

MONOPOLY PRICING OF EXPERIENCE GOODS

By

Dirk Bergemann and Juuso Välimäki

July 2004

COWLES FOUNDATION DISCUSSION PAPER NO. 1463



COWLES FOUNDATION FOR RESEARCH IN ECONOMICS

YALE UNIVERSITY

Box 208281

New Haven, Connecticut 06520-8281

<http://cowles.econ.yale.edu/>

Monopoly Pricing of Experience Goods*

Dirk Bergemann[†]

Juuso Välimäki[‡]

First Version: May 2003

This Version: June 2004

Abstract

In this paper, we develop a model of experience goods pricing with independent private valuations. We show that the optimal paths of sales and prices take qualitatively different shapes for different products. If the buyers are initially optimistic, then the prices are declining over time. If the buyers are initially pessimistic, then the optimal prices are initially low followed by higher prices that extract goodwill from the buyers with a high willingness to pay.

We also characterize the optimal commitment price path for the monopolist and the steady state equilibrium when there is turnover on the buyer side of the market.

KEYWORDS: Monopoly, dynamic pricing, learning, experience goods, continuous time, Markov perfect equilibrium.

JEL CLASSIFICATION: D81, D83

* We would like to thank Steve Berry, Patrick Bolton, Sven Rady, Mike Riordan, Klaus Schmidt and Ferdinand von Siemens for very helpful comments and Colin Stewart for excellent research assistance. We benefited from discussions at the IO Day at New York University and seminars at the University of Darmstadt, EUI, LSE, University of München, University of Nürnberg, IUI, University of Oulu, and UCL. The first author gratefully acknowledges financial support from NSF Grant SES 0095321 and DFG Mercator Research Professorship at the Center of Economic Studies at the University of Munich. The second author gratefully acknowledges support from ESRC through grant R000223448.

[†]Department of Economics, Yale University, New Haven, CT 06520-8268, U.S.A., dirk.bergemann@yale.edu.

[‡]Helsinki School of Economics and University of Southampton, valimaki@hkkk.fi.

1 Introduction

Since the original contribution by Nelson (1970), experience goods have been defined as those products whose true quality is only learned upon consuming the good. As a result, any consumer choice model of experience goods must combine the perishable goods nature of repeat purchases with the durable information resulting from consumption. In this paper, we develop a simple tractable model of optimal pricing for a monopolist that sells an experience good over time to a population of potential buyers. We find that different types of experience goods induce qualitatively different long run outcomes depending on whether the short run perishable attributes or the long run informational features of the product are the most important in the purchasing decisions.

In an experience goods market, the seller is facing simultaneously two different submarkets. The demand curve in the part of the population that has already learned its preferences is similar to the standard textbook treatment. Those buyers that are uncertain about the true quality of the product must behave in a more sophisticated manner. Each current consumption decision incorporates a decision of information acquisition that is relevant for repeat purchases at future dates. The value of this information is endogenously determined in the market. If future prices are very high, current purchases are unlikely to yield information that results in future consumer surpluses. If future prices are low, then it may be in the buyers best interest to forego purchases in the current period as future prices are attractive regardless of the true value of the product. As a result, current and future prices determine simultaneously the sales in the informed segment of the market, and they also determine the value of information for the uninformed segment.

The model in this paper is an infinite horizon, continuous time model of monopoly pricing. There is a continuum of ex ante identical consumers that have a unit demand per period for the purely perishable good. At each instant of time, the monopolist offers a spot price and the buyers decide whether to purchase or not. In the beginning of the game, all buyers are uncertain about their true preferences. If they purchase the product, they observe with positive probability a perfectly informative signal. We assume for analytical convenience that these signals arrive according to a Poisson process. The true preferences are modeled by a single parameter that represents the buyer's willingness to pay for the product, and we assume that the aggregate distribution of preferences in the population is

common knowledge between all players. The key feature of the model is then that different buyers become informed at different times. As a result, the market is segmented at each point in time and the degree of segmentation depends on the prices that the monopolist sets. Hence the model incorporates elements of demand management and market building. We treat separately two specifications for the buyer side of the market. In the first model, we assume that the same set of buyers is present in all periods. In our view this is the right assumption for analyzing the launching phase of new products. The second model deals with a changing set of buyers using a conventional steady state analysis in a market where the buyers enter and exit at exogenously given rates. This is a more appropriate modeling choice for mature markets.

To fix ideas, consider the pharmaceutical market. In that industry, each new drug undergoes an extensive period of pre-launch testing to ascertain its performance with respect to the overall population. The aggregate uncertainty relating to the product has hence been reduced to a large extent at the moment of introduction. Yet many drugs differ in their effectiveness and incidence of side effects across agents. The uncertainty about the idiosyncratic value of the drug then provides a strong motive for the individual agent to experiment. In fact, a recent empirical study by Crawford & Shum (2003) regarding the dynamic demand behavior in pharmaceutical markets documents the important role of idiosyncratic uncertainty and learning in explaining demand.¹ For a data set of anti-ulcer descriptions, they observe substantial uncertainty about the idiosyncratic effectiveness of the individual drugs and high precision in the signals received through consumption experience. We model the effectiveness of the new treatment to an individual new patient as a random event. The time of response to the drug is random and the response may be either positive or negative (successful recovery from the illness or severe side effects).

Our main result is that the qualitative features of the equilibrium market outcome depend on a rather simple intertemporal comparison. In the full information benchmark where all of the buyers have learned their true preferences for the good, the monopolist sets

¹The empirical literature on learning based models in pharmaceutical demand is growing rapidly. Ching (2002) provides structural, dynamic demand estimates when there is learning among patients about a new (generic) pharmaceutical with common values. Coscelli & Shum (forthcoming) estimate the impact of uncertainty and learning for the introduction of a new drug. The role of information is also central in Bhattacharya & Vogt (2003), where a model and preliminary estimates regarding informative marketing for new pharmaceutical products are presented.

the price of the good at the textbook monopoly price. Consider the incentives of a single new consumer that enters the market in its long run full information equilibrium. A new consumer does not know if she likes the product sufficiently much to purchase the product in the long run at the monopoly price. She must calculate the expected consumer surplus from potential future purchases and compare that to the (possible) short run losses that may result from purchases at the monopoly prices. If the *uninformed* buyer is willing to purchase the good at the long run monopoly price, we call the market optimistic, if she is not willing to buy the product, we call the market pessimistic. Theorem 1 shows that in the optimistic case, the long run equilibrium price converges to the full information price and the long run sales also converge to full information sales levels. In the pessimistic case, long run prices also converge to full information levels, but sales levels fall short of the full information monopoly sales. We also show that the equilibrium price paths take quite different shapes in these two cases. In the optimistic case, the monopolist skims the more attractive part of the market (i.e. the uninformed buyers) as long as profitable. In the pessimistic case, the monopolist offers low initial prices to capture a larger share of the uninformed at the expense of targeting the more attractive informed segment of the market. When new buyers enter the market, we show that neither long run prices nor long run quantities will converge to full information levels in general. In the optimistic case, the optimal steady state price is constant, but in the pessimistic case, for some parameter values, the optimal price path exhibits price dispersion.

We also examine the welfare properties of such markets. Since information on product quality is a durable good, we ask in the Coasian tradition how a monopolist's power to commit to future prices affects the market outcomes. In the optimistic case, commitment can be shown to be welfare reducing as a longer period of skimming the market becomes feasible. In the pessimistic case, however, commitment is beneficial from the welfare point of view. Under commitment, the optimal long run prices are below the full information monopoly price and the resulting consumer surplus is extracted from the buyers through higher initial prices. The welfare losses in the early periods are more than offset by the gains in the late periods.

To our knowledge, the current paper is the first to address the issue of experimental consumption in a fully dynamic model with a population of heterogenous buyers. One of the contributions of this paper is then to provide a tractable analytical framework that is

suitable for the study of similar issues in related contexts. In the conclusion to this paper, we suggest some avenues for further research within the same modelling framework.

Monopoly models dealing with issues of dynamic pricing include Milgrom & Roberts (1986), Farrell (1986) and Tirole (1988). All of these models make the assumption that the perceived quality is either high or otherwise of no value and the main emphasis is on vertical differentiation between the buyers. Furthermore these models have only a two period horizon. We view both of these restrictions as unnecessary and unrealistic in many situations. Our model allows for the possibility that the monopolist discriminates intertemporally in the market in a more flexible manner and as a result, our conclusions are quite different from those in the earlier literature. In our model, it is possible that the marginal buyer in the later periods might have a lower willingness to pay for quality than the marginal buyer in the earlier periods and as a result, buyers have an incentive to engage in experimental consumption. Cremer (1984) considers a model with initially identical buyers and idiosyncratic experience to explain coupons and entry fees for shopping clubs. In a recent contribution, Villas-Boas (2004a) considers the equilibrium in a duopoly model with horizontal and vertical differentiation with two periods. The horizontal differentiation is known whereas the vertical differentiation is uncertain and idiosyncratic and learned by experience. Villas-Boas (2004b) provides an extension to an infinite horizon model.

Finally, conditions for initially high prices have been obtained in asymmetric information models of entry. In those papers, the monopolist is assumed to know the true value of the product, and the prices chosen serve as signals of the true quality. A prominent example of such models is Bagwell & Riordan (1991) where high and declining prices serve as signals of high product quality. Judd & Riordan (1994) consider a model with initially symmetric information where private signals are received by the monopolist and the buyers after first period choices. The firm then faces a signaling problem in the second period. The results in these models depend on the details of the information revelation mechanism and the cost structure. In our model, the results depend only on the quality difference between the products which can in principle be inferred directly from the realized prices.

The paper is organized as follows. Section 2 sets up the basic model and discusses the appropriate solution concepts. Section 3 represents the intertemporal decision problems of buyers and seller by dynamic programming equations and introduces the relevant option values. Section 4 analyzes the market with a single cohort of buyer. Section 5 considers

the case where the monopolist can commit to a sequence of future prices. Section 6 deals with the steady-state analysis of the market where the buyers enter and exit the market. Section 7 concludes and discusses future extensions of the current work. The proofs of all the results are collected in the Appendix.

2 Model

We consider a continuous time model with $t \in [0, \infty)$ and a positive discount rate $r > 0$. A monopolist with a zero marginal cost of production offers a single product for sale in a market consisting of a continuum of consumers. For analytical simplicity, we assume that the buyers have unit demand for the product within periods, and that the product is not storable. We also abstract from the possibility of price differentiation within periods. At each instant, the monopolist starts the game by setting the spot price. Upon seeing the price, all consumers decide whether to purchase or not.

Each consumer is characterized by her idiosyncratic willingness to pay for the product θ . The good is an experience good and therefore the true value of each individual θ is initially unknown to the buyers as well as to the seller. The ex ante distribution of buyer types is given by a continuously differentiable distribution function $F(\theta)$ with support $[\theta_l, \theta_h] \subset \mathbb{R}$. This distribution is assumed to be common knowledge and this reflects our assumption that there is no aggregate uncertainty in the model. As the focus in this paper is on private individual experiences, we abstract from possible common sources of uncertainty. To simplify the analysis, we also require that $\theta[1 - F(\theta)]$ be strictly quasiconcave in θ . This assumption guarantees that the full information problem is well behaved.

All buyers are ex ante identical, and their utility from consuming the product prior to learning their type is given by v where

$$v \triangleq \int_{\theta_l}^{\theta_h} \theta dF(\theta).$$

Throughout the paper, we assume that a perfectly informative signal (e.g. the emergence of side effects in a drug therapy) arrives at a constant Poisson rate λdt for all buyers that purchase the product in a time instant of length dt .² In this case, the posterior distribution

²It might be natural to allow for cases where λ depends on t . In the pharmaceutical example, such a time-varying arrival rate might reflect e.g. the decline in the probability of a treatment being eventually

on θ remains constant at the prior until the signal is observed.³ The most important analytical consequence of this assumption for our analysis is that conditional on not having observed a signal, the buyers are identical. Upon the arrival of the signal, buyers become heterogenous and the monopolist's key objective is to manage the endogenous composition of these two market segments.

