Accessible Pareto-Improvements:
Using Market Information to Reform Inefficiencies

by

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Abstract:

We study Pareto improvements whose implementation requires knowledge of only market prices and traded quantities, not utility and demand functions. Quantity stabilization gives agents the right to repeat the net trades they previously conducted, but requires policymakers to have records of those trades. While reasonable in some partial equilibrium contexts, such an assumption is implausible in general equilibrium. To diminish informational requirements further, we also consider price stabilization, which holds constant the relative prices that consumers face. Although price stabilizations do not achieve first-best efficiency, they lead to Pareto-improvements and production efficiency. Moreover, the production efficiency advantage persists under price stabilization but not under quantity stabilization when some firms are not profit-maximizers; this difference can be critical in transition policies for planned economies. In addition to planning, we consider several other applications of quantity and price stabilization, both partial equilibrium and general equilibrium: removal of rent controls, deregulation of a cross-subsidizing public utility, and the entry of an autarkic economy into world trade.

1. Introduction

The fact that the standard policy prescriptions of economics inevitably harm some agents has long troubled welfare theory. Economists often turn to the idea of a potential Pareto improvement to justify policy recommendations, but limits on information mean that policymakers cannot calculate the transfers needed to translate potential improvements into actual ones. This paper considers two schemes to achieve actual Pareto improvements and that can be instituted using only information about market transactions, the type of data policymakers have at their disposal. Since the schemes do not use a revelation procedure in which agents communicate their characteristics, agents do not have to understand a game form imposed by policymakers, know how to play that game optimally, or concur on what equilibrium and solution concept they are part of.

A policymaker encounters an economy with some preexisting inefficiency (say the state directs production in some industries) and observes agents' net trades. By giving agents the right to repeat their ex ante trades and the option of making further trades, the policymaker can ensure that a policy reform leaves no agent worse off. One way to view such policies, which we call *quantity stabilizations*, is as a set of lump-sum transfers that ensure that agents can afford their ex ante trades. Since the transfers are lump sum, Pareto optimality is achieved as well if markets are complete and competitive. The second welfare theorem also provides a way to achieve a Pareto-improving optimum, but the theorem supposes that policymakers specify an optimal allocation that they wish to implement; and this information is inaccessible in the extreme. Quantity stabilization aims only to achieve *some* Pareto-improving optimum, and the transfers become simple to calculate.

When policymakers do not know agents' ex ante quantity choices but do know the prices that agents face ex ante, a dual approach is available. A *price stabilization* leaves the relative prices facing consumers unchanged and does not diminish any agent's endowment. Once again, no agent can be worse off, although here, since consumer relative prices are fixed, a Pareto optimum is not reached. The feasibility of these two schemes is the primary theory issue we will consider.

Lau, Qian, and Roland's (1997, 2000) theory of dual-track reforms for planned economies – quantity stabilizations in our terminology – marks an important advance in the treatment of Pareto improvements: they rightly stress that plausible policies must not rely on detailed information. Their model gives agents the right to repeat at the plan's prices the purchases and sales they previously made under planning, and then allows unconstrained trading. For the reasons we have outlined, such reforms are Pareto-improving and reach a Pareto optimum.

The feasibility of quantity stabilizations partly depends on policymakers knowing the trades agents execute prior to reform. In partial equilibrium, that level of knowledge may be modest, as we illustrate in a simple model of rent control. But in general equilibrium, the required knowledge can be considerable; thus, even if quantity stabilizations require less information than the second welfare theorem demands, they can still ask too much. For example, a state planner will know agents' ex ante trades only if it dictates all purchases and sales, denying consumers the latitude to make even trivial consumption decisions. This assumption poorly describes actual planned economies. But if the state does not know consumer trades under planning, consumers under reform must be allowed to buy or sell arbitrary quantities at the plan prices to ensure a Pareto improvement; this leeway can make quantity stabilizations infeasible. And, as we will see, even with complete information about prereform trades, the feasibility of quantity stabilizations remains problematic.

Price stabilizations can overcome the informational obstacles that stand in the way of quantity stabilizations. Consumers determine their purchases at relative prices that either remain unchanged or that change in ways that cannot harm them (e.g., consumption good prices fall). As with quantity stabilizations, consumers can always execute the trades they made ex ante and hence cannot be worse off; if their wealth levels increase or if consumption goods prices fall, consumers are better off. The limitation on consumer price changes means that equilibria usually will not be fully efficient. But excise taxes can allow firms and consumers to face distinct prices, and hence production efficiency and partial allocative efficiency can be attained (in accord with Diamond and Mirrlees (1971) on optimal taxation). Price stabilizations thus disentangle production efficiency from the distributional consequences of reforms; if production inefficiencies are an economy's primary difficulty, then a price stabilization may be attractive. Mandler (1999) analyzes a tatonnement adjustment scheme for tax rates that uses only market information.

We illustrate price stabilization with two examples: (1) a partial equilibrium model that allows a regulated public utility to achieve production efficiency without harming the customers whom the utility was previously required to subsidize, and (2) a general equilibrium model that allows an autarkic economy to enjoy the benefits of international trade without harming the agents who would normally be hurt when domestic relative prices equilibrate at world levels.

