

Supplemental Material to
Weak Identification of Long Memory with
Implications for Inference

By

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Online Supplement to
“Weak Identification of Long Memory with
Implications for Inference”

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Abstract

This online supplement contains two sections. Section [S1](#) provides details of the datasets employed in our empirical analysis. Section [S2](#) presents various robustness checks for the empirical findings in the main paper. Section [S3](#) provides additional empirical results with economic and climate data.

S1 Data Description

This section details the data series used in our empirical analysis. Tables [S1](#) and [S2](#) describe the two datasets for realized volatility (RV) measures employed in the empirical analysis of Section [4.1](#) of the main text. This includes ten ETFs and 30 Dow Jones Industrial Average stocks obtained from Dacheng Xiu’s Risk Lab (see Table [S1](#)) and 31 internal stock market indices obtained from the Oxford–Man Realized Library (see Table [S2](#)). Table [S3](#) provides summary statistics for the trading volume series employed in Section [4.2](#) of the main text.

Table S1: Summary Statistics for Log Realized Variances of U.S. Equities

Ticker	Start date	Nob	Mean	Std.	Skew.	Kurto.
S&P 500 market ETF (SPY)	03-Jan-1996	6164	-2.19	0.55	-0.20	8.62
Industry ETF: Material (XLB)	05-Jan-1999	5161	-1.98	0.65	-1.41	12.90
Industry ETF: Energy (XLE)	23-Dec-1998	5392	-1.81	0.62	-1.77	14.53
Industry ETF: Financial (XLF)	23-Dec-1998	5403	-1.95	0.65	-0.37	8.05
Industry ETF: Industrial (XLI)	05-Jan-1999	4943	-2.17	0.69	-1.64	14.19
Industry ETF: Technology (XLK)	23-Dec-1998	5403	-1.93	0.60	-0.20	8.33
Industry ETF: Consumer staples (XLP)	23-Dec-1998	5380	-2.40	0.70	-2.01	16.23
Industry ETF: Utilities (XLU)	24-Dec-1998	5108	-2.10	0.64	-1.67	14.36
Industry ETF: Health care (XLV)	05-Jan-1999	5013	-2.25	0.64	-2.58	28.33
Industry ETF: consumer discretionary (XLY)	07-Jan-1999	4948	-2.17	0.71	-1.70	15.31
Dow Jones 30						
Apple Inc (AAPL)	03-Jan-1996	6174	-1.35	0.55	-0.34	9.40
Honeywell International Inc (ALD or HON)	03-Jan-1996	6152	-1.60	0.56	-0.51	9.32
Amgen Inc (AMGN)	03-Jan-1996	6172	-1.47	0.42	0.54	4.15
American Express Co (AEXP or AXP)	03-Jan-1996	6173	-1.56	0.58	0.04	8.99
Boeing Co (BA)	03-Jan-1996	6174	-1.53	0.46	0.49	3.90
Verizon Communications Inc (BEL or VZ)	03-Jan-1996	6145	-1.72	0.47	0.34	5.39
Caterpillar Inc (CAT)	03-Jan-1996	6173	-1.48	0.45	-0.35	12.10
Chevron Corp (CHV or CVX)	03-Jan-1996	6174	-1.68	0.43	0.12	8.35
Salesforce.Com Inc (CRM)	24-Jun-2004	4110	-1.27	0.45	0.37	3.71
Cisco Systems Inc (CSCO)	03-Jan-1996	6173	-1.45	0.47	0.47	3.08
Walt Disney Co (DIS)	03-Jan-1996	6172	-1.60	0.47	0.53	3.29
Goldman Sachs Group Inc (GS)	05-May-1999	5359	-1.48	0.47	1.01	4.46
Home Depot Inc (HD)	03-Jan-1996	6174	-1.56	0.47	0.58	3.29
International Business Machines Corp (IBM)	03-Jan-1996	6174	-1.72	0.47	0.64	3.28
Intel Corps (INTC)	03-Jan-1996	6172	-1.43	0.43	0.58	3.31
Johnson & Johnson (JNJ)	03-Jan-1996	6173	-1.88	0.45	0.60	3.47
JPMorgan Chase & Co (JPM)	03-Jan-1996	6153	-1.50	0.55	0.23	7.69
Coca-Cola Co (KO)	03-Jan-1996	6174	-1.84	0.46	0.60	3.40
McDonald's Corp (MCD)	03-Jan-1996	6173	-1.75	0.47	0.47	3.50
3M Co (MMM)	03-Jan-1996	6173	-1.75	0.47	0.21	5.27
Merck & Co Inc (MRK)	03-Jan-1996	6172	-1.65	0.42	0.69	4.08
Microsoft Corp (MSFT)	03-Jan-1996	6172	-1.60	0.42	0.52	3.31
Nike Inc (NIKE)	03-Jan-1996	6174	-1.55	0.48	-0.13	11.20
Procter & Gamble Co (PG)	03-Jan-1996	6172	-1.83	0.46	0.76	3.96
Travelers Companies Inc (SPC or TRV)	03-Jan-1996	6153	-1.73	0.65	-1.05	11.17
UnitedHealth Group Inc (UNH)	03-Jan-1996	6156	-1.48	0.48	-0.09	9.40
Visa Inc (V)	20-Mar-2008	3183	-1.68	0.48	1.00	4.26
Walgreens Boots Alliance Inc (WAG or WBA)	03-Jan-1996	6141	-1.53	0.43	-0.29	9.54
Walmart Inc (WMT)	03-Jan-1996	6139	-1.73	0.49	0.22	5.99
Exxon Mobil Co (XOM)	03-Jan-1996	6345	-1.70	0.45	0.55	3.79

