Supplemental Material for
ENTRY GAMES UNDER PRIVATE INFORMATION

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Online Appendix

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A Examples of Non-Linear Profit Functions

The proposed framework accommodates a wide variety of post-entry competition models used in practice. These applications range from reduced form models, such as those presented in Table 2 in the paper, to traditional micro-founded oligopoly models. To exemplify microfounded models let assume that $c_i(v_i)$ and $K_i > 0$ represent the marginal cost and the fixed cost of production, respectively. Also, assume that $c_i(v_i)$ is decreasing and differentiable in v_i and that $K_i \in (\mathbb{E}[\pi_i(\overline{v}_i)|n), \pi_i(\underline{v}_i))$ for some finite values of $\underline{v}_i < \overline{v}_i$, so that A3 is always satisfied. Using this cost structure, common applications of the model are:

Example (Homogeneous-good Cournot competition)**.** Consider the (inverse) demand function $P = 1 - Q$ where $Q = \sum_{i \in e} q_i$ and assume $c_i(v_i) \in (0, 1]$. For any particular realization of v_e there is a corresponding vector of marginal costs $(c_1, c_2, \ldots, c_{n_e})$. Let $\bar{c}_e = \sum_{j \in e} c_j / n_e$ be the average marginal costs. Given v_e, the equilibrium profits are equal to $\pi_i(v_e) = q_i(v_e)^2 - K_i$ where $q_i(v_e) = (1 + n_e(\bar{c}_e - c_i) - c_i)/(n_e + 1)$ which satisfies A1 and a strict version of A2. Let $\mu_i(v_e) = (p^*(v_e) - c_i(v_i))/c_i(v_i)$ be the mark-up of firm i under v_e where $p^*(v_e) = (1 + n\bar{c}) / (n + 1)$ is the equilibrium price. Using a version of (4) in Corollary 2 (see main text) for n firms, we obtain the following sufficient condition:

$$\frac{-c_i(v_i)}{c_i(v_i)} \Bigg/ \frac{f_i(v_i)}{F_i(v_i)} > \frac{n+1}{2n} \mu_i(v_e).$$

Example (Nash-Bertrand under logit demand)**.** Suppose firms compete in price in a market characterized by a logit demand. Let $(\lambda_i)_{j=1}^n$ be a vector of positive constants. In this context, firm's i profits can be characterized as $\pi_i(v_e) = \mathcal{M}_i(v_e)(p_i^* - c_i(v_i)) - K_i$ where $\mathcal{M}_i(v_e) = \exp(\lambda_i - \alpha p_i^*)/(1 + D_e)$ is the fraction of consumers entering in the market and consuming good i, $D_e = \sum_{j \in e} \exp(\lambda_j - \alpha p_j^*)$, and p_i^* is the equilibrium price which depends on v_e. Then $\pi_i(v_e)$ satisfies A1 and a strict version of A2. Let $\mu_i(v_e) = (p^*(v_e) - c_i(v_i))/c_i(v_i)$ be the mark-up of firm i and $p^*(v_e)$ is the equilibrium price under market structure v_e. Using a version of (4) in Corollary 2 (see main text) for n firms, we obtain the following sufficient condition:

$$\frac{-c_i(v_i)}{c_i(v_i)} \Bigg/ \frac{f_i(v_i)}{F_i(v_i)} > \rho_i(v_e)\mu_i(v_e).$$

where $\rho_i(v_e) = 1 - \mathcal{M}_i(v_e) \sum_{j \in e \setminus i} \frac{1+D_e}{1+D_{e \setminus j}} \mathcal{M}_j(v_e)^2 \in (0, 1)$.

B Microfoundation of Examples in Table 2

Case A – No interactions CES models with atomistic agents under the traditional trade assumption that firms do not incorporate strategic interactions when deciding prices falls into this category.

Table 3: Numeric Example

		Firm 1	
		In	Out
Firm 2	In	$v_1 - \frac{1}{2}, v_2 - \delta\frac{1}{2}$	$0, v_2$
	Out	$v_1, 0$	$0, 0$

Case B – Type-independent extensive margin Any model in which the private information corresponds to entry costs and every characteristic of the market is public and commonly known satisfy this structure. In this case, we can index $\delta_{i,j,e}$ to represent the profit loss of firm i when firm firm j enters under market structure e. If, for instance, firms are symmetric we can index δ_n to represent the profit loss of entry by n competitors.

Case C – Type-dependent extensive margin Let the demand for firm i as a monopolist be $D(p_i; v_i)$ where D is decreasing in p_i and increasing in v_i. When firm i faces competition from j its demand is $D(p_i, p_j; v_i) = \gamma D(p_i; v_i)$; i.e., entry by j reduces the market demand uniformly by γ. Then, j's entry does not affect pricing decisions and $\pi(v_i, v_j) = \gamma\pi(v_i)$.

C Numeric Example

Consider the entry game in Table 3. Assume $v_1 \sim U[0,1]$ and $v_2 \sim U[0,\alpha]$ with $\alpha \leq 1$. With a low α, firm 2 becomes less likely of drawing a high type. Therefore, a lower α leads to a *stronger* firm 1 becomes. Conversely, the lower is δ the smaller is loss that firm 2 suffers from firm 1 entry. Therefore, a lower δ implies a *stronger* firm 2. If we set $\alpha = \delta = 1$ then the game is symmetric.

