

## Uses of Prices \*

[Proceedings of the Conference on Operations Research in Production and Inventory Control, Cleveland, 1954 pp. 90–96]

*Economists have long believed that commodity prices serve adequately to guarantee efficient distribution and allocation of goods. Dr. Koopmans shows that this belief can be confirmed mathematically under certain limiting assumptions, but points out that when these assumptions do not hold we have no knowledge of the reliability of a competitive market in performing the function of allocation. Although much more research is needed to develop the theory of efficient resource allocation, important uses are already indicated in the use of prices for decisions pertaining to allocation of resources within a plant or an industry.*

In the explanations printed in the program for this conference, Dr. Arnoff has been careful to insert the following sentence: “The emphasis of the Conference will be on specific production and inventory problems, Operations Research methods and results, not on theoretical discussions”. In all sessions so far, this injunction has been heeded with admirable consistency. In the relaxed atmosphere of an after-dinner talk, I wonder if I may ask for some indulgence, some relaxation of this rule. It so happens that my own association with problems of efficient allocation of resources has been largely on the theoretical side. I therefore ask permission to offer some general remarks on problems of efficiency in production operations.

The main intent of my remarks is already expressed in the listing of today’s program. Let me read to you three successive items which will help me make my point. In the afternoon’s program we first find: “Uses

\* I am indebted to R. L. Cardwell and A. Harberger for valuable comments on the written version of this presentation.

of Automatic Computing Equipment in Industry". Next comes: "Uses of Automatic and Semi-automatic Processing Equipment". Then we come to the evening, and it says: "Uses of Prices". This nicely suggests that prices can be looked upon as one of several alternative or supplementary tools that can be used in solving problems of efficient utilization of resources in production.

### Allocation and Distribution

The discussions in this conference have been mostly concerned with problems arising within the plant or within the establishment. There are other problems of allocation that go beyond the individual establishment. How are various scarce materials and other resources best apportioned to different firms, to different industries? The idea that this group of problems is handled through a price system is so axiomatic with us that we have stopped giving much thought to this matter. It is obvious, indeed, that business decisions in response to prices actually determine how the various materials flow through industry, and which of the available methods of production are used in their transformation.

One difficulty in fully understanding this use of the price system is that prices at the same time also have another role, of which the general public is much more conscious. This is the role of prices (including wage rates and interest rates) in influencing the distribution of income between occupations, between geographical areas, between farmers, labor, business and investors, or between any other groups with opposite interests in the level of some price.

In the public discussion about government policies, one finds many examples where the considerations that determine policies with respect to prices are essentially distributive. The support price legislation for farm products is one instance. Another example is public utility rate regulation, guided mostly by a concern with what is considered fair in dealing with different groups in the country. The regulation of wage rates such as took place through the War Labor Board is a third example. In such cases, there has been little concern with the best allocation of resources. The dominating considerations have been those of security for the individual, of social justice and social welfare for the various groups in the economy.

**Conflict between Price Uses**

There is some possibility and some actuality of conflict between these two uses of the price system. The allocative function, as I said, is to guide the use of resources. If the manufacturer chooses between two materials which may serve his purpose equally well — leading to the same quality of product — he will choose the material that has the lowest price, or better, the lowest price-plus-manufacturing cost. If he chooses between two different processes of making steel, he will again be guided by profitability comparisons based on the prices of the goods and services that are inputs to or outputs from the production processes in question. Finally, when he chooses the rate of production — the amount to be produced of any given article in a given period — then he is again guided by his estimate of how much can be sold at the price he charges.

Thus, all along the line from raw material to finished product, price is regarded as a label, a signal, a piece of information that is attached to the good or service traded. This information expresses simultaneously the ultimate usefulness to consumers of this good, and the foregone usefulness of other goods or services that could have been produced alternatively from the resources absorbed in making this good. Choices about methods of production and about amounts to be produced are based on this information; if these choices are to be good choices, the information used has to be accurate.