Table S2: Summary Statistics for Log Realized Variances of Stock Market Indices

Name (Ticker)	Start date	Nob	Mean	Std.	Skew.	Kurto.
AEX index (AEX)	03-Jan-2000	5459	-9.68	1.02	0.53	3.40
All Ordinaries (AORD)	04-Jan-2000	5409	-10.47	0.96	0.71	4.18
Bell 20 Index (BFX)	03-Jan-2000	5457	-9.81	0.93	0.53	3.41
S&P BSE Sensex (BSESN)	03-Jan-2000	5308	-9.44	0.98	0.60	3.69
PSI All-Share Index (BVLG)	15-Oct-2012	2193	-10.23	0.76	0.66	4.03
BVSP BOVESPA Index (BVSP)	03-Jan-2000	5268	-9.21	0.83	0.62	4.52
Dow Jones Industrial Average (DJI)	03-Jan-2000	5366	-9.86	1.12	0.43	3.52
CAC 40 (FCHI)	03-Jan-2000	5461	-9.48	0.99	0.39	3.28
FTSE MIB (FTMIB)	01-Jun-2009	3042	-9.50	0.88	0.36	3.59
FTSE 100 (FTSE)	04-Jan-2000	5400	-9.65	1.01	0.58	3.66
DAX (GDAXI)	03-Jan-2000	5427	-9.36	1.04	0.42	3.22
S&P/TSX Composite index (GSPTSE)	02-May-2002	4772	-10.30	1.10	0.90	4.47
HANG SENG Index (HSI)	03-Jan-2000	5245	-9.69	0.84	0.68	3.95
IBEX 35 Index (IBEX)	03-Jan-2000	5426	-9.33	0.95	0.17	3.04
Nasdaq 100 (IXIC)	03-Jan-2000	5370	-9.68	1.10	0.44	3.09
Korea Composite Stock Price Index (KS11)	04-Jan-2000	5269	-9.64	1.03	0.47	3.05
Karachi SE 100 Index (KSE)	03-Jan-2000	5216	-9.78	1.04	0.18	4.79
IPC Mexico (MXX)	03-Jan-2000	5370	-9.94	0.92	0.67	3.88
Nikkei 225 (N225)	02-Feb-2000	5205	-9.70	0.94	0.30	3.43
NIFTY 50 (NSEI)	03-Jan-2000	5300	-9.66	1.05	0.42	3.90
OMX Copenhagen 20 Index (OMXC20)	03-Oct-2005	3888	-9.59	0.89	1.10	5.15
OMX Helsinki All Share Index (OMXHPI)	03-Oct-2005	3929	-9.82	1.01	0.91	4.15
OMX Stockholm All Share Index (OMXSPI)	03-Oct-2005	3929	-9.95	1.03	0.81	3.97
Oslo Exchange All-share Index (OSEAX)	03-Sep-2001	4917	-9.56	0.97	0.78	4.11
Russel 2000 (RUT)	03-Jan-2000	5367	-10.07	1.00	0.33	7.28
Madrid General Index (SMSI)	04-Jul-2005	4055	-9.48	0.96	0.27	3.45
S&P 500 Index (SPX)	03-Jan-2000	5369	-9.89	1.15	0.38	3.39
Shanghai Composite Index (SSEC)	04-Jan-2000	5171	-9.38	1.06	0.40	3.06
Swiss Stock Market Index (SSMI)	04-Jan-2000	5363	-9.94	0.90	1.07	4.61
Straits Times Index (STI)	03-Jan-2000	3425	-10.05	0.68	0.56	3.91
EURO STOXX 50 (STOXX50E)	03-Jan-2000	5458	-9.38	1.05	0.01	5.70

