COMPETITIVE SCREENING AND MARKET SEGMENTATION

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ABSTRACT*

We characterize competitive equilibrium in markets (financial etc.) where price taking Bayesian decision makers *screen* to accept or reject applicants. Unlike signaling models, equilibrium fails to resolve imperfect information. In classical statistics terminology, some qualified applicants are rejected (type I error) and some unqualified applicants are accepted (type II error). We report three new results: i. optimal firm behavior is deduced to be a Bayesian variant of the Neyman-Pearson theorem; ii. competitive equilibrium entails screening if and only if (net of screening costs) the cost of type II errors exceed the cost of type I errors, i.e. contrary to signaling (where buyers identify more qualified applicants who self screen to differentiate themselves e.g. Stiglitz 1975), price taking firms screen to avoid lower quality sellers; iii. equilibrium groups the least attractive applicants into a single high risk assignment pool.

Depending on costs of screening, the unique equilibrium may involve complete pooling (all applicants trade at one price) or partial separation (there are m separate pools with successive pools supported by a single (rising) price and a subset of agents of different screen levels trading at that price). A screening equilibrium has $m \geq 2$ and the mth secondary market entails no screening, as the most adversely selected agents are assigned to the high risk pool.

Screening induces market segmentation. Invariably secondary markets contain individuals who with better or different screening mechanisms could be accepted in the primary market. What roles traits such as ethnicity, gender, and race might assume in such decision making is relegated to subsequent research to explore the statistical theory of discrimination.

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Under what circumstances do firms invest resources to purchase information about potential transaction partners, i.e. when do competitive firms screen? Signaling models suggest that competitive firms confronting imperfect information about traders screen to identify and capture gains from trade with higher quality agents.[1] But that intuition can be misleading. In the setting examined here, when the gains from trade with higher quality agents are sufficiently high, firms do not screen! Competitive firms screen only to avoid relatively costly low quality agents. This paper examines the properties of competitive equilibrium in markets where price taking Bayesian decision makers use a *screening mechanism* to accept or reject applicants. We consider market settings where signaling behavior is inappropriate. Consumers seeking credit have every intention of repaying their loans and retaining the properties (homes, autos, consumer durables) purchased with their loans. Each purchaser of insurance is optimistic about her prospects of not presenting an insurance claim. In general, applicants are not (in any statistical sense) informed with respect to their relative merit vis a vis other applicants.[2]

Firms offer the existing market price to applicants who pass the screen and reject those who do not. Market segmentation is a natural outcome of the screening process. Because rejected applicants are not permitted to participate in the primary market, their presence is the basis for the formation of secondary markets. Screening equilibria are generally composed of $m > 1$ segmented markets where applicants face increasingly adverse terms of trade as they descend down the hierarchy of segmented markets. The increasingly adverse terms of trade occur in secondary markets because prices are determined by the characteristics of applicants whose qualifications are characterized by a distribution truncated from above by the loss of previously accepted applicants who screen more favorably and hence trade in more advantageous markets.

[1] The discussion in Stiglitz 1975, pp. 286 and 290 suggests this intuition.
[2] There exists no systematic asymmetry of information that could serve as foundation for a self-selected signal that is adequately correlated with applicants' true relative quality.

The interpretation of the segmented markets depends on the institutional setting. For example, in the setting examined here (financial markets) the same firm may operate in different segments of the market and applicants may simply be thought to be offered one of the firm's loan packages or insurance policies. In other institutional settings where up or down decisions are common practices such as education and job markets it is more appropriate to see institutions operating in only one of the segmented markets. Acceptance decisions by schools and employers are important examples of the competitive screening processes examined in this paper and many of the market properties deduced for financial markets apply to them as well. However, school and job markets present complexities that are not explored in this paper.

What we shall call binary choice competitive screening is quite appropriate for investigating economic phenomena such as statistical discrimination. One reason for this suitability is that binary choice screening decisions occur within an economic context that does not resolve the imperfect information inherent to the market transaction.[3] Even in equilibrium, firms' accept and reject decisions are inherently prone to error. In the terminology of classical statistics, some qualified applicants are rejected (type 1 error) and some unqualified applicants are accepted (type 2 error). This implies that invariably secondary markets will contain individuals who if firms had used better or simply different screening mechanisms could have been accepted in the primary market. What role traits such as ethnicity, gender, and race assume in such decision-making and the allocation of resources becomes all the more important in light of a salient implication of binary choice screening. We show (Remark 1) that competitive equilibrium entails screening only if the costs of making type II errors are high relative to the costs of making type I errors. In effect, competitive firms screen to identify and avoid lower quality sellers, a motivation rife with discriminatory dangers when different social groups are subject to

[3]Allocation decisions are not predicated on a screening process whose equilibrium prices generate perfect information as in common models of signaling (cf. Spence 1973 and Stiglitz 1975).

stereotyping. Subsequent research will examine the implications of such screening for understanding market phenomena such as statistical discrimination and so called predatory lending practices in markets where a wide degree of segmentation is actually observed. This paper characterizes some important properties of competitive equilibrium with binary choice screening.

The equilibrium concept used is the Nash equilibrium. It is assumed that firms maximize expected profits without regard to competitors' strategies and that there is free entry of firms so that only zero profit equilibria are considered. Several interesting properties of such equilibria are demonstrated. First, under similar conditions where Nash equilibrium often fails to exist Rothschild-Stiglitz (1976), Wilson (1977), Jaynes (1979), and Riley (1979) the binary choice screening model studied has a Nash equilibrium under standard assumptions of economic competition. We show a unique competitive equilibrium exists. Furthermore, depending on the costs of screening relative to the value of its information, that equilibrium may involve complete pooling (all heterogeneous applicants trade at one price) or partial separation (there are m separate pools with each pool supported by a price and a subset of agents of different screen levels trading at that price). The least desirable secondary market entails no screening, as the most adversely selected agents are assigned to the high-risk pool.