To bring out some of the difficulties of quantity stabilization and to compare it with price stabilization, we study transitions from planning in some detail. We show that when the state under planning dictates exact trades, which is necessary if quantity stabilization is to be feasible, consumers will generically accumulate money balances, causing a fiscal deficit under planning. Under reform, the state must raise taxes to close this deficit, and those taxes can undermine the Pareto-improvement goal. Price stabilization, on the other hand, can be applied even if the government under planning does not dictate all trades. Lastly, we compare price and quantity stabilization with respect to production efficiency. Under quantity stabilization, the state must continue to pay firms the subsidies they received under planning (so that they can still afford their prereform trades). But the subsidies can also shelter inefficient non-profit-maximizing firms from competition, a chronic problem in the reform of planned economies. Price stabilizations do not suffer from this drawback.

We do not mean to exaggerate the benefits of quantity or price stabilization. Even though they achieve Pareto improvements, the stabilization schemes may not be particularly desirable from a welfare point of view. A non-stabilized competitive equilibrium may well be normatively superior. But a Pareto improvement has the distinct advantage that it can overcome (buy off) the vested interests that may otherwise block all change from a grossly inefficient status quo. If only a "big bang" competitive equilibrium is offered as an alternative, in contrast, the status quo may prevail.

2. Quantity stabilization

An economy suffers from some status quo inefficiency, e.g., production is planned by the state or housing rents are controlled, that calls for reform. Policymakers observe a set of net trades, $\bar{z}_i$ for each agent i, that occur at prices $\bar{p}$. The value of i's net trades is nonnegative, that is, they satisfy the budget constraint $\bar{p} \cdot \bar{z}_i \leq 0$. And the sums of the net trades is feasible: $\Sigma_i\ \bar{z}_i \leq 0$.

Quantity stabilizations give agents the right to repeat their prior trades and the obligation to repeat prior trades when other parties insist. Agents may resell any goods they acquire through these obligated trades, but resales must occur at whatever new market prices are induced by the policy change. When agent i is a net purchaser of good k and the new postreform price of k, $p(k)$, rises relative to its prereform price $\bar{p}(k)$, that is, if

$$\bar{z}_i(k) > 0 \text{ and } p(k) > \bar{p}(k),$$

i will want to repeat the trade, earning $(p(k) - \bar{p}(k))\,\bar{z}_i(k)$ in arbitrage profits. These are i's profits even if i does not consume any of good k since after buying $\bar{z}_i(k)$ units of k at $\bar{p}(k)$, i may sell at price $p(k)$. When i is a net seller of k and the price of k rises relative to its prereform price,

$$\bar{z}_i(k) < 0 \text{ and } p(k) > \bar{p}(k),$$

the agents who bought from i will invoke their right to repeat their trades. So i experiences a loss of $(p(k) - \bar{p}(k))\,\bar{z}_i(k)$. The cases when $p(k) < \bar{p}(k)$ are similar. In sum, agent i receives arbitrage profits equal to

$$\Sigma_k\ (p(k) - \bar{p}(k))\,\bar{z}_i(k) = (p - \bar{p}) \cdot \bar{z}_i,$$

and i's budget constraint is

$$p \cdot z_i \leq (p - \bar{p}) \cdot \bar{z}_i. \tag{2.1}$$

Since $\bar{p} \cdot \bar{z}_i \leq 0$, agent i can replicate his or her prereform net trades: $z_i = \bar{z}_i$ always satisfies the budget constraint 2.1. Moreover, since the right hand side of 2.1 amounts to a lump-sum transfer, Pareto efficiency can still be achieved: each agent's marginal decisions about current trades and consumption are valued at the same prices p. Seen in this light, quantity stabilization amounts to an informationally parsimonious way to design transfers that ensure that agents can afford their ex ante consumption bundles.

Quantity stabilizations can be instituted in two ways: (1) as a set of private trades that agents have the right or obligation to repeat, (2) as a set of lump-sum transfers issued by the state. The advantage of (1) is that quantity stabilization can remain feasible even if policymakers have no *immediate* knowledge of the $\bar{z}_i$. Of course, if the government has no access to the $\bar{z}_i$, then agents will repudiate their obligations to trade. But if agents can establish the validity of claims about prereform trades in court (agents sue their trading partners if they refuse to honor their obligations to repeat trades), then decentralized quantity stabilization may still be possible. The mere threat of court action by itself might be sufficient to enforce the repetition of trade if we suppose that judges assign court costs to the losers of suits over trade claims. Note also that method (1) does not have to proceed via a physical transfers of goods; agents i for whom $(p(k) - \bar{p}(k))\,\bar{z}_i(k)$ is negative could hand over $|(p(k) - \bar{p}(k))\,\bar{z}_i(k)|$ to their previous traders in good k to discharge their obligations.

Under method (2), the government in effect transfers to agents the appropriate Slutsky compensations, $(p - \bar{p}) \cdot \bar{z}_i$ to each agent i. If complementary slackness obtains ex ante (i.e., $\sum_i \bar{z}_i(k) < 0 \Rightarrow \bar{p}(k) = 0$), the sum of the transfers is nonpositive: complementary slackness implies $\bar{p} \cdot \sum_i \bar{z}_i = 0$, and so $\sum_i (p - \bar{p}) \cdot \bar{z}_i = \sum_i p \cdot \bar{z}_i = p \cdot \sum_i \bar{z}_i \leq 0$. Under mild conditions, therefore, a government can afford quantity-stabilization transfers.

