RISK AND WEALTH IN A MODEL OF SELF-FULFILLING CURRENCY ATTACKS

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September 2003

COWLES FOUNDATION DISCUSSION PAPER NO. 1433

COWLES FOUNDATION FOR RESEARCH IN ECONOMICS

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Risk and Wealth in a Model of Self-Fulfilling Currency Attacks

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July 2003
revised September 2003

Abstract

We analyze the effect of risk aversion, wealth and portfolios on the behavior of investors in a global game model of currency crises with continuous action choices. The model generates a rich set of striking theoretical predictions. For example, risk aversion makes currency crises significantly less likely; increased wealth makes crises more likely; and foreign direct investment (illiquid investments in the target currency) make crises more likely. Our results extend linearly to a heterogeneous agent population.

Keywords: Currency crisis, sunspots, global games, risk aversion, wealth, portfolio.
Jel Classification: F3, D8

1 Introduction

Currency crises are often self-fulfilling. Agents have an incentive to sell a currency short when they anticipate that others will short the currency and to go long in the currency when expecting others to do so. Many authors have developed coordination game models of currency crises.[1] In a complete information setting, such models yield multiple equilibria. Removing the assumption of common knowledge of fundamentals and knowledge of others' actions in equilibrium, Morris and Shin (1998) developed a "global games" model of currency crises with a unique prediction of when a currency crisis would occur. That work assumed that agents were risk neutral and were making a binary decision whether to attack or not. In this paper, we ask how risk aversion, wealth and portfolio composition of agents affect the likelihood of an attack. We derive a rich set of theoretical predictions.

We first assume that there is a continuum of homogenous agents, characterized by the degree of relative risk aversion, the composition of their portfolio of dollar and peso-denominated assets and their propensity to consume in dollar and peso denominated goods. They earn an interest rate premium from holding pesos. However, there is a possibility that the exchange rate will be devalued by a known amount. Each investor will choose an optimal portfolio given his beliefs about the likelihood of devaluation. Devaluation occurs if the aggregate net sales of pesos exceeds some stochastic threshold (the "fundamentals"). Each agent has a different noisy signal of fundamentals. We derive a closed-form solution for the unique equilibrium of this model. The equilibrium is characterized by the critical threshold where the currency is devalued. We examine how the critical threshold varies as parameters change.

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[1]Obstfeld (1996).

The critical threshold will always be in the range where, if there was complete information, there would an equilibrium with devaluation and an equilibrium without devaluation. The threshold implicitly determines what we call the "sunspot" probability: the probability of devaluation conditional on fundamentals being in the multiple equilibrium range of the complete information model. The sunspot probability is a measure of the likelihood of a self-fulfilling attack when fundamentals do not require it. By showing how the sunspot probability depends on the parameters of the model, we develop comparative static predictions that would not arise in a complete information model.

Some key findings are:

1. **Risk Aversion**. Under the "one way bet" assumption - i.e., the interest rate differential from holding pesos is much smaller than the capital loss from holding pesos in the event of a devaluation - risk aversion has a very large effect. If agents are risk neutral, the sunspot probability is close to 1, but if agents' constant relative risk aversion is greater than 1, the sunspot probability is less than $\frac{1}{2}$ and, in the limiting case of infinite risk aversion, is close to 0. The intuition is that for a risk averse agent, increased returns to devaluation reduce the size of the long position he must take to hedge against the possibility of devaluation.

2. **Wealth**. The probability of a crisis is *increasing* in wealth (the opposite of the conventional wisdom underlying some contagion stories). Here it is key that agents can both short and go long in the peso. Increased wealth allows agents to short the currency more, increasing the likelihood of a successful attack. If there are incomplete markets and agents cannot short the currency, we regain the conventional result that increased wealth reduces the likelihood of crises.

3. **Portfolio**. If agents' wealth is shifted from dollar denominated assets to peso denominated assets, then a risk averse marginal agent who is uncertain whether a devaluation will occur or not will have a hedging incentive to go long in dollars. Thus foreign direct investment may, *ceteris paribus*, increase the likelihood of a currency crisis.

4. **Ownership**. Home residents consume more peso denominated goods than foreign residents. In the case of relative risk aversion coefficient greater than 1, if agents become foreign rather than domestic, the likelihood of an attack increases. Although the returns to attacking are higher for domestic residents, this leads to a lower probability of crisis because of risk aversion.

5. **Devaluation Size**. For reasonable levels of risk aversion, increasing the size of devaluation may increase or decrease the likelihood of crisis.

6. **Interest Rate Defense**. For sufficiently high risk aversion, devaluation size and interest rate differential, increasing the interest rate differential may increase the likelihood of attack. However, for reasonable parameter values, an interest rate defense will reduce the likelihood of crisis.

A common theme to many of the comparative static results is that when relative risk aversion is above 1, intuitive comparative statics may be reversed. As risk aversion increases, consumption in the two states (devaluation, no devaluation) become complements rather than substitutes. Thus income effects come to dominate substitution effects.

None of these results would arise in a complete information model, where risk aversion and ownership have no effect at all on the set of equilibria. Possibly one could derive related comparative statics in a complete information model with symmetric uncertainty. However, to make risk aversion matter in such a model, a large amount of uncertainty about fundamentals would be required. Our results continue to hold even when uncertainty about fundamentals is arbitrarily small.

Our results thus imply a rich set of empirical predictions from a global games model of currency crises that would probably be hard to replicate with other models that do not build on agents' strategic uncertainty in equilibrium. Unfortunately, the type of data required to directly test the predictions are surely not available. We leave the task of confronting our model to data to future work.

Our analysis focusses on the representative agent with known characteristics. However, we show that our analysis extends to an arbitrary distributions of characteristics. The devaluation threshold is *linear* in the distribution of characteristics in the population.

The analysis builds an approach to modelling currency crises due to Morris and Shin (1998), building on the global games analysis of Carlsson and van Damme (1993). Morris and Shin (1998) and other applied papers using these techniques (surveyed in Morris and Shin (2003)) make heavy use of the assumption that each player faces a binary choice (to attack the currency or not). Frankel, Morris and Pauzner (2003) showed how existence and uniqueness results can be extended to global games with many actions. The model in this paper is a tractable example of a global game with many actions where closed form solutions can be obtained. A theoretical contribution is that it identifies another setting where there is "noise independent selection": the threshold as noise becomes small does not depend on the shape of the noise. By allowing for a continuum of actions, we are able to endogenize the amount of "hot money" available in currency attacks and endogenize whether attacking or defending the currency is the riskier action. These have been arbitrary modelling choices in existing models.

This paper is related to an important work on contagion by Goldstein and Pauzner (2001). They model the idea that catastrophic losses in Russia, say, may reduce the wealth of investors. If those same investors are also investing in Brazil, and those investors have decreasing absolute risk aversion, then they will reduce their risky exposure to Brazil, thus generating a wealth contagion mechanism. Goldstein and Pauzner emphasize that risk aversion has a large impact on the unique equilibrium even though there may be an arbitrarily small amount of uncertainty about fundamentals.[2] The same mechanism underlies our results. We are able to allow for continuous rather than binary investment choices and extend this type of comparative statics to the range of other issues discussed above. Our results show how the Goldstein-Pauzner model — and the underlying intuition about contagion — rely on a (perhaps empirically plausible) incomplete markets modelling assumption that people who lost money in Russia were unable to short the Brazilian real. In a complete markets model, their loss of wealth in Russia should reduce their ability to short the real, and under the one way bet assumption and plausible risk aversion, this would actually decrease the likelihood of a Brazilian crisis.

We describe and solve our basic representative agent model in section 2. Comparative statics with respect to risk aversion, wealth, portfolios, ownership, devaluation size and interest rate differentials are analyzed in section 3. In section 4, we highlight the key modelling assumptions that allow us to get a closed form solution and also noise independence, noting how this result implies that our representative agent analysis immediately extends to a heterogeneous agent world in a linear way; we also examine the robustness to various assumptions: the form of asymmetric information about fundamentals, the known size of the potential devaluation, the constant relative risk aversion assumption and the size of agents. We conclude in section 5.

[2] Calvo and Mendoza (2000) modelled this type of contagion using an informational story. Kyle and Xiong (2001), like Goldstein-Pauzner, modelled a wealth effect version of the contagion story, but the mechanism is different, relying on a significant amount of uncertainty in equilibrium. These papers also rely on explicit or implicit assumptions that "attacking" (selling pesos) rather than "defending" (buying pesos) is the safe action.

2 Basic Model

2.1 Setup

A continuum of agents (measure 1) will realize wealth w_D denominated in dollars and wealth w_P denominated in pesos next period. Each agent must decide his net demand for dollars today, y, with $-y$ being the dollar value of the agent's net demand for pesos.[3] Dollar investments earn an interest rate normalized to zero. Pesos investments earn an interest rate of r. The initial peso/dollar exchange rate is fixed at e_0, but there is a possibility that the exchange rate will be devalued next period. Thus the exchange rate next period (e_1) will be either e_0 or $E > e_0$.[4] Thus the agent's final period wealth (denominated in dollars) is given by

$$\begin{aligned} \widetilde{w}(y, e_1) &= w_D + y + \left(\frac{w_P}{e_1} - y \frac{e_0}{e_1} (1+r) \right) \\ &= w_D + \frac{w_P}{e_1} + y \left(1 - \frac{e_0}{e_1} (1+r) \right). \end{aligned}$$

The agent may consume both foreign goods (x_D, denominated in dollars) and domestic goods (x_P, denominated in pesos). The agent's von-Neumann Morgenstern utility function over foreign and domestic goods is Cobb-Douglas,

$$u(x_D, x_P) = x_D^{\alpha} x_P^{1-\alpha},$$

with $\alpha \in [0,1]$. Letting q_D and q_P be the constant prices of dollar and peso denominated goods, respectively, indirect vNM utility is

$$\begin{aligned} & \left(\frac{\alpha \widetilde{w}(y, e_1)}{q_D} \right)^{\alpha} \left(\frac{(1-\alpha) e_1 \widetilde{w}(y, e_1)}{q_P} \right)^{1-\alpha} \\ = & \left(\frac{\alpha}{q_D} \right)^{\alpha} \left(\frac{1-\alpha}{q_P} \right)^{1-\alpha} e_1^{1-\alpha} \widetilde{w}(y, e_1). \end{aligned}$$

Dividing through by the constant

$$\left(\frac{\alpha}{q_D} \right)^{\alpha} \left(\frac{1-\alpha}{q_P} \right)^{1-\alpha} e_0^{1-\alpha},$$

we have normalized indirect vNM utility

$$v(y, e_1) = \left(\frac{e_1}{e_0} \right)^{1-\alpha} \widetilde{w}(y, e_1).$$

We will assume that the net return to attacking the currency by buying a dollar (and going short in pesos to do so) if there is a devaluation is positive, so

$$v_A = \frac{dv(y, E)}{dy} = \left(1 - (1+r) \frac{e_0}{E} \right) \left(\frac{E}{e_0} \right)^{(1-\alpha)} > 0;$$

and the net return to defending the currency by selling a dollar (and purchasing pesos) if there is no devaluation is

$$v_D = -\frac{dv(y, e_0)}{dy} = r > 0.$$

[3] For simplicity, we assume that the agent has zero liquid assets in the current period. If the agent had positive liquid assets, we could simply add their current dollar value to w_D, and our analysis would be unchanged.