1. Screens and Screening Mechanisms

In a variety of market settings, firms sort products and services into the binary categories qualified and unqualified. Denote qualified and unqualified applicants by $q_1 = 1$ and $q_0 = 0$ respectively. It is assumed firms cannot determine whether an applicant is qualified or unqualified until after she is accepted. However, for a cost a firm may observe a numerical indicator x $[0 \le x \le 1]$ that is positively correlated with applicant qualifications and therefore may be used to predict an applicant's subsequent value to the firm. A screen assigns to each applicant just such a numerical indicator.

Generally, a screen is a function $h: R^N \to R$ where z an element of R^N is a vector of observable characteristics (e.g. debt, late payments, past insurance claims, age, gender, income, etc). An example of a screen is a regression model whose independent variables may be termed the screening criteria and whose dependent variable is used to rank applicants. Less formally (and more commonly used) a screen is any scalar-valued ranking of applicants that is based on an objective or subjective probability model.

Associated with a screen is a joint probability density $f(x,q)$ that gives the probability that an applicant of quality q is screened x. In terms of the conditional probabilities computable from the joint probability density, a screen is associated with a probability density $f(x|q_i)$ that gives the conditional probability that an applicant of true qualification i will be screened x. A screening process is a screen h(.) mapping applicant characteristics to a scalar x and an associated conditional probability function $f(x|q_i)$ indicating the probability an applicant of quality q_i is screened x. Hereafter, the respective conditional probability densities will be denoted $f_i(x)$ and their cumulative distributions by $F_i(x)$. We refer to such probability functions as screening probabilities and we restrict the screening probabilities considered to those belonging to a subset of the continuously differentiable probability densities defined on the unit interval.

For any screen screening process and $f_0(x) > 0$, consider the likelihood ratio $R(x) = \frac{f_1(x)}{f_0(x)}$. R(x) is the relative likelihood that an applicant screened x is qualified. We say a screen is *efficient* if its likelihood ratio satisfies the monotone likelihood ratio condition (see Milgrom, 1981),

$$R'(x) = \frac{f_0(x) \cdot f_1'(x) - f_1(x) \cdot f_0'(x)}{f_0(x)^2} > 0 \quad \forall x.$$

Eliminating the denominator of $R'(x)$ shows efficient screens satisfy condition

$$\text{P1.} \qquad \frac{f_1'(x)}{f_1(x)} > \frac{f_0'(x)}{f_0(x)} \quad \forall x.$$

An increase in x increases the proportion of qualified applicants more than it increases the proportion of unqualified applicants. Therefore, any increase in the observed screening value increases the relative likelihood that an applicant is qualified. If this condition were not satisfied, all kinds of perverse results could occur and the underlying screening process would not be a very good one. We only consider efficient screens.

A *screening mechanism* is composed of a screen and a decision criterion that accepts or rejects applicants on the basis of the indicator x assigned to the applicant by the screen. A decision criterion is a discrete function $d(\cdot)$ that maps the unit interval, the space of possible screen values, to the binary set q of possible qualification values so that 0 and 1 also signify reject and accept. Thus, the value d(x) (equal 0 or 1) indicates whether an applicant screened x is accepted or rejected. We exploit certain properties of efficient screens. The most important of these properties is that a rational decision maker who has chosen to accept applicants who are screened at some indicator x_1 will accept all applicants for whom $x > x_1$. Thus, the screening mechanism will have the property that there exists an x^* such that the decision criterion $d(\cdot)$ satisfies:

$$d(x) = \begin{matrix} 1 & x \geq x^* \\ 0 & x < x^*. \end{matrix}$$

Another important property of efficient screens is that the conditional probability densities they generate can cross at most one point. Since R(x) is strictly increasing, once the densities cross at some screen value x' (referred to as the switch point), $f_1 > f_0$ must be true for all greater values of the screen. It follows that if both densities are to integrate to 1 on the unit interval, then P2 must be true:

$$\text{P2:} \qquad f_0(0) > f_1(0).$$

2. Competitive Screening Markets

Consider a market where a very large number of applicants N (each with the identical reservation price $R \geq 0$ apply at no cost to one of many identical risk neutral firms. Successful applicants receive a reward valued at $r > 0$ and unsuccessful applicants receive nothing. All agents and firms are price takers, applicants accept any offer that equals or exceeds the reservation price and they reject any offer below the prevailing market price. $N_q < N$ of the applicants are qualified and the remainder are not. The returns to firms are normalized so that for each accepted applicant a firm receives an economic value $v > 0$ for each qualified and 0 for each unqualified applicant.

Decision makers base their choices on Bayesian expectations of applicants' qualifications. Assume that any necessary learning process has taken place and that agents know the screening process and the distributions it generates. Let $\hat{q} = \frac{N_q}{N}$ denote the true proportion of qualified applicants. Then $\hat{q}$ is the decision maker's prior expectation of the probability that any given applicant is qualified. If a firm elects to screen applicants, Bayes' Law ensures that the posterior expectation an applicant is qualified is $p(q_1/x) = \frac{f_1(x)}{g(x)} \cdot \hat{q}$, *where* $g(x) = \hat{q} \cdot f_1(x) + (1-\hat{q}) \cdot f_0(x)$.

Ignoring screening costs, the expected return from accepting an applicant screened at x is simply the expected value added minus the certain reward payment.

$$\pi(x) = p(q_1/x) \cdot (v - r) - [1 - p(q_1/x)] \cdot r = p(q_1/x) \cdot v - r.$$

Given a decision rule $d(\cdot)$ that accepts all applicants screened at or above some $\bar{x}$ and rejects all others (neglecting screening costs) the expected return from n screened applications is

$$\hat{\pi}(\bar{x}, d) = n \cdot \int_0^1 \pi(x) \cdot d(x) g(x) dx = n \cdot [1 - F_1(\bar{x})] \cdot \hat{q} \cdot v - n \cdot [1 - G(\bar{x})] \cdot r.$$

The first four terms after the second equal sign represent expected value added from accepted applicants and the last three terms are expected payments to applicants who pass the screen. Setting $\bar{x} = 0$ and assuming free entry of firms will drive expected returns to zero so that $\hat{\pi} = 0$ we find $\hat{q} \cdot v = \hat{r}$. This is the *no screening pooling equilibrium.* As would be expected, in competitive equilibrium with free entry, if there is no screening of applicants and firms are risk neutral the expected return from a random applicant (as determined by firms' prior $\hat{q}$) will just equal the reward paid to each accepted applicant.