Notice how easy Slutsky compensations are to devise compared to Hicksian compensations, which would transfer to each i exactly the amount of income that would keep i at his or her ex ante utility level. Slutsky compensations will generally not leave agents at the same utility levels, but the errors always overshoot: no agent's utility falls.

If agents prior to reform spend all their wealth ($\bar{p} \cdot \bar{z}_i = 0$ for each i), then the lump-sum transfers are very simple: $p \cdot \bar{z}_i$ to each agent i. The government thus does not even need to know the prereform prices $\bar{p}$ to calculate quantity-stabilization transfers. (Also, since complementary slackness obtains when all agents satisfy their budget constraints with equality, the government can necessarily afford the transfers in this case.) We will see in our application to planning, however, that the case where $\bar{p} \cdot \bar{z}_i < 0$ for some i arises naturally.

The above treatment is general equilibrium. Partial equilibrium quantity stabilizations are analyzed similarly. Suppose prior to reform that a policymaker knows only the price of good 1 and the trades in good 1: $\bar{p}(1)$ and the $\bar{z}_i(1)$. And assume, in the spirit of partial equilibrium analysis, that the government is contemplating a reform that will affect only the price of good 1: for $k \neq 1$, $p(k) = \bar{p}(k)$.

The necessary transfer to each agent *i* then reduces to

$$(p(1) - \bar{p}(1))\,\bar{z}_i(1).$$

Our previous conclusion still holds: each agent can afford his/her prereform trades and cannot be worse off. In this case, however, the government does not have the knowledge to calculate the simple transfers $p \cdot \bar{z}_i$ that entirely ignore prereform prices. Transferring $p(1)\,\bar{z}_i(1)$ to each *i* will *not* allow an agent *i* with $\bar{z}_i(1) < 0$ to afford his or her preform trades. We have allowed just one price to change in this case, but the same conclusions carry over if some subset of the economy's prices were to change. We consider a multiple-good partial equilibrium case of quantity stabilization in our analysis of rent control in section 4.

We have assumed so far that all agents can insist that all of their prereform transactions are repeated, which we call a system of *strong obligations*. But when a consumption good *k*'s prereform price $\bar{p}(k)$ exceeds $p(k)$, then prereform buyers suffer losses when forced to repeat their purchases and so have an incentive to lie about their earlier purchases. Since there are far fewer sellers than buyers of consumption goods, it would presumably be less costly to compel sellers to honor their prereform trades than to compel buyers. Whether for this reason or on political grounds, a policy of forcing only sellers to repeat trades may be preferable. *Weak obligations*, under which buyers can compel sellers to repeat prereform exchanges but not vice versa, are therefore important. It turns out in general equilibrium that the conditions under which quantity stabilizations with strong obligations exist also ensure the viability of a Pareto-improving quantity stabilization with weak obligations: simply let the postreform price of *every* good be greater than its prereform price, thus eliminating any incentive for buyers disavow prereform purchases (section 7, Theorem 3). In partial equilibrium, in contrast, most goods are not part of the reform and so this trick is unavailable. Partial-equilibrium quantity stabilizations with weak obligations can therefore necessarily fail to be Pareto improving, since sellers may not be able to replicate their prereform trades (section 4).

With regard to the feasibility of quantity stabilizations, however, it is the partial equilibrium model that has the advantage. The conditions that ensure the existence of general-equilibrium quantity stabilizations are strong: policymakers need to know all of the prereform trades. Perhaps the only natural context for such an assumption is in a planned economy, where the state dictates trades ex ante. Even here, though, the assumption remains demanding, and can conflict with other requirements for a successful quantity stabilization (section 7).

The theory of quantity stabilization thus presents a trade-off to policymakers. Government knowledge of ex ante trades is vastly more plausible in partial equilibrium settings than in general equilibrium, but in partial equilibrium policymakers may have to must force buyers to repeat purchases to ensure a Pareto improvement.

The existence of quantity stabilizations in a general equilibrium model raises some technical points. From the second-welfare-theorem point of view, finding an equilibrium to support a Pareto improvement is easy. A policymaker chooses an *interior* optimal allocation and then calculates supporting prices and transfers that leave agents positive incomes equal to the value of their assigned consumption bundle. With limited information, on the other hand, a policymaker might not be able to specify any optimal allocation, let alone an interior one. By using quantity-stabilization transfers, a policymaker guarantees that any equilibrium is Pareto-improving – but the transfers can leave agents with zero income at some price vectors, and that can mean that no equilibria exist. (A quantity stabilization in effect gives each agent an endowment equal to his or her ex ante consumption bundle, and that point may well be on the boundary.) In reference to Grandmont and McFadden's (1972) design of Pareto-improving transfers for autarkic economies entering world trade, this problem was pointed out by Cordella, Minelli, and Polemarchakis (1999). We defer this problem to section 7, but the reader is forewarned that we will impose assumptions that are somewhat stronger than normal.

