

**INFORMATION LIMITS OF AGGREGATE DATA**

**By**

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# Information Limits of Aggregate Data

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## Abstract

This paper uses a small model in the Cowles Commission (CC) tradition to examine the limits of aggregate data. It argues that more can be learned about the macroeconomy following the CC approach than the reduced form and VAR approaches allow, but less than the DSGE approach tries to do.

## 1 Introduction

It is obvious that there are severe limits as to how much can be learned about economic behavior from aggregate macroeconomic data. One way of distinguishing among approaches to using aggregate data is to consider how much theory is imposed on the data before analyzing it. VAR and reduced form approaches use very little theory; the DSGE approach imposes tight theoretical restrictions. An in between approach is the Cowles Commission (CC) approach, which goes back at least to Tinbergen (1939). Theory is used to guide the choice of left hand side and right hand side variables in structural equations, but the equations are then generally estimated without further restrictions.

In this paper I want to make a Goldilocks argument regarding the use of aggregate data, namely that the reduced form and VAR approaches are too cold, the DSGE approach is too hot, and the CC approach is just right. I am going to do

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this by way of an example. I have developed a multicountry econometric (MC) model that is in the CC tradition. I have gathered my research in macroeconomics in one document, *Macroeconometric Modeling, November 11, 2013 (MM)*, on my website, and this document contains a complete description and listing of the MC model. The MC model is large, as is just the United States (US) model alone. For example, for the US model the National Income and Product Accounts (NIPA) and Flow of Funds data have been integrated, which requires many equations and variables. Not everyone is willing to wade through all this, and in this paper I have reduced the US model to a more manageable size. This version, called “US mini,” is a good approximation to the overall US model in a number of important respects, although a number of features have been left out. The income side has been substantially reduced; there is no labor sector; and there is no wage-price sector, just a price equation. US mini is thus not a replacement for the US model, but, as will be seen, it is useful for making a number of points about the use of macro data.

To preview some of the results. The GDP identity is an important theoretical restriction to impose, which is not done in VAR and reduced form work. Using this identity and estimated consumption and investment equations allows the government spending multiplier to be computed by solving a simultaneous set of equations. This solution incorporates all the theoretical restrictions in the model. It uses much more information than directly estimating reduced form equations. All the exogenous and lagged endogenous variables are used, not just a subset, which is usually done when reduced form equations are directly estimated. A similar argument can be made regarding computing tax multipliers. Incorporating these restrictions is likely to narrow the range of uncertainty of the multiplier estimates. This means that dynamic scoring of various Congressional tax and spending proposals may not be as problematic as some suggest. If what seem to be sensible theoretical restrictions are imposed, the range of uncertainty is not that large. Some examples are given below.

US mini is useful for examining wealth effects. I have argued in Fair (2014), using the entire MC model, that much of the 2008–2009 recession can be explained by the decrease in household financial and housing wealth. This will be shown below. These kinds of wealth effects are missing from both VAR and DSGE models. The theory behind these effects is simply that changes in aggregate financial and housing wealth affect changes in aggregate consumption and investment, assuming that most of the changes are unanticipated. These effects can be picked up in the aggregate data.

There is an estimated Fed rule in the US model, and this has been carried over to US mini. Monetary policy is thus endogenous. Interest rates affect consumption and investment. These estimated effects are not large enough to allow monetary policy to eliminate business-cycle fluctuations, or even to come close. In many DSGE models, on the other hand, monetary policy can completely control the economy. This is an example in my view where the theoretical restrictions are too tight. The restrictions are not supported by the aggregate data.

In US mini U.S. exports are exogenous, unlike in the MC model. However, imports are endogenous—there is an estimated import demand equation. This import equation has a large effect on the properties of the model, for example, on the size of the government spending and tax multipliers. The marginal propensity to import is large, so the multipliers are considerably smaller than they would be if all of the change in demand was a change in domestic production. Exports are less important in this regard, since they are driven by demand in other countries, which is only modestly affected by changes in U.S. government spending and taxes. Import demand is typically ignored in both VAR and DSGE models, and so important theoretical information is being ignored. This is a case in which DSGE models impose less theory than does the CC approach.

Ten estimated equations have been taken from the US model. With a few exceptions to meet the constraints of US mini, the equations have not been changed. There are three consumption equations, three investment equations, a demand for

imports equation, a price equation, the Fed's interest rate rule, and a term structure equation explaining the mortgage rate. The equations are estimated by two-stage least squares (2SLS) with account taken, when necessary, of serial correlation of the residuals. The estimation period is 1954:1–2014:4 except for the Fed rule, where the period ends in 2008:3. .

The data construction and identities are discussed in Section 2. Section 3 then discusses the estimated equations and their various strengths and weaknesses. There is a practical tone to this discussion, where I am trying to convince the reader that while the equations may not be perfect, they are probably capturing most of what one can get out of the aggregate data. In other words, the main empirical regularities in the data are probably being accounted for. After the model is estimated in Section 3, it is analyzed in Sections 5 through 9.

## **2 Tables 1 and 2: Variable Construction and the Identities**

Table 1 lists all of the variables in the model and their construction. Nominal variables are denoted with a \$ at the end. Table 2 lists the identities.

Most of Table 1 is self explanatory. The data sources are given at the bottom of the table. The stock variables are summed from flows, where the base-quarter value is given in the table. The summation when relevant is both forward and backward. The variables constructed from peak-to-peak interpolations are on straight lines between the peaks. The capital gain or loss variable,  $CG\$$ , is constructed from Flow of Funds data. Likewise, the construction of  $PHOUSE$ , the price of housing relative to the GDP deflator, uses Flow of Funds data.

The depreciation variables,  $DEPD$ ,  $DEPH$ , and  $DEPK$ , require some explanation. Consider  $DEPD$ . Given quarterly observations on durable expenditures,  $CD$ , quarterly observations for the stock of durables,  $KD$ , can be constructed using equation I-10 in Table 2 once a base-quarter value and values for  $DELD$  are

**Table 1**  
**The Variables in Alphabetical Order**

<b>Variable</b>	<b>Eq.</b>	<b>Description</b>	<b>Used in Equations</b>
<i>AA</i>	I-18	Total wealth, B2009\$. Definition I-18.	1, 2, 3
<i>A1</i>	I-16	Financial wealth, B2009\$. Definition I-16.	I-18
<i>A1\$</i>	I-15	Financial wealth, B\$. Definition I-15. Base quarter 1971.4; benchmark value 2,862.6.	I-16
<i>A2</i>	I-17	Housing wealth, B2009\$. Definition I-17.	4, I-18
<i>AG1</i>	exog	Percent of 16+ population 26-55 minus percent 16-25. BLS data.	1, 2, 3
<i>AG2</i>	exog	Percent of 16+ population 56-65 minus percent 16-25. BLS data.	1, 2, 3
<i>AG3</i>	exog	Percent of 16+ population 66+ minus percent 16-25. BLS data.	1, 2, 3
<i>BETA</i>	exog	Ratio of DEP\$ to Y\$.	I-6
<i>CD</i>	3	Consumer expenditures for durable goods, B2009\$. NIPA 1.1.3, line 4.	7, I-1, I-8, I-10
<i>CDA</i>	exog	Peak-to-peak interpolation of CD/POP. Peaks are 1953:1, 1955:3, 1960:2, 1963:2, 1965:4, 1968:3, 1973:2, 1978:4, 1985:1, 1988:4, 1994:1, 1995:4, 2000:3, 2007:2, 2012:1, 2013:4.	3
<i>CG\$</i>	exog	Capital gains(+) or losses(-) on the financial assets of the household sector, B\$. FF, Financial assets of households and nonprofit organizations, F101 and L101, FF code 154090005. <i>CG\$</i> is the change in the stock (L101) minus the flow (F101). The stock includes capital gains and losses and the flow does not.	I-15
<i>CN</i>	2	Consumer expenditures for nondurable goods, B2009\$. NIPA, 1.1.3, line 5.	7, I-1, I-8
<i>C2</i>	exog	time varying constant term.	1, 2, 3, 4, 5, 6, 7, 8
<i>CS</i>	1	Consumer expenditures for services, B2009\$. NIPA, 1.1.3, line 6.	7, I-1, I-8
<i>D593</i>	exog	1 in 1959:3; 0 otherwise.	6
<i>D594</i>	exog	1 in 1959:4; 0 otherwise.	6
<i>D601</i>	exog	1 in 1960:1; 0 otherwise.	6
<i>D691</i>	exog	1 in 1969:1; 0 otherwise.	7
<i>D692</i>	exog	1 in 1969:2; 0 otherwise.	7
<i>D714</i>	exog	1 in 1971:4; 0 otherwise.	7
<i>D721</i>	exog	1 in 1972:1; 0 otherwise.	7
<i>DELD</i>	exog	Physical depreciation rate of the stock of durable goods, rate per quarter. See text.	3, I-10
<i>DELH</i>	exog	Physical depreciation rate of the stock of housing, rate per quarter. See text.	4, I-11
<i>DELK</i>	exog	Physical depreciation rate of the stock of capital, rate per quarter. See text.	I-12

**Table 1 (continued)**

<b>Variable</b>	<b>Eq.</b>	<b>Description</b>	<b>Used in Equations</b>
<i>DEP</i> \$	I-6	Capital depreciation, B\$. NIPA, 1.7.5, line 5.	I-5
<i>EX</i>	exog	Exports, B2009\$. NIPA, 1.1.3, line 16.	I-1
<i>G</i>	exog	Government purchases of goods and services, B2009\$. NIPA, 1.1.3, lines 23 plus 26.	I-1
<i>GAP</i>	I-14	Percentage output gap. Definition I-14.	8, 9
<i>IH</i>	4	Residential (housing) investment, B2009\$. NIPA, 1.1.3, line 13.	7, I-1, I-11
<i>IHA</i>	exog	Peak-to-peak interpolation of IH/POP. Peaks are 1955:2, 1963:4, 1978:3, 1986:3, 1994:2, 2004:2, 2006:2, 2007:4, flatend.	4
<i>IK</i>	I-12	Non-residential fixed investment, B2009\$. NIPA, 1.1.3, line 9.	I-1
<i>IM</i>	7	Imports, B2009\$. NIPA, 1.1.3, line 19.	I-1
<i>IV</i>	I-2	Inventory investment, B2009\$. NIPA, 1.7.6, line 1.	I-3
<i>KD</i>	I-10	Stock of durable goods, B2009\$. Definition I-10. Base quarter 1952:1; benchmark value 255.5.	3
<i>KH</i>	I-11	Stock of housing, B2009\$. Definition I-11. Base quarter 1952:1; benchmark value 3,121.7.	4, I-16
<i>KK</i>	5	Stock of capital, B2009\$. Definition I-12. Base quarter 1952:1; benchmark value 2,914.9.	I-12
<i>KKMIN</i>	I-13	Amount of capital required to produce Y, B2009\$. Definition I-13.	5
<i>MUH</i>	exog	Amount of output capable of being produced per unit of capital. Peak-to-peak interpolation of $Y/KK$ . Peaks are flatbeg, 1953:2, 1955:3, 1959:2, 1962:3, 1965:4, 1969:1, 1973:3, 1977:3, 1981:1, 1984:2, 1988:4, 1993:4, 1998:1, 2006:1, 2013:4.	I-13
<i>P</i>	8	GDP deflator. $Y\$/Y$ .	I-4, I-8, I-9, I-16, I-19
<i>PCP</i>	I-19	Percentage change in P, annual rate, percentage points. Definition I-19.	9
<i>PIM</i>	exog	Price deflator for IM. NIPA, 1.1.5, line 19 divided by <i>IM</i> .	7, 8
<i>POP</i>	exog	Noninstitutional population 16+, millions. BLS data.	1, 2, 3, 4, 7
<i>PHOUSE</i>	exog	Ratio of the price of housing to P. Price of housing is FF, nominal value of real estate of households and nonprofit organizations, FF code 155035005, B.101, divided by <i>KH</i> .	I-17
<i>RM</i>	10	Mortgage rate, percentage points. BOG, quarterly average.	3, 4
<i>RS</i>	9	Three-month Treasury bill rate, percentage points. BOG, quarterly average.	1, 2, 10
<i>SH</i> \$	I-8	Financial saving of household sector B\$. Definition I-8.	I-15
<i>STATP</i>	exog	Statistical discrepancy relating to the use of chain type price indices, B2009\$. Definition I-1.	I-1