Credit and insurance markets are two institutional specifications of the model.

Credit Market: Here r is the size of the loan received by borrowers, and v is the loan repayment (principal plus interest) required by the lender. The stipulated gross return is $\rho = \frac{v}{r}$ and the rate of interest is $i = \frac{v-r}{r} \bullet 100\% = (\rho - 1) \bullet 100\%$. In the pooling equilibrium, $\rho = \frac{1}{\hat{q}}$ and $i = \frac{1-\hat{q}}{\hat{q}} \bullet 100\%$.

Insurance Market: Here r is the insurance premium paid by the insured and v is the indemnity paid by the insurer in the event of a claim.[4]

Returning to the general model, we note that if $\hat{q} \geq \frac{R}{v}$ there always exists a unique allocation that pools all applicants at the same price and involves no screening. Throughout the remainder of the paper it is assumed that this condition is satisfied. The pooling allocation is the competitive equilibrium if screening is not cost effective. If screening is cost effective, the competitive equilibrium is defined by the allocation (screening versus no screening) offering applicants the greater reward. Below it will be shown that whether or not screening is present at the competitive equilibrium depends on the net value of the information obtainable from screening. To examine this issue firms' optimal behavior must be considered in more detail.

[4] For the insurance market, variables must be redefined and the profit function rewritten to reflect the nature of the assets being traded.

All firms have access to the same screening technology. Let $c(n)$ equal the cost of screening n applicants. We assume that the screen requires some kind of formal interview or application process followed by a checking of successful applicant's references and background materials. Under these conditions, assume that average screening costs conform to the classic case of the competitive firm whose average costs decrease initially and then increase after reaching the optimal scale n^*. During the rest of the paper screening costs are assumed to conform to this case.

Considering screening costs, the objective function of a firm utilizing a screening mechanism with the acceptance screen equal to x and screening n applicants is $\hat{\pi}(x,d) - c(n)$ or:

$$\hat{q} \cdot v \cdot n \cdot [1 - F_1(x)] - r \cdot n \cdot [1 - G(x)] - c(n).$$

This objective function warrants discussion. Again the first four terms are expected revenues, the prior probability ($\hat{q}$) that an applicant is qualified times the expected gain from qualified applicants screened x or above. The middle two terms represent the expected total rewards paid to accepted applicants (the expected number screened at or above the rejection index). The last term of course is total screening cost. Unlike the no screening case, some applicants are now rejected. Reading from right to left there is increasing attrition with respect to the three occurrences of n within the objective function. All n applicants screened contribute to screening costs, but a smaller number ($n \cdot [1 - G(x)]$, those passing the screen) are accepted and paid the reward, and a still smaller number ($n \cdot [1 - F_1(x)]$, those accepted and qualified) add value to the firm.

3. The Optimal Screening Rule and the Neyman-Pearson Theorem

An optimal strategy for a firm entails its choices of the number of applicants to screen and an acceptance policy or decision rule defined by the minimum x^* required for an applicant to be accepted. Assuming an interior solution (i.e. the firm indeed screens):

$$\frac{d\hat{\pi}}{dn} = \hat{q} \cdot v \cdot [1 - F_1(x^*)] - r^* \cdot [1 - G(x^*)] - c'(n^*) = 0 \qquad \text{(A)}$$

$$\frac{d\hat{\pi}}{dx} = -\hat{q} \cdot v \cdot f_1(x^*) + r^* \cdot g(x^*) = 0 \qquad \text{(B)}$$

Condition (A) states that at the optimum the expected net value added of the last applicant screened should just equal the marginal cost of screening her. (B) is an important condition that will aid understanding of screening equilibria. With a little manipulation of the equation:

$$\frac{f_1(x^*)}{g(x^*)} \cdot \hat{q} \cdot v = p(q_1/x^*) \cdot v = r^*. \qquad \text{(C)}$$

The optimizing firm sets its decision rule so that the expected value added of the minimally acceptable applicant is just equal to the reward paid those accepted. Note that the expectation is made in terms of the decision maker's posterior probability that the applicant is qualified conditional on the screen.

Two aspects of the optimal decision rule are worthy of fuller discussion. First, we note that with more rearrangement of terms equation (B) can be rewritten as

$$\frac{f_1(x^*)}{f_0(x^*)} = \frac{(1-\hat{q}) \cdot r^*}{\hat{q} \cdot (v - r^*)} = k. \qquad \text{(C')}$$

The left hand side of this equation is the relative likelihood an applicant screened x* is qualified. Since this likelihood function is monotone increasing in x, all rejected applicants have a relative likelihood less than k. And thus, the optimal decision rule takes the form of the Neyman-Pearson likelihood ratio test (Degroot, 1969; Christian, 1994; Bernardo and Smith, 1994). Moreover, because the optimal decision rule requires that an applicant be rejected if and only if the likelihood function [R(x)] is less than a constant k of this particular form (e.g. k is the relative expected cost of type II and type I errors), the optimal decision rule is equivalent to a Bayesian variant of the Neyman-Pearson theorem of classical statistics.

