optimal growth paths for the two models coincide and the increased flexibility will not be exploited.

In CFP 313, Karl Shell (University of Pennsylvania), M. Sidrauski and Stiglitz investigated the role played by changing prices of capital goods, relative to one another and relative to those of consumer goods (capital gains and losses), on individual savings and on portfolio choices in a number of simple neoclassical growth models. Dynamic behavior in models of this sort is shown to differ substantially from those in which capital gains play no role. In a particular example in which the nominal supply of money is assumed to change at a constant rate, the economy—starting with a fixed
endowment of capital and labor, and a given money supply—will approach a long run balanced growth only for certain specifications of initial prices.

In CFP 319, Stiglitz considered a two-sector economy in which there was embodied technical change, so that newer machines are "more efficient" than previously produced machines. The central problem posed was how would an economy allocate the different kinds of machines between the two sectors; it was shown that all the newer machines would be allocated to the labor intensive sector, assuming one sector to be unambiguously more labor intensive than the other.

In the analysis of many of the conventional problems in economics, static assumptions are made with the hope that the dynamic effects may be safely ignored. The development of neoclassical growth theory in recent years, however, has made available tools by which some of these problems may be reexamined in an explicitly dynamic setting. That is the object of the two papers about to be described.

In "Factor Price Equalization in a Dynamic Economy," Stiglitz examines several classical trade propositions, using a model with two factors of production—capital and labor—in each of two countries. Alternative descriptions of savings behavior are examined: the Marxian savings hypothesis that a given fraction of profits are saved and the "rational" savings hypothesis that each country acts as if it were maximizing an intertemporal utility function with a constant discount rate. It is shown that under certain conditions at least one of the two countries specializes so that factor prices and interest rates are never equalized; indeed, under the normal capital intensity hypothesis, factor price differentials actually increase over time.

In the second of these papers (CFP 320), the major determinants of economic inequality were examined in the context of a growth model. Attention was focused on four factors: (a) savings behavior, (b) inheritance policies, (c) differences in factor productivities, and (d) human reproduction rates; an attempt was made to identify those factors which tended to increase or decrease equality in the distribution of wealth. The methodology of growth theory was also applied by Newbery (CFDP 278) to a model of an underdeveloped country containing a modern industrialized sector alongside of a traditional sector engaged, for example, in agricultural production at a subsistence level. The implicit prices for investment, labor, and consumption are determined by a programming problem in which a discounted sum of utilities for workers in both sectors is maximized.

A production plan consistent with a specific technology is "efficient" if its inputs cannot be decreased without entailing a corresponding decrease in at least one of the outputs. In a model involving a finite number of pro-
duction periods, an efficient plan will maximize the present value of the stream of profits when compared to all alternative feasible plans—for some appropriate selection of prices and interest rates implicit in the efficient plan. This simple relation between efficiency and the maximization of present value may however be lost in those models typically discussed in growth theory in which an infinite sequence of decision periods are permitted. In the paper, "Present Values Playing the Role of Efficiency Prices in the One-Good Growth Model," Cass and Menahem Yaari analyze this relationship in the context of a one-sector growth model of the type introduced by Ramsey. It turns out that present values do have a maximality property, thereby providing a price interpretation of efficiency; but the interpretation in the infinite horizon model is weaker and somewhat more subtle than the customary one.

VI Macroeconomics and Monetary Theory

As in the case of descriptive growth theory, short-run macroeconomics—including its monetary aspects—is distinguished from general equilibrium theory by less formal consideration of the rational behavior of economic agents in a large number of markets. Aggregation across large numbers of decision-making units remains, by definition, an essential characteristic of macroeconomics. However, macroeconomic research has increasingly broken away from the simplifications of early Keynesianism in two respects. First, more care has been taken in formulating aggregate analogues to reasonable assumptions about the behavior of individual consumers or producers. Thus, aggregate capital investment functions are more frequently related to profit maximizing conditions for competitive firms, and money demand functions are similarly related to the portfolio choices of individual households. Second, more explicit consideration is given to market equilibria (or disequilibria) for a larger number of goods and financial assets, and to such macroeconomic identities as the summation of assets and liabilities to total wealth.

These trends in macroeconomic theory have entailed costs by making models more complicated to solve—resulting in greater interest in computer simulation techniques—and by making aggregate relations considerably more difficult to estimate empirically—requiring the application of more elaborate econometric procedures and raising new questions about econometric techniques. The resulting benefits are richer implications, and in some cases, modifications of earlier policy conclusions from macroeconomic theory.

