ORGANIZATIONAL STRUCTURE AND PRICING:
EVIDENCE FROM A LARGE U.S. AIRLINE

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Abstract
We study how organizational boundaries affect pricing decisions using comprehensive data from a large U.S. airline. We document that the firm’s advanced pricing algorithm, utilizing inputs from different organizational teams, is subject to multiple biases. To quantify the impacts of these biases, we estimate a structural demand model using sales and search data. We recover the demand curves the firm believes it faces using forecasting data. In counterfactuals, we show that correcting biases introduced by organizational teams individually have little impact on market outcomes, but coordinating organizational outcomes leads to higher prices/revenues and increased deadweight loss in the markets studied.

JEL Classification: C11, C53, D22, D42, L10, L93
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1 Introduction

Dramatic decreases in the cost of computation and data storage, along with algorithmic innovations, have increasingly allowed firms to develop data-driven decision optimization systems. Data and algorithms now play a key role in driving firm decisions across industries. This is especially relevant in the airline context where firms must match fixed flight capacity with dynamically evolving demand. To solve this difficult allocation problem, airlines have developed sophisticated pricing systems over the last several decades. These systems depend on inputs from multiple organizational teams. How airlines allocate decision rights across teams within the firm is not unique to the industry. Hotels, cruises, car rentals, entertainment venues, and retailers have all adopted features of the airline pricing model. Given the investments firms have made into these decision machines and their wide use across industries, we may expect that prices are close to optimal.

In this paper, we study how organizational boundaries affect pricing decisions by leveraging a data partnership with a large international air carrier based in the United States.\textsuperscript{1} The granularity of the data allow us to understand the firm’s incentives to adjust prices without needing to assume prices are optimally set. We show that the pricing at a sophisticated firm—one that employs advanced optimization techniques and has a heavy reliance on automation—does not appear to react to some important market fundamentals. This includes not internalizing consumer substitution to other products, using persistently biased forecasts, and not responding to changes in opportunity costs driven by scarcity. We show that these “frictions” are introduced by separate teams within the firm. What happens to prices and allocative efficiency if the firm does not face pricing frictions? Using a new technique to estimate demand and detailed forecasting data to infer the firm’s beliefs about the demand it faces, we find that correcting pricing frictions introduced by teams individually does little to affect market outcomes. However, we also show that improving organizational outcomes through the coordination of pricing inputs can result in increased price targeting and higher revenues, but also higher dead-weight loss for the routes studied. Our results highlight how non-coordinating teams with complementary functions can have significant

\textsuperscript{1}The airline has elected to remain anonymous.
consequences on firm performance.

We begin by providing an empirical glimpse under the hood of dynamic pricing solutions used by airlines. In addition to observing prices and quantities, we also observe granular demand forecasts, outputs of the pricing and allocation algorithms, the optimization code itself, and clickstream data that detail all consumer interactions on the airline’s website. The core data cover hundreds of thousands of flights spanning hundreds of domestic origin-destination pairs. We document the main organizational details of how pricing decisions are made within the firm and provide insights on the incentives to adjust prices over time in Section 2 and Section 3, respectively. We show that all major airlines have similar organizational structures. Therefore, we believe our discussion and subsequent empirical findings likely hold for the airline industry broadly and perhaps for other industries that have adopted similar pricing technologies and organizational structures.

In Section 4, we discuss data patterns that suggest the airline could be doing more with its data. We show that prices do not necessarily adjust when the value of remaining capacity changes. This is caused by coarse pricing, or the use of “fare buckets.” However, the fact opportunity costs may adjust by hundreds of dollars without triggering a price adjustment suggests a mismatch between the fares chosen by one organizational team and demand fundamentals. We establish that the forecasts maintained by a separate team respond to demand “surprises” too little and too late. These demand forecasts are biased upward in two years of data. We establish that the pricing algorithm itself is biased by showing that cross-price elasticities are not considered when setting prices. Finally, we show that the firm chooses prices on the inelastic side of the demand curve—according to their own estimates of demand—in more than half of the data sample. These frictions affect all routes, regardless of market structure, and are even more pronounced in competitive markets (larger forecast bias and more frequent “inelastic prices” based on the firm’s beliefs of (residual) demand). Due to the additional complexity of modeling the competitive interaction, our subsequent analysis focuses on routes where our carrier is the only airline providing nonstop service.

In the second stage of our analysis, we quantify the impacts of these observed pricing frictions on welfare. To do this, we estimate a structural model of consumer demand using a
recently proposed demand methodology (Hortaçsu, Natan, Parsley, Schwieg, and Williams, 2021). In Section 5, we consider a model in which “leisure” and “business” travelers arrive according to independent and time-varying Poisson distributions in discrete time. Consumers know their preferences and solve discrete choice maximization problems. Each consumer chooses among the available flight options or an outside option. We provide evidence to motivate some of our demand assumptions, including that consumers do not appear to be betting on price and consumer arrivals are not endogenous to price.

We estimate the model using consumer search and bookings data. Aggregate search counts calculated from the clickstream data inform the overall arrival process, and we identify the price coefficient using instrumental variables (see Section 6). The estimates presented in Section 7 reveal meaningful variation in demand, with a general increase in search for travel as the departure date approaches and substantial changes in the overall price sensitivity of consumers over time. We discuss similarities and differences in model estimates across routes.

Given the demand estimates, we then ask: what does the firm believe its demand curves look like? We call these “firm beliefs” and we recover them in Section 8 using detailed forecasting data and output from an algorithm that classifies search and bookings as coming from a “leisure” or “business” traveler. Relative to our baseline demand estimates, we find that the firm’s beliefs about their demand has more compressed demand elasticities both within and across routes, more elastic demand near departure, and consumer types that are “closer together” in terms of preferences. Using these recovered demand curves, we confirm our descriptive finding that prices are often too low: nearly 30% of observed prices are below the optimal price even if capacity costs were zero.

In Section 9, we perform counterfactual exercises using a pricing model that closely follows the heuristic the firm uses. First, we isolate pricing frictions individually—removing forecast bias and mismatched fare choices to the forecast, separately. We show that outcomes are largely unchanged because the pricing heuristic commonly defaults to the lowest filed fare because it expects that future demand can be satisfied with remaining capacity. However, we also show internalizing complementarities across teams—correcting the forecast and inputting fares into the algorithm tailored to this forecast—yields very different
outcomes. Coordination guarantees that fares never drop below the optimal price if capacity costs were zero. This raises the distribution of fares offered and allows the firm to target business travelers with higher fares. Revenues increase substantially—upward of 17% for some markets. Dead-weight loss also increases by over 10%. The fact that that the firm may be able to extract additional surplus but has chosen not to do so is puzzling. We argue under-experimentation across organizational teams may play a role. We quantify the use of experiments in the data. Although the firm may have long-run demand in mind when pricing, we argue that observed forecast bias as well as bias in the composition of tickets sold is inconsistent with this hypothesis. We hypothesize the firm may consider the implicit cost of regulatory oversight or long-term competitive responses as alternative explanations. Finally, we acknowledge that the observed pricing decisions may be optimal if the cost of organizational change is sufficiently high.

1.1 Related Literature

This paper contributes to a growing literature in behavioral industrial organization that examines pricing frictions, including DellaVigna and Gentzkow (2019) and Hitsch, Hortacsu, and Lin (2021) in retailing, Huang (2021) in peer-to-peer markets, Ellison, Snyder, and Zhang (2018) in online retailing, and Cho and Rust (2010) in rental cars. We confirm pricing frictions also impact firms that have pioneered advanced pricing algorithms. Our work also contributes to research on miscalibrated firm expectations, as forecasts are persistently biased (Massey and Thaler, 2013; Akepanidtaworn, Di Mascio, Imas, and Schmidt, 2019; Ma, Ropele, Sraer, and Thesmar, 2020). Our work complements existing work, including Blake, Nosko, and Tadelis (2015), who use a large-scale experiment to show sponsored search resulted in negative returns at eBay, and Dubé and Misra (2021), who provide an example where a firm (ZipRecruiter) has not priced optimally and pricing to the correct demand curve greatly increases revenue. In our setting, pricing to the correct demand curves is insufficient to greatly impact revenues if other pricing inputs are not realigned to this change.

Our work also contributes to the literature in organizational economics. The adoption
of information technology (IT) can increase productivity when complementary organizational and management practices are implemented alongside these investments (Bresnahan, Brynjolfsson, and Hitt, 2002; Bloom, Sadun, and Van Reenen, 2012). However, organizations may not adopt technologies that increase productivity or revenues. Atkin, Chaudhry, Chaudry, Khandelwal, and Verhoogen (2017) study barriers to the adoption of cost-saving technology. Sacarny (2018) study the slow take-up of revenue generating activities at hospitals. We estimate significantly higher revenues with alternative pricing inputs, but this outcome requires coordination on complementary tasks that can be difficult in practice (Milgrom and Roberts, 1990, 1995; Siggelkow, 2001). Dessein and Santos (2006) use a team-theoretic model and show that depending on the cost of communication, a firm may choose to be rigid and specialized, e.g., airlines maintaining both pricing and revenue management departments. We estimate the impacts of this decentralized organizational structure and complement recent work by Aguirregabiria and Guiton (2020) and Filippas, Jagabathula, and Sundararajan (2021).

Finally, this paper quantifies the effectiveness of pricing heuristics proposed in operations research using airline data (Littlewood, 1972; Belobaba, 1987, 1989; Brumelle, McGill, Oum, Sawaki, and Tretheway, 1990; Belobaba, 1992; Wollmer, 1992).

2 Organizational Structure and Division Responsibilities

We study the US airline industry, an industry that directly supports over two million jobs and contributes over $700 billion to the US economy. In 2019 alone, 811 million passengers flew within the United States. In addition to being an important industry in its own right, airlines have influenced the development of pricing technologies that are now used in other sectors—for example, in hospitality, retailing, and entertainment and sports events. Although the sophistication of these technologies has improved, many of the original yield

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2Brynjolfsson and Milgrom (2012) provide an overview of this and related work.
3For additional theoretical work on organizational teams, see Che and Yoo (2001), Siemsen, Balasubramanian, and Roth (2007), Alonso, Dessein, and Matouschek (2008, 2015).
management ideas described in McGill and Van Ryzin (1999) and Talluri and Van Ryzin (2004) remain in place today.

Fares at our air carrier depend on the actions of managers in several distinct departments, and these departments do not explicitly coordinate their actions. Generally, decisions become increasingly granular, taking all previous departments’ decisions as given. First, network planning decides the network, flight frequencies, and capacity choices. Second, the pricing department sets a menu of fares and fare restrictions for all possible itineraries. Finally, the revenue management (RM) department decides the number of seats to sell for every fare-itinerary combination.

The RM group maintains the demand forecasting model and pricing algorithm, but does not have control over the fares inputted into these algorithms. The pricing department sets fares, but does not use the forecasting information. The forecasting model incorporates historical information and current bookings information to predict flight-level sales. The pricing algorithm allocates remaining inventory given the fare menu. A commonly used pricing heuristic in the industry is Expected Marginal Seat Revenue (EMSR-b), which closely approximates the algorithm used by the firm. We provide additional details of the EMSR-b in Online Appendix A and outline the algorithm here. EMSR-b belongs to a class of static optimization solutions. Dynamics are removed because it assumes all future demand will arrive tomorrow. The key trade-off, therefore, is to offer seats today versus reserve them for tomorrow. Given all pricing inputs, it calculates the opportunity cost of a seat and then assigns the number of seats it is willing to sell at all price levels. Lowest priced units are assumed to sell first. If expected future demand is high (low), it will restrict (not restrict) inventory at lower prices today.

We demonstrate how inputs impact prices for an example flight at our airline in Figure 1. On the vertical axis, we show the anonymized fare buckets decided by the pricing department, with bucket one being the least expensive and bucket twelve being the most expensive. The bottom right of the graph shows that the pricing department restricts the

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6Each filed fare contains an origin, destination, filing date, class of service, routing requirements, and other ticket restrictions. A common fare restriction decided by the pricing department is an advance purchase discount, which specifies an expiration date for a discounted fare to be purchased by. These discounts are commonly observed seven, 14, and 21 days before departure.
availability of the lowest fares close to the departure date. There is relatively little variation in prices for a given bucket over time. Given fares and the forecast (not shown), the white line marks to lowest available price (LAP) offered to consumers. This is an output of the algorithm maintained by the RM group. All flights, regardless of market structure, flight frequencies, etc., are priced using the same algorithm.