Recall that the Neyman-Pearson theorem refers to a test of a simple null hypothesis (e.g. $q = q_1$) against a simple alternative ($q = q_0$). It states, if there exists a positive constant k and a region C of the set of possible decision variables x with $\int_C f_1(x)dx = \alpha$ such that $R(x) \geq k \text{ } for \text{ } x \notin C \text{ } and \text{ } R(x) < k \text{ } for \text{ } x \in C$ then C is a most powerful critical region of size α (equal probability of type 1 error) for rejecting the null hypothesis. Alternatively, k is equivalent to the inverse of the Lagrange multiplier determined by the optimization problem choose x to minimize the probability of a type II error subject to the constraint that the probability of a type I error equals α.

A well known criticism of this classical theory is its arbitrary choice of α. Since the value of the Lagrange multiplier may be interpreted as the shadow price of type II error probability in terms of type I error probability, the question becomes how do you determine an optimal tradeoff between the two error types? The Bayesian firm maximizing expected profit eliminates the arbitrary choice of α by evaluating this tradeoff in terms of the relative costs of the two types of errors. Equation (B) and its variants (C) and (C') are easily seen to be the solution to the alternative optimization program: minimize the weighted sum of the probabilities of type I and type II errors. That is, choose x to minimize the function

$$a \cdot F_1(x) + b \cdot [1 - F_0(x)].$$

Where $F_1(x)$ and $1-F_0(x)$ are the conditional probabilities of making type I and type II errors respectively when x is the minimum screen acceptable. The weights a and b (whose ratio equals k) are equal to the expected losses incurred from the respective error types, $\hat{q} \cdot (v - r) \text{ } and \text{ } (1 - \hat{q}) \cdot r$. It follows from this discussion that the minimum x^* determining the optimal decision criterion is a function of the relative costs of type II and type I errors. Below this will prove important to our discussion of the equilibrium properties of the model.

The second point worthy of fuller discussion is the implication from optimization condition C that the optimal decision rule accepts applicants (those at or slightly above the optimal x^*) who are not expected to contribute enough value to cover the marginal cost of accepting them, $r^* + c'(n)$. This is because screening costs are a sunk cost. Once an applicant is screened, if she is expected to contribute more than r^* to the firm, accepting her is optimal because she contributes something to the payment of the fixed screening costs.

With free entry competition should drive firms' returns to zero so a third condition is needed to characterize equilibrium with screening.

$$\hat{q} \cdot v \cdot n^* \cdot [1 - F_1(x^*)] - r^* \cdot n^* [1 - G(x^*)] - c(n^*) = 0 \qquad \text{(D)}$$

Together conditions (A) and (D) require the standard competitive equilibrium result that average and marginal screening costs be equal and that therefore n* equals the optimal (low cost) number of applicants to screen. Obviously, if there is no screening in equilibrium the strict inequality (<) obtains in equations (A) and (B).

We are now in a position to clarify the concept of competitive equilibrium used in this paper. Let η_i and N_i denote respectively the number of firms and applicants in the ith market. Define a competitive screening equilibrium as a sequence of m markets characterized by a set $\{r_i^*, x_i^*, n^*, \eta_i^*, N_i^*, N\}$; i = 1….m where $r_i^* > r_{i+1}^*$, $x_i^* > x_{i+1}^*$, $n^* = \dfrac{N_i^*}{\eta_i^*}$, $N_i^* \leq N$ and $r_i^* \geq R$; $x_i^* \geq 0$.

If $x_i^* = 0$, competitive equilibrium entails no screening in market i. In that case, condition (D) holds with c(0) = 0. Letting $\hat{q}_i$ denote the proportion of applicants in segmented market i who are qualified, we have $r_i^* = \hat{q}_i \cdot v$ and conditions (A) and (B) are satisfied with the strict inequality (<) holding. In this case, it is obvious that i = m. If $x_i^* > 0$ competitive equilibrium entails screening in market i and each of conditions (A) (B) and (D) are satisfied by the equality signs.

4. CHARACTERIZATION OF EQUILIBRIA

We begin by discussing the primary market. For convenience the subscript i =1 designating the primary market is omitted. Condition (B) characterizing the firm's optimal screening decision rule defines x as an implicit function of r, v, and $\hat{q}$. Suppressing the parameters v and $\hat{q}$, denoting that function as $x(r)$, and differentiating implicitly:

$$\frac{dx}{dr} = \frac{g(x)^2}{\hat{q} \cdot v \cdot [g(x) \cdot f_1'(x) - f_1(x) \cdot g'(x)]} > 0$$

because the monotone likelihood condition ensures that the value within the brackets is positive. Increasing the reward to accepted applicants intensifies the screening criterion. An increase in r holding v and $\hat{q}$ constant raises the relative cost of a false positive acceptance decision vis a vis a false negative and induces the firm to raise its indicator of quality to reduce the likelihood of accepting unqualified applicants.

Let r_0 *and* r_1 satisfy respectively, $i = x(r_i), i = 0, 1$ and define the function

$$\Phi(r) = \hat{q} \cdot v \cdot n^* \cdot [1 - F_1(x(r))] - r \cdot n^{*\cdot} \cdot [1 - G(x(r))] - c(n^*).$$

From (D) we see that for each reward price r, $\Phi(r)$ defines the expected return to firms who have optimized with respect to conditions (A) and (B). Therefore, any values of r that set Φ(r) = 0 will satisfy all three of the equilibrium conditions (A), (B), and (D) and will define an equilibrium with screening for the appropriate number of firms.

Φ(r) is continuously differentiable on the open interval (r_0, r_1) with a first derivative

$$\Phi'(r) = n^* \cdot [r \cdot g(x(r)) - \hat{q} \cdot v \cdot f_1(x(r))] \cdot \frac{dx}{dr} - n^* \cdot [1 - G(x(r))].$$

By the optimal screening rule (condition (B)) the terms in the first set of brackets vanish everywhere on the interval (r_0, r_1). Therefore,

$$\Phi'(r) = -n^* \cdot [1 - G(x(r))] < 0 \ \ for\ r_0 < r < r_1. \qquad \text{Moreover,}$$

$$\Phi''(r) = n^* \cdot g(x(r)) \cdot \frac{dx}{dr} \geq 0.$$

Furthermore, $\Phi(r_1) = -c(n^*) < 0$. Therefore, if we find necessary and sufficient conditions for $\Phi(r_0)$ to be positive we will have demonstrated that Φ(r) is a strictly decreasing convex continuous function on the interval $[r_0, r_1]$ that attains positive and negative values at the respective endpoints and therefore must vanish at some unique r^* contained within the closed interval. That unique r^* would be a candidate to be a competitive equilibrium with screening.