Table S3: Summary Statistics: De-trended Log Trading Volume

Ticker	Start date	Nob	Mean	Std.	Skew.	Kurto.
S&P 500 market ETF (SPY)	01-Feb-1993	7137	-0.16	0.51	-0.66	6.86
Industry ETF: Material (XLB)	23-Dec-1998	5647	-0.30	0.81	-0.99	6.18
Industry ETF: Energy (XLE)	23-Dec-1998	5647	-0.25	0.66	-0.58	4.35
Industry ETF: Financial (XLF)	23-Dec-1998	5647	-0.24	0.66	-0.66	4.85
Industry ETF: Industrial (XLI)	23-Dec-1998	5646	-0.35	0.87	-1.04	5.81
Industry ETF: Technology (XLK)	23-Dec-1998	5647	-0.17	0.55	0.09	3.75
Industry ETF: Consumer staples (XLP)	23-Dec-1998	5647	-0.27	0.68	-0.12	4.47
Industry ETF: Utilities (XLU)	23-Dec-1998	5647	-0.28	0.73	-0.79	5.79
Industry ETF: Health care (XLV)	23-Dec-1998	5647	-0.32	0.76	-0.44	5.33
Industry ETF: consumer discretionary (XLY)	23-Dec-1998	5647	-0.38	0.89	-1.09	6.13
Dow-Jones 30						
Apple Inc (AAPL)	01-Feb-1993	7137	-0.13	0.47	0.39	4.40
Honeywell International Inc (ALD or HON)	01-Feb-1993	7137	-0.10	0.42	0.30	5.04
Amgen Inc (AMGN)	01-Feb-1993	7137	-0.11	0.43	0.54	6.58
American Express Co (AEXP or AXP)	01-Feb-1993	7137	-0.10	0.42	0.31	4.26
Boeing Co (BA)	01-Feb-1993	7137	-0.12	0.46	0.12	4.81
Verizon Communications Inc (BEL or VZ)	01-Feb-1993	7137	-0.09	0.39	0.54	6.02
Caterpillar Inc (CAT)	01-Feb-1993	7137	-0.10	0.42	0.29	4.20
Chevron Corp (CHV or CVX)	01-Feb-1993	7137	-0.07	0.34	0.27	4.67
Salesforce.Com Inc (CRM)	23-Jun-2004	4267	-0.15	0.51	0.48	4.94
Cisco Systems Inc (CSCO)	01-Feb-1993	7137	-0.09	0.40	0.11	6.99
Walt Disney Co (DIS)	01-Feb-1993	7137	-0.10	0.41	0.71	5.17
Goldman Sachs Group Inc (GS)	04-May-1999	5558	-0.12	0.44	0.32	4.92
Home Depot Inc (HD)	01-Feb-1993	7137	-0.09	0.40	0.48	4.50
International Business Machines Corp (IBM)	01-Feb-1993	7137	-0.09	0.39	0.58	4.67
Intel Corps (INTC)	01-Feb-1993	7137	-0.08	0.39	0.08	5.69
Johnson & Johnson (JNJ)	01-Feb-1993	7137	-0.07	0.36	0.41	5.03
JPMorgan Chase & Co (JPM)	01-Feb-1993	7137	-0.10	0.40	0.22	3.99
Coca-Cola Co (KO)	01-Feb-1993	7137	-0.07	0.36	0.41	4.42
McDonald's Corp (MCD)	01-Feb-1993	7137	-0.09	0.39	0.52	4.44
3M Co (MMM)	01-Feb-1993	7137	-0.09	0.39	0.56	4.74
Merck & Co Inc (MRK)	01-Feb-1993	7137	-0.09	0.39	0.65	4.99
Microsoft Corp (MSFT)	01-Feb-1993	7137	-0.08	0.39	0.35	4.68
Nike Inc (NIKE)	01-Feb-1993	7137	-0.12	0.46	0.48	4.76
Procter & Gamble Co (PG)	01-Feb-1993	7137	-0.09	0.39	0.72	6.51
Travelers Companies Inc (TRV)	01-Feb-1993	7137	-0.12	0.46	0.03	4.48
UnitedHealth Group Inc (UNH)	01-Feb-1993	7137	-0.13	0.47	0.41	5.03
Visa Inc (V)	19-Mar-2008	3326	-0.11	0.42	0.72	5.66
Walgreens Boots Alliance Inc (WAG or WBA)	01-Feb-1993	7137	-0.10	0.41	0.52	4.54
Walmart Inc (WMT)	01-Feb-1993	7137	-0.08	0.38	0.58	4.66
Exxon Mobil Co (XOM)	01-Feb-1993	7137	-0.06	0.33	0.31	5.12