2.1 Potential Pricing Bias with Uncoordinated Inputs

Pricing heuristics can be sensitive to algorithm inputs, which we demonstrate with a simple example. Consider a firm selling 15 units over two sequential markets. Demand in the first period is equal to \( Q_1(p_1) = 10 - 10p_1 \), and demand in the second period is \( Q_2(p_2) = 10 - p_2 \). If the firm maximizes total revenues subject to the capacity constraint, the capacity constraint will not bind, and optimal prices are equal to \( (p_1, p_2) = (0.5, 5) \). This outcome can be also obtained using the pricing and revenue management roles and the pricing algorithm EMSR-b described above if the pricing department assigns prices to be \( \{0.5, 5\} \) and \( \{5\} \), and the RM group “forecasts” demand to be the functions above.

EMSR-b decides the number of seats that can be sold at each input price to ensure that future demand can be satisfied. Lower prices are restricted only in situations where future demand cannot be satisfied. In this case, five seats are needed for period two, and
the algorithm will appropriately allocate all seats to 0.5 in the first period. Suppose instead that the pricing department did not coordinate with the RM group and set prices equal to \{0.2, 0.5, 5\} and \{5\}. Note that all first-period prices leave sufficient capacity available for the second period, which means EMSR-b will allocate all seats at the lowest price, 0.2. Consequently, the heuristic will choose a suboptimal price even though the optimal price, 0.5, is included in the choice set.

2.2 All US Airlines have the same Organizational Structure

Our description of airline pricing is not unique to our airline—all airlines have the same organizational structure and use similar pricing techniques. We show this by collecting job postings information for all the major carriers in the U.S.\(^7\) We confirm that Alaska, American, Delta, JetBlue, Southwest, and United have a network planning, pricing, and revenue management department. As an example, JetBlue Airlines job postings show that the firm has three teams related to pricing: Future Schedules, Revenue Management-Pricing, and Revenue Management-Inventory. Job details delineate team responsibilities. The Revenue Management department at JetBlue has two separate teams, Pricing and Inventory. The Pricing team has ownership over fares by “monitoring industry pricing changes filed through a clearinghouse throughout the day, and determining and executing JetBlues response.”\(^8\) The Inventory team uses “inventory controls to determine the optimal fare to sell at any given moment in time to maximize each flights revenue.”\(^9\) American Airlines managers describe how inventory controls are implemented in Smith, Leimkuhler, and Darrow (1992)—they outline EMSR-b. Because all carriers have the same organizational structure and use similar algorithms, we believe our analysis characterizes the entire industry, rather than the perspective from a single firm.

\(^7\)Screenshots of the job postings are available on request.


3 Data and Summary Analysis

We use data provided by a large international air carrier based in the United States. To maintain anonymity, we exclude some data details. In Online Appendix B, we describe our route selection criteria.

3.1 Data Tables

We combine several data sources, which we commonly refer to as: (1) bookings, (2) inventory, (3) search, (4) fares, and (5) forecasting data.

(1) Bookings data: The bookings data contain details for each purchased ticket, regardless of booking channel, e.g., the airline’s website, travel agency, etc. Key variables included in these data are the fare paid, the number of passengers involved, the particular flights included in the itinerary, the booking channel, and the purchase date.\(^{10}\) Our analysis concentrates on nonstop bookings and economy class tickets.

(2) Inventory data: The inventory data contain the decisions made by the RM group. Inventory allocation is conducted daily. The data include the number of seats the airline is willing to sell for each fare class in economy and aircraft capacity. We also observe output from the pricing algorithm, including the opportunity cost of a seat.

(3) Search data: We observe all consumer interactions on the airline’s website for two years. The clickstream data include search actions, bookings, and referrals from other websites. Tracking occurs regardless as to whether an individual has a consumer loyalty account or is logged in.

(4) Filed fares data: The filed fares data contain the decisions made by the airline’s pricing department. A filed fare contains the price, fare class, and all ticket restrictions, including any advance purchase discount requirements.

(5) Forecasting data: The air carrier forecasts future demand at granular levels. We observe these predictions down to the flight-passenger type-price level. In addition to the baseline forecast, we also observe all managerial adjustments to the forecasts.

\(^{10}\) We document facts using nonstop bookings, however, our measure of remaining capacity adjusts for all tickets sold, e.g., connections, reward tickets, and consumers altering tickets, etc.\(\ldots\)
3.2 Data Summary

Table 1 provides a basic summary of the nearly 300,000 flights in our cleaned sample. We focus on the last 120 days before departure due to the overwhelming sparsity of search and sales observations earlier in the booking horizon.

<table>
<thead>
<tr>
<th>Data Series</th>
<th>Variable</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Median</th>
<th>5th pctile</th>
<th>95th pctile</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fares</td>
<td>One-Way Fare ($)</td>
<td>201.3</td>
<td>139.4</td>
<td>163.3</td>
<td>88.0</td>
<td>411.1</td>
</tr>
<tr>
<td></td>
<td>Num. Fare Changes</td>
<td>9.3</td>
<td>4.2</td>
<td>9.0</td>
<td>3.0</td>
<td>17.0</td>
</tr>
<tr>
<td></td>
<td>Fare Changes Inc.</td>
<td>50.4</td>
<td>73.0</td>
<td>31.2</td>
<td>2.2</td>
<td>164.5</td>
</tr>
<tr>
<td></td>
<td>Fare Changes Dec.</td>
<td>-53.0</td>
<td>75.5</td>
<td>-32.2</td>
<td>-175.2</td>
<td>-4.3</td>
</tr>
<tr>
<td>Bookings</td>
<td>Booking Rate-OD</td>
<td>0.2</td>
<td>0.7</td>
<td>0.0</td>
<td>0.0</td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td>Booking Rate-All</td>
<td>0.6</td>
<td>1.4</td>
<td>0.0</td>
<td>0.0</td>
<td>3.0</td>
</tr>
<tr>
<td></td>
<td>Load Factor (%)</td>
<td>82.2</td>
<td>21.4</td>
<td>90.0</td>
<td>36.0</td>
<td>102.0</td>
</tr>
<tr>
<td>Searches</td>
<td>Search Rate</td>
<td>1.9</td>
<td>4.8</td>
<td>0.0</td>
<td>0.0</td>
<td>9.0</td>
</tr>
</tbody>
</table>

Summary statistics for the data sample. Fares are for nonstop flights only. The initial load factor is the percentage of the number of seats occupied 120 days before departure. The booking rates are for non-award, direct travel on nonstop flights and for all traffic on nonstop flights, respectively. The number of passengers denotes the number of passengers per booking. The ending load factor includes all bookings, including award and connecting itineraries. The search rate is for origin-destination queries at the daily level. The number of passengers is the number of passengers per request.

Average flight fares in our sample are $201, with large dispersion across routes and over time. Typically, prices for a particular flight adjust nine times and double in 120 days. Many adjustments occur at specified times, such as after expiration of advance purchase discount opportunities. However, over 60% of price adjustments occur before the first AP fares expires. This is because inventory (and therefore, prices) is re-optimized daily.

In our sample, the average load factor is 82.2%. Although overselling is possible, we abstract from this possibility because we do not observe denied boarding/no show information. Our notion of capacity will be actual plane capacity plus the number of seats the airline is willing to sell over capacity (if any)—the observed “authorized” capacity.

3.3 Empirical Facts that Motivate Demand Assumptions

We summarize search and purchase patterns to motivate some of our demand assumptions.
The bookings data suggest that unit demand is a reasonable assumption. The average passengers per booking is 1.3. In addition, the bookings data confirm that overwhelmingly, consumers purchase the lowest available fare even though several fares may be offered at any point in time. We find that 91% of consumers purchase the lowest available fare. Using a separate data base that contains an indicator for corporate bookings under special fares, we find that corporate discounts are not a concern for the routes studied.

Bookings and searches are sparse, which motivates using a model that accounts for low daily demand. 60-80% of observations involve zero observed searches. The fraction of zero sales is even higher (80% zeros). Zeros are not just present because we focus on nonstop demand. The fraction of zero sales for any itinerary involving a particular flight ranges between 40-80%.

We adopt a two-type consumer model, corresponding to “leisure” and “business” travelers, because that is how the firm considers demand. The airline maintains separate forecasts for these consumer types, and an algorithm classifies every search and booking into these two categories.\textsuperscript{11} We explore the predictions of this algorithm in Section 8.

Figure 2: Search and Booking Facts to Motivate Demand Model

(a) CDF of Same Itin. Searches
(b) CDF of Similar Itin. Searches
(c) Channel Booking Distributions

(a) Empirical CDF of the number of days from departure searchers appear for a given itinerary. (b) Empirical CDF of the number of departure dates a given searcher looks for. (c) Percentage of Bookings, across days from departure, for each Channel. Direct refers to bookings that occur on the air carrier’s website, OTAs is purchases made on online travel agencies, and Agency are bookings made through travel agencies.

Figure 2-(a) and (b) motivate our assumption that consumers solve static optimization problems. We investigate the tendency for consumers to return to search for tickets for consumers who were not referred to the airline from other websites. Panel (a) shows the CDF

\textsuperscript{11}The airline does consider additional types of passengers, but these categorizations are very small relative to the two we consider. If we observe any searches or sales from other categories, we reassign them to be leisure travelers.
of number of times that consumers search for the same itinerary across days. 90% of consumers search for an itinerary (OD-DD pair) once. Panel (b) shows the CDF for the number of different departure dates (for the same OD) that consumers search for within a cookie. 82% of customers search a single departure date. The average time lag between searches for different departure dates is 45 days, which may suggest entirely different purchasing opportunities (different trips).

Figure 2-(c) motivates adjusting our model for non-observed searches differently over time. The figure shows the distributions of bookings across booking channels over time. OTAs, or online travel agencies, closely follows the distribution of bookings via the direct channel. However, the agency curve—which includes corporate travel bookings—is more concentrated closer to departure. We discuss this adjustment in Section 6.1. Note that Figure 2-(c) shows some bunching in bookings immediately before advance purchase opportunities expire. Although this may suggest consumers strategically time their purchasing decisions—they are forward looking—we find evidence that supports certain days before departure simply have higher demands. Using the search data, we split the sample into two groups, one that includes routes that never have 7-day AP requirements, and one that includes these requirements. We find that that search activity (and purchases) bunch at the 7-day AP requirement, regardless of their existence. Because arrivals increase regardless of price changes, we maintain the commonly used assumption that the market size is not endogenous to price. Instead, we flexibly estimate arrivals as a function of time and the departure date that allows for this bunching.

4 Pricing Frictions Across Organizational Teams

In this section, we document several pricing frictions that suggests that the firm does not price optimally.
4.1 Coarse Pricing and Not Responding to All Available Information

Figure 3 demonstrates that the firm has access to, and indeed generates, payoff relevant information that its pricing algorithms do not respond to. In panel (a), we plot the fraction of flights that experience changes in price or marginal costs (the shadow value on the capacity constraint as reported by the pricing algorithm) over time. The figure shows that costs change at a much higher frequency than do prices. This occurs because of the industry practice of using a discrete set of fares (fare buckets). That is, it is possible that marginal costs change by $1 but the next fare is $20 more expensive. Our analysis suggests this friction is significantly more important. In panel (b), we run a flexible regression of the change in costs on an indicator function of a price adjustment occurring. As the figure shows, changes in marginal costs exceeding $150 only lead to price adjustments with 50% probability. This may suggest alternative fares could lead to higher revenues.

![Figure 3: Fare Adjustments in Response to Opportunity Cost Changes](image)

Figure 3: Fare Adjustments in Response to Opportunity Cost Changes
(a) Fare vs. Shadow Price Changes
(b) Probability of Fare Change

Note: (a) The fraction of flights that experience changes in the fare or the opportunity cost of capacity over time. (b) The probability of a fare change, conditional on the magnitude in the change in the shadow value.

