INTERTEMPORAL PRICE DISCRIMINATION IN SEQUENTIAL QUANTITY-PRICE GAMES

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Intertemporal Price Discrimination in Sequential Quantity-Price Games

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Abstract

When firms first choose capacity and then compete on prices in a series of advance-purchase markets, we show that strong competitive forces prevent firms from utilizing intertemporal price discrimination. We then enrich the model by allowing firms to use inventory controls, or sales limits assigned to individual prices. We show that firms will choose to set inventory controls in order to engage in intertemporal price discrimination, but only if demand becomes more inelastic over time. Thus, although typically viewed as a tool to manage demand uncertainty, we show that inventory controls can also facilitate price discrimination in oligopoly.

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1 Introduction

In many oligopoly markets, such as the airline, entertainment, hotel, and sports industries, firms choose capacity long before they set price. Seminal research by Kreps and Scheinkman (1983) and Davidson and Deneckere (1986) analyzes sequential quantity-price games and characterizes when the Cournot model is a valid prediction in these games. However, an important limitation of this research is that it considers only one pricing or sales period; yet, in all of the aforementioned industries, consumers purchase in advance and firms adjust their prices over time.

In this paper, we develop an oligopoly model of sequential quantity-price games with multiple sales periods and a perishability date. We use the model to characterize the pricing behavior of firms competing in multiple advance-purchase markets, and, more specifically, we explore the conditions required for intertemporal price discrimination to arise in an oligopoly setting.

We highlight two contributions. First, absent additional commitments made by firms, strong competitive forces drive equilibrium prices—advance-purchase prices—to be flat over time. That is, extending the findings of Kreps and Scheinkman (1983) and Davidson and Deneckere (1986) to multiple sales periods creates a costless arbitrage opportunity, resulting in intense price competition, which prevents firms from utilizing intertemporal price discrimination. This is true regardless of whether demand becomes more or less elastic over time, though, we highlight some important asymmetries between these two cases. Second, we enrich the model by incorporating inventory controls, or sales limits assigned to set prices. We show that firms will choose to set inventory controls in order to engage in intertemporal price discrimination, but only if demand becomes more inelastic over time. Inventory controls allow for increasing prices as well as higher profits for firms, even when there is no uncertainty about demand. Thus, although inventory controls have been studied extensively in the context of demand uncertainty, we show that they can also be used for a different purpose: to facilitate price discrimination in oligopoly markets.

In our baseline model, firms sell a homogeneous good and have no private information. Firms first choose capacity levels—a capacity constraint common across periods—and then compete on price in a series of sequential markets. The timing of the game is such that remaining capacities are observed, and then firms simultaneously choose prices. Transactions are observed and the game repeats with updated remaining capacities. After the final period, no further sales can take place, and unsold inventory is scrapped.

We assume that there is a continuum of consumers who are each assigned to one of the sequential markets. We allow the elasticity of demand to change over time, both increasing and decreasing; we emphasize several results in which demand becomes more inelastic over time. This case is appealing because prices tend to rise over time in many applications, including entertainment and sporting events, airlines, trains, and hotels. Also, when prices are increasing, consumers do not have an incentive to wait until later periods to purchase. We employ more-restrictive assumptions under the scenario of increasing price elasticity.

For tractability, we analyze a model with two advance-purchase sales periods, although we do discuss extending the analysis to any finite number of periods in an extension. The challenge in solving our game, as well as the sequential quantity-price games studied by Kreps and Scheinkman (1983) and Davidson and Deneckere (1986), is that quantity-constrained price games often have mixed-strategy equilibria. Solving our game is even more challenging because we consider more than one sales period. We make the analysis simpler and more intuitive by focusing on sufficiently high costs of capacity—a scenario also considered in the prior literature on sequential quantity-price games. Doing so ensures that equilibrium capacity choices will be sufficiently small and that the equilibrium strategies in the price game will be pure strategies both on and off the equilibrium path.

Our first main result is that there exist strong competitive forces that prevent intertemporal price discrimination. Equilibrium prices are flat over time, even when consumers who arrive later have a higher willingness to pay. Equilibria with increasing prices do not exist because individual firms have an incentive to raise their price in the early period—shifting early sales to its competitors— in order to sell more in the later period, when consumers are less price-sensitive and the equilibrium price is higher. Similarly, equilibria with decreasing prices do not exist because individual firms have an incentive to lower price in the early period, shifting high-priced sales to itself and low-priced sales to its rivals later.

Because firms can costlessly shift their capacity across periods (or markets), our result may not seem surprising. But recall that if firms could choose how much of their capacity to allocate to each market, then they would equate the marginal revenues, and not the prices, in the two markets. We show that with more than one pricing period, the sequential capacity-then-price game is different from the Cournot model, even though it is the same as the Cournot model with just one pricing period.

We characterize sufficient conditions under which uniform prices arise as the unique pure-strategy equilibrium outcome. With a uniform price, sales are equal to the Cournot quantity associated with aggregate demand—the sum of demands over time. Although the flattening of prices occurs whether the elasticity of demand is increasing or decreasing over time, we show that there exist important asymmetries in the sufficient conditions for uniform pricing in the two scenarios.

We then enrich the model by allowing firms to implement unit-sales limits, or inventory controls, in conjunction with price setting. Inventory controls have been studied extensively in the context of demand uncertainty as a means to more efficiently allocate scarce capacity (Talluri and Van Ryzin 2006, McGill and Van Ryzin 1999). We show that they can be used to facilitate intertemporal price discrimination. We analyze a game that differs from the baseline model in the price stage. In the game with inventory controls, firms simultaneously decide sales limits as well as prices in each of the sequential markets. We show that firms commit to the use of inventory controls in order to increase profits, but only when the elasticity of demand decreases over time. In this case, equilibria exist in which prices are increasing because inventory controls curtail firms' abilities to shift demand to their competitors in the early, lower-priced period. Firms sell their Cournot output in each period, and so prices rise over time because demand becomes less elastic. In contrast, when the elasticity of demand increases over time, inventory controls do not change equilibrium prices.

We also discuss a number of extensions. The first extension is to consider product differentiation. When products are differentiated, prices are no longer uniform across time, as firms benefit from the inability to shift all of the demand using very small price changes. However, the strategic incentives explored in this paper are still present. We provide an example that shows that products must be highly differentiated for prices to increase substantially across periods. Our results suggest that inventory controls are particularly valuable when products are close substitutes; they allow firms to target the market in which consumers are less price-sensitive. We also discuss generalizing the model to include aggregate demand uncertainty and more than two sales periods.

1.1 Related Literature

This paper contributes to three strands of the economics literature.

First, we analyze a model of price competition with capacity constraints (Levitan and Shubik 1972, Allen and Hellwig 1986, Osborne and Pitchik 1986, Klemperer and Meyer 1986, Acemoglu, Bimpikis, and Ozdaglar 2009). As in Kreps and Scheinkman (1983) and Davidson and Deneckere (1986), our firms choose capacity and then price, but unlike earlier research, we consider more than one pricing period. Our results are also related to Van den Berg, Bos, Herings, and Peters (2012), who consider a two-period quantity game with capacity constraints, with and without commitment. However, our main focus is on the way firms use prices to shift rivals' sales from the higher-price to the lower-price period, which does not happen in their sequential quantity game because quantity decisions do not affect the way that their rivals' capacity is allocated across periods, as price decisions do in our model.[1]

Second, we analyze intertemporal price discrimination. Stokey (1979) is a seminal paper that shows that monopoly intertemporal price discrimination is not always feasible. Several more-recent papers find that price adjustments over time are profitable in environments with deadlines and limited capacity (Gallego and van Ryzin 1994, Su 2007, Board and Skrzypacz 2016, Dilmé and Li 2018). This is particularly true when consumers learn their preferences over time, as in Akan, Ata, and Dana (2015) and Ata and Dana (2015).

Important empirical contributions to the literature on intertemporal price discrimination and, more generally, price adjustments over time, include Nair (2007) on video games, Sweeting (2012) on stadium seats, and Hendel and Nevo (2013) on storable goods. Much of our focus is on prices that increase as a deadline draws closer (such as for a scheduled event or a scheduled departure time), a pattern found in many airline studies (Puller, Sengupta, and Wiggins 2012, Lazarev 2013, McAfee and te Velde 2007, Williams 2018).[2]

Finally, our work is related to the literature on inventory controls (see Littlewood (1972), Belobaba (1987), Belobaba (1989) and Weatherford and Bodily (1992), and surveys by Talluri and Van Ryzin (2006) and McGill and Van Ryzin (1999)). While prior research views inventory controls as a tool for managing aggregate demand uncertainty, we establish that they also facilitate intertemporal price discrimination in oligopoly markets when firms prefer to set prices that increase over time. Airlines, hotels, theaters, and trains utilize inventory controls.

2 The Model

Consider an oligopoly with n firms selling a homogeneous good to a continuum of consumers in a series of advance-purchase sales markets. For tractability, we consider just

[1]Also see Benassy (1989) and Reynolds and Wilson (2000) for related pricing games and De Frutos and Fabra (2011) and Aguirre (2017) for quantity games.

[2]Fare increases in the final weeks before departure are consistent with intertemporal price discrimination, and also with theoretical models of demand uncertainty (Prescott 1975, Eden 1990, Dana 1999).

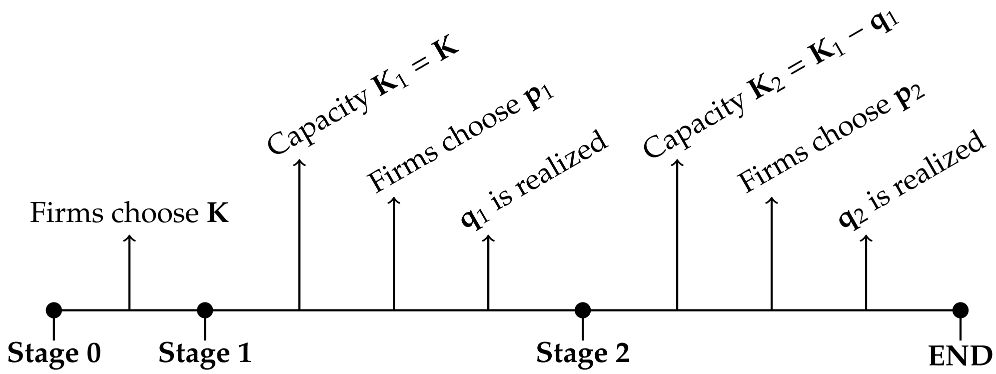
two selling periods, $t = 1, 2$, and describe adding more selling periods in an extension. We assume that consumers cannot postpone their purchasing decisions but, instead, are exogenously assigned to purchase in either the first or the second sales period. However, this assumption can be relaxed by assuming that some consumers learn their preferences in period one and some in period two, and consumers choose when to purchase (Dana 1998, Akan, Ata, and Dana 2015).[3] But, this would imply that firms never set prices that decline over time since consumers would prefer to wait and buy when prices are lower. Although our results under increasing price elasticity are unchanged by relaxing this assumption, consumers' ability to wait would curtail intertemporal price discrimination if demand were to become more elastic.