As I said already, there are instances of clear conflict between the allocative and the distributive uses of the price system. Our present surpluses of farm products are the result of one such conflict. The supply of wheat exceeds the amount that can find an outlet at the current price. It would be economical, from the point of view of efficient allocation of resources, for some of this wheat to be fed to chickens, and thus to be turned into eggs; but this doesn't happen to the economically desirable extent, because at the current price it is not sufficiently profitable. Tariffs that prevent the best international division of labor are another example of conflict between allocative and distributive considerations. It is not difficult to spot further examples. Although this conflict, or possible conflict, between the two uses of the price system, is not my main topic tonight, I have mentioned it because, whenever we try to solve our distributive problems through measures that obstruct the allocative function of prices, a loss in efficiency of the economy is incurred. I fully recognize that it is a worthy and necessary objective to give those of us who raise farm products some protection from the wide fluctuations in income due

to weather variability and other causes. More generally, any increase in economic security that legislative and economic ingenuity can provide without affecting the efficiency of our economy is so much gain. Serious thought should be given, however, to ways of safeguarding the allocative function of the price system, and of achieving these important objectives in some more efficient way.

After these general remarks, let me now invite you to join me in a closer look at our main topic: the allocative uses of the price system. I have referred to price as a signal, a piece of information that guides decisions. This is in line with the traditional belief of economists that competitive markets in goods and services lead to the formation of prices that guide producers to an efficient allocation of resources. The belief has been so traditional, and is so inherent in all our thinking, that maybe we have stopped wondering at the fact that this should be so. But it is really a remarkable thing. Let us take just one example: Aluminum may be substituted for copper in transmission wires. The copper may be substituted for steel in car radiators. The steel may again be substituted for aluminum in furniture, or in restaurant equipment, or in a number of other things. Now it may be worthwhile to carry out this round of substitution to some extent. It is conceivable that one could in this way end up with a net saving of some metal somewhere (without spending more of some other metal elsewhere) and come out with products of the same quantity, quality and usefulness. There is nobody in our form of society (unless he should happen to be simultaneously a manufacturer of transmission wires, of car radiators, and of furniture or restaurant equipment) who actually makes this particular calculation all around the circle. And of course one could construct many other and more complicated loops by which one gets back to where one starts, and ask in each case: "Are our materials and our other resources used most efficiently in current practice?"

#### **Mathematics and a Belief of Economists**

The contention that is contained in the traditional belief of economists is that if we associate with each commodity one number, called its price, we have a sufficient summary measure of the whole range of opportunities that are foregone by not using this commodity (or the resources that went into making it) in some other way. So formulated, it is really surprising that this contention should be true. It implies that there is no need for all possible shifts of resources to be explored one by one in all their ramifications. Instead, one price for each commodity is sufficient to measure the loss sustained by other purposes or uses if the commodity

is made and used for one particular purpose. This is truly a remarkable phenomenon in the condensation of information! It looks to me at times like an Indian rope trick! Where is the point of support? How is circular reasoning avoided?

Nevertheless, the traditional belief of economists, I believe, is well-founded and has been confirmed under definite though somewhat limited assumptions by mathematical proof. These assumptions constitute what one might call a model. I am using the word "model" here in the sense of a set of postulates about some part of reality, like in Niels Bohr's model of the atom. The proof, of course, is not concerned with the reality but with the model of reality used, and is no better than that model. Let the model be formulated as follows<sup>1</sup>: There are a number of production processes; each process is described fully by stating the ratios between the inputs to the process (such as materials, labor, managerial effort, use of equipment) and the outputs (which may be one, or more than one, product). The further postulate is made — and this is a rather drastic simplification of reality — that this process has the characteristic of proportionality. If you double its level, and use double the inputs, you will get double the outputs. So far this seems a natural assumption since, if the necessary material and human resources are available, you can always duplicate any process. But the assumption is also made in reverse: If you want to carry out the process at half the previous level, you can do with half the previous inputs and obtain half the outputs. At this point I think you already start with me to doubt the adequacy of the model. The assumption just stated involves a neglect of indivisibilities of equipment and of the human agent. Let us face the fact that this inadequate model is so far the only basis on which the necessary mathematical reasoning has been completed; therefore I must honestly state its assumption. Another assumption is that the outcome of production processes is completely predictable, which is of course at variance particularly with the facts of agricultural production. It is further assumed that there are fixed limits to the availability of basic resources, such as land, labor, minerals, and the use of capital. Finally, the assumption of additivity of production processes is made: The net outputs that would result from each of two processes if engaged in one at a time will add up to those of the two processes if engaged in simultaneously.