**Table 1 (continued)**

<b>Variable</b>	<b>Eq.</b>	<b>Description</b>	<b>Used in Equations</b>
<i>T</i>	exog	1 in 1952:1, 2 in 1952:2, etc.	8
<i>TAU</i>	exog	Ratio of TAX\$ to Y\$.	I-7
<i>TAX\$</i>	I-7	Net taxes, B\$. NIPA, 3.1, lines 1 minus 17 plus 18.	I-5
<i>V</i>	I-3	Stock of inventories, B2009\$. Definition I-3. Base quarter 1996:4; benchmark value 1,517.3, from NIPA 5.8.6A, line 1.	6
<i>X</i>	I-1	Total sales, B2009\$. $Y + IV$ .	12
<i>Y</i>	6	Gross Domestic Product, B2009\$. NIPA, 1.1.3, line 1.	I-2, I-4, I-14
<i>Y\$</i>	I-4	Gross Domestic Product, B\$. NIPA, 1.1.5, line 1.	I-6, I-7
<i>YD</i>	I-9	Personal disposable income, B2009\$. Definition I-9.	1, 2, 3, 4
<i>YD\$</i>	I-5	Personal disposable income, B\$. Definition I-5.	I-8
<i>YS</i>	exog	Potential output, B2009\$. Computed from peak-to-peak interpolation of $\log Y$ . Peaks are 1953:1, 1960:1, 1969:1, 1978:4, 1990:2, 2000:3, 2007:4.	I-14

- B\$ = Billions of dollars.
- B2009\$ = Billions of 2009 dollars.
- First line extended back and last line extended forward for peak-to-peak interpolations unless flatbeg or flatend. For flatbeg the first peak is extended back horizontally, and for flatend the last peak is extended forward horizontally.
- NIPA: National Income and Product Accounts.
- FF: Flow of Funds Accounts.
- BLS: Bureau of Labor Statistics.
- BOG: Board of Governors of the Federal Reserve System.



**Table 2**

<b>Identities</b>		
<b>Eq.</b>	<b>LHS Variable</b>	<b>Explanatory Variables</b>
I-1	$X =$	$CS + CN + CD + IH + IK - IM + G + EX + STATP$ [Total sales]
I-2	$IV =$	$Y - X$ [Inventory investment]
I-3	$V =$	$V_{-1} + IV$ [Stock of inventories]
I-4	$Y\$ =$	$P \cdot Y$ [Nominal GDP]
I-5	$YD\$ =$	$Y\$ - DEP\$ - TAX\$$ [Nominal personal disposable income]
I-6	$DEP\$ =$	$BETA \cdot Y\$$ [Nominal depreciation]
I-7	$TAX\$ =$	$TAU \cdot Y\$$ [Nominal net taxes]
I-8	$SH\$ =$	$YD\$ - P(CS + CN + CD + IH)$ [Nominal household financial saving]
I-9	$YD =$	$YD\$/P$ [Real personal disposable income]
I-10	$KD =$	$(1 - DELD)KD_{-1} + CD$ [Stock of durable goods]
I-11	$KH =$	$(1 - DELH)KH_{-1} + IH$ [Stock of housing]
I-12	$IK =$	$KK - (1 - DELK)KK_{-1}$ [Non-residential fixed investment]
I-13	$KKMIN =$	$Y/MUH$ [Capital stock required to produce $Y$ ]
I-14	$GAP =$	$1 - Y/Y_S$ [Percentage output gap]
I-15	$A1\$ =$	$A1\$_{-1} + SH\$ + CG\$$ [Nominal financial wealth]
I-16	$A1 =$	$A1\$/P$ [Real financial wealth]
I-17	$A2 =$	$PHOUSE \cdot KH$ [Real housing wealth]
I-18	$AA =$	$A1 + A2$ [Real total wealth]
I-19	$PCP =$	$100((P/P_{-1})^4 - 1)$ [Percentage change in $P$ at an annual rate]

chosen. End of year estimates of the stock of durable goods are available from the BEA Fixed Assets Table 9.1. Given, say, the value of  $KD$  at the end of 1952 and given quarterly values of  $CD$  for 1953:1–1953:4, a value of  $DELD$  can be computed such that the predicted value from equation I-10 for 1953:4 matches within a prescribed tolerance level the published BEA value for the end of 1953. This value of  $DELD$  can then be used to compute quarterly values of  $KD$  for 1953:1, 1953:2, and 1953:3. This process can be repeated for each year, which results in a quarterly series for  $KD$ . The values of  $DELD$  are different for each year, but the same for the four quarters within a year. Values for  $DELH$  and  $DELK$  are constructed in a similar fashion, also using the BEA Fixed Assets Table 9.1.

There are 19 identities in Table 2. Equation I-1 defines total sales as consumption plus investment plus exports plus government spending minus imports. Equation I-2 defines inventory investment as production (real GDP) minus sales. Equation I-3 is an updating equation for the stock of inventories.

Equation I-4 defines nominal GDP as the GDP deflator times real GDP. Equation I-5 defines nominal disposable income,  $YD\$$ , as nominal GDP minus nominal depreciation and nominal net taxes. This is where the income side is missing in US mini (unlike in the US model).  $YD\$$  is approximately, but not exactly, nominal disposable income in the NIPA.  $BETA$  in equation I-6 is constructed as nominal depreciation divided by nominal GDP. It is taken as exogenous. Equation I-6 defines nominal depreciation as  $BETA$  times nominal GDP. So nominal depreciation is endogenous because nominal GDP is endogenous. A similar procedure is followed for nominal net taxes in equation I-7, where  $TAU$  is the ratio of nominal net taxes to nominal GDP and is taken as exogenous. Nominal net taxes are all taxes minus all government transfers, both federal and state and local.

Equation I-8 defines nominal household financial saving as nominal disposable income minus nominal spending on consumption and housing investment. It is only an approximation to nominal household financial saving in the Flow of

Funds accounts because of lack of an income side and because (unlike in the US model) separate price deflators are not used for the three consumption categories and housing investment. Equation I-9 defines real disposable income as nominal disposable divided by the GDP deflator.

Equations I-10, I-11, and I-12 relate three physical stocks to three flows: stocks of durable goods, housing, and capital. The three depreciation rates are exogenous and were chosen as discussed above. Equation I-12 has the flow on the left hand side, and this is explained below in the discussion of equation 5. In equation I-13  $MUH$  is constructed from peak-to-peak interpolations of output divided by capital, and equation I-13 defines the minimum amount of capital required to produce the output in the quarter as output divided by  $MUH$ . This procedure assumes a fixed proportions technology in the short run, with long-run technical change reflected in the change in  $MUH$  between the peaks. Equation I-14 defines the output gap,  $GAP$ , as one minus the ratio of actual output to potential output. Potential output,  $YS$ , is exogenous and is constructed from peak-to-peak interpolations of actual output.

Equations I-15–I-18 define wealth variables. In equation I-15 nominal financial wealth equals last quarter's value plus nominal household financial saving plus nominal capital gains or losses on stocks. Real financial wealth is defined in equation I-16 as nominal financial wealth divided by the GDP deflator. Real housing wealth is defined in equation I-17. It is equal to the physical stock of housing times the price of housing relative to the GDP deflator,  $PHOUSE$ . Equation I-18 defines real total wealth,  $AA$ , as real financial wealth plus real housing wealth.

Finally, equation I-19 defines the inflation rate as the percentage change in the GDP deflator at an annual rate.

### 3 Estimated Equations

The 10 estimated structural equations are presented in Tables 3.1 through 3.10. The coefficient estimates are presented along with results of various  $\chi^2$  tests. The estimation technique is two-stage least squares (2SLS) under the assumption, in some cases, of serial correlation of the residuals. The structural coefficients are estimated along with the serial correlation coefficients. The estimation period is 1954:1–2014:4, 244 quarterly observations, except for the Fed rule, where the period ends in 2008:3 because of the zero lower bound constraint. The first stage regressors used for each equation are presented in Table A in the appendix.

The  $\chi^2$  tests consist of adding one or more variables to the equation and seeing if it or they are significant. The tests include adding lagged values of the explanatory variables, adding a serial correlation assumption (if it is not already used), adding a linear time trend, and adding led values where appropriate. A test of overidentifying restrictions for 2SLS is also performed.

Adding lagged values, called the “Lags test,” is a test of the dynamic specification, as is adding the assumption of serial correlation, called the “RHO test.” Adding a time trend, called the “T test,” is a way of testing for spurious correlation from common trending variables. When led values are added, Hansen’s (1982) method is used for the estimation. Adding led values is a way of testing the rational expectations assumption. For the leads test, two sets of led values are tried per equation. For the first set the values of the relevant variable or variables led once are added. For the second set the values led one through eight quarters are added, where the coefficients for each variable are constrained to lie on a second degree polynomial with an end point constraint of zero. The test in each case is a  $\chi^2$  test that the additional variables are significant. The two tests are called “Leads +1” and “Leads +8.” This test is discussed in Fair (1993) and in *MM* (Section 2.8.5). For some of the tests additional first stage regressors from those listed in Table A were used.

**Table 3.1: Equation 1**  
**LHS Variable is  $\log(CS/POP)$**

RHS Variable	Equation	Coef.	t-stat.	Test	$\chi^2$ Tests		
					$\chi^2$	df	<i>p</i> -value
<i>C</i> <sup>2</sup>		0.0300	5.87	Lags	4.03	3	0.258
<i>C</i>		-0.0695	-2.84	T	8.88	1	0.003
<i>AG</i> <sub>1</sub>		-0.163	-4.80	Leads +1	3.93	1	0.047
<i>AG</i> <sub>2</sub>		-0.393	-7.29	Leads +8	3.99	2	0.136
<i>AG</i> <sub>3</sub>		0.394	5.12				
$\log(CS/POP)_{-1}$		0.897	44.26				
$\log(YD/POP)$		0.0358	1.80				
<i>RS</i>		-0.000980	-4.01				
$\log(AA/POP)_{-1}$		0.0378	6.23				
<i>RHO</i>		0.197	3.15				
SE	0.00370						
R <sup>2</sup>	0.999						
DW	2.05						

overid test (df = 7, *p*-value = 0.094).  $\chi^2$  (AGE) = 51.13 (df = 3, *p*-value = 0.000).

Lags test adds  $\log(CS/POP)_{-2}$ ,  $\log(YD/POP)_{-1}$ , and *RS*<sub>-1</sub>.

Leads tests are for  $\log(YD/POP)$ .

Estimation period is 1954.1-2014.4.

**Table 3.2: Equation 2**  
**LHS Variable is  $\log(CN/POP)$**

RHS Variable	Equation	Coef.	t-stat.	Test	$\chi^2$ Tests		
					$\chi^2$	df	<i>p</i> -value
<i>C</i> <sup>2</sup>		0.002	0.40	Lags	1.94	3	0.585
<i>C</i>		-0.300	-5.25	RHO	10.93	1	0.001
<i>AG</i> <sub>1</sub>		-0.052	-1.14	<i>T</i>	0.29	1	0.590
<i>AG</i> <sub>2</sub>		-0.177	-2.78	Leads +1	0.76	1	0.383
<i>AG</i> <sub>3</sub>		0.053	0.51	Leads +8	1.28	2	0.526
$\log(CN/POP)_{-1}$		0.781	21.57				
$\Delta \log(CN/POP)_{-1}$		0.153	2.69				
$\log(AA/POP)_{-1}$		0.0524	5.45				
$\log(YD/POP)$		0.0853	3.53				
<i>RS</i>		-0.000580	-1.76				
SE	0.00635						
R <sup>2</sup>	0.999						
DW	1.98						

overid test (df = 1, *p*-value = 0.003).  $\chi^2$  (AGE) = 16.68 (df = 3, *p*-value = 0.001).