As we analyze the dynamic behavior of the market, it is natural to use dynamic programming tools to derive the equilibrium conditions for the model. For consistency with the rest of the modeling, we assume that the only observable variables in each period are the prices and aggregate quantities. This is in line with the assumption that each individual buyer is small and has no strategic impact on the aggregate outcomes. The state variable of the model at each instant t is the fraction of agents that have become informed. We denote this fraction by $\alpha(t) \in [0, 1]$. Even though $\alpha(t)$ is not directly observable to the players, it can be calculated from the equilibrium purchasing strategies.⁴

A pure Markovian behavior strategy of the seller is given by

$$p : [0, 1] \rightarrow \mathbb{R}.$$

The price is thus a function of the state variable $\alpha \in [0, 1]$ only. A Markovian strategy of an uninformed buyer is given by

$$d^u : [0, 1] \times \mathbb{R} \rightarrow \{0, 1\}.$$

In this decision rule, 0 indicates no purchases and 1 indicates a purchase. This decision depends on the current price as well as the current state of the system.

The Markovian strategy of the informed buyer is denoted by

$$d^i : [\theta_l, \theta_h] \times [0, 1] \times \mathbb{R} \rightarrow \{0, 1\},$$

and it depends on the price and the state as well as her private valuation of the object.

successful given a number of unsuccessful trials. We have analyzed this possibility, but given that the qualitative features of the model remain the same, we report only the constant case.

³This assumption is made for the ease of exposition only. We have also computed the model for posteriors with positive and negative drift. Again, the qualitative features of the solution are the same as in the constant case.

⁴Here we are assuming a law of large numbers for our continuum population case. This is not problematic since independence is not required in our model with anonymity among the buyers.

The evolution of the market state variable, conditional on the uninformed buying in period t , is quite simply:

$$\frac{d\alpha(t)}{dt} = \lambda(1 - \alpha(t)) dt$$

as in period t a fraction λdt of the currently uninformed, of which there are $1 - \alpha(t)$, become informed in a time interval of length dt .

The quantity of sales in period t is denoted by $q(t)$. We can write the intertemporal objective function for the monopolist as:

$$\int_0^\infty e^{-rt} p(t) q(t) dt.$$

For the informed buyer with valuation θ , we have the payoffs given by:

$$\int_0^\infty e^{-rt} d^i(t) (v - p(t)) dt.$$

Finally, the payoffs to an uninformed buyer are given by:

$$\int_0^\infty e^{-rt} d^u(t) (v(t) - p(t)) dt,$$

as long as she remains uninformed and as above for the informed once she observes the signal.

In the absence of aggregate uncertainty, the price and aggregate sales process are deterministic. The individual buyer however faces uncertainty regarding the random time at which she will receive new information.

3 Value and Option Value

We are interested in the Markov perfect equilibria of the dynamic game between the monopolist and the buyers. In this section we first describe the intertemporal decision problems by the agents in terms of familiar dynamic programming equations. We then focus on the learning problem of the uninformed buyers and decompose his intertemporal decision into a flow value due to consumption and an option value due to experimental learning.

3.1 Value Functions

We start by describing the decision problem of the informed buyers. These buyers have complete information about their true valuation θ of the object. The value function, $V^\theta(\alpha(t))$,

of the informed agent with valuation θ , is represented by dynamic programming equation which poses only a static decision problem for the buyer:

$$rV^\theta(\alpha(t)) = \max\{\theta - p(\alpha(t)), 0\} + \frac{dV^\theta}{d\alpha(t)} \frac{d\alpha(t)}{dt}. \quad (1)$$

The decision whether to buy or not to buy is solved by the myopic decision rule: buy whenever θ exceeds the current price $p(\alpha(t))$. The only intertemporal component in this equation (the second term) reflects the effect of a change in the composition of the market segments, $\alpha(t)$ and $1 - \alpha(t)$, on the future utilities. The future utilities of the informed buyers are affected by the change in the size of the segments as the future prices of the seller responds to changes in the aggregate demand. Yet these changes are beyond the control of any single (informed) buyer and hence the myopic rule characterizes optimal behavior. The current size of the informed market segment, $\alpha(t) \in [0, 1]$, is the state variable of the model. We shall omit the indexation with respect to the time index t and simply write all value functions as a function of α rather than $\alpha(t)$, or:

$$rV^\theta(\alpha) = \max\{\theta - p(\alpha), 0\} + \frac{dV^\theta}{d\alpha} \frac{d\alpha}{dt}.$$

In contrast, for the uninformed buyer, a purchase of the new product represents a bundle, consisting of the flow of consumption and the flow of information. The value function $V^u(\alpha)$ of the uninformed buyer is given by:

$$rV^u(\alpha) = \max\left\{v - p(\alpha) + \lambda \left(\mathbb{E}_\theta V^\theta(\alpha) - V^u(\alpha)\right), 0\right\} + \frac{dV^u}{d\alpha} \frac{d\alpha}{dt}. \quad (2)$$

The additional element in value function, compared to the one of the informed buyer with valuation θ , represents the value of information. A purchase in the current period t generates an inflow of information at rate λ . Conditional on receiving the signal, the uninformed becomes informed. In consequence the new value function becomes $V^\theta(\alpha)$ for some $\theta \in [\theta_l, \theta_h]$. Yet, from the point of a currently uninformed, there is uncertainty about his true valuation θ and thus to assess the expected gain from the information, we take the expectation with respect to θ and write $\mathbb{E}_\theta V^\theta(\alpha)$. The informational gain attached to a current purchase is thus given by:

$$\lambda \left(\mathbb{E}_\theta V^\theta(\alpha) - V^u(\alpha)\right).$$

The above difference is always weakly positive as the informed buyer can always do at least as well as the uninformed one. As a result, the maximal price that the uninformed buyer is willing to pay in any given period is (weakly) above the myopic value v .

The value function of the firm is denoted by $V(\alpha)$. We describe the firm's dynamic programming equation in two parts to separate the intertemporal considerations as cleanly as possible. The basic trade-off facing the firm is that sales are made at a single price in two separate market segments. If the firm decides to sell to the uninformed buyers as well as some informed ones, the relevant equation is given by:

$$rV(\alpha) = \max_{p(\alpha) \in \mathbb{R}_+} \{p(\alpha) [1 - \alpha + \alpha(1 - F(p(\alpha)))]\} + \frac{dV}{d\alpha} \frac{d\alpha}{dt}, \quad (3)$$

subject to

$$p(\alpha) \leq v + \lambda \mathbb{E}_\theta \left(V^\theta(\alpha) - V^u(\alpha) \right).$$

Here $(1 - \alpha)$ is the share of uninformed buyers in the population and $\alpha(1 - F(p(\alpha)))$ is the fraction of informed buyers that are willing to buy at prices $p(\alpha)$. The constraint on the price $p(\alpha)$ guarantees that the uninformed buyers are indeed willing to purchase at prices $p(\alpha)$.

If the monopolist gives up on the uninformed buyers and sells to the informed segment only, then her value function satisfies:

$$rV(\alpha) = \max_{p(\alpha) \in \mathbb{R}_+} \{p(\alpha) \alpha(1 - F(p(\alpha)))\}. \quad (4)$$

In this latter case, the size of the informed segment, α , remains constant and $d\alpha/dt = 0$, since the flow of information to the uninformed buyers has stopped. The Markovian prices in this regime must remain constant.

We shall therefore approach the problem of optimal intertemporal pricing as one of optimal stopping. The monopolist's task is to decide how long to sell the product in the uninformed market segment. With these preliminaries, we can define:

Definition 1 (Markov Perfect Equilibrium)

A Markov Perfect Equilibrium of the dynamic game is a pair $(d, , p)$ such that the problems (1)-(4) are simultaneously solved for all α and θ .

3.2 Option Value

The qualitative behavior of the market and the nature of the optimal pricing policy can be characterized dichotomously in terms of an option value. More precisely, we shall argue that the key question is whether the uninformed are willing to buy at the static monopoly prices. We denote by \hat{p} the optimal monopoly price in the static model where each buyer knows his valuation for the object. We refer to this as the *perfect information model*. In other words, \hat{p} solves the problem:

$$\hat{p} \in \arg \max_{p \in \mathbb{R}_+} \{p(1 - F(p))\}.$$

We recall that the revenue function $p(1 - F(p))$ is assumed to be strictly quasiconcave in p and thus the above optimization problem has a unique solution \hat{p} . The static equilibrium quantity, denoted by \hat{q} , is given by $\hat{q} = 1 - F(\hat{p})$.

The key element in the analysis is the willingness to pay of the uninformed relative to the static equilibrium price \hat{p} . As the segment of informed agents is growing, the segment of uninformed is decreasing and will eventually cease to be the critical factor for the pricing policy. There are two possible reasons why the uninformed buyers might not be the marginal buyer in the market: either they may be priced out of the market, or they may become inframarginal buyers. The question as to which of these two events will occur in equilibrium will be determined by the option value the uninformed assigns to a last, marginal, purchase of the new good.

We can compute the option value explicitly on the basis the (maximal) *willingness to pay* of the uninformed, denoted by $w(\alpha)$. The willingness to pay can be obtained from the dynamic programming equations of the informed and uninformed buyer, (1) and (2), respectively. As we compute the willingness to pay of the uninformed, we do not pursue the optimal policy of the seller and hence do not solve his value function. We simply want to ask how much the uninformed would be willing to pay if the seller would sell to the uninformed in every period. The value function of the uninformed is then given by:

$$rV^u(\alpha) = \max \left\{ v - w(\alpha) + \lambda \left(\mathbb{E}_\theta V^\theta(\alpha) - V^u(\alpha) \right), 0 \right\} + \frac{dV^u}{d\alpha} \frac{d\alpha}{dt}. \quad (5)$$

The maximal willingness to pay $w(\alpha)$ at $\alpha(t) = \alpha$, is the price at which the uninformed is indifferent between buying and not buying, or solving for indifference in (5):

$$w(\alpha) = v + \lambda \left(\mathbb{E}_\theta V^\theta(\alpha) - V^u(\alpha) \right).$$

If the uninformed is charged his maximal willingness to pay in every period, then he is indifferent between buying and not buying in every period. The value function of the uninformed is then constant at $V^u(\alpha) = 0$ for all α and naturally $dV^u/d\alpha = 0$. In consequence, we can insert these values into the dynamic programming equation (5) to obtain

$$0 = v - w(\alpha) + \lambda \left(\mathbb{E}_\theta V^\theta(\alpha) - 0 \right), \quad \forall \alpha \in [0, 1]. \quad (6)$$

The willingness to pay, $w(\alpha)$, is now given by the same condition in every period, and hence permits a constant solution $w(\alpha) \triangleq w$ for all $\alpha \in [0, 1]$. The continuation value of the informed buyer is the expected discounted value of the purchases at a price equal to the willingness to pay of the uninformed, or

$$\mathbb{E}_\theta V^\theta(\alpha) = \frac{1}{r} \mathbb{E}_\theta \max \{ \theta - w, 0 \}. \quad (7)$$

The maximal willingness to pay, w , is given as the solution to equation (6) after using (7):

$$w = v + \frac{\lambda}{r} \mathbb{E}_\theta \max \{ \theta - w, 0 \}. \quad (8)$$

The equation (8) has a unique solution as the left hand side is strictly increasing and the right hand side is strictly decreasing in w . The willingness to pay w is then defined on the basis of the primitives of the model itself, namely r, λ , and $F(\theta)$. The excess willingness to pay over and above the expected value of the product, $w - v$, represents the option value of the current purchase for the uninformed buyer if all future prices are given by the willingness to pay of the uninformed customers.