Each firm's strategy consists of three choices, capacity and two prices, denoted by K^i, p_1^i, and p_2^i, where subscripts denote time, and, when necessary, superscripts denote the individual firm. The game proceeds in three stages or periods (see Figure 1). First, in stage zero, firms simultaneously choose their capacities, $K^i \geq 0$. The vector of initial capacities is denoted by $\mathbf{K}$. The common cost per unit of capacity is $c \geq 0$. We assume that the marginal cost of production for each unit sold is zero. Then, in stage one, firms simultaneously choose prices denoted by the vector $\mathbf{p}_1$. Consumers who purchase in stage one then make their purchase decisions. Sales, $\mathbf{q}_1 \geq \mathbf{0}$, are constrained only by the firms' first-period capacities, $\mathbf{K}_1 = \mathbf{K}$, and firms' remaining capacities, $\mathbf{K}_2 = \mathbf{K} - \mathbf{q}_1 \geq \mathbf{0}$, are carried forward to the next period. That is, the capacity constraint is common across periods. In stage two, firms simultaneously choose prices denoted by the vector $\mathbf{p}_2$, and then consumers who purchase in stage two make their purchase decisions. Sales, $\mathbf{q}_2 \geq \mathbf{0}$, are constrained only by the firms' residual capacities, $\mathbf{K}_2$. Capacity not used in stage two, $\mathbf{K}_2 - \mathbf{q}_2$, is scrapped with value zero. We ignore discounting.

Because we assume a continuum of heterogeneous consumers with unit demands, and

[3] In this version of the model, some consumers prefer to purchase in the second period because they do not know their demand until the second period, while other consumers prefer to purchase in the first period, even with the option to wait, because they know their demands early and anticipate that the firms' prices will be higher if they wait.

Figure 1: Timing of the Game



goods are homogeneous, we can represent preferences using market demand functions. The market demands in each period, denoted by $D_1(p_1)$ and $D_2(p_2)$, respectively, are strictly decreasing and differentiable functions. And we let $p_1(q)$ and $p_2(q)$ denote the inverse demands associated with $D_1(p)$ and $D_2(p)$. We let $D_{Tot}(p) = D_1(p) + D_2(p)$ denote the total demand when the price is the same in both periods, and $p_{Tot}(q)$ denote the associated inverse total demand when q units of total output are sold at a uniform price. We also assume that the market demand functions are associated with concave revenue functions. That is, we assume that $p_t''(q_t)q_t + 2p_t'(q_t) < 0, \forall t = 1, 2$, where q_t denotes total sales in period t. Let $\eta_t(p) = D_t'(p)p/D_t(p)$ denote the price elasticity of demand in period t.

We now describe how firm i's individual demand is derived from these market demand functions, given the prices of all firms. Products are homogeneous, so consumers purchase at the lowest price available, as long as their valuation exceeds the price. If firms set different prices, then a firm with a higher price can have positive sales only if all of the firms with lower prices have sold all of their capacity. If two or more firms charge the same price, then those firms divide the sales equally, subject to their capacity constraints. How much the firm with the higher price sells—that is, the residual demand after the lower-priced firm sells all of its capacity—depends on the rationing rule. The residual demand function is written $RD_t\left(p_t^i; \mathbf{p}_t^{-i}, \mathbf{K}_t^{-i}\right)$, where the arguments are firm i's own price,

p_t^i, and vectors of all of the other firms' prices and current capacities, denoted $\mathbf{p}_t^{-i}$ and $\mathbf{K}_t^{-i}$, respectively. This is the total residual demand at price p_t^i, which is split among all firms charging p_t^i.

Our results hold for both the efficient rationing rule and the proportional rationing rule. The efficient rationing rule specifies that the lowest-price unit for sale (after other lower-priced units are gone) goes to the remaining consumer with the highest willingness to pay. Equivalently, under the efficient rationing rule, the residual demand function is

$$RD_t(p_t^i; \mathbf{p}_t^{-i}, \mathbf{K}_t^{-i}) = D_t(p_t^i) - \sum_{j:\ p_t^j < p_t^i} K_t^j, \forall t = 1, 2, \tag{1}$$

where the summation is over all firms j that charge a lower price than firm i, and where $-i$ denotes other firms. The proportional rationing rule specifies that lowest-price unit for sale (after all other lower-priced units are gone) is equally likely to be sold to every remaining consumer whose willingness to pay exceeds the price. Equivalently, under the proportional rationing rule, the residual demand function is

$$RD_t(p_t^i; \mathbf{p}_t^{-i}, \mathbf{K}_t^{-i}) = D_t(p_t^i)\left[1 - \sum_{j: p^j < p^i} \frac{K_t^j}{D_t(p_t^j)}\right], \forall t = 1, 2. \tag{2}$$

In both cases, the residual demand describes demand at price p^i. If only firm i charges price p^i, then firm i's demand at p^i is given by the residual demand function, and its sales are the smaller of residual demand and its capacity. If more than one firm charges p^i, then the residual demand is divided among the firms, subject to firms' capacity constraint.

Note that the rationing rules determine how sales are allocated to different firms within each period, but not how sales are allocated across periods. If early buyers have higher valuations, then it is natural to think that rationing is efficient and that demand becomes more elastic over time. Similarly, when demand is less elastic in period two, then proportional rationing may be more compelling. However, we do not assume a link between rationing rules and the change in the demand elasticity. Instead, we derive results that are independent of the rationing rule, but depend on whether demand becomes more or less elastic over time.

If there were just one pricing period, and the game ended at the end of stage one, then we know from Kreps and Scheinkman (1983), who analyze efficient rationing, and Davidson and Deneckere (1986), who analyze proportional rationing, that the pricing subgame has a unique Nash equilibrium. Both of these papers characterize the equilibrium profits of the pricing subgame for all capacity choices, including subgames in which the equilibrium prices were in mixed strategies.

Kreps and Scheinkman (1983) and Davidson and Deneckere (1986) both show that if firms choose sufficiently small capacity, then pricing subgame has a unique pure-strategy equilibrium in which all prices equal the market-clearing price. They also show that for sufficiently high capacity costs, every pricing subgame in which firms earn positive profits has a pure-strategy equilibrium. Far from being a special case, the sufficient conditions are just that the cost of capacity is large enough that firms never find it profitable to choose so much capacity that the marginal revenue function is negative. That is, firms collectively choose capacities smaller than the revenue-maximizing capacity.

A particularly enlightening example is the case of constant elasticity demand, or $p(q) = q^{1/\epsilon}$. In this case, marginal revenue is strictly positive for all q if $|\epsilon| > 1$, because $p(q)+p'(q)q = (1 + 1/\epsilon)q^{1/\epsilon}$, which is positive and decreasing for all q when $|\epsilon| > 1$ and is negative for all q when $|\epsilon| < 1$. This means that in both the models by Kreps and Scheinkman (1983) and Davidson and Deneckere (1986), with constant elasticity demand and $|\epsilon| > 1$, the equilibrium of every pricing subgame is the market-clearing price, regardless of the firms' capacity choices in the first stage. The pricing subgame never has a mixed-strategy equilibrium, so the unique equilibrium of the quantity-price game with either efficient or proportional rationing is the Cournot equilibrium. This is true regardless of the rationing rule, as long as demand is sufficiently elastic, or $|\epsilon| > 1$, for every price (Madden 1998).

Because we have two pricing periods, characterizing the equilibria of the pricing subgame is considerably more challenging than in Kreps and Scheinkman (1983) and Davidson and Deneckere (1986), and simply bounding profits without solving for the equilibrium is also more challenging. To simplify our analysis, we focus solely on the case in which capacity costs are sufficiently large so that all of the pricing subgames have pure-strategy equilibria.

Besides making it easy to derive the equilibrium prices and profits in all of our subgames, another benefit of assuming high capacity costs are that we can easily derive identical results for both the efficient and proportional rationing rules. Recall that in both Kreps and Scheinkman (1983) and Davidson and Deneckere (1986), when firms' capacities are small—specifically, smaller than the revenue-maximizing capacity or, equivalently, smaller than a monopolist's output if capacity were free—the price is always equal to the market-clearing price and this is independent of the rationing rule. This is because the marginal revenue is positive in the pricing stage even when firms act as monopolists, which implies that marginal revenue must be positive for each individual firm. Thus, firms can never increase their profits by setting a price above the market-clearing price.

We can focus on pure-strategy equilibria of the pricing subgames by making two assumptions. The first assumption guarantees that marginal revenue is positive in the second pricing period, even if every firm chooses not to sell any of its capacity in the first period. We guarantee this by assuming that $D_{Tot}(c)$, the total demand at the competitive price—that is, the demand when firms set price equal to the marginal cost of capacity—is less than the output sold by a zero-cost monopolist in the second period. This is clearly true if the cost of capacity, c, is sufficiently large.

Assumption 1. *The total competitive output with capacity costs c is smaller than the capacity that maximizes industry revenue, or, equivalently, smaller than the second-period monopoly output* *when the firm has zero capacity costs. That is,*

$$\operatorname*{argmax}_{q} p_2(q)q > D_{Tot}(c).$$

We provide two examples to highlight the potential restrictiveness, or lack of restrictiveness, of Assumption 1.

Example: With constant elasticity demand—$p(q) = q^{1/\epsilon}$ each period and $|\epsilon| > 1$—a monopolist with zero cost of capacity produces an infinite output, so Assumption 1 is trivially satisfied. Marginal revenue is strictly positive for all q, so marginal revenue is positive in period two for $q = D_{Tot}(c)$, for any value $c > 0$. Formally, this is because $p(q) + p'(q)q = (1 + 1/\epsilon)q^{1/\epsilon}$, which is strictly positive for all $\epsilon < 1$ and for all finite $q > 0$.

Example: To see that Assumption 1 can be restrictive, but not unreasonably so, consider the case of linear demand, $p(q) = a - bq$, and assume that demand is the same in both periods. Then, $D_{Tot}(c) = 2 \cdot ((a - c)/b)$, and the monopoly output in the second period (with zero capacity costs) is $a/(2b)$. Thus, Assumption 1 holds if, and only if, $c > (3/4)a$.

The second assumption places a mild and intuitive restriction on firms' strategies. We assume that the firms' total capacity does not exceed the capacity that would be produced if the market were perfectly competitive—that is, total capacity is less than $D_{Tot}(c)$.

Assumption 2. *Firms' capacities are less than total demand at the perfectly competitive price, or* $\sum_i K^i \leq D_{Tot}(c)$.

The interpretation of Assumption 2 is similar to that of an equilibrium refinement. That is, we characterize the unique equilibrium within this restricted strategy set and show that it is in the interior of the set—$\sum_i K^i < D_{Tot}(c)$. While we do not formally show that every deviation outside of this set isn't profitable, we do show that profits are zero and decreasing in capacity on the border of the set.

Assumption 2 can be difficult to relax because characterizing firms' profits and their subgame equilibrium mixed strategies in a model with two pricing periods is difficult.