Lags test adds  $\log(CN/POP)_{-3}$ ,  $\log(YD/POP)_{-1}$ , and *RS*<sub>-1</sub>.

Leads tests are for  $\log(YD/POP)$ .

Estimation period is 1954.1-2014.4.

**Table 3.3: Equation 3**  
**LHS Variable is  $CD/POP - (CD/POP)_{-1}$**

RHS Variable	Equation			Test	$\chi^2$ Tests		
	Coef.	t-stat.			$\chi^2$	df	p-value
$C2$	0.0628	3.24		Lags	1.39	3	0.709
$C$	0.0143	0.29		RHO	0.00	1	0.968
$AG1$	-0.232	-2.57		$T$	16.54	1	0.000
$AG2$	1.215	3.44		Leads +1	1.12	1	0.291
$AG3$	-0.669	-2.21		Leads +8	2.08	2	0.353
$a$	0.118	2.85					
$(KD/POP)_{-1}$	-0.0155	-4.23					
$YD/POP$	0.0219	3.41					
$RM \cdot CDA$	-0.00538	-2.64					
$(AA/POP)_{-1}$	0.000573	3.70					
SE	0.0146						
$R^2$	0.209						
DW	2.02						

<sup>a</sup>Variable is  $DEL D(KD/POP)_{-1} - (CD/POP)_{-1}$ .  
overid test (df = 1, p-value = 0.706).  $\chi^2$  (AGE) = 17.40 (df = 3, p-value = 0.001).  
Lags test adds <sup>a</sup> lagged once,  $(YD/POP)_{-1}$ , and  $(RM \cdot CDA)_{-1}$ .  
Leads tests are for  $YD/POP$ .  
Estimation period is 1954.1-2014.4.

**Table 3.4: Equation 4**  
**LHS Variable is  $IH/POP - (IH/POP)_{-1}$**

RHS Variable	Equation			Test	$\chi^2$ Tests		
	Coef.	t-stat.			$\chi^2$	df	p-value
$C2$	0.128	0.72		Lags	1.56	3	0.668
$C$	2.170	3.15		$T$	1.52	1	0.218
$a$	0.441	8.04					
$(KH/POP)_{-1}$	-0.0584	-5.48					
$YD/POP$	0.0955	3.10					
$RM_{-1} \cdot IHA$	-0.0208	-5.65					
$(A2/POP)_{-1}$	0.00570	5.00					
RHO1	0.707	9.92					
RHO2	0.270	3.77					
SE	0.0172						
$R^2$	0.503						
DW	2.02						

<sup>a</sup>Variable is  $DEL H(KH/POP)_{-1} - (IH/POP)_{-1}$ .  
overid test (df = 15, p-value = 0.018).  $\chi^2$  (AGE) = 10.66 (df = 3, p-value = 0.014).  
Lags test adds <sup>a</sup> lagged once,  $(YD/POP)_{-1}$ , and  $(RM_{-1} \cdot IHA)_{-1}$ .  
Estimation period is 1954.1-2014.4.

**Table 3.5: Equation 5**  
**LHS Variable is  $\Delta \log KK$**

RHS Variable	Equation		Test	$\chi^2$ Tests		
	Coef.	t-stat.		$\chi^2$	df	p-value
$C_2$	-0.00026	-3.31	Lags	3.26	3	0.354
$C$	0.00046	2.57	RHO	0.75	1	0.388
$\log(KK/KKMIN)_{-1}$	-0.00653	-2.57	$T$	1.39	1	0.238
$\Delta \log KK_{-1}$	0.909	67.13	Leads +1	0.03	1	0.865
$\Delta \log Y$	0.0315	3.74	Leads +8	0.08	2	0.960
$\Delta \log Y_{-1}$	0.0079	1.87				
$\Delta \log Y_{-2}$	0.0001	0.04				
$\Delta \log Y_{-3}$	0.0071	2.20				
$\Delta \log Y_{-4}$	0.0058	1.75				
$a$	0.000784	3.85				
SE	0.000375					
$R^2$	0.981					
DW	1.87					

$a$ Variable is  $(CG\$_{-2} + CG\$_{-3} + CG\$_{-4}) / (P_{-2}YS_{-2} + P_{-3}YS_{-3} + P_{-4}YS_{-4})$ .  
overid test (df = 3, p-value = 0.045).

Lags test adds  $\log(KK/KKMIN_{-2})$ ,  $\Delta \log KK_{-2}$ , and  $\Delta \log Y_{-5}$ .

Leads tests are for  $\Delta \log Y$ . Estimation period is 1954.1-2014.4.

**Table 3.6: Equation 6**  
**LHS Variable is  $\log Y$**

RHS Variable	Equation		Test	$\chi^2$ Tests		
	Coef.	t-stat.		$\chi^2$	df	p-value
$C$	0.184	3.08	Lags	2.69	2	0.260
$\log Y_{-1}$	0.229	4.81	$T$	1.30	1	0.255
$\log X$	0.939	17.21	Leads +1	0.20	1	0.658
$\log V_{-1}$	-0.206	-8.66	Leads +8	3.05	2	0.217
$D593$	-0.00788	-2.66				
$D594$	-0.00225	-0.76				
$D601$	0.00855	2.89				
RHO1	0.489	5.90				
RHO2	0.326	4.57				
RHO3	0.130	1.78				
SE	0.00323					
$R^2$	0.999					
DW	2.04					

overid test (df = 9, p-value = 0.089).

Lags test adds  $\log Y_{-2}$  and  $\log X_{-1}$ . Leads tests are for  $\log X$ . Estimation period is 1954.1-2014.4.

**Table 3.7: Equation 7**  
**LHS Variable is  $\log(IM/POP)$**

RHS Variable	Equation		Test	$\chi^2$ Tests		
	Coef.	t-stat.		$\chi^2$	df	p-value
$C^2$	0.044	2.56	Lags	27.63	3	0.000
$C$	-1.101	-4.56	$T$	8.59	1	0.003
$\log(IM/POP)_{-1}$	0.761	15.89	Leads +1	2.62	1	0.105
$a$	0.492	4.49	$\log P$	9.57	1	0.002
$\log(P/PIM)$	0.100	4.45				
$D691$	-0.113	-4.17				
$D692$	0.139	4.98				
$D714$	-0.088	-3.26				
$D721$	0.104	3.77				
$RHO$	0.226	2.92				
SE	0.0268					
$R^2$	0.999					
DW	2.04					

<sup>a</sup>Variable is  $\log[(CS + CN + CD + IH + IK)/POP]$ .  
overid test (df = 5, p-value = 0.439 ).  
Lags test adds <sup>a</sup> lagged once,  $\log(IM/POP)_{-2}$ , and  $\log(P/PIM)_{-1}$ .  
Leads test is for <sup>a</sup>.  
Estimation period is 1954.1-2014.4.

**Table 3.8: Equation 8**  
**LHS Variable is  $\log P$**

RHS Variable	Equation		Test	$\chi^2$ Tests		
	Coef.	t-stat.		$\chi^2$	df	p-value
$C^2$	0.0161	1.41	Lags	8.29	3	0.040
$C$	-0.0611	-2.24				
$C^2 \cdot T$	-0.000135	-5.14				
$T$	0.000354	4.62				
$\log P_{-1}$	0.940	59.12				
$\log PIM$	0.0301	7.75				
$GAP$	-0.0541	-3.34				
$RHO$	0.491	8.26				
SE	0.00258					
$R^2$	0.999					
DW	2.25					

overid test (df = 5, p-value = 0.000).  
Lags test adds  $\log P_{-2}$ ,  $\log PIM_{-1}$ , and  $GAP_{-1}$ .  
Estimation period is 1954.1-2014.4.



**Table 3.9: Equation 9**  
LHS Variable is  $RS$

RHS Variable	Equation		Test	$\chi^2$ Tests		
	Coef.	t-stat.		$\chi^2$	df	$p$ -value
$C$	0.110	1.39	Lags	2.81	3	0.421
$RS_{-1}$	0.950	47.27	RHO	0.00	1	0.970
$PCP$	0.0655	2.98	$T$	0.58	1	0.445
$GAP$	-4.56	-1.20	Leads +1	0.51	2	0.775
$\Delta GAP$	-18.07	-0.84	Leads +4	3.08	4	0.544
$\Delta GAP_{-1}$	-12.79	-1.59	$p_4$	0.61	1	0.435
$\Delta RS_{-1}$	0.376	4.90	$p_8$	2.96	1	0.085
$\Delta RS_{-2}$	-0.216	-3.21				
SE	0.427					
$R^2$	0.979					
DW	1.93					

overid test (df = 3,  $p$ -value = 0.023).  $\chi^2$  (GAP) = 26.42 (df = 3,  $p$ -value = 0.000).  
Lags test adds  $PCP_{-1}$ ,  $\Delta GAP_{-2}$ , and  $\Delta RS_{-3}$ . Leads tests are for  $PCP$  and  $GAP$ .  
 $p_4$  is the four-quarter rate of inflation.  $p_8$  is the eight-quarter rate of inflation.  
Estimation period is 1954.1-2008.3.

**Table 3.10: Equation 10**  
LHS Variable is  $RM - RS_{-2}$

RHS Variable	Equation		Test	$\chi^2$ Tests		
	Coef.	t-stat.		$\chi^2$	df	$p$ -value
$C$	0.369	5.48	<sup>a</sup> Restriction	0.05	1	0.832
$RM_{-1} - RS_{-2}$	0.882	43.70	Lags	0.94	3	0.816
$RS - RS_{-2}$	0.205	2.62	RHO	0.65	4	0.420
$RS_{-1} - RS_{-2}$	0.017	0.17	$T$	1.59	1	0.207
			Leads +1	0.07	1	0.797
			Leads +8	0.83	2	0.661
			$p_4$	0.78	1	0.378
			$p_8$	0.68	1	0.411
SE	0.350					
$R^2$	0.905					
DW	1.90					

<sup>a</sup> $RS_{-2}$  added.  
overid test (df = 6,  $p$ -value = 0.365).  
Lags test adds  $RM_{-2}$ ,  $RS_{-2}$ , and  $RS_{-3}$ . Leads tests are for  $RS$ .  
 $p_4$  is the four-quarter rate of inflation.  $p_8$  is the eight-quarter rate of inflation.  
Estimation period is 1954.1-2014.4.

The overidentification test is simply the standard test of regressing the 2SLS residuals on the first stage regressors and computing the  $R^2$ . Then  $T \cdot R^2$  is distributed as  $\chi_q^2$ , where  $T$  is the number of observations and  $q$  is the number of first stage regressors minus the number of explanatory variables in the equation being estimated. The null hypothesis is that all the first stage regressors are uncorrelated with the error term. If  $T \cdot R^2$  exceeds the specified critical value, the null hypothesis is rejected, and one would conclude that at least some of the first stage regressors are not predetermined. This test is denoted “overid” in the tables.