Let this be our model, for whatever it is worth as a basis for analysis.

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<sup>1</sup> For a more detailed description, see T. C. Koopmans, "Efficient Allocation of Resources", *Econometrica*, October, 1951, Vol. 19, pp. 455 — 465.

Let us now define the notion of an efficient use of resources. We will say that a productive system efficiently utilizes its resources if a situation has been reached where, if anyone wanted further to increase the output of one particular final good, the only way in which this could be done would be at the cost of a decrease in the output of another final good. One might say that in such a situation the system has reached the limit of what can be done with given primary resources and given technological possibilities. Working from this concept of efficiency, it is a matter of straightforward mathematical proof<sup>2</sup> to arrive at a theorem which, in honor of Adam Smith, I shall call the Invisible Hand Theorem. For the validity of this theorem, it does not matter whether an efficient use of resources has been achieved by sheer luck, by hard calculation, or through the action of a market with a price system. The theorem merely says that if efficiency has somehow been achieved, then there must exist a set of prices on all goods and services such that all production processes in use break even; and such that if anyone were to engage in a production process that does not fit in with an efficient use of resources, he would sustain a loss. By "existence" of these prices I do not necessarily mean that these are quoted in some markets or even that anyone should have a knowledge of any of these prices. I mean "existence" in the mathematical sense. If a man from Mars with unlimited powers of observation and calculation would study the efficient economy in question he would be able to calculate a set of prices and satisfy himself that it endows the various processes of production with the profitability features indicated. In application to a competitive market economy, of course, the main significance of the theorem is that it gives us some confidence in market prices as a guide to decisions. Once our model economy has reached an efficient state, competitive prices will keep it efficient. The processes that are needed to maintain the efficient use of resources just break even. The processes that would be at variance with it would lead to a loss.

Thus the statement that I originally described as surprising is actually proved for one particular and admittedly somewhat narrow model. Another surprise is that the mathematical reasons for its validity are by no means simple or trivial. Most literary economists have believed the theorem all along, and in fact, have tried to argue it verbally at great length. For them there is therefore little novelty in our proposition. But

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<sup>2</sup> See T. C. Koopmans, "Analysis of Production as an Efficient Combination of Activities", Chapter III of *Activity Analysis of Production and Allocation*, Cowles Commission Monograph 13, John Wiley and Sons, 1951.

perhaps the new aspect of recent research is that we have perceived better what mathematical concepts and tools are involved in really proving it. The essential step in this respect was made not by an economist, but by the mathematician von Neumann (unless one regards him, with justification, as an economist as well). The mathematical tools involved are the properties of systems of linear inequalities — or of the convex sets defined by such inequalities — the same tools that happen to be used also in the theory of games. There is little substantive similarity between the two bodies of theory, but it so happens, for formal reasons, that they draw on the same mathematical theories.

Since we have already recognized that our model is a highly simplified image of reality, any inferences from this model to our actual economy must be regarded as extrapolations. When uncertainty actually enters, or when indivisibilities are important, we just do not know at present whether or to what extent the price system of a competitive market economy is a reliable guide to the allocation of resources. This must be regarded as an open question, which at some future time may be resolved through somebody's ingenuity in constructing more realistic models accessible to mathematical analysis. While some progress has been made recently toward the recognition of uncertainty<sup>3</sup> in models, the incorporation of indivisibilities has proved extremely difficult except in a few very simple cases, to which I shall return further on.

