COMPETITION AND PUBLIC INFORMATION: A NOTE

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Abstract

We study price discrimination in a market in which two firms engage in Bertrand competition. Some consumers are contested by both firms, and other consumers are "captive" to one of the firms. The market can be divided into segments, which have different relative shares of captive and contested consumers. It is shown that the revenue-maximizing segmentation involves dividing the market into "nested" markets, where exactly one firm may have captive consumers.

KEYWORDS: Price Competition, Bertrand Competition, Price Count, Price Quote, Information Structure, Bayes Correlated Equilibrium.

JEL CLASSIFICATION: D41, D42, D43, D83.

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1 Introduction

Consider a market where two firms compete in prices in selling a homogenous good, in a market where some consumers are "captive" (can only buy from one of the firms) and some are "competitive" (buy from the firm offering the highest price). Armstrong and Vickers (2019) have shown that the expected price is higher if firms have public signals about the composition on the market. We characterize the public splitting of the market that generates the highest expected price. It is optimal to split the market into two "nested" markets (Armstrong and Vickers, 2019) where each market has captive consumers of only one firm. The ratio of captive to contested consumers are equalized across markets.

In Bergemann, Brooks, Bergemann, and Morris (2020), we characterize the highest distribution of prices when firms observe private signals about the market. We use the result in this note to contrast public and private signals. The result is a simple and cute application of concavification in a three state model.

2 Result

A market is described by the proportion of captive consumers of each firm, and the proportion of contested consumers. We write γ_i for the proportion of captive consumers of firm i. The set of possible markets is then

$$\Gamma = \left\{ (\gamma_1, \gamma_2) \in [0,1]^2 \, | 0 \leq \gamma_1 + \gamma_2 \leq 1 \right\},$$

where given (γ_1, γ_2), $1 - \gamma_1 - \gamma_2$ is the proportion of contested consumers. Let (μ_1, μ_2) be the original market. A public information structure is a distribution η over markets such that for all $i = 1, 2$,

$$\mu_i = \sum_{(\gamma_1, \gamma_2)} \eta (\gamma_1, \gamma_2) \, \gamma_i.$$

Theorem 1. *The optimal public information structure puts proportion $\mu_1 / (\mu_1 + \mu_2)$ of consumers in market $(\mu_1 + \mu_2, 0)$ and proportion $\mu_2 / (\mu_1 + \mu_2)$ in market $(0, \mu_1 + \mu_2)$.*

Proof of Theorem 1. Narasimhan (1988) and Lemma 1 in Armstrong and Vickers (2019) reports that revenue in the unique equilibrium is

$$R (\gamma_1, \gamma_2) = \frac{\max \{\gamma_1, \gamma_2\}}{1 - \min \{\gamma_1, \gamma_2\}} (2 - \gamma_1 - \gamma_2).$$

A market (γ_1, γ_2) is *nested* if $\min \{\gamma_1, \gamma_2\} = 0$. Observe that revenue in a nested market, where there is a proportion $\gamma \in [0,1]$ consumers are captive to one firm,

$$\widehat{R}(\gamma) = R(\gamma, 0) = R(0, \gamma) = \gamma(2-\gamma),$$

which is concave in γ. An immediate implication is that if there are two nested markets $(\gamma_1, 0)$ and $(\gamma_1', 0)$, with proportions η and η' respectively, the expected price weakly increases if we merge these markets into a single market with proportion $\eta+\eta'$ of consumers and distribution $(\gamma_1\eta/(\eta+\eta') + \gamma_1'\eta'/(\eta+\eta'), 0)$.

Next, we claim that the expected price is weakly increased by splitting market (γ_1, γ_2) into two nested signals with posteriors $(\gamma_1+\gamma_2, 0)$ and $(0, \gamma_1+\gamma_2)$, with weights $\gamma_1/(\gamma_1+\gamma_2)$ and $\gamma_2/(\gamma_1+\gamma_2)$, respectively, i.e., $R(\gamma_1,\gamma_2) \leq \widehat{R}(\gamma_1+\gamma_2)$. To see this, observe that

$$\begin{aligned}
R(\gamma_1,\gamma_2) &= \frac{\max\{\gamma_1,\gamma_2\}}{1-\min\{\gamma_1,\gamma_2\}}(2-\gamma_1-\gamma_2) \\
&\leq \frac{\max\{\gamma_1,\gamma_2\} + (1-\gamma_1-\gamma_2)\min\{\gamma_1,\gamma_2\}}{1-\min\{\gamma_1,\gamma_2\}}(2-\gamma_1-\gamma_2) \\
&= \frac{(\gamma_1+\gamma_2)(1-\min\{\gamma_1,\gamma_2\})}{1-\min\{\gamma_1,\gamma_2\}}(2-\gamma_1-\gamma_2) \\
&= (\gamma_1+\gamma_2)(2-\gamma_1-\gamma_2) \\
&= \widehat{R}(\gamma_1+\gamma_2).
\end{aligned}$$

Thus, we know that for any market splitting, there is another that puts all consumers into nested markets. Moreover, the preceding observation implies that any market splitting with nested signals has a weakly lower expected price than the split where we put all captive consumers of firm i into the same market. Thus, it is without loss to restrict attention to information structures with exactly two nested markets.