An attempt is made in some equations to try to pick up a time varying relationship. It is hard with macro data to do much, but some significant estimates of a time varying constant term have been picked up. The assumption made, for a sample from 1 through  $T$ , is that the constant term is the same up to some observation  $T1$ , then changes linearly up to some observation  $T2$ , and is then unchanged at the  $T2$  value through  $T$ . The estimate of  $C2$  in the tables for an equation is the estimate of the slope. The estimate of  $C$  is the estimate of the constant term up to  $T1$ . If the estimate of  $C2$  is significant, this is evidence in favor of time variation of the constant term. After some experimentation,  $T1$  was taken to be 1969:4 for all the equations and  $T2$  was taken to be 1988:4. For more discussion see *MM* (Section 2.3.2).

Finally, age distribution effects are tested for by adding the age variables,  $AG1$ ,  $AG2$ , and  $AG3$ , to the household expenditure equations. These tests are discussed in Fair and Dominguez (1991) and *MM* (Section 3.6.2).

The theory behind the following specifications is not discussed here. The theory is standard household and firm maximization. A complete discussion is in *MM* (Section 3). Remember that under the CC approach theory is used to choose the left hand side and right hand side variables. There is sometimes, however, “extra” theorizing regarding the dynamics, and this is discussed below. Also, lagged dependent variables are often used as explanatory variables. These can be justified as picking up partial adjustment effects and/or adaptive-expectations

effects.

For ease of discussion, a coefficient estimate and variable will be said to be “significant” if the t-statistic is greater than 2 in absolute value. A test will be said to be “rejected” if the p-value is less than 0.01 and “passed” if it is greater than or equal to 0.01. The null hypothesis for a  $\chi^2$  test is that whatever is added has a zero effect, and if a significance level of 0.01 is used, the null hypothesis is rejected for a p-value smaller than this.

It will be seen that some variables are not significant and some tests are not passed. Not all equations are perfect. When, say, lagged values are added and they are significant, the resulting equation may not have sensible dynamic properties. This is where the smoothness of the aggregate data can be a problem; there may be too much collinearity for the number of coefficients estimated when lagged values are added. The specifications that were chosen are those that seemed to work best from experimenting with different specifications, but it is always an open question whether more can be done. More will be said about this in the Conclusion.

**Table 3.1: Equation 1. *CS*, consumer expenditures: services**

Equation 1 is in real, per capita terms and is in log form. The explanatory variables include income, an interest rate, lagged wealth, the age variables, and the lagged dependent variable. It is estimated under the assumption of first order serial correlation of the error term.

The age variables are highly jointly significant, and all the other variables are significant except for income, which has a t-statistic of 1.80. The overid test is passed. For the lags test the lagged values of income, the interest rate, and lagged consumption (i.e.,  $\log(CS/POP)_{-2}$ ) were used. They are not jointly significant, with a p-value of 0.258. . This is my view is a fairly strong test. As discussed above, aggregate data are smooth, and the ability to distinguish among lagged values is not always easy. On the other hand, when the time trend is added, it is significant. The trend effects have not been completely captured. For the leads

tests the income variable was used. The led values are not significant at the 1 percent level, although Leads +1 has a p-value of 0.047.  $C2$  is significant, which suggests that there has been some change in the constant term over time.

The interest rate,  $RS$ , is the short-term interest rate. It is in nominal terms. Tests of nominal versus real interest rates in the household expenditure equations are discussed at the end of this section.

**Table 3.2: Equation 2.  $CN$ , consumer expenditures: nondurables**

The specification of equation 2 is similar to that of equation 1. The two differences are that the assumption of serial correlation is not used and the change in the lagged dependent variable is added.

The age variables are jointly significant. The other variables are significant except for  $C2$  and the interest rate. The interest rate has a t-statistic of -1.76. The lagged values are not significant, nor are the led values. The time trend is not significant. On the negative side, the overid test fails, and when the equation is estimated under the assumption of first order serial correlation of the error term, the estimate of the serial correlation coefficient is highly significant. When RHO is added, the estimates of some of the other coefficients are not sensible, and so RHO was not included in the final specification. This is an example of problems associated with the smoothness of aggregate data.

**Table 3.3: Equation 3.  $CD$ , consumer expenditures: durables**

Equation 3 is in real, per capital terms. The explanatory variables include income, an interest rate, lagged wealth, the age variables,  $DEL D(KD/POP)_{-1} - (CD/POP)_{-1}$ , and  $(KD/POP)_{-1}$ .  $KD$  is the stock of durable goods, and  $DEL D$  is the depreciation rate of the stock.

It turns out when experimenting with different estimates of consumer durable equations that both lagged expenditures,  $CD_{-1}$ , and the lagged stock,  $KD_{-1}$ , are

significant. How can one make sense of this? The following is one way, which is used for the current specification.

Let  $KD^{**}$  denote the stock of durable goods that would be desired if there were no adjustment costs of any kind. If durable consumption is proportional to the stock of durables, then the determinants of consumption can be assumed to be the determinants of  $KD^{**}$ :

$$KD^{**} = f(\dots), \quad (1)$$

where the arguments of  $f$  are the determinants of consumption. Two types of partial adjustments are then postulated. The first is an adjustment of the durable stock:

$$KD^* - KD_{-1} = \lambda(KD^{**} - KD_{-1}), \quad (2)$$

where  $KD^*$  is the stock of durable goods that would be desired if there were no costs of changing durable expenditures. Given  $KD^*$ , desired durable expenditures,  $CD^*$ , is postulated to be

$$CD^* = KD^* - (1 - DELD)KD_{-1}, \quad (3)$$

where  $DELD$  is the depreciation rate. By definition  $CD = KD - (1 - DELD)KD_{-1}$ , and equation (3) is merely the same equation for the desired values. The second type of adjustment is an adjustment of durable expenditures,  $CD$ , to its desired value:

$$CD - CD_{-1} = \gamma(CD^* - CD_{-1}) + \epsilon. \quad (4)$$

This equation is assumed to reflect costs of changing durable expenditures. Combining equations (1)–(4) yields:

$$\begin{aligned} CD - CD_{-1} &= \gamma(DELD \cdot KD_{-1} - CD_{-1}) - \gamma\lambda KD_{-1} \\ &\quad + \gamma\lambda f(\dots) + \epsilon. \end{aligned} \quad (5)$$

This specification of the two types of adjustment is thus a way of adding to the durable expenditure equation both the lagged dependent variable and the lagged

stock of durables. Otherwise, the explanatory variables are the same as they are in the other expenditure equations.<sup>1</sup> The interest rate used in equation 3 is the mortgage rate,  $RM$ , multiplied by a scale variable,  $CDA$ .  $CDA$  is exogenous in the model. It is constructed from a peak-to-peak interpolation of  $CD/POP$ .

The age variables are jointly significant, and all the other variables are significant. The estimate of  $\gamma$ , the coefficient of  $DELD(KD/POP)_{-1} - (CD/POP)_{-1}$ , is 0.118. This is the partial adjustment coefficient for  $CD$ . The estimate of  $\gamma\lambda$ , the coefficient of  $(KD/POP)_{-1}$ , is 0.0155, which gives an implied value of  $\lambda$ , the partial adjustment coefficient for  $KD^*$ , of 0.131.  $KD^*$  is thus estimated to adjust to  $KD^{**}$  at a rate of 0.131 per quarter.  $C2$ , the time varying constant term, is significant.

All the tests are passed except for adding the time trend, where the time trend is highly significant. When the time trend was added, some of the other coefficients were not sensible, again showing that estimates of equations using aggregate data can be fragile.

#### **Table 3.4: Equation 4. $IH$ , housing investment**

The same partial adjustment model is used for housing investment as was used above for durable expenditures, which adds  $DELH(KH/POP)_{-1} - (IHH/POP)_{-1}$ , and  $(KH/POP)_{-1}$  to the housing investment equation.  $KH$  is the stock of housing, and  $DELH$  is the depreciation rate of the stock. The wealth variable used in equation 4 is housing wealth, not total wealth. The financial wealth part of total wealth was not significant. It also does not include the age variables because they only had a p-value of 0.014. The equation is estimated under the

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<sup>1</sup>Note in Table 3.3 that  $CD$  is divided by  $POP$  and  $CD_{-1}$  and  $KD_{-1}$  are divided by  $POP_{-1}$ , where  $POP$  is population. If equations (1)–(4) are defined in per capita terms, where the current values are divided by  $POP$  and the lagged values are divided by  $POP_{-1}$ , then the present per capita treatment of equation (4) follows. The only problem with this is that the definition used to justify equation (2) does not hold if the lagged stock is divided by  $POP_{-1}$ . All variables must be divided by the same population variable for the definition to hold. This is, however, a minor problem, and it has been ignored here. The same holds for equation 4.

assumption of a second order autoregressive process for the error term. The interest rate used in equation 4,  $RM_{-1}$ , is multiplied by a scale variable,  $IHA$ .  $IHA$  is exogenous in the model. It is constructed from a peak-to-peak interpolation of  $IH/POP$ .

All the variables are significant in equation 4 except for  $C2$ . The lagged values are not significant, nor is the time trend. The overid test has a p-value of 0.018. The estimate of  $\gamma$ , the partial adjustment coefficient for  $IH$ , is 0.441. The estimate of  $\gamma\lambda$  is 0.0584, which gives an implied value of  $\lambda$ , the partial adjustment coefficient for  $KH^*$ , of 0.132. The estimates of  $\lambda$  are thus essential the same for  $CD$  and  $IH$ , but the estimate of  $\gamma$  is much larger for  $IH$ .

**Table 3.5: Equation 5.  $KK$ , stock of capital**

Equation 5 explains the stock of capital,  $KK$ . Given  $KK$ , non-residential fixed investment,  $IK$ , is determined by identity I-12:

$$IK = KK - (1 - DELK)KK_{-1}, \quad (I - 12)$$

where  $DELK$  is the depreciation rate. Equation 5 can be considered to be an “investment” equation, since  $IK$  is determined once  $KK$  is.

The estimated equation for  $KK$  is based on the following two equations:

$$\begin{aligned} \log(KK^*/KK_{-1}) = & \alpha_0 \log(KK_{-1}/KKMIN_{-1}) + \alpha_1 \Delta \log Y \\ & + \alpha_2 \Delta \log Y_{-1} + \alpha_3 \Delta \log Y_{-2} + \alpha_4 \Delta \log Y_{-3} \\ & + \alpha_5 \Delta \log Y_{-4} + \alpha_6 r, \end{aligned} \quad (6)$$

$$\begin{aligned} \log(KK/KK_{-1}) - \log(KK_{-1}/KK_{-2}) = & \lambda [\log(KK^*/KK_{-1}) - \\ & - \log(KK_{-1}/KK_{-2})] + \epsilon, \end{aligned} \quad (7)$$

where  $r$  is some measure of the cost of capital.  $KKMIN$ , under the assumption of a short-run putty-clay technology, is an estimate of the minimum amount of capital required to produce the current level of output,  $Y$ .  $KK_{-1}/KKMIN_{-1}$  is thus the ratio of the actual capital stock on hand at the end of the previous period to the

minimum required to produce the output of that period.  $\log(KK_{-1}/KKMIN_{-1})$  will be referred to as the amount of “excess capital” on hand.

$KK^*$  in equation (6) is the value of the capital stock the firm would desire to have on hand in the current period if there were no costs of changing the capital stock. The desired change,  $\log(KK^*/KK_{-1})$ , depends on 1) the amount of excess capital on hand, 2) five change-in-output terms, and 3) the cost of capital. The lagged output changes are meant to be proxies for expected future output changes. Other things equal, the firm desires to increase the capital stock if the output changes are positive. Equation (7) is a partial adjustment equation of the actual capital stock to the desired stock. It states that the actual percentage change in the capital stock is a fraction of the desired percentage change.