#### Value of a Model

Even though our theory of efficient resource allocation thus has not really progressed far enough toward its goal, I think the model I have described is already of considerable help in pushing forward in another direction. I have described how the allocation of resources between industries and between establishments is guided through prices. The allocation of resources within the plant or business establishment is mostly guided through direct managerial decision, sometimes aided by systematic calculation — and this conference is doing a great deal toward clarifying and spreading the use of techniques for making such decisions. However, the allocation problem within the firm is not essentially different from

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<sup>3</sup> Kenneth J. Arrow, "The Role of Securities in the Optimal Allocation of Risk Bearing"; *International Colloquium on Econometrics, 1952*; published by Centre National de la Recherche Scientifique, Paris, 1953.

Gerard Debreu, "An Economic Theory of Uncertainty", unpublished manuscript.

that between firms. It remains the problem of getting the most out of given physical resources when we have a choice of processes of production. By inference, one would expect that the notion of prices as a guide to the allocation of resources could also have useful applications to some of the problems that arise within the plant. This is in fact so. With the help of models such as I have described, one can attach valuations, or prices if you like, to resources within the plant, even though these resources are not being traded.

#### Imputed Prices as a Guide to Decision

I would like to refer you to one example of this use of imputed prices. This example has been provided in a recent article<sup>4</sup> by Robert Dorfman, of the University of California. It is so well chosen and so clearly developed through a sequence of diagrams, that I would not be able to think of a better or more effective one. Let me therefore quote from Dorfman's analysis, which effectively illustrates the idea of extending the price system that already exists outside the firm to certain resources inside the firm, as a guide to decisions. The example concerns a plant which makes both automobiles and trucks. It has available to it four types of capacity: metal stamping, engine assembly, automobile assembly, and truck assembly capacity. The assumption is made that the equipment representing any one of these capacities cannot be shifted from its natural task to that of any other of the four capacities. Automobile and truck production share the use of the metal stamping and engine assembly capacities, utilizing fractions of these capacities that are proportional to the rates of output of automobiles and trucks, respectively. In particular, the production of an automobile gives rise to a larger claim on metal stamping capacity, and a smaller claim on engine assembly capacity, than does the production of a truck. In addition, of course, the production of a given amount of each type of vehicle utilizes a proportional fraction of the vehicle assembly capacity specific to it.

The numerical values of the four capacities, and of their rates of utilization per vehicle of each type produced monthly, the sales prices of the two vehicles, and their direct manufacturing costs (before charging for use of capacities) are chosen by Dorfman in such a manner (see his Figure 1), that the most profitable utilization of available capacities turns out to consist in production of both vehicle types at rates such that each of the

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<sup>4</sup> Robert Dorfman, "Mathematical, or 'Linear', Programming", *American Economic Review*, December 1953, pp. 797-825.



two shared capacities is fully utilized. It turns out further that the specific vehicle assembly capacities are not fully taken up by these rates of output. In his Section V (Figure 9), he then imputes values or implicit prices to the unit of each capacity as follows: The two vehicle assembly capacities are priced at zero because they are not fully used. This does not imply that the unused excess of each of these two capacities is without value as a reserve against future fluctuations in prices or manufacturing costs that may make a different composition of output more profitable. It merely means that in a model that does not recognize uncertainty, these capacities are evaluated on the basis of the temporary uselessness of their unused excesses over best current utilization rates.

The two remaining capacities that currently represent bottlenecks are each assigned a positive price. The levels of these implicit prices are computed from two conditions (two linear equations in two unknowns). These conditions require the prices "paid" for the unit of each of these two capacities to be such that, for each type of vehicle, the excess of sales price over manufacturing cost per vehicle just suffices to "pay" for the amounts of utilization of each capacity that arise from the production of one vehicle of that type.