Prices are coarse both within a day and across days. At any point in time, each route sees roughly a dozen different fare classes (such as the example flight in Figure 1). Over time, the median number of unique prices used for a particular fare is two (mean is three).
4.2 Reacting to Surprises with Delay

The spikes in Figure 3-(a) occur at seven day intervals because firm reforecasts demand for future flights on a weekly interval. Outside of these periods, the firm reoptimizes inventory given the forecast. The firm’s decision not to update their forecasts continuously may be another source of pricing friction. We demonstrate via an example that information exists which could improve/tune forecasting models. With the current system, reactions to “surprises” occur too little and too late. In particular, demand forecasts maintained by RM respond to demand surprises with delay, leading to missed opportunities both for the flight in question, but also for future flights which are mistakenly thought to be over-(or under-) demanded.

Figure 4: Reacting to Surprises: A Conference Example

(a) Average Load Factor
(b) Fares
(c) Forecasted Demand

Note: All plots contain data series for flights departing the date of a large conference (which moves both location and dates each year) and the corresponding flights in the surrounding weeks of the conference. (a) Shows the average load factor across flights. (b) Contains the average lowest available fares across flights. (c) Has the average total expected seats to sell at a given point in time unconstrained by the remaining capacity.

In Figure 4, we show average load factors, fares, and forecasted demand for a particular route-departure date. This departure date is special because it involves a conference which alternates both date and location each year. In addition to flights on the conference date, we include information for flights on this route one week before and after the conference date for comparison. As shown in panel (a), as soon as the location and date of the conference is announced, around 200 days from departure, there is a sudden jump in load factor. The firm’s revenue management software responds with delay (over a month) to the sudden jump in bookings. Prices eventually increase dramatically as seen in panel (b). Panel (c) shows that the forecasting algorithm, having observed the conference shock, then inflates the forecast for the following week—to higher levels than the conference date. That is, the
algorithm incorrectly believes the next week will now also involve a conference. However, in panel (a) we see that the flights a week later contain no surprises—bookings follow a similar pattern as other dates. Consequently, fares are too high for the non-conference flights and too low for a conference flights.

4.3 Using Persistently Biased Forecasts

The forecasting model maintained by RM systematically not only reacts to surprises with delay, it also persistently overstates future demand. We observe biased forecasts over two years of data. In Figure 5, we plot the firms’ forecast against realized future sales. On average, the firm’s forecasts are biased upwards from the true distribution of future sales for nearly the entire booking horizon. For the median observed forecast, the forecast is 10% higher than the actual future demand, which is equivalent to predicting an extra 2.5 seats will be sold. The forecast becomes more accurate toward departure because total remaining demand declines. Although the average forecast is biased upward, suggesting prices may be too high, the forecasts appear misaligned with observed demand at different prices (panel b). Low-fare transactions are underforecasted by 20%, and high-fare transactions are overforecasted by 10%. This suggests the forecasting model may not accurately reflect the actual composition of demand.

Figure 5: Firm Forecasting and Realized Sales

(a) Forecasting Bias Time Series
(b) Forecasting Bias over Fare Levels

Note: Forecasts and future sales are normalized by the aircraft’s total coach capacity. Plots show 7-day moving average to smooth across strong day of week effects in the forecast sample. (a) Business traveler forecasts and realized sales. (b) Leisure traveler forecasts and realized sales.

We also observe all managerial adjustments to the forecasts, which is also plotted in
Figure 5-(a). We find that user adjustments improve forecast accuracy, but the improvement is small in magnitude relative to the total bias.

Forecasts are biased for all routes in the sample. Biases are slightly larger, both in percentage terms and in levels, for markets with competition. This may reflect the additional complexity the firm faces when predicting residual demand.

4.4 Not Accounting for Cross-Price Elasticities

Dynamic pricing is computationally and theoretically complicated. Research in operations research have offered heuristics (including EMSR-b) to solve such models, but this typically comes at the cost of abstracting from key market features. We show one important market feature not captured by the pricing algorithm are cross-price elasticities.

![Figure 6: Shadow Value and Price Response to Bookings with Multiple Flights](image)

(a) Shadow Value

(b) Prices

Note: (a) The orange line denotes the average change in shadow value for a flight when a sale at a given intensity occurs. The blue line is the average change to shadow value when a sale occurs to another flight at a given intensity. (b) This panel depicts the same as panel a, but instead of changes in shadow value it depicts changes in price.

To show that pricing does not internalize substitutes, we subsample our data. We extract observations that satisfy the following conditions: (i) the firm offers two flights a day; (ii) we include periods where demand is not being reforecasted (the observed as spikes in Figure 3); (iii) the total daily booking rate is low (less than 0.5); and (iv) one flight receives bookings and the other flight does not. By considering markets where the total booking rate is low, the following intuition uses theoretical results in continuous time. In Figure 6-(a), we plot the average change in shadow values for the flights that receive bookings and for the flights that do not receive bookings (the substitute option) using flexible regressions.
In standard continuous time dynamic pricing models, every time a unit of capacity is sold, prices jump, e.g., in Gallego and Van Ryzin (1994). This is also true in environments with multiple products—any sale causes all prices to increase. Figure 6-(a) confirms substitute opportunity costs are unaffected by bookings. Panel (b) shows there is no price response. This necessarily means that prices are not directly affected by competitor pricing decisions in markets with competition.

4.5 Pricing on the Inelastic Side of the Demand Curve

We provide model-free evidence that the firm may be systematically underpricing according to its own expectations about demand. This bias is the result of incompatibilities in pricing inputs and the pricing algorithm. We use granular forecasting data (EQ) that report the expected sales at the level of flight (j), departure date (d), days from departure (t), forecasting period (s), and price (p). The difference between s and t defines how far in advance the forecast is constructed. For this exercise, we select observations such that s = t. Because we observe the forecast for multiple prices, we calculate the elasticity of demand, \( ((Q_1 - Q_2)/Q_2)/((P_1 - P_2)/P_2) \), using the observed flight price as the base price along with the next highest price. We also compute the arc elasticity, \( ((Q_1 - Q_2)/(Q_1 + Q_2)/2)/((P_1 - P_2)/(P_1 + P_2)/2) \).

We find that 34% to 52% of flight observations are priced on the inelastic side of the demand curve using the firm’s forecasting data, depending on how elasticities are computed.

We consider whether differences in managerial ability influence outcomes (Goldfarb and Xiao, 2011, 2016; Hortaçsu, Luco, Puller, and Zhu, 2019) by comparing the incidence of pricing on the inelastic side of the demand curve across teams. We observe the identity of the revenue and pricing analysts involved in managing pricing inputs, and estimate regressions of the form \( \mathbb{I} (\text{elasticity} > -1)_{r,i} = X_{r,i} \beta + u_{r,i} \), where X contains team identifiers as well as route characteristics. We find statistically significant differences across teams, however, even the best teams are associated with setting “inelastic prices” 30% of the time. Higher traffic markets (in terms of nonstop passengers) have a larger percentage of inelastic prices, and single-carrier markets see roughly 5% more frequent inelastic prices than
competitor markets, holding passenger count constant.

5 Empirical Model of Air Travel Demand

In order to quantify the impacts of the pricing biases just discussed, we need to estimate a model of air travel demand. We utilize both the demand model and estimation approach of Hortaçsu, Natan, Parsley, Schwieg, and Williams (2021). We consider the demand for nonstop flights for a particular origin-destination pair departing on a particular departure date. The definition of a market is an origin-destination \((r)\), departure date \((d)\), and day before departure \((t)\) tuple. The booking horizon for each flight \(j\) leaving on date \(d\) is \(t \in \{0, ..., T\}\). The first period of sale is \(t = 0\), and the flight departs at \(T\). In each of the sequential markets \(t\), arriving consumers choose flights from the choice set \(J(r, t, d)\) that maximize their individual utilities, or select the outside option, \(j = 0\).

5.1 Utility Specification

Arriving consumers are one of two types, corresponding to leisure \((L)\) travelers and business \((B)\) travelers. An individual consumer is denoted as \(i\) and her consumer type is denoted by \(\ell \in \{B, L\}\). The probability that an arriving consumer is a business traveler is equal to \(\gamma_t\).

We incorporate two assumptions to greatly simplify the demand system. First, we assume that consumers are not forward looking and do not strategically choose flights based on remaining capacity, \(C_{j,t,d}\). This avoids the complication that consumers may choose a less preferred option in order to increase the chances of securing a seat. Second, we assume that when demand exceeds remaining capacity for a particular flight, random rationing ensures the capacity constraint is not violated.

We assume that the indirect utilities are linear in product characteristics and given by (suppressing the \(r\) subscript)

\[
 u_{i,j,t,d} = \begin{cases} 
 X_{j,t,d} \beta - p_{j,t,d} a_{\ell(i)} + \xi_{j,t,d} + \epsilon_{i,j,t,d}, & j \in J(t, d) \\
 \epsilon_{i,0,t,d}, & j = 0 
\end{cases}.
\]
In the specification, \( X_{j,t,d} \) denote product characteristics other than price \( p_{j,t,d} \). Consumer preferences over product characteristics and price are denoted by \((\beta, \alpha_l)_{l \in \{B,L\}}\). For notational parsimony, we commonly refer to the collection \( \{\alpha_B, \alpha_L\} \) as \( \alpha \). The term \( \epsilon_{i,j,t,d} \) is an unobserved random component of utility and is assumed to be distributed according to a type-1 extreme value distribution. All consumers solve a straightforward utility maximization problem; consumer \( i \) chooses flight \( j \) if, and only if,

\[
u_{i,j,t,d} \geq u_{i,j',t,d}, \forall j' \in J \cup \{0\}.
\]

The distributional assumption on the idiosyncratic error term leads to analytical expressions for the individual choice probabilities of consumers (Berry, Carnall, and Spiller, 2006). In particular, the probability that consumer \( i \) wants to purchase a ticket on flight \( j \) is equal to

\[
s_{j,t,d} = \frac{\exp \left( X_{j,t,d} \beta - p_{j,t,d} \alpha_{l(i)} + \xi_{j,t,d} \right)}{1 + \sum_{k \in J \cup \{d\}} \exp \left( X_{k,t,d} \beta - p_{k,t,d} \alpha_{l(i)} + \xi_{k,t,d} \right)}.
\]

Since consumers are one of two types, we define \( s_{j,t,d} \) to be the conditional choice probability for a leisure consumer (and \( s_{j,t,d}^B \) for a business consumer). Integrating over consumer types, we have

\[
s_{j,t,d} = \gamma_t s_{j,t,d}^B + (1 - \gamma_t) s_{j,t,d}^L.
\]

### 5.2 Arrival Processes and Integer-Valued Demand

We assume both consumer types arrive according to time-varying Poisson distributions. By explicitly modeling consumer arrivals, we can rationalize low or even zero sale observations. Specifically, we assume: (i) arrivals are distributed Poisson with rate \( \lambda_{t,d} \), (ii) arrivals are independent of price (as argued in Section 3.3); (iii) consumers have no knowledge of remaining capacity; (iv) consumers solve the above utility maximization problems. With these assumptions, conditional on prices and product characteristics, demand for flight \( j \) is equal to

\[
\tilde{q}_{j,t,d} \sim \text{Poisson} \left( \lambda_{t,d} \cdot s_{j,t,d} \right).
\]
With a random rationing assumption, demand may be censored, i.e., \( q_{j,t,d} = \min\{\bar{q}_{j,t,d}, C_{j,t,d}\} \).

6 Estimation

6.1 Empirical Specification

Because consumer arrivals are observed at the \( t, d \) level, we cannot estimate the arrival process at the same granularity. Instead, we estimate the arrival process assuming a multiplicative relationship between day before departure and departure dates using the following specification,

\[
\lambda_{t,d} = \exp(\lambda_t + \lambda_d).
\]

We pursue this parameterization because searches tend to increase over time (\( \lambda_t \)) but there are also strong departure-date effects (\( \lambda_d \)). These parameters are route-specific.