[4] The assumption that the size of a potential devaluation is known is discussed in section 4.3.2.

We will often want to make the "one way bet" assumption[5] that

$$v_A > v_D$$

or

$$\left(1-(1+r)\frac{e_0}{E}\right)\left(\frac{E}{e_0}\right)^{(1-\alpha)} > r.$$

Writing

$$t = \frac{v_D}{v_A + v_D} = \frac{r}{\left(1-(1+r)\frac{e_0}{E}\right)\left(\frac{E}{e_0}\right)^{(1-\alpha)} + r},$$

the one way bet assumption is equivalent to the requirement that

$$t < \frac{1}{2}.$$

The agent has constant relative risk aversion ρ over his vNM index. So he chooses y to maximize the expected value of

$$\frac{1}{1-\rho}\left(\left(\frac{e_1}{e_0}\right)^{1-\alpha}\widetilde{w}\left(y, e_1\right)\right)^{1-\rho}$$

The agent's optimal portfolio choice will thus depend of the probability he attaches to devaluation. We assume that devaluation occurs if the aggregate net demand for dollars exceeds a stochastic threshold θ.[6] We assume that θ is uniformly distributed on the real line and that agents don't know θ but each agent i observes a signal x_i, $x_i = \theta + \varepsilon_i$, where the ε_i are distributed in the population according to probability density function f.[7]

The Inada condition implies non-negative wealth next period; now $\widetilde{w}\left(y, E\right) \geq 0$ implies that

$$y > \underline{y} = -\frac{w_D + \frac{w_P}{E}}{1-(1+r)\frac{e_0}{E}}; \tag{1}$$

and $\widetilde{w}\left(y, e_0\right) \geq 0$ implies

$$y < \overline{y} = \frac{w_D + \frac{w_P}{e_0}}{r}. \tag{2}$$

We will assume that the agent must choose $y \in \left[\underline{\theta}, \overline{\theta}\right]$. This implies that if there was complete information, there would be a tripartite division of fundamentals. If $\theta < \underline{\theta}$, there must be devaluation. If $\underline{\theta} \leq \theta \leq \overline{\theta}$, there are multiple equilibria. If $\theta > \overline{\theta}$, the peg must be maintained.

Most of our analysis will concern the "complete markets" model, with no limits on an agent's ability to go long or short in dollars and pesos, so that $\left[\underline{\theta}, \overline{\theta}\right] = \left[\underline{y}, \overline{y}\right]$. Note that $\underline{y}$ and $\overline{y}$ depend on some parameters of the model. In other cases, we will look at various "incomplete markets" scenarios, where there are exogenous limits on the position the agent can take. In this case, we will have

$$\underline{y} < \underline{\theta} < \overline{\theta} < \overline{y}.$$

[5] Betting in favor of a devaluation is often seen as a one-way bet because the opportunity cost of taking a temporary short position in the currency is small relative to the potential gains from devaluation.

[6] This assumption should be understood as a reduced form description of an optimizing decision by the government whether to abandon the peg. Morris and Shin (1998) had a slightly more detailed modelling of government behavior - the government pays an exogenous reputational cost of abandoning the peg - that would give the same results in this setting.

[7] The assumption of a uniform prior is standard in the global games and used for convenience. As discussed in section 4.3.1, the results continue to hold for any prior if the noise is small.

2.2 Solution

This game is an example of a global game with strategic complementarities and continuous actions spaces, studied by Frankel, Morris and Pauzner (2003). There will be a unique equilibrium where each agent's dollar position will be a decreasing function of the signal he observes.[8] This implies that there will be a critical θ^* such that above that θ^*, the peg will survive and below that θ^*, there will be a devaluation.

To calculate θ^*, first let $y^*(\pi)$ be the dollar position of an agent who believes that the peg will be maintained with probability π,

$$y^*(\pi) = \underset{y \in [\underline{\theta}, \overline{\theta}]}{\arg\max} \left[\pi \left(\widetilde{w}(y, e_0) \right)^{1-\rho} + (1-\pi) \left(\left(\frac{E}{e_0} \right)^{1-\alpha} \widetilde{w}(y, E) \right)^{1-\rho} \right]. \tag{3}$$

An agent who observes signal x attaches probability $F(x - \theta^*)$ to the peg being maintained (i.e., $\theta \geq \theta^*$) and thus holds $y^*(F(x - \theta^*))$. Thus

$$\begin{aligned} \theta^* &= \int_{\varepsilon=-\infty}^{\infty} y^*\left(F\left((\theta^* + \varepsilon) - \theta^*\right)\right) f(\varepsilon)\, d\varepsilon \\ &= \int_{\varepsilon=-\infty}^{\infty} y^*\left(F(\varepsilon)\right) f(\varepsilon)\, d\varepsilon \\ &= \int_{\pi=0}^{1} y^*(\pi)\, d\pi, \text{ by change of variables } \pi = F(\varepsilon). \end{aligned}$$

This is key observation in the paper. As discussed in section 4.1, this observation implies that we have noise independent selection: the critical threshold does not depend on the nature of the uncertainty. What is the intuition for this important observation? Whatever the critical threshold θ^* turns out to be and whatever the distribution of noise, we know that at θ^*, the distribution in the population of their posterior that the θ is greater than θ^*, is uniform on the interval $[0, 1]$.

2.3 The "Sunspot Probability" with complete markets

It is both interesting and analytically convenient to introduce the variable

$$\widehat{\theta} = \frac{\theta^* - \underline{\theta}}{\overline{\theta} - \underline{\theta}}.$$

Since $[\underline{\theta}, \overline{\theta}]$ is the range of fundamentals where there are multiple equilibria, we have that $\widehat{\theta}$ can be interpreted as the probability of a "bad sunspot". If we interpreted the data using a complete information model, the probability of a bad sunspot would be the proportion of times that a self-fulfilling run occurred when this was consistent with fundamentals but not required by fundamentals.

In the special case of complete markets, we can provide a very simple closed form characterization of $\widehat{\theta}$ which we will use extensively in our analysis. The first order conditions for (3) imply that

$$\pi \left(\widetilde{w}(y, e_0) \right)^{-\rho} v_D = (1-\pi) \left(\left(\frac{E}{e_0} \right)^{1-\alpha} \widetilde{w}(y, E) \right)^{-\rho} v_A.$$

[8] The background theory is discussed in more detail in section 4.

Thus

$$\left(\frac{\left(\frac{E}{e_0}\right)^{1-\alpha}\widetilde{w}\left(y,E\right)}{\widetilde{w}\left(y,e_0\right)}\right)^{-\rho} = \left(\frac{\pi}{1-\pi}\right)\left(\frac{t}{1-t}\right),$$

$$\frac{\widetilde{w}\left(y,E\right)}{\widetilde{w}\left(y,e_0\right)} = \left(\frac{1-\pi}{\pi}\right)^{\frac{1}{\rho}}\left(\frac{1-t}{t}\right)^{\frac{1}{\rho}}\left(\frac{e_0}{E}\right)^{1-\alpha},$$

and

$$y^*\left(\pi\right) = \frac{\left(\frac{1-\pi}{\pi}\right)^{\frac{1}{\rho}}\left(\frac{1-t}{t}\right)^{\frac{1}{\rho}}\left(\frac{e_0}{E}\right)^{1-\alpha}\left(w_D+\frac{w_P}{e_0}\right)-\left(w_D+\frac{w_P}{E}\right)}{\left(\frac{1-\pi}{\pi}\right)^{\frac{1}{\rho}}\left(\frac{1-t}{t}\right)^{\frac{1}{\rho}}\left(\frac{e_0}{E}\right)^{1-\alpha}r+1-\frac{e_0}{E}\left(1+r\right)}.$$

But observe that

$$\underline{y} = y^*\left(1\right) \text{ and } \overline{y} = y^*\left(0\right),$$

so

$$\begin{aligned}
\widehat{y}\left(\pi\right) &= \frac{y^*\left(\pi\right)-\underline{y}}{\overline{y}-\underline{y}} \\
&= \frac{y^*\left(\pi\right)-y^*\left(1\right)}{y^*\left(0\right)-y^*\left(1\right)} \\
&= \frac{\frac{\left(\frac{1-\pi}{\pi}\right)^{\frac{1}{\rho}}\left(\frac{1-t}{t}\right)^{\frac{1}{\rho}}\left(\frac{e_0}{E}\right)^{1-\alpha}\left(w_D+\frac{w_P}{e_0}\right)-\left(w_D+\frac{w_P}{E}\right)}{\left(\frac{1-\pi}{\pi}\right)^{\frac{1}{\rho}}\left(\frac{1-t}{t}\right)^{\frac{1}{\rho}}\left(\frac{e_0}{E}\right)^{1-\alpha}r+1-\frac{e_0}{E}\left(1+r\right)}+\frac{\left(w_D+\frac{w_P}{E}\right)}{1-\frac{e_0}{E}\left(1+r\right)}}{\frac{w_D+\frac{w_P}{e_0}}{r}+\frac{\left(w_D+\frac{w_P}{E}\right)}{1-\frac{e_0}{E}\left(1+r\right)}} \\
&= \frac{1}{1+\frac{1-\frac{e_0}{E}\left(1+r\right)}{r}\left(\frac{\pi}{1-\pi}\right)^{\frac{1}{\rho}}\left(\frac{t}{1-t}\right)^{\frac{1}{\rho}}\left(\frac{E}{e_0}\right)^{(1-\alpha)}} \\
&= \frac{1}{1+\left(\frac{\pi}{1-\pi}\right)^{\frac{1}{\rho}}\left(\frac{1-t}{t}\right)^{1-\frac{1}{\rho}}}.
\end{aligned}$$

Thus

$$\begin{aligned}
\widehat{\theta} &= \frac{\int\limits_{\pi=0}^{1} y^*\left(\pi\right)d\pi-\underline{y}}{\overline{y}-\underline{y}} \\
&= \int\limits_{\pi=0}^{1}\widehat{y}\left(\pi\right)d\pi \\
&= \int\limits_{\pi=0}^{1}\frac{1}{1+\left(\frac{\pi}{1-\pi}\right)^{\frac{1}{\rho}}\left(\frac{1-t}{t}\right)^{1-\frac{1}{\rho}}}d\pi
\end{aligned} \tag{4}$$

A convenient feature of this expression is that it depends only on the determinants of t (r, $\frac{E}{e_0}$ and α) and risk aversion ρ, and not on portfolio variables (w_D and w_P).