What good does such a fictitious price calculation do to the manufacturer? Dorfman emphasizes its usefulness, particularly in more complicated cases, in providing shortcuts to the computation of a most profitable production program. I should like to add, on my own responsibility but, I believe, in the spirit of Dorfman's analysis, that the calculation of prices implicit in a proposed program also helps in making decisions about alternative ways of using these same resources. Suppose there is a question of producing some third vehicle, say a jeep, which again lays different claims on some or all of the four capacities. Will it be worthwhile to shift at least some capacity to the production of jeeps? Here our principle of representing each scarce resource by a single price (or unit value) comes into play. To answer the question raised, all we need to do is to charge for the use of a unit of each of the two scarce capacities for the production of jeeps at the same rates that are already charged (fictitiously) against trucks and automobiles. Then, if the price at which the jeep is sold, less its direct manufacturing cost, is less than the "charges" for use of these two capacities, then the diversion of capacity to jeep production is unprofitable. If there is just enough to meet these charges, the use of some capacity to jeeps is a matter of indifference as far as profits are concerned. But if there is a net excess, then the production of jeeps, at some yet to be determined rate of output, along with cars and trucks

will be more profitable than to reserve all scarce capacity for the production of cars and trucks only. It is then worthwhile to set up a new program calculation to determine the most profitable rate of output for jeeps.

The calculation of implicit prices on the use of capacities thus enables the manufacturer to play a game of market within his own house. He sets "competitive" prices on his scarce resources. Then he splits himself up into different and competing personalities, and bids against himself for the use of these scarce resources for alternative purposes. An efficient use of resources can be secured if he guides his decisions in this way. The effect is the same as if it all happened in an actual competitive market.

#### Other Applications and Uses

I would like briefly to mention some other applications of this pricing principle. I once worked out one myself that relates to transportation systems<sup>5</sup>. I believe that the transportation problem was discussed in some previous sessions today or yesterday. In this problem one looks for the cheapest way to ship a certain good from many origins to many destinations, if freight rates are given. The case I was interested in was parallel, but the good to be shipped was itself an empty vehicle (truck, railroad car, airplane, ocean ship or other movable piece of transportation equipment). The purpose was to move these empty "vehicles" from the places where they end up after unloading to the other places where they are needed for the next loading, in such a way as to minimize the average number of "vehicles" that is tied up in these empty movements. The same pricing device that I have described for an automobile firm is applicable here. One works out certain "prices" on the vehicles in question, that vary with the location of the vehicle. Only the differences in vehicle "prices" between various locations enter into the analysis. Once these prices have been determined, the rule of action is that the movement of an empty vehicle from one place to another will be justified if the difference in price on that vehicle between the destination and the origin of the empty voyage just suffices to pay for the time or other cost of moving that vehicle

<sup>5</sup> T. C. Koopmans, "Optimum Utilization of the Transportation System", *Proceedings of the International Statistical Conference Held in Washington, D. C. September 6 - 18, 1941*, Vol. 5, 1949.

T. C. Koopmans and S. Reiter, "A Model of Transportation", Chapter XIV of *Activity Analysis of Production and Allocation*, Cowles Commission Monograph 13, John Wiley and Sons, 1951.

in that way. Here again a price system, such as has been in operation for a long time in the tramp shipping market in London, can also be used, in the absence of a market, as a computing device for allocating transportation equipment efficiently.