In an ideal world, we observe all searches and estimate arrival rates using the sum of all leisure and business searches, i.e., \( A_{t,d}^L + A_{t,d}^B \). However, we do not observe all searches—for example, a consumer who searches and purchases through a travel agency will result in an observed purchase without an observed search. Figure 2-(a) suggests that we should adjust for unobserved searches differently over time. We use the distributions of bookings and searches by passenger type as determined by the passenger-type classifier. Using properties of the Poisson distribution, we assign

\[
A_{t,d}^L \sim \text{Poisson}(\lambda_{t,d}(1-\tilde{\gamma}_t)\xi_t^L)
\]

\[
A_{t,d}^B \sim \text{Poisson}(\lambda_{t,d}\tilde{\gamma}_t\xi_t^B),
\]

where \( \tilde{\gamma}_t \) is the firm’s beliefs over the probability of business (see Section 8 for more details) and \( \xi_t^L \) is the fraction of bookings that do not occur on the direct channel for each consumer type.\(^{12}\) That is, we use the relative fraction of \( L \) (\( B \)) sales and searches across channels to scale up \( L \) (\( B \)) arrivals. This logic follows the simpler case with a single consumer type:

\(^{12}\)We use time intervals early on because of sparsity in searches and sales. The largest time window is composed of 14 days. Closer to the departure date, the intervals become length one. We smooth the calculated fractions using a fifth order polynomial approximation.
if searches account for 20% of total sales, and we assume unobserved searches follow the
same underlying demand distributions, we can scale up estimated arrival rates by 5×. As
we are concerned about the accuracy of this assignment algorithm, we conduct robustness
to this specification in Section 7.

We assume consumer utility is given by

\[ u_{i,j,t,d} = \beta_0 - \alpha_{t(i)} p_{j,t,d} + \text{FE(Time of Day } j) + \text{FE(Week)} + \text{FE(DoW)} + \xi_{j,t,d} + \epsilon_{i,j,t,d}, \]

where "FE" denotes fixed effects for the variable in parentheses. This flexibility in the
utility and arrivals allows for rich substitution patterns, including seasonality effects, day-
of-week effects, etc.

We parameterize the probability an arrival is of the business type as

\[ \gamma_t = \frac{\exp(f(t))}{1 + \exp(f(t))}, \]

where \( f(t) \) is an orthogonal polynomial basis of degree five with respect to days from de-
parture. This specification allows for non-monotonicites while producing values bounded
between zero and one.

Finally, we allow for the relationship between the unobserved demand shock \( \xi \) and
prices to change over the booking horizon using four blocks of time. For each block,
we assume the two unobservables are jointly normal. This captures varying managerial
intervention in pricing over time that we observe in the data.\(^{13}\)

6.2 Estimation Procedure

We use a hybrid-Gibbs sampler to estimate route-specific parameters. With Poisson ar-
rivals, we can rationalize zero sale observations while maintaining a Bayesian IV corre-
lation structure between price and \( \xi \). Our approach builds upon the estimation procedure
developed by Jiang, Manchanda, and Rossi (2009) by incorporating search, Poisson de-

\(^{13}\)If we instead estimate the model with \( \xi \) as a pure random effect, we estimate demand to be slightly more
inelastic compared to this specification that allows for price endogeneity.
mand, and censored demand. Additional details on the estimation procedure can be found in Online Appendix C. A complete treatment can be found in Hortaçsu, Natan, Parsley, Schwieg, and Williams (2021).

6.3 Identification and Instruments

One difficulty in estimating a model with aggregate demand uncertainty is separably identifying shocks to arrivals from shocks to preferences. We address this complication by using search data. Conditional on market size, preference parameters are identified using the same variation commonly cited in the literature on estimating demand for differentiated products using market level data. The flight-level characteristic parameters are identified from the variation of flights offered across markets, and the price coefficients are identified from exogenous variation introduced by instruments.

We use the carrier’s shadow price of capacity as reported by the revenue management software, advance purchase indicators, and total number of inbound or outbound bookings from a route’s hub airport as our demand instruments. The shadow price informs the opportunity cost of capacity. The advance purchase indicators account for that fact that prices typically adjust in situations where the opportunity cost is not observed to change (see Figure 3). The total number of inbound or outbound bookings to a route’s hub airport captures the change in opportunity cost for flights that are driven by demand shocks in other markets. For example, for a flight from A to B, where B potentially provides service elsewhere and is a hub, we use all traffic from B onward to other destinations C or D. We assume demand shocks are independent across markets, so shocks to $B \rightarrow C$ and $B \rightarrow D$ are unrelated to demand for $A \rightarrow B$. Thus, a positive shock to onward traffic, out of hub B, will raise the opportunity cost of serving $A \rightarrow B \rightarrow C$ or $A \rightarrow B \rightarrow D$. This propagates to price set on the $A \rightarrow B$ leg.

---

14 For a route with origin $O$ and destination $D$, where $D$ is a hub, the total number of outbound bookings from the route’s hub airport is defined as the following: $\sum_{i=1}^{K} Q_{D,D'}$. Where $Q_{D,D'}$ is the total number of bookings in period $t$, across all flights, for all $K$ routes where the origin is the original route’s destination. If the route’s origin is the hub, we calculate the total number of inward bound bookings, which would be; $\sum_{i=1}^{K} Q_{O,D}$. Where $Q_{O,D}$ is the total bookings from all $K$ routes where the original routes origin is the destination.
7 Demand Estimates

We select a subset of routes for estimation (39 ODs) where our air carrier is the only airline providing nonstop service. Our estimation sample includes routes with varying market characteristics, including flight frequency, importance of seasonality, and percentage of nonstop and non-connecting traffic. See Online Appendix B for additional information.

Figure 7: Model Estimates for Example Route

(a) Model vs. Data Search
(b) Model vs. Empirical Sales
(c) Pr(Business) over Time
(d) Demand Elasticities over Time

Note: The horizontal axis of all plots denotes the negative time index, e.g., zero corresponds to the last day before departure. (a) Normalized model fit of searches with data searches. (b) Model fit of product shares with empirical shares. (c) Fitted values of $\gamma_t$ over time, along with the probability a consumer is business conditional on purchase. (d) Mean product elasticities over time, along with the least and most elastic flights.

We first present key findings for a single route and then summarize our results across routes. For our example route, 88% percent of observations have zero product sales. It is not unusual to have so many zeros. The number of nonstop flights varies over the calendar year; typically, A single flight is offered. In Figure 7-(a), we show that our arrival rates closely match the scaled up arrival data. The fit is very good because our specification
includes $d$- and $t$-specific parameters. Arrival rates are increasing toward the deadline. Panel (b) shows model and data bookings over time. Model bookings closed follow the data and show a common pattern that purchases increase when prices rise. This suggests demand becomes more inelastic, which we confirm in the bottom panels. Panel (c) reports our estimates of the probability of a business-type consumer. We commonly find a significant change in the composition of arriving consumers over time, starting with a very low probability of business and ending close to one. This pattern is consistent with the demand estimates in Williams (2021). Recall that consumer types describe preferences, but not necessarily the reason for travel. In panel (d), we plot average flight elasticities. Elasticities start at -2.1 and increase past -1.0 closer to the departure date.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Median</th>
<th>25th Pctile.</th>
<th>75th Pctile.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monday Arrivals</td>
<td>3.653</td>
<td>2.882</td>
<td>2.645</td>
<td>1.484</td>
<td>5.432</td>
</tr>
<tr>
<td>Tuesday Arrivals</td>
<td>3.001</td>
<td>2.260</td>
<td>2.030</td>
<td>1.352</td>
<td>4.827</td>
</tr>
<tr>
<td>Wednesday Arrivals</td>
<td>3.274</td>
<td>2.433</td>
<td>2.075</td>
<td>1.472</td>
<td>5.127</td>
</tr>
<tr>
<td>Thursday Arrivals</td>
<td>3.785</td>
<td>2.760</td>
<td>2.650</td>
<td>1.685</td>
<td>5.685</td>
</tr>
<tr>
<td>Friday Arrivals</td>
<td>4.395</td>
<td>3.432</td>
<td>2.995</td>
<td>2.007</td>
<td>6.119</td>
</tr>
<tr>
<td>Saturday Arrivals</td>
<td>3.085</td>
<td>2.412</td>
<td>2.175</td>
<td>1.285</td>
<td>4.490</td>
</tr>
<tr>
<td>Sunday Arrivals</td>
<td>4.286</td>
<td>3.393</td>
<td>3.426</td>
<td>1.764</td>
<td>6.466</td>
</tr>
<tr>
<td>Day of Week Spread</td>
<td>32.53</td>
<td>19.61</td>
<td>28.19</td>
<td>17.55</td>
<td>39.81</td>
</tr>
<tr>
<td>Flight Time Spread</td>
<td>74.99</td>
<td>59.29</td>
<td>45.45</td>
<td>34.70</td>
<td>95.95</td>
</tr>
<tr>
<td>Week Spread</td>
<td>52.35</td>
<td>61.90</td>
<td>35.12</td>
<td>21.98</td>
<td>56.62</td>
</tr>
<tr>
<td>Intercept</td>
<td>-1.095</td>
<td>1.274</td>
<td>-0.777</td>
<td>-1.405</td>
<td>-0.509</td>
</tr>
<tr>
<td>$\alpha_B$</td>
<td>0.286</td>
<td>0.167</td>
<td>0.277</td>
<td>0.165</td>
<td>0.376</td>
</tr>
<tr>
<td>$\alpha_L$</td>
<td>1.764</td>
<td>0.736</td>
<td>1.834</td>
<td>1.169</td>
<td>2.199</td>
</tr>
</tbody>
</table>

Note: Spread refers to the dollar amount a leisure consumer would pay to move from the least preferred time or day offered to the most preferred time or day of week. Arrival parameters refer to the variation in search across flight departure day of week.

In Table 2, we report variation in demand estimates across routes. The top panel shows average arrival rates for different days of the week. The interquartile ranges across routes confirm that average arrivals tend to be low. Friday and Sunday tend to be the busiest travel days for the markets in our estimation sample. The next panel describes the spread in willingness to pay (in dollars) for a leisure consumer to switch between the most and least-preferred option (day of the week, time of day, week of year). Time of day preferences tend
to be stronger than day of the week preferences. Consumers generally prefer morning and late afternoon departure times. We estimate that some weeks have systematically higher demands than other weeks. This is not true for all routes, and it does not always reflect seasonal variation in demand.

In Figure 8-(a), we plot arrival rates for the mean route as well as the interquartile range over routes over time. Although levels of arrivals vary (the interquartile range spans more than a doubling of arrivals), overall more consumers search as the departure date approaches. In addition, demand tends to become significantly more inelastic over time, even though prices tend to price. This is shown in panel (b), which shows average own-price elasticities for the mean, median, and interquartile range of markets. The drop off in elasticities close to the departure date mostly reflect very significant price increases after crossing advance purchase discount opportunities.

![Figure 8: Aggregate Arrivals and Elasticities](image)

(a) Estimated Arrivals aggregated over all 39 routes. (b) Estimated Own Price Elasticity of demand aggregated over all 39 routes

Over all markets observed, we estimate a mean elasticity of -1.05. We find that 56% of markets have inelastic demand. We further decompose these elasticities by route. We find that 82% of our estimated routes feature at least 10% of markets (departure date, days before departure) with inelastic demand. Just above half of the routes have inelastic demand on average. Inelastic demand tends to occur close to the departure date. We find that 85% of routes have inelastic demand in the final ten days before departure. We find no correlation between elasticity and number of searches for the route; in fact, although many of the
inelastic routes tend to be routes from a large city to smaller regional cities, we find that the smallest and largest routes by search volume have elastic demand. We find a correlation (26%) between estimated elasticities and the probability of business, as calculated from the firm’s passenger assignment algorithm. That is, routes that tend to have a greater fraction of business arrivals have less elastic demand than other routes.