3 Comparative Statics

For the complete markets model, we obtained a very convenient characterization of the unique equilibrium in the previous section. For equation (4), we know how the sunspot probability $\widehat{\theta}$ depends on risk aversion ρ and the payoff parameter, t, which in turn depends on the interest rate differential r, the devalued exchange rate E and the preference parameter α. The actual threshold is then given by

$$\theta^* = \left(1 - \widehat{\theta}\right) \underline{y} + \widehat{\theta}\overline{y},$$

where $\underline{y}$ and $\overline{y}$ are given by (1) and (2); $\underline{y}$ and $\overline{y}$ depend on wealth and portfolio composition (w_D and w_P), r and E.

Thus we analyze the effect of risk aversion (ρ) and ownership (α) exclusively by looking at their effect on $\widehat{\theta}$ ($\underline{y}$ and $\overline{y}$ are independent of ρ and α); we then analyze the effect of wealth and portfolio composition (w_D and w_P) exclusively by looking at the their effect on $\underline{y}$ and $\overline{y}$ ($\widehat{\theta}$ is independent of w_D and w_P); finally, when analyzing the effect of r and E, we must take both kinds of effects into account.

As well as analyzing our benchmark complete markets scenario, we also look at a number of incomplete markets scenarios, to see how very different comparative statics conclusions may result under reasonable market restrictions.

3.1 Risk Aversion

3.1.1 Complete Markets

We are interested in comparative statics with respect to ρ. Here $\widehat{\theta}$ depends on ρ, but $\underline{y}$ and $\overline{y}$ are independent of ρ. When $\rho \to 0$,

$$\widehat{y}(\pi) \to \begin{cases} 0 & \text{if} \quad \pi > 1 - t \\ 1 & \text{if} \quad \pi < 1 - t \end{cases}$$

An almost risk-neutral agent will bet virtually all his future consumption unless his signal is arbitrarily close to $1 - t$. We get:

$$\hat{\theta} \to 1 - t$$

$$\theta^* \to t\underline{y} + (1 - t)\overline{y}$$

In applications, t is often considered to be close to 0. That implies $\hat{\theta}$ close to 1 in the risk neutral case — conditional on θ being in the multiple equilibrium region, the probability of a "bad sunspot" is very high. For this reason, it has been said that global-games currency-crisis models tend to select the 'bad' equilibrium.[9] But with risk aversion, $\hat{\theta}$ drops dramatically.

When $\rho = 1$ (log utility), we get that:

$$\widehat{y}(\pi) = 1 - \pi$$

With logarithmic utility, the proportion invested in dollars is equal to the probability of a devaluation and does not depend on any other thing — prices are irrelevant. Then,

$$\hat{\theta} = \frac{1}{2}$$

[9] See, for example, Chamley (2003).

$$\theta^* = \frac{1}{2}\underline{y} + \frac{1}{2}\overline{y}$$

Note the dramatic impact of risk aversion on $\hat{\theta}$. For example: if $t = 0.05$, the probability of a "bad sunspot" is 95% in the risk neutral case but equals only 50% if agents have logarithmic utility function.

When $\rho \to \infty$:

$$\hat{y}(\pi) \to t$$

$$\hat{\theta} \to t$$

When there is very little uncertainty (i.e., f puts most probability close to 0) and ρ is large, agents will be almost always choosing either $\underline{y}$ or $\overline{y}$, but $\hat{\theta}$ will be close to t anyway. Now

$$\theta^* \to (1-t)\underline{y} + t\overline{y} = \frac{w_P\left(\frac{1}{e_0} - \frac{1}{E}\right)}{\left(1 - (1+r)\frac{e_0}{E}\right)\left(\frac{E}{e_0}\right)^{(1-\alpha)} + r}.$$

Note that when $w_P = 0$ and ρ tends to ∞, θ^* tends to 0. With no hedging demand because of peso exposure, risk averse agents take zero positions.

Figure 1 shows $\hat{\theta}$ as a function of $\log(\rho)$ and t. Under the one way bet assumption ($t < \frac{1}{2}$), risk aversion reduces $\hat{\theta}$ and makes investors less willing to attack the currency. The opposite holds when $t > \frac{1}{2}$. The sunspot probability ($\hat{\theta}$) equals $\frac{1}{2}$ whenever $t = \frac{1}{2}$ or $\rho = 1$.[10]

Agents' expectations on others' actions play a crucial role in determining the outcome of the game. Risk aversion influences other agents' positions, which determines θ^*. What drives the results is the impact of risk aversion on what *all others* will do — not on what *a single individual* will do. The impact of a tiny fraction of agents with different levels of risk aversion on θ^* is negligible, as we show at section 4.2.

Figure 2-a cuts figure 1 at some given values of ρ. We can see that the impact of t on the sunspot probability depends crucially on risk aversion. Interestingly, for $\rho > 1$, $\hat{\theta}$ is *increasing* in t: with complete markets, for empirically plausible levels of risk aversion, a higher cost of attacking the currency leads to a higher probability of a "bad sunspot". This result may sound counter-intuitive at first. The intuition is that although the gains from a successful currency attack are decreasing in t, the incentives for attacking are not increasing in the gains from attacking: depending on the level of risk aversion, hedging motivations may dominate the prospects of higher gains. Moreover, factors that influence t may also affect $\underline{y}$ and $\overline{y}$, so the overall effects of prices (E and r) on θ^* are not totally captured by figures 1 and 2-a.

Figure 2-b cuts figure 1 at some given values of t and shows that $\hat{\theta}$ reacts strongly to changes in risk aversion for low values of ρ. If t is small, the sunspot probability for empirically plausible degrees of risk aversion is completely different from the risk neutral case.

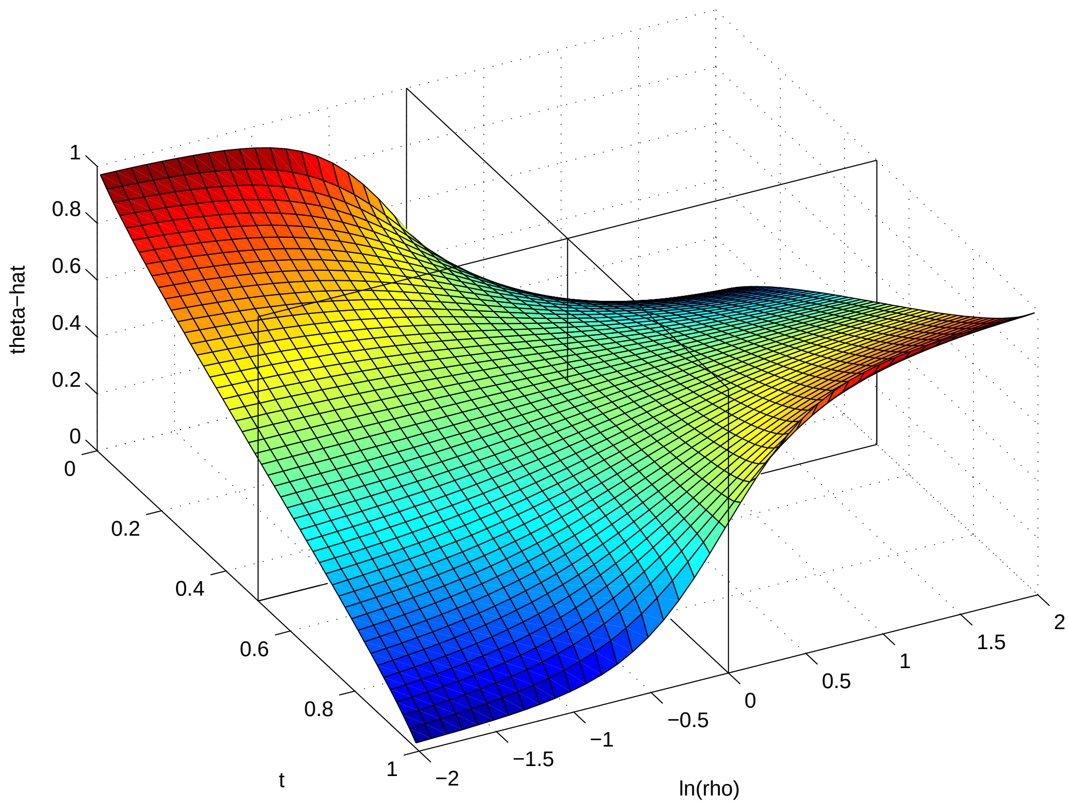
In sum, when agents are free to short any amount of dollars and pesos, risk aversion has huge impacts on $\hat{\theta}$ (and thus θ^*). Next, we check what happens when agents positions are restricted and compare results.