We now relate the willingness to pay of the uninformed to the *marginal willingness to pay*, denoted by \hat{w} . The marginal situation we consider is a final time t at which the uninformed can buy the new good, after which the seller offers the product only to the informed agents. (The argument supposes that the seller can discriminate between informed and uninformed buyers. This simply facilitates the representation of the problem but the possibility on the basis of the identity is not used in the equilibrium analysis.) For this arbitrary marginal time t , with $\alpha(t) = \alpha$, we now compute the willingness to pay by the uninformed. We observe that after period t , the seller will optimally offer his product at price $p = \hat{p}$ as this will maximize the revenue from the informed agents. We can now again use the value function (5) to compute the marginal willingness to pay, \hat{w} , at $\alpha(t) = \alpha$:

$$rV^u(\alpha) = \max \left\{ v - \hat{w} + \lambda \left(\mathbb{E}_\theta V^\theta(\alpha) - V^u(\alpha) \right), 0 \right\} + \frac{dV^u}{d\alpha} \frac{d\alpha}{dt}.$$

As the continuation value of the uninformed is, by hypothesis, $V^u(\alpha) = 0$, and the value function of the informed is, similar to (7), given by

$$\mathbb{E}_\theta V^\theta(\alpha) = \frac{1}{r} \mathbb{E}_\theta \max\{\theta - \hat{p}, 0\},$$

the marginal willingness to pay is given as a solution to:

$$v - \hat{w} + \frac{\lambda}{r} \mathbb{E}_\theta \max\{\theta - \hat{p}, 0\} = 0,$$

or after rearranging:

$$\hat{w} = v + \frac{\lambda}{r} \mathbb{E}_\theta \max\{\theta - \hat{p}, 0\}. \quad (9)$$

The marginal willingness to pay \hat{w} is again defined on the basis of the primitives of the model itself, namely $r, \lambda, F(\theta)$ and the optimal static price \hat{p} . The marginal willingness to pay, \hat{w} , and the willingness to pay, w , can be related to the optimal static price \hat{p} through (8) and (9) as follows: either (i) $\hat{w} < w < \hat{p}$ or (ii) $\hat{w} = w = \hat{p}$, or (iii) $\hat{w} > w > \hat{p}$. We shall refer henceforth to case (i) as a *low option value* environment and to (ii) (by convention) and (iii) as a *high option value* environment. It is natural to think of the high option value case as a market where the buyers are initially optimistic about the product. The low option value case is then a market with a relatively more pessimistic clientele.

The ranking of the marginal willingness to pay relative to the willingness to pay and the optimal static price is quite intuitive. If the willingness to pay is below the optimal static price, then the marginal willingness \hat{w} must satisfy $\hat{w} < w$ as the option value in the computation of the marginal willingness to pay arises from the relatively high optimal static prices rather than the relatively lower willingness to pay. Similarly, the ranking is reversed when the willingness to pay exceeds the optimal static price.

An immediate consequence of the definitions is the following Lemma.

Lemma 1 (Option Value)

The (marginal) willingness to pay, w and \hat{w} , are increasing in λ and decreasing in r .

4 Equilibrium

In the following equilibrium analysis we employ the dichotomy between low and high option value to represent the optimal launch strategy as the solution to a specific stopping

problem. In Subsection 4.1, we begin with the strategy for the low option value and continue in Subsection 4.2 with the high option value. The equilibrium prices and quantities are analyzed in Subsection 4.3. Finally we show in Subsection 4.4 that the unique Markov perfect equilibrium is essentially the only subgame perfect equilibrium of the game.

4.1 Low Option Value

We start the equilibrium analysis with the low option value case, where the willingness to pay is below the static optimal price, or $\hat{w} < \hat{p}$. All buyers are uninformed at $t = 0$ and it is clearly in the monopolist's best interest to sell to the uninformed segment. Over time, many of the uninformed buyers become informed and the market begins to look more like the complete information market. Eventually, the monopolist will then find it advantageous to charge a price close to the static monopoly price. If the option value is low, this implies that the uninformed segment stops buying because the equilibrium price \hat{p} is higher than the willingness to pay \hat{w} . In consequence, the monopolist has to decide how long he wishes to serve the uninformed market segment before he stops selling to the uninformed. The problem for the monopolist then becomes a stopping problem, in which he has to determine the critical market size for the informed segment. At the stopping point, denoted by $\hat{\alpha}$, the optimal price policy switches from \hat{w} to \hat{p} .

Next we describe the marginal conditions which characterize the stopping point $\hat{\alpha}$. Suppose that at $\hat{\alpha}$ the monopolist is indifferent between charging \hat{w} or \hat{p} . By selling dt additional units of time to the uninformed, the profit to the monopolist is given by:

$$\begin{aligned} & ((1 - \hat{\alpha}) \hat{w} + \hat{\alpha} \hat{w} (1 - F(\hat{w}))) dt \\ & + \frac{e^{-rdt}}{r} (\hat{\alpha} + \lambda(1 - \hat{\alpha}) dt) ((1 - F(\hat{p})) \hat{p}). \end{aligned}$$

Since $p[1 - F(p)]$ is strictly quasiconcave and since $\hat{w} < \hat{p}$, it is never optimal to charge a price below \hat{w} . If it is optimal to stop selling to the uninformed at $\hat{\alpha}$, the profit from this strategy must equal the profit from stopping at $\hat{\alpha}$, which is given by

$$\frac{\hat{\alpha} (1 - F(\hat{p})) \hat{p}}{r}.$$

Requiring the equality of these two expressions yields:

$$(1 - \hat{\alpha}) \hat{w} + \hat{\alpha} \hat{w} (1 - F(\hat{w})) = \left(\hat{\alpha} - \frac{\lambda(1 - \hat{\alpha})}{r} \right) (1 - F(\hat{p})) \hat{p}. \quad (10)$$

If we denote by $\pi(p, \alpha)$ the flow profit to the monopolist from price p when α is the fraction of informed buyers, then the above equation can be written as

$$\pi(\hat{p}, \hat{\alpha}) - \pi(\hat{w}, \hat{\alpha}) = \frac{\lambda(1 - \hat{\alpha})(1 - F(\hat{p}))\hat{p}}{r}. \quad (11)$$

This equality has a simple economic intuition. The left hand side represents the differential gains from extracting surplus from the informed agents and the right hand side represents the benefits from building up future demand. The later is the long term gain from an additional inflow of $\lambda(1 - \alpha)$ informed buyers of whom $[1 - F(\hat{p})]$ are willing to purchase at complete information monopoly prices. As the right hand side is positive for all interior values of \hat{p} , we conclude that with low option values, the monopolist sacrifices current profits to build up future demands.

By the optimality of the complete information price \hat{p} , we have at $\alpha = 1$:

$$\pi(\hat{p}, 1) - \pi(\hat{w}, 1) > 0.$$

By continuity, it then follows immediately from equation (11) that the optimal stopping point $\hat{\alpha}$ satisfies $\hat{\alpha} < 1$ and in equilibrium a positive proportion of buyers, $(1 - \hat{\alpha})$ will remain uninformed forever and will eventually be priced out of the market. Since both \hat{p} and \hat{w} are independent of α , the comparative statics of the optimal stopping point determined by equation (11) are straightforward to calculate.

Lemma 2 (Stopping Point with Low Option Value)

If $\hat{w} < \hat{p}$, then:

1. $\hat{\alpha} < 1$;
2. $\hat{\alpha}$ is increasing in λ and decreasing in r .

4.2 High Option Value

We now consider the case of a high option value where $\hat{w} \geq \hat{p}$. As the willingness to pay exceeds the static complete information price, the uninformed buyers continue to make purchases even if the price is set optimally with respect to the informed buyers. The high option value by the uninformed buyers provides a motive for the monopolist to offer higher prices than the complete information price \hat{p} . Initially, as the informed segment does not

yet exist, he will then offer prices which will leave the uninformed agents just indifferent between buying and not buying. The monopolist can thus extract initially all the surplus from current purchases of the uninformed agents. As the size of the informed segment grows, any price which leaves the uninformed indifferent, will lead to a substantial shortfall in revenue from the informed segment relative to the revenue resulting from the optimal price \hat{p} for the informed segment. The monopolist therefore has to determine at which point he start to leave surplus to the uninformed buyers and set his price below their willingness to pay in order to attract a larger share of informed buyers.

To start the analysis in this case, we denote the current willingness to pay by the uninformed buyers by $w(\alpha)$ where α is the current fraction of informed buyers. By the same argument as in the previous subsection, $w(\alpha)$ satisfies the following equation

$$v - w(\alpha) + \lambda \left(\mathbb{E}_\theta V^\theta(\alpha) - V^u(\alpha) \right) = 0$$

as long as indifference between purchasing and not purchasing holds. The flow revenue of the seller is then given by

$$\pi(p, \alpha) = (1 - \alpha)p + \alpha(1 - F(p))p, \quad (12)$$

provided that $p \leq w(\alpha)$ and is given by

$$\pi(p, \alpha) = \alpha(1 - F(p))p$$

if $p > w(\alpha)$. In other words as long as the price does not exceed the willingness to pay of the uninformed, the seller sells to both segments, and only at a price higher than $w(\alpha)$, the uninformed segment drops out. As the option value is high, it follows that $p(\alpha) > w(\alpha)$ is never optimal as $w(\alpha) > \hat{p}$ by hypothesis, and thus the only reason to charge $p(\alpha) > \hat{p}$ is the marginal revenue from the uninformed. The monopolist does not deviate downwards (locally) from the *full extraction* prices $p = w(\alpha)$ as long as:

$$\frac{\partial \pi(w(\alpha), \alpha)}{\partial p} \geq 0,$$

or

$$1 - \alpha \geq \alpha(w(\alpha) f(w(\alpha)) + F(w(\alpha)) - 1). \quad (13)$$

As the static revenue function $p(1 - F(p))$ is assumed to be strictly quasiconcave, it follows that the local condition is a global condition as well. It is also clear that for each fixed $w(\alpha)$,

this equation holds for α small. As a result, the optimal sales path starts always with initial sales to the uninformed at the full extraction price. Once the inequality in (13) is reversed, the optimal price setting just follows from solving:

$$1 - \alpha + \alpha(1 - F(p) - pf(p)) = 0. \quad (14)$$

The second order conditions for maximization imply as usual that the equilibrium price is decreasing as the number of informed buyers is increasing. As $\alpha \rightarrow 1$, the equilibrium price converges to the myopic equilibrium price \hat{p} . We observe that even though this has the flavor of a static optimization problem, the dynamics of the model still enter into the determination of prices through the evolution of α .

The problem of the monopolist is not quite solved yet since the variable $w(\alpha)$ in (13) is endogenous. It is computed on the basis of the equilibrium continuation values and thus relies not only on local but also on global information about the equilibrium price path. We can then compute the stopping point by matching the prices resulting from full extraction and the optimal price when the marginal buyer is the informed buyer.

Lemma 3 (Stopping Point with High Option Value)

If $\hat{w} \geq \hat{p}$, then:

1. $\hat{\alpha} \leq 1$;
2. $\hat{\alpha}$ is decreasing in λ and increasing in r .

If we compare the comparative static results in Lemma 2 and 3, then it is worth observing that the stopping points move in opposite directions as a function of r and λ . Consider therefore an initial distribution of valuations so that the same model generates both low and high option values for different values of the discount rate r . For very high discount rates, the learning stops very early and very few buyers are informed. As the discount rate decreases the seller becomes more patient and the buyers value the information more, so that the initial price will be higher and learning will proceed for some time. Eventually r will become so high that the option value exactly equals the static optimal price, or $w = \hat{w} = \hat{p}$ and the price will be constant for all t and $\hat{\alpha} = 1$. As r decreases even further, the optimal pricing strategy changes in nature. The option value for the uninformed buyers increases so that the price at which the uninformed buyer is still willing to buy increases as

well. If the seller then maintains the uninformed buyer as the marginal buyer, he sets the price too high to receive substantial sales from the informed buyers. In consequence, he will drop the price below the option value as soon as there are a sufficient number of informed buyers. As r decreases, the problem gets worse and it is optimal to have the informed buyers as marginal customers earlier. The stopping point $\hat{\alpha}$ then drops again below 1, but this time the uninformed customers become inframarginal rather than excluded buyers. The comparative statics are simply reversed for λ . In Figure 1, the increasing part of the graph corresponds to the high option value case and the decreasing part belongs to the low option value case.