7.1 The Impact of the Scaling Factor on Demand Estimates

We consider alternative specifications on our scaling factor $\zeta$ in order to understand how changes in imputed market size affect our demand estimates. Our biggest concern is that our scaling factor may understate the presence of price-sensitive consumers who primarily shop with online travel agencies. For each route, we adjust our leisure scaling factor by multiplying the original scaling factor by 1.5, 2, 3, 5 and 10. We find that between 1.5 to 3 times the original scaling factor, our demand estimates are largely unchanged. For larger scaling factors—between 5 and 10—we find that demand becomes less price sensitive far from departure and more price sensitive close to departure. The parameters most affected by this scaling are the parameters governing the probability of business, $\gamma$. As we scale up the leisure arrival process, our estimated probability of business falls. The change in consumer types over time is reduced, however, we still estimate average elasticities to be similar to the baseline model.

8 Firm Beliefs about Demand

We ask, *What does the firm believe demand looks like?* To answer this question, we use detailed forecasting data and our demand model to infer the firm’s beliefs about demand.

We proceed in two steps. First, we recover firm beliefs on the arrival processes. We assume the firm uses same model of consumer arrivals and that the total intensity of demand is the same as our estimates, i.e., $\lambda_{t,d} = \lambda_t \lambda_d$. However, we allow the composition of
arriving customers to vary. For every route, we calibrate $\gamma_t$ as

$$\gamma_t^{\text{beliefs}} = \frac{\sum \text{Arrivals}_t^B}{\sum \text{Arrivals}_t^B + \sum \text{Arrivals}_t^L},$$

where $\text{Arrivals}_t^B$ is the total number of arrivals classified as business for route $r$ ($L$ is similarly defined) using the passenger classification algorithm (we directly observe the algorithm as well as its outputs). With these estimates, firm beliefs on the arrival process are $\lambda_t^B\gamma_t^{\text{beliefs}}$ for business passengers, and $\lambda_t^L(1 - \gamma_t^{\text{beliefs}})$ for leisure traffic. We label these Poisson distribution rates $\tilde{\lambda}_t^B$ and $\tilde{\lambda}_t^L$.

Second, we recover firm beliefs on preferences using the forecasting data, $EQ$. Recall that we observe the forecast for period $t$, $s$ days before departure. As an example, we observe the forecast for a flight three days before departure, forecasted 100 days before departure. Therefore, the forecast was constructed $(100 - 3)$ periods in advance. Define $\Delta$ to be all combinations of differences between $s$ and $t$, i.e.,

$$\Delta = \{(s - t) | \forall t \leq s \leq T \text{ and } 0 \leq t \leq T\} \in \mathbb{N}.$$

Whereas our previous analysis used the aggregate forecast (see Section 4.5), here we use the forecasts at the consumer-type level, $\ell \in \{L, B\}$.

We assume the firm also uses a Poisson demand model, with the same specification as ours. Because the firm considers single-product demand, we consider a single-product setting when recovering beliefs. We transform the forecasting data to a cumulative forecast that provides a direct analogue to our model,

$$\hat{Q}_{j,k,t,d}^\ell := \sum_{k' \geq k} EQ_{j,k',t,d}^\ell.$$

This is the forecast at fare buckets greater than or equal to $k$ for consumer type $\ell$. In addition, we assume the forecasting model assigns $\tilde{\lambda}_{t,d}^\ell$ as the arrival process for each flight $j \in J_d$.\footnote{Instead, we could assume arrivals are $\tilde{\lambda}_{t,d}^\ell/J$, so that each flight receives $1/J$ of arrivals. This increases product shares and results in consumers estimated to be more price insensitive.}

Our assumptions imply that the unconstrained forecast is simply the Poisson
demand rate for the given index, i.e.,

$$\tilde{Q}^f_{t,k,t,d} = \tilde{\lambda}_{t,d} s^f_{j,k,t,d}(\cdot).$$

If we take logs of the equation above and subtract the log of the outside good, we can use the inversion of Berry (1994) to obtain the following estimation equation

$$\log\left(\frac{\tilde{Q}^f_{j,k,t,d}}{\tilde{\lambda}_{t,d}}\right) - \log(s_{0,t,d}^f) = \log(s_{j,k,t,d}^f) - \log(s_{0,t,d}^f) = \tilde{\delta}^f_{j,k,t,d}. \quad (1)$$

This is only possible because the forecasting data is at the consumer-type level, which allows us to avoid using the contraction mapping in Berry, Levinsohn, and Pakes (1995) and Berry, Carnall, and Spiller (2006). The inversion allows us to impose similar restrictions imposed in our model, i.e., the only difference in mean utility across consumer types is on the price coefficient.\footnote{We must also confront a data limitation in that our forecasting data is not necessarily at the $t$ level, but rather, at a grouping of $t$s the firm uses for decision making. The number of days in a grouping varies. We address this data feature in the following way. Note that our demand model does not have $t$-specific parameters—preferences do not vary by day before departure. Therefore, if $\tilde{Q}^f$ is the forecast for consumer type $i$ for multiple periods, the model analogue to this is}

Defining the left-hand side of Equation 1 above as $\tilde{\delta}$, we obtain a linear estimating equation of the form

$$\tilde{\delta} = X\tilde{\beta} - \tilde{a}p + \tilde{\xi} + u,$$

We can simply sum over the relevant time indices for arrival rates because the time-index does not enter within-consumer type shares, and the forecasting data assumes a constant price within a grouping of time. This is important because we can then define consumer-type product shares as

$$\frac{\tilde{Q}^f_{t^*,i}}{\sum_{t \in t^*} \tilde{\lambda}_{t,d}^i} = s^f(\cdot).$$

Thus, we obtain the following inversion,

$$\log\left(\frac{\tilde{Q}^f_{t^*,i}}{\sum_{t \in t^*} \tilde{\lambda}_{t,d}^i}\right) - \log(s_{0^*}^f) = \log(s^f) - \log(s_{0^*}^f) = \tilde{\delta}^f. \quad (2)$$
where $\hat{\beta}, \tilde{\alpha}_B, \tilde{\alpha}_L$ are recovered firm beliefs about demand. We include our estimated $\xi$ in the model, which is the mean of the posterior for that observation taken from our estimates. Thus, this approach also estimates a firm "$\xi$" that also differs across consumer types through $u$. We set these residuals equal to zero after recovering firm beliefs. These assumptions do not greatly impact our findings.

Figure 9: Firm Beliefs on Demand

(a) Market Shares
(b) Probability of Business
(c) Expected demand
(d) Flight Elasticities

Note: (a) Comparison of product shares across consumer types, over time. (b) Estimates of $\gamma$ versus those calculated using the passenger assignment algorithm. (c) Forecasted demand across consumer types, over time. (d) Comparison of own-price elasticities over time. (b) and (d) contain the 25th and 75th percentiles. Results are reported averaging over all observations in the data.

In Figure 9, we provide a visual summary comparing the model predictions using our demand estimates (Model E) from those recovered from the forecasting data (Model B). In panel (a), we plot product shares for both passenger types over time. Both Models B and E produce similar preferences for business travelers, however, Model B results in consumer types being "closer together" than under Model E, with leisure travelers being more price inelastic than under our estimates. In panel (b), we plot the probability that an
arriving customer is a business traveler. Model E places more mass on business travelers and produces larger changes in the types of consumers arriving over the booking horizon. Model B produces a small drop in business consumer arrivals very close to departure. Panel (c) depicts expected demand. The significant differences in $\gamma_t$ create a sizeable gap in business passenger demand close to departure. Model E results in more purchases close to departure. Finally, in panel (d), we plot own-price elasticities over time. Model E produces elasticities that are increasing (toward zero) as $\gamma_t$ increases, whereas Model B results in mostly constant elasticities that then drop close to departure. This is due to the probability of business being relatively flat and consumer types being closer together in terms of preferences.

Overall, the two models are quite different. Model B yields more compressed demand elasticities where aggregate demand is slightly more inelastic well in advance of the departure date than compared to Model E. This is driven by the upward bias in the forecasting data along with reduced variance in the forecasts across flights (relative to bookings). Model E suggests there is more heterogeneity in demand across both flights and routes, with a more pronounced change in arriving consumer preferences over time.

We also estimate more flexible specifications to examine whether the firm is learning about future demand via their forecast, but we find that learning does not play a large role in shaping the firm’s beliefs. We allow for the firm’s beliefs $(\hat{\beta}, \hat{\alpha})$ to be specific to $\Delta$ or to $\Delta \times T$. We do not find evidence of learning about consumer preferences, as deviations between the belief estimates and model estimates do not converge across $\Delta$ or $\Delta \times T$. These findings are consistent with Section 4.2, which suggests that the firm may be updating beliefs about demand incorrectly based on realized sales. This is likely due to oversmoothing in the demand forecast methodology. We will abstract from both the firm’s exact forecast methodology and from any learning about demand in our counterfactual simulations. Instead, we will use the baseline firm belief estimates discussed above.
8.1 How compatible are fares with the firm’s beliefs about demand?

Using Model B demand estimates, we assess the compatibility of prices with the firm’s beliefs about willingness to pay. To do so, we consider a simple scenario: Suppose capacity were sufficiently large so that capacity costs are zero. In this scenario, the firm can solve a static pricing problem. What would be the revenue maximizing price? The optimal price sets marginal revenue equal to zero, or \( MR^{\text{Model B}}(p) = 0 \). This price identifies the lowest price the firm should ever charge under Model B demand. We perform this calculation for every flight-day before departure combination.

We find a significant mismatch between prices and Model B demand. Only 49.6% of observations involve fare menus, filed by the pricing department, where the minimum menu price exceeds the price that solves \( MR^{\text{Model B}}(p) = 0 \). Although we find that 85.1% of filed fares are higher than the lowest price that should ever be charged (higher fare classes are more expensive), 29.8% of observed prices are below the optimal price if capacity were unconstrained. This implies that the pricing algorithm frequently selects prices that are incompatible with the demand estimates recovered using the forecasting data. As the simple example in Section 2.1 demonstrates, the pricing heuristic can be sensitive to inputs, and our results suggest fare menus often contain prices that are too low, consistent with our descriptive analysis in Section 4.5.

9 Counterfactual Analysis of Pricing Frictions

9.1 Counterfactual Models

We quantify the impacts of pricing frictions on welfare through several counterfactuals. Our baseline model approximates the firm’s current practices. We use the demand estimates calibrated from the biased forecasts managed by the RM department (Model B), the ESRMb pricing heuristic, and the observed fares filed by the pricing department.

Next, we correct a single pricing bias and leave others uncorrected. We substitute the Model B demand estimates for the Model E demand estimates. This corrects for persistently biased forecasts. We leave the set of fares used in the heuristic fixed.
In the third counterfactual, we use the biased forecasts (Model B) but alter the fares inputted to the algorithm. We adjust the pricing menus so that they are tailored to the demand forecast. Using the insights from Section 8.1, we set the minimum price of the menu to be the price that solves $MR = 0$. We then increase fares by scaling prices from $1 \times$ to $2.5 \times$ the minimum price spanning the number of observed buckets for each route. This counterfactual simulates coordination between the fares filed by the pricing departure—a group that does not forecast demand, with the revenue management department—a group that forecasts demand but does not select fares.

Finally, we address frictions introduced by both the RM and pricing departments. We use the unbiased forecasts (Model E) along with fares coordinated with Model E demands using the procedure just outlined.

EMSR-b is a heuristic and is itself biased (Wollmer, 1992) because it does not consider substitute products. To account for substitutes, we also consider counterfactuals where prices are determined by solving a dynamic pricing problem. We follow the dynamic pricing (DP) problem in Williams (2021), where a firm selects a price for each flight from a discrete set of prices that maximizes its current and expected future profits. We assume that the firm solves

$$V_t(C_t, p_t) = \max_{p \in P_t} \left( R^e_t(C_t, p_t) + EV_{t+1}(C_{t+1}, p_{t+1} | C_t, p_t) \right),$$

where $C_t$ is the vector of remaining capacity for each flight offered in that time period, $p_t$ is the vector of prices the firm selects, and $R^e_t(C_t, p_t)$ is the firm’s expected flow revenue. These value functions are specific to a route and departure date.