We note $\Phi(r_0) = n^* \cdot [\hat{q} \cdot v - r_0] - c(n^*)$. Recall $x(r_0) = 0$ and note further that when there is screening and the reward is r_0, each firm plans to accept every applicant just as in the no screening allocation with complete pooling of applicants. Therefore because every qualified applicant is hired in either case, the expected value added from an accepted applicant $(\hat{q} \cdot v)$ is the same in both cases. Thus the difference between expected returns per applicant in the two cases is just the difference in total costs per applicant. This cost differential is given by $[\hat{r} - (r_0 - \frac{c(n^*)}{n^*})]$ and when it is positive, $\Phi(r_0) > 0$. To see this observe:

$$\frac{\Phi(r_0)}{n^*} = \hat{q} \cdot v - r_0 - \frac{c(n^*)}{n^*} = \hat{r} - r_0 - \frac{c(n^*)}{n^*} \geq 0 \Leftrightarrow \hat{r} - r_0 \geq \frac{c(n^*)}{n^*}. \qquad \text{(E)}$$

(E) says that if firms choose to move from the no screening allocation where all applicants are rewarded $\hat{r}$ to the allocation where firms screen and reward successful applicants r_0, it must be true that the savings in the reward paid each accepted applicant $[(\hat{r} - r_0)]$ exceeds the average cost of screening. Obviously, this cannot always be true. There are two general cases to consider.

Case I: equilibrium with no screening and no segmented markets.

Here we reverse the final inequality in (E) and assume $\Phi(r_0) \le 0$. Since Φ(r) is strictly decreasing on the relevant interval, expected returns with screening are everywhere negative except possibly at r_0 where such returns may equal zero. However, even if $\Phi(r_0) = 0$, with the reward equal to r_0 applicants would be paid less than under the no screening case.[5] So r_0 cannot support a competitive equilibrium since an entering firm could offer a reward between r_0 and $\hat{r}$ not screen and expect a positive return. In this case, screening costs are too high so the no screening complete pooling equilibrium with zero firm profit is the unique competitive equilibrium.

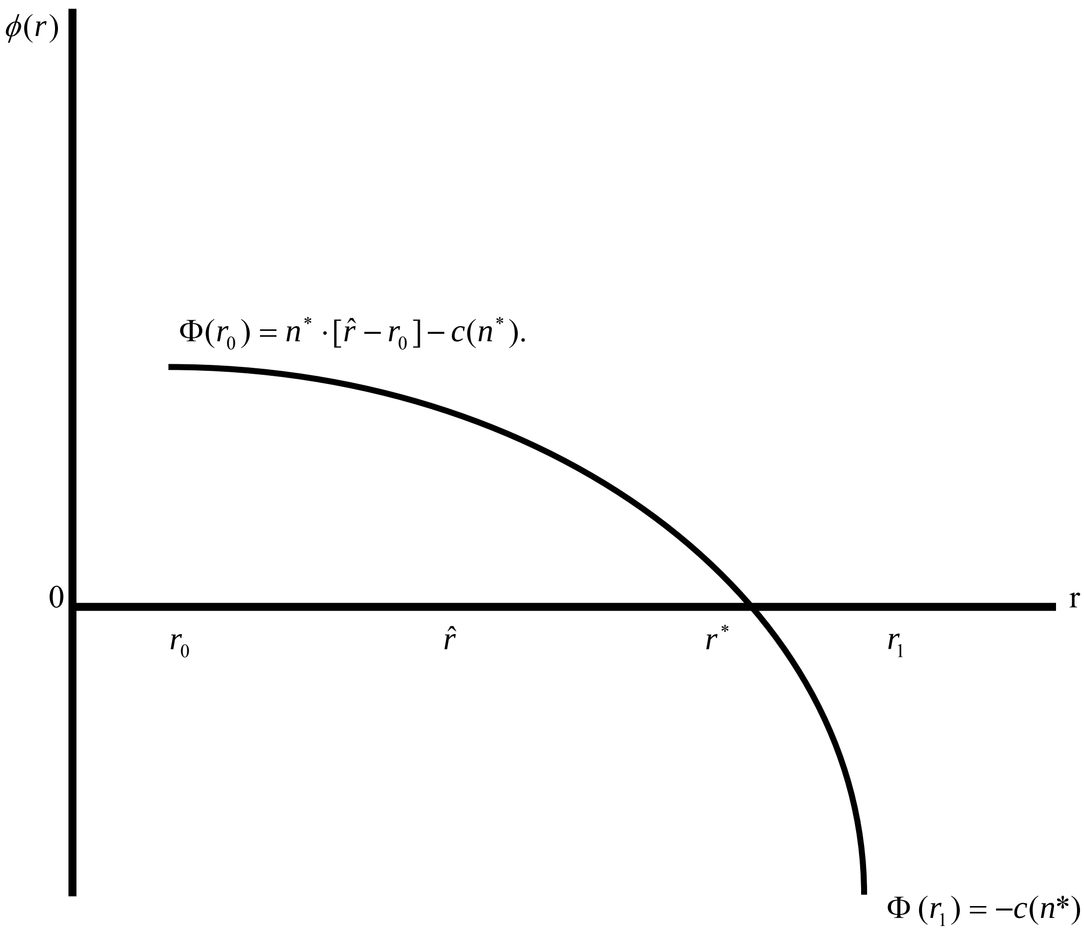
Alternatively, assume the inequality in (E) holds strictly at r_0. This implies that Φ(r) vanishes uniquely at some reward r^*. There are three sub cases to consider. In the first, $r^* < \hat{r}$ and again the unique competitive equilibrium is at $\hat{r}$ with no screening.