INSERT HERE FIGURE 1: STOPPING POINT FOR VARYING $\frac{\lambda}{r}$.

4.3 Equilibrium Price

Next we describe the dynamics of the prices and quantities directly in terms of the time coordinate t . This allows us to discern the intertemporal patterns of the two cases in a more transparent manner. Consequently, the equilibrium variables are written here as function of time t rather than the state variable $\alpha(t)$. Yet, in equilibrium, the state variable $\alpha(t)$ is a deterministic and monotone function of time t , and hence the mapping from $\alpha(t)$ to t is immediate. With either a low or a high option value, the uninformed buyers are indifferent between purchasing and not purchasing in the initial periods until $\alpha(t) = \hat{\alpha}$. From the value functions we can deduce the differential equation that governs the prices prior to $\hat{\alpha}$. This leads us to the characterization of the intertemporal properties of the price path and the path of sales.

Proposition 1 (Evolution over Time)

1. $p(t)$ satisfies for all t with $\alpha(t) \leq \hat{\alpha}$:

$$\dot{p}(t) = r(p(t) - v) - \lambda \mathbb{E}_\theta \max\{\theta - p(t), 0\}. \quad (15)$$

2. $p(t)$ is decreasing and concave in t for all $\alpha(t) < \hat{\alpha}$;
3. $q(t)$ is initially decreasing and convex in t .

Our interpretation of the two qualitatively different price path goes as follows. In the high option value case, the monopolist skims the high valuation buyers in the market (the uninformed buyers) with a high and declining price. In the low option value case, the monopolist makes introductory offers to increase the number of goodwill customers once the price is raised. It should be noted that with low and high option value, the prices do not change by large amounts before $\hat{\alpha}$ and as a result, adjustment costs to changing prices might well force the monopolist to adopt a two price regime with low initial prices followed by higher prices in the low option value, and high prices followed by low prices with high option value.

The intertemporal pricing policies are graphically depicted in Figures 2 and 3 for low and high option value, respectively. The regular lines describe the equilibrium price and the dotted line the commitment price as a function of time. The commitment price path is discussed in detail in Section 5. With the low option value, the introductory price slowly decreases until it reaches a value equal to the marginal option value and at that point, the seller ceases to pursue new customers and sells only to informed customers with sufficiently high valuations. With the high option value, the discount factor r is very small, and hence the option value for the uninformed buyer is almost constant. In consequence, the price declines very slowly (and in the graph it almost appears as a constant) until the seller begins to seek sales more aggressively from the informed customers. At this point, the price begins to decrease more rapidly and eventually converges to the static monopoly price.

INSERT HERE FIGURE 2: EQUILIBRIUM PRICE FOR LOW OPTION VALUE

INSERT HERE FIGURE 3: EQUILIBRIUM PRICE FOR HIGH OPTION VALUE

Notice also that our model provides a theoretical predictions for the joint movements of prices and equilibrium quantities. These effects should be taken into account when estimating the demand for new products. If one estimates a static demand function for a product using data that includes observations of prices for $\alpha < \hat{\alpha}$, it is clear that the resulting estimators are biased. Hence the dynamics of the market demand ought to be taken into account when estimating e.g. the consumer surplus from recently introduced products.

The equilibrium characterization also allows for a fairly complete picture of the asymptotic behavior of the equilibrium.

Proposition 2 (Asymptotic Behavior)

1. $\lim_{t \rightarrow \infty} p(t) = \hat{p}$,
2. $\lim_{t \rightarrow \infty} q(t) = \hat{q}$ if $\hat{w} \geq \hat{p}$,
3. $\lim_{t \rightarrow \infty} q(t) < \hat{q}$ if $\hat{w} < \hat{p}$.

This proposition shows immediately how the model with forward looking behavior differs from the myopic model. Even though the long run behavior of the prices is similar in the myopic model, it is clear that the convergence to the myopic quantities is more likely with forward looking behavior since $\hat{w} > v$. In the low option value case, the long run sales with myopic buyers would, in fact, be always lower than in the current forward looking model.

The following corollary shows that the optimistic (pessimistic) case is characterized by prices above (below) the perfect information monopoly price level at all points.

Corollary 1

1. If $\hat{w} < \hat{p}$, then $p(t) \leq \hat{p}$ for all t .
2. If $\hat{w} > \hat{p}$, then $p(t) \geq \hat{p}$ for all t .

4.4 Uniqueness of Equilibrium

In this subsection, we show that the equilibrium characterized in the previous sections remains the only sequential equilibrium outcome of the game, even after removing the earlier Markovian restriction on the strategies.⁵ The key to understanding this result is our informational assumption that only own past purchases and the aggregate market data are available to the players. The implication of this is that the continuation paths of play are independent of the choices of any individual buyer. Hence trigger strategy equilibria of the type where all buyers refuse high prices as long as all buyers have refused high prices in the past are not consistent with equilibrium.

⁵Note that the game is formally one of incomplete information. Hence if we follow the canonical model of incomplete information, the resulting extensive form game has no proper subgame and the distinction between subgame perfect and Nash equilibrium disappears. In a sequential equilibrium, the behavior of all players is optimal after all conceivable (private) histories.

Before we argue the uniqueness result formally, it might be helpful to see how the restriction to aggregate purchase decision affects the determination of the equilibrium policies. For the informed buyer the myopic decision rule to buy if and only if the value of the object is larger than the current price is a strictly dominant strategy in the game for arbitrary beliefs about future price paths. This follows from the fact that the pricing policy cannot depend on individual purchases.

Given the decision rule of the informed buyers, a sufficient statistic for the monopolist's pricing in any period t is the maximal willingness to pay of the uninformed buyers. The current willingness to pay may be quite arbitrary if the monopolist's strategy conditions future prices on the current price in a nontrivial manner. As long as the uninformed buyers are the marginal buyers, the monopolist already extracts the maximal surplus from the buyers in the Markov Perfect Equilibrium. The only remaining room for different equilibrium values and policies to arise is therefore after the uninformed buyers cease to be the marginal buyers. But again, in this region, the monopolist makes the optimal intertemporal decision given the size of the informed market segment.

The precise argument to establish uniqueness uses again the distinction between low and high option value. We present the argument fully for the low option value case and comment on the differences for the optimistic case.

Lemma 4 *For low option value, $\hat{w} < \hat{p}$, the maximal willingness to pay for the uninformed buyers (over all future continuation price paths) is strictly below \hat{p} .*

In the proof of this result, it is also shown that the maximal willingness to pay for the uninformed buyers obtains when the price path makes the uninformed indifferent between buying and not buying at all future points. As the size of the uninformed segment is decreasing over time and their willingness to pay is below \hat{p} , we next establish that for all subgame perfect equilibria, there is a maximal size of the informed segment, $\tilde{\alpha} < 1$, such that for all $\alpha \geq \tilde{\alpha}$, the monopolist will find it optimal to set the price $p(t) = \hat{p}$.

Lemma 5 *For the low option value,*

1. *there exists $\tilde{\alpha} < 1$ such that $p(t) = \hat{p}$ in any equilibrium if $\alpha(t) > \tilde{\alpha}$;*
2. *there exists $T < \infty$ such that $p(t) = \hat{p}$ for all $t \geq T$.*

The upper bound $\tilde{\alpha}$ on the informed market segment naturally translates into a finite upper bound T on the time horizon such that after T the seller is guaranteed to charge the perfect information price \hat{p} . With this result in place, the infinite horizon game can be analyzed as a finite horizon game and it is now easy to prove the uniqueness claim.

Proposition 3 (Uniqueness)

Every sequential equilibrium has the same equilibrium path as the unique Markov Perfect equilibrium path.

The argument for the case of a high option value is very similar and hence omitted. The central step is again a truncation argument. Only this time, it shows that there is a finite upper bound T at which the uninformed cease to be the marginal buyers, but stay in the market, and the equilibrium pricing rule will again revert to a myopic pricing rule, derived earlier in (14), but one which still relies on the size of market segments, α and $1 - \alpha$, respectively. Using a backward induction argument, it can again be shown that the stopping point must coincide for all subgame perfect equilibria with the one derived for the Markov Perfect equilibrium.

5 Commitment and Welfare

The optimal equilibrium policies of the monopolist evidently lead to distortions relative to the efficient allocation. The inefficiency of the equilibrium allocation per se does not come as a surprise as already the static equilibrium leads to distortions due to the marginal revenue considerations of the monopolist. Yet, two new sources of inefficiencies emerge in the intertemporal analysis. First, in the low option value, a substantial proportion of the uninformed buyers are eventually priced out of the market and do not have a sufficient amount of time to learn their true valuations. Second, in the initial phase of the game, the seller raises prices above the static optimum so as to extract the option value of the purchases accruing to the uninformed buyers. This leads to further inefficiencies among the informed buyers.

These new distortionary effects emerge from the intertemporal trade-offs faced by the seller. For this reason, we now ask whether these inefficiencies can be overcome when the seller has the ability to commit himself to an entire future price path at the beginning of

the game, at $t = 0$. The objective in this section is then to determine the commitment policy of the seller and compare it with the earlier, time-consistent, equilibrium policies.

The qualitative features of the solution in the model with commitment are quite different from the equilibrium solution in the previous section for the low option value case. For high option value case, the difference is much smaller and as a result, we provide the full analysis of the low option value case and simply comment on the solution for the high option value. At the outset of the game, the monopolist commits to a price path $\bar{p}(t)$, where we use the upper bar to make the distinction to the sequentially rational case. Each $\bar{p}(t)$ induces a choice path for the uninformed as well as for the informed consumers. Again, it is clear that there is an optimal Markovian strategy for the monopolist and to conform with the notation of the previous sections, we denote the commitment price path by $\bar{p}(\alpha)$.

Since we are not allowing for price discrimination elsewhere in this model, we do not consider other forms of commitment such as committing to an intertemporal two part tariff or differential prices to first time buyers and existing clientele. Nonetheless we would like to point out that with the initial homogeneity among the buyers, a simple two part tariff would allow the seller to extract all the surplus from the consumers and implement the efficient allocation. The seller could simply charge the consumer up-front with an initial fee equal to their surplus and in exchange offer them a commitment to provide the product at a price equal to marginal cost in all future periods. This solution clearly relies heavily on the initial homogeneity and the ability of the consumer to pay large up front fees.

We start the analysis of the commitment policy by showing that in the low option value case, the buyers receive no surplus in equilibrium. The following lemma states that in the early phase of the game, the uninformed buyers must be indifferent between purchasing and not purchasing. This is quite intuitive as all the buyers are uninformed in the beginning and since the share of the informed increases continuously in time.

Lemma 6 (Indifference) *There exists a $\tau > 0$ such that for the optimal price path $\bar{p}(\alpha)$,*

$$V^u(0) = e^{-r\tau} V^u(\alpha(\tau)).$$

With this lemma in place, it is easy to show that in the low option value case, the expected payoff of the uninformed buyer in equilibrium is initially equal to zero. This result follows from the fact that in the low option value case, $w < \hat{p}$ and hence an increase

in the current price results in a higher current revenue from both market segments (if at the original prices, the uninformed prefer strictly to make a purchase). We can extend this result to show that along the optimal price path, the uninformed buyer is always indifferent. By the arguments of the previous section, we then know that this implies a constant price path and hence the solution is quite different compared to the equilibrium solution of the previous section.