Most macroeconomic and monetary models have, for convenience, treated
portfolio choice and saving as separate decisions. The typical individual is assumed to decide first how much wealth he desires and how fast to accumulate it, and second, how to apportion it among various assets. This is neither realistic nor analytically satisfactory. In CFP 288, Tobin considers the optimal rates of growth of "outside" money and of financial intermediation needed to place an economy on an efficient growth path. The solution to this problem is shown to involve both the amount of saving and the division of savers' portfolios between capital and monetary assets. Thus, in the second part of the paper, Tobin sets forth a preliminary analysis in which the two decisions are integrated; amounts saved depend on the menu of assets available, and on the probability distributions of their returns. Ebel worked further on the same subject while at the Cowles Foundation.

Work in both theoretical and empirical monetary economics at the Cowles Foundation has been relevant to a number of monetary controversies that have intensified in the past three years. The issues concern: the relative importance of monetary and fiscal policies in affecting short-run economic fluctuations and determining long-run trends; the proper indicators and targets of monetary policy; the impact—stabilizing or destabilizing—of active discretionary government policy intended to stabilize the economy. The general spirit of the work at Cowles is opposed to the "monetarist" school. Papers expressing the point of view developed at the Cowles Foundation have been noted in earlier reports (CFP 224, 1964; CFP 229, 1965; CFP 257, 1967). In the period covered by the current Report, Tobin completed and published work (1967 Report, p. 33) showing how misleading, as indicators of causation, can be temporal leads and lags between time series of money stock and money income. Indeed an "ultra-Keynesian" model in which the money stock has no causal importance is shown to generate lead-lag patterns exactly like those exhibited by Friedman and Schwartz in support of their "monetarist" position. Friedman's own "permanent income" model of money demand, a theory which attributes business fluctuations to cycles in the rate of growth of the money stock, is shown to imply lead-lag observations quite different from the observed pattern (CFP 323). In CFP 296, Tobin and Craig Swan examined the statistical fit of the same "permanent-income" model of U.S. data and its ability to forecast money income from money stock series. The model was not successful.

Research on monetary theory and policy at Cowles has been inclusive and eclectic. Many assets—not just those dubbed as "money"—have monetary importance. Many institutions—not just commercial banks—play significant roles. Monetary policy is important, but there is no iron mechanical
link from monetary variables to business activity. Tobin published an expository article describing this "general equilibrium" approach (which was described briefly in the previous Report, p. 33) in the first issue of the new Journal of Money, Credit, and Banking ("A General Equilibrium Approach to Monetary Theory," Vol. 1, February 1969, pp. 15–29).

As mentioned in the 1967 Report, Brainard and Tobin conducted simulations of "general equilibrium" financial models and their adjustment paths. Their results were published in CFP 279. In addition to the substantive theory embodied in the models used in the simulations, this paper makes two main methodological points. One point similar to that of CFP 323 (cited above), but more complex and general, is that temporal sequences of peaks and troughs are without value as indicators of causation. The other is that both equilibrium and disequilibrium models must explicitly respect Walras' law. In this context the law is that, since the demands for various assets are always constrained to sum to total wealth, anything which affects the demand for one asset without affecting total wealth must have equal and opposite effects on the demand for at least one other asset. Econometric models typically ignore this important principle, and the paper presents simulations that illustrate the unhappy consequences of doing so.

The model of CFP 279 is theoretical and the simulations are illustrative. But the same framework is the basis for an empirical model of the financial sector of the U.S. economy which is being estimated by students of the macroeconomics workshop under the direction of Brainard and Tobin.

Although research has shifted away from sole emphasis on the role of commercial banks in monetary phenomena, these financial institutions continue to play an important role in the adjustments of portfolios, interest rates and credit flows. During the summer of 1968, Hester continued work on a forthcoming monograph, Bank Management and Portfolio Behavior, which he is preparing in collaboration with James L. Pierce of the Board of Governors of the Federal Reserve System. This monograph both extends the application of the portfolio theoretic approach to commercial banks and includes fairly extensive empirical investigation of their behavior.