Case II: equilibrium with screening and segmented markets.

In subcase II.a $r^* = \hat{r}$ and the unique equilibrium at $\hat{r}$ could have all firms screening or no firms screening. Only in sub case II.b (Figure 1) where $r^* > \hat{r}$ are the costs of screening low enough to accommodate a competitive equilibrium with every firm screening. In what follows, the boundary case ($r^* = \hat{r}$) is ignored so that discussion of screening equilibrium assumes $r^* > \hat{r}$.

[5] By (C) $r_0 = p(q_1/x_0) \cdot v$ and $\hat{r} = \hat{q} \cdot v$ making it clear that $r_0 < \hat{r}$ because the conditional probability that the lowest screened applicant is qualified must be less than the proportion of all applicants who are qualified.

Figure 1 (Case IIb)



What can be deduced about the properties of equilibrium in the primary market when screening costs are not too high? From the final inequality exhibited in (E) we have surmised that if screening is to occur it must be true that $n^* \cdot (\hat{r} - r_0) \geq c(n^*)$. In moving from the reward price at a no screening allocation $(\hat{r})$ to the lowest reward price with screening (r_0), the savings in reduced reward payments must exceed the cost of screening. In effect, if screening is to be adopted by competitive firms, it must pay for itself. And to do that the introduction of a screen must lower a firm's non-screening costs. The major result of the paper gives this intuition a more specific economic interpretation.

Proposition 1:

> *If the direct costs of screening are positive and the market screen is efficient, there is a unique competitive equilibrium, it may or may not entail screening. For the competitive equilibrium to entail screening it is necessary and sufficient that at the unique no screening allocation the introduction of an efficient screen would show that the reduction in the expected cost of accepting unqualified applicants (type II error) exceeds the increase in the expected cost of rejecting qualified applicants (type I error) by at least the cost of screening.*

A proof of proposition 1 is given directly below. Prior to giving the proof, in order to appreciate the result it will be illuminating to discuss the nature of the full costs of screening. As remarked earlier, whether or not competitive firms invest in screening depends on the value of information obtained from the screening mechanism. In order to examine this information, consider starting from an allocation with no screening and zero profits. If a firm were to introduce screening at this allocation it could anticipate increasing its return because the screen, by reducing the number of unqualified applicants accepted, would lower the firm's total reward bill without affecting its value added. However, this gain would also entail two costs. The first addition to costs would be the direct cost of screening c(n). The second addition to costs would be the indirect cost associated with the fact that screening implies that some qualified applicants who were accepted under no screening will now be rejected. It follows that if screening is to increase net return above the no screening allocation the expected gains from rejecting unqualified applicants must exceed the direct costs of screening plus the expected opportunity costs of rejecting qualified applicants. If screening does pay for itself in this way, it is obvious that a firm active at the no screening allocation with zero profit that takes the price $\hat{r}$ as given and begins to screen would raise its profits so $\Phi(\hat{r}) > 0$. It is clear that a screening equilibrium exists.

To prove proposition one let $\hat{x} = x(\hat{r})$ (so $\hat{x}$ equals a firm's optimal acceptance policy at $\hat{r}$) and assume that the expected gain from rejecting unqualified applicants does exceed the sum of the direct costs of screening plus the expected losses from rejecting qualified applicants, i.e.:

$$(1-\hat{q})\cdot\hat{r}\cdot F_0(\hat{x}) > \hat{q}\cdot(v-\hat{r})\cdot F_1(\hat{x}) + \frac{c(n^*)}{n^*}. \Rightarrow$$

$(1-\hat{q})\cdot\hat{r}\cdot F_0(\hat{x}) - \hat{q}\cdot(v-\hat{r})\cdot F_1(\hat{x}) - \frac{c(n^*)}{n^*} + [\hat{q}\cdot v - \hat{r}] > 0$ because $[\hat{q}\cdot v - \hat{r}] = 0$. Thus, recalling that $G(x) = \hat{q}\cdot F_1(x) + (1-\hat{q})\cdot F_0(x)$, we have

$\hat{q}\cdot v\cdot n^*\cdot[1-F_1(\hat{x})] - \hat{r}\cdot[1-(\hat{q}\cdot F_1(\hat{x}) + (1-\hat{q})\cdot F_0(\hat{x}))]\cdot n^* - c(n^*) = \Phi(\hat{r}) > 0$. And there must exist an equilibrium r^* *such that* $\hat{r} < r^* < r_1$. This proves sufficiency. To show necessity, suppose that a competitive equilibrium with screening exists. This means there is a unique $r^* > \hat{r}$ such that $\Phi(r^*) = 0$ and therefore because $\Phi(r)$ is monotone decreasing in r, $\Phi(\hat{r}) > 0$. Reversing the steps used to show sufficiency, the result follows immediately.

In an important sense, screening in competitive markets is all about identifying and rejecting unqualified applicants. In the framework under consideration, firms have two options, they may screen or not screen. Clearly, screening will be chosen if and only if the value of the information it produces is worth its cost. In determining the net value of information from screening, the status quo against which the judgment to screen is made is to accept every applicant. Since the baseline of no screening already accepts all qualified applicants firms cannot improve their returns by increasing the acceptance rates of qualified applicants. The only way to increase returns is to use screening to reduce the proportion of unqualified applicants accepted.

The essentials of this point are contained within Proposition 1 from which it trivially follows:

Remark 1:

If competitive firms invest in screening the equilibrium expected cost of a type II error $(1-\hat{q})\cdot r^*$ strictly exceeds the expected cost of a type I error $\hat{q}\cdot(v-r^*)$.