Proposition 4 (Commitment with Low Option Value)

In the low option value case,

1. *the expected value of the uninformed buyer is $V^u(t) = 0$;*
2. *the optimal commitment price is constant for all $\alpha \in [0, 1]$:*

$$\bar{p}(\alpha) = w.$$

The above argument is not valid for the high option value case. It is easy to see that the first result in Proposition 4 cannot hold generally for the high option value case. While the central ideas in the high option value case are the same as in the low option value case, the arguments are slightly more cumbersome. For this reason, we only provide a summary of the results for that case.

Proposition 5 (Commitment with High Option Value)

In the high option value case, the optimal commitment price path is

$$\bar{p}(\alpha) = w \text{ for all } \alpha,$$

or

$$\bar{p}(\alpha) \rightarrow \hat{p} \text{ as } \alpha \rightarrow 1.$$

We conclude this section by discussing the welfare implications of the above proposition. In the high option value case, the commitment price is always above the equilibrium price. To see this, notice first that by Lemma 6, we know that initially along the optimal commitment price path, the uninformed buyers are indifferent between purchasing and not purchasing. It is easy to show that it is never optimal to commit to leaving surplus to the uninformed buyer at an earlier date than in the equilibrium solution. Consider next the

points where the monopolist prices according to the myopic pricing rule. Since the myopic price is chosen optimally, there are no first order instantaneous losses from charging a higher price. Since the myopic price is below the uninformed buyers' maximum willingness to pay, an increase of dp in current prices results in a higher willingness to pay in the earlier periods. The monopolist can thus extract extra surplus from the uninformed through higher prices in the earlier periods. Since the prices are higher than in the equilibrium model, total surplus in the model decreases as a result of the commitment possibility. The commitment price path is represented as a dotted line in Figure 2 and 3 and illustrates the systematic difference between time consistent and commitment policies.

In the low option value case, ex ante expected consumer surplus is zero in the commitment model and in the equilibrium model. Hence the monopolist extracts full social surplus in both cases. A revealed preference argument is then sufficient to show that commitment is good for the social surplus.

Proposition 6 (Welfare)

The monopolist's ability to commit to a price path decreases social welfare with a high option value and increases social welfare with a low option value.

6 Stationary Model

So far, we have been concerned with the dynamic demand behavior of infinitely lived agents who learn over time their true valuation for a new product. We have described the optimal pricing policy for the introduction of a new product. While the buyers are ex ante identical in their expected valuation of the new product, their personal experiences eventually lead them to have heterogeneous and idiosyncratic valuations.

In this section, we consider a market with a constant inflow and outflow of consumers. This naturally expands the scope of our analysis as many products face a constant renewal in their customer base, either because of the ageing of the customers or other systematic and age-related changes to the agents' preferences. In the following, we analyze the steady state equilibrium and omit the description of the transition path (this is obtained by combining the prior analysis with the steady state). A market of this type can be thought of as a market for an established experience good. The steady state equilibrium consists of a

constant proportion of informed buyers, denoted by α^* , and a, possibly random, price policy p^* .

We model the change in the population by constant entry rate λ of new customers and equal exit rate γ of old customers. The new customers are all initially uninformed and become informed according to the same information technology as in the previous sections. The old customers, informed and uninformed, leave the market at rates proportional to the current population, or $\alpha^*\gamma$ and $(1 - \alpha^*)\gamma$ respectively.

The characterization of the equilibrium price policy can be given again using the notion of option value. The willingness to pay w of the uninformed agent is naturally being adjusted by exit rate γ which increases the effective discount rate from r to $r + \gamma$:

$$w = v + \frac{\lambda}{r + \gamma} \mathbb{E}_\theta \max \{\theta - w, 0\}.$$

The uninformed buyers are willing to buy at any constant price p^* provided that $p^* \leq w$. The option value is again said to be high if $\hat{p} \leq w$ and conversely is said to be low if $\hat{p} > w$.

When the option value is high, it is profitable to sell to the uninformed in all periods. The equilibrium policy of the seller can be described as the solution to a static optimization problem

$$p^* = \arg \max_{p \in \mathbb{R}_+} \{p((1 - \alpha^*) + \alpha^*(1 - F(p)))\}, \quad (16)$$

subject to $p^* \leq w$ and the steady state proportion α^* is given by

$$\alpha^* = \frac{\lambda}{\gamma + \lambda}.$$

By the high option value property, the uninformed are always willing to pay more than the static monopoly price \hat{p} . For this reason, the seller seeks to find the optimal balance between extracting surplus from the uninformed and maintaining a high sales volume from the informed. The optimal price p^* is the unique solution (by the quasiconcavity property of the static monopoly problem) in this trade-off. It can be verified that p^* is decreasing in α^* as a larger α^* leads the monopolist to give more weight to the informed customers and hence lower the price p^* to bring it closer to \hat{p} .

For the low option value environment, the analysis is more subtle and may involve a random price policy by the seller. The low option value property suggests that the seller may not want to sell to the uninformed in all periods but rather extract surplus from the informed agents. With a constant inflow of new customers, the seller cannot abandon the

new customers altogether as this would imply a diminishing customer base. If the inflow of new customer occurs only at a moderate level, then the optimal pricing policy will randomize between a low price p^* , at which the uninformed buyers are indifferent between buying and not buying, and a high price \hat{p} which optimally extracts surplus from the informed agents. We denote the respective steady state probabilities by x^* and $1 - x^*$.⁶ The resulting steady state proportion of informed customers is given by:

$$\alpha^* = \frac{\lambda x^*}{\gamma + \lambda x^*}. \quad (17)$$

With the random price policy, the low price p^* is determined by the indifference condition of the uninformed buyer. The indifference condition is simply the option value augmented by the fact that the seller alternates between a low price p^* and a high price \hat{p} :

$$p^* = v + \frac{\lambda}{r + \gamma} (x^* \times \mathbb{E}_\theta \max \{\theta - p^*, 0\} + (1 - x^*) \times \mathbb{E}_\theta \max \{\theta - \hat{p}, 0\}). \quad (18)$$

If new customers enter at a high rate, then most customers will be uninformed in steady state and it will never be optimal to charge the optimal static price \hat{p} as too few informed customers exists. It is then in the monopolist's best interest to give up on extracting surplus from the informed agents and offer in all periods the low price p^* to the uninformed agents. In this case, the probability of high price is zero, or $1 - x^* = 0$, and the low price is then simply determined by the maximal willingness to pay of the uninformed, or:

$$p^* = v + \frac{\lambda}{r + \gamma} \mathbb{E}_\theta \max \{\theta - p^*, 0\},$$

and hence $p^* = w$. We can summarize our findings in the following proposition. The complete set of equilibrium conditions, including the equilibrium value functions of the seller are detailed in the proof.

Proposition 7 (Steady State)

The steady state equilibrium is characterized:

1. *in the low option value environment by a (possibly) random price choice $\{p^*, \hat{p}\}$ with $p^* < \hat{p}$ and all uninformed agents buying only at the lower price p^* ;*

⁶Our notion of random prices in a continuous time model can be thought of as being the limit of a model where each time interval of length dt is split into subintervals of lengths x^*dt and $(1 - x^*)dt$ where the prices are set to p^* and \hat{p} respectively. A similar construction can be found in e.g. Keller & Rady (2003)

2. *in the high option value environment by a constant price $p^* \in (\hat{p}, w]$ and all uninformed agents buying in all periods.*

The comparative static behavior of the steady equilibrium variables follows the logic in the trade-off for the seller. The equilibrium price for the uninformed agents, p^* , the probability of its occurrence, x^* , and the steady state proportion, α^* , are all decreasing in r . In Figure 4, the equilibrium probability x^* of making a price offer p^* is denoted by a solid line. The equilibrium prices p^* and \hat{p} which are in the support of the random pricing policy are represented by the dotted lines.

INSERT HERE FIGURE 4: STEADY STATE POLICIES DEPENDING ON r

The behavior in λ and γ is more subtle as a change in γ or λ affects the steady state fraction of informed buyers directly. It can be shown that x^* is increasing in γ , but α^* and p^* are decreasing in γ and the opposite behavior can be established for λ .⁷

In interpreting the mixed strategy equilibria of the low option value case, it may be helpful to think about the implications for a discrete time model. With discrete time, the equilibria would be in pure strategies and they would take the form that below the steady state level α^* of informed buyers, the monopolist sells to both market segments and above α^* , she extracts rent from the informed segment at full information monopoly prices. Hence the qualitative prediction of price dispersion is valid for that model as well.

The steady prices of the high option value case are in accordance with the typical predictions for perishable goods monopolies. The dispersed prices of the low option value case are more common in models of durable goods with entry of new buyers. For example Sobel (1991) shows that equilibria in the durable goods case take the form that occasionally, the good is sold to a large part of the clientele and in the intervening periods, the price is set to skim the surplus from the high valuation buyers. In this light, our model indicates that when buyers are initially optimistic about the quality of an experience good (the high option value case), the market outcomes are similar to those with perishable search goods.

⁷The last result holds if the expected value of the object is positive, or $v \geq 0$. If $v < 0$, then an increase in the entry and exit rate, will eventually force the willingness to pay of the uninformed below zero. In consequence, the seller wishes to sell to the informed eventually at an increasing rate, further lowering the probability of sale at a low price, and in equilibrium no sales will occur at all even though it would be socially efficient to transact.

When buyers are more pessimistic, the durable nature of the information purchased in combination with the good becomes more important and the market outcomes share some characteristics with durable goods markets.

Finally, we briefly comment on the role of commitment in the steady state environment. As in the earlier section, we again label the relevant commitment variables as $\bar{\alpha}$, \bar{p} and \bar{x} to distinguish them from the equilibrium variables. The optimal steady state commitment policy is given as the solution to:

$$\max_{(\bar{x}, \bar{p})} \{ \bar{x}\bar{\alpha}(1 - \bar{\alpha}F(\bar{p})) + (1 - \bar{x})\bar{\alpha}\hat{p}(1 - F(\hat{p})) \},$$

subject to

$$\bar{p} \leq v + \frac{\lambda}{r + \gamma} (\bar{x} \times \mathbb{E}_\theta \max \{ \theta - \bar{p}, 0 \} + (1 - \bar{x}) \times \mathbb{E}_\theta \max \{ \theta - \hat{p}, 0 \}), \quad (19)$$

and

$$\bar{\alpha} = \frac{\lambda\bar{x}}{\gamma + \lambda\bar{x}}.$$

The constraints on the revenue maximization simply represent the willingness to pay and the steady state proportion of informed buyers.

In the high option value environment, it is obvious that the seller cannot do better than pursuing the time consistent price policy. While he could extract more revenue from the uninformed, he prefers not to increase the price so as to maintain a reasonably high sales volume from the informed sellers. And indeed the solution to the time consistent price policy was found by solving the static optimization problem (16) with the solution of $x^* = \bar{x} = 1$ and consequently $p^* = \bar{p} = w$. However, in the low option value environment, he would like to commit to higher probability \bar{x} of offering low prices to the uninformed and the equilibrium price \bar{p} would then be determined by an equality in the condition (19). A higher probability of sales \bar{x} relative to x^* would translate into a higher willingness to pay by the uninformed, or $p^* < \bar{p}$. The reason that the seller cannot maintain the commitment policy in a time consistent equilibrium is that he would be tempted to offer high prices to informed more often than planned and this lowers the willingness to pay by the uninformed as the frequency of favorable prices is low. In consequence, the commitment solution would display $\bar{\alpha} > \alpha^*$, and thus share the same welfare enhancing properties commitment had in the low option value environment with a constant group of buyers.

7 Conclusion

In this paper, we have shown that the optimal sales policy of a monopolist in a model of experience goods is qualitatively different depending on whether the buyers are initially optimistic or pessimistic about the good. With pessimistic buyers, it is in the monopolist's best interest to build up a sufficient base of goodwill clientele. To achieve this, the monopolist sacrifices current profit by pricing low in order to find new future buyers. The durability of information about the product quality thus plays a key role in this situation. In the optimistic case, managing information is less important as the uninformed buyers have a high option value and are willing to buy at the static optimal prices. In this case the monopolist's optimal price path can be seen as an attempt to skim the uninformed segment of the market until the informed segment becomes overwhelming large.