An application of the general equilibrium-portfolio approach to a question of policy is provided by Tobin's paper CFP 322, criticizing the reliance of policy-makers on deposit interest ceilings for commercial banks and thrift institutions. The basis of this criticism is the allocative and distributive effects of such ceilings in periods of restrictive monetary policy. An investigation by Hester of credit flows from financial to real sectors during the unusual period of the 1966 "credit crunch" leads to conclusions closely
related to these arguments of Tobin. In CFP 311, Hester suggests that output of the housing industry was cut back in 1966 by forcing suppliers of mortgage loans into a position from which they could not successfully compete for new funds. This was brought about by Federal Reserve open market operations which forced market interest rates up while, simultaneously, the Federal Home Loan Bank Board imposed effective ceilings on the interest rates which savings and loan associations could pay on savings deposits and shares.

Another policy application was given by Tobin in a paper ("Monetary Semantics") questioning the value of the monetarists' search for a single indicator and target of monetary policy and in particular challenging the suitability of the quantity of money (currency plus bank deposits) for this role.

Turning to the real sectors of the economy, macroeconomic models characteristically include two basic building blocks, a set of one or more investment relations and a set of one or more consumption functions. Bischoff has been concerned with examining the behavior of business fixed investment, and particularly with examination of the effects of monetary and fiscal policy on this form of investment. The "putty-clay" model of production relations, in which factor proportions are variable only up to the point at which new machines are installed implies, under several simplifying assumptions, that the short-run elasticity of investment demand with respect to changes in the quasi-rental cost of investment goods will not exceed the long-run elasticity. In contrast, the initial demand response to a change in output will exceed the long-run response. In an empirical application to the demand for equipment (CFDP 250), Bischoff found that the estimated dynamic response patterns conform to the suggestions of the theoretical model: factors affecting the quasi-rents, including the investment tax credit, the long-term interest rate, and the yield of equities, are found to have substantial long-run effects but smaller short-run effects.

In "A Model of Nonresidential Construction in the United States" (CFP 325), Bischoff examines the demand for investment in structures. The estimated response patterns to changes in quasi-rents do not conform to the predictions of the "putty-clay" model; as with the response to output changes, the short-run elasticity exceeds that for the long run. This is consistent with the intuitive observation that building is likely to adapt more flexibly than machines to various factor proportions. The results also suggest that equity yields exert substantial effects on construction demands in the short run, and to a lesser extent in the long run.

One central aspect of the "general-equilibrium" monetary theory discussed
above is the importance it assigns to discrepancies between market valuations of old capital goods and the cost of new capital goods. In particular, securities markets continuously re-value the plant and equipment of corporations; the relation of these valuations to the costs of new plant and equipment determine how large a claim on their future earnings corporations must give up in order to finance new investments. The hypothesis that this relation is a significant factor in corporate investment decisions is being tested by the macroeconomics workshop and has performed well in preliminary tests by Bischoff.

The value of the long-run elasticity of investment demand with respect to the quasi-rental cost of the services of investment goods has been the subject of a controversy between Dale Jorgenson and Robert Eisner. In CFP 301, Bischoff shows that the conclusions drawn are critically dependent on the precise assumption made about the stochastic process which generates the disturbances in the equation being studied.

"The Lag Between Orders and Production of Machinery and Equipment: A Re-examination of the Kareken-Solow Results," reconsiders a portion of a study of lags in monetary policy and shows that the lag in the machinery industry between orders and production is considerably shorter than the earlier study indicated. Bischoff's reexamination relies on more sophisticated methods for estimating distributed lags and serially correlated errors than the original authors used. Bischoff has recently surveyed contemporary innovations in specification and estimation of distributed lag models.

Aggregate consumption functions received extensive treatment in early econometric literature because of the concurrent Keynesian emphasis on the importance of the consumption multiplier. A number of alternative theories of aggregate consumption behavior have been developed from underlying microeconomic theories of rational consumer behavior. A standard result of these theories, in a static monetary economy, is that a consumer's demand functions for commodities are homogeneous of degree zero in prices, money income, and money wealth. This condition has been defined as the absence of money illusion. Aggregating over all commodities and all consumers, this absence-of-money illusion result implies that the economy's aggregate real consumption should be a function of aggregate real income and aggregate real wealth, but not the price level. The world in which consumers make their decisions and take their actions is, however, quite different from the world of traditional consumer theory where rationality and perfect information always prevail. This observation suggests the following basic question to which research by Kleverick and William Branson of Princeton University was directed: If one estimates a short-run consumption function taking account of distributed-lag adjustments, simul-
taneous-equation relationships and the like, will the resulting short-run relationship show that money illusion is present?