3.1.2 Incomplete Markets

With complete markets, we can sign the effect of risk aversion on both $\widehat{\theta}$ and θ^*. In the complete markets analysis, it is endogenous whether attacking the currency (high y) or defending the currency (low y) is the riskier action.

[10] Analytically, we are able to show that for $t < \frac{1}{2}$, $\hat{\theta}$ is decreasing in ρ for $\rho \geq 1$ — we couldn't prove it for $\rho < 1$ although we believe it also holds in this case. Analogously, for $t > \frac{1}{2}$, we can show that $\hat{\theta}$ is increasing in ρ only for $\rho \leq 1$.

Figure 1: $\hat{\theta}$ as a function of t and $\log(\rho)$



Some of our intuition about the effect of risk aversion on currency crises comes from situations where we know that either defending or attacking is riskier for the investor. This intuition depends on some incompleteness of markets. Next, we present scenarios where risk aversion will unambiguously increase the probability of attacks or unambiguously reduce it, independent of the one-way bet assumption (i.e., the size of t).

INCOMPLETE MARKETS SCENARIO 1.

Foreign investors with all their wealth in dollars cannot go short in either currency. Thus, $w_D > 0$, $w_P = 0$, $\alpha = 1$, $\underline{\theta} = -w_D$ and $\overline{\theta} = 0$ (if $y^* = -w_D$, all his wealth is invested in pesos and if $y^* = 0$, all his wealth is invested in dollars). In this case, $\hat{\theta}$ is given by:

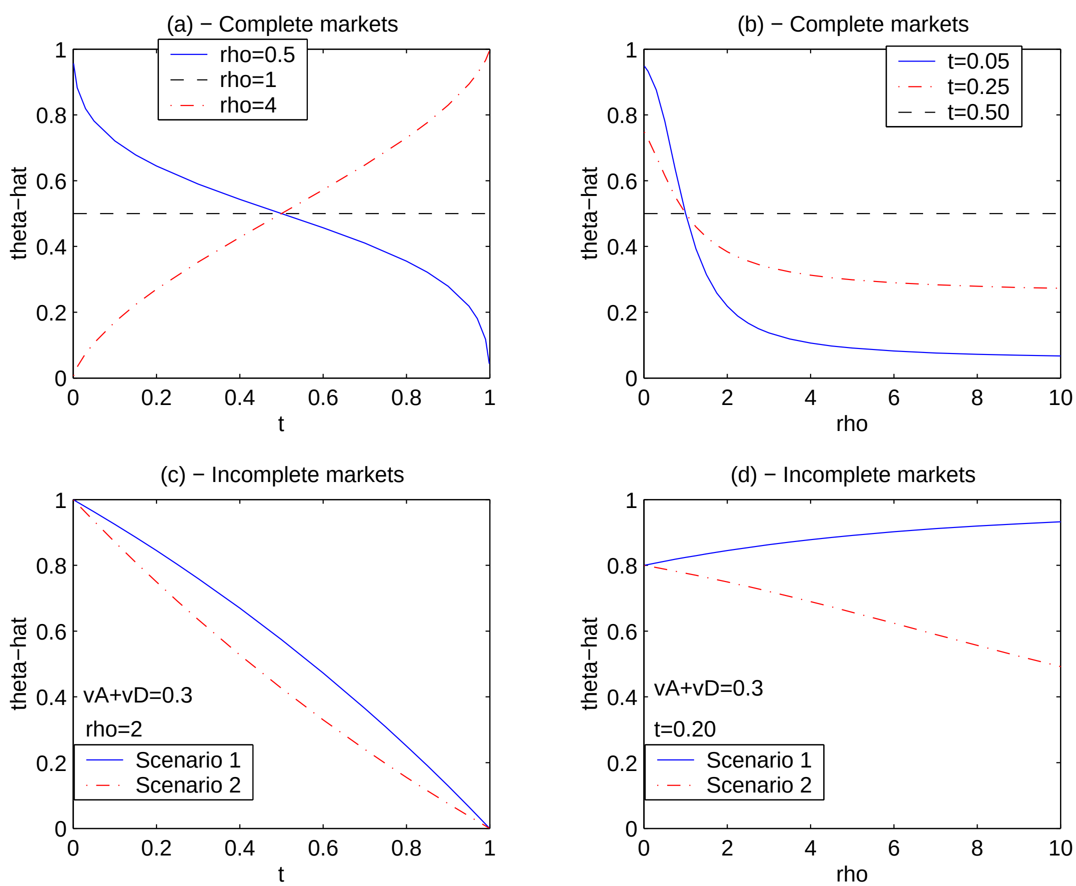
$$\hat{\theta} = \frac{\theta^* + w_D}{w_D}$$

For such an investor, the safe action is to hold his wealth in dollars (i.e., to attack the currency) and the risky action is to hold pesos (i.e., to defend the currency). In particular, this investor's problem is:

$$y^*(\pi) = \underset{y \in [-w_D, 0]}{\arg\max} \left[\pi (w_D - ry)^{1-\rho} + (1-\pi) \left(w_D + y \left(1 - \frac{e_0}{E}(1+r) \right) \right)^{1-\rho} \right]. \tag{5}$$

For any $\rho \in (0, \infty)$, if $\pi < 1 - t$, we will have that the investor would like to short pesos and

Figure 2: Effects of t and ρ in $\hat{\theta}$



go long in dollars, but cannot, so $y^*(\pi) = 0$; if $\pi > 1-t$, the investor will hold less than his whole portfolio in dollars, so $y^*(\pi) < 0$.

Now if $\rho \to 0$, we will have

$$y^*(\pi) \to \begin{cases} -w_D & \text{if} \quad \pi > 1-t \\ 0 & \text{if} \quad \pi < 1-t \end{cases}, \ \hat{\theta} \to 1-t \text{ and } \theta^* \to -tw_D$$

As $\rho \to \infty$, we will have

$$y^*(\pi) \to 0, \ \hat{\theta} = 1 \text{ and } \theta^* = 0.$$

Thus risk aversion increases the probability of attacks (independent of t), because attacking is the safe action by assumption.

INCOMPLETE MARKETS SCENARIO 2. A domestic investor with all wealth in pesos who cannot go short in either currency. Thus $w_D = 0$, $w_P > 0$, $\alpha = 0$, $\underline{\theta} = 0$ and $\overline{\theta} = \frac{w_P}{e_0}$ (if $y^* = \frac{w_P}{e_0}$ all his wealth is invested in dollars). In this case, $\hat{\theta}$ is given by:

$$\hat{\theta} = \frac{\theta^* e_0}{w_P}$$

For this investor, the safe action is to hold his wealth in pesos (i.e., to defend the currency) and the risky action is to hold dollars. The investor's problem is

$$y^*(\pi) = \underset{y \in \left[0, \frac{w_P}{e_0}\right]}{\arg\max} \left[\pi \left(\frac{w_P}{e_0} - ry \right)^{1-\rho} + (1-\pi) \left(\frac{w_P}{e_0} + y \left(\frac{E}{e_0} - (1+r) \right) \right)^{1-\rho} \right]. \qquad (6)$$

For any $\rho \in (0, \infty)$, if $\pi > 1-t$, the investor would like to short dollars and go long in pesos, but cannot, so $y^*(\pi) = 0$; if $\pi < 1-t$, the investor will hold a positive amount of dollars, $y^*(\pi) > 0$.

Now if $\rho \to 0$, we will have

$$y^*(\pi) \to \begin{cases} 0 & \text{if} \quad \pi > 1-t \\ \frac{w_P}{e_0} & \text{if} \quad \pi < 1-t \end{cases}, \ \hat{\theta} \to 1-t \text{ and } \theta^* \to (1-t)\frac{w_P}{e_0}$$

As $\rho \to \infty$, we will have

$$y^*(\pi) \to 0, \ \hat{\theta} = 0 \text{ and } \theta^* = 0.$$

Thus risk aversion reduces the probability of attacks (independent of t), because defending is the safe action by assumption.

If we included both the investors of scenario 1 and the investors of scenario 2, then the heterogeneous agent argument of section 4.2 shows that threshold would move linearly between the two results as a function of the proportion of investors of both types.

Figures 2-c and 2-d show numerical results for $\hat{\theta}$ in both scenarios. As shown above, when $\rho \to 0$, $\hat{\theta}$ approaches $(1-t)$ — which is the result in a model with risk neutral agents. The effect of risk aversion in the sunspot probability depends on which is the risky action and its sign is independent of t.

The impacts of risk aversion with short-selling constraints (figures 2-c and 2-d), although not at all negligible, are not as huge as in the complete market case. When agents are free to take any position they want, they may end up shorting large amount of dollars or pesos and, therefore, facing the risk of getting almost no consumption if their bet goes wrong. Therefore, impacts of risk aversion on their decisions are potentially big. On the other hand, if investors' positions are limited, so are the effects of risk aversion.

With incomplete markets, when $v_A + v_D$ is small, there is little risk and ρ has not much impact on agent's decision. With complete markets, $\hat{\theta}$ does not depend on $v_A + v_D$ because investors choose the amount of risk they will face.

3.2 Ownership

We are interested in comparative statics with respect to the parameter α. We focus on the case of complete markets. Our interpretation is that a high α corresponds to foreign investors (who will use terminal wealth to purchase dollar denominated goods) and a low α corresponds to domestic investors. Here $\widehat{\theta}$ in independent depends on α, but $\underline{y}$ and $\overline{y}$ are independent of α.

All the impact of α on the threshold goes through t. As:

$$\frac{d\widehat{\theta}}{d\left(\frac{1-t}{t}\right)} = -\int_0^1 \frac{\left(\frac{\rho-1}{\rho}\right)\left(\frac{1-t}{t}\right)^{-\frac{1}{\rho}}}{\left(1+\left(\frac{\pi}{1-\pi}\right)^{\frac{1}{\rho}}\left(\frac{1-t}{t}\right)^{\frac{\rho-1}{\rho}}\right)^2} d\pi$$

$\frac{d\hat{\theta}}{d\left(\frac{1-t}{t}\right)}$ is positive for $\rho < 1$ and negative for $\rho > 1$. Also, we have that:

$$\frac{d\left(\frac{1-t}{t}\right)}{d\alpha} = -\frac{1}{r}\left(1-(1+r)\frac{e_0}{E}\right)\left(\frac{E}{e_0}\right)^{1-\alpha}\ln\left(\frac{E}{e_0}\right) < 0$$

So, the effect of α on θ^* depends on ρ, as shown at table 1.