We have kept the model as simple as possible in order to highlight the dynamics of price setting. The modeling strategy of the current paper could be used to investigate models with more general specifications for either the buyers' valuations of the product or the strategic environment. An interesting instance of a more general demand structure would be one where the buyers differ in their willingness to pay for quality, but the perceived quality is idiosyncratic and must be learned over time. Regarding the competitive structure of the model, the natural next step would be consider the role of idiosyncratic learning in a strategic environment against either a known or similarly unknown product. An interesting variation of the current model and of special importance for the pharmaceutical market would be to consider optimal pricing when a competitor will only appear in T periods hence, induced by the expiration of the patent, see Berndt, Ling & Kyle (2003) for an empirical analysis of this situation.

An interesting direction for further research would be to analyze a multi-product monopolist directly in the steady state model. In many industries, different versions of the same product are offered in a given market. For example in the software industry, some of these varieties are meant to be offered to first time users and contain restrictions such as time limitation, limited computing power and alike. The steady state with monopolistic discrimination, in terms of either quantity or quality, could then analyze the optimal product menu in the presence of new and old customers. The regular adverse selection problem would then contain a novel dynamic element as some consumer do not yet know their true

valuation for the product.

8 Appendix

The appendix collects the proofs of all the results in the main body of the paper. For notational convenience, we shall adopt a standard notation from probability theory by writing:

$$(\theta - p)^+ \triangleq \max\{\theta - p, 0\}.$$

Proof of Lemma 2. (1.) The optimal stopping point is given as a solution to (10) by:

$$\hat{\alpha} = \frac{\hat{w} + \frac{\lambda}{r}\hat{p}(1 - F(\hat{p}))}{\hat{w}F(\hat{w}) + \hat{p}(1 - F(\hat{p}))\left(1 + \frac{\lambda}{r}\right)}. \quad (20)$$

For simplicity we define $\sigma \triangleq \frac{\lambda}{r}$, and with this we rewrite equation (20) as:

$$\hat{\alpha} = \frac{\hat{w} + \sigma\hat{p}(1 - F(\hat{p}))}{\hat{w} + (1 + \sigma)\hat{p}(1 - F(\hat{p})) - \hat{w}(1 - F(\hat{w}))}, \quad (21)$$

which shows that $\hat{\alpha} < 1$ since $\hat{p}(1 - F(\hat{p})) - \hat{w}(1 - F(\hat{w})) > 0$.

(2.) After replacing \hat{w} by its explicit expression given in (9), we rearrange equation (21) to get:

$$\begin{aligned} & \hat{\alpha} \left((v + \sigma\mathbb{E}(\theta - \hat{p})^+) F(v + \sigma\mathbb{E}(\theta - \hat{p})^+) + \hat{p}(1 - F(\hat{p})) (1 + \sigma) \right) \\ &= v + \sigma\mathbb{E}(\theta - \hat{p})^+ + \sigma\hat{p}(1 - F(\hat{p})). \end{aligned} \quad (22)$$

Differentiating the equality (22) implicitly with respect to σ yields:

$$\begin{aligned} & \frac{d\hat{\alpha}}{d\sigma} (\hat{w}F(\hat{w}) + \hat{p}(1 - F(\hat{p})) (1 + \sigma)) + \hat{\alpha} ((\hat{w}f(\hat{w}) + F(\hat{w}))\mathbb{E}(\theta - \hat{p})^+ + \hat{p}(1 - F(\hat{p}))) \\ &= \mathbb{E}(\theta - \hat{p})^+ + \hat{p}(1 - F(\hat{p})), \end{aligned}$$

and hence

$$\frac{d\hat{\alpha}}{d\sigma} = \frac{\mathbb{E}(\theta - \hat{p})^+ (1 - \hat{\alpha}F(\hat{w}) - \hat{\alpha}\hat{w}f(\hat{w})) + (1 - \hat{\alpha})\hat{p}(1 - F(\hat{p}))}{\hat{w}F(\hat{w}) + \hat{p}(1 - F(\hat{p})) (1 + \sigma)}.$$

The denominator is clearly positive. For the numerator, we observe that $1 - \hat{\alpha}F(\hat{w}) - \hat{\alpha}\hat{w}f(\hat{w})$ is the derivative of the profit function $p(1 - \hat{\alpha} + \hat{\alpha}(1 - F(p)))$ evaluated at \hat{w} , which is positive by the assumed quasiconcavity of $p(1 - F(p))$ together with the fact that $\hat{w} < \hat{p}$. Therefore, the numerator is also positive, as needed. ■

Proof of Lemma 3. (1.) Suppose that for all $\alpha \geq \hat{\alpha}$, the marginal buyer is an informed buyer. The equilibrium price $p(\alpha)$ is then given as the solution of to the static revenue

maximization problem (14). The value function of an informed buyer at $\alpha(t) = \alpha$ is:

$$V^\theta(\alpha) = \int_t^\infty e^{-r(\tau-t)} (\theta - p(\alpha(\tau)))^+ d\tau,$$

The expected value of the informed buyer at $\alpha(t) = \alpha$ is:

$$\mathbb{E}_\theta [V^\theta(\alpha)] = \int_{\theta_l}^{\theta_h} \left[\int_t^\infty e^{-r(\tau-t)} (\theta - p(\alpha(\tau)))^+ d\tau \right] dF(\theta).$$

In contrast the value function of the uninformed informed buyer for a particular realization $T \geq t$ of the signal arrival time, is given by:

$$\int_t^T e^{-r(\tau-t)} (v - p(\alpha(\tau))) d\tau + \int_{\theta_l}^{\theta_h} \left[\int_T^\infty e^{-r(\tau-t)} (\theta - p(\alpha(\tau)))^+ d\tau \right] dF(\theta). \quad (23)$$

The value function of the uninformed buyer is obtained from (23) by taking the expectation over all signal arrival times $T \geq t$, or $V^u(\alpha)$ is given by:

$$\int_t^\infty \int_{\theta_l}^{\theta_h} \left[\int_t^T e^{-r(\tau-t)} (v - p(\alpha(\tau))) d\tau + \int_T^\infty e^{-r(\tau-t)} (\theta - p(\alpha(\tau)))^+ d\tau \right] dF(\theta) \lambda e^{-\lambda T} dT.$$

The willingness to pay for all $\alpha \geq \hat{\alpha}$ is given by:

$$w(\alpha) = v + \lambda \left[\mathbb{E}_\theta V^\theta(\alpha) - V^u(\alpha) \right].$$

The difference in the value functions, $\mathbb{E}_\theta V^\theta(\alpha) - V^u(\alpha)$, can therefore be written, using the above expressions as:

$$\mathbb{E}_\theta V^\theta(\alpha) - V^u(\alpha) = \int_t^\infty \int_{\theta_l}^{\theta_h} \int_t^T e^{-r(\tau-t)} (p(\alpha(\tau)) - \theta)^+ d\tau dF(\theta) \lambda e^{-\lambda T} dT. \quad (24)$$

The gain of the informed vis-a-vis the uninformed buyer, arises in all those instances where the uninformed buyer accepts the offer by the seller even though his true valuation is below the equilibrium price. It follows that $w(\alpha)$ is decreasing in α (and t) as $p(\alpha)$ is decreasing in α (and t). We can then run $w(\alpha)$ backwards as long as $w(\alpha) \geq p(\alpha)$, where we recall that $p(\alpha)$ is determined by (14).

The stopping point $\hat{\alpha}$ is the smallest α at which

$$w(\alpha) = p(\alpha). \quad (25)$$

We next argue that there is a unique stopping point $\hat{\alpha}$, by showing that $w(\alpha)$ and $p(\alpha)$ are single crossing. By hypothesis of $\hat{w} > \hat{p}$, we have

$$w(1) > p(1). \quad (26)$$

Observe further that

$$\lim_{\alpha \downarrow 0} p(\alpha) = +\infty.$$

The maximal willingness to pay, $w(\alpha)$, is a constant v and a discounted average over future prices $p(\alpha)$, represented by (24). It therefore follows that, provided $p(\alpha)$ is monotone,

$$|p'(\alpha)| > w'(\alpha),$$

which together with (26) is sufficient to guarantee existence and uniqueness of the stopping point.

(2.) We observe first that $p(\alpha)$ is independent of r . It follows that r affects the expression, $\mathbb{E}_\theta V^\theta(\alpha) - V^u(\alpha)$, only through discounting. As an increase in r decreases $w(\alpha)$, it follows that the intersection (25) is reached later and thus at a higher value of $\hat{\alpha}$. The argument for λ is similar except for the obvious reverse in the sign. ■

Proof of Proposition 1. (1.) The differential equation (15) for the full extraction prices has a unique rest point, $\dot{p}(t) = 0$, at $p(t) = w$ as w uniquely solves:

$$0 = r(w - v) - \lambda \mathbb{E}_\theta \max\{\theta - w, 0\}.$$

(2.) We show the monotonicity of $p(t)$ separately for $\hat{p} < \hat{w}$ and $\hat{p} \geq \hat{w}$. We start with the later case and argue by contradiction. Thus suppose that $p(0) > w$, then $p(t) > w > \hat{w}$ for all t . It follows that at $\alpha = \hat{\alpha}$, we have $p(\hat{\alpha}) > w > \hat{w}$, but at $a(t) = \hat{\alpha}$ we have to have $p(\hat{\alpha}) = \hat{w}$ for the uninformed buyer to be willing to buy and this leads to the desired contradiction.

Consider then $\hat{p} < \hat{w}$ and consequently $\hat{p} < w < \hat{w}$. Suppose that $p(0) > w$ and hence by the differential equation $p(t) > w$ for all t with $\alpha < \hat{\alpha}$. We also recall that as the equilibrium price path is continuous, it follows that at α such that $p(\alpha) = w > \hat{p}$, we have $w(\alpha) > p(\alpha)$. From Lemma 3 we recall that the equilibrium during the full extraction phase satisfies

$$p(\alpha) = v + \lambda \left[\mathbb{E}_\theta V^\theta(\alpha) - V^u(\alpha) \right],$$

or more explicitly:

$$p(\alpha) = v + \lambda \int_t^\infty \int_{\theta_t}^{\theta_h} \int_t^T e^{-r(\tau-t)} (p(\alpha(\tau)) - \theta)^+ d\tau dF(\theta) \lambda e^{-\lambda T} dT.$$

For notational ease we shall denote by $R(t)$ the foregone utility benefit from being uninformed in period t (i.e. the regret) or

$$R(t) \triangleq \int_{\theta_l}^{\theta_h} (p(\alpha(t)) - \theta)^+ dF(\theta),$$

and the equilibrium price is then given by

$$p(t) = v + \lambda \int_t^\infty \int_t^T e^{-r(\tau-t)} R(\tau) d\tau \lambda e^{-\lambda T} dT.$$

By hypothesis $p(t)$ is strictly increasing until $t = \hat{t}$ where $\alpha(\hat{t}) = \hat{\alpha}$ and decreasing thereafter. It is immediate that $R(t)$ shares the monotonicity properties with $p(t)$. We next show that $p(t)$ cannot be monotone increasing for all $t < \hat{t}$. After integrating with respect to T , we get

$$p(t) = v + \lambda \int_t^\infty e^{-(r+\lambda)(\tau-t)} R(\tau) d\tau.$$

Differentiating with respect to t we get

$$p'(t) = \lambda \left(-R(t) + (r + \lambda) \int_t^\infty e^{-(r+\lambda)(\tau-t)} R(\tau) d\tau \right),$$

which has to turn negative as $t \uparrow \hat{t}$ by the hypothesis of an increasing price for all $t < \hat{t}$ and the continuity of the price path. This delivers the desired contradiction. The concavity in t follows immediately from $p(0) < w$ and the differential equation (15).