S2 Empirical Robustness Checks

S2.1 Subsample analysis for international stock market indices

We perform a subsample analysis for the realized volatility (RV) measures of the 31 international stock market indices obtained from the Oxford–Man Realized Library. This complements the full-sample analysis shown in Figure 8 of the main text. The full sample is divided into two subsamples using the first day of 2010 as the cutoff. The confidence sets for α and d are plotted in the top (resp. bottom) row of Figure S1 for the 2000–2009 (resp. 2010–2021) subsample. Due to data availability, the first subsample for the FTSE MIB index (FTMIB) is very short, containing only seven months of data, and results in highly inaccurate estimates. In addition, the PSI all-share index (BVLG) is unavailable for the first subsample as the data only starts from 2012.

We find similar patterns as those in the full-sample analysis shown in Figure 8 of the main text. Indeed, we again observe that the confidence sets of some indices contain two disjoint regions, but many of them have only a single region associated with long-memory. As expected, the bifurcation, or weak identification, phenomenon is more severe for the shorter subsamples. The number of bifurcated confidence sets increases from 9 to 18 (resp. 15) for the first (resp. second) subsample.

S2.2 Analysis using alternative realized volatility measures

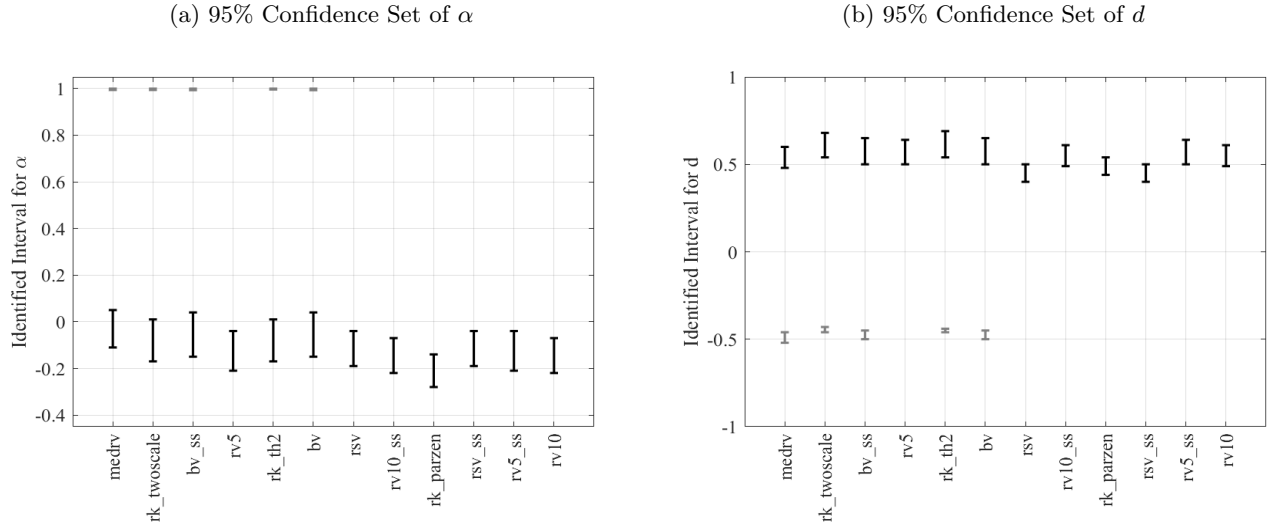
Our main empirical analysis (see Section 4.1 of the main text) on the international stock market indices are based on the classical 5-minute RV from the Oxford–Man Realized Library. In this section, we further check whether the weak identification issue is specific for this particular measure by considering several alternative RV measures that are also available in the Realized Library. For brevity, we focus on the S&P 500 index (SPX). The list of RV measures and their summary statistics are reported in Table S4.

Figure S2 reports the projected one-dimensional confidence sets for the autoregressive parameter α and the memory parameter d , computed separately for each RV measure. These results are in line with those reported in the main text. Some confidence sets bifurcate with two disjoint regions, suggesting the presence of weak identification. When the confidence set contains only one region, the evidence points to long-memory. It is also interesting to note that the long-memory regions of the confidence sets are relatively stable across different RV measures.