ρ	< 1	$= 1$	> 1
$\frac{d\hat{\theta}}{d\alpha}$	< 0	$= 0$	> 0

Table 1: $\frac{d\hat{\theta}}{d\alpha}$

This result may sound counter intuitive — shouldn't a higher α imply a lower threshold? It is true that a higher α implies a higher t — i.e., a higher cost of attacking. A higher t turns investors less inclined to attack the currency if they are not very risk averse but also turn them more interested in holding dollars if $\rho > 1$ by increasing demand for hedging. When we have log utility, α does not impact the threshold: hedging motivations are just enough to offset the prospects of higher gains, as pointed by figures 1 and 2-a and the discussion at section 3.1.

3.3 Wealth

3.3.1 Complete Markets

It is often said that a negative wealth shock may threaten a currency peg because investors are forced to withdraw their money. For example, when Russia defaulted its debt in 1998, Brazil experienced a large capital outflow.

We first examine how a wealth shock could effect the coordination of agents and thus change θ^* even with complete markets. It is important to note that with complete markets, a decrease in wealth will decrease the size of the position that both attacking and defending agents can take (consistent with the Inada condition). We want to find out which effect is more important. For simplicity, we focus on the case where $w_P = 0$, $w_D > 0$ and $\alpha = 1$, so that all wealth belongs to foreigners.[11] We are interested in comparative statics with respect to wealth, w_D. Here the sunspot probability $\widehat{\theta}$ in independent of wealth, but $\underline{y}$ and $\overline{y}$ depend on wealth.

[11] The result also holds if all money belong to domestic residents.

We have:

$$
\begin{aligned}
\theta^* &= \hat{\theta}\overline{y} + \left(1-\hat{\theta}\right)\underline{y} \\
&= w_D\left[\frac{\hat{\theta}}{r} - \frac{1-\hat{\theta}}{1-\frac{e_0}{E}(1+r)}\right] \\
\frac{d\theta^*}{dw_D} &= \frac{\hat{\theta}}{r} - \frac{1-\hat{\theta}}{1-\frac{e_0}{E}(1+r)} \\
&= \left(\frac{1-\hat{\theta}}{r}\right)\left(\frac{\hat{\theta}}{1-\hat{\theta}} - \frac{t}{1-t}\right)
\end{aligned}
$$

If $t < \frac{1}{2}$, $\hat{\theta} > t$ for any ρ. So

$$\frac{d\theta^*}{dw_D} > 0.$$

Thus increased wealth reduces the probability of a successful currency attack (while the sunspot probability remains unchanged). Note that θ^* is linear in w_D. Our result, with complete markets, is totally different from the usual intuition. If $t < \frac{1}{2}$, most agents are selling *Pesos* short. As the utility function is concave, the amount of risk one agent chooses to face depends positively on his wealth. So, a negative wealth shock reduces the munition an agent is willing to use to attack the currency.

As we show now, with incomplete markets, this result may not hold.

3.3.2 Incomplete Markets

Consider again scenario 1 above. The investor will choose:

$$
y^* = \begin{cases} -w_D & \text{if } y^{foc} \le -w_D \\ y^{foc} & \text{if } y^{foc} \in (-w_D, 0) \\ 0 & \text{if } y^{foc} \ge 0 \end{cases}
$$

where

$$
y^{foc} = w_D \frac{1 - \left(\frac{\pi\ r}{(1-\pi)v_A}\right)^{-\rho}}{r + v_A\left(\frac{\pi\ r}{(1-\pi)v_A}\right)^{-\rho}}.
$$

As in the complete markets case, y^* is given by a linear function of w_D. However, in the present example, the investor will always choose a negative y^*. Thus, increasing wealth will always increase the size of his position in absolute value, increasing the region of a successful currency attack.

This result is similar to those obtained in the contagion model by Goldstein and Pauzner (2001).[12] Short selling constraints are implicitly assumed in their paper and agents decide to invest or not one unit of money in the country. We show here that their conclusion still holds when agents have a continuum of actions but depends on positive investments in the country — agents need to be investing, not attacking.

[12] Their result is in a more general model: in particular, they allow for any decreasing absolute risk aversion utility function (not just constant relative risk aversion); and they allow for general strategic complementarities in investment in each country.

3.4 Portfolio

3.4.1 Complete Markets

We are interested in comparative statics as we shift wealth between w_D and w_P. Again, $\widehat{\theta}$ is independent of such portfolio reallocations, but $\underline{y}$ and $\overline{y}$ depend on the portfolio. Suppose that we increase w_D by e_0 units and reduce w_P by 1 unit. That is a change in the initial allocation of portfolio without changing agents' wealth. Then, we have:

$$\frac{1}{e_0}\frac{d\theta^*}{dw_D}-\frac{d\theta^*}{dw_P}=\frac{E\left(1-\hat{\theta}\right)}{E-e_0(1+r)}\left(\frac{e_0-E}{e_0.E}\right)\ <\ 0$$

We could also increase w_D by E units and reduce w_P by 1 unit. We still get:

$$\frac{1}{E}\frac{d\theta^*}{dw_D}-\frac{d\theta^*}{dw_P}=\frac{\hat{\theta}}{r}\left(\frac{e_1-E}{e_1.E}\right)\ <\ 0$$

An increase in the share of peso assets creates incentives for agents to *reduce* y_D for they are risk averse and search for hedge. This result says that investors who are exposed to exchange rate risk (say, through foreign direct investment), will be more likely to attach the currency at the margin because of hedging motives.

Figure 3 shows that the quantitative importance of portfolio effects, with complete markets, is small.[13] The threshold θ^* is very sensitive to ρ but seems to be almost unaffected by changes in portfolio allocation.

Next, we investigate whether portfolio effects still occur in the presence of short selling constraints and if their magnitude is higher in that case.

3.4.2 Incomplete Markets

With complete markets, we are allowing the investor to borrow against his peso wealth. A more interesting scenario might be when investors' peso wealth is illiquid and short sales are restricted. Next, we study an example in which agents can sell their dollar wealth but not their peso wealth and can't go short in either currency.[14]

INCOMPLETE MARKETS SCENARIO 3. A foreign investor has all his liquid wealth in dollars but may have illiquid investments in pesos. Thus $w_D > 0$, $w_P > 0$, $\alpha = 1$, $\underline{\theta} = -w_D$ and $\overline{\theta} = 0$. In this case,

$$y^*(\pi)=\underset{y\in[-w_D,0]}{\arg\max}\left[\pi\left(w_D+\frac{w_P}{e_0}-ry\right)^{1-\rho}+(1-\pi)\left(w_D+\frac{w_P}{E}+y\left(1-\frac{e_0}{E}(1+r)\right)\right)^{1-\rho}\right].$$

Figure 3 shows that, with incomplete markets, higher illiquid investment in pesos also implies a higher threshold θ^* and, therefore, a larger set of states in which devaluation occurs.[15] Such effect is increasing in ρ and vanishes as $\rho \to 0$.

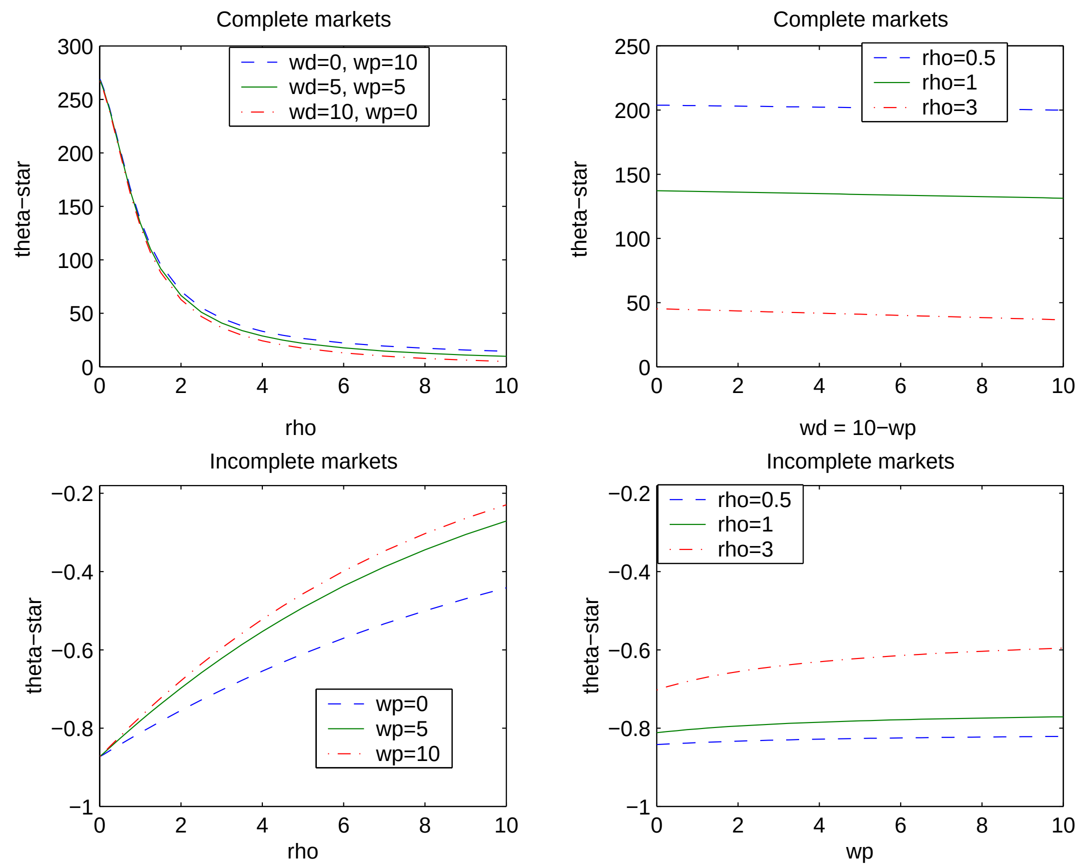
As in the complete market case, risk aversion is much more important than portfolio allocation in determining θ^*.