Branson and Kleverick estimated such a "money-illusion consumption function" for the United States based on quarterly data for the sample period 1955-I-1965-IV. Using an Ando-Modigliani-Brumberg life-cycle hypothesis model, they estimated a consumption function that allowed the general price level to play an independent role in determining the level of per capita real consumption. The estimates and the tests to which they are subjected lead to the conclusion that the price level does indeed have an independent effect on real consumption.

To the extent that consumers are subject to money illusion, it is reasonable to expect them to have unfavorable attitudes toward general price increases even if matched or exceeded by increases in nominal income. In a paper presented in 1968 to the American Political Science Association, Lepper extended previous work of Kramer's (see p. 20 of preceding Report) on voters' responses in Congressional elections to economic fluctuations. Experimentation, at the national level of aggregation, with a number of different specifications and sample periods indicated that changes in the consumer price index might have some independent influence on voters' satisfaction with economic conditions—in addition to the influence of unemployment (or change in real income)—but the statistical evidence was very weak and was not replicated in experiments with county data. In this same paper, she elaborated the model of voter choice underlying her own, and Kramer's earlier work and speculated on the implications of the regression analysis for policy makers' choice of a national unemployment target.

Wage, price and productivity relations, which were frequently omitted from early aggregate econometric models, have proven to be among the most difficult to estimate econometrically. In regard to prices, this generalization is one of the major conclusions of Nordhaus' review of "Recent Developments in Price Dynamics" (CFDP 296) in which he compares and evaluates nine econometric studies of price behavior. In the same paper, Nordhaus develops a model of long-run profit maximizing behavior for firms, which provides rules for price behavior. He is then able to compare the specification of price relations implied by this theoretical model—as modified by qualitative arguments concerning short-run price adjustments—with the existing econometric studies. In most cases he finds that omission of theoretically relevant variables is a reasonable explanation for the deviation of estimated coefficients from values that would be expected on theoretical grounds.

In the macroeconomic workshop directed by Brainard and Tobin, Nord-
haus has engaged in theoretical and empirical work on the problems of explaining cyclical productivity movements. His approach is based on dynamic models of producers' choice which incorporate uncertainty and fixed costs in changing the labor force.

Assumptions about wage and price adjustments and the nature of expectations about future prices, wages, and sales play crucial roles in explaining unemployment. On a theoretical level, Nordhaus has also explored the determinants of unemployment in a decentralized economy subject to uncertainty in demand. In a preliminary paper, he shows that rational producers will choose to have excess capacity (capital), and that some secular unemployment of labor will be normal, in an economy subject to random fluctuations in demand. Furthermore, the average capital-labor ratio will be biased downward. The amount of unemployment of capital and labor, and the size of the bias in the capital-labor ratio depend on the variability of demand. These conclusions follow from the assumption of a conventional "putty-clay" production function and the absence of short-run cyclical adjustments in prices and wages, even when long-run behavior conforms to all the conventional neo-classical assumptions.

In CFP 290, "Output, Wages and Employment in the Short Run," Stiglitz and R. M. Solow of the Massachusetts Institute of Technology consider the determination of unemployment in the short-run, i.e., when the capital stock is fixed. The authors present a model of short-run "equilibrium" in which money wages are not rigid yet neither do real wages rise in a depression nor do firms sell all they would like at the given prices and wages. Two types of short-run equilibrium are identified, "demand constrained" and "supply constrained." Income shares, in the first of these, are determined according to the so-called Cambridge theory of distribution but, in the latter, marginal productivity theory applies.

VII. Econometric Methodology

Econometric investigations have long been concerned with the analysis of economic relationships using as statistical evidence a sequence of observations over time. But it is only within the last five or ten years that the use of spectral techniques for the analysis of economic time series has been widely advocated. This methodology, which has had a long history of application in engineering and the physical sciences, can be applied to any stochastic process which is stationary in the sense that the correlation between any two observations—made at two distinct points in time—depends only on the time difference. Given this assumption, and some mild addi-
tional conditions, these correlations and other statistical aspects of the process can be estimated by a single sequence of observations, if it is sufficiently long.