(3.) The equilibrium sales are given by:

$$q(t) = (1 - \alpha) + \alpha(1 - F(p(t)))$$

as long as the uninformed buyers are participating. The equilibrium sales are governed by the following differential equation:

$$q'(t) = -(1 - \alpha) F(p(t)) \lambda - \alpha f(p(t)) p'(t).$$

It follows that, even though $p'(t) < 0$, for all α sufficiently small, $q'(t) < 0$. The second derivative is given by

$$q''(t) = (1 - \alpha) F(p(t)) \lambda^2 - 2(1 - \alpha) \lambda f(p(t)) p'(t) - \alpha \left[f'(p(t)) (p'(t))^2 + f(p(t)) p''(t) \right]$$

and again for all α sufficiently close to zero the convexity of the sales follows directly from the decreasing price. ■

Proof of Corollary 1. It follows directly from differential equation (15) that if $\hat{p} < \hat{w}$, then $\hat{p} < w < \hat{w}$ and likewise if $\hat{p} > \hat{w}$, then $\hat{p} > w > \hat{w}$. The asserted inequalities then follow from the monotonicity behavior of the prices as established in Proposition 1. ■

Proof of Lemma 4. Recall that the maximal willingness to pay, \tilde{w} , by the uninformed buyers for an arbitrary continuation price path $\tilde{\mathbf{p}} = (\tilde{p}(s))_{s=0}^{\infty}$ is given by

$$\tilde{w} = v + \lambda \mathbb{E}_{\theta} \left(V^{\theta}(\tilde{\mathbf{p}}) - V^u(\tilde{\mathbf{p}}) \right). \quad (27)$$

We claim that the right hand side of this equation is maximized if $\tilde{\mathbf{p}}$ is such that

$$\tilde{p}(t) = v + \lambda \mathbb{E}_{\theta} \left(V^{\theta}(\tilde{\mathbf{p}}(t)) - V^u(\tilde{\mathbf{p}}(t)) \right), \quad (28)$$

for all t , where $\tilde{\mathbf{p}}(t)$ denotes the continuation price path from t onwards. To see this, observe that if $p(t) > \tilde{p}(t)$, then $p(t)$ could be reduced for an instant dt , thus increasing $V^{\theta}(\tilde{\mathbf{p}}(t))$ while leaving $V^u(\tilde{\mathbf{p}}(t))$ unchanged (and thus increasing $V^u(\tilde{\mathbf{p}}(t))$ by less than $V^u(\tilde{\mathbf{p}}(t))$). A similar argument shows that an increase in $p(t)$ reduces $V^u(\tilde{\mathbf{p}}(t))$ by more than $V^{\theta}(\tilde{\mathbf{p}}(t))$ if $p(t) < \tilde{p}(t)$.

Along this path, only a constant $\tilde{p}(t) = \tilde{p}$ for all t solution is consistent with (15). By assumption of the low option value case,

$$\hat{p} > w,$$

we have $\hat{p} < \tilde{p}$, which concludes the proof. ■

Proof of Lemma 5. (1.) By Lemma 4, the maximal price from the uninformed is strictly below \hat{p} . By the strict quasiconcavity of $p[1 - F(p)]$, there is an $\tilde{\alpha}$ such that for all $\alpha > \tilde{\alpha}$, we have:

$$\tilde{p} = \arg \max_{p \in \mathbb{R}_+} \left\{ \alpha p(1 - F(p)) + (1 - \alpha) \mathbb{I}_{p < p'} \right\},$$

where \mathbb{I}_A denotes the indicator function of the event A and p' is an arbitrary price such that $p' < \tilde{p}$. The right hand side of the equation is an upper bound on the revenue from a market with $\alpha > \tilde{\alpha}$. Since the maximum of this upper bound is achieved at $p = \tilde{p}$, the claim is proved.

(2.) In light of the first part of the current lemma, we only need to show that it is never optimal to sell to the uninformed buyers at far away dates. From the previous Lemma, we

know that if the uninformed buyers buy infinitely often, then there must be an $\alpha' < \alpha^* < 1$ such that

$$\lim_{t \rightarrow \infty} \alpha(t) = \alpha'.$$

Consider then period t_ε such that $\alpha(t_\varepsilon) = \alpha' - \varepsilon$. By the same logic as in the previous part, it must be the case that $p(t) = \tilde{p}$, for all $t \geq t_\varepsilon$ is optimal for ε small enough. ■

Proof of Proposition 3. By Lemma 5, there is a smallest time $T < \infty$ such that $p(t) = \tilde{p}$ for all $t \geq T$.⁸ For all periods, $t < T$, the choices for the monopolist can be calculated by backwards induction as for the Markov perfect equilibrium. Since at T , the monopolist must be indifferent between selling to the uninformed and not selling to them in all sequentially rational equilibria, the point of stopping must be the same as in the Markovian problem. ■

Proof of Lemma 6. As a starting point, observe that there is a $\tau' > 0$ such that for all $t < \tau'$, the uninformed buy along the optimal path. Suppose now that the claim in the Lemma is not true. Then, for all $\varepsilon > 0$, we can find a $\tau_\varepsilon < \varepsilon$ such that the uninformed buyers have a strict preference to buy. Fix ε and consider a δ such that $\varepsilon > \delta > 0$. Define ε_δ by:

$$\alpha(\varepsilon_\delta) = \delta.$$

As in the previous section, the instantaneous profit to the monopolist can be written as:

$$(1 - \alpha) \min\{\bar{w}(\alpha), \bar{p}(\alpha)\} + \alpha[1 - F(\bar{p}(\alpha))]\bar{p}(\alpha),$$

where $\bar{w}(\alpha)$ is the maximal willingness of the uninformed to pay. It is thus clear that for a small enough δ , the optimal instantaneous price is $\bar{w}(\alpha)$ whenever $t < \varepsilon_\delta$. But then if δ is small enough and the uninformed buyers strictly prefer to buy at some $\tau < \tau_{\varepsilon_\delta}$, it must be the case that $\bar{p}(\alpha) < \bar{w}(\alpha)$ contradicting optimality. ■

Proof of Proposition 4. (1.) Let τ be as in the previous lemma. Consider a moment $t > \tau$ such that the uninformed buyers strictly prefer to buy for all $t' \in (\tau, t)$. In the low option value case, we know then that $\bar{p}(\alpha(t)) < w < p^m$. By the quasiconcavity of the full information profit function, the revenue from the informed buyers increases if the price in t is increased slightly. Since the uninformed strictly prefer the purchase at price $\bar{p}(\alpha(t))$, they are also willing to purchase at slightly higher prices. Furthermore, the

⁸If the smallest such time does not exist, we can use the infimum over times T that satisfy Lemma 5.

decision of the uninformed buyers in previous periods is unchanged since by the argument in Lemma 4, the option value of the previous purchases is increased by the price raise. Hence a change to $\bar{p}(\alpha(t)) + dp$ in period t results in an increase in the profit of the monopolist contradicting the assumed optimality of $\bar{p}(\alpha)$.

(2.) According to the first part of this Proposition, $V^u(\alpha) = 0$ for all α . The revenue to the monopolist is thus equal to the expected consumer surplus accruing to an individual consumer when informed. Profit maximization thus implies that it is optimal for the monopolist to maximize consumer surplus for the informed subject to the constraint that the uninformed get an equilibrium payoff of zero. But this implies constant pricing at w . ■

Proof of Proposition 7. (1.) A steady state (Markov perfect) equilibrium for the model can be calculated from the following three conditions. First of all, steady state implies a balanced flow to the pool of informed buyers:

$$\lambda(1 - \alpha^*)x^* = \gamma\alpha^*$$

or

$$\alpha^* = \frac{x^*\lambda}{\gamma + x^*\lambda}.$$

The uninformed buyers must be indifferent between buying and not buying at price p^* :

$$p^* = v + \frac{\lambda}{r + \gamma} \{x^*\mathbb{E}_\theta \max\{\theta - p^*, 0\} + (1 - x^*)\mathbb{E}_\theta \max\{\theta - \hat{p}, 0\}\}. \quad (29)$$

Finally, the value function of the seller is given by

$$rV(\alpha^*) = \max \left\{ \begin{array}{l} p^*(1 - \alpha^* + \alpha^*(1 - F(p^*))) + \frac{dV}{d\alpha^*} \frac{d\alpha^*}{dt^+}, \\ \hat{p}\alpha^*(1 - F(\hat{p})) + \frac{dV}{d\alpha^*} \frac{d\alpha^*}{dt^-} \end{array} \right\} \quad (30)$$

where

$$\frac{d\alpha^*}{dt^+} = (1 - \alpha^*)\lambda - \gamma\alpha^* > 0,$$

and

$$\frac{d\alpha^*}{dt^-} = -\gamma\alpha^* < 0.$$

The first term in the maximand (30) represents sales at a low price in which the uninformed agents buy and accompanied by an inflow and outflow into the group of informed agents:

$$\frac{d\alpha^*}{dt^+} = (1 - \alpha^*)\lambda - \gamma\alpha^* > 0.$$

The second term in the maximand (30) represents sales at a high price. The uninformed agents will not buy at the high price and hence there is only an outflow from the segment of informed agents:

$$\frac{d\alpha^*}{dt^-} = -\gamma\alpha^* < 0.$$

If $x^* \in (0, 1)$, then the seller is indifferent between pursuing either of the two pricing strategies and hence the values of the two policies have to be equal:

$$p^*(1 - \alpha^*F(p^*)) + \frac{dV}{d\alpha^*} \frac{d\alpha^*}{dt^+} = \hat{p}\alpha^*(1 - F(\hat{p})) + \frac{dV}{d\alpha^*} \frac{d\alpha^*}{dt^-},$$

which allows us to obtain the derivative of the value function as:

$$\frac{dV}{d\alpha^*} = \frac{\hat{p}\alpha^*(1 - F(\hat{p})) - p^*(1 - \alpha^*F(p^*))}{(1 - \alpha^*)\lambda},$$

which simply states that the marginal value of an informed agent is given by the difference between the revenue from an informed and an uninformed agents divided by the rate at which uninformed agents become informed. (The derivative $V'(\alpha^*)$ can be positive or negative in a steady state with a random price policy. If it is positive, it indicates that sales to the informed are of more value than to the uninformed and hence leads to fewer sales to the uninformed. If it is negative, it enhances sales to the uninformed.)

In equilibrium the value function of the seller is given by the expectation over the revenues:

$$rV(\alpha^*) = x^*p^*(1 - \alpha^*F(p^*)) + (1 - x^*)\hat{p}\alpha^*(1 - F(\hat{p})) + \frac{dV}{d\alpha} \frac{d\alpha^*}{dt}.$$

With the randomized strategy

$$\frac{d\alpha^*}{dt} = (1 - \alpha^*)\lambda x^* - \gamma\alpha^* = 0,$$

and hence the equilibrium is exactly equal to the expected revenue. We then obtain a second expression for the derivative of the value function of the seller and this time it involves the probabilities x^* by which the seller randomizes by differentiating both sides with respect to α^* , or:

$$rV'(\alpha^*) = (1 - x^*)\hat{p}(1 - F(\hat{p})) - x^*p^*F(p^*) + \frac{d^2V}{(d\alpha)^2} \frac{d\alpha^*}{dt} + \frac{dV}{d\alpha} \frac{d\alpha^*}{d\alpha} \quad (31)$$

The second term on the rhs vanishes as $\frac{d\alpha^*}{dt} = 0$ and the derivative of $\frac{d\alpha^*}{d\alpha}$ with respect to $d\alpha$ is given by

$$\frac{\frac{d\alpha^*}{dt^-}}{d\alpha} = \frac{d((1 - \alpha^*)\lambda x^* - \gamma\alpha^*)}{d\alpha} = -\lambda x^* - \gamma,$$

and hence we can rewrite (31) as:

$$V'(\alpha^*) = \frac{(1-x^*)\widehat{p}(1-F(\widehat{p})) - x^*p^*F(p^*)}{r + \gamma + \lambda x^*}.$$

By requiring that the marginal value of an informed agent is equalized in equilibrium, we obtain

$$\frac{(1-x^*)\widehat{p}(1-F(\widehat{p})) - x^*p^*F(p^*)}{r + \gamma + \lambda x^*} = \frac{\widehat{p}\alpha^*(1-F(\widehat{p})) - p^*(1-\alpha^*F(p^*))}{(1-\alpha^*)\lambda} \quad (32)$$

which together with the indifference condition of the uninformed buyer, (29), jointly determine the equilibrium price and probability (p^*, x^*) .