[13] Parameters in this example: $E = 1.2$, $e_0 = 1$, $r = 0.03$ and $\alpha = 0.5$.

[14] Allowing agents to short some amount of either currency (say, at most twice their dollar wealth) has little impact on our results.

[15] Parameters in this example: $E = 1.2$, $e_0 = 1$, $r = 0.03$ and $w_D = 5$.

Figure 3: Portfolio effects



3.5 The Size of Devaluation

We are interested in comparative statics with respect to the parameter E. An increase in E represents an increase in the size of the devaluation, if it occurs.

Observe first that an increase in E increases $\underline{y}$, since agents must then take a smaller peso position and thus a smaller (negative) dollar position; $\overline{y}$ does not depend on E. Thus an increase in E shifts the complete information multiple equilibria range unambiguously in the direction of more attacks.

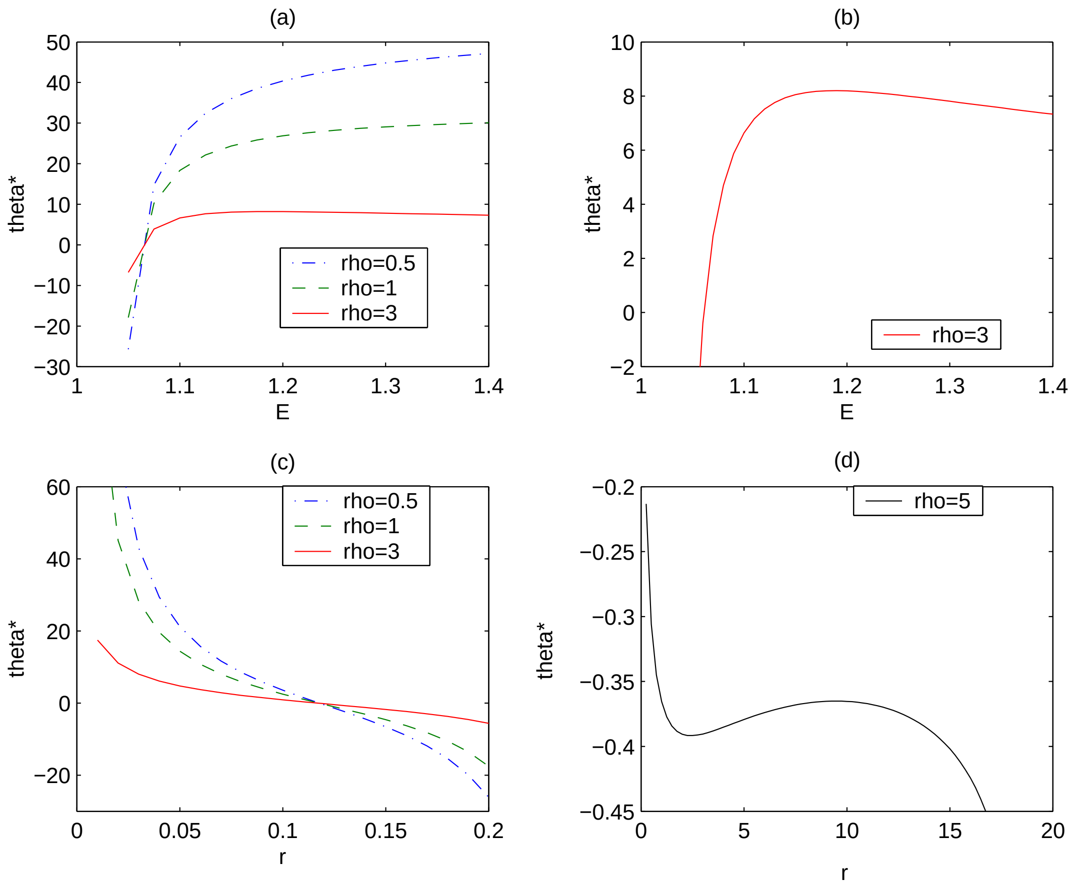
An increase in E decreases t. As we have already observed at section 3.1, $\widehat{\theta}$ is decreasing in t if $\rho < 1$ but increasing in t if $\rho > 1$. Thus for empirically plausible levels of risk aversion, a higher value of E leads to a *higher* probability of a bad sunspot. The intuition is simply that a higher E leads to an increase in the gains from a successful currency attack and, for $\rho > 1$, hedging motivations are the dominant forces.

Combining the two effects, we know that for $\rho \leq 1$, we must have θ^* increasing in E. But for $\rho > 1$, the combined effect may go either way for reasonable values of the parameters. Figure 4-a shows θ^* as function of E for different levels of risk aversion.[16] We can see that for $\rho \leq 1$, θ^* is increasing in E: higher rewards for a successful attack make it more likely. For $\rho = 3$, however, θ^*

[16]Parameters of this example: $w_D = 1$, $w_P = 1$, $r = 0.03$, $e_0 = 1$, $\alpha = 0.5$. The non-monotonicity of θ^* as function of E holds for empirically plausible values of parameters regardless of the values of α, w_D and w_P.

is decreasing in E for sufficiently high values of E (i.e., for sufficiently low values of t). Figure 4-b plots the same function θ^* using different scale and shows clearly that, for $\rho = 3$, an increase E may turn a devaluation less likely.

Figure 4: Effects of E and r



3.6 Interest Rate Defense

We are interested in comparative statics with respect to the parameter r. How does an increase in r affect the likelihood of currency crises?[17] Again, $\widehat{\theta}$ depends on r (through t), but so do $\underline{y}$ and $\overline{y}$.

Observe first that an increase in r reduces $\underline{y}$, since the reduced return to attacking the currency allows investors to take a larger peso position and thus a larger (negative) dollar position; an increase in r also reduces $\overline{y}$, since the largest dollar position consistent with being able to pay the interest differential on the short peso position is reduced. Thus an increase in r shifts the complete information multiple equilibria range unambiguously in the direction of less attacks.

An increase in r increases t. As we have already observed, this increases $\widehat{\theta}$ if $\rho < 1$ but decreases $\widehat{\theta}$ if $\rho > 1$. Again, for empirically plausible levels of risk aversion, an interest rate defense paradoxically

[17] In our model, the choice of r does not serve a signal of private information of the central bank. In reality, the interest rate may be a signal. See Angeletos, Hellwig and Pavan (2003) for a global games model with interest rate signalling by the central bank.

increases the probability of a bad sunspot — a higher interest rate means that the full hedging demand for dollars is increased.

Which effect wins out overall? For $\rho \leq 1$, we must have θ^* decreasing in r. We find that this comparative static is maintained for $\rho > 1$ for reasonable values of the parameters. Figure 4-c shows θ^* for different levels of risk aversion.[18] Regardless of the value of ρ, an increase in r turns a devaluation less likely.

However, it is possible to construct examples where θ^* is increasing in r. Figure 4-d shows an example of such curious behavior of θ^*. The parameters in such example ($e_0 = 1$, $E = 20$ and $r \in (200\%, 900\%)$) are too unrealistic in the context of currency crisis.[19] However, in other applications in which the difference between agent‘s utility in the 2 possible states is too big, we may find this perverse effect of an interest rate defense.[20]

4 Model Robustness

4.1 A General Noise Independence Result for Two State Global Games

Consider a game played by I continua of agents, where each agent with index i chooses an action $a \in \left[\underline{\theta}, \overline{\theta}\right]$. Let λ_i be the mass of agents with index i. Let $u_i : A \times \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$ be agent i's utility function, so that

$$u_i(a, \overline{a}, \theta)$$

is agent i's utility if he chooses action a, the average action in the population is given by $\overline{a}$ and the state is θ. Assume that the payoff function takes the following special form:

$$u_i(a, \overline{a}, \theta) = \begin{cases} \underline{v}_i(a), \text{ if } \theta \geq \overline{a} \\ \overline{v}_i(a), \text{ if } \theta < \overline{a} \end{cases}$$

Further assume that $a' > a$ implies

$$\overline{v}_i(a') - \overline{v}_i(a) > \underline{v}_i(a') - \underline{v}_i(a). \tag{7}$$

Let

$$\widetilde{a}_i(\pi) = \arg\max_a (1 - \pi)\overline{v}_i(a) + \pi \underline{v}_i(a).$$

Assumption (7) ensures that $\widetilde{a}_i(\pi)$ is weakly decreasing in π. Let $\widetilde{a}_i(0) = \lim\limits_{\pi \rightarrow 0} \widetilde{a}_i(\pi)$ and $\widetilde{a}_i(1) = \lim\limits_{\pi \rightarrow 0} \widetilde{a}_i(\pi)$.

We maintain the information assumptions: thus we assume that θ is uniformly distributed on the real line and that each class of agents doesn't know θ but observe a signal x, $x = \theta + \varepsilon$, where the ε are distributed in the population according to probability density function f.

These assumptions ensure that key supermodularity and limit dominance properties of Frankel, Morris and Pauzner (2003) are satisfied. As a consequence, there will be an essentially unique equilibrium where each type of agent has a non-increasing strategy $s_i : \mathbb{R} \rightarrow A$. Corresponding to this strategy profile $s = (s_i)_{i=1}^I$, there is an average action

$$\widehat{a}(\theta) = \sum_{i=1}^{I} \lambda_i \int_{\varepsilon=-\infty}^{\infty} s_i(\theta + \varepsilon)\, d\varepsilon.$$

[18] Parameters of this example: $w_D = 1$, $w_P = 1$, $E = 1.25$, $e_0 = 1$, $\alpha = 0.5$. We could not find an example of θ^* increasing in r for any reasonable values of E, e_0 and r.

[19] Other parameters are: $w_P = 1$, $w_D = 1$, $\alpha = 0.5$.

[20] One could extend our model to the case of debt crises. If there is risk of total default, could an interest rate defense make matters worse?