3. Price stabilization

We now suppose that the economy necessarily suffers from a *production* inefficiency ex ante. For example, the state may be dictating the organization of production, or regulators may be sheltering an industry from competition, thus discouraging the adoption of new technologies, or the state may be directing an industry to sell at specific prices that do not reflect marginal costs. But now, rather than knowing individual trades, policymakers know only the initial prices $\bar{p}$. If the laws or regulations that fostered inefficiency were removed, and if prices p could somehow be stabilized at $\bar{p}$ and no agent's endowment of resources were to fall, then no agent could be worse off. If the government can distribute money or physical wealth to agents, then some or all agents could be made better off. Alternatively, a policymaker could ensure that agents are better off by lowering the price of goods which agents are net purchasers of or by raising the price of goods which agents are net sellers of. Call prices *stabilized* if only price changes such as these occur.

Keeping prices stabilized will generally be inconsistent with a big-bang deregulation or privatization. And if a government were to enforce a single set of stabilized prices by fiat, then some or all of the production inefficiency might persist. For instance, if $\bar{p}$ had earlier been set by a planner, the factor prices in $\bar{p}$ may have been chosen to serve political goals (e.g., boosting the income of low marginal-productivity factors) rather than efficiency considerations. Or a regulated monopolist might have been rent-sharing with its workers, thus raising the price of the labor it uses. In both cases, keeping factor prices at their $\bar{p}$ levels (or higher) would be a barrier to efficiency

Price stabilization as a reform strategy separates the production inefficiencies caused by regulation from the distributional consequences of regulation. The former is reformed by letting producers face a distinct set of prices, say q, that can differ from the consumer prices $\bar{p}$. In models of general equilibrium (section 6), a price-stabilized equilibrium therefore is a $q > 0$, a distribution of wealth to agents, and a feasible y_j for each firm j such that markets clear and each y_j maximizes firm j's profits calculated at prices q. As we will see, if production is initially productively inefficient, there will exist a price stabilization that is production efficient and that Pareto improves on the initial equilibrium. If we view $\bar{p} - q$ as a vector of commodity taxes, the production efficiency conclusion accords with the Diamond and Mirrlees (1971) results on optimal taxation.

Fixing consumer prices at $\bar{p}$ comes at the cost that consumer and producer prices do not equalize, leading at least a residual inefficiency to remain. Price stabilization proceeds on the assumption that elimination of production inefficiencies is the more important objective, or at least that not harming any agent trumps any other goal.

Price stabilization places modest informational demands on the government compared to either second-welfare-theorem compensations or quantity stabilizations. To institute a price-stabilized equilibrium, a policymaker need know only how to set the tax rates $\bar{p} - q$ and how much wealth to distribute. The former can be determined by adjusting producer prices according to supply and demand: raise $q(k)$ when market demand for k at $\bar{p}(k)$ is greater than market supply at $q(k)$ and lower $q(k)$ when demand is less than supply. Mandler (1999) shows that under decreasing returns to scale (so that supply functions are well-defined) the tax rates will converge to values that support a price-stabilized equilibrium. Convergent adjustment mechanisms for consumer wealth also exist, and they can run concurrently with the adjustment of tax rates. Moreover, the adjustment mechanism for consumer wealth employs only information about how aggregate consumer demand responds at the planning prices $\bar{p}$ to changes in aggregate consumer income; no information about individual utilities or wealth effects is needed.

Partial equilibrium price stabilizations, which affect only a few goods, are also possible (section 5). In this case, a natural mechanism to ensure that no consumer is worse off and some are better off is to lower the consumer prices of the goods in question. The government's feasibility constraint is then to find subsidy and tax rates that are revenue neutral. A natural policy adjustment rule converges in plausible cases, similarly to the general equilibrium case.

We now turn to several example of quantity and price stabilization, beginning the simpler partial equilibrium cases.

4. Reforming rent control: quantity stabilization in partial equilibrium

Economists have long viewed rent control as a paradigmatic inefficiency: since tenants with rent-control leases will often have reservation prices for their apartments that are lower than the valuations of other agents, Pareto improvements are possible. Quantity stabilization achieves those

Pareto improvements by giving tenants the right both to continue their rent-control leases and to relet their apartments at market prices. From the Coasean point of view, the defect of rent control is that neither current tenants nor owners can reassign properties freely; quantity stabilization clarifies property rights by assigning them to tenants, allowing Pareto improvements to move forward. The feasibility of quantity stabilizations depends on whether or not policymakers (or courts) can verify the quantities that agents exchange ex ante. Since preexisting leases provide a written record of the needed information, rent control is thus a plausible candidate for quantity stabilization.

Let A be a set of n apartments, with each apartment $a_k \in A$ having a landlord $l(a_k)$ and a tenant $t(a_k)$. The lease for a_k specifies that $t(a_k)$ may live in a_k as long as he or she desires for a fixed rental price of $\bar{p}(a_k)$ per period. For any agent i, let $T(i)$ indicate the set of apartments of which i is the tenant and let $L(i)$ indicate the set of apartments that i owns. (Thus, $a_k \in T(t(a_k))$ and $a_k \in L(l(a_k))$.) For simplicity, suppose that each agent is the tenant of at most one apartment. An agent i's budget constraint initially therefore is

$$x_i + \bar{p}(T(i))h_i \leq I_i + \sum_{a_k \in L(i)} \bar{p}(a_k),$$

where x_i is i's non-housing consumption (which has price 1), h_i is 1 if i chooses to rent $T(i)$ and 0 if i chooses not to rent, and I_i is i's nonrental income. To cover nontenants, define $\bar{p}(\varnothing)$ to equal 0. The fact that a tenant may choose $h_i = 0$ means that tenants are not forced to rent apartments (in the language of section 7, rationing is partial, not complete).