Combining equations (6) and (7) yields:

$$\begin{aligned} \Delta \log KK &= \lambda\alpha_0 \log(KK_{-1}/KKMIN_{-1}) + (1 - \lambda)\Delta \log KK_{-1} \\ &+ \lambda\alpha_1 \Delta \log Y + \lambda\alpha_2 \Delta \log Y_{-1} + \lambda\alpha_3 \Delta \log Y_{-2} \\ &+ \lambda\alpha_4 \Delta \log Y_{-3} + \lambda\alpha_5 \Delta \log Y_{-4} + \lambda\alpha_6 r + \epsilon. \end{aligned} \quad (8)$$

Equation 5 is the estimated version of equation (8).

The cost of capital variable in equation 5 is a function of stock price changes. It is the ratio of capital gains or losses on the financial assets of the household sector (mostly from corporate stocks) over three quarters to nominal potential output. This ratio is a measure of how well or poorly the stock market is doing. If the stock market is doing well, for example, the ratio is high, which should in general lower the cost of capital to firms. The variable is lagged two quarters.

The variables are significant in equation 5 except for some of the change in output variables, and the equation passes all the tests. The estimate of  $1 - \lambda$  is 0.909, and so the implied value of  $\lambda$  is 0.091. The capital stock is thus estimated to adjust 9.1 percent of the way to the desired stock each quarter. The estimate of  $\lambda\alpha_0$  is  $-0.00653$ , and so the implied value of  $\alpha_0$  is  $-0.072$ . This says that 7.2 percent of excess capital is eliminated each quarter, other things being equal.



**Table 3.6: Equation 6. Y production**

This equation is in effect an inventory investment equation. Given sales,  $X$ , from identity I-1 and given production,  $Y$ , from equation 6, inventory investment,  $IV$ , is from identity I-2  $Y - X$ . The theory behind equation 6 is that production is smoothed relative to sales because of various costs of adjustment, which include costs of changing employment, costs of changing the capital stock, and costs of having the stock of inventories deviate from some proportion of sales. If a firm were only interested in minimizing inventory costs, it would produce according to the following equation (assuming that sales for the current period are known):

$$Y = X + \gamma X - V_{-1}, \quad (9)$$

where  $Y$  is the level of production,  $X$  is the level of sales,  $V_{-1}$  is the stock of inventories at the end of the previous period, and  $\gamma$  is the inventory-sales ratio that minimizes inventory costs. Since by definition  $V - V_{-1} = Y - X$ , producing according to equation (9) would ensure that  $V = \gamma X$ . Because of the other adjustment costs, it is generally not optimal for a firm to produce according to equation (9), and so further specification is needed.

The estimated production equation is based on the following three assumptions:

$$\log V^* = \beta \log X, \quad (10)$$

$$\log Y^* = \log X + \alpha(\log V^* - \log V_{-1}), \quad (11)$$

$$\log Y - \log Y_{-1} = \lambda(\log Y^* - \log Y_{-1}) + \epsilon, \quad (12)$$

where \* denotes a desired value. (In the following discussion all variables are assumed to be in logs.) Equation (10) states that the desired stock of inventories is proportional to current sales. Equation (11) states that the desired level of production is equal to sales plus some fraction of the difference between the desired stock of inventories and the stock on hand at the end of the previous period. Equation (12) states that actual production partially adjusts to desired production each period.

Combining equations (10)–(12) yields

$$\log Y = (1 - \lambda) \log Y_{-1} + \lambda(1 + \alpha\beta) \log X - \lambda\alpha \log V_{-1} + \epsilon. \quad (13)$$

Equation 6 is the estimated version of equation (13). The equation is estimated under the assumption of a third order autoregressive process of the error term, and three dummy variables are added to account for the effects of a steel strike in the last half of 1959.

The estimate of  $1 - \lambda$  is 0.229, and so the implied value of  $\lambda$  is 0.771, which means that actual production adjusts 77.1 percent of the way to desired production in the current quarter. The estimate of  $\lambda\alpha$  is 0.206, and so the implied value of  $\alpha$  is 0.267. This means that (in logs) desired production is equal to sales plus 26.7 percent of the desired change in inventories. The estimate of  $\lambda(1 + \alpha\beta)$  is 0.939, and so the implied value of  $\beta$  is 0.816.

The  $C2$  variable is not included in the equation because it was not significant. As with equation 5, equation 6 passes all the tests, and so the results are fairly strong for both equations.

### **Table 3.7: Equation 7. *IM*, Imports**

The import equation is in per capita terms and is in log form. The explanatory variables include per capita expenditures on consumption and investment, the GDP deflator relative to the import price deflator, and four dummy variables to account for two dock strikes. The equation is estimated under the assumption of first order serial correlation of the error term.

The coefficient estimate of the relative price term is positive, as expected, since an increase in domestic prices relative to import prices should lead to a substitution toward imports. This equation is fragile in that it does not do well in the tests. The added lags are significant, as is the time trend. The last  $\chi^2$  test adds  $\log P$  to the equation, which is a test of the restriction that the coefficient of  $\log P$  is equal to the negative of the coefficient of  $\log PIM$ . The  $\log P$  variable is significant, and so

the restriction is rejected. The coefficient estimate of  $\log P$  is positive (not shown), so the GDP deflator is estimated to get more weight than the import price deflator.  $C2$  is significant.

**Table 3.8: Equation 8.  $P$ , GDP deflator**

The price equation is in log form. The price level is a function of the lagged price level, the price of imports, the GAP variable, and the time trend. The GAP variable is taken as a measure of demand pressure. The lagged price level is meant to pick up expectational effects, and the import price variable is meant to pick up cost effects.

An important feature of the price equation is that the price *level* is explained by the equation, not the price *change*. This treatment is contrary to the standard Phillips-curve treatment, where the price (or wage) change is explained by the equation. It is also contrary to the standard NAIRU specification, where the change in the change in the price level (i.e., the change in the inflation rate) is explained. The tests that I have run—Fair (2000) and *MM* (Section 3.13)—support the level over the change specification.

The time trend in the equation is meant to pick up any trend effects on the price level not captured by the other variables. Adding the time trend to an equation like 8 is similar to adding the constant term to an equation specified in terms of changes rather than levels. The constant term in the equation is assumed to be time varying, so  $C2$  is added. In addition, the coefficient of  $T$  is assumed to be time varying, with the same beginning and ending quarters as for  $C2$ . The additional variable added is  $C2 \times T$ . The equation is estimated under the assumption of first order serial correlation of the error term.

The main feature of equation 8 is that the price of imports has a positive effect on the price level and GAP has a negative effect. The coefficient estimate of  $\log P_{-1}$  is less than one, and the coefficient estimate of the time trend is time varying ( $T \times T$  is significant). The overid test fails, and for the lags test the p-value is 0.040.

Equation 8 is not as good as the price equation in the US model. In the US model there is a wage-price sector, and the wage rate is an explanatory variable in the price equation. Also, the price variable is the private non farm deflator, not the GDP deflator. The private non farm deflator is a better measure of prices set by the firm sector. And the unemployment rate is used instead of  $GAP$ , which dominates  $GAP$  when both are included in the equation. Equation 8 does, however, pick up the effects of cost shocks and demand pressure on the price level. The import price deflator,  $PIM$ , is highly significant and is an important force explaining the high inflation in the 1970s.

**Table 3.9: Equation 9.  $RS$ , three-month Treasury bill rate**

Equation 9 is the estimated Fed rule, where the target variable is taken to be the three-month Treasury bill rate,  $RS$ . The explanatory variables include the rate of inflation, current and lagged values of  $GAP$ , and lagged values of  $RS$ . Although not shown in the table, quarterly dummy variables are used for the quarters between 1979:4 and 1982:3, when the Fed announced that it was putting more weight on monetary aggregates. The estimation period ends in 2008:3, after which the zero lower bound was in effect.

Inflation is significant in the equation, and the  $GAP$  variables are jointly significant. The equation does very well in the tests. The leads tests are tests whether the Fed has rational expectations regarding future values of inflation and the  $GAP$ , and the results suggest no. The last two tests add the four-quarter and eight-quarter rates of inflation to see if they are proxies for expected future inflation, and again the results suggest no.

The estimated Fed rule in the US model is somewhat better than the equation in Table 3.9. In the US model the unemployment rate is used instead of  $GAP$ , which dominates  $GAP$  when both are included together. Also, the lagged growth of the money supply is used as an explanatory variable in the equation in the US model. Quarterly dummy variables are not used and instead the different behavior between

1979:4 and 1982:3 is handled by adding a variable that is the lagged growth of the money supply multiplied by a dummy variable that is 1 between 1979:4 and 1982:3 and 0 otherwise. This variable is highly significant and has a coefficient estimate much larger than the coefficient estimate of the lagged growth of the money supply in other periods. This is consistent with the Fed putting more weight on monetary aggregates in this period.

The estimated Fed rule in the US model is stable in the following sense. The hypothesis was tested that the equation's coefficients are the same before 1979:4 as they are after 1982:3 (though 2008:3). This was done using a Wald test, and the hypothesis of stability was not rejected.

Fed rules are usually called Taylor rules, after Taylor (1993), but they go back much further. The first example of an estimated interest rate rule is in Dewald and Johnson (1963), followed by Christian (1968). An equation like equation 9 was first estimated in Fair (1978).

**Table 3.10: Equation 10.  $RM$ , mortgage rate**

Equation 10 explains the mortgage rate,  $RM$ . It is based on the expectations theory of the term structure of interest rates states, where the expected future short-term rates are proxied by current and lagged values of  $RS$  and the lagged value of  $RM$ . The equation is estimated under the restriction that, say, a one point increase in  $RS$  leads eventually to a one point increase in  $RM$ . This restriction is tested in the first  $\chi^2$  test and is not rejected.

The equation does very well in the tests. The leads tests show that there is no evidence of rational expectations regarding future short-term rates. The last two tests add the four-quarter and eight-quarter rates of inflation, under the assumption that they might be proxies for expected future inflation. The variables are not significant.

## Comments

Some of the main conclusions from the estimation results are the following. The age variables are significant in 3 of the 4 household expenditure equations—evidence that the age distribution matters for aggregate spending. Total wealth is significant in the 3 consumption equations, and housing wealth is significant in the housing investment equation. Wealth appears to have important effects on aggregate spending. More will be said about this later. Significant time varying effects (variable  $C2$ ) occur in 4 of the equations—equations 1, 3, 5, and 7. In addition,  $C2 \times T$  is significant in the price equation 8.

Regarding the tests, only for the import equation 7 are the added lags significant at the 1 percent level. Adding the time trend leads to significant results in only 2 of 9 cases—equations 1 and 7. The overidentification tests fails in only 2 of 10 cases—equations 2 and 8. None of the 16 leads tests is significant at the 1 percent level. One is significant at the 5 percent level—Leads +1 for equation 1. The rational expectations hypothesis is strongly rejected using this test.

The three-month Treasury bill rate is significant in equation 1, has a t-statistic of -1.76 in equation 2, and the mortgage rate is significant in equations 3 and 4. Interest rates were not significant in the non-residential investment equation 5, where the cost of capital variable is a stock-market variable. Fed policy thus affects aggregate spending by affecting consumption and housing investment.

The interest rates in the four household expenditure equations are nominal interest rates. One can test for real interest rates by adding an estimate of the expected rate of inflation to the equations. If real interest rates matter, the coefficient on the inflation variable should be positive and equal to the negative of the coefficient on the nominal interest rate. To test for this, three proxies for expected future inflation were tried: the rate in the current quarter, the rate in the previous four-quarter period, and the rate in the previous eight-quarter period. Both current and one-quarter-lagged values were tried. This gave 6 regressions per equation—24 in all. In all but 6 regressions the inflation variable was not significant. In 5 it was

significant at the 5 percent level, and in 1 it was significant at the 1 percent level. In only 4 of the regressions was the coefficient estimate of the inflation variable positive (none significant). The nominal interest rate specification thus dominates the real interest rate specification. Why this is the case is an interesting question. One possibility is that the expected rate of inflation is simply a constant, so that the nominal interest rate specification is also the real interest rate specification (with the constant absorbed in the constant term of the equation). If, for example, agents think the monetary authority is targeting a fixed inflation rate, this might be a reason for the expected rate of inflation being constant. Whatever the case, the proxies for the expected rate of inflation used here are not significant.