Any appropriately regular function of time can be represented, over a finite interval, by its Fourier Series: a representation in terms of elementary periodic functions of differing frequencies and with amplitudes which measure the contribution of that frequency. When the function of time is generated by a stochastic process the amplitude associated with a given frequency will itself be a random variable. If the process is stationary, in the sense previously defined, the decomposition may be taken to have the property that the random amplitudes at two differing frequencies are mutually orthogonal—or in the special case of amplitudes with a normal distribution, independent of each other. The sum (or in some cases, the integral) of the variances of these amplitudes, for all frequencies less than a given frequency, is defined to be the spectrum of the process.

The spectrum fully determines the covariances of the process and as such can be used as an alternative description, at least in so far as the second order moments are concerned. One of the major virtues of the spectral description of a stationary process is that a number of elementary transformations of the process—such as smoothing or filtering of the observations—can be represented by a simple modification of the spectrum. On the other hand the spectrum itself may have less of an intuitive appeal for economic time series than in other applications where it may be capable of a concrete physical interpretation. Other possible drawbacks are that spectral techniques may require either a longer series of observations than is typically available in economics, or the assumption of stationarity over a longer horizon than is justified.

One of the major applications of spectral techniques has been to the appraisal of the seasonal adjustment procedures developed by the Bureau of the Census. This work, originally initiated by Nerlove, led to the development of several informal criteria for good seasonal adjustment. These criteria were expressed in terms of the spectral distributions of the series both before and after seasonal adjustments, and on the properties of the cross spectrum (based on the covariance between lagged observations in two series) between the adjusted and unadjusted series. Though apparently plausible, these criteria were essentially ad hoc and were not developed with reference to any specific purpose of seasonal adjustment. Grether and Nerlove (CFDP 261) considered several possible definitions of "optimal" seasonal adjustment—in terms of the series themselves—and showed that even under ideal circumstances each of the definitions led to seasonal ad-
justment procedures which violated some or all of the spectral criteria. This work therefore casts some doubt on the spectral criteria and shows the need for specifying the objectives of seasonal adjustment directly.

The same basic statistical theory was applied by Grether (CFDP 279) to the problem of deriving distributed lag models when the economic agent's behavior depends upon his forecast of the value of an economic time series or upon an estimate of some component of a time series (e.g. the seasonal component). Distributed lags of the so-called rational type were derived and it was shown how the order of the lag depends upon the forecast horizon and on the properties of the series being forecast. As an example these results were applied to a simple model of inventory adjustment and production smoothing.

Research at the Cowles Foundation on spectral techniques was considerably strengthened by the visit of Professor E. J. Hannan, of the Australian National University, for the second semester of 1969–1970. During Hannan's visit final revisions were made on a volume entitled *Multiple Time Series* which was published in the fall of 1970. The volume is a comprehensive survey of spectral techniques applied to stationary time series in which the observation at each moment of time is a vector rather than a single number.

Many econometric models make the simplifying assumption that the residuals, or the incremental stochastic input in the current observation, are independent of each other at successive moments of time. Not only is this assumption implausible for a variety of models, but it cannot be maintained under a number of transformations which are customarily used to simplify the analysis. For example, if one series of observations is obtained from a second by means of a distributed lag plus an error term, then the first series of observations may in many cases also be represented in an auto-regressive form. If the errors are assumed to be independent of each other in one of these formulations then they will be serially correlated in the other.

During his visit to Cowles, Hannan investigated a number of basic econometric problems, under the general assumption that the residuals form a stationary stochastic process. In CFDP 294 "Time Series Regression with Linear Constraints," written jointly with R. D. Terrell (Australian National University), optimal procedures are given for estimating a system of regressions with linear constraints on the coefficients; asymptotically efficient estimates are obtained for the regression coefficients and the parameters of the residual process, and an application to the estimation of systems of demand equations is discussed.

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A second paper CFDP 298 "Non-Linear Time Series Regression" is concerned with the estimation of regression parameters which occur in a non-linear fashion—a problem which was previously considered by Jennrich* under the restrictive assumption that the residuals are identically distributed, independent random variables.

In CFDP 291, Hannan and D. F. Nicholls (Australian National University) have treated the problem of estimating the parameters of an equation with lagged dependent variables, exogenous variables and moving average disturbances. The presence of the moving average structure in the errors greatly complicates estimation due to the essential non-linear nature of the problem. Hannan and Nicholls suggest a computational procedure for jointly estimating all the parameters of the equation and prove that their method is asymptotically efficient. In addition to treating the general model they also treat the case in which the equation arises from a transformation of a rational distributed lag model.