Each agent maximizes utility subject to the above budget constraint. In equilibrium, it must be utility-maximizing for each tenant i to choose $h_i = 1$ (otherwise i vacates $T(i)$ and some other agent becomes its tenant). Another way to express this requirement is that for each apartment a_k the reservation price of agent $t(a_k)$ for a_k must be greater than or equal to $\bar{p}(a_k)$. Due to the lack of competitive markets, for many apartments a_k, there will be some agent j with a reservation price for a_k that is larger than $t(a_k)$'s reservation price. Leases are not transferable, however, and so this inefficiency can persist. Tenants, we assume, have enough political power to block repeal of rent control.

Quantity stabilization can maneuver around this roadblock. Each agent i who continues to rent $T(i)$ is now permitted to sublet a_k at the going market price. If each i must continue to rent $T(i)$, then obligations are *strong*, while if i can abandon a rent-control lease, obligations are *weak*.

Letting $p = (\ ..., p(a_k), ...\ )$ be the market prices, the typical agent i subject to strong obligations has the budget constraint

(4.1) $$x_i + p \cdot c_i \leq I_i + p(T(i)) - \bar{p}(T(i)) + \Sigma_{a_k \in L(i)}\, \bar{p}(a_k).$$

where c_i is an n-vector of 1's and 0's whose kth entry indicates that i rents (if 1) or does not rent (if 0) a_k. The term $p(T(i)) - \bar{p}(T(i))$ are the profits or losses i earns on subletting $T(i)$, the apartment for which he or she has a rent-control lease.[1]

When subject to weak obligations, i's budget constraint is

(4.2) $$x_i + p \cdot c_i \leq I_i + \max[p(T(i)) - \bar{p}(T(i)), 0] + \Sigma_{a_k \in L(i)} \min[p(a_k), \bar{p}(a_k)].$$

Since obligations are weak, the tenant of a_k will abandon his or her lease if $p(a_k) < \bar{p}(a_k)$ and the landlord of a_k therefore earns only $p(a_k)$ in this case. So i's subletting profits equal $\max[p(T(i)) - \bar{p}(T(i)), 0]$ and i's profits as a landlord are $\Sigma_{a_k \in L(i)} \min[p(a_k), \bar{p}(a_k)]$.

To complete the model, assume that each agent i maximizes the utility function $u_i(x_i, c_i)$ subject to either 4.1 or 4.2, generating an n-vector of housing demands as a function of p, say $c_i(p)$. A *quantity-stabilized equilibrium* (with either strong or weak obligations) occurs at a p such that demand is no greater than supply, $\Sigma_i\, c_i(p) \leq (1, ..., 1)$: at most one tenant chooses to rent each apartment.[2]

If each agent i has a utility function that is linear in nonhousing consumption, $u_i(x_i, c_i) = x_i + v_i(c_i)$, then, just as with standard competitive markets, consumer surplus (the sum of utilities) is maximized at a quantity-stabilized equilibrium. Pareto inefficiency is eliminated.

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[1] Agent i's prereform excess demand for $T(i)$ equals 1, and so $p(T(i)) - \bar{p}(T(i))$ in 4.1 has the same form as the transfer term for $T(i)$ in the budget inequality 2.1 in section 2. Agent i's excess demand for the apartments i owns but does not rent is -1, but since for these apartments c_i is a demand rather than an excess demand (c_i equaling 0 in these coordinates), $\Sigma_{a_k \in L(i)}\, \bar{p}(a_k)$ rather than $(\Sigma_{a_k \in L(i)}\, p(a_k) - \Sigma_{a_k \in L(i)}\, \bar{p}(a_k))(-1)$ appears on the right hand side of 4.1.

[2] For n-vectors x and y, $x \geq y$ means $x(i) \geq y(i)$ for each coordinate i, $x > y$ means $x \geq y$ and not $y \geq x$, and $x \gg y$ means $x(i) > y(i)$ for each i.

With regard to achieving a Pareto improvement, rent-control quantity stabilizations exhibit the trade-off discussed in section 2. To necessarily be Pareto-improving, a quantity stabilization must employ strong obligations. An agent *i* who owns no apartments cannot be worse off under either strong or weak obligations since the x_i selected prior to reform and c_i equal to 1 in the $T(i)$ component and 0 elsewhere is affordable: this (x_i, c_i) satisfies both 4.1 and 4.2. But under weak obligations if there are apartments a_k such that $p(k)$ falls below $\bar{p}(k)$, then the landlord $l(a_k)$ will be worse off unless he or she enjoys offsetting gains on other properties or as a tenant. For $p(k)$ to fall below $\bar{p}(k)$, it must be that the reservation price of $t(a_k)$, which originally had to be above $\bar{p}(k)$, falls below $\bar{p}(k)$ under reform. (Of course, the other agents must then also have reservation prices for a_k that are below $\bar{p}(k)$.) This possibility is by no means pathological. Reservation prices for a_k depend on the set of available apartments and on the rental prices of other apartments. Prior to reform, a tenant *i* rents only the $T(i)$ (say an apartment in Queens) for which he or she has a lease. The new availability of other apartments (in Manhattan) might well drive down the value that *i* and other agents place on $T(i)$.