Because it illuminates the relationship between Bayesian decision theory and the economics of firm optimization, an alternative demonstration of Remark 1 is provided. The result follows from the firm's choice of an optimal screening rule. From (C) we have that in a competitive equilibrium with screening;

$$p(q_1/x^*)\cdot v = r^* > \hat{r} = \hat{q}\cdot v \Rightarrow f_1(x^*) > f_0(x^*)$$

and the desired result follows immediately from (C').

We note that in the statement of Remark 1 the expected costs of type I and II errors are based on prior probabilities. As a consequence, the previous equation allows us to arrive at the intuitively appealing result that for a competitive equilibrium to unambiguously entail screening the posterior probability that a marginal acceptee is qualified must exceed the prior probability that a marginal acceptee is qualified ($p(q_1/x^*) > \hat{q}$). And it follows that if equilibrium entails screening it must be true that $x^* > x'$ (the switch point where the likelihood ratio of the two densities equals one). Equivalently, in the border case where $x^* = x(\hat{r})$ we have, $p(q_1 / \hat{x}) \cdot v = \hat{r}$. For an allocation with screening to be a competitive equilibrium, the screening mechanism must signify that the screening index assigned to the marginal accepted applicant is more likely to have been generated from the probability density for qualified applicants. Thus, a necessary condition for screening to exist in equilibrium is that the value of information be positive.

5. SECONDARY MARKETS

Given an equilibrium with screening represented by the descriptive parameters $\{r^*, x^*\}$ the primary market has rejected $G(x^*) \cdot N$ applicants. However, $F_1(x^*) \cdot N_q$ of these applicants are in fact qualified. The existence of these unselected but qualified applicants gives firms incentives to contract with members of the pool of rejected applicants, and thus a secondary market may form. The formal structure of the equilibrium in this secondary market is completely analogous to that in the primary market. Let $\hat{q}(x^*) = \frac{F_1(x^*) \cdot N_q}{G(x^*) \cdot N} = \frac{F_1(x^*)}{G(x^*)} \cdot \hat{q}$ denote the proportion of qualified applicants in the pool of rejected applicants. The analysis of equilibrium in secondary markets proceeds as it did in the primary market. We first note that $\hat{q}(x^*) \geq \frac{R}{v}$ is a sufficient condition for the formation of a secondary market. For in this case, denoting secondary market variables by the subscript i = 2,…m, it is possible that there exists an equilibrium without screening in the secondary market that is defined by $\hat{q}(x^*) \cdot v = \hat{r}_2 \geq R.$ To determine if there is an equilibrium with screening we return to Proposition One.

From Proposition One, there will be an equilibrium with screening in any secondary market if and only if, starting from the market's no screening pooling allocation, the expected reduction in type II error costs exceed the expected increase in type I error costs by more than the cost of screening. The cost of a type II error $(\hat{r}_i)$ is decreasing while type I error cost $(v-\hat{r}_i)$ is increasing in i. However, since the probability of making a type II error is increasing while the probability of making a type I error is decreasing in i, the respective costs of type I and type II error appear to be indeterminate without further restrictions on the model. In general there will be $m \geq 1$ markets formed. It is trivial to show that each such market has the property that its succeeding market has a smaller proportion of qualified applicants and a lower reward to applicants. Moreover, if there is a final pool of rejected applicants failing to gain admittance to any market, it must be true that $\hat{q}_{m+1} \cdot v < R.$

We make one final observation about the structure of screening equilibria and their segmented markets. In markets where the value added of qualified applicants is large relative to applicants' reservation price, there is a tendency for the mth secondary market to pool applicants without screening. This tendency is formalized in the following proposition. Recall that x' the switch point is defined by $f_1(x') = f_0(x').$

Proposition 2:

> *If* $\hat{q}(x') \cdot v \geq R$*, then the mth secondary market entails no screening and is a pooling equilibrium with every remaining applicant accepted at the terms of trade* $\hat{r}_m = \hat{q}(x^*_{m-1}) \cdot v.$

To prove this proposition note if there is no screening in the primary market the proposition is shown. Thus, assume there is screening in at least the primary market. To see that market m must have a pooling equilibrium, assume otherwise. We have shown that any market (j) that has equilibrium with screening must satisfy the condition that $f_1(x^*_j) > f_0(x^*_j)$. The last inequality implies $x^*_j > x'$ for every market j that has a screening equilibrium. Note that x^*_m is the acceptance criterion in market m. So if market m has a screening equilibrium, $x^*_m > x'$ and $\hat{q}(x^*_m)$ is the proportion of qualified applicants among those who remain rejected. But $\hat{q}(x_m^*) > \hat{q}(x') \geq \frac{R}{v}$. This implies that the pooling allocation without screening defined by $\hat{r}_{m+1} = \hat{q}(x_m^*) \cdot v$ is feasible and there exists an equilibrium in an (m+1)th market, a contradiction. The mth market must be a pooling equilibrium without screening and $\hat{r}_m = \hat{q}(x_{m-1}^*) \cdot v$.

Robustness of Equilibrium

Some comments about this model have suggested an alternative specification with firms offering applicants a continuous reward function r(x) giving a specific reward tailored to each value of the screen. It is not difficult to show there exist such schedules which support a Nash equilibrium; e.g. let $r(x) = p(q_1 / x) \cdot v - \frac{c(n^*)}{n^*}$. Firms could of course make distinctions in applicants' rewards based on infinitesimal differences in screen scores but the objective of this paper is to examine the properties of competitive equilibrium within the context of frequently observed firm behavior in a number of important market settings. Moreover, it is straightforward to show that the competitive screening equilibrium cannot be undermined by a firm offering a reward schedule r(x) that varies with applicants' screen values.