Some current research is concerned with extending, or attempting to extend, this method of pricing to situations that go beyond the model I have described, particularly situations in which indivisibilities are present. What has been done in this direction, so far, covers just a few very simple cases. These studies seem to indicate that in some problems with indivisibilities one can extend the notion of implicit prices, and in some one cannot. As an example where one can, I would like to mention the personnel assignment problem which I had the privilege to discuss in more detail here in Cleveland at a meeting of the Operations Research Society in May 1953, also at Case Institute of Technology. I shall therefore be very brief now. It is a problem of matching, say, operators and pieces of equipment<sup>6</sup>. The data are as follows: For each combination of one out of  $n$  persons with one out of  $n$  pieces of equipment, we are given a measurement of the value or productivity of that particular operator-equipment combination. If there are ten operators and ten machines, this gives us a hundred numbers. In the end, of course, each person is to be matched up with only one piece of equipment. The problem is how to assign people to equipment so as to make the sum of the productivities associated with the ten matched pairs as large as possible.

Of course the problem is not limited to operators and pieces of equipment. It can refer to plants and locations, or to desks and cabinets in an office, or to a variety of other particular realizations; but the same structure of the problem recurs in many situations. Staying with pieces of equipment and operators, one can again develop a price system<sup>7</sup>. There is a set of wage rates, say, for the operators, and rentals for the equipment, that have the following properties. If you take a pair that is matched in

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<sup>6</sup> R. L. Thorndike, "The Problem of Classification of Personnel", *Psychometrika*, Vol. 15 (1950), pp. 215 – 235.

J. von Neumann, "A Certain Zero-Sum Two-Person Game Equivalent to the Optimal Assignment Problem", pp. 5 – 12, in *Contributions to the Theory of Games*, Vol. II, 1953, *Annals of Mathematical Studies* Number 28, H. W. Kuhn and A. W. Tucker, Editors.

<sup>7</sup> This result can be obtained through an economic interpretation of certain aspects of von Neumann's solution (see footnote 6), developed jointly by M. Beckmann and myself.

an optimal assignment, then the value produced by that pair will be just enough to pay for the wage of the operator, and for the rental of the equipment. But if you make any wrong assignments, if you put an operator with a piece of equipment so as to prevent someone else who in view of all available alternatives was really more suitable for it, then the value of that operator-equipment combination would be insufficient to pay the wage and the rental on that combination. Thus we again have an application of the notion of imputed prices that sustains an optimal solution, and helps one find it.

Another problem involving indivisibilities was presented, along with its solution, by Dr. Selmer Johnson of the RAND Corporation before the Christmas 1953 meeting of the Econometric Society in Washington. This elegant piece of analysis relates to a simple situation arising in production scheduling. There are a number of items that have to be processed successively on two machines, A and B, the two machine operations occurring in that same order for all items. The process time (including set-up time) on each machine is known for each item. These times are independent of the order in which the items go through either machine. The problem is to put the items through each machine in such a sequence that the total time period elapsing between the placement of the first item on the first machine and the completion of the last item on the second machine is made as short as possible.

Since Johnson's analysis will soon be published<sup>8</sup>, I will not here anticipate the nature of his surprisingly simple solution. I will only describe one aspect of the solution which permits me to add an equally simple price interpretation to it.

Let us add to the data of the problem that each minute by which we can reduce the total processing time is worth one dollar to us. Consider an arbitrarily given sequence in which the items are processed through machine A. (Johnson shows that once the sequence on A is fixed — optimally or not — no retrievable time is lost if the items are then processed through machine B in the same sequence.) Now consider the question of what it costs us, through postponement of the time of availability of all fully processed items, if a particular one of the machines, at some given instant separating process times of successive items, is diverted for

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<sup>8</sup> Selmer Johnson, "Optimal Two- and Three-Stage Production Schedules with Setup Times Included", *Naval Research Logistics Quarterly* (1954), 61 — 68.

one minute<sup>9</sup> to some use outside the processing programs considered. The answer implied in Johnson's analysis is that the entire process period falls into two parts. During the first part, diversion of machine A for one minute costs us a dollar, diversion of machine B is without cost. During the second part the situation is reversed. Machine A can spare a minute between items without delaying the program, but the schedule on machine B is absolutely tight. The item that comes off A at the time point when the pressure shifts from machine A to machine B must be started on B immediately when it comes off A if the time of completion of the program is not to be delayed. We shall call this the critical item, associated with the sequence arbitrarily assigned to the items on each of the two machines.