We hypothesize that dropping Assumption 2 does not alter the equilibrium strategies, or introduce any new equilibria. That is, no equilibrium exists in which firms choose more capacity than they would choose in a perfectly competitive market. If firms produced that much capacity and sold all of it, total profits would be negative. And it is also difficult to see how firms could benefit from such a large capacity if it is not all sold. Of course, unused capacity might be an effective off-the-equilibrium path instrument for punishment; however, that is not the case here. Finally, note that there are alternatives to Assumption 2. For example, we could assume that each firm's capacity is less than $D_{Tot}(c)/n$ and obtain the same results. This alternative, and others, are stronger, but they have the advantage that we are not placing restrictions on firms' strategies that depend on other firms' actions.

Together, Assumptions 1 and 2 imply that the firms' total capacity is always less than $\operatorname{argmax}_q p_2(q)q$, which implies that the firms' remaining capacity in period two is always less than $\operatorname{argmax}_q p_2(q)q$, and that the equilibrium price in the final pricing period is always the market-clearing price. This means that both on and off of the equilibrium path, every final-period subgame has a unique pure-strategy equilibrium.

For several reasons, it is helpful to distinguish between the case in which demand becomes more inelastic over time and the case in which demand becomes more elastic over time. While we establish results for both scenarios, for convenience, we state the former case as the following explicit assumption.

Assumption 3. *Demand becomes more inelastic over time, so* $|\eta_2(p)| < |\eta_1(p)|$ *for all* $p > 0$.

Assumption 3 is reasonable in advance-purchase markets when consumers with less-elastic demand purchase later. This happens, for example, when consumers' purchase decisions require planning, and planning is less costly for consumers with more-elastic demands. As a consequence, the demand elasticity decreases over time—this has empirical support for the airline and theater industries, for example.

3 A Benchmark Result

Before characterizing the equilibrium of the game, we consider a useful benchmark.

Imagine that firms set price just once or, more precisely, that firms are constrained to set the same price in stage two as in stage one—that is, $p_2^i = p_1^i, \forall i$. So, firms choose $\mathbf{K}$ in stage zero and $\mathbf{p}_1$ in stage one, and the price in stage two is constrained to also be $\mathbf{p}_1$. Then, $\mathbf{K}$ must be the symmetric Cournot output (the Cournot output when demand is $D_1 + D_2$). This is because Assumptions 1 and 2 imply that the equilibrium price in the pricing subgame is always equal to the market-clearing price, so the stage zero capacity game reduces to a standard Cournot model. We show this in the following lemma. The proof is in the Appendix.

Lemma 1. *Under either the efficient or the proportional rationing rule, if Assumption 1 holds, and given any capacities that satisfy Assumption 2, the equilibrium price chosen by all firms in the pricing subgame is the market-clearing price—that is, the price such that $D_1(p) + D_2(p) = \sum_i K^i$.*

The lemma implies that firms must choose the Cournot output.[4] So, as in the Cournot model, as the number of firms goes to infinity, the price converges to the cost of capacity, c, and the total capacity and sales converges to $D_1(c) + D_2(c)$. Hence, we refer to c as the competitive price.

In the next section, we consider the model when firms can set different prices in each period, and when the elasticity is higher (or lower) in stage 2, so that setting different prices generates higher profits.

4 Equilibrium Characterization

We now solve the full model as described in Section 2. We solve the three-stage game (the capacity decision in stage zero, the price decision in stage one, and the price decision in stage two) by backwards induction. All proofs appear in the Appendix.

[4] We need to also assume that $p_{Tot}(q)\, q$ is concave in order for the Cournot equilibrium to be unique.

4.1 The Pricing Subgames

We first characterize equilibrium prices in the second pricing period. Lemma 2 states that in the second period, firms set prices to clear the market.

Lemma 2. *Under either the efficient or the proportional rationing rule, if Assumption 1 and Assumption 2 hold, then in any subgame perfect equilibrium (SPE) of the three-stage game, the price in the second selling period clears the market.*

Lemma 2 allows us to easily characterize all of the pure-strategy subgame-perfect equilibria of the pricing subgame. Note that we typically state that an equilibrium is unique when all equilibria have the same outcomes and payoffs for all players. Also, Lemma 1 shows that the unique equilibrium of the second stage is a uniform price, so no mixed-strategy equilibria of the subgame exist; however, we simplify the solution of the game by focusing on pure-strategy equilibria of the three-stage game.

Next, we characterize pure-strategy equilibria of the two-period pricing subgame. Note that no symmetric equilibrium exists in the pricing subgame in which firms charge different prices in the two periods, p_1 and p_2. If $p_1 < p_2$, a firm could profitably deviate to a slightly higher price in period one. The firm's period-one sales would fall discretely (possibly to zero); its period-two sales would rise discretely; and its transacted prices would change by an arbitrarily small amount. Its profits would be higher. Similarly, if $p_1 > p_2$, then a firm could profitably deviate to a slightly lower price in period one. The firm's period-one sales would rise discretely (possibly rise to its capacity); its period-two sales would fall discretely; and its transacted prices would change by an arbitrarily small amount. Its profits would be strictly higher.

Proposition 1, below, shows that there are two types of pure-strategy subgame perfect equilibria in the pricing subgame. In a uniform-price equilibrium, prices are the same across firms and periods. Since the market clears in the second period (Lemma 2), any uniform-price equilibrium must satisfy $D_1(p^*) + D_2(p^*) = \sum_i K^i$, so the uniform price is unique. In asymmetric-price equilibria, a single firm sells in the first period; the first-period price is lower than the second-period price; and all other firms sell only in the second period.

Asymmetric-price equilibria have a particular form. Only one firm, firm i, sells in the first period. Let p_1^i and q_1^i denote its first-period price and quantity, where

$$p_1^i = \operatorname*{argmax}_{p \in [p_1(K^i), \infty]} pD_1(p) + p_2\left(\sum_i K^i - D_1(p)\right)\left(K^i - D_1(p)\right), \tag{3}$$

or, equivalently,

$$q_1^i = \operatorname*{argmax}_{q \in [0, K^i]} p_1(q)q + p_2\left(\sum_i K^i - q\right)\left(K^i - q\right). \tag{4}$$

In both expressions, the firm's output is constrained so that first-period sales do not exceed K^i. The second-period price is higher than p_1^i and is given by

$$p_2 = p_2\left(\sum_i K^i - D_1(p_1^i)\right). \tag{5}$$

Proposition 1. *Under either the efficient or the proportional rationing rule, if Assumptions 1 and 2 hold, then every pure-strategy SPE of the pricing subgame is either a uniform-price equilibrium or an asymmetric-price equilibrium satisfying Equations (3), (4) and (5). When a uniform-price equilibrium exists, it is the unique pure-strategy SPE. And*

1. *when $\eta_1(p) = \eta_2(p), \forall p > 0$, a uniform-price equilibrium exists and is the unique pure-strategy SPE;*

2. *when $|\eta_1(p)| < |\eta_2(p)|, \forall p > 0$, or when demand becomes more elastic over time, a uniform-price equilibrium may exist, and an asymmetric-price equilibrium never exists; and*

3. *when $|\eta_1(p)| > |\eta_2(p)|, \forall p > 0$, or when demand becomes less elastic over time, either a uniform-price or an asymmetric-price equilibrium exists, but not both; at most, n asymmetric-price equilibria exist.*

Intuitively, asymmetric-price equilibria exist because a lower price in the first pricing period increases sales in the first period, leading to less output sold and a higher second-period price. But a firm can increase its profit in this way only if the elasticity is decreasing and only if it has sufficient capacity to meet all of the first-period demand, plus some additional capacity to sell at the higher second-period price. Other firms free ride and sell only in the second-pricing period at the higher price.

Asymmetric-price equilibria are more likely to exist when one firm chooses more capacity than its rivals in stage zero. And the uniform-price equilibrium is less likely to exist when one firm chooses significantly more capacity than its rivals. The incentive to deviate to a lower price is increasing in the deviating firm's capacity, decreasing in the rival firms' capacity, increasing in the elasticity of first-period demand, and decreasing in the size of first-period demand.

Proposition 1 holds whether the elasticity is increasing or decreasing. That is, in either case, price competition puts pressure on firms to equalize prices across the two markets. We will emphasize the important fundamental asymmetries between increasing- and decreasing-elasticity models later in the paper.

Although Proposition 1 shows that asymmetric-price equilibria of the pricing subgame may exist, Assumption 4 below implies that only a uniform-price equilibrium exists when the demand elasticity is decreasing over time. This relatively weak assumption implies that no firm has enough capacity to profitably deviate from the symmetric uniform-price equilibrium.

Assumption 4 requires that demand in the second pricing period not be too inelastic relative to demand in the first pricing period. Demand in the second pricing period is less elastic than in the first pricing period when Assumption 3 holds; Assumption 4 limits how inelastic demand can be in the final period.

Assumption 4. *The elasticities of demand and capacities satisfy*

$$\frac{\eta_2(p)}{\eta_1(p)} > \frac{K^i}{\sum_{j=1}^{n} K^j}, \forall p > 0, i = 1, ..., n. \tag{6}$$

The next proposition, Proposition 2, shows that under our additional assumptions, Assumption 3 and Assumption 4, the unique subgame perfect equilibrium of the two-period pricing subgame is the uniform-price equilibrium.

Proposition 2. *When Assumptions 1-4 hold, the unique subgame-perfect pure-strategy equilibrium of the pricing subgame is a uniform-price equilibrium.*

Intuitively, when the demand elasticity is decreasing, deviating to a lower price from a uniform price is profitable for a monopolist if it raises the second-period profit by more than it lowers the first-period profit. However, since rivals free ride and sell only in period two, an oligopoly firm that deviates from a uniform price, by lowering its first-period price, earns, at most, $1/n$th of the second-period industry profits. The oligopoly firm that deviates cannot increase its profit unless it can increase the second-period industry profits by at least n times the decrease in its first-period profit. For such a deviation to be profitable, the first-period demand must be at least n times more elastic than the second-period demand. Assumption 4 guarantees that such a deviation is not profitable.

Strong competitive pressures also exist when demand becomes more elastic over time; however, the existence of a unique uniform pricing equilibrium is more nuanced. Proposition 3 describes two sufficient conditions for the existence of a uniform-price equilibrium:

Proposition 3. *When Assumptions 1-2 hold, and when demand becomes more elastic over time (the opposite of Assumption 3), then the unique subgame-perfect pure-strategy Nash equilibrium of the pricing subgame is a uniform-price equilibrium when*

$$D_1(p) < \sum_{j \neq i} K^j, \forall i, \tag{7}$$

under both proportional or efficient rationing, or, alternatively, when

$$\frac{\eta_1(p)}{\eta_2(p)} > \max_{i=1,\ldots,n} \frac{K^i}{\sum_{j\neq i}^n K^j}, \forall p > 0, i = 1, .., n, \tag{8}$$

under efficient rationing.

The first condition in Proposition 3 guarantees that no firm is sufficiently large to act as a residual monopolist in the first period, and, hence, a price increase by one firm cannot affect any other firm's profit—it merely shifts sales to other firms at the first-period price. The second condition in Proposition 3 is analogous to Assumption 4 and guarantees that no profitable deviation exists, but it holds only under efficient rationing. Under proportional rationing, a uniform-price equilibrium may not exist when first-period demand is large relative to second-period demand. If equation (7) does not hold, then a firm with enough capacity can profitably deviate to a higher price in period one. If a uniform-price equilibrium does not exist, then no pure-strategy equilibrium of the pricing subgame exists.

