Supplemental Material to
Recursive Preferences, the Value of Life, and Household Finance

By

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Electronic Supplementary Material for "Recursive Preferences, the Value of Life, and Household Finance"

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This Electronic Supplementary Material (ESM) contains two parts. Section 1 provides a detailed account of the computational method we use to solve for the quantitative model of the main paper. Section 2 contains the graph for the deterministic age-productivity profile taken from Harenberg and Ludwig (2019).

1 Details on the computational implementation

There is no analytical solution to the agents' problem outlined in Section 4 of the main paper. We solve the model and obtain decision rules numerically and then use those decision rules to simulate the agents' behavior. The next two subsections describe the solution of the model and the simulation of decision rules.

1.1 Model solution

While alive, the agent maximizes her intertemporal utility by choosing a feasible allocation $(c_t, b_t, a_t, s_t, \eta_t)_{t\geq 0}$ in the set $\mathcal{A}$. The utility V_t of the alive agent at age t depends on five state variables: the beginning-of-period holdings in bonds b_{t-1}, annuities A_{t-1} and stocks s_{t-1}; the stochastic component of labor income, ζ_{t-1}; and the stock market participation status, $\eta_{t-1} \in \{0, 1\}$. The last of these is discrete, while the first four are continuous. Given that annuity purchase may only occur in period $T_R - 1$, we have $A_t = 0$ for all $t < T_R$ and $A_t = a_{T_R-1}$ for $t \geq T_R$. Since there exists a maximal age for the agent, T_M, we solve the model by iterating

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on the value function, starting from the last period of life. Utility maximization involves solving:

$$V_{T_M}(b_{T_m-1}, A_{T_m-1}, s_{T_m-1}, \eta_{T_m-1}, \zeta_{T_m-1}) = \max_{(c_{T_M}, b_{T_M}, a_{T_M}, s_{T_M}, w_{T_M}, \eta_{T_M}) \in \mathcal{A}} (1-\beta)u(c_{T_m}) + E_t\left[e^{-k(1-\beta)v(x_{T_m+1})}\right], \tag{1}$$

subject to the constraints, which we don't restate here, outlined in Section 4 of the main paper. Due to the presence of stocks in bequest, the continuation utility in case of death is uncertain. Both the instantaneous utility function for period $T_m(u(x_{T_m}))$ and the utility obtained when dying and bequeathing $x_{T_{M+1}}(v(x_{T_{M+1}}))$ are known and the model is solved for a discrete set of points on a grid. This gives us knowledge of V_{T_m} at a subset of the points in the state space and allows us to approximate V_{T_m} as $\hat{V}_{T_m}$ at all points. With this approximation in hand we solve an approximation to problem (1) for period $T_M - 1$ and then, iteratively, for all preceding periods t:

$$V_t(b_{t-1}, A_{t-1}, s_{t-1}, \eta_{t-1}, \zeta_{t-1}) = \max_{(c_t, b_t, a_t, s_t, w_t, \eta_t) \in \mathcal{A}} (1-\beta)u(c_t) - \frac{\beta}{k}\log\left(\pi_t E_t\left[e^{-k\hat{V}_{t+1}(b_t, A_t, s_t, \eta_t, \zeta_t)}\right] + (1-\pi_t)E_t\left[e^{-k(1-\beta)v(x_{t+1})}\right]\right). \tag{2}$$

Note that the only difference between the maximand in (2) and that in our household's problem is that the continuation value function is an approximate value function. We will now, briefly, discuss four features of the numerical procedure: i) the discretization of the continuous variables, ii) the integration of the value function, iii) the approximation method for evaluating the value function at points outside the discretized state space and iv) how the optimization is carried out.

Discretization of the continuous variables. We define a variable, total liquid wealth, which is the sum of bond and stock holdings at the start of a period. We define a grid of 54 points from \$0 to \$10m such that the gaps between successive grid points are smaller at lower levels of wealth, where the curvature of the value function will be greater. We define a grid of 36 points for annuity income from \$0 to \$800,000 such that the gaps between successive grid points are smaller at lower levels of income. Earnings are placed on a grid of 7 points each year using the procedure introduced by Tauchen (1986).

Integration. There are three risks facing households: mortality, earnings and financial risks. The risks are independent. Realizations for the first of these are naturally discrete and integration involves a simple weighted average. For the latter two, we define a discrete set of possible realizations and integrate over outcomes using the Tauchen (1986) procedure.

Approximation. To evaluate the value function at points other than those in the discrete sub-set of points, we use linear interpolation in multiple dimensions.