Table S4: Summary Statistics: log volatility estimators of SPX (Realized Library)

Name (Ticker)	Nob	Mean	Std.	Skew.	Kurto.
Median realized variance (5-min, <i>medrv</i>)	5369	-10.81	1.22	0.37	3.34
Realized kernel variance (Two-Scale/Bartlett, <i>rk_twoscale</i>)	5369	-10.00	1.10	0.43	3.46
Bipower variation (5-min sub-sampled, <i>bv_ss</i>)	5369	-10.09	1.14	0.44	3.46
Realized variance (5-min, <i>rv5</i>)	5369	-9.89	1.15	0.38	3.39
Realized kernel variance (Tukey-Hanning(2), <i>rk_th2</i>)	5369	-9.99	1.11	0.43	3.42
Bipower variation (5-min, <i>bv</i>)	5369	-10.09	1.14	0.44	3.46
Realized semi-variance (5-min, <i>rsv</i>)	5369	-10.69	1.24	0.33	3.26
Realized variance (10-min sub-sampled, <i>rv10_ss</i>)	5369	-9.90	1.17	0.35	3.35
Realized kernel variance (non-flat Parzen, <i>rk_parzen</i>)	5369	-10.01	1.22	0.29	3.25
Realized semi-variance (5-min sub-sampled, <i>rsv_ss</i>)	5369	-10.69	1.24	0.33	3.26
Realized variance (5-min, sub-sampled, <i>rv5_ss</i>)	5369	-9.89	1.15	0.38	3.39
Realized variance (10-min, <i>rv10</i>)	5369	-9.90	1.17	0.35	3.35

Figure S2: Confidence Sets for Alternative Realized Volatility Measures of SPX



S3 Additional Empirical Results

There is evidence that weak identification of the type studied in the present work is present in other economic data and certain climate data. To illustrate we studied two additional time series: US 10-year Treasury Inflation Indexed Security downloaded from the FRED¹ at the daily frequency from January 2, 2003, to June 13, 2022, containing 4,867 observations; and annual global land surface temperature (1880 - 2021) obtained from Berkeley Earth².

We applied identification-robust inference to these two (demeaned) data series. Figure S3 depicts the dynamics of the two data series on the left column and the identified confidence sets on the right column. For both data sets the method identifies two disjoint confidence sets for the model parameters, revealing the presence of weak identification.

¹<https://fred.stlouisfed.org/>

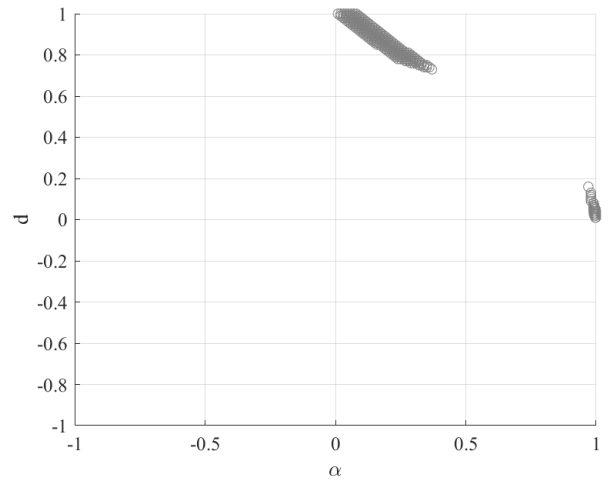
²<http://berkeleyearth.org/>

Figure S3: Time Series Dynamics and 95% Confidence Sets: the US 10-year Treasury Inflation Indexed Security and the Global Land Surface Temperature

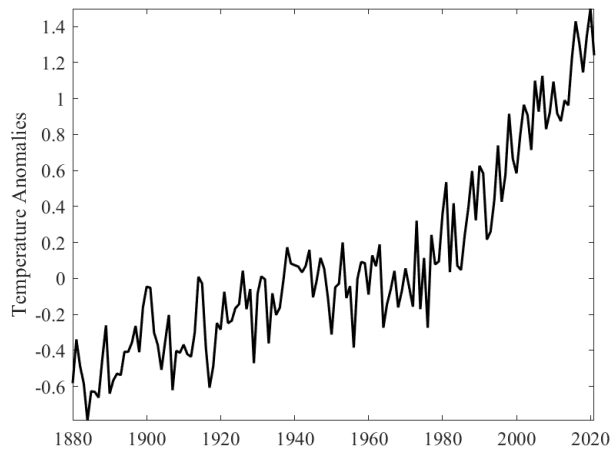
(a) 10-year Treasury Inflation Indexed Security



(b) Confidence Set: 10-year Treasury Inflation Indexed Security



(c) Global Land Surface Temperature



(d) Confidence Set: Global Land Surface Temperature