During the period of this report, Nerlove continued to explore the important area of estimating dynamic economic relationships from a time series of cross-section data. The methodology had previously been introduced by Pietro Balestra and Nerlove in their study of the demand for natural gas, and has also been used in work originally undertaken with Donald Hester on estimating the rates of return on investments in individual common stocks. A condensed summary of this later project was reported by Nerlove in the paper "Factors Affecting Differences Among Rates of Return on Investments in Individual Common Stocks."

Since the general problem of determining the properties of alternative estimation techniques for time series of cross-section data has proved to be rather intractable analytically, Monte Carlo methods have been used extensively. Two series of experiments have been performed so far and are reported in the two papers by Nerlove, CFDP 266 and CFDP 257. The second paper shows quite clearly the relevance of large-sample properties, especially consistency and efficiency, in small-sample situations.

In CFDP 271, Nerlove explored the properties of certain estimates which arise in connection with a specific model in the study of cross section data over time. At each moment of time, each observation in the cross-section is assumed to be a function of a number of independent variables, with an error term depending both on the time and the individual in the cross-section. An analysis of the least squares estimates is carried out under the assumption that the error process can be decomposed into three indepen-

dent parts: one specific to the individual, one related to the time of observation, and a remainder independent of the other two processes.

The previous three year report described in detail the important work by J. Kadane on the comparison of estimators for simultaneous equation models, under the assumption that the parameters of the error process are small. Asymptotic moments for these estimates can be derived for finite samples—as the variance of the errors tend to zero—instead of the customary asymptotic procedure which requires the number of observations to tend to infinity. Two applications of this work are reported in the papers, "Testing Overidentifying Restrictions when the Residuals are Small" (CFP 326), and "Comparison of k-Class Estimators when the Disturbances are Small" (CFDP 269), both by Kadane. In the second of these papers, involving models with no lagged endogenous variables, it is shown that for equations in which the degree of overidentification is no larger than six, the two stage least squares estimator uniformly dominates the limited information maximum likelihood estimator.
GUESTS

The Cowles Foundation is pleased to have as guests scholars and advanced students from other research centers in this country and abroad. Their presence contributes stimulation and criticism to the work of the staff and aids in spreading the results of its research. To the extent that its resources permit, the Foundation has accorded office, library, and other research facilities to guests who are in residence for an extended period. The following visited or were associated with the organization in this manner during the past three years.

HELmut Frisch (University of Vienna), January–May, 1969. Sponsored by the Ford Foundation.

WAhidul Haqur (University of Toronto), June–August, 1967.


COWLES FOUNDATION SEMINARS
AND CONFERENCES

Seminars
In addition to periodic Cowles Foundation staff meetings, at which members of the staff discuss research in progress or nearing completion, the Foundation also sponsors a series of Cowles Foundation Seminars conducted by colleagues from other Universities or elsewhere in Yale. These speakers usually discuss recent results of their research on quantitative subjects and methods. All interested members of the Yale community are invited to these Cowles Foundation Seminars, which are frequently addressed to the general economist including interested graduate students. The following seminars occurred during the past three years.

July 1, 1967–June 30, 1970

1967

November 30. Roy Radner, University of California, Berkeley, "The Role of Prices as Information Signals in the Allocation of Resources under Uncertainty."

1968
January 2. Hiroyuki Ueda, Osaka University, "Income Distribution and Growth in a Monopolist Economy."

March 27. T.N. Srinivasan, Indian Statistical Institute and Visiting Professor of Economics at Stanford University, "Optimal Savings under Uncertainty."

April 5. Vernon L. Smith, Brown University, "Economics of Production from Natural Resources."


May 3. Amartya Sen, Yale and University of Delhi, "Interpersonal Aggregation and Social Choice."

May 15. Jacques Drèze, University of Chicago and University Catholique de Louvain, "Two Certainty Equivalents Theorems for Savings under Uncertainty."

May 24. Merton Miller, University of Chicago, "Portfolio Theory and the Structure of Interest Rates."

May 31. Benedikt Korda, Higher School of Economics, Prague, "Recent Economic Events in Czechoslovakia."


October 24. Harold Watts, University of Wisconsin, "The Negative Income Tax Experiment in New Jersey."
December 6. CHRISTOPHER A. SIMS, Harvard University, "Some Pitfalls of Approximate Specification in Distributed Lag Estimation."