How do the 10 equations compare to those in the US model? The specification issues regarding the price equation and the Fed rule have already been mentioned. One other issue concerns disposable income,  $YD\$$ . In US mini this is determined by equation I-5, but this is only an approximation to disposable income in NIPA (and in the US model). This then means that  $SH\$$  in equation I-8 and  $A1\$$  in equation I-15 are only approximations to the “true” values in the US model. Likewise for  $YD$ ,  $A1$ , and  $AA$ . The different values for disposable income and wealth obviously affect the estimates of equations 1–4 compared to those for the US model.

### **Equations for $CG\$$ and $PHOUSE$**

$CG\$$  and  $PHOUSE$  are exogenous in the model, but for some of the results below equations for them are needed. For the first equation, equation 11,  $CG\$/ (P \cdot YS)$  is regressed on a constant:

$$\frac{CG\$}{P \cdot YS} = 0.0978 \quad (5.95)$$

$$SE = 0.257, DW = 1.81, 1954.1 - 2014.4$$

11

For the second equation, equation 12,  $\Delta PHOUSE$  is regressed on a constant:

$$\Delta PHOUSE = 0.00263$$

$$(2.51)$$

$$SE = 0.0164, DW = 0.38, 1954.1 - 2014.4 \quad 12$$

To get a sense of these variables,  $A1/Y_S$  is plotted in Figure 1 and  $PHOUSE$  is plotted in Figure 2, each for the 1990:1–2014:4 period. In Figure 1 one can see the booms and busts in stock prices since 1995, and in Figure 2 one can see the huge increase and then decrease in housing prices. The reason for the low Durbin-Watson statistic in equation 12 is obvious from Figure 2. The change in  $PHOUSE$  is obviously not a random walk. This regression is simply used to get some measure of the variability of  $PHOUSE$  for use below. The change in stock prices as measured by  $\frac{CG\$}{P.Y_S}$  is closer to a random walk.

## 4 Flow Chart

A flow chart of US mini is presented in Figure 3. There are two exogenous fiscal policy variables, government spending,  $G$ , and the net tax rate,  $TAU$ , two foreign variables, exports,  $EX$ , and the price of imports,  $PIM$ , and two financial variables, capital gains and losses,  $CG\$$ , and housing prices,  $PHOUSE$ .

$G$  and  $EX$  directly affect aggregate demand.  $TAU$  affects demand by affecting disposable income, which affects consumption and housing investment. The financial variables affect wealth, which affects consumption and housing investment. In addition,  $CG\$$  affects non-residential investment through the cost of capital variable. The price of imports affects demand by affecting imports, and it affects the domestic price level,  $P$ , through equation 8—a cost shock variable. The change in  $P$  affects the short-term interest rate—the Fed rule—which affects the mortgage rate,  $RM$ , and both affect demand. Demand directly affects  $GAP$ , which in turn affects  $P$  and  $RS$ .



Figure 1  
A1/YS: 1990:1--2014:4



Figure 2  
PHOUSE: 1990:1--2014:4

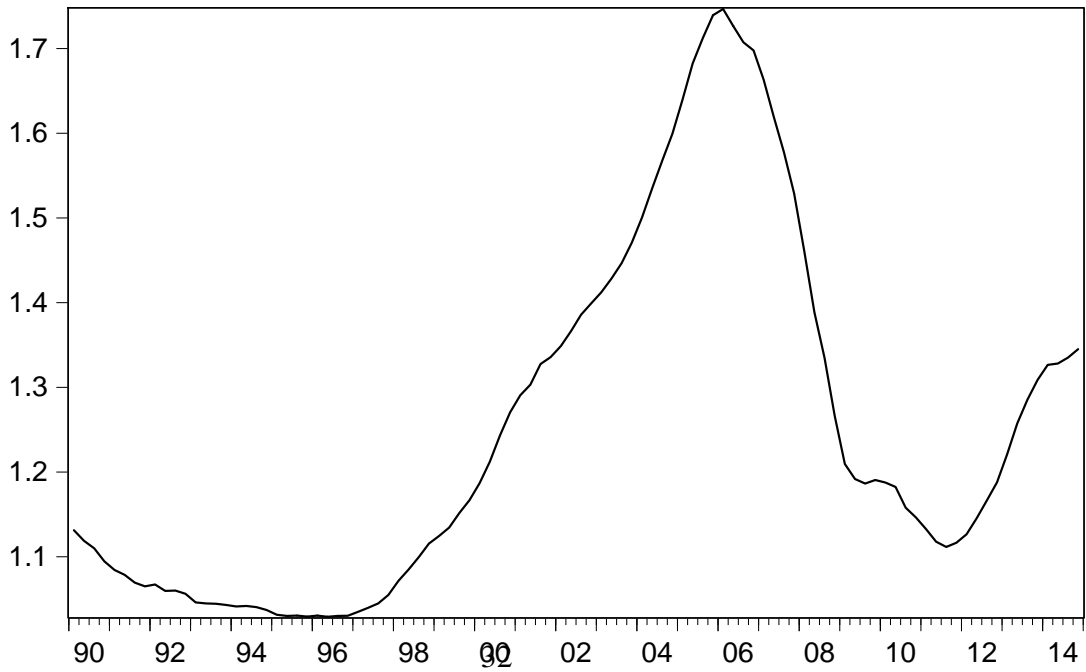
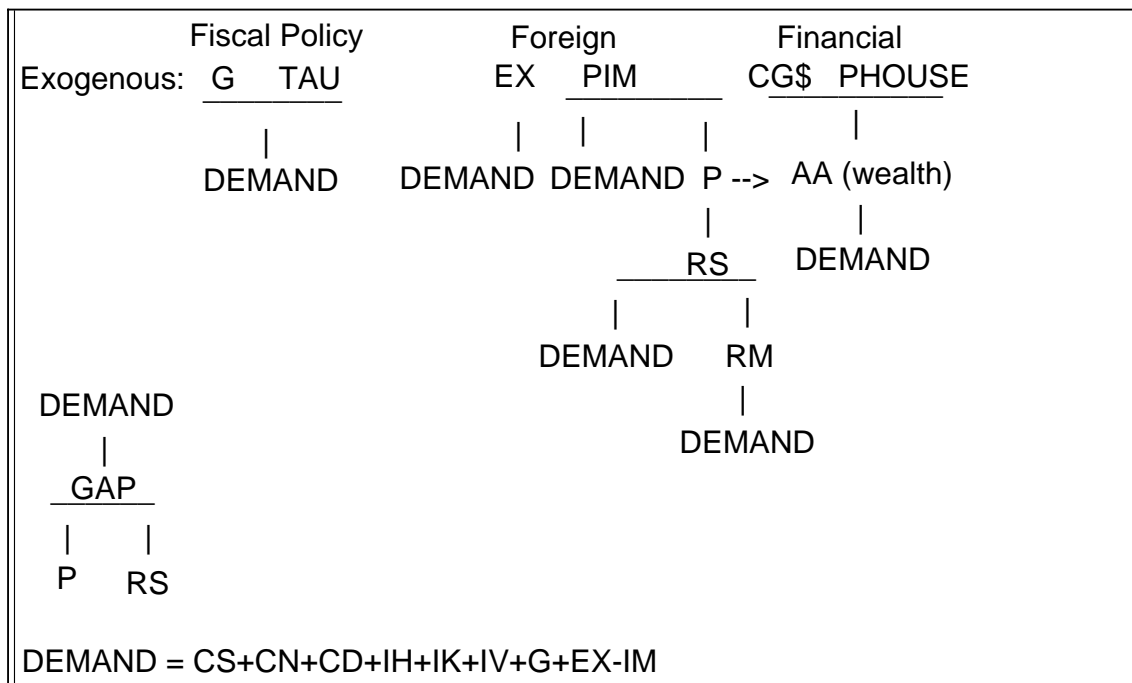


Figure 3  
Flow Chart of US Mini



Net taxes include transfer payments that are tied to the state of the economy, and so some components that are used in measuring  $TAU$  are endogenous. Less important, some components of  $G$  may be endogenous. The financial variables are not endogenous in the sense that I cannot find any macroeconomic variables that affect them—Fair (2014). In the 2SLS estimation,  $G$  and  $TAU$  have not been used as first stage regressors because of possible endogeneity issues.

## 5 Overall Fit of the Model

### Root Mean Squared Errors

In Table 1 there are 28 exogenous variables and 29 endogenous variables (10 estimated equations and 19 identities). There are also lagged endogenous variables in the model. Conditional on the exogenous variables, the model can be solved. For the 1954:1–2014:4 period there are 244 one-quarter-ahead predictions that can be made, 243 two-quarter-ahead predictions, and so on. Root mean squared errors are presented in Table 4 for  $Y$ ,  $P$ , and  $RS$  for the one-, four-, and eight-quarter-ahead predictions. Two sets of results are presented, one taking  $CG\$$  and  $PHOUSE$  as exogenous, and one using equations 11 and 12 for these.

As expected, the predictions for  $Y$  are sensitive to the treatment of  $CG\$$  and  $PHOUSE$ . The eight-quarter-ahead RMSE is 1.56 percent when they are known and 2.32 percent when they are not. For  $RS$  the respective numbers are 1.31 and 1.63 percentage points. For  $P$  the difference is not large: 1.29 percent versus 1.34 percent.

In an actual ex ante forecasting situation there is obviously more uncertainty than reflected in the RMSEs in Table 4. The exogenous variables must be forecast ahead of time, and the coefficients can only be estimated using observations up to the first quarter of the forecast. This can be adjusted for by using rolling regressions and by specifying autoregressive equations for each of the exogenous variables.

**Table 4**  
**Root Mean Squared Errors**

Qtr. Ahead	<i>CG\$ and PHOUSE</i> known			Equations 11 and 12		
	<i>Y</i>	<i>P</i>	<i>RS</i>	<i>Y</i>	<i>P</i>	<i>RS</i>
1	0.55	0.25	0.41	0.55	0.25	0.41
4	1.31	0.73	1.08	1.51	0.74	1.14
8	1.56	1.29	1.31	2.32	1.34	1.63

Errors are in percentage points.  
Sample period: 1954:1–2014:4.  
Number of observations: 244 for 1-quarter-ahead,  
241 for 4-quarter-ahead, 237 for 8-quarter-ahead.

Also, the RMSEs are not estimates of variances because for nonlinear models variances are not constant across time. This can be adjusted for by using stochastic simulation. These extensions are not pursued here, although stochastic simulation is done next.

### Forecastability Limits

The RMSEs in Table 4 show that the prediction errors are sensitive to the treatment of *CG\$* and *PHOUSE*. If these variables are not forecastable and if they affect, say, *Y*, this provides a bound on the ability to forecast *Y*. Stochastic simulation can be used to estimate this bound, which is done here.

Consider the model as including equations 11 and 12. Estimate the model for the 1954:1–2014:4 period and record the residuals. There are 244 vectors of 12 residuals. The stochastic simulation will draw from this set of vectors. Consider solving the model for the 2008:1–2009:4 period. Draw randomly with replacement 8 vectors from the 244 vectors and use these residuals in the solution of the model. Record the solutions. Do this, say, 10,000 times, which gives 10,000 solution values for each endogenous variable and quarter, from which variances can be

computed. Let  $\sigma^2$  denote the variance of a particular variable. For the work here the two variables used are the growth rate of  $Y$  over eight quarters at an annual rate and the growth rate of  $P$  over eight quarters at an annual rate.