Finally, we can determine whether or not the equilibrium involves randomization by verifying that the seller has no profitable deviation to switch at $x^* = 1$ by rewriting (32) as:

$$p^*(1-\alpha^*F(p^*)) \geq \widehat{p}\alpha^*(1-F(\widehat{p})) + p^*F(p^*)\frac{(1-\alpha^*)\lambda}{r + \gamma + \lambda}, \quad (33)$$

where p^* is given as solution to (29) under the hypothesis that $x^* = 1$.

(2.) With the equilibrium conditions established, it remains to verify that the pure strategy $p = p^*$ is a steady state equilibrium provided that $w \geq \widehat{p}$. Clearly, a sufficient condition for (33) to hold is that it holds at $p = \widehat{p}$:

$$\widehat{p}(1-\alpha^*F(\widehat{p})) \geq \widehat{p}\alpha^*(1-F(\widehat{p})) + p^*F(\widehat{p})\frac{(1-\alpha^*)\lambda}{r + \gamma + \lambda},$$

or cancelling terms:

$$\widehat{p} \geq \widehat{p}\alpha^* + \widehat{p}F(\widehat{p})\frac{(1-\alpha^*)\lambda}{r + \gamma + \lambda}.$$

For a given α^* , the inequality is most difficult to satisfy with $r = \gamma = 0$, and it thus suffices to verify that

$$\widehat{p} \geq \widehat{p}\alpha^* + \widehat{p}F(\widehat{p})(1-\alpha^*),$$

which clearly holds as $F(\widehat{p}) \leq 1$. ■

References

- Bagwell, K. & M. Riordan. 1991. "High and Declining Prices Signal Product Quality." *American Economic Review* 81:224–239.
- Berndt, E.R., D. Ling & M.K. Kyle. 2003. The Long Shadow of Patent Expiration: Do Rx to OTC Switches Provide an Afterlife. In *Scanner Data and Price Indexes*, ed. R. Feenstra & M. Shapiro. Chicago: University of Chicago Press.
- Bhattacharya, J. & W.B. Vogt. 2003. "A Simple Model of Pharmaceutical Price Dynamics." *Journal of Law and Economics* 46:599–626.
- Ching, A.T. 2002. Consumer Learning and Heterogeneity: Dynamics of Demand for Prescription Drugs After Patent Expiration. Technical report Ohio State University.
- Coscelli, A. & M. Shum. forthcoming. "An Empirical Model of Learning and Patient Spillovers in New Drug Entry." *Journal of Econometrics* .
- Crawford, G.S. & M. Shum. 2003. Uncertainty and Learning in Pharmaceutical Demand. Technical report University of Arizona and Johns Hopkins University.
- Cremer, J. 1984. "On the Economics of Repeat Buying." *Rand Journal of Economics* 15:396–403.
- Farrell, J. 1986. "Moral Hazard as an Entry Barrier." *Rand Journal of Economics* 17:440–449.
- Judd, K. & M. Riordan. 1994. "Price and Quality in a New Product Monopoly." *Review of Economic Studies* 61:773–789.
- Keller, G. & S. Rady. 2003. "Price Dispersion and Learning in a Dynamic Differentiated-Goods Duopoly." *Rand Journal of Economics* 34:138–65.
- Milgrom, P. & J. Roberts. 1986. "Prices and Advertising Signals of Product Quality." *Journal of Political Economy* 94:796–821.
- Nelson, P. 1970. "Information and Consumer Behaviour." *Journal of Political Economy* 78:311–329.

Sobel, J. 1991. “Durable Goods Monopoly with Entry of New Consumers.” *Econometrica* 59:1455–1485.

Tirole, J. 1988. *The Theory of Industrial Organization*. Cambridge: MIT Press.

Villas-Boas, J.M. 2004a. “Consumer Learning, Brand Loyalty, and Competition.” *Marketing Science* 23:134–145.

Villas-Boas, J.M. 2004b. Dynamic Competition with Experience Goods. Technical report University of California at Berkeley.

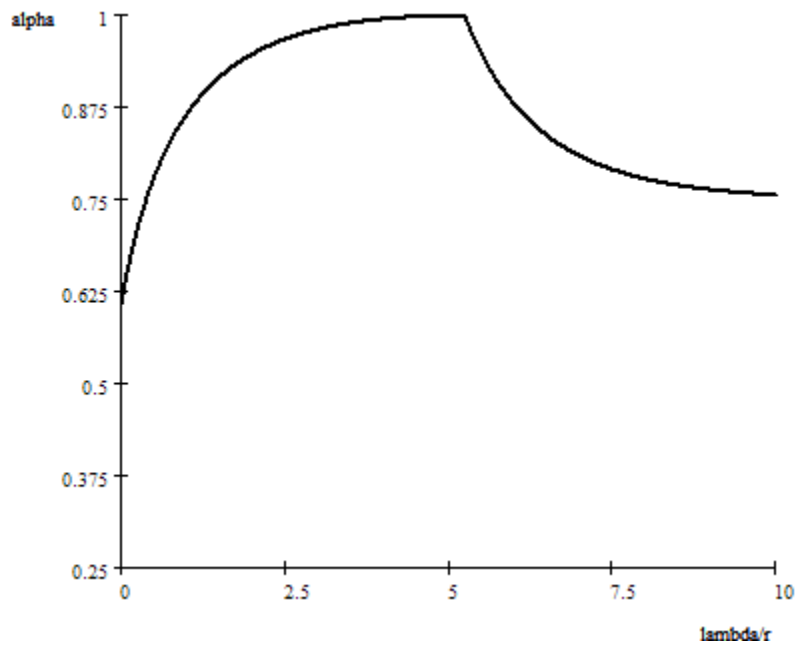


Figure 1: Figure 1: Stopping Point for Low Option Value ($\frac{\lambda}{r} < 5\frac{1}{4}$) and High Option Value ($\frac{\lambda}{r} \geq 5\frac{1}{4}$) for Varying $\frac{\lambda}{r}$.

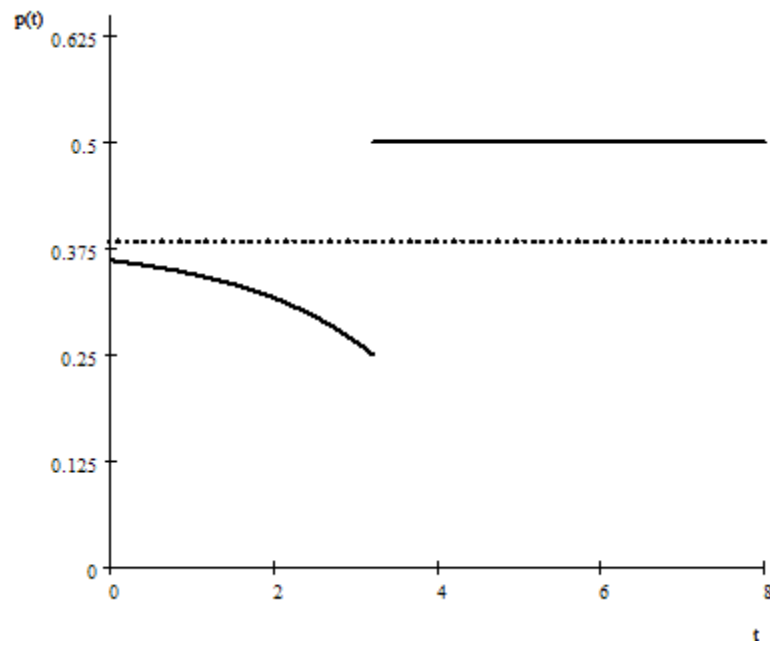


Figure 2: Figure 2: Intertemporal Price Path for Low Option Value
 (— equilibrium price path, ••• commitment price path)

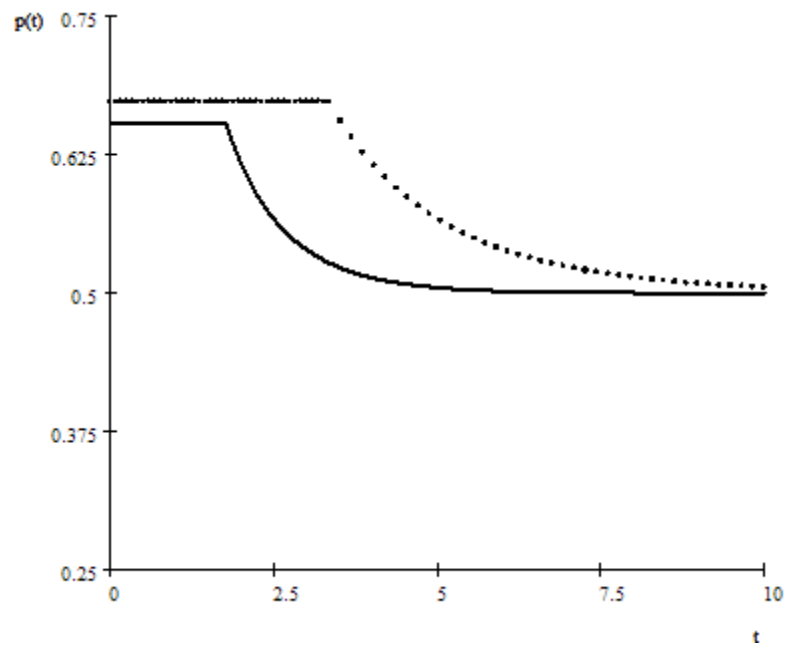


Figure 3: Figure 3: Intertemporal Price Path for High Option Value
 (— equilibrium price path, ••• commitment price path)

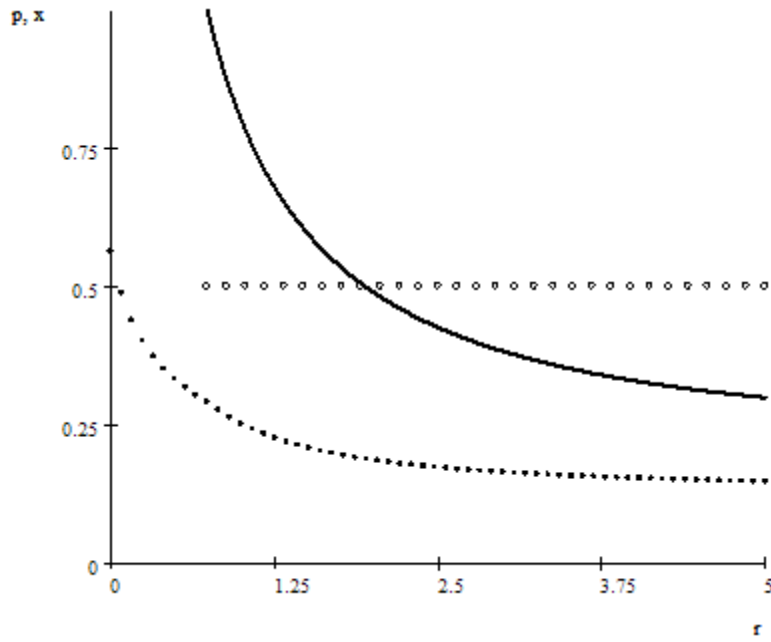


Figure 4: Figure 4: Steady State Policies for Varying Discount Rate r
 (— probability x^* , ●●● low price p^* , ○○○ high price \hat{p})