Optimization. Every period households make up to four choices. They decide on how much to consume, how much to save in each of bonds and the risky asset, whether to pay the participation cost (if they have not previously done so) and how much of an annuity income stream to purchase (in the period before retirement). Our problem is not globally concave, so our optimization of the household's decision problem cannot fully rely on local approaches. We therefore start by discretizing the choice variables. We define three grids on the unit interval, S_a^t, S_s, S_c, which will represent shares of available resources dedicated to purchases of annuities, portfolio shares in stocks and consumption respectively. All grids have equally spaced nodes from 0 to 1 (except that in periods other than period T_{R-1}, the grid for annuities has only one element, 0, as no annuity purchase is possible in those periods). We evaluate the household's objective function at each combination of $(s_{a,i}^t)_{i=1,\dots,I^t}$ in S_a^t, $(s_{s,j})_{j=1,\dots,J}$ in S_s, and $(s_{c,k})_{k=1,\dots,K}$ in $S_{c,k}$. Defining the total available resources as $R_t = y_t + w_t$ (the sum of income and wealth) in time t, we set the annuity choice to $s_{a,i}^t R_t$, the level of saving in the risky asset set to $s_{s,j}(1 - s_{a,i}^t)R_t$, the level of consumption as $s_{c,k}(1 - s_{s,j})(1 - s_{a,i}^t)R_t$ and the savings in the bond as $(1 - s_{c,k})(1 - s_{s,j})(1 - s_{a,i}^t)R_t$. We find the combination of the points that yields the maximum value to the household. This is our candidate optimal decision. We then do a further local search for the split between consumption and the bond using golden section search. Formally, taking the candidate maximum as indexed by s_{a,i^*}^t in S_a^t, s_{s,j^*} in S_s, s_{c,k^*} in $S_{c,k}$, we look for the utility-maximizing split between consumption and the bond in the interval for consumption of $[s_{c,\max\{1,k^*-1\}}(1 - s_{s,j^*})(1 - s_{a,i^*}^t)R, s_{c,\min\{K,k^*+1\}}(1 - s_{s,j^*})(1 - s_{a,i^*}^t)R]$.

For points where individuals have not already paid the participation charge we implement this procedure twice, once assuming they pay the charge, once assuming they do not. The maximum of these indicates the decision rule.

This procedure yields decision rules as a function of the vector of state variables $\boldsymbol{X}_t$: $\hat{c}(\boldsymbol{X}_t)_t, \hat{b}(\boldsymbol{X}_t)_t, \hat{s}(\boldsymbol{X}_t)_t, \hat{a}(\boldsymbol{X}_t)_t, \hat{\eta}(\boldsymbol{X}_t)_t$.

1.2 Simulation of Profiles

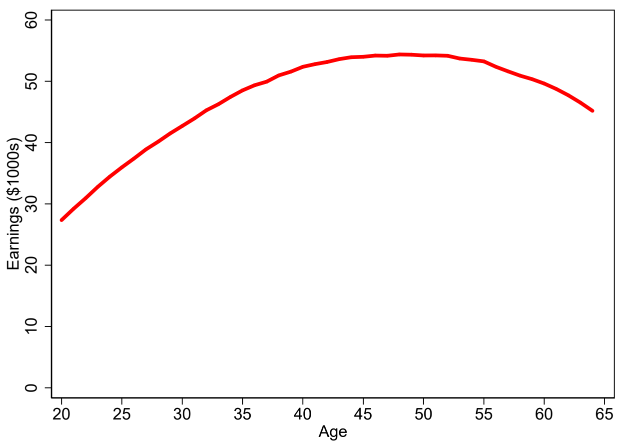
Once decision rules $\hat{c}(\boldsymbol{X}_t)_t, \hat{b}(\boldsymbol{X}_t)_t, \hat{s}(\boldsymbol{X}_t)_t, \hat{a}(\boldsymbol{X}_t)_t, \hat{\eta}(\boldsymbol{X}_t)_t$ are obtained, we simulate a data set for 3,000 individuals. We do this as follows:

1. Initial values for wealth are set to 0.
2. Earnings draws for the first period of economic life are drawn randomly for each individual. Using these values of the state variables and the decision rules we can obtain optimal behavior in the first period.
3. We draw a equity price shock to apply to any equity holdings held at the end of the period.
4. Optimal behavior, the rate of return and the inter-temporal budget constraint yield the state variables for period 2.
5. We repeat steps (2) to (4) to obtain optimal behavior and subsequent state variables for each age up to 100. In most time periods, individuals will have realizations of the continuous state variables that are off this grid. Our approach here is to solve the individual's problem for decision rules as we do in the solution stage.

2 Lifecycle productivity

The shape of the deterministic age-productivity profile is taken from Harenberg and Ludwig (2019), who compute it from PSID data using the method in Huggett et al. (2011) US$.[1] Figure 1 displays the average earnings profile.

Figure 1: Average earnings over the lifecycle



[1]Note that we multiply labor income with $\bar{y}$, thereby making sure that average labor income in 2016 is matched.

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