In general equilibrium, a quantity stabilization can always be designed so that each price $p(k)$ is greater than the prereform price $\bar{p}(k)$, ensuring both that all agents invoke their right to buy at $\bar{p}$ and that no seller is worse off (see section 7). But this option is unavailable in the current partial equilibrium setting.

While as a matter of theory, quantity stabilizations under weak obligations may fail to be Pareto improving, this outcome is less likely if the components of $\bar{p}(k)$ are uniformly low, as is often the case with rent control laws. Policymakers can also restore the Pareto improvement property by various routes. First, even if policymakers are prohibited from directly imposing strong obligations – that is, requiring tenants to honor their rent control leases – they can impose taxes and subsidies that accomplish the same end. For any a_k such that $p(a_k) < \bar{p}(a_k)$, tax the rent-control tenant $t(a_k)$ the sum $\bar{p}(a_k) - p(a_k)$ and give the proceeds to the rent-control landlord $l(a_k)$. Just as with strong obligations, this equilibrium Pareto improves on the original rent-control equilibrium. Such taxes may be politically tricky, but no purely economic barriers stand in the way. Second, revenue to compensate landlords could instead be raised by taxing subletting profits: rather than taxing the tenants $t(a_k)$ with

$p(a_k) < \bar{p}(a_k)$, the government may tax the $t(a_k)$ with $p(a_k) > \bar{p}(a_k)$. Although the receipts from a subletting profits tax might not cover subsidy payments, it is plausible that they would. The tax schemes are either directly or effectively lump sum (since if subletting profits are taxed proportionally $t(a_k)$ will still decide to continue his/her rent-control lease if and only if $p(a_k) > \bar{p}(a_k)$) and hence cause no efficiency loss. Also, the taxes can be implemented using only information from preexisting leases; so the overall reform plan remains informationally accessible.

The Pareto-improvement problem notwithstanding, quantity stabilization is well-suited to the problem of rent control. First, rent-control leases provide clear evidence of tenants' ex ante trades and hence of obligations under a quantity stabilization. Second, there are no producers in the rent-control example whose rights to subsidies are preserved under the reform. The problem of sheltering inefficient producers from market pressure, which we discuss in section 7, therefore does not arise. Lastly, price-stabilized reforms of rent control are effectively impossible. If each agent had the right to rent each apartment at its rent control price, demand for apartments would markedly outstrip supply. (In terms of the model of section 7, the discontinuity of demand problem that can affect price stabilizations would be sizable.)

5. Public utility deregulation: price stabilization in partial equilibrium

To begin our look at price stabilization, consider a simple model of the deregulation of a public utility – telephone service to be concrete. Ex ante, two inefficiencies are present. First, the phone company, since entry is prohibited or regulated, does not produce efficiently (rationales for this are discussed in section 7). Second, regulators require the phone company to subsidize some of its customers. A conventional big-bang deregulation, which simply opens the telephone market to pure competition, correspondingly has two effects: competition induces technological progress, lowering costs, and cross subsidization ends. While the first effect potentially benefits all consumers, the second can be large enough to lead previously subsidized consumers to be worse off overall. Price stabilization in contrast solves the production inefficiency, but allows the cross subsidization to remain intact, at least in the short run. The scheme thus rests on a presumption that the production inefficiency, rather than the distributional goals, is the more serious defect of regulation.

Proceeding to the model, we assume that residences and businesses pay the same price for phone service ex ante even though the cost of providing phone service to businesses is lower – equivalently residences could pay a lower price even though they incur the same or greater cost than businesses. Under regulation the phone company provides service at price $\bar{p}$ to residences and businesses, with demand functions for telephone service given by $x_r(p)$ and $x_b(p)$ respectively. Telephone service to businesses initially costs $\bar{c}_b$ per unit, while telephone service to residences costs $\bar{c}_r$ per unit, where $\bar{c}_r > \bar{c}_b$. We assume that under regulation the phone company breaks even:

$$\bar{p}\,[x_r(\bar{p}) + x_b(\bar{p})] - \bar{c}_r\,x_r(\bar{p}) - \bar{c}_b\,x_b(\bar{p}) = 0.$$

So $\bar{c}_r > \bar{p} > \bar{c}_b$. The regulatory regime's entry restrictions lead to production inefficiency: the costs $c_b < \bar{c}_b$ and $c_r \le \bar{c}_r$ are assumed to be achievable when phone service is instead delivered by competitive firms.