Strength In the context of two firms, and for an arbitrary firm i, the *strength* of firm i, s_i, is given by:

$$\pi_i(s_i)F_j(s_i) + \int_{s_i}^{\infty} \pi_i(s_i, v)dF_j(v) = 0.$$

In the example above the firms' *strength* are given by the solution to

$$\left(s_1 - \frac{1}{2}\right)\frac{s_1}{\alpha} + (s_1 - 1)\left(1 - \frac{s_1}{\alpha}\right) = 0, \qquad \left(s_2 - \frac{1}{2}\right)s_2 + \left(s_2 - \frac{1+\delta}{2}\right)(1 - s_2) = 0.$$

Solving above we obtain $s_1 = 2\alpha/(1+2\alpha)$ and $s_2 = (1+\delta)/(2+\delta)$. Observe that if firms are equally strong (i.e., $s_1 = s_2$) if and only if $\delta = 2\alpha - 1$.

Equilibrium We solve for equilibrium making use of equation (2) in the main text for the two firms scenario; i.e.,

$$\pi_i(x_i)F_j(x_j) + \int_{x_j}^{\infty} \pi_i(x_i, v)dF_j(v) = 0.$$

In our example the equilibrium is given by the solution to the following system of equations:

$$\left(x_1 - \frac{1}{2}\right)\frac{x_2}{\alpha} + (x_1 - 1)\left(1 - \frac{x_2}{\alpha}\right) = 0, \quad \left(x_2 - \frac{1}{2}\right)x_1 + \left(x_1 - \frac{1+\delta}{2}\right)(1 - x_2) = 0.$$

Solving we get $x_1 = (4\alpha - \delta - 1)/(4\alpha - \delta)$ and $x_2 = 2\alpha/(4\alpha - \delta)$.

Uniqueness It is easy to verify that no firm enters if its type is below the entry cost $v_i < 1/2$. Therefore, we have that $\underline{v}_1 = 0.5$ and $\underline{v}_2 = 0.5$. The entry game has a unique equilibrium if (3) in the main text holds for all $v_i > \underline{v}_i$. In our example the sufficient condition holds if $v_1 > 0.5$ (always true) and $v_2 > \delta/2$. The latter holds whenever $\delta \leq 1$.

Probability of Entering Since the distribution of types is uniform, the firms probability entering the market are $p_1 = 1 - x_1$ and $p_2 = 1 - \frac{x_2}{\alpha}$

Ex-ante Expected Profits Equation (5) in the main text for the two-firm scenario becomes:

$$\bar{H}_i(x_i, x_j) = \int_{x_i}^{\infty} \quad \pi_i(x)F_j(x_j) + \int_{x_j}^{\infty} \pi_i(x,v)dF_j(v) \quad dF_i(x)$$

In the example above $\bar{H}_1$ and $\bar{H}_2$ become:

$$\int_{x_1}^{1}\left((x - \frac{1}{2})\frac{x_2}{\alpha} + \int_{x_2}^{\alpha}\frac{x-1}{\alpha}dv\right)dx, \quad \frac{1}{\alpha}\int_{x_2}^{\alpha}\left(\left(x - \frac{1}{2}\right)x_1 + \int_{x_1}^{1}\frac{1}{\alpha}(x - \frac{1+\delta}{2})dv\right)dx$$

Solving we obtain: $\bar{H}_1 = 1/(2(4\alpha - \delta)^2)$ and $\bar{H}_2 = \alpha(4\alpha - \delta - 2)^2/(2(4\alpha - \delta)^2)$.

Examples Now we solve particular examples of the game when the value of *strength* is the equal for both firms.

1. **Symmetric Game** When the firms are symmetric ($\alpha = \delta = 1$) the unique equilibrium of the game consist of cutoff equal to the firms' strength; in this case $x_i = s_i = 2/3$. In this scenario, each firm enters the market with identical probability $p_i = 1/3$ and the expected profit of each firm is equal to $\bar{H}_i = 1/18$.

2. **Firm one dominance** Suppose $\alpha = 9/10$ and $\delta = 8/10$. A lower α gives a small comparative advantage to firm 1. This advantage is offset by a lower δ. By construction, firms are equally strong and the unique equilibrium is given by the firms' strength; i.e., $x_i = s_i = 9/14$. Unlike the previous example firms do not enter with the same probability nor have the same expected profits. The probabilities of each firm entering the game are $p_1 = 5/14$ and $p_2 = 0.2/7$. That is, even if both firms are using the same entry strategy, because their distributions of types are different, they will enter the game with different probabilities. The firms' expected profits are $\bar{H}_1 = 25/392$ and $\bar{H}_2 = 9/245$. Notice that firm 1 enters with slightly higher probability than firm 2 but its profits are almost twice as big.

3. **Reversal** Suppose instead $\alpha = 9/10$ and $\delta = 5/9$. In this example we keep α as before but lower δ even more. We show that firm 2 is stronger, plays the lowest cutoff, enters with higher probability but still gets lower expected payoffs than firm one. In particular, $s_1 = 9/14 > 14/23 = s_2$; the unique equilibrium is given by $(x_1, x_2) = (92/137, 81/137)$; the probabilities of each firm entering the game are $(p_1, p_2) = (45/137, 47/112)$, and; the firms' expected profits are given by $(\bar{H}_1, \bar{H}_2) = (0.0539, 0.053)$.