1969

January 24. EDWARD J. HANNAN, Australian National University, "Mixed Moving Average Auto-regressive Processes."

February 14. KENNETH J. ARROW, Harvard University, "Existence of Temporary Equilibrium."


April 22. LAWRENCE KLEIN, University of Pennsylvania, "The Theory of Economic Prediction."

April 25. DONALD TUCKER, The Urban Institute, "Money Demand and Market Disequilibrium."

May 16. ASSAR LINDBECK, University of Stockholm and Columbia University, "Stabilization Policy in 'Narrow Band' Economies."

1970

January 16. ROBERT E. HALL, University of California, Berkeley, "Inflationary Bias in Labor Markets."

March 6. DUNCAN FOLEY, Massachusetts Institute of Technology, "Economic Equilibrium with Marketing Costs."

March 11. ALAN S. MANNE, Stanford University, "A Dynamic Multi-Sector Model for Mexico, 1968-80."

April 10. JAMES BUCHANAN, Virginia Polytechnic Institute, "Notes on the Theory of Supply."

April 17. ZVI GRILICHES, Harvard University, "Estimating Production Functions from Micro-Data."


May 8. ARNOLD ZELLNER, University of Chicago, "Bayesian Inference in the Analysis of Log-Normal Distributions and Regressions."

Conferences

The Cowles Foundation was also the host for two conferences in the Fall of 1968. The first of these, held on September 20–21, was concerned with the Federal Reserve Board-M.I.T. forecasting model and more general considerations of model building and policy simulation. The second was a Symposium on Economic Growth Theory, which took place on November 22–24.
FINANCING AND OPERATION OF THE
COWLES FOUNDATION

The Cowles Foundation relies largely on gifts, grants and contracts to finance its research activities. Yale University contributes to the Cowles Foundation the use of a building at 30 Hillhouse Avenue which provides office space, a seminar room, and related facilities. The University also supports the Foundation’s research and administration through paying or guaranteeing part or all of the non-teaching fractions of the salaries of three permanent staff members.

The gifts of the Cowles family are the cornerstone of the financial support of the Cowles Foundation. These gifts provide a permanent source of untied funds that assure the staff continuing research support, that permit the staff freedom to shift the balance of their time among various subjects of research, and that provide for general operating expenses not appropriately chargeable to grants and contracts for work on specific topics. In addition, a growing amount of financial support has come from grants and contracts from the National Science Foundation, the Ford Foundation, the Office of Naval Research and other, usually private, sources. The amount of this support varies from time to time and much of it has, in the past, been tied to specific research projects. For two of the past three years, however, the Cowles Foundation has been fortunate in having sizeable institutional grants from the National Science, and Ford Foundations. The National Science Foundation grant is a “continuing” grant providing annual funding for the period July 1968 through June 1973. Additional funds for support of the general program of the Cowles Foundation and for a program of visiting staff members were generously provided by the Ford Foundation for the same period. This Ford visitors program is intended specially to facilitate visits by Eastern European economists, and also by scholars in disciplines other than economics but related to interests of Cowles Foundation staff. These guests are regular members of the Cowles Foundation staff for the period of their stay—generally four months or longer.

The major part of Cowles Foundation expenditures is accounted for by research salaries (and associated fringe benefits). The rest of the budget consists of office and library salaries, overhead expenses such as the costs of preparing and distributing manuscripts, and the costs of computing services (the Yale Computer Center currently makes available the services of a direct coupled IBM 7040–7094 system, an IBM time-sharing computer and necessary auxiliary equipment).

The pattern of Cowles Foundation income and expenditures in recent years is outlined in the table below.
ANNUAL INCOME AND EXPENDITURES
OF THE COWLES FOUNDATION

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During the period of this report, the research staff of the Cowles Foundation included 18 or 19 members in faculty ranks. This size was determined by an interplay of considerations including financial constraints, limitations of space at 30 Hillhouse Avenue, and opportunities to bring to the Foundation colleagues who would complement or supplement current research activities. The balance among ranks of the staff in residence varied from year to year depending largely upon leaves of absence and the opportunities to compensate for such absences by visiting appointments. Excluding staff members on such visiting appointments, the staff included six tenured faculty, of the Departments of Economics, Administrative Science and Political Science, and 9 to 11 faculty on term appointments. On average, both the permanent and the younger members of the staff devoted about half of their professional effort in the academic year, and up to two full months in the summer, to their research and to seminars and discussions with their colleagues.