Big-bang deregulation ends cross subsidization, so phone companies will now charge average cost to each type of customer, and achieves the costs c_b and c_r. But if c_r is strictly higher than $\bar{p}$, residences will be worse off under deregulation. The price stabilization alternative imposes taxes and subsidies on telephone companies to ensure that p_r, the deregulated price for residential telephone service, does not rise above $\bar{p}$. Specifically, regulators set a subsidy for residential telephone service, $s > 0$, and a tax rate on business telephone service, $\tau > 0$, to as to satisfy the conditions,

(5.1) $$\bar{p} = c_r - s,$$

(5.2) $$p_b = c_b + \tau,$$

(5.3) $$s\,x_r(\bar{p}) = \tau\,x_b(p_b),$$

where p_b is the post-regulation price for business telephone service.

Under this price stabilization scheme, businesses are still better off relative to regulation and residences are no worse off. The firms producing telephone service break even under both regimes and so their welfare does not change. Price stabilization is not first-best efficient (it does not maximize consumer surplus) since price does not equal marginal cost in either sector. But note that if c_r falls through time to a value below $\bar{p}$, the taxes and subsidies would be self-extinguishing and first-best efficiency would be restored in the long run.

If policymakers know c_r and c_b and the function $x_b(p_b)$, then s and τ may be directly set at values consistent with conditions (5.1) - (5.3). The more plausible case, of course, occurs when policymakers do not have all of this information. Suppose first that a policymaker knows c_r but not c_b or $x_b(p_b)$. The subsidy rate s may then be set to satisfy (5.1). A natural adjustment rule for τ is to raise τ when revenue from taxation falls below expenditures on subsidies and to lower τ when expenditure exceeds revenue. This rule will lead τ to equilibrium if the demand for business telephone services is sufficiently inelastic. To see this, set

$$\dot{\tau} = s\, x_r(\bar{p}) - \tau\, x_b(p_b) = s\, x_r(\bar{p}) - \tau\, x_b(c_b + \tau).$$

Then the stability condition that $\dot{\tau}$ is decreasing in τ will be satisfied when tax revenue $T(\tau)$ = $\tau\, x_b(c_b + \tau)$ is increasing in the tax rate τ. Assuming $x_b(p_b)$ is differentiable, $T'(\tau) = x_b(p_b) +$ $\tau\, x_b'(p_b)$ and stability therefore indeed obtains if $x_b'(p_b)$ is small. The market adjusts p_b so that the equilibrium condition $p_b = c_b + \tau$ is satisfied. Policymakers can thus infer c_b from observations of p_b.

One might argue that a policymaker does not in fact need to know c_r in order to set s: since $\bar{p}$ $= c_r - s$ must hold, if s were set at any value other than $c_r - \bar{p}$, the market will generate boundless supply or no supply at all. But more realistically, we should understand (5.1) and (5.2) as long-run requirements, and suppose that supplies of residential and business phone service are in the short-run increasing functions of $p_r - c_r + s$ and $p_b - c_b - \tau$, respectively (where p_r is the price of residential phone service). To finish the model so that policy adjustment does not rely on *any* nonprice information, let us suppose (in addition to the adjustment rule for τ above) that the policymaker raises or lowers s according to whether p_r is greater than or less than $\bar{p}$. Under these conditions and again assuming x_b' is small, p_r will converge to $\bar{p}$ and s and τ will converge to values that solve (5.1) - (5.3). We omit the details, which are a little cumbersome but routine (see also Mandler (1999)).

Proponents of deregulation frequently fold together the cross-subsidization and the production inefficiency defects of government regulation. Price stabilization uses a simple scheme that relies only on publicly available information to untie the two problems; by eliminating only the production inefficiency, a Pareto improvement (though not a full optimum) becomes feasible.

6. Opening an autarkic economy: price stabilization in general equilibrium

A small closed economy wants to open its markets to international trade, but doing so without redistributions will harm some agents. Although maintaining autarky is Pareto dominated by some free-trade equilibrium with lump-sum redistributions (Samuelson (1939, 1956)), it is implausible to imagine that the government could calculate the required transfers. Quantity stabilization transfers in particular are impracticable since policymakers cannot verify ex ante consumption levels or trades. Price information, on the other hand, is widely available, and so price stabilizations offer a way to capture some of the gains from trade while keeping each agent at least as well off as under autarky.

Let there be a finite set of N goods and finitely many consumers, each consumer i having the locally nonsatiated, strictly quasiconcave utility u_i: $R_+^N \to R$. We assume that production sets are constant returns to scale and therefore describe the production side of the economy by an aggregate production set, the closed convex cone Y_A, which is assumed to contain the negative orthant and to intersect the positive orthant only at 0.

We assume that the economy has a money or credit with price equal to 1. Money does not enter utility functions, is not produced or used as an input, and can be printed in arbitrarily large quantities by the state. In terms of notation, the commodity and price vectors below are N-vectors whose coordinates correspond to the nonmoney goods. Agent i's endowment is e_i and p is the price vector of nonmoney goods. So i's income is $p \cdot e_i + m_i$, where m_i is i's endowment of money. Let $z_i(p, p \cdot e_i + m_i)$ denote i's excess demands.

An *autarky equilibrium* is a $\bar{p} \geq 0$ and $y \in Y_A$ such that $\sum_i z_i(\bar{p}, p \cdot e_i + m_i) = y$ and such that $\bar{p} \cdot y \geq \bar{p} \cdot y'$ for all $y' \in Y_A$.