We consider two versions of the DP. We first simulate pricing for each flight independently, assuming other flights will be priced at the lowest priced fare. This is analogous to how we proceed with EMSR-b. We then consider a multi-product DP and limit ourselves to $|J| = 2$ due to the dimensionality of the more complicated environments. Our DP results are thus based on a selected set of routes (and departure dates). We use the coordinated fare menus derived under Model B and Model E as inputs. As before, these fares may not be optimal, especially when the firm prices substitute products.
9.2 Counterfactual Implementation

For each counterfactual, we simulate flights based on the empirical distribution of observed remaining capacity 120 days before departure. For each vector of initial remaining capacities, we then draw preferences and arrival rates given our demand estimates (Model E). We simulate 10,000 flights for each initial capacity, demand combination. Like our demand model, we do not endogenize connecting (or flow) bookings. Therefore, we handle connecting bookings through exogenous decreases in remaining capacity, based on Poisson rates estimated using changes in observed remaining capacity not due to nonstop bookings.\footnote{Alternatively, we could subtract off observed connecting bookings from the initial capacity condition. However, this constrains initial capacity and results in higher prices than what we observe in the data.} Consumers are assumed to arrive in a random order within a period. If demand exceeds remaining capacity, consumers are offered seats in the order they arrive. That is, if the lowest-priced fare has a single seat and is sold immediately, the next arriving consumer within a period is offered the next least-expensive fare. Note that this differs from our demand model where all consumers are assumed to pay the same price within a period. However, because arrival rates are low, consumers very rarely pay different prices in our simulations. This is consistent with the data as well.\footnote{We remove seven markets from our analysis that are estimated to have inelastic demand throughout time. These markets feature very low arrival rates and a very high percentage of zeros (over 95%). Our results are robust to including these routes, though the average revenue gains are over 5% higher with the inelastic routes included.}

9.3 Welfare Comparison and Addressing Organizational Frictions

We report our main counterfactual results in Table 3. Our baseline model—used to approximate present day airline pricing practices—is shown in the first row. We normalize this baseline to 100% for all welfare measures (consumer surplus leisure and business, revenues, and welfare). Rows two through four present counterfactuals in which a single or multiple pricing biases are corrected. Results are reported in percentage differences.

We find that correcting a single pricing friction but not others leaves outcomes largely unchanged. As the table shows, in rows two and three, all numbers are close to 100% (except scenario 2, $C_{SL}$). Consider using the unbiased forecasts with observed fares (scenario...
Table 3: Counterfactual Estimates and a Comparison to Present Practices

<table>
<thead>
<tr>
<th>Counterfactual</th>
<th>$CS_L$</th>
<th>$CS_B$</th>
<th>$Rev$</th>
<th>$W$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) Observed Fares and Biased Forecasts</td>
<td>100.0</td>
<td>100.0</td>
<td>100.0</td>
<td>100.0</td>
</tr>
<tr>
<td>2) Coordinated Fares Given Biased Forecast</td>
<td>72.5</td>
<td>103.4</td>
<td>94.1</td>
<td>98.3</td>
</tr>
<tr>
<td>3) Unbiased Forecasts Given Observed Fares</td>
<td>99.8</td>
<td>99.9</td>
<td>101.6</td>
<td>100.5</td>
</tr>
<tr>
<td>4) Coordinated Fares and Unbiased Forecasts</td>
<td>118.0</td>
<td>65.6</td>
<td>117.2</td>
<td>87.1</td>
</tr>
</tbody>
</table>

Note: In counterfactual (1), we approximate current pricing practices. Counterfactual (2) and (3) address a single organizational team bias, but leave others in place. Finally, in counterfactual (4), we consider a scenario in which RM and pricing department decisions are coordinated.

3). Although Model E demands differ greatly from Model B, the reason that the counterfactuals produce similar outcomes is because EMSR-b generally expects that future demand can be accommodated with remaining capacity and both data generating processes. Because the opportunity cost of capacity is estimated by the algorithm to be low, EMSR-b typically allocates units to the lowest filed fare. This is also observed in the data. The lowest filed fare in scenarios (1) and (3) coincide, hence, outcomes are very similar. Scenario (2) produces similar results because the coordinates fares based on the incorrect forecast assign the lowest filed fare to be reasonably close to observed fares. In particular, because firm beliefs place consumer types closer together and the optimal static price early on is higher than observed fares. This lowers leisure consumer surplus. On the other hand, the fare grid closer to departure contains fares slightly lower than observed, causing business surplus to increase slightly. The combined effects of pricing according to the incorrect demand curves results in revenues that are lower than baseline.

When the unbiased forecast is used with price menus coordinated to that unbiased forecast, outcomes are very different. This is shown in row (4). Recall that passenger types are farther apparent in terms of preferences according to Model E estimates. Because capacity is often not constrained, coordinates fares early on tend to be lower, and coordinated fares later on tend to be much higher. This leads to lower transacted prices among leisure consumers, increases output and leisure consumer surplus. Business consumers are made significantly worse off in scenario (4) because the filed fares are increasing over time, tailored to capture increasing willingness to pay, as shown in Figure 9. Revenues increase by
17% due to increased price targeting; overall, dead-weight loss also rises in the markets studied. Correcting both sources of mispricing has a complementary effect.

One concern with Model E is that it assumes the firm knows preferences and arrivals rates in advance. To address how sensitive our results are to these assumptions, we consider an alternative behavioral model where the firm has “persistently average beliefs,” or where the firm prices according to $\text{avg}(\lambda)$ and $\text{avg}(\beta)$. The firm knows $\gamma$ and $\alpha$ in this counterfactual. We find that even with average beliefs, results are similar to scenario (4) in Table 3, with a 15% increase in revenues. Essentially, knowing the average change in willingness to pay over time is very important, much more so than day of the week variation in preferences and arrival rates. Correcting beliefs about the average shape and evolution of demand—when paired with coordinated fare options—is sufficient to improve revenues.

<table>
<thead>
<tr>
<th>Counterfactual</th>
<th>$CS_L$</th>
<th>$CS_B$</th>
<th>Rev</th>
<th>W</th>
</tr>
</thead>
<tbody>
<tr>
<td>4) Pricing heuristic EMSR-b</td>
<td>100.0</td>
<td>100.0</td>
<td>100.0</td>
<td>100.0</td>
</tr>
<tr>
<td>5) Single-J Dynamic Pricing</td>
<td>95.0</td>
<td>99.0</td>
<td>100.6</td>
<td>99.6</td>
</tr>
<tr>
<td>6) Multi-J Dynamic Pricing</td>
<td>93.3</td>
<td>97.1</td>
<td>100.9</td>
<td>98.8</td>
</tr>
</tbody>
</table>

Note: In counterfactual (4) prices are set using EMSR-b with Model E and the coordinated price menus. Counterfactual (5), endogenously sets prices for each flight independently using the DP. Finally, counterfactual (6) we jointly set prices of all products in the same market using the DP.

In Table 4, we compare EMSR-b under scenario (4) to models of dynamic pricing that also use these inputs. We report two rows after our EMSR-b results corresponding to the situation where the firm optimizes flight prices individually and one in which the firm prices flights jointly. Outcomes are normalized to ESMR-b. We estimate marginally lower consumer surplus and slightly higher revenues under dynamic pricing. These results are not due to the discrete nature of prices—implementing a continuous-price version for single flight markets yields quantitatively similar results.
9.4 Discussion

Our descriptive evidence and counterfactual results suggest the firm has the ability to extract additional surplus by raising prices but has chosen not to do so. Note that the firm’s current prices practices are rational if the costs of adjusting the pricing inputs are sufficiently high, and our estimates of the revenue gains provide a lower bound of these costs. Our analysis suggests coordinating fares to an unbiased forecast results in the largest revenue gains—more so than changing the pricing technology or moving to continuous prices. Implementing these changes would require revising demand estimates firm-wide and addressing span of control to allow coordination in fares chosen by the pricing department to the forecast maintained by the RM department.

On the other hand, our results may provide evidence of “behavioral firms.” The fact that optimal prices are significantly higher than current ones may suggest experiments could fail to reveal consumers’ willingness to pay, or the firm may be experimenting too little. We find some support for the latter possibility. Our search data allow us to see experiments run on the airline’s website. We can confirm there has been very limited experimentation during the sample period. Although it is possible that the firm has long-run demand considerations in mind when determining prices (supporting lower prices), recall that the forecasts are persistently biased upward and in general understate (overstate) the number of low (high) priced tickets sold. Moreover, the firm reacts to observed demand shocks which affect forecasted demand for neighboring departures. Our interpretation of the data is the firm’s pricing technologies aim to maximize short-run revenues.

Recall that our analysis focuses on routes where the air carrier is the only airline providing nonstop service. It may be the case that the firm is choosing to offer lower prices in these markets as a strategic response to the threat of an additional entrant (Goolsbee and Syverson, 2008). We have abstracted away from such dynamic considerations. The fact that our documented biases also occur in markets with direct competitors may suggest this hypothesis is not correct.

Finally, the firm may be concerned about potential regulatory oversight with optimal prices. Our demand estimates suggest more pronounced change in demand elasticity across
the booking period, so optimal prices lead to increased segmentation across consumers. It may be that the firm is concerned about backlash of “price gouging” as observed in other contexts, including in ride share and retailing.

10 Conclusion

In this paper, we how organizational boundaries affect pricing decisions at a large U.S. airline. We first document several ways in which a sophisticated pricing system does not react to some key market fundamentals. For example, the pricing system uses persistently biased forecasts and frequently sets price on the inelastic side of the demand curves the firm believes it faces. We attribute these pricing frictions to the different organizational teams responsible for managing pricing inputs.

We estimate a structural demand model and conduct counterfactual experiments using a pricing heuristic that closely approximates what the firm uses in practice. We show that the current pricing algorithm is effective at filling seats, but could extract more revenue. Addressing pricing frictions individually does not substantially change market outcomes. However, if teams coordinate on algorithm inputs, we estimate strong complementary effects on organizational outcomes. The firm can more effectively optimize on price and doing so may increase dead-weight loss.

Beyond airlines, our results highlight the difficulty firms may have in designing organizational structures around data and algorithms. When algorithms are complex and require numerous inputs, it may be infeasible for a single organizational team to monitor and manage all inputs. When firms delegate tasks to teams, and teams have distinct boundaries, this may prohibit potential complementarities in data-driven decision making.

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Online Appendix
Organizational Structure and Pricing: Evidence from a Large U.S. Airline
by Hortaçsu, Natan, Parsley, Schwieg, and Williams

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</table>
A Details on the Pricing Heuristic, EMSR-b

We approximate the solution to a dynamic pricing (DP) problem using a well-known heuristic in operations research, Expected Marginal Seat Revenue-b or EMSR-b (Belobaba, 1987). The heuristic was developed in order to avoid solving highly complex dynamic pricing problems. The heuristic simplifies the firm’s decision in each period by aggregating all future sales before the deadline into a single future period. It also simplifies the demand system to be for only a single product, so competitive effects cannot be considered. We describe this process below and show how to incorporate Poisson demand in EMSR-b. It is important to note that EMSR-b provides an allocation over a given finite set of prices, instead of providing the optimal price itself given any state of the world. EMSR-b associates each price with a fare-class then chooses a maximal number of sales that can be made to each fare-class. This means that consumers may face different prices within a single pricing period when one class is closed and a higher priced class opens.

A.1 Littlewood’s Rule

EMSR-b is a generalization of Littlewood’s rule, which is a simple case where a firm prices two time periods uses two fare classes. A firm with a fixed capacity of goods (seats) wants to maximize revenue across two periods, where leisure (more elastic) consumers arrive in the first period and business (less elastic) consumers arrive in the second period. The firm sets a cap on the number of seats $b$ it is willing to sell in the first period to leisure passengers. This rule returns a maximum number of seats for leisure when the price to both leisure and business customers has already been decided; it does not determine optimal pricing.