Now do the same thing but excluding equations 11 and 12, which assumes that  $CG\$$  and  $PHOUSE$  are known. Let  $\sigma_a^2$  denote this variance. Then  $\sigma^2 - \sigma_a^2$  is an estimate of how much of the overall variance is due to not knowing  $CG\$$  and  $PHOUSE$ . Results are presented in Table 5 for three eight-quarter periods: 2008:1–2009:4, 2010:1–2011:4, and 2012:1–2013:4. They show that the variance of eight-quarter output growth is about 22 percent higher from the addition of equations 11 and 12. To the extent that  $CG\$$  and  $PHOUSE$  are unpredictable, this part of the variance can never be eliminated. The effects on the eight-quarter growth rate of  $P$  are much smaller. The increase in variance is about 2.5 percent.

When experiments like this are run for the MC model the increase in the variance of output growth is higher, around 33 percent. In the MC model there are more asset prices, namely, exchange rates and oil prices, which which leads to larger differences when these are used. The increase in the variance of growth rate of  $P$  is also larger, also about 33 percent. Exchange rates and oil prices affect the aggregate price level, which accounts for the much larger increase.

## 6 Policy Multipliers

Multiplier experiments are easy to perform. Solve the model without some change; solve the model with the change; and compare the two different solutions. The process is, however, more involved if standard errors of the multipliers are to be computed. This can be done as follows. Consider changing  $G$  for the period 2000:1–2004:4. Using the 244 vectors of residuals discussed in the previous section, draw with replacement 244 vectors, add the vectors of residuals to the model for the 1954:1–2014:4 period, and solve the model for this period. Take the solution values as the new data set and estimate the model using these data for the

**Table 5**  
**Estimated Variances**

Period	(1) $\sigma^2$	(2) $\sigma_a^2$	(3) $\sigma^2 - \sigma_a^2$	(4) $\frac{\sigma^2 - \sigma_a^2}{\sigma^2}$
<b>Output Growth over Eight Quarters, annual rate</b>				
2008:1–2009:4	1.042	0.791	0.251	0.241
2010:1–2011:4	1.053	0.833	0.220	0.209
2012:1–2013:4	1.022	0.816	0.206	0.202
Average				0.217
<b>Inflation over Eight Quarters, annual rate</b>				
2008:1–2009:4	0.529	0.515	0.014	0.026
2010:1–2011:4	0.532	0.519	0.013	0.024
2012:1–2013:4	0.529	0.518	0.011	0.021
Average				0.024

- $\sigma^2$  = total forecast-error variance.
- $\sigma_a^2$  = forecast-error variance, asset-price errors not used (equations 11 and 12).
- 10,000 trials each experiment.
- Same draws for each experiment.
- Historical errors between 1954:1 and 2014:4 drawn.
- Values are in percentage points.

1954:1–2014:4 period using 2SLS. Using this “new” model and the new data, solve the model for the 2000:1–2004:4 period. Record the solution values. Then solve it again using the change in  $G$  and record the solution values. Finally, record the difference between the solution values for the endogenous variables and quarters of interest. This is one trial. Do this, say, 1,000 times, where each trial begins with a draw of 244 vectors and reestimation of the model. This procedure takes into account the uncertainty from the coefficient estimates because the model is reestimated for each trial. From the 1,000 multipliers, averages and measures of dispersion can be computed.<sup>2</sup>

<sup>2</sup>The values presented below are as follows. Rank the 1,000 values of a given multiplier by size. Let  $m_r$  denote the value below which  $r$  percent of the values lie. The median,  $m_{.5}$ , is used for the

Again, this procedure does not require any distributional assumptions because the draws are from the historical errors.

### **Fiscal Policy Multipliers**

Results are presented in Table 6 for a change in  $G$  and for a change in  $G$  with the import equation dropped. Results are presented in Table 7 for a change in  $TAU$ . Values for  $Y$ ,  $P$ , and  $RS$  are presented.  $G$  was increased by 1 percent of the historical value of  $Y$  for each quarter, and  $TAU$  was decreased by 0.01 for each quarter. The solution period is 2000:1–2004:4, and the procedure discussed above was used to compute the standard errors. The number of trials was 1,000.

The multipliers peak after 3 or 4 quarters for output. After 4 quarters the multiplier for output is 1.66 for the model with the import equation and 1.94 without. This shows the importance of accounting for import demand. DSGE models that do not have a foreign sector are missing an important link. The multipliers for  $TAU$  in Table 7 are smaller. The multiplier for output after 4 quarters is 0.54. It is smaller for the standard introductory-economics reason. An increase in  $G$  adds directly to aggregate demand, whereas a decrease in  $TAU$  only indirectly affects demand by affecting disposable income of households. Some of the tax decrease is saved.

The standard errors are fairly small relative to the size of the multipliers. This is a common result for these kinds of models. Most of the uncertainty is from the coefficient estimates. For the trials the additive errors are the same for both the base solution and the solution with the policy variable changed, and so they tend to cancel out. (They exactly cancel out for a linear model.) Coefficient-estimate uncertainty is generally fairly modest, which leads to modest estimates of the standard errors.

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multiplier, and  $(m_{.8413} - m_{.1587})/2$  is used as the measure of dispersion. For a normal distribution this measure of dispersion is one standard error. This measure will be called a standard error.

**Table 6**  
**G Multipliers**

<b>qtr</b>	<b>Y</b>		<b>P</b>		<b>RS</b>	
<b>Imports Endogenous</b>						
1	1.19	(0.09)	0.06	(0.02)	0.40	(0.28)
2	1.68	(0.12)	0.15	(0.04)	0.87	(0.38)
3	1.73	(0.17)	0.23	(0.06)	1.08	(0.33)
4	1.66	(0.20)	0.31	(0.08)	1.10	(0.27)
8	1.23	(0.24)	0.49	(0.13)	0.99	(0.28)
12	1.01	(0.24)	0.57	(0.17)	0.91	(0.31)
16	1.02	(0.24)	0.61	(0.19)	0.87	(0.35)
<b>Imports Exogenous</b>						
1	1.23	(0.12)	0.07	(0.02)	0.39	(0.24)
2	1.79	(0.17)	0.16	(0.04)	0.87	(0.33)
3	1.93	(0.23)	0.25	(0.06)	1.12	(0.30)
4	1.94	(0.30)	0.33	(0.08)	1.17	(0.27)
8	1.52	(0.39)	0.58	(0.15)	1.18	(0.26)
12	1.15	(0.38)	0.67	(0.20)	1.05	(0.24)
16	1.02	(0.38)	0.71	(0.23)	0.94	(0.30)

- $G$  increased by 1 percent of historic values of  $Y$ .
- Values are in percentage points.
- Estimated standard errors in parentheses.
- Number of trials = 1,000.
- Simulation period: 2000:1–2003:4.

As discussed in the Introduction, the multipliers computed here use the GDP identity and take into account all the restrictions on the reduced form coefficients. Much more information is being used than is used from the reduced-form approach. It is thus not surprising that the range of multiplier estimates from the reduced-form approach is much larger than would be expected from the estimates of the standard errors here. Using the reduced-form approach Hall (2009) estimates the government spending to be 0.55 after four quarters and Barro and Redlick (2011) estimate it to be 0.44. Neither of these seem sensible using the current approach. The initial spending injection is 1.0, and it would take odd expenditure equations



**Table 7**  
*TAU Multipliers*

<b>qtr</b>	<b>Y</b>		<b>P</b>		<b>RS</b>	
1	0.19	(0.04)	0.01	(0.00)	0.06	(0.05)
2	0.37	(0.07)	0.03	(0.01)	0.17	(0.09)
3	0.49	(0.09)	0.05	(0.02)	0.26	(0.11)
4	0.54	(0.11)	0.08	(0.02)	0.32	(0.11)
8	0.47	(0.12)	0.15	(0.05)	0.35	(0.12)
12	0.38	(0.11)	0.19	(0.06)	0.31	(0.12)
16	0.37	(0.10)	0.21	(0.07)	0.29	(0.14)

- *TAU* decreased by 0.01 from its historic values.
- Values are in percentage points.
- Estimated standard errors in parentheses.
- Number of trials = 1,000.
- Simulation period: 2000:1–2003:4.

to have the multiplier be about half of this after four quarters. Likewise, the tax multiplier of 3.08 after 10 quarters of Romer and Romer (2010) does not seem sensible. Again, this would require odd expenditure equations. These seemingly unrealistic estimates are likely due to having ignored theoretical restrictions on the reduced form coefficients.

### **Monetary Policy Multipliers**

Monetary policy is endogenous, so equation 9 has to be dropped to perform any experiments. For the results in Table 8 equation 9 was dropped and *RS* was increased by 1 percentage point. In this case the multiplier for output peaks at about 8 quarters, where it is -0.75. After 4 quarters the multiplier is -0.46. Remember that changes in *RS* directly affect consumption of services and non durables and indirectly affect consumption of durables and housing investment through its effect on the mortgage rate, *RM*. These effects are clearly important, but they are not close to being large enough for monetary policy to be able to come close to

**Table 8**  
*RS Multipliers*

qtr	Y		P	
1	-0.07	(0.02)	-0.00	(0.00)
2	-0.19	(0.04)	-0.01	(0.00)
3	-0.33	(0.06)	-0.03	(0.01)
4	-0.46	(0.08)	-0.05	(0.01)
8	-0.75	(0.12)	-0.16	(0.04)
12	-0.76	(0.13)	-0.27	(0.07)
16	-0.69	(0.12)	-0.33	(0.09)

- Equation 9 dropped.
- *RS* increased by 1.0 from its historic values.
- Values are in percentage points.
- Estimated standard errors in parentheses.
- Number of trials = 1,000.
- Simulation period: 2000:1–2003:4.

controlling the economy. DSGE models in which monetary policy is able to do this seem far from what is estimated from the aggregate data.

In the model there is no effect of monetary policy on asset prices. If, say, quantitative easing affects both interest rates and asset prices, the results in Table 8 have underestimated the effects of monetary policy since asset prices affect aggregate demand.

## 7 Effects of a Price Shock

A typical feature of DSGE models is that a positive price shock with the nominal interest rate held constant is expansionary (usually explosive) because of the fall in the real interest rate. This is not supported by the aggregate data. First, as noted in Section 3, the data support the use of nominal interest rates rather than real interest

rates in expenditure equations. So with a price shock there is no real interest rate effect on expenditures. Second, an increase in prices lowers real wealth, which has a negative effect on aggregate demand. This can be seen in Table 9, where the price equation was dropped and  $P$  was increased by 1 percent. After 8 quarters output is down by 0.26 percent. This is from the fall in real financial wealth,  $A1$ . From equation I-16  $A1$  is equal to  $A1\$/P$ , and in this experiment  $A1\%$  does not change much.  $A1$  thus decreases by roughly one percent.

This negative price effect is more pronounced in the US model, which has a wage-price sector. Nominal wages lag nominal prices, and so a positive price shock leads to an initial fall in real wages and thus real income, which has a contractionary effect on expenditures. This channel is missing in US mini, but it is still the case that a price shock is contractionary because of the real wealth effect.

## 8 Wealth Effects

How large is the estimated wealth effect in the model? It is possible to isolate the wealth effect in the four household expenditure equations (equations 1–4) by taking  $YD$ ,  $RS$ ,  $RM$ ,  $A1$ , and  $A2$  to be exogenous. Then  $A1$  or  $A2$  can be changed to see the effect on total household expenditures:  $CS + CN + CD + IH$ . Table 10 presents results of an increase in  $A1$  of \$1,000 billion and an increase in  $A2$  of \$1,000 billion. After 4 years expenditures are up about \$60 billion in the first case and about \$70 billion in the second. The increase is larger in the second case because  $IH$  is not directly affected by  $A1$ , whereas  $A2$  affects all four. The estimated long-run effect is thus about 6 or 7 cents on the dollar per year from a sustained increase in wealth of a dollar, other things being equal. These effects are somewhat larger than those computed using the US model— $MM$  (Section 5.7.4)—which are about 4 and 6 cents. They may thus be a little large, possibly because of the use of poorer data on disposable income.