Note that our results reveal an asymmetry between the case in which demand becomes less elastic and the case in which demand becomes more elastic over time. When demand becomes more elastic, the only potentially profitable deviation from a uniform-price equilibrium is to lower price in the first period; when demand becomes less elastic, the only potentially profitable deviation is to increase price in the first period. However, in the former case, deviating is profitable only when a firm's capacity is sufficiently larger than that of rival firms so it can have significant market share in the second period when prices are higher. In the latter case, deviating is profitable only if it induces all rival firms to sell all of their capacity in the first period.

Another asymmetry is the existence of pure-strategy equilibrium. We have shown that when demand becomes less elastic over time, the non-existence of a uniform-price equilibrium implies the existence of an asymmetric-price equilibrium. However, there is no analogy when demand becomes more elastic over time. If a uniform-price equilibrium does not exist, then firms prefer to set higher prices in the first period, but no pure-strategy equilibrium exists with declining prices.

4.2 The Initial Capacity Choice

Proposition 2 is an important result of the paper. It specifies that for any allocation of initial capacity satisfying Assumption 4, oligopoly firms cannot price discriminate when the demand conditions are such that a monopolist clearly would price discriminate.

We now consider what happens when firms choose their initial capacity optimally. We replace Assumption 4, which is a restriction on firms' capacities, with Assumption 5, which is a restriction on the elasticities of demand. Assumption 5 is weaker: it is equivalent to Assumption 4 when the firms' capacities are symmetric. Proposition 4 establishes that Assumption 5 is sufficient to guarantee that in the full game—that is, when the firms choose capacities in stage zero—the unique subgame perfect equilibrium is a uniform-price equilibrium.

Assumption 5. *The elasticity of demand satisfies*

$$\frac{\eta_2(p)}{\eta_1(p)} > \frac{1}{n}, \forall p > 0.$$

Proposition 4. *Under Assumptions 1-3 and Assumption 5, and when the total revenue function, $p_{Tot}(q)q$, is concave, the unique pure-strategy subgame perfect Nash equilibrium of the full game is a uniform-price equilibrium, and equilibrium capacity and profits are equal to the Cournot capacity and profits given demand $D_1(p) + D_2(p)$.*

Proposition 4 implies that intertemporal price discrimination is impossible when demand becomes more inelastic over time. Unless the decrease in the magnitude of the demand elasticity is very large, firms will choose symmetric capacities, and prices will be uniform over time.

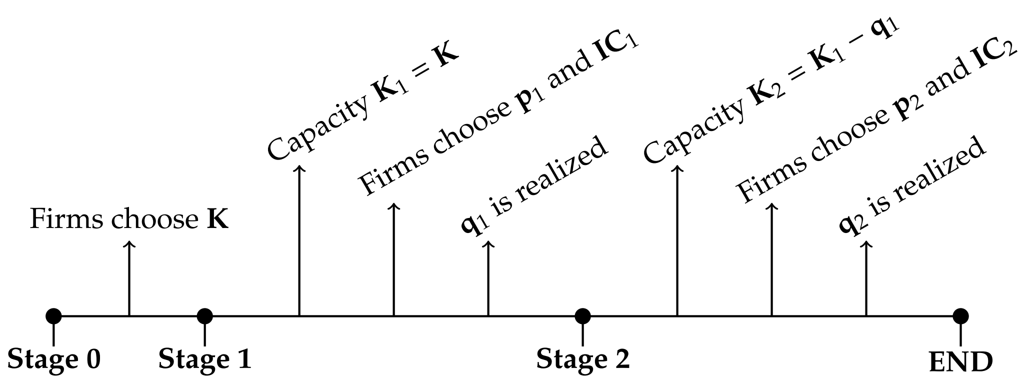
If demand becomes more elastic over time, then the equilibria need not have uniform prices, even under Assumptions 1, 2, and 5. The sufficient conditions for a uniform-price equilibrium to exist are significantly stronger because firms can no longer easily break an increasing price equilibrium by deviating to a higher price and forcing rivals to sell more low-priced units.

5 Inventory Controls

In the previous section, we showed that when demand becomes more inelastic over time, firms produce the same Cournot capacity and set the Cournot price in both periods, as if there were just one period with demand $D_1(p) + D_2(p)$. This is true even though profits would be higher if firms could price discriminate.

We now show that inventory controls make it possible for firms to price discriminate and earn higher profits, but only if demand becomes more inelastic over time. We model inventory controls as an upper bound on quantities sold, and we allow firms to set inventory controls when they set their price. That is, firms first choose their initial capacity, and then, in each of the two subsequent periods, simultaneously choose both their price and an inventory control. The game is shown in the figure below.

Figure 2: Timing of the Game with Inventory Controls



Inventory controls allow firms to limit the number of units available so that its availability exactly matches the number of units it expects to sell in period one. Thus, by utilizing inventory controls, firms can ensure that if a rival deviates to a higher price in period one, their own sales will not increase.

The logic above holds only when demand becomes more inelastic over time because inventory controls place a cap, not a floor, on sales. Thus, inventory controls highlight another natural asymmetry that arises between increasing and decreasing elasticity of demand: Inventory controls can prevent a rival from increasing a firm's sales by deviating to a higher price, but they cannot prevent a rival from lowering a firm's sales by deviating to a lower price.

Proposition 5. *Under Assumptions 1-3 and Assumption 5, then under either the efficient or the proportional rationing rule, a subgame perfect Nash equilibrium of the model with inventory controls exists in which all firms set the Cournot price and set inventory controls equal to the Cournot quantity in each selling period. Profits are strictly higher in this equilibrium than in the uniform-price equilibrium.*

In the equilibrium described in Proposition 5 firms commit to inventory controls that are equal to each firm's equilibrium sales in each period. Inventory controls do not restrict output on the equilibrium path, but they do act as a strategic commitment device because they constrain the firm's off-the-equilibrium-path output. In equilibrium, firms sell their Cournot output in each period, and so prices rise over time because demand becomes less elastic. Without inventory controls, firms set a constant price across the two periods, and sales are equal to the Cournot quantity associated with the aggregate demand, $D_1(p) + D_2(p)$. In the equilibrium with inventory controls, firms set prices equal to the Cournot quantities in each period—that is, the Cournot output associated with demand $D_1(p_1)$ in period one and the Cournot output associated with demand $D_2(p_2)$ in period two.

The model with inventory controls does have other equilibria. In particular, the symmetric capacity, uniform-price equilibrium characterized in Proposition 4 may still be a subgame perfect equilibrium of the inventory control game. Even when it is not, there are many different increasing price paths that inventory controls can support. We think that it is natural for firms to coordinate on the Cournot quantities, but the point is that they can price discriminate and earn higher profits using inventory controls.

Example: We illustrate the impact of inventory controls on prices and profits in an example with linear demand, $p_t = a_t - b_t q_t$, and constant cost per unit of capacity, c. Suppose that the firms can choose capacity independently for each period (as if the two periods were separate markets). Then, the Cournot profits with price discrimination are given by

$$\Pi_{\text{discr.}} = \frac{(b_2(a_1 - c)^2 + b_1(a_2 - c)^2)}{(b_1 b_2 (n+1)^2)},$$

and the Cournot profits with uniform pricing are

$$\Pi_{\text{uniform}} = \frac{(b_2(a_1 - c) + b_1(a_2 - c))(\frac{b_2 a_1 + b_1 a_2}{b_1 + b_2} - c)}{(b_1 b_2 (n+1)^2)}.$$

And profits are higher with discrimination (see the Appendix for more details),

$$\begin{aligned}
\Pi_{\text{discr.}} - \Pi_{\text{uniform}} &= b_2(a_1 - c)^2 + b_1(a_2 - c)^2 - (b_2(a_1 - c) + b_1(a_2 - c))\left(\frac{b_2 a_1 + b_1 a_2}{b_1 + b_2} - c\right) \\
&= \frac{1}{b_1 + b_2}\left(b_2(a_1 - c)(b_1(a_1 - a_2) + b_1(a_2 - c)b_2(a_2 - a_1)\right) \\
&= \frac{b_1 b_2}{b_1 + b_2}\left((a_1 - c)(a_1 - a_2) + (a_2 - c)(a_2 - a_1)\right) = \frac{b_1 b_2}{b_1 + b_2}(a_1 - a_2)^2 > 0.
\end{aligned}$$

6 Model Extensions

6.1 Alternative Timing of Inventory Controls

Inventory controls can also be modeled other ways, including allowing firms to commit to inventory controls before setting price. If firms can commit to inventory controls before announcing prices, inventory controls serve two functions. First, they prevent rival firms ($-i$ firms) from raising their price in order to increase firm i's sales when the price is low. And, second, they limit firm i's sales in period one. The latter is important and impacts equilibrium strategies, but, collectively, firms want to set increasing prices and sell *more* in period one than they do in the uniform-price equilibrium, so commitment does not help firms to unilaterally increase profits in obvious ways.

If firms could announce and commit to their inventory controls in each period before any firm sets price, then Proposition 4 would still hold. In this case, if each firm set an inventory control equal to the Cournot output, this would result in the Cournot prices, and no unilateral inventory control deviation would affect the subsequent prices. But this timing may also eliminate the uniform-price equilibrium. In a duopoly model, a unilateral inventory control would curtail the rival's incentives to raise price and cause the rival to equate marginal revenue across the two periods, even when the rival had not set an inventory control itself.

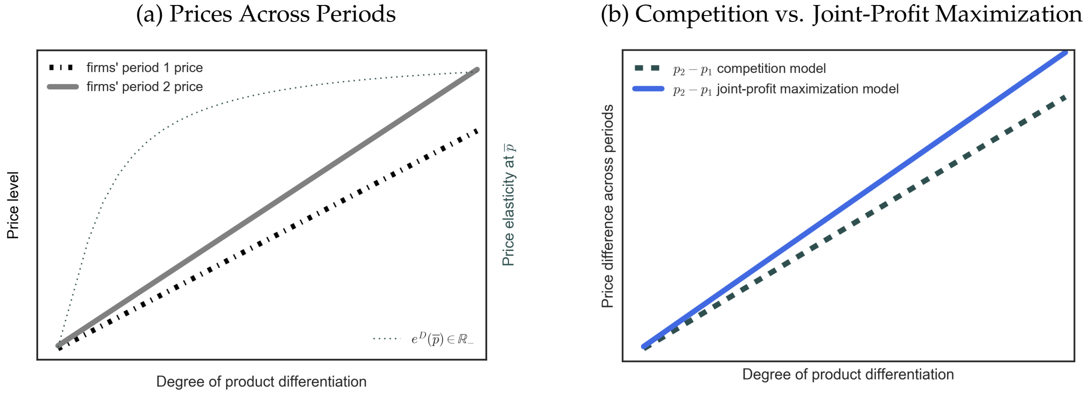
6.2 Product Differentiation

Differentiation does not alter firms' incentive to attempt to shift demand to competitors in the early period. However, product differentiation makes it more costly to shift demand. With undifferentiated products, a small price change shifts all of the demand. With differentiated products, the firm's price increase must be larger, and have a first-order effect on its profits in order to significantly impact a rival's sales.