The solution equates the price of a seat sold in the first period (to leisure travelers) to the opportunity cost of lowering capacity for sales in the second period (business travelers). Given prices $p_L$, $p_B$, capacity $C$, and the arrival CDF of business travelers $F_B$, Littlewood’s rule equates the fare ratio to the probability that business class sells out. The fare ratio is
the marginal cost of selling the seat to leisure (the lower revenue $p_L$) which is set equal to the marginal benefit—the probability that the seat would not have sold if left for business customers only. Littlewood’s rule is given by

$$1 - F_B(C - b) = \frac{p_L}{p_B}.$$

This equation can then be solved for $b$, the maximum number of seats to sell to leisure customers in period one. This solution is exact if consumers arrive in two separate groups and there are only two time periods and two consumer types.

### A.2 EMSR-b Algorithm

The EMSR-b algorithm (Belobaba, 1987) extends Littlewood’s rule to multiple fare levels or classes. For each fare class, all fare classes with higher fares are aggregated into a single fare-class called the “super-bucket.” Once this bucket is formed, Littlewood’s rule applies, and can be done for each fare class iteratively. Rather than just comparing leisure and business classes, the algorithm now weights the choice of selling a lower fare-class ticket against an average of all higher fare classes.

We apply the algorithm for $K$ sorted fare-classes such that $p_1 > p_2 > ... > p_K$. Each fare class has independent demand with a distribution $F_k$. Under our specification, the demand for each fare class is distributed Poisson with mean $\mu_k$ that is given by future arrivals times the share of the market exclusive to that bucket.

The super-bucket is a single-bucket placeholder for a weighted average of all higher fare-class buckets. Independent Poisson demand simplifies this calculation, as the sum of independent Poisson distributions is itself Poisson. The mean of the super-bucket is the sum of the mean of each higher fare-class bucket. The price of the super-bucket is a weighted average of the price of each higher-fare class, using the means as the weight.

For each fare class, Littlewood’s Rule is then applied with the fare-class taking the place of leisure travel, and the super-bucket in place of business travel. It is assumed that
all future arrivals appear in a single day. The algorithm then describes a set of fare-class limits $b_k$ that define the maximum number of sales for each class before closing that fare class. We denote the remaining capacity of the plane at any time by $C$. The algorithm uses the following pseudo-code:

```plaintext
for $t > 2$ do
    for $k \leftarrow K$ to 1 by $-1$ do
        i) Compute un-allocated capacity $C_{k,t} = C - \sum_{i=k}^{K} b_i$,
        ii) Construct the super-bucket
            $$\mu_{s_b} = \sum_{i=1}^{k-1} \mu_i, \quad p_{s_b} = \frac{1}{\mu_{s_b}} \sum_{i=1}^{k-1} p_i \mu_i, \quad F_{s_b} \sim \text{Poisson}(\mu_{s_b}),$$
        iii) Apply Littlewood’s Rule using the super-bucket distribution as the demand for business
            $$C_{k,t} - b_k = \min \left\{ F_{s_b}^{-1} \left( 1 - \frac{p_k}{p_{s_b}} \right), C_{k,t} \right\}.$$
    end
end
```

In the case where $t = 1$, dynamics are no longer important, so there is no longer a need to trade off based on the opportunity cost. As a result, we limit the fare of the highest revenue class to all remaining capacity, and set limits of all other classes to zero.

### A.2.1 Fare Class Demand

What remains is computing the mean $\mu_k$ for each fare class bucket. We detail the process in this section. Demand in each market is an independent Poisson with arrival rate $\exp(\lambda_i + \lambda_d) s_j(p)$. Note that this $p$ is a vector of the prices of all flights in the market. We assume that the firm believes other flights will be priced at their historic average over the departure date and day before departure. This allows us to construct a residual demand function $s_j(p_j)$ that is a function of the price of the current flight only. We will treat this as the demand for
the flight at a given bucket’s price for the remainder of this section.

Each fare class has a set price $p_k$, at any time $t$, departure date $d$ we will see $\exp(\lambda^i_t + \lambda^d_d)$ arrivals, of which $s(p_k)$ are willing to purchase a fare for bucket $k$. However, $s(p_{k-1})$ are willing to purchase a fare for bucket $k-1$ as well, since they will buy at the higher price $p_{k-1}$. Only $\exp(\lambda^i_t + \lambda^d_d)[s_t(p_k) - s_t(p_{k-1})]$ are added by the existence of this fare class with price $p_k < p_{k-1}$. Note that this is a flow quantity—the amount of purchases in time $t$, but EMSR-B requires stock quantities: How many will purchase over the remaining lifetime of the sale?

What is the distribution of future purchases then? Each day $t$ is an independent Poisson process split by the share function. Independent split Poisson processes are still Poisson, so we may compute the mean of purchases solely in a fare class by summing arrivals over future time $t$, and taking the difference in shares between price $p_k$ and $p_{k-1}$. For time $t$ and departure date $d$, the stock demand for fare-class $k$ is given by

$$\sum_{i=1}^{t} \exp(\lambda^i_t + \lambda^d_d)[s_t(p_k) - s_t(p_{k-1})],$$

where $s_t(p_0) = 0$ for notational parsimony.

This demand distribution is only used to compute the super-bucket demand distribution. Note that we only include future stock demand in the super bucket, and thus only sum arrivals until time $t-1$. For fare-class $k$. The super bucket’s stock demand is given by

$$\mu_{s,b} = \left( \sum_{i=1}^{t-1} \exp(\lambda^i_t + \lambda^d_d)s_t(p_{k-1}) \right)$$

$$p_{s,b} = \frac{1}{\mu} \sum_{j=1}^{k-1} p_j \sum_{i=1}^{t-1} \exp(\lambda^i_t + \lambda^d_d)[s_t(p_j) - s_t(p_{j-1})].$$

The updated pseudo-code for the EMSR-b algorithm is:
for $t > 2$ do
  for $k \leftarrow K$ to 1 by $-1$ do
    i) Compute un-allocated capacity $C_{k,t} = C - \sum_{i=k}^{K} b_i(t)$,
    ii) Construct the super-bucket
        \[
        \mu_{sb} = \left( \sum_{i=1}^{t-1} \exp(\lambda_i^t + \lambda_d^d) \right) s_i(p_{k-1}),
        \]
        \[
        p_{sb} = \frac{1}{\mu_{sb}} \sum_{j=1}^{k-1} p_j \sum_{i=1}^{t-1} \exp(\lambda_i^t + \lambda_d^d) \left[ s(p_j) - s(p_{j-1}) \right],
        \]
        \[ F_{sb} \sim \text{Poisson}(\mu_{sb}), \]
    iii) Apply Littlewood’s Rule using the super-bucket distribution as the demand for business
        \[
        C_{k,t} - b_k(t) = \min \left\{ F_{sb}^{-1} \left( 1 - \frac{p_k}{p_{sb}} \right), C_{k,t} \right\}.
        \]
  end
end

For $t = 1$ we continue to allocate the highest revenue fare class to the entire remaining capacity. Note that for this allocation rule, $b_k(t, d)$ is a function of time since the arrivals are changing over time. This policy can be computed for each time $t$ and remaining capacity $c$, for all departure dates $d$ and arrival rates $\lambda$.

**B Route Selection**

We use publicly available data to select markets to study. The DB1B data are provided by the Bureau of Transportation Statistics and contain a 10% sample of tickets sold. The DB1B does not include the date purchased nor the date traveled and is at the quarterly level. Because the DB1B data contain information solely for domestic markets, we limit
our analysis to domestic markets as well. Furthermore, we use the air carrier’s definition of markets to combine airports within some geographies.

Figure 10: Nonstop, One-stop and Connecting Traffic

Destination

Connecting

Connecting

One-stop

Origin

Note: We use the term nonstop to denote the sold black line, or passengers solely traveling between (Origin, Destination). Unless otherwise noted, we will use directional traffic, labeled $O \rightarrow D$. Non-directional traffic is specified as $O \leftrightarrow D$. The blue, dashed lines represent passengers flying on $O \leftrightarrow D$, but traveling to or from a different origin or destination. Finally, one-stop traffic are passengers flying on $O \rightarrow D$, but through a connecting airport.

We consider two measures of traffic flows when selecting markets: traffic flying nonstop and traffic that is non-connecting. Both of these metrics are informative for measuring the substitutability of other flight options (one-stop, for example) as well as the diversity of tickets sold for the flights studied (connecting traffic). Figure 10 provides a graphical depiction of traffic flows in airline networks that we use to construct the statistics. We consider directional traffic flows from a potential origin and destination pair that is served nonstop by our air carrier. The first metric we calculate is the fraction of traffic flying from $O \rightarrow D$ nonstop versus one or more stops. This compares the solid black line to the dashed orange line. Second, we calculate the fraction of traffic flying from $O \rightarrow D$ versus $O \rightarrow D \rightarrow C$. This compares the solid black line to the dashed blue line.

Figure 11 presents summary distributions of the two metrics for the markets (ODs) we select. In total, we select 407 ODs for departure dates between Q3:2018 and Q3:2019. The top row measures the fraction of nonstop and connecting traffic for tickets sold by our our carrier. The left plot shows that, conditional on the air carrier operating nonstop flights between OD, an overwhelming fraction of consumers purchase nonstop tickets instead of purchasing one-stop connecting flights. The right panel shows that fraction of consumers who are not connecting to other cities either before or after flying on segment OD. There is
Figure 11: Route Selection Using Bureau of Transportation Statistics Data

(a) Within Airline Fraction Nonstop

(b) Within Airline Fraction Non-Connecting

(c) All Airlines Fraction Nonstop

(d) All Airlines Fraction Non-Connecting

Note: Density plots over the fraction of nonstop traffic and the fraction of non-connecting traffic for the selected routes using DB1B data. "Within" means passengers flying on our air carrier. "Total" means all air carriers on a given origin-destination pair. Within nonstop and total nonstop coincide if our carrier is the only carrier flying nonstop.

significant variation across markets, with the average being close to 50%.

Figure 12: DB1B Comparison

(a) DB1B OD Traffic Comparison

(b) DB1B OD Fare Comparison

Note: (a) A scatter plot of the fraction nonstop and fraction non-connecting for all origin-destination pairs served by our air carrier. The blue dots show selected markets; the orange dots show non-selected markets. (b) Kernel density plots of all fares in the DB1B data for our air carrier; the blue line shows the density for our selected markets.
The bottom panel repeats the statistics but replaces the denominator of the fractions with the sum of traffic flows across all air carriers in the DB1B. Both distributions shift to the left because of existence of competitor connecting flights and sometimes direct competitor flights. In nearly 75% of the markets we study, our air carrier is the only firm providing nonstop service. Our structural analysis will only consider single carrier markets.

In Figure 12-(a), we show a scatter plot of the fraction of nonstop traffic and the fraction of non-connecting traffic for all origin-destination pairs offers by our air carrier in the DB1B. The orange dots depict routes non-selected markets and the blue dots show the selected markets. We see some dispersion in selected markets, however this is primarily on non-connecting traffic. An overwhelming fraction of the selected markets have high nonstop traffic, although this is true in the sample broadly. Essentially, conditional on the air carrier providing nonstop service, most passengers choose nonstop itineraries. In Figure 12-(b) we show the distribution of purchased fares in the DB1B for our carrier along with our selected markets. The distribution of prices for the selected sample are slightly shifted to the right, which makes sense since we primarily select markets where the air carrier is the only airline providing nonstop service.

**B.1 Estimation Sample Comparison**

Our estimation sample contains 39 markets. Compared to the overall sample, these routes tend to be smaller in terms of total number of passengers, larger in terms of percentage of nonstop and non-connecting passengers, and nonstop service is provided only by our air carrier. We report percentage differences between our estimation routes and the entire sample for key characteristics below in Table 5. Figure 13 shows a two-way plot of the fraction of nonstop and non-connecting traffic for the routes selected for estimation relative to the entire sample.
Table 5: Estimation Routes Comparison

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Percentage Difference from Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Nonstop Passengers</td>
<td>-38.8%</td>
</tr>
<tr>
<td>Total Number of Passengers</td>
<td>-33.4%</td>
</tr>
<tr>
<td>Number of Local Passengers</td>
<td>-37.7%</td>
</tr>
<tr>
<td>Fraction of Traffic Nonstop</td>
<td>1.02%</td>
</tr>
<tr>
<td>Fraction of Traffic Non-Connecting</td>
<td>5.91%</td>
</tr>
</tbody>
</table>

Note: Statistics calculated using the DB1B data for the years 2018-2019.