These activities were supported by the services of five secretaries and manuscript typists who, under the direction of Miss Althea Strauss, prepared and circulated Cowles Foundation Papers and Discussion Papers. A varying number of student research assistants and two part-time computer programmers, Mrs. Elizabeth Bockelman and Mrs. Marilyn Hurst, assisted directly in the research studies.

A small library, most recently under the supervision of Mrs. Patricia Graczyk, is maintained in the building of the Cowles Foundation. It makes research materials readily available to the staff and supplements the technical economics and statistics collections of other libraries on the Yale campus. The collection includes a permanent collection of some 5,200
books and 180 journals primarily in the fields of general economics, mathematical economics, econometric studies and methods, statistical methods and data; numerous pamphlets from Government sources and international organizations; series of reprints from 22 research organizations at other universities in the United States and abroad; and a rotating collection of recent unpublished working papers. Although the library is oriented primarily to the needs of the staff, it is also used by other members of the Yale faculty and by students of the University.
MONOGRAPHS
1934–1970*

The monographs of the Cowles Commission (Nos. 1–15) and Cowles Foundation (Nos. 16–22) are listed below:


No. 5. The Variate Difference Method, by GERHARD TINTNER. 1940. Evanston, Ill.: Principia Press. 175 pages. (Out of print.)


No. 8. Price Flexibility and Employment, by OSCAR LANGE. 1944. Evanston, Ill.: Principia Press. 114 pages. A clarification of important concepts that have had much currency in the practical discussion of depression and of economic stabilization generally but had yet to be formalized in a complete theoretical structure.


*Orders for Monographs 12, 14, 16, and 22 should be sent to Yale University Press, 92A Yale Station, New Haven, Connecticut 06520. Orders for, or inquiries concerning, all other Monographs should be sent to the Cowles Foundation for Research in Economics at Yale University, Box 2125 Yale Station, New Haven, Connecticut 06520.


No. 20. Studies in Portfolio Behavior, edited by Donald D. Hester and James Tobin. 1967. New York: John Wiley and Sons. 258 pages. Six essays applying the theoretical developments of Monograph 19 to the circumstances and objectives of various economic units such as households, nonfinancial corporations, banks and life insurance companies.

No. 21. Financial Markets and Economic Activity, edited by Donald D. Hester and James Tobin. 1967. New York: John Wiley and Sons. 256 pages. These essays are concerned with the conditions of general equilibrium in economy-wide financial markets. The micro-economic principles discussed in Monographs 19 and 20 are assumed to guide the behavior of economic units in supplying assets and debts in these markets.

No. 22. Economic Theory of Teams, by Jacob Marschak and Roy Radner, forthcoming 1971. New Haven: Yale University Press. This monograph emphasizes the informational aspect of the problem of designing efficient organizations. After an introduction to decision-making under uncertainty and to the economics of information, a wide variety of models is treated within a unifying conceptual framework.
COWLES FOUNDATION PAPERS
July 1, 1967–June 30, 1970


*Single copies available on request.


*Single copies available on request.


*Single copies available on request.

* Single copies available on request.
COWLES FOUNDATION DISCUSSION PAPERS
July 1, 1967—June 30, 1970

No. 235 GAVIN WRIGHT, An Econometric Study of Cotton Production and Trade before 1860.
236 MARTIN SHUBIK and DAVID H. STERN, Some Experimental Non-Constant-Sum Games Revisited, Part I.
237 MARTIN SHUBIK, A Note on Taxation, Increasing Returns Due to Set-up Costs and Competitive Equilibrium.
238 J. E. STIGLITZ, Distribution of Income and Wealth among Individuals.
239 MARTIN WEITZMAN, Iterative Multi-Level Planning with Production Targets.
240 MARTIN SHUBIK and DAVID H. STERN, Some Experimental Non-Constant-Sum Games Revisited, Part II.
242 JOSEPH E. STIGLITZ, A Re-Examination of the Modigliani-Miller Theorem.
243 JOSEPH B. KADANE, Quiz Show Problems.
244 WILLIAM C. BRINNAB and JAMES TOBIN, Pitfalls in Financial Model-Building.
245 JAMES TOBIN, Notes on Optimal Monetary Growth.
246 JAMES W. FRIEDMAN, On Experimental Research in Oligopoly.
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- Papers: CFP 317
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**YAARI, MENAHEM**

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