Product differentiation also introduces increased complexity, so we focus our attention on two firms in a symmetric environment, and give intuition instead of analyzing the equilibrium of the model. We also maintain the assumption that capacity is sufficiently small that firms always set market-clearing prices in the second period.

Product differentiation results in equilibrium subgame prices that are no longer uniform over time; however, prices are flatter—as a function of the degree of product

Figure 3: Intertemporal Price Discrimination as a Function of Product Differentiation



Notes: Example constructed using a random utility model (logit) with two firms and two periods. Product differentiation is increasing towards the right of the plots. (a) The light dashed line corresponds to the own-price elasticity for a constant price offered by both firms. As products become increasingly differentiated, the difference between p_1 and p_2 increases. (b) Shows the change in price ($p_2 - p_1$) of competition model versus the joint-profit maximization model. Prices are flatter in the competition model, as the gap between the two models grows with the degree of differentiation.

differentiation—than joint-profit-maximizing prices (for example, see Figure 3, where the left plot shows increasing differences in prices across periods as product differentiation increases). To see this, consider two firms, A and B, and let the inverse demand functions be $P_1^A(q_1^A, q_1^B)$, $P_1^B(q_1^A, q_1^B)$, $P_2^A(q_2^A, q_2^B)$, and $P_2^B(q_2^A, q_2^B)$. Joint-profit-maximizing firms would set marginal revenue equal to the shadow cost of capacity in each of the four product markets, so $\frac{\partial p_i^j(q_t^j, q_t^{-j})}{\partial q_t^j} q_t^j + p_t^j(q_t^j, q_t^{-j}) = \lambda, \forall t = 1, 2; j = A, B$. Suppose that the joint-profit-maximizing prices are increasing over time.

Contrast these prices with the prices that would be set by two competing firms given the same initial capacity. If Firm A sets a higher price than the joint-profit-maximizing firm, it will sell less in the first period and, hence, more in the second period. Sales for Firm B are higher in the first period, and it has less to sell in the second period; thus, in the second period, its price is higher and Firm A's demand is higher. Because it ignores the loss for Firm B, Firm A has an incentive to set a higher first-period price than the joint-profit-maximizing monopolist. Firm B has a similar incentive, and, in equilibrium, both firms' prices will be flatter relative to joint-profit-maximizing prices (see the right panel in Figure 3). It is also worth noting that prices might still be perfectly flat if sufficiently many consumers were indifferent between the firms – a symmetric increasing price equilibrium does not exist because either firm could strictly increase profits with an arbitrarily small price increase.

6.3 Aggregate Demand Uncertainty

Inventory controls are generally described as a tool for managing demand uncertainty, so it is important to describe how the model can be extended to include such uncertainty. To generate intuition, we describe an extension in which just first-period demand is uncertain. A monopolist sets the first-period price before learning the first-period demand and sets a second-period price to clear the market.

A simple way to add uncertainty to the model is assume that realized demand can be high or low in the first period, but is known to be high in the second period. In this case, a monopolist choosing capacity optimally would set a lower price (based on expected demand) in the first period.

However, the monopoly prices are not an equilibrium with competing firms, even if the firms have the same capacity as the monopolist. Because the monopoly prices increase in expectation, competing firms prefer to sell more of their capacity in the second period, when the expected price is higher. And any firm can shift a discrete amount of its first-period sales to its rival through an arbitrarily small price increase in period one. Thus, expected prices must be equal in the two periods in any symmetric pure-strategy equilibrium.

6.4 Many Periods

An obvious limitation of the paper is that we consider only two pricing periods. One challenge to extending Propositions 2 and 3 to many periods is that it requires stronger assumptions than Assumptions 1 and 2 to ensure that firms play pure strategies for all histories of the game. Another limitation is the potential for multiple equilibria.

One way to consider many periods would be to focus on a duopoly model with constant elasticity demand, but to allow the demand elasticity to become more inelastic over time. Thus, sufficient conditions could be derived under which the last m periods have a uniform-price equilibrium for all capacities. Then, going from m to $m + 1$ periods is similar to going from one to two periods in our analysis above, because the m-period-subgame price is always the market-clearing price. The only challenge is that those sufficient conditions are no longer on the initial capacities.

In particular, consider a three-period model. Here, it is possible to describe conditions under which a uniform price is the unique outcome: on the equilibrium path, the final two pricing periods of the three-period game are equivalent to our analysis above, so prices must be equal in the final two periods as long as capacities are sufficiently equal. More importantly, the intuition that firms can profit from shifting lower-priced sales to their rivals still holds, which is why sustaining increasing prices is difficult.

7 Conclusion

We establish that inventory controls can facilitate intertemporal price discrimination in oligopoly. We consider an advance-purchase, sequential-pricing model with complete information. When a single firm is in the market, and demand becomes more inelastic over time, the firm can clearly charge higher prices to late-arriving consumers. However, in our oligopoly model, strong competitive forces arise. Individually, firms have an incentive to move their capacity to the period with the highest price. That is, firms have an incentive to shift sales to their rivals in early periods, when consumers have lower willingness to pay, in order to capture increased sales in later periods, when consumers have higher willingness to pay. Consequently, we find that firms will compete on price until prices are equalized across the selling periods, even though firms have market power and benefit from increasing prices.

In order for firms to coordinate price increases when late-arriving consumers have higher willingness to pay, they must shield themselves from these strong competitive forces. By committing to a cap on their sales in each of the sequential markets, which they can do with inventory controls, firms are able to implement increasing prices, even in an oligopoly. There is extensive research in economics and operations research on the use of inventory controls as a tool to manage uncertain demand, but here, we show that they are also a tool to facilitate intertemporal price discrimination.

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A Appendix

Proof of Lemma 1:

Proof. Suppose not. Then some firm is charging a price not equal to the market-clearing price (and trade takes place at that price).

First, suppose, instead, that some firm is charging a price strictly below the market-clearing price. Let p_L be the lowest price offered by any firm. Clearly, any firm offering to sell at p_L sells all of its capacity. It either sells all of its capacity in period one, or its sales in period 1 are equal to $D_1(p_L)$; then, in period two the demand for its product is $D_2(p_L)$, and since p_L is below the market clearing price, $D_1(p_L) + D_2(p_L)$ exceeds all firms' capacity by Assumptions 1 and 2, which clearly exceeds each individual firm's capacity. But then there must exist a strictly higher price at which the same firm sells all of its capacity and earns strictly higher profits, which is a contradiction.

Now suppose, instead, that some firm sets a price strictly above the market-clearing price with strictly positive probability. Let p_H be the highest price offered in equilibrium with positive probability, which implies that at least one firm offering to sell at p_H does not sell all of its capacity.

If two or more firms set a price of p_H with strictly positive probability, then a firm that does not sell off of its capacity can decrease its price to $p_H - \epsilon$ and strictly increase its sales and profits, which is a contradiction.

If, at most, one firm is charges the price p_H with strictly positive probability, then any firm charging p_H earns profit equal to $p_H RD_1(p_H; \mathbf{p}^{-i}, \mathbf{K}_1^{-i}) + D_2(p_H)$ if its sales are positive in period one, and it earns a profit equal to $p_H RD_2(p_H; \mathbf{p}^{-i}, \mathbf{K}_2^{-i})$ otherwise, where $\mathbf{p}^{-i}$ and $\mathbf{K}^{-i}$ are the other firms' prices and capacities, and $\mathbf{K}_2^{-i}$ is the other firms' remaining capacity at the start of stage 2. Clearly, the firm will not sell all of its capacity in either case, because p_H exceeds the market-clearing price, and, as shown above, other firms are all setting prices above the market-clearing price, so total consumption must be less than available capacity.

Consider for a moment the firm's profits if rivals are playing pure strategies.

Under the efficient rationing rule, if the firm charging p_H has positive sales in period one, then the derivative of profit with respect to price is $RD_1(p_H; \mathbf{p}^{-i}, \mathbf{K}^{-i}) + p_H D_1'(p_H) + D_2(p_H) + p_H D_2'(p_H)$, which is negative because $RD_1(p; \mathbf{p}^{-i}, \mathbf{K}^{-i}) < D_1(p)$, because $pD_1'(p) + D_1(p) < 0$, and because $pD_2'(p) + D_2(p) < 0$. The second and third statements are true because, by Assumption 1, $D_1(p_H) + D_2(p_H)$ is less than the revenue-maximizing output (marginal revenue is positive). So, lowering price below p_H increases profit, which is a contradiction.

Under the efficient rationing rule, if the firm charging p_H has zero sales in period one, then the derivative of profit with respect to price is $D_2(p_H) + p_H D_2'(p_H)$, which is negative because, by Assumption 1, $D_2(p_H)$ is less than the revenue-maximizing output (marginal revenue is positive). So, lowering price below p_H increases profit, which is a contradiction.

However, rival firms may be playing mixed strategies, so the firm's expected profit is a weighted average of the profit functions above, all of which are higher at a lower price, so we have a contradiction.

Under the proportional rationing rule, if the firm charging p_H has positive sales in period one, then the derivative of profit with respect to firm i's price is

$$\begin{aligned} RD_1(p; \mathbf{p}^{-i}, \mathbf{K}^{-i}) + p_H RD_1'(p_H; \mathbf{p}^{-i}, \mathbf{K}^{-i}) + p_H D_2(p_H) + D_2'(p_H) = \\ \Big(p_H D_1'(p_H) + D_1(p_h)\Big)\left[1 - \sum_{j \neq i} \frac{K^j}{D_2(p^j)}\right] + \Big(pD_2'(p) + D_2(p)\Big), \quad (9) \end{aligned}$$

which is negative because $p_H D_1'(p_H) + D_1(p_H) < 0$ and $p_H D_2'(p_H) + D_2(p_H) < 0$. These are both true because, by Assumption 1, $D_1(p_H) + D_2(p_H)$ is less than the revenue-maximizing output. So, lowering price below p_H increases profit, which is a contradiction.

Under the proportional rationing rule, if the firm charging p_H has zero sales in period one, then the derivative of profit with respect to firm i's price is

$$RD_2(p; \mathbf{p}^{-i}, \mathbf{K}_1^{-i}) + p_H R D_2'(p_H; \mathbf{p}^{-i}, \mathbf{K}_1^{-i}) = \\ \Big(p_H D_1'(p_H) + D_1(p_h)\Big)\left[1 - \sum_{j \neq i} \frac{K^j}{D_2(p^j)}\right] + \Big(p D_2'(p) + D_2(p)\Big) \quad (10)$$

which is negative because $p_H D_1'(p_H) + D_1(p_H) < 0$ and $p_H D_2'(p_H) + D_2(p_H) < 0$. This is true because $D_1(p_H) + D_2(p_H)$ is less than the revenue-maximizing output. So lowering price below p_H increases profit, which is a contradiction.