**Table 9**  
**Effects of a Positive Price Shock**  
*RS* Exogenous

qtr		Y
1	-0.02	(0.00)
2	-0.07	(0.01)
3	-0.12	(0.02)
4	-0.16	(0.02)
8	-0.26	(0.03)
12	-0.27	(0.03)
16	-0.24	(0.03)

- Price equation 8 dropped.
- $P$  increased by 1 percent from its historic values.
- Equation 9 dropped.
- Values are in percentage points.
- Estimated standard errors in parentheses.
- Number of trials = 1,000.
- Simulation period: 2000:1–2003:4.

**Table 10**  
**Effects on  $CS + CN + CD + IH$**   
**of a Change in Real Wealth**

Year	A1 Changed	A2 Changed
1	12.2	36.3
2	37.0	72.4
3	51.3	73.3
4	59.6	71.8
5	62.5	69.5
6	62.2	67.1

- Units are billions of 2009 dollars.
- $A1$  and  $A2$  each changed by 1,000.
- $YD$ ,  $RS$ ,  $RM$ ,  $A1$  and  $A2$  taken to be exogenous.

## 9 Analyzing the 2008–2009 Recession

$A1/Y\$$  is plotted in Figure 1. From 2007:4 to 2009:4  $A1$  fell by \$5.7 trillion.  $PHOUSE$  is plotted in Figure 2.  $A2$ , which is  $PHOUSE \cdot KH$ , fell by \$5.1 trillion from 2007:4 to 2009:4. The effects of this huge fall in wealth is examined in this section. How much of the 2008–2009 recession can be explained by this fall in wealth? The period examined is 2008:1–2011:4.

The experiment is as follows. First, the values of  $PHOUSE$  for the 2008:1–2011:4 period were taken to be such that the value of  $A2$  was equal to its actual value in 2007:4 using the historical values of  $KH$ . (Remember that  $A2 = PHOUSE \cdot KH$ .) This means that the huge fall in housing prices shown in Figure 2 is avoided. Second, the values of  $CG\$$  for the 2008:1–2011:4 period were taken to be such that the value of  $A1$  was equal to its actual value in 2007:4 using the historical values of  $SH\$$  and  $P$ . This means that the huge fall in financial wealth in Figure 1 is avoided. Third, the Fed rule, equation 9, was dropped and  $RS$  was taken to be equal to its historical values. If the equation were retained, the Fed would be predicted to increase  $RS$  from its historical values in the more robust economy. For simplicity it seemed best not to compound the effects of wealth changes and interest rate changes, and so  $RS$  was taken to be exogenous. The model is solved with and without the changes, and the differences in the solution values are recorded. The same procedure was followed here as was followed for the multiplier experiments to compute the standard errors. The number of trials was 1,000, and reestimation was done for each trial. Results are presented in Table 11.

The GAP was 1.3 percent in 2008:1, and it reached a peak of 7.9 percent in 2009:3. Most of this is eliminated in the experiment. The decrease in the GAP in 2009:3 is 6.8 percentage points, leaving a gap of only 1.1 percent. In 2010:4 the decrease is 5.6 percentage points, leaving a gap of 1.6 percent. The estimated standard errors peak at about 0.8 percentage points. The price level is higher as expected since GAP is smaller. By 2010:4  $P$  is higher by 2.2 percent.

**Table 11**  
**Effects on *GAP* and *P***  
**No Fall in Real Wealth from 2007:4 On**

Qtr.	<i>GAP</i>				<i>P</i>	
	Act.	Pred.	Dif.	SE	Dif.	SE
2008.1	1.3	1.3	-0.0	0.00	0.0	0.00
2008.2	1.4	1.0	-0.4	0.05	0.0	0.00
2008.3	2.4	1.2	-1.2	0.14	0.1	0.02
2008.4	5.1	2.8	-2.3	0.25	0.2	0.04
2009.1	6.9	3.1	-3.8	0.40	0.4	0.09
2009.2	7.6	2.2	-5.4	0.57	0.7	0.14
2009.3	7.9	1.1	-6.8	0.71	1.0	0.21
2009.4	7.5	0.1	-7.4	0.79	1.3	0.28
2010.1	7.7	0.4	-7.3	0.80	1.6	0.35
2010.2	7.3	0.5	-6.8	0.77	1.9	0.41
2010.3	7.3	1.1	-6.2	0.74	2.1	0.45
2010.4	7.2	1.6	-5.6	0.68	2.2	0.49
2011.1	8.2	3.2	-5.0	0.63	2.4	0.52
2011.2	8.0	3.8	-4.2	0.59	2.4	0.54
2011.3	8.4	4.9	-3.5	0.54	2.4	0.55
2011.4	7.9	4.9	-3.0	0.53	2.4	0.55

- Equation 9 dropped.
- *RS* taken to be exogenous.
- Values are in percentage points.
- SE is the standard error of the difference.

Much of the 2008-2009 recession is thus attributed to the fall in real wealth. Had real financial wealth and real housing wealth remained the same as their values in 2007:4, there would have been no recession. A similar experiment was performed in Fair (2014) using the entire MC model. The gap measure used was the unemployment rate, and the results show that much, but not all, of the increase in unemployment would not have taken place had real wealth not fallen. The present results using US mini may thus be a little too strong—estimated wealth effects that are a little large—but the basic conclusion is similar.

## 10 Conclusion

Any model in the CC tradition is judged by the quality of its estimated equations. Can better versions of the equations in Tables 3.1–3.10 be found? One key issue is trying to account for possible structural change. The use of  $C2$  may account for some change, but probably not all. The problem with doing more is the smoothness of the aggregate data—there are a limited number of recessions and high inflation periods. But possibly more could be done. However, focusing on just one episode, like the 2008–2009 recession and its aftermath, and arguing that this episode is different is problematic, since it is just one observation. The 2008–2009 recession may be different in some ways, but the aggregate data are unlikely to show it. The results in the previous section suggest that most of the recession was just standard wealth effects at work.

Although US mini is not meant to replace the US or MC models, it is useful for examining the limits of aggregate data. What the aggregate data do seem to show are the following. Age effects on household expenditures can be picked up. Also, wealth effects can be picked up. They are large and can account for much of the 2008–2009 recession. The existence of wealth effects provides an upper bound on forecasting accuracy. Nominal interest rates dominate real interest rates in household expenditure equations. Gap effects and inflation effects can be picked up in an estimated Fed rule. The Fed can affect demand by changing interest rates, but by no means does it have complete control over output. There is no evidence of rational expectations using a test that adds future values to the estimated equations. Aggregate government spending and tax multipliers can be estimated with reasonable accuracy. They are smaller than would otherwise be the case because imports are endogenous. Positive price shocks are contractionary because real wealth falls and there is no offset from falling real interest rates because nominal interest rates dominate.

It was mentioned in the Introduction that multipliers computed from models like US mini may be useful for dynamic scoring. The estimated standard errors of the multipliers are fairly small. There is a caveat, however, in that it is probably not possible with aggregate data to distinguish among different categories of government purchases of goods and services or of different categories of government transfer payments or different types of taxes. A change in  $G$  affects total output, which affects disposable income, which in turn affects household expenditures. A change in  $TAU$  directly affects disposable income, which affects expenditures. Everything feeds through aggregate disposable income. Given the smoothness of the aggregate data, it is unlikely that different categories of disposable income can be used in the expenditure equations to estimate separate effects.<sup>3</sup>

Dynamic scoring could thus be done as follows. Take a proposed bill and figure out first how the bill translates into changes in  $G$  and  $TAU$  for each quarter of interest. Then the model can be solved with these changes to see the effects on the economy. For a model like the MC model a few more aggregate policy variables (other than  $G$  and  $TAU$ ) are relevant, but only a few. The main constraint is the limited number of explanatory variables that can be added to aggregate expenditure equations.

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<sup>3</sup>In the US model aggregate tax rates affect labor supply, so a little more disaggregation can be done when there is a labor market.



**Table A**  
**First Stage Regressors**

Eq.	First Stage Regressors
1	$C2, C, AG1, AG2, AG3, \log(CS/POP)_{-1}, \log(YD/POP)_{-1}, RS_{-1}, \log(AA/POP)_{-1}, C2_{-1}, AG1_{-1}, AG2_{-1}, AG3_{-1}, \log(CS/POP)_{-2}, \log(AA/POP)_{-2}, \log(EX/POP)_{-1}$
2	$C2, C, AG1, AG2, AG3, \log(CN/POP)_{-1}, \Delta \log(CN/POP)_{-1}, \log(AA/POP)_{-1}, \log(YD/POP)_{-1}, RS_{-1}, \log(EX/POP)_{-1}$
3	$C2, C, AG1, AG2, AG3, (KD/POP)_{-1}, DELD(KD/POP)_{-1} - (CD/POP)_{-1}, (YD/POP)_{-1}, (RM \cdot CDA)_{-1}, (AA/POP)_{-1}, (EX/POP)_{-1}$
4	$C2, C, (KH/POP)_{-1}, (YD/POP)_{-1}, RM_{-1}IHA, C2_{-1}, C2_{-2}, (KH/POP)_{-2}, (KH/POP)_{-3}, YD/POP)_{-2}, (RM_{-1}IHA)_{-1}, (RM_{-1}IHA)_{-2}, DELH(KH/POP)_{-1} - (IH/POP)_{-1}, DELH_{-1}(KH/POP)_{-2} - (IH/POP)_{-2}, DELH_{-2}(KH/POP)_{-3} - (IH/POP)_{-3}, \Delta(IH/POP)_{-1}, \Delta(IH/POP)_{-2}, (A2/POP)_{-1}, (A2/POP)_{-2}, (A2/POP)_{-3}, (EX/POP)_{-1}, GAP_{-1}$
5	$C2, C, \log KK_{-1}, \log KK_{-2}, \log Y_{-1}, \log Y_{-2}, \log Y_{-3}, \log Y_{-4}, \log Y_{-5}, \log(KK/KKMIN)_{-1}, (CG\$_{-2} + CG\$_{-3} + CG\$_{-4})/(P_{-2}YS_{-2} + P_{-3}YS_{-3} + P_{-4}YS_{-4}), \log(EX/POP)_{-1}, GAP_{-1}$
6	$C, \log Y_{-1}, \log V_{-1}, D593, D594, D601, \log Y_{-2}, \log Y_{-3}, \log Y_{-4}, \log V_{-2}, \log V_{-3}, \log V_{-4}, D601_{-1}, D601_{-2}, D601_{-3}, \log(EX/POP)_{-1}$
7	$C2, C, \log(IM/POP)_{-1}, \log[(CS+CN+CD+IH+IK)/POP]_{-1}, \log(P/PIM)_{-1}, D691, D692, D714, D721, C2_{-1}, D692_{-1}, D721_{-1}, \log(IM/POP)_{-2}, \log(EX/POP)_{-1}$
8	$C2, C, C2 \cdot T, T, \log P_{-1}, GAP_{-1}, C2_{-1}, C2_{-1} \cdot T_{-1}, \log P_{-2}, GAP_{-2}, \log(EX/POP)_{-1}, \log PIM_{-1}$
9	$C, RS_{-1}, PCP_{-1}, GAP_{-1}, GAP_{-2}, GAP_{-3}, RS_{-2}, RS_{-3}, RS_{-4}, \log PIM_{-1}, \log(EX/POP)_{-1}, \text{plus 12 quarterly dummy variables for 1979:4-1982:3}$
10	$C, RM_{-1}, RS_{-1}, RS_{-2}, \log(EX/POP)_{-1}, PCP_{-1}, GAP_{-1}, GAP_{-2}, RS_{-3}, RM_{-2}$

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