This answer to our question suggests placing a price of \$ 1 a minute on the use of each machine during its tight period, and a price of zero during its period of slack. Would this price system help in finding and/or sustaining an optimal sequencing of the items? It does to a certain extent. Consider a change in the placement of one item (which is not to be the critical item) within a given sequence of all other items. In particular consider only such changes in placement of an item that leave the identity of the critical item unaffected. Then it is easily seen that any such change will shorten, leave unchanged, or lengthen the total process period, depending on whether the charge on that item increases, remains constant, or decreases as a result of the change. This suggests a procedure for stepwise improvement of an arbitrarily given ordering of items that may prove useful in more complicated cases than that already solved by Johnson. The simple rule given applies unless the change in placement considered affects the identity of the critical item, in which case charges on other items change also, and these changes need to be taken into account as well. In the optimal sequence there are no changes in placement of any item that diminish the charge on that item without affecting the identity of the critical item. This interpretation is simplest if, as we shall assume, the given table of process times obliges the analyst through the absence of certain kinds of ties that could arise from "accidental" equalities.

Another use of this pricing procedure presents itself, if it is possible, by prior work that does not involve the use of machines A or B, to reduce the process time of some item on one of the machines. It will be worthwhile to spend up to one dollar for each minute taken off the process

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<sup>9</sup> It is assumed that a change in any one process time by at most one minute cannot produce ties of the kind referred to.

time on one of the machines if in an optimal sequence the item incurs a charge for the use of that machine. It will not be worth a penny to reduce a process time for which the charge is zero. These statements are universally valid only for changes in process time that are small enough so as not to affect the identity of the critical item.

We have here an example of a situation involving indivisibilities, where one can sustain allocation by a price rule valid within certain limits. In each application one must verify whether these limits are not exceeded. Further research is needed to clarify in what class of problems involving indivisible resources such price rules can be found, and what the limits of their applicability may be.

### Summary

The examples on which I have touched illustrate in various ways the use of imputed prices as a tool in problems of production. This tool can be used as an alternative to, in addition to, or in combination with the more direct methods of computation that have been discussed in this conference. The use in combination with direct computation deserves particular emphasis. Often it may be useful first to determine the broad outlines of a production program through computations based on a relatively simple model, next to find imputed "prices" for the resources in question that follow from that model, in order finally to use these "prices" to consider the advantage or disadvantage of several possible adjustments to or modifications of the initial program, before or without calculating through more complicated models that incorporate these modifications. The basic idea underlying this technique is that the relation between prices on the one hand and choices of methods of production or of rates of output on the other is essentially the same, whether this relation is secured by a competitive market or is adhered to by optimizing decisions within the establishment.

One may also turn around, and look upon the processes of price formation in the market through bidding and selling between business firms as the equivalent of a vast computing machine on a national or even international scale, on which the utilization of our resources is being "computed". It may be surmised that, from the point of view of productive and allocative efficiency, this machine works all right if the markets in question are in fact competitive, and provided distributive objectives are pursued by means other than the regulation of prices. With some exceptions already noted, this last proviso may be met by and large in our modern economy in peace-time. It has not been and probably could

not have been met in war economies of recent experience, because any price changes large enough to induce the drastic shifts in the use of resources needed by war would have had extremely unequal distributive effects, unwanted particularly in war. For this reason, direct allocation by such bodies as the War Production Board was substituted for price-guided allocation. In such a situation, it would still make sense for a WPB to work out through computation the imputed prices that express the actual scarcity of each important material or product, in order to facilitate consistency of the various decisions that have to be made.

Finally, no matter which problem of utilization of resources one looks at, it is likely that a determination of the proper kind of prices will help in making good decisions.