The final step is to argue that the proposed split of the contested consumers is optimal. We can parametrize the two markets as market $(\mu_1/x, 0)$ and $(0, \mu_2/(1-x))$ with probabilities x and $1-x$, respectively, for $x \in [\mu_1, 1-\mu_2]$. The resulting revenue is

$$\begin{aligned}
\tilde{R}(x) &= x\frac{\mu_1}{x}\left(2-\frac{\mu_1}{x}\right) + (1-x)\frac{\mu_2}{1-x}\left(2-\frac{\mu_2}{1-x}\right) \\
&= \mu_1\left(2-\frac{\mu_1}{x}\right) + \mu_2\left(2-\frac{\mu_2}{1-x}\right).
\end{aligned}$$

This function is strictly concave, with the first-order condition for an optimum

$$0 = \frac{\mu_1^2}{x^2} - \frac{\mu_2^2}{(1-x)^2},$$

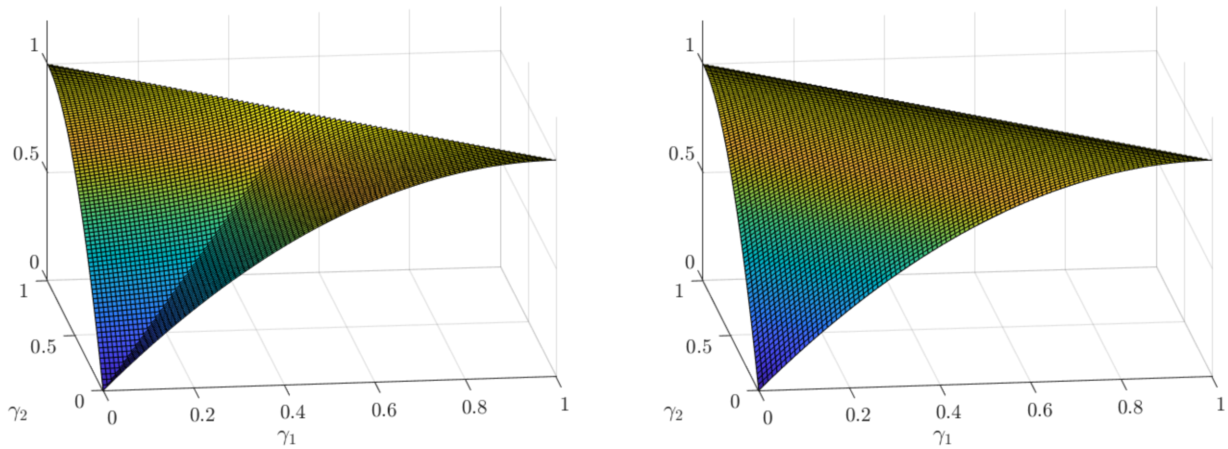

Figure 1: Revenue, left, and its concavification, right.

which has a unique solution $x = \mu_1/(\mu_1 + \mu_2)$, as desired. □

The proof of Theorem 1 essentially computes the concavification of revenue when there are three states. Revenue is depicted in Figure 1. The concavification of this function is obtained by splitting along the minus forty-five degree line, to place all mass on the coordinate axes.

A natural conjecture is that nested signals also maximize revenue among public information structures when there are more than two firms. The proof of Theorem 1 uses the known functional form for equilibrium revenue as a function of the distribution of quotes. Armstrong and Vickers (2020) characterize equilibrium revenue with asymmetric quote distributions for three firms, and in principle this could be used to generalize Theorem 1, although the case with more than three firms is not solved in general.

In Bergemann, Brooks, Bergemann, and Morris (2020), we compare this optimal revenue with public signals to the higher revenue attainable with private signals, in the case where the distribution of consumers is symmetric across firms. Bergemann, Brooks, and Morris (2013) reports some conjectures for the optimal revenue with private signals in the two firm case with asymmetric distributions of captive consumes across firms.

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