Again, if rivals are playing mixed strategies, then the firm's expected profit is a weighted average of the pure-strategy profit functions above, all of which are higher at a price below p_H, so we have a contradiction. ■

Proof of Lemma 2:

Proof. Suppose not, so some firm is charging a price not equal to the market-clearing price. First, suppose, instead, that some firm is charging a price strictly below the market-clearing price with positive probability. Let p_L be the lowest price offered in equilibrium with positive probability. Clearly, any firm offering to sell at p_L sells all of its capacity (because p_L is below the market-clearing price), but then there must exist a strictly higher price at which the same firm sells all of its capacity and earns strictly higher profits, which is a contradiction.

Now suppose, instead, that some firm charges a price strictly above the market-clearing price with positive probability. Let p_H be the highest price offered in equilibrium with positive probability. Clearly, at least one firm offering to sell at p_H does not sell all of its capacity (because p_H is above the market-clearing price).

If two or more firms charge p_H with strictly positive probability, then at least one of the firms does not sell all of its capacity, and that firm can decrease its price to $p_H - \epsilon$ and strictly increase its sales and profits, which is a contradiction.

If zero or one firm is charging the price p_H with strictly positive probability, and if other firms are playing pure strategies, then a firm charging p_H earns profits equal to $pRD_2(p; \mathbf{p}^{-i}, \mathbf{q}^{-i})$, where $\mathbf{p}^{-i}$ and $\mathbf{q}^{-i}$ are the other firms' prices and remaining capacities.

Under the efficient rationing rule, the derivative of profit with respect to price is $RD_2(p; \mathbf{p}^{-i}, \mathbf{q}^{-i}) + pD_2'(p)$, which is negative because $RD_2(p; \mathbf{p}^{-i}, \mathbf{q}^{-i}) < D_2(p)$ and because $pD_2'(p) + D_2(p) < 0$. This is true because, by Assumption 1 and Assumption 2, $D_2(p)$ is less than the revenue-maximizing output. So, lowering price below p_H increases profit, which is a contradiction.

Under the proportional rationing rule, the derivative of profit with respect to firm i's price is $RD_2(p; \mathbf{p}^{-i}, \mathbf{q}^{-i}) + pRD_2'(p; \mathbf{p}^{-i}, \mathbf{q}^{-i}) = \left(pD_2'(p) + D_2(p)\right)\left[1 - \sum_{j \neq i} \frac{q^j}{D_2(p^j)}\right]$, which is negative because $pD_2'(p) + D_2(p) < 0$. This is true because $D_2(p)$ is less than the revenue-maximizing output. So, lowering price below p_H increases profit, which is a contradiction.

Finally, because the firm charging p_H earns high profits at a lower price regardless of the prices that the rivals set, the argument above also holds when rivals are playing mixed strategies. ∎

Proof of Proposition 1:

Let $p_L = \min_i p_1^i$ denote the lowest equilibrium price offered in period one.

By Lemma 1, under Assumption 1 and Assumption 2, for any history of the game, all firms with positive remaining capacity in the final period charge the market-clearing price in the second-period subgame.

The proof of the proposition proceeds as a series of eight claims.

1) In any pure-strategy equilibrium of the pricing subgame, $p_L \leq p_2$.

Suppose not, so $p_L > p_2$. It follows that every firm has positive expected sales in equilibrium, because firms with zero sales earn profits equal to $p_2 K^i$, and they can earn strictly higher profits by deviating to $p_L - \epsilon$ in period one.

Suppose that $p_L > p_2$ and that every firm has positive sales in period one. We claim that it follows that every firm with positive sales must be charging p_L. Otherwise, some firm j has positive sales and is charging a price $p^j > p_L$. Let $\hat{K} = \sum_{i|p_i=p_L} K^i$ denote the total capacity at price p_L, where, clearly, $D_1(p_L) > \hat{K}$, because firm j's residual demand is positive at $p^j > p_L$. But this implies that there exists a strictly positive ϵ such that a firm i charging p_L can deviate to a higher price, $p_L + \epsilon$, and still sell all of its capacity, which is a contradiction.

However, if $p_L > p_2$, and all firms with positive sales in period one are charging p_L, then for sufficiently small ϵ, any firm with excess capacity in period one can strictly increase its profit by deviating to a first-period price of $p_L - \epsilon$. The deviating firm sells strictly more at a first-period price that is arbitrarily close to p_L and sells less at a second-period price that is arbitrarily close to p_2, and $p_L > p_2$, so its profits are strictly higher. So, $p_L \leq p_2$.

2) In any pure-strategy equilibrium of the pricing subgame, when p_L is offered by two or more firms in period one, then $p_L = p_2$.

Suppose not, so $p_L < p_2$, and p_L is offered by two or more firms. Let firm i be one of these firms. Then, firm i's profit can be written as $p_L x_i + p_2 \left(K^i - x_i\right)$, where $x_i = \min\left\{RD_1\left(p_L; p_L, \sum_{j\neq i|p^j=p_L} K^j\right), K^i\right\}$ is firm i's sales at p_L.

If firm i deviates to a slightly higher price, $p_L + \epsilon$, its profit is

$$(p_L + \epsilon)\min\left\{RD_1\left(p_L + \epsilon; p_L, \sum_{j\neq i|p^j=p_L} K^j\right), K^i\right\} \\ + \hat{p}_2(\cdot)\max\left\{K^i - RD_1\left(p_L + \epsilon; p_L, \sum_{j\neq i|p^j=p_L} K^j\right), 0\right\}, \quad (11)$$

where $\hat{p}_2(\cdot)$ is the market-clearing price in period two, which is a continuous and decreasing function of the capacity remaining after period one.

If $x^i = K^i$, then firm i's profit is clearly higher since $p_L + \epsilon > p_L$ and $\hat{p}_2(\cdot) > p_L$, so all of firm i's sales are at a higher price, and its sales volume doesn't change.

If, on the other hand, $x^i < K^i$ and $RD_1(p_L; p_L, \sum_{j \neq i | p^j = p_L} K^j) < K^i$, then the same deviation is still profitable for firm i because

$$\lim_{\epsilon \to 0} RD_1^i \left(p_L + \epsilon; p_L, \sum_{j \neq i | p^j = p_L} K^j \right) \leq RD_1^i(p_L; p_L, \sum_{j \neq i | p^j = p_L} K^j) < K^i,$$

since RD is decreasing in price (for either rationing rule), and so the limit of (11) as ϵ goes to 0 is

$$p_L \lim_{p \downarrow p_L} RD_1^i(p; p_L, \sum_{j \neq i | p^j = p_L} K^j) + p_2 \left(K^i - \lim_{p \downarrow p_L} RD_1^i(p; p_L, \sum_{j \neq i | p^j = p_L} K^j) \right).$$

Profits are higher because the firm sells more units at p_2 and fewer units at p_L and $p_2 > p_L$. A deviation is profitable, which is a contradiction, so either $p_L = p_2$, or only one firm charges p_L.

3) If $p_L = p_2$, then the equilibrium is a uniform-price equilibrium.

Suppose not, so some firm j sets a price $p^j > p_L = p_2$ in period one and has strictly positive sales. The residual demand at p^j is strictly positive, which implies that firms charging p_L sell all of their capacity, and that the residual demand in a neighborhood of p_L must also be strictly positive. Therefore, for sufficiently small ϵ, if a firm charging p_L deviates to $p_L + \epsilon$, it is still able to sell all of its capacity (for either rationing rule), so its profits are higher, which is a contradiction.

4) There exists, at most, one uniform-price equilibrium of the pricing subgame (the total sales and the transaction prices in each period are unique).

Given the capacity, the price and volume of sales in a uniform-price equilibrium are uniquely defined, because only one price satisfies $D_1(p) + D_2(p) = \sum_i K^i$.

5) If $|\eta_1(p)| = |\eta_2(p)|, \forall p$, then a uniform-price equilibrium exists and is the unique pure-strategy subgame perfect equilibrium.

Consider the unique candidate uniform-prime equilibrium. So, $p_1 = p_2 = p$, and no firm has positive sales at any other price in either period. This equilibrium clearly exists when no firm has an incentive to deviate in period one.

Deviating to a higher price in period one is profitable only if the demand is strictly more elastic in the second pricing period. When elasticity does not change, deviating to a higher first-period price lowers industry profit – given the industry capacity and Assumption 4, industry profit is clearly lower when the first-period price is higher than the second-period price – and the deviating firm's share of first-period revenue falls, and its share of second-period revenue rises, so the change in revenue for the deviating firm must be smaller than for other firms, and so the deviating firm's profit must fall.

Deviating to a lower price in period one can be profitable only if the demand is strictly less elastic in period two. When elasticity does not change, then deviating to a lower first-period price lowers industry profits. And the deviating firm's market share in the first period rises relative to other firms', so the deviating firm's profit must fall.

6) If $|\eta_1(p)| < |\eta_2(p)|, \forall p$, so demand becomes more elastic over time, then any pure-strategy subgade perfect equilibrium is a uniform-price equilibrium, but a pure-strategy equilibrium may not exist.

As above, consider the unique candidate uniform-prime equilibrium.

As above, deviating to a lower price in period one can be profitable only if the demand is strictly less elastic in period two. When elasticity does not change, then deviating to a lower first-period price lowers industry profits. And the deviating firm's market share in the first period rises relative to other firms', so the deviating firm's profit must fall.

If deviating to a higher price is profitable in period one, then no uniform-price equilibrium exists.

And, clearly, no other pure-strategy equilibrium can exist because, by claim 1, no pure-strategy equilibrium exists in which prices strictly decline for any firm. And when the demand becomes more elastic over time, no pure-strategy equilibrium can exist in which prices strictly increase for any firm because, charging the lowest period-one price can increase industry profits and increase the firm's own profits by raising its price.

7) If $|\eta_1(p)| > |\eta_2(p)|, \forall p$, so demand becomes less elastic over time, then all pure-strategy subgame perfect equilibria are either uniform-price or asymmetric-price, and either a uniform-price equilibrium or an asymmetric price equilibrium, exists, but not both.

As above, consider the unique candidate uniform-prime equilibrium.

As in claim 5 above, deviating to a higher price in period one is profitable only if the demand is strictly more elastic in the second pricing period. When demand becomes less elastic, then deviating to a higher first-period price lowers industry profit; the deviating firm's share of first-period revenue falls, and its share of second-period revenue rises, so the change in revenue for the deviating firm must be smaller than for other firms, and, thus, the deviating firm's profit must fall. So, a uniform-price equilibrium exists if and only if no deviation to a lower first-period price is profitable for any firm.

Suppose that a deviation to a lower price is profitable for some firm; then, it is clearly also profitable for the firm with the largest capacity. Let i denote the firm with the largest capacity; let p_1^i denote the firm's profit-maximizing deviation in period one; and let $\hat{p}_2$ denote the resulting second-period market-clearing price.

Then, p_1^i and $\hat{p}_2$ represent an asymmetric-price equilibrium. Firm i sells in both periods, and all other firms sell only in period two. Clearly, firm i has no incentive to deviate since, by construction, p_1^i is its best response. And if any other firm can increase its profits by charging a price less than p_1^i, then it follows that firm i can also increase its profit by deviating to that same price, in which case p_1^i is not firm i's profit-maximizing price, which is a contradiction.

8) There exist, at most, n asymmetric-price equilibria.