This is non-increasing in θ. Thus there will be a unique θ^* solving $\theta = \widehat{a}(\theta)$. Now recall that an agent who observes signal x attaches probability $F(x - \theta^*)$ to $\theta \geq \theta^*$. Thus $s_i(x) = \widetilde{a}_i(F(x - \theta^*))$. Thus

$$\begin{aligned} \theta^* &= \sum_{i=1}^{I} \lambda_i \int_{\varepsilon=-\infty}^{\infty} \widetilde{a}_i \left(F\left((\theta^* + \varepsilon) - \theta^*\right)\right) f(\varepsilon) \, d\varepsilon \\ &= \sum_{i=1}^{I} \lambda_i \int_{\varepsilon=-\infty}^{\infty} \widetilde{a}_i \left(F(\varepsilon)\right) f(\varepsilon) \, d\varepsilon \\ &= \sum_{i=1}^{I} \lambda_i \int_{\pi=0}^{1} \widetilde{a}_i(\pi) \, d\pi. \end{aligned}$$

Thus we have a closed form characterization of the unique equilibrium: an agent of type i observing signal x chooses action $\widetilde{a}_i(F(x - \theta^*))$, where

$$\theta^* = \sum_{i=1}^{I} \lambda_i \int_{\pi=0}^{1} \widetilde{a}_i(\pi) \, d\pi. \tag{8}$$

Frankel, Morris and Pauzner (2003) established the existence of a unique equilibrium in a global game with a uniform prior.[21] However, some games satisfy a "noise independence property," where the structure of equilibrium is independent of the shape of the noise. In particular, the limiting behavior of the unique equilibrium as the noise goes to zero is independent of the distribution of the noise. Such a noise independence property holds in symmetric games with a continuum of agents and binary actions, and Morris and Shin (2003) argue how the simple "Laplacian" characterization of the unique equilibrium in this case is useful in applications. Frankel et al. (2003) and Morris and Ui (2002) give other examples of games where the noise independence property holds. However, the noise independence property does not always hold, as shown by an example in Frankel et al. (2003) and the currency crisis application of Corsetti, Dasgupta, Morris and Shin (2003). The above argument identifies another sufficient condition for noise independent selection. It relies on the fact that there even though there are a continuum of actions, others' actions and the state θ only enter each agent's utility via a binary classification. This is a fairly restrictive condition. However, it allows for arbitrary action sets and arbitrary asymmetry (consistent with monotonicity properties) among agents' payoff functions. The sufficient condition could clearly be weakened a little bit - for example, the binary classification might depend on an increasing aggregate statistic of all agents' actions (rather than a linear function). With this general argument, we immediately see how we could introduce heterogeneous agents into the earlier model and the only impact on the results would threshold θ^* would be a weighted sum of the thresholds that would have arisen in a homogenous agent model, with the weights of each type equal to their proportion in the population.

4.2 Heterogeneous Agents

For notational convenience, we will analyze the complete markets model with a finite set of possible types. However, the incomplete markets models will generalize in a similar way and we could deal with a continuum of types essentially by replacing summations with integrals.

An agent of type i is characterized by a coefficient of constant relative risk aversion ρ_i, dollar-denominated wealth w_{Di}, peso-denominated wealth w_{Pi}, and preference parameter α_i. If this agent

[21] The result is actually proved for an arbitrary prior and small noise. However, a step in the argument involves showing uniqueness for a uniform prior and arbitrary noise.

assigned probability π to the peg being maintained, this agent demand for dollars would be

$$y_i^*(\pi) = \underset{y\in[\underline{\theta},\overline{\theta}]}{\arg\max} \left[\begin{array}{l} \pi\left(w_{Di} + \frac{w_{Pi}}{e_0} - yr\right)^{1-\rho_i} \\ +(1-\pi)\left(\left(\frac{E}{e_0}\right)^{1-\alpha_i}\left(w_{Di} + \frac{w_{Pi}}{E} + y\left(1 - \frac{e_0}{E}(1+r)\right)\right)\right)^{1-\rho_i} \end{array} \right]. \tag{9}$$

If there was a homogenous continuum of agents of type i, we know that the critical threshold would be

$$\theta_i^* = \int_{\pi=0}^{1} y_i^*(\pi)\, d\pi.$$

But if there was a heterogeneous population, with proportion λ_i of type i, then the argument of the previous section and equation (8) imply that:

$$\begin{aligned} \theta^* &= \sum_{i=1}^{I} \lambda_i \theta_i^* \\ &= \sum_{i=1}^{I} \lambda_i \int_{\pi=0}^{1} y_i^*(\pi)\, d\pi. \end{aligned}$$

4.3 Assumptions

At this point, it is useful to review the role of some of the assumptions made in our analysis.

4.3.1 The Uniform Prior Assumption

We made the convenient assumption that θ was uniformly distributed on the real line. This is a standard simplifying assumption in the global games literature (see Morris and Shin (2003)). If we bounded the support of the noise distribution f, we could have had θ uniform on a bounded interval, with no change in the analysis. In addition, if θ were drawn from a smooth, but non-uniform, prior and we let the variance of the noise shrink to zero (i.e., the support f shrinks to zero), then the limiting equilibrium threshold is equal to threshold identified under the uniform prior assumption. Intuitively, if the noise is small, variation in the density of the prior becomes irrelevant. Thus our results should be understood as applying if either uncertainty is small or uncertainty is large but there is not too much prior or public information about θ.

4.3.2 The Known Devaluation Assumption

A crucial assumption in our model is that the exchange rate at period 1, conditional on the occurrence of a devaluation, is common knowledge and constant (equal to E). The size of the devaluation is independent of θ, which represents the ability of the government to defend the peg. A more realistic assumption would be that if a devaluation occurred in state θ, then the new exchange rate would be $e_1(\theta)$, where e_1 is a decreasing function of θ.[22] Both our noise independence property and our ability to get a closed form solution would go away in this model. In particular, we heavily exploited the fact that we were always evaluating two state gambles. In general, there would be a complicated interaction between the binary uncertainty about whether will be a devaluation or not, and the richer uncertainty about the size of the devaluation.

[22] This is essentially the model analyzed in Morris and Shin (1998). In that risk neutral incomplete markets model, the uniqueness result continues to hold.

However, there is some hope of extending our results if there was a small amount of uncertainty about θ. In this case, agents' uncertainty about the size of the devaluation would go away even as strategic uncertainty about others' actions remained (this is the key insight of the global games approach). Thus if we restricted the noise to have finite support and let the support shrink to zero, then the existing analysis might apply.

If there is no uncertainty about the size of the devaluation, a bit of algebra on the results of section 2.3 shows that θ^* is the unique value of θ solving:

$$\begin{aligned}\theta &= \frac{w_D + \frac{w_P}{e_0}}{r}\left(\int_{\pi=0}^{1}\left[1+\left(\frac{\pi}{1-\pi}\right)^{\frac{1}{\rho}}\left(\frac{\left(1-(1+r)\frac{e_0}{E}\right)}{r}\right)^{1-\frac{1}{\rho}}\right]^{-1} d\pi\right)\\ &+ \frac{w_D + \frac{w_P}{E}}{1-(1+r)\frac{e_0}{E}}\left(1-\int_{\pi=0}^{1}\left[1+\left(\frac{\pi}{1-\pi}\right)^{\frac{1}{\rho}}\left(\frac{\left(1-(1+r)\frac{e_0}{E}\right)}{r}\right)^{1-\frac{1}{\rho}}\right]^{-1} d\pi\right)\end{aligned}$$

If agents' demand for dollars was increasing in E (implying that the right hand side is increasing in E), then with uncertainty about the size of a devaluation, our candidate solution would be the unique value of θ solving:

$$\begin{aligned}\theta &= \frac{w_D + \frac{w_P}{e_0}}{r}\left(\int_{\pi=0}^{1}\left[1+\left(\frac{\pi}{1-\pi}\right)^{\frac{1}{\rho}}\left(\frac{\left(1-(1+r)\frac{e_0}{e_1(\theta)}\right)}{r}\right)^{1-\frac{1}{\rho}}\right]^{-1} d\pi\right)\\ &+ \frac{w_D + \frac{w_P}{e_1(\theta)}}{1-(1+r)\frac{e_0}{e_1(\theta)}}\left(1-\int_{\pi=0}^{1}\left[1+\left(\frac{\pi}{1-\pi}\right)^{\frac{1}{\rho}}\left(\frac{\left(1-(1+r)\frac{e_0}{e_1(\theta)}\right)}{r}\right)^{1-\frac{1}{\rho}}\right]^{-1} d\pi\right)\end{aligned} \tag{10}$$

A unique solution will exist since the left hand side is increasing in θ and the right hand side is decreasing in θ.

Our results at section 3.5 imply that, if $\rho \leq 1$, agents' demand for dollars is increasing in E and, therefore, the right hand side of equation (10) is increasing in E. In this case, we have a unique equilibrium, as shown at figure 5-a. The decreasing function is the (exogenous) relation between fundamentals (θ) and the exchange rate (the inverse of $e_1(\theta)$). The increasing function is what θ^* would be if E was a known constant. The intersection of both curves gives the unique equilibrium.[23]