Figure 13: Route Estimation Selection using DB1B Data

Note: A scatter plot of the fraction nonstop and fraction non-connecting for all origin-destination pairs served by our air carrier. The blue dots show markets used for estimation; the orange dots show non-selected markets.

C Demand Estimation Procedure

We provide an overview on the implementation details of each stage the MCMC routine for demand parameter estimation. Simultaneously drawing from the joint distribution of our large parameter space is infeasible, therefore, we use a Hybrid Gibbs sampling algorithm. The algorithm steps are shown below. At each step of the posterior sampler, we sequentially draw from the marginal posterior distribution groups of parameters, conditional on other parameter draws. Where conjugate prior distributions are unavailable, we use the Metropolis-Hastings algorithm, a rejection sampling method that draws from an approximating candidate distribution and keeps draws which have sufficiently high likelihood. Additional detail can be found in Hortaçsu, Natan, Parsley, Schwieig, and Williams (2021).
1: for $c = 1$ to $C$ do 
   2: Update arrivals $\lambda$ (Metropolis-Hastings) 
   3: Update shares $s(\cdot)$ (Metropolis-Hastings) 
   4: Update price coefficients $\alpha$ (Metropolis-Hastings) 
   5: Update consumer distribution $\gamma$ (Metropolis-Hastings) 
   6: Update linear parameters $\beta$ (Gibbs) 
   7: Update pricing equation $\eta$ (Gibbs) 
   8: Update price endogeneity parameters $\Sigma$ (Gibbs) 
9: end for 

Algorithm 1: Hybrid Gibbs Sampler 

Sampling Arrival Parameters

We start the sampling procedure by drawing from the posterior distribution of arrival parameters, $\lambda_{t,d}$. The posterior is derived by defining the joint likelihood of arrivals for each consumer type and quantities sold, conditional on product shares. Recall that arriving consumers have likelihood based on their type:

$$A^L_{t,d} \sim \text{Poisson}(\lambda_{t,d}(1 - \tilde{\gamma}_t)\zeta^L_t),$$
$$A^B_{t,d} \sim \text{Poisson}(\tilde{\lambda}_{t,d} \tilde{\gamma}_t \zeta^B_t),$$

where $\tilde{\gamma}_t$ is the probability a consumer is of the business type as derived from the passenger assignment algorithm, and $\zeta^L_t$ is the fraction of bookings that do not occur on the direct channel for each consumer type (leisure and business). The purchase likelihood is a function of shares and arrivals and is equal to

$$\tilde{q}_{j,t,d} \sim \text{Poisson} \left( \lambda_{t,d} \cdot s_{j,t,d} \right),$$
$$q_{j,t,d} = \min \left\{ \tilde{q}_{j,t,d}, C_{j,t,d} \right\}.$$

This directly accounts for censored demand due to finite capacity. Since arrivals are restricted to be non-negative, we restrict the set of fixed effects by transforming the multiplicative fixed effects to be of the form $\lambda_{t,d} = \exp(W_{t,d} \tau)$. We select a log-Gamma prior.
for $\tau$. We sample from the posterior distribution by taking a Metropolis-Hastings draw from a normal candidate distribution.

**Sampling Shares and Utility Parameters**

**Updating shares.** We treat product shares as unobserved, since the market size may be very small and lead to irreducible measurement error. We use data augmentation to treat shares as a latent parameter that we estimate. Conditional on all other parameters $(\lambda, \alpha, \gamma, \beta, \eta, \Sigma)$, product shares are an invertible function of the demand shock, $\xi$. If we conditioned additionally on $\xi$, shares would be a deterministic function of data and other parameter draws. Instead, we leverage the stochastic nature of $\xi$, which we explicitly parameterize. The distribution of unobserved $\xi$ is the source of variation for constructing a conditional likelihood for shares:

$$\begin{align*}
\xi_{j,t,d} &= f^{-1}(s_{j,t,d} | \beta, \alpha, \gamma, X) \\
v_{j,t,d} &= p_{j,t,d} - Z'_{j,t,d} \eta
\end{align*}$$

such that $\Sigma_k = \begin{pmatrix} \sigma_{k,11}^2 & \rho_k \\ \rho_k & \sigma_{k,22}^2 \end{pmatrix}$.

Here, $\kappa$ is a mapping from days to departure $t$ to an interval (block) of time. That is, the pricing error and the demand shock have a block-specific joint normal distribution. Conditional on the pricing shock $v$, the distribution of $\xi$, $f_{\xi, j, t, d}(\cdot)$, is

$$\xi \mid v, \kappa = k \sim \mathcal{N}\left(\frac{\rho_k v}{\sigma_{k,11}^2}, \frac{\sigma_{k,22}^2}{\sigma_{k,11}^2} - \frac{\rho_k^2}{\sigma_{k,11}^2}\right).$$

The density of shares is then given by the transformation $f_{s_{j,t,d}}(x) = f_{\xi_{j,t,d}}\left(f^{-1}(x)\right) |_{\text{Jacobian of model shares}}^{-1}$, where $\text{Jacobian of model shares}$ is the Jacobian matrix of model shares with respect to $\xi$. To produce the full joint conditional likelihood of shares, we also include the mass function for sales, which are a product of shares and arrivals:
where \( \phi(\cdot) \) is the standard normal density function. We draw from the posterior based on a uniform prior distribution and normal candidate Metropolis-Hastings draws.

**Updating price coefficients, \( \alpha_B, \alpha_L \).** We construct the conditional likelihood (and thus the conditional posterior distribution) for \( \alpha = (\alpha_B, \alpha_L) \) in a similar manner to the product shares. For any candidate value of price sensitivity, we recover a residual \( \xi \), invert the demand system, and recover a likelihood. Conditional on \( \lambda \), shares, \( \eta, \beta \), and \( \Sigma \), we compute the distribution of \( \xi \) and determine the likelihood of a particular draw of \( \alpha \), given by

\[
\prod_{t} \prod_{d} \prod_{j=1}^{J(t,d)} \left[ \phi \left( \frac{f^{-1}(s_{j,t,d}) - \frac{\rho_{t,d} \psi}{\sigma_{t,11}}}{\sqrt{\sigma_{k,22}^2 - \frac{\rho_{t,d}^2}{\sigma_{k,11}^2}}} \right) \right] \cdot \left| \mathcal{J}_{\xi_{-s}} \right|^{-1},
\]

where \( \phi(\cdot) \) is the standard Normal density function. We impose a log-Normal prior on \( \alpha \), and impose \( \alpha_B < \alpha_L \) to avoid label-switching. To draw from the conditional posterior, we take a Metropolis-Hasting step using a normal candidate distribution.

**Updating the distribution of consumer types, \( \gamma \).** We allow for the mix of consumer types to change over the booking horizon \( t \). We define \( \gamma \) from a sieve estimator of the booking horizon \( t \), and we sample the sieve coefficients, \( \psi \), according to

\[
\gamma_t = \text{Logit}(G(t)\psi),
\]

where \( G(t) \) is a vector of Bernstein polynomials. The logistic functional form ensures that the image of \( \gamma \) in the interval \((0,1)\). The inversion procedure used to construct the likelihood
is similar to $\alpha$ and shares. It yields a likelihood for sieve coefficients $\psi$ of the form

$$
\prod_t \prod_d \prod_{j=1}^{J(t,d)} \phi \left( \frac{f^{-1}(s_{j,t,d}) - \frac{\rho_k \psi_j}{\sigma_{k,11}}}{\sqrt{\sigma_{k,22}^2 - \frac{\rho_k^2}{\sigma_{k,11}^2}}} \right) \cdot J(\hat{\xi} - \mu)^{-1}.
$$

We use a uniform prior on $\psi$, and we sample from the posterior with a Metropolis-Hastings step using a normal candidate draw.

**Updating remaining preferences, $\beta$.** To sample the remaining preferences that are common across consumer types, we impose a normal prior on $\beta$, with mean $\bar{\beta}_0$ and variance $V_0$. We adjust for price endogeneity to conduct a standard Bayesian regression. Define $\delta_{j,t,d} = X_{j,t,d} \beta + \xi_{j,t,d}$, which is evaluated at the $\xi$ computed in the prior step. We normalize each component of $\delta$ by subtracting the expected value of $\xi$ and dividing by its standard deviation. The normalized equations have unit variance and are thus conjugate to the normal prior. Let $\sigma_{k,21} = \sqrt{\sigma_{k,22}^2 - \frac{\rho_k^2}{\sigma_{k,11}^2}}$ be the variance of $\xi$ conditional on $\nu$ and $\Sigma$.

We center and scale $\delta$:

$$
\frac{\delta_{j,t,d} - \frac{\rho_k \psi_j}{\sigma_{k,11}} \nu}{\sigma_{K,21}} = \frac{1}{\sigma_{K,21}} X_{j,t,d} \hat{\beta} + U^\beta J(t,d),
$$

where $U^\beta \sim \mathcal{N}(0,1)$. Then, the posterior distribution of $\beta$ is $\mathcal{N}(\beta_N, V_N)$, where

$$
\beta_N = (\hat{X}' \hat{X} + V_0^{-1})^{-1} \left( V_0^{-1} \beta_0 + \hat{X}' \hat{\delta} \right),
$$

$$
V_N = (V_0^{-1} + \hat{X}' \hat{X})^{-1},
$$

$$
\hat{X}_{j,t,d} = \frac{X_{j,t,d}}{\sigma_{K,21}},
$$

$$
\hat{\beta} = \frac{\delta_{j,t,d} - \frac{\rho_k \psi_j}{\sigma_{k,11}} \nu}{\sigma_{K,21}}.
$$

Given this normalization, we can draw directly from the conditional posterior distribution of $\beta$ using a Gibbs step.
Sampling Price-Endogeneity Parameters

Updating pricing equation, $\eta$. We use a linear pricing equation of the form

$$p_{j,t,d} = Z_{j,t,d} \eta + \nu_{j,t,d},$$

Conditional on shares, $\lambda, \gamma, \alpha$, and $\beta, \xi$ is known. Therefore, we use the conditional distribution of $\nu$ given $\xi$ to perform another Bayesian linear regression in a similar manner to $\beta$. We impose a Normal prior and normalize prices. Define $\sigma^2_{K_t,11} = \frac{\sigma^2_{K_t,12}}{\sigma^2_{K_t,22}}$. It follows that

$$\frac{p_{j,t,d} - \rho_{K_t} \xi_{j,t,d}}{\sigma^2_{K_t,11}} = \frac{1}{\sigma^2_{K_t,11}} X_{j,t,d} \tilde{\eta} + U_{j,t,d},$$

where $U_{\tilde{\eta}} \sim \mathcal{N}(0, 1)$. Just as we did for $\beta$, we can draw from the posterior of $\eta$ from a linear regression with unit variance. This step allows us to directly sample from the posterior of $\eta$ rather than using a Metropolis-Hastings step.

Updating the price endogeneity parameters, $\Sigma$. We flexibly model the joint distribution of $\xi$ and $\nu$ by allowing for a route-specific, time-varying correlation structure. We divide the booking horizon into four equally sized 30-day periods, and each block is indexed $k$. We restrict the price endogeneity parameters $\Sigma$, which determine the joint distribution of $\xi, \nu$, to be identical within these blocks. Within each block, the pricing and demand residual follow the same joint distribution. We draw the variance of this normal distribution with a typical Inverse-Wishart parameterization. Our prior for $\Sigma_k$ is $IW(v, V)$ where $k$ refers to the block. Define the vector $Y_k = (\nu, \xi)$ to be the collection of residual pairs conditional on block $k$, and $Y_k \sim \mathcal{N}(0, \Sigma_k)$. The posterior for the covariance matrix $\Sigma_k$ is then

$$\Sigma_k \sim IW(v + n_k, V + Y_k' Y_k).$$

Block $k$ has $n_k$ observations. This Gibbs step is repeated for each block $k$, and we sample directly from the conditional posteriors of $\Sigma$. 

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