We show that there exists, at most, one asymmetric-price equilibrium in which firm i is the low-priced firm in period one (or, more strictly speaking, such equilibria differ only in the prices of firms with zero sales).

In an asymmetric-price equilibrium, if firm i is the low-priced firm, then it is the only firm with positive sales in period one. Let p denote firm i's price.

If $p > p_1^i$, then firm i can profitably deviate to p_1^i. If $p < p_1^i$, then, because $\pi(p)$ is concave and maximized at p_1^i, it follows that firm i is strictly better off increasing its price. So, p does not describe a situation in which an asymmetric-price equilibrium exists.

Therefore, the only asymmetric-price equilibrium that exists in which firm i is the low-priced firm in the first period is given by (3) and (5).

Proof of Proposition 2:

Proof. Let K^i denote each firm's capacity, and let $\tilde{p}$ denote the unique uniform price defined by $D_{tot}(\tilde{p}) = D_1(\tilde{p}) + D_2(\tilde{p}) = \sum_{i=1}^{n} K^i$.

Consider a deviation to a lower price in the first pricing period. If $D_1(\tilde{p}) \geq \max_i K^i$, then a deviation to a lower price is not profitable, because any firm that cuts its price in period one will sell all of its capacity at the lower deviation price and, hence, earn strictly lower profits.

If $D_1(\tilde{p}) < \max_i K^i$, then for any firm i such that $K^i \leq D_1(\tilde{p})$, a deviation to a lower price will not be profitable by the same argument. When $K^i > D_1(\tilde{p})$, then a deviation to a lower price can increase the market-clearing price in period two, and can increase the firm's profits, but only if demand is becoming less elastic over time, so firms jointly prefer to set prices that increase over time.

Let firm i be the deviating firm, and let $p_2(\cdot)$ denote the second-period market-clearing price as a function of remaining capacity. Firm i's problem is to choose a price $p^i < \tilde{p}$ or, equivalently, a quantity $q^i = D_1(p^i)$ to maximize

$$\hat{\pi}^i(q^i;\tilde{p},\mathbf{K}) = q^i p_1(q^i) + p_2\left(\sum_{i=1}^{n} K^i - q^i\right)\left(K^i - q^i\right), \tag{12}$$

subject to $q^i \in \left(D_1(\tilde{p}), K^i\right]$ – higher output levels are not feasible, and lower output levels are inconsistent with a lower first-period price. The first-order condition is

$$\frac{d\hat{\pi}(q^i;\tilde{p},\mathbf{K})}{dq} = p_1(q^i) + q^i p_1'(q^i) - p_2\left(\sum_{i=1}^{n} K^i - q^i\right) - p_2'\left(\sum_{i=1}^{n} K^i - q^i\right)(K^i - q^i) = 0, \tag{13}$$

or

$$\begin{aligned} \frac{d\hat{\pi}(q^i;\tilde{p},\mathbf{K})}{dq} &= p_1(q^i)\left(1 + \frac{1}{\eta_1(p_1(q^i))}\right) \\ &\quad - p_2\left(\sum_{i=1}^{n} K^i - q^i\right)\left(1 + \frac{1}{\eta_2\left(p_2\left(\sum_{i=1}^{n} K^i - q^i\right)\right)}\frac{K^i - q^i}{\sum_{i=1}^{n} K^i - q^i}\right) = 0. \end{aligned} \tag{14}$$

Clearly, the objective function, equation (12), is concave, so (14) implies that a deviation to a lower price is profitable if and only if $\lim_{q\downarrow D_1(\tilde{p})} \frac{d\hat{\pi}(q;\tilde{p},\mathbf{K})}{dq} > 0$ or, equivalently, $\lim_{p\uparrow\tilde{p}} \frac{d\hat{\pi}(D_1(p);\tilde{p},\mathbf{K})}{dq} > 0$. But, clearly,

$$\begin{aligned} \lim_{p\uparrow\tilde{p}} \frac{d\hat{\pi}(D_1(p);\tilde{p},\mathbf{K})}{dq} &< p_1(D_1(\tilde{p}))\left(1 + \frac{1}{\eta_1(p_1(D_1(\tilde{p}))}\right) \\ &\quad - p_2\left(\sum_{i=1}^{n} K^i - D_1(\tilde{p})\right)\left(1 + \frac{1}{\eta_2\left(p_2\left(\sum_{i=1}^{n} K^i - D_1(\tilde{p})\right)\right)}\frac{K^i}{\sum_{i=1}^{n} K^i}\right) \end{aligned}$$

because $\frac{K^i - q}{(\sum_{i=1}^{n} K^i - q)} < \frac{K^i}{\sum_{i=1}^{n} K^i}$. Since $p_1(D_1(\tilde{p})) = p_2\left(\sum_{i=1}^{n} K^i - D_1(\tilde{p})\right) = \tilde{p}$, it follows that a deviation to a lower price is not profitable if

$$\frac{1}{\eta_1(p_1(D_1(\tilde{p})))} - \frac{1}{\eta_2\left(p_2\left(\sum_{i=1}^{n} K^i - D_1(\tilde{p})\right)\right)}\frac{K^i}{\sum_{i=1}^{n} K^i} < 0 \iff \frac{\eta_2(\tilde{p})}{\eta_1(\tilde{p})} > \frac{K^i}{\sum_{i=1}^{n} K^i}, \tag{15}$$

or, equivalently, if Assumption 4 holds. If demand in the second period is too much more inelastic, a deviation will be profitable.

Now consider a deviation to a higher price. If $D_1(\tilde{p}) < \sum_{j \neq i} K^j$, for all i, then no firm's deviation to a higher price can have any effect on first- or second-period sales. The firms that do not deviate can meet all of the demand at the price $\tilde{p}$.

If, on the other hand, $D_1(\tilde{p}) > \sum_{j \neq i} K^j$, for some i, then some firm or firms can deviate to a higher price and have positive sales; however, even a monopolist would not find such a deviation profitable when demand is becoming less elastic over time, so no firm will deviate to a higher price. ∎

Proof of Proposition 3

Let K^i denote each firm's capacity, and let $\tilde{p}$ denote the unique uniform price defined by $D_{tot}(\tilde{p}) = D_1(\tilde{p}) + D_2(\tilde{p}) = \sum_{i=1}^{n} K^i$.

Suppose that Assumptions 1 and 2 hold, and suppose that demand becomes more elastic over time.

First, consider a deviation to a lower price in the first period. When demand becomes more elastic over time, even a monopolist does not find this profitable, so no firm can increase its profits by selling at a lower price, even if it raises the second-period price for a portion of its sales.

Now consider a deviation to a higher price in the first period. If $D_1(\tilde{p}) \leq \sum_{j \neq i} K^j$, then if firm i deviates to a higher price, it has zero sales in period one and still sells all of its output at the uniform price in the second pricing period, so its profits are the unchanged.

So, $D_1(\tilde{p}) \leq \sum_{j \neq i} K^j \forall i$, is a sufficient condition for a uniform-price equilibrium to exist.

If, on the other hand, $D_1(\tilde{p}) > \sum_{j \neq i} K^j$, for some i, then, when firm i deviates to a higher price, its rivals sell all of their output at a price $\tilde{p}$, so its rivals' profits are unchanged. In this case, firm i acts as a residual monopolist. We can think of the firm as a residual monopolist selling to consumers who are not served at price $\tilde{p}$. So, firm i's problem is to choose a price $p^i > \tilde{p}$ to maximize

$$\hat{\pi}^i(p^i;\tilde{p},\mathbf{K}) = p^i RD_1(p^i,\tilde{p},K^{-i}) + p_2\left(K^i - RD_1(p^i,\tilde{p},K^{-i})\right)\left(K^i - RD_1(p^i,\tilde{p},K^{-i})\right), \tag{16}$$

or, equivalently, to choose a quantity q^i to maximize

$$\hat{\pi}^i(q^i;\tilde{p},\mathbf{K}) = q^i p_1^r\left(q^i, \sum_{j\neq i} K^j\right) + p_2\left(K^i - q^i\right)\cdot\left(K^i - q^i\right), \tag{17}$$

where p_1^r is the inverse of the residual demand function.

The objective function is concave for both rationing rules, so a price increase is profitable if and only if

$$\lim_{p^i\downarrow\tilde{p}} \left.\frac{d\hat{\pi}^i(q^i;\tilde{p},\mathbf{K})}{dq}\right|_{q^i=D_1(p^i)-\sum_{j\neq i}K^j} < 0. \tag{18}$$

This implies that profits increase as the firm restricts its output and drives price up above $\tilde{p}$. This derivative is given by

$$\frac{d\hat{\pi}^i(q^i;\tilde{p},\mathbf{K})}{dq} = p_1^r(q^i) + q^i\frac{dp_1^r(q^i)}{dq} - p_2\left(K^i - q^i\right) - p_2'\left(K^i - q^i\right)(K^i - q^i), \tag{19}$$

or

$$\frac{d\hat{\pi}^i(q;\tilde{p},\mathbf{K})}{dq} = p_1^r(q^i)\left[1 + \frac{q^i}{p_1^r(q^i)}\frac{dp_1^r(q^i)}{dq}\right] - p_2\left(K^i - q^i\right)\left[1 + p_2'\left(K^i - q^i\right)\frac{K^i - q^i}{p_2\left(K^i - q^i\right)}\right], \tag{20}$$

so for efficient rationing, which implies that $\frac{dp_1^r(q^i)}{dq} = \frac{dp_1(q^i)}{dq}$, it follows that

$$\lim_{p^i\downarrow\tilde{p}} \left.\frac{d\hat{\pi}(q^i;\tilde{p},\mathbf{K})}{dq}\right|_{q^i=D_1(pi)-\sum_{j\neq i}K^j} = \tilde{p}\left[\frac{1}{\eta_1(\tilde{p})}\frac{D_1(\tilde{p}) - \sum_{j\neq i}K^j}{D_1(\tilde{p})} - \frac{1}{\eta_2(\tilde{p})}\right]. \tag{21}$$

Hence, if $\frac{\eta_1(\tilde{p})}{\eta_2(\tilde{p})} > \frac{D_1(\tilde{p})-\sum_{j\neq i}K^j}{D_1(\tilde{p})}$, then no deviation is profitable.

And, clearly, $\frac{\eta_1(\tilde{p})}{\eta_2(\tilde{p})} > \frac{K^i}{\sum_{j\neq i} K^j}$ is sufficient for a uniform-price equilibrium to exist. That is, if demand in the first period is too inelastic compared to demand in the second period, a deviation will be profitable.

Proof of Proposition 4:

Under Assumptions 1, 2 and 5, if a subgame perfect equilibrium exists in which every firm chooses K^* units of capacity, then, by Proposition 2, the unique subgame perfect equilibrium of the pricing subgame is a uniform-price equilibrium. Moreover, for all firm capacities in a neighborhood of K^*, Assumption 5 and Proposition 2 imply that the unique subgame perfect equilibrium of the pricing subgame is a uniform-price equilibrium, so the first-stage profit function for firm i can be written as

$$\Pi^u(K^i; K^{-i}) = \left(p_{tot}\left(\sum_j K^j\right) - c\right)K^i, \tag{22}$$

where K^{-i} is the capacity of the other firms.