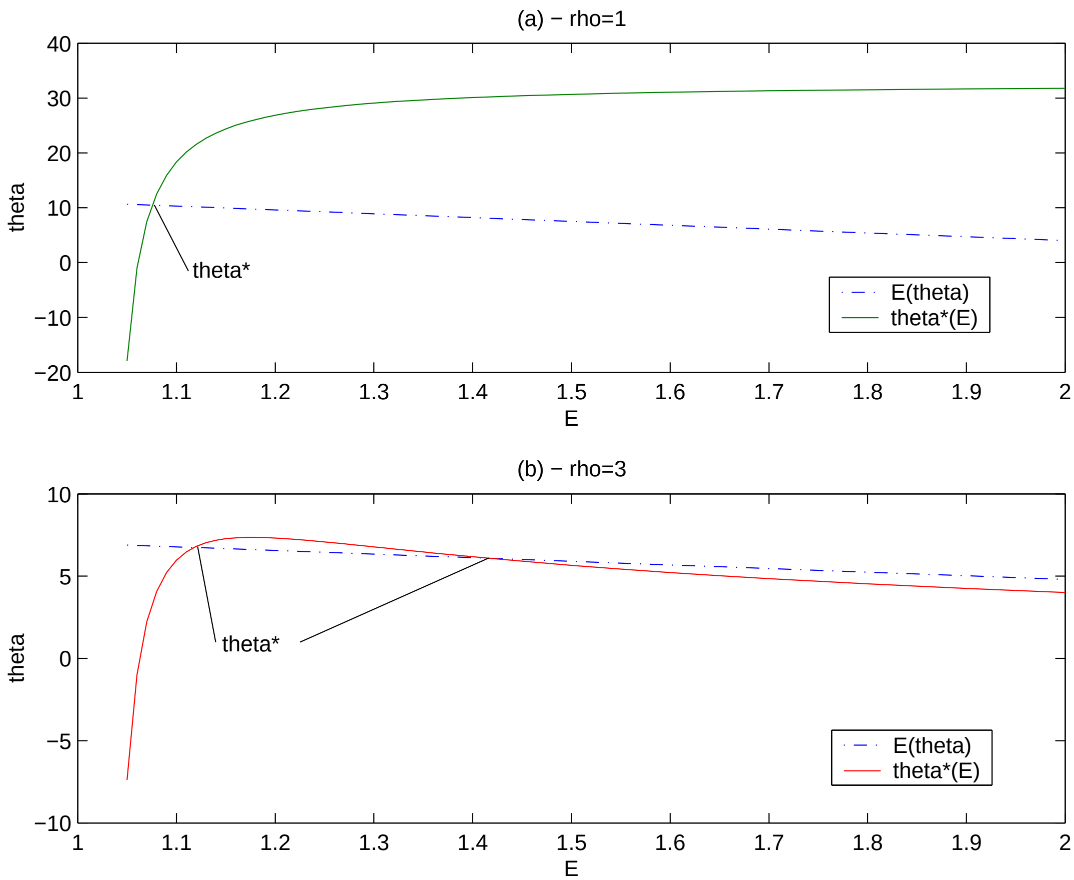
However, for $\rho > 1$, agents' demand for dollars may be decreasing in E, so there might be multiple solutions to equation (10) and thus multiple equilibria in the game. Figure 5-b shows an example for $\rho = 3$ in which the (exogenous) relation between fundamentals and the exchange rate crosses the function $\theta^*(E)$ more than once. In this case, we would require additional restrictions — e.g., upper bounds on the slope of $e_1(.)$ — to restore uniqueness.

4.3.3 The Constant Relative Risk Aversion Assumption

We assumed throughout that agents had constant relative risk aversion. This allowed simple solutions. Suppose instead that the agent has utility function $U(\cdot)$ over his vNM index. We report how some of our results would vary with more general utility functions. Clearly, we would have

[23] Parameters used for the graphs at 5-a and 5-b: $w_D = 1$, $w_P = 1$, $e_0 = 1$, $r = 0.03$ and $\alpha = 0$.

Figure 5: Equilibria



$$y^*(\pi) = \underset{y \in [\underline{\theta}, \overline{\theta}]}{\arg\max} \left[\pi U \left(\widetilde{w}(y, e_0) \right) + (1-\pi) U \left(\left(\frac{E}{e_0} \right)^{1-\alpha} \widetilde{w}(y, E) \right) \right]. \tag{11}$$

and

$$\theta^* = \int_{\pi=0}^{1} y^*(\pi) \, d\pi.$$

Hyperbolic Absolute Risk Aversion Consider the three parameter family of "hyperbolic absolute risk aversion" utility functions, with

$$U(w) = \zeta \left(\eta + \frac{w}{\rho} \right)^{1-\rho}.$$

The special case where $\eta = 0$ corresponds to constant relative risk aversion coefficient ρ and the special case with $\eta > 0$ and $\rho \to \infty$ corresponds to constant absolute risk aversion coefficient $\frac{1}{\eta}$.[24] In

[24] For derivations of these results and more on this class of utility functions, see Gollier (2001); these utility functions are defined only when $w > -\eta\rho$; concavity of U requires that we have $\frac{\zeta(1-\rho)}{\rho} > 0$.

this case, the Inada conditions no longer hold (for $\eta \neq 0$) so the comparison between the incomplete information game and the complete information game is harder to interpret. However, limiting optimal demands are

$$y^*(1) = -\frac{w_D + \frac{w_P}{E} + \eta\rho}{1 - \frac{e_0}{E}(1+r)} \text{ and } y^*(0) = \frac{w_D + \frac{w_P}{e_0} + \eta\rho}{r}$$

The first order conditions imply the same formula for

$$\widehat{y}(\pi) = \frac{y^*(\pi) - y^*(1)}{y^*(0) - y^*(1)} = \frac{1}{1 + \left(\frac{\pi}{1-\pi}\right)^{\frac{1}{\rho}} \left(\frac{1-t}{t}\right)^{1-\frac{1}{\rho}}}.$$

Thus we have a general formula for the hyperbolic absolute risk aversion case:

$$\theta^* = \left(1 - \widehat{\theta}\right)\left(-\frac{w_D + \frac{w_P}{E} + \eta\rho}{1 - \frac{e_0}{E}(1+r)}\right) + \widehat{\theta}\left(\frac{w_D + \frac{w_P}{e_0} + \eta\rho}{r}\right)$$

$$\text{where } \widehat{\theta} = \int_{\pi=0}^{1} \frac{1}{1 + \left(\frac{\pi}{1-\pi}\right)^{\frac{1}{\rho}} \left(\frac{1-t}{t}\right)^{1-\frac{1}{\rho}}} d\pi.$$

In the special case where $w_P = 0$, this simplifies to

$$\begin{aligned}
\theta^* &= (w_D + \eta\rho)\left(\left(1 - \widehat{\theta}\right)\left(-\frac{1}{1 - \frac{e_0}{E}(1+r)}\right) + \widehat{\theta}\left(\frac{1}{r}\right)\right) \\
&= \frac{w_D + \eta\rho}{r + 1 - \frac{e_0}{E}(1+r)}\left(\left(1 - \widehat{\theta}\right)\left(-\frac{1}{1-t}\right) + \widehat{\theta}\left(\frac{1}{t}\right)\right)
\end{aligned}$$

If $\alpha = 1$, one can show that as $\rho \to \infty$,

$$\begin{aligned}
\rho\left(\widehat{\theta} - t\right) &= \rho\left(\int_{\pi=0}^{1} \frac{1}{1 + \left(\frac{\pi}{1-\pi}\right)^{\frac{1}{\rho}} \left(\frac{1-t}{t}\right)^{1-\frac{1}{\rho}}} d\pi - t\right) \\
&\to -t(1-t)\ln\frac{t}{1-t}.
\end{aligned}$$

Thus as $\rho \to \infty$,

$$\begin{aligned}
\theta^* &\to \frac{w_D + \eta\rho}{r + 1 - \frac{e_0}{E}(1+r)}\left(\begin{array}{c} \left(1 - t + \frac{t(1-t)}{\rho}\ln\frac{t}{1-t}\right)\left(-\frac{1}{1-t}\right) \\ + \left(t - \frac{t(1-t)}{\rho}\ln\frac{t}{1-t}\right)\left(\frac{1}{t}\right) \end{array}\right) \\
&= \frac{w_D + \eta\rho}{\left(r + 1 - \frac{e_0}{E}(1+r)\right)\rho}\ln\frac{t}{1-t}\left(t(1-t)\left(-\frac{1}{1-t}\right) - t(1-t)\left(\frac{1}{t}\right)\right) \\
&= -\frac{\eta}{r + 1 - \frac{e_0}{E}(1+r)}\ln\frac{t}{1-t}.
\end{aligned}$$

Thus under the one way bet assumption with constant absolute risk aversion case, we have that θ^* is increasing in the coefficient of constant absolute risk aversion, going from $-\infty$ as the absolute risk aversion coefficient $(\frac{1}{\eta})$ tends to 0 to 0 as the coefficient goes to ∞.

The $t = \frac{1}{2}$ case We mention one case where we can solve for a general utility function $U(\cdot)$ that provides some useful intuition for our earlier results. If $\alpha = 1$, $w_P = 0$ and $t = \frac{1}{2}$ (i.e., $r = 1 - \frac{e_0}{E}(1+r)$), then, by symmetry, $y^*(\pi) = -y^*(1-\pi)$. This implies that $\widehat{\theta} = \frac{1}{2}$ and $\theta^* = 0$.

Decreasing Absolute Risk Aversion Most of arguments do not generalize to an arbitrary decreasing absolute risk aversion utility function. However, when we performed comparative statics of wealth in the incomplete markets case earlier, we were only using the DARA property of our CRRA utility function (as in Goldstein and Pauzner (2001)).

4.3.4 The Infinitessimal Agent Assumption

We assumed a continuum of agents. With a finite set of agents, each agent would anticipate his own impact on whether a crisis would occur. The direction of this effect would vary across the scenarios we considered. If agents were risk neutral, $\alpha = 1$ and $w_P = 0$, then each agent has a private interest in having a successful attack. So attacks would be more likely with large traders (this is the case analyzed by Corsetti, Dasgupta, Morris and Shin (2003)). But if $w_P > 0$ (e.g., the agent has foreign direct investment in the country) then an agent has a lot to lose from devaluation. In a continuum model, this does not influence his best response. But with large traders, this would make attacks less likely.

5 Conclusion

We built a 'global-games' model of currency crisis in order to analyze the impact on agents behavior of issues related to risk and wealth. While our analysis concerns currency crises, the modelling may be relevant to a wide array of macroeconomic issues. The analysis of risk and wealth is central to macro. Self-fulfilling beliefs and strategic complementarities play an important role in many macroeconomic settings. In the marriage of these two strands in this paper, risk, wealth and portfolio effects play a central role in determining how strategic complementarities translate into economic outcomes.

Under our naive, static, complete markets model of agents' portfolio choices, we were able to derive a number of striking predictions about the likelihood of currency crises. However, our conclusions were sensitive to the market assumptions: plausible sounding incomplete market restrictions can have a dramatic impact on comparative statics. Real currency markets reflect the transaction, hedging and speculative demands of many private traders, the policy interventions of central banks and the strategies of large institutions such as hedge funds that may be hard to explain and model as the aggregation of individual utility maximizing behavior. One message of this paper is that if currency crises are self-fulfilling, the motives and strategies of market participants may be important in a way they are not in models where an arbitrage condition (and not strategic considerations) pins down the equilibrium.

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