Suppose a firm were to make such an offer in a market in competitive equilibrium at a reward price r*. We note only applicants offered a reward greater than their competitive equilibrium offer will accept. This requires r(x) > r* for all who accept the new offer. Furthermore, no profit maximizing firm will offer any applicant a reward that exceeds the applicant's expected value to the firm $p(q_1 / x) \cdot v$. Because $p(q_1 / x)$ is increasing in x and $p(q_1 / x^*) \cdot v = r^*$ by first order condition (C), it must be true that $r(x) \leq r^* all x \leq x^*$. Therefore, $x \geq x^*$ for all applicants accepting r(x). Moreover, the optimal number of applicants to screen will still equal n* where screening costs are minimized. Thus, for any such r(x) and n the new firm's expected profit per applicant will equal:

$$\int_{x^*}^{x_1}[p(q_1/x)\cdot v - r(x)]g(x)dx - \frac{c(n)}{n} < \int_{x^*}^{x_1}[p(q_1/x)\cdot v - r^*]g(x)dx - \frac{c(n^*)}{n^*} = 0 \qquad \text{(F)}$$

since r(x) > r* for all x > x*.

It is also true an entering firm could not make a profit by offering a reward schedule only acceptable to applicants screened above x*; i.e. for some $\bar{x} > x^*$

$$r(x) \begin{cases} < r^* \text{ for } x < \bar{x} \\ > r^* \text{ for } x \geq \bar{x}. \end{cases}$$

To see this note that the inequality of integrals in (F) must hold for all x and not just x*. Moreover, for any x other than x* the second integral must have a negative value. Otherwise the firms earning zero profit in competitive equilibrium at r* could have chosen a different minimum acceptable x and earned a positive profit. Since x* is the profit optimal screen value this is a contradiction.

A second means by which a competitive screening equilibrium might be undermined is by an outside firm raiding an active firm's pool of clients. In equilibrium, each successful applicant to a firm receives a reward $r^* = \hat{q}(x^*)\cdot v - c(n^*)/n^*$. It has been suggested an inactive firm could (without cost) identify <u>all</u> of the clients of an active firm and offer a reward slightly higher than r* to every such client and make a profit because the poaching firm would not incur screening costs. The problem here is how does the poacher uncover the information without cost? In order to ensure that it is hiring the exact client base of an existing firm, the poacher must incur the same sorts of costs as did the active firms. The poacher must make its offer known to the pool of applicants, and it must screen all interested parties to ensure that it is contracting only with the clients of the firm it is poaching. It is clear that in most market settings the poacher cannot obtain such information without cost. In particular, for the financial markets under discussion here it is unreasonable to assume that a poacher could identify another firm's geographically dispersed client pool except through some kind of industrial espionage gaining access to the firm's internal database of clients. Furthermore, client poaching of this kind does not seem reasonable in markets where firms and clients are protected by contracts that obligate them to perform certain actions for the other party. Even in labor markets, where contracts interpreted as indentured servitude will not be enforced by courts, there are service jobs where employers protect trade secrets from poaching by having new hires sign non-compete agreements. These non-compete agreements prevent employees from doing the same kind of work for another employer for a defined length of time after severing employment with the original employer. Such contracts are enforced in U.S. courts.[1]

Generally, the poaching argument will not work if poaching costs are at least as expensive as the original screening costs. That assumption seems reasonable in cases where the original screening costs are largely confined to interviewing and verification of information given by applicants as has been assumed here. In the case where screening costs entail more substantial and costly actions so that poaching costs are less expensive, the payment of a single reward to all successful applicants may be vulnerable to poaching. Therefore, the results of this paper apply to markets where firms can enter legally binding contracts with applicants that require the delivery of applicants' services or payments, or where screening costs are no larger than the costs that would necessarily be incurred by a would be poacher.

CONCLUSIONS

Why do organizations screen? This question can be answered in at least two ways. Some people claim that the motivation behind screening is to identify the best applicants. Others assert that screening is a mechanism whose primary function is to identify and keep out undesirables. An economist might answer that these two explanations amount to the same thing. That is neoclassical economic intuition suggests that in equilibrium the two assertions are likely to be indistinguishable. However, the results of this paper provide support for the view that, at least in competitive markets where, unlike signaling models information remains imperfect, screening is a gate keeping device. Competitive firms' motivation to screen is based on their incentive to cut costs by identifying and rejecting applicants perceived to be of lower quality.

However, in competitive equilibrium free entry of firms drives the applicant reward price upward until firm profits equal zero. As a consequence, all of the benefits provided by firms' investment in better information ultimately accrue to successful applicants. Moreover, because the information provided by the screen is imperfect, firms make errors in their acceptance decisions. Unavoidably, applicants who screen well are rewarded more than those who do not screen well even in cases where the underlying quality of the former is lower. In terms of social efficiency this result is disturbing. Indeed, we have seen (Proposition 3) that under general conditions, the mth secondary market will entail no screening and as a consequence ultimately all applicants are accepted by some market. Thus, screening may merely redistribute income among applicants without any improvement in social efficiency. In equilibrium, screening does not ultimately allocate resources any better than does pooling of applicants without screening.

The model developed here provides a natural setting for investigating questions relating to statistical discrimination in labor and credit markets: questions concerning who gets screened out, under what conditions, and why. Most importantly, the relationship between screening and the formation of segmented markets strongly suggests that reanalysis of the relationship between ethnic and gender relations (beliefs about phenotypically identifiable groups and behavioral interactions among them) and disparate economic treatment is likely to have starkly different implications than those suggested by standard models of statistical discrimination that are similar to signaling models (Aigner and Cain, 1977; Phelps, 1972). However, a review of the economics literature suggests that only Coate and Loury (1993) (who model the effects of affirmative action on promotion decisions in internal labor markets) have utilized Bayesian decision making to analyze questions related to those considered here. The model of binary choice screening and market segmentation developed in this paper can be used to analyze a broad array of problems concerning resource allocation and equity in markets where probabilistic decision making may well have significant ramifications for the differential treatment of groups.

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[1] Actually, the threat that legal fees will be incurred defending against a law suit alleging a new employee is engaged in an activity that she contracted not to do can deter such poaching.