Firm i's capacity, K^i, maximizes firm i's profits only if $K^i = K^*$ is the solution to

$$\frac{\partial \Pi^u(K^i; K^*)}{\partial K^i} = p_{tot}((n-1)K^* + K^i) - c + p'_{tot}((n-1)K^* + K^i)K^i = 0, \tag{23}$$

which is concave and has a unique solution, $K^i(K^*)$, which is decreasing in K^*. So, (23) uniquely defines a symmetric solution K^*, and it is easy to see that K^* must be exactly equal to the Cournot quantity associated with n firms, production cost c, and demand $D_{tot}(p)$. So, we have shown that $K^i = K^*$ is local best response. Next, we show that $K^i = K^*$ is the global best response when rival firms choose K^*.

Suppose that $K^i < K^*$. If a uniform-price equilibrium exists when firm i chooses K^i and other firms choose K^*, then firm i's profits are given by (22), and so firm i's profits at K^i are strictly lower than at K^*.

If a uniform-price equilibrium does not exist, then, by Proposition 1, an asymmetric-price equilibrium must exist. Under Assumption 5, firm i cannot profit by deviating from the uniform-price equilibrium even if its capacity is K^*, so firm i is not the low-priced firm in the first period. The only asymmetric-price equilibrium that can exist is one in which one of firm i's rivals is the firm that sells at the low price in the first period. There are $n-1$ such equilibria because any of the $n-1$ firms with capacity K^* can set the low price in the first period.

Firm i's profit in all of these asymmetric-price equilibria is

$$\Pi^a(K^i;K^*) = \left[p_2\left((n-1)K^* + K^i - D_1(p_1)\right) - c\right]K^i, \tag{24}$$

where p_1 is the price charged in the first period, and so p_1 maximizes

$$D_1(p_1)p_1 + p_2\left((n-1)K^* + K^i - D_1(p_1)\right)\left(K^* - D_1(p_1)\right). \tag{25}$$

Firm i's first-order-condition is

$$p_2'\left((n-1)K^* + K^i - D_1(p_1)\right)\left(1 - D_1'(p_1)\frac{dp_1}{dK^i}\right) + p_2\left((n-1)K^* + K^i - D_1(p_1)\right) - c = 0. \tag{26}$$

Because $p_1 < p_2$, $D(p_1)$ is greater than first-period sales at the uniform price. This implies that $n-1$ firms are each selling less than $K^* - D(\tilde{p})/n$ in period two, where $\tilde{p}$ is the uniform price. In this case, ignoring the impact of K^i on p_1, firm i's best response is greater than $K^* - D(\tilde{p})/n$, which implies that $K^i > K^*$, which is a contradiction. And, as K^i increases, the optimal first-period price falls ($dp_1/dK^i < 0$). Thus, ignoring the impact of K^i on p_1 does not alter the result. Deviating to a lower K^i is still not profitable.

Now suppose that $K^i > K^*$. Again, the equilibrum of the pricing subgame may be an asymmetric-price equilibrium or a uniform-price equilibrium. If it is a uniform-price equilibrium, then, by the same argument, profits are strictly lower.

If it is an asymmetric-price equilibrium, then it must be an asymmetric-price equilibrium in which firm i sets a low price in the first period. This is because an asymmetric-price equilibrium exists only if a firm wants to deviate from the uniform-price equilibrium, and equation (15) tells us that a firm wants to deviate only if $\eta_2(p)/\eta_1(p)$ exceeds its share of capacity. But, by Assumption 5, this happens only if the capacity share exceeds $1/n$ and only if firm i's share of capacity exceeds $1/n$.

So, if firm i deviates to $K^i > K^*$, then its profit must be

$$\max_{p_1} D_1(p_1)\, p_1 + p_2\Big((n-1)K^* + K^i - D_1(p_1)\Big)\Big(K^i - D_1(p_1)\Big).$$

Rewriting this as a function of quantity yields

$$\max_{q_1} p_1(q_1)q_1 + p_2\Big((n-1)K^* + K^i - q_1\Big)\Big(K^i - q_1\Big). \tag{27}$$

Thus, the firm's profit in stage one is

$$\max_{q_1} p_1(q_1)q_1 + p_2\Big((n-1)K^* + K^i - q_1\Big)\Big(K^i - q_1\Big) - cK^i, \tag{28}$$

and its maximized profit in stage one is

$$\max_{q_1, K_1} p_1(q_1)q_1 + p_2\Big((n-1)K^* + K^i - q_1\Big)\Big(K^i - q_1\Big) - cK^i, \tag{29}$$

which we can rewrite using a change of variables ($q_2 = K^i - q_1$) as

$$\max_{q_1, q_2} p_1(q_1)q_1 - cq_1 + p_2\left((n-1)K^* + q_2\right) q_2 - cq_2. \tag{30}$$

Therefore, q_1 is the first-period monopoly output. and q_2 is the second-period best response to $(n-1)K^*$. But this is not an equilibrium unless $p_1 < p_2$ or, equivalently, the Lerner index in the first period is smaller than the Lerner index in period 2, or

$$\frac{p_1'(q_1)q_1}{p_1(q_1)} < \frac{p_2'\left((n-1)K^* + q_2\right)q_2}{p_2\left((n-1)K^* + q_2\right)} \tag{31}$$

$$\frac{1}{|\eta_1(p_1)|} < \frac{1}{|\eta_2(p_2)|}\frac{q_2}{((n-1)K^* + q_2} \tag{32}$$

or

$$\frac{\eta_2(p_2)}{\eta_1(p_1)} < \frac{q_2}{((n-1)K^* + q_2}, \tag{33}$$

which violates Assumption 5 because $q_2 < K^*$. So, this is a contradiction. Hence, there exists no profitable deviation for any firm.

Proof of Proposition 5:

Proof. Let k_t^i denote the inventory control for firm i in period t.

Consider an equilibrium in which, on the equilibrium path, firms choose capacity equal to the sum of the Cournot capacity in each period, $q_1^C + q_2^C$, then set the Cournot price, p_t^C in each period, and set $k_t^i = q_t^C$ in each period, so inventory controls are set equal to the Cournot output in each period.

Off of the equilibrium path, firms set the market-clearing price in the last period. In the first period, given capacity, firms set the price p_1^i and the inventory control k_1^i, which correspond to the Cournot price and quantity. That is, firms allocate their capacity across periods one and two as if they were playing a Cournot game in each period with a capacity constraint across the two periods. Frims set price equal to the Cournot price associated with the firms' allocations, and they set the inventory control equal to that allocation. More formally, firms equate the marginal revenue of output across the two periods.

Clearly, no deviation is profitable in the final period. That is, in every subgame, firms set the market-clearing price and a non-binding inventory control. This is because Lemma 2 holds, so any second-period price not equal to the market-clearing prices are not sequentially optimal – the presence of inventory controls does not change this result.

Now consider a deviation by firm i to a lower price in the first selling period. Assumption 3 implies that $p_1^C < p_2^C$, so a small decrease in firm i's first-period price discontinuously increases its first-period sales, decreases its second-period sales, and decreases its profits. More generally, if firm i had a profitable deviation to a lower price in period one, then that price would define an asymmetric-price equilibrium, but by Proposition 2, an asymmetric-price equilibrium does not exist. So, no deviation to a lower price is profitable.

Suppose, instead, that firm i deviates to a higher price in the first period. Under the efficient rationing rule, the residual demand function facing the deviating firm is $RD_1^i(p^i; p - i_1, q - i_1) = D_1(p) - (n-1)q_1^C$. This is because rival firms' inventory controls, $k_1^j = q_1^C$ (if any firm deviates in stage zero, then k_1^j equals then adjust Cournot output given the new capacity constraint.

Since the shadow cost of capacity is c on the equilibrium path (and, more generally, is equalized across both periods), firm i's first-period profit function is $(D_1(p^i)-(n-1)q_1^C)(p^i-c)$ or, equivalently, $(p_1((n-1)q_1^C+q^i)-c)q^i$ where p_1 is the first period inverse demand function. Thus, the optimal price deviation is given by the first-order condition, which is

$$p_1'\left((n-1)q_1^C + q\right)q + p_1\left((n-1)q_1^C + q\right) = c.$$

But this implies that $q = q_1^C$ and that the optimal price and quantity are the first-period Cournot output (or, more generally, the output that equalizes the marginal revenue across the two periods), so no deviation to a higher price is profitable.

Under proportional rationing, the deviating firm's residual demand function is

$$RD_1^i(p^i; p_1^C, q_1^C) = D_1(p^i)\left[1 - \frac{(n-1)q_1^C}{D_1(p_1^C)}\right] = \frac{1}{n}D_1(p^i),$$

since $D_1(p_1^C) = nq_1^C$. The shadow cost of capacity is c on the equilibrium path (and, more generally, is equalized across the two periods), so firm i's first-period profit function is

$\frac{1}{n}D_1(p)(p-c)$, or, equivalently, $p_1(nq)-c)q$. The first-order condition is $p_1(nq)+p_1'(nq)\,q = c$, which implies that $q = q_1^C$, so no deviation to a higher price is profitable.

In stage zero, firms choose capacity expecting to equalize marginal revenue across periods one and two, and it is easy to see that $K^i = q_1^C + q_2^C$ is a best response to $K^j = q_1^C + q_2^C$ for all $j \neq i$. ■

Cournot Model with linear demand and with and without discrimination

First, suppose that $p = a - bq$; firms have constant cost c; and there are n firms. Cournot output for each of n firms is $(a-c)/b(n+1)$, so the total Cournot output is $(a-c)n/b(n+1)$; the Cournot price is $(a+nc)/(n+1)$, and the Cournot profit of each firm is $(a-c)^2/b(n+1)^2$.

Now consider two markets and suppose that firms sell in both markets and that demands are $p_1 = a_1 - b_1q_1$ and $p_2 = a_2 - b_2q_2$. Then, if the demands are combined into one with the same price, demand is $q^{Tot} = a_1/b_1 + a_2/b_2 - p\left(\frac{1}{b_1} + \frac{1}{b_2}\right)$ or $b_1b_2q^{Tot} = b_2a_1 + b_1a_2 - p\,(b_1 + b_2)$ or $p = \frac{b_2a_1+b_1a_2}{b_1+b_2} - \frac{b_1b_2}{b_1+b_2}q^{Tot}$, so Cournot profit is $(\frac{b_2a_1+b_1a_2}{b_1+b_2} - c)^2(b_1+b_2)/(b_1b_2(n+1)^2)$ or $(b_2a_1 + b_1a_2 - (b_1+b_2)c)(\frac{b_2a_1+b_1a_2}{b_1+b_2} - c)/(b_1b_2(n+1)^2)$ or $(b_2(a_1-c) + b_1(a_2-c))(\frac{b_2a_1+b_1a_2}{b_1+b_2} - c)/(b_1b_2(n+1)^2)$

If the markets are separate and firms set different quantities (and prices) in each market, then the Cournot profits are $(a_1-c)^2/(b_1(n+1)^2) + (a_2-c)^2/(b_2(n+1)^2)$ or, equivalently, $\left(b_2(a_1-c)^2 + b_1(a_2-c)^2\right)/(b_1b_2(n+1)^2)$.