Let the first n of the N nonmoney goods be traded internationally, let $T \subset R^N$ be the set of N-vectors whose last $N - n$ coordinates are constrained to equal 0, and let $\pi \in T$ be the vector whose first n coordinates are the world prices of the traded goods. We assume that $\pi > 0$ and that the economy is small; so π is unaffected by the economy's trades with the rest of the world. The set of feasible international trades is therefore $T_I = \{\gamma \in T\colon \pi \cdot \gamma = 0\}$. If an economy participates in world trade, it will have at its disposal a trade-modified production set $Y_T = Y_A + T_I$. This closed, convex set indicates the commodity vectors that, through production or international trade, can be traded to consumers. We assume that Y_T does not intersect the positive orthant, which will be satisfied if all produced internationally traded goods require non-traded goods as inputs.

Since $Y_A \subset Y_T$, free trade expands the set of feasible aggregate production vectors relative to autarky. Due to profit maximization under autarky, aggregate production under autarky is on the boundary of Y_A, but not necessarily on the boundary of Y_T.

Price stabilization requires that consumers face the prices $\bar{p}$ and that producers face distinct prices q. A *price-stabilized equilibrium* is therefore a q, a distribution of money to each consumer i $\{m_{di}\}$, and a $y \in Y_T$ such that (1) $\sum_i z_i(\bar{p},\ p \cdot e_i + m_i + m_{di}) = y$ and (2) $q \cdot y \geq q \cdot y'$ for all $y' \in Y_T$.

To state precisely the advantages of price stabilization, we categorize types of efficiency as follows. *Production efficiency* obtains at a price-stabilized equilibrium $(q, \{m_{di}\}, y)$ if there does not exist a $y' \in Y_T$ such that $y' \gg y$. A set of net trades z_i (or the equilibrium that generates those trades) is *partially allocatively efficient* if there does not exist a reallocation of $\sum_i z_i$ that benefits at least one of the consumers and harms none of them, i.e., a set of $z_i' \geq 0$ that satisfies $\sum_i z_i' = \sum_i z_i$ such that $u_i(z_i' + e_i) \geq u_i(z_i + e_i)$ for all i, and with strict equality for some i. We say "partially" because it could be that marginal rates of substitution do not equal marginal rates of transformation even though an equilibrium is both production efficient and partially allocatively efficient.

If production under autarky is in the interior of Y_T, then price stabilization can be successfully applied:

Theorem 1. If y under autarky is in the interior of Y_T, then there exists a price-stabilized equilibrium that is production and partially allocatively efficient and that Pareto improves on the autarky equilibrium.

The proof of Theorem 1 and other proofs are in the appendix.

Given that we are treating international trade as a type of production, the producer prices for the n internationally traded goods must be proportional to π. Up to the exchange rate, the producer and world prices of these n goods coincide.[3] To see this, let q_T be the projection of q onto T. For any $y \in T_I$, $q_T \cdot y = 0$, since it must be that $-y \in T_I$ and therefore both $q_T \cdot y \leq 0$ and $q_T \cdot (-y) \leq 0$. Since $\pi > 0$ and $q_T \geq 0$ are both orthogonal to every $y \in T_I$ and T_I has dimension $n - 1$, $q_T = \alpha\pi$ for some scalar $\alpha \geq 0$. It may be that $q_T = 0$, but for any equilibrium q, there is another equilibrium vector of producer prices, say q', that differs only with respect to the first n coordinates such that q_T' is a strictly positive multiple of π, e.g., $q' = q + \pi$.

Because of the wedge between consumer prices $\overline{p}$ and producer prices q, some goods can be subsidized at a price-stabilized equilibrium. In particular, the domestic consumer price of some goods may be below their world price. Many nations employ such subsidies; for example, underdeveloped countries often subsidize grain or bread prices, and oil producers frequently subsidize domestic gasoline prices. In the former case, international lending agencies have traditionally criticized these subsidies, asserting that domestic prices of goods should be aligned with international prices, and can make aid contingent on the elimination of subsidies. The counter argument is that subsidies deliver indispensable political stability. Price stabilization adds an economic theory rationale for the latter position: subsidies can be an efficient device for shielding agents from the losses that would accompany participation in international markets. Bread purchasers, for example, might suffer considerable losses if the price of bread rose to its international level. Lump-sum transfers to bread purchasers would in principle provide a better way to achieve the same end; but governments will not be able to identify the quantities purchased ex ante.

Price-stabilized subsidies must not be directed to domestic producers alone, however – this could shelter them from lower-cost competitors abroad, thus causing production inefficiency. Subsidies should at most establish a wedge between consumer and producer prices; and the producer price of a subsidized good must (after accounting for the exchange rate) equal its international price. If foreign producers are the sole minimum cost producers of a subsidized good, then those firms and only

[3] See Dixit and Norman (1980, ch. 6) for a classical treatment, from the optimal taxation point of view. I am grateful to Roger Guesnerie for pointing out this reference to me.

those firms would produce the good under a price stabilization. Thus, even if the aim of not harming any agent might conceivably justify domestic protectionism, such sacrifices are unnecessary when price stabilizations are possible.

7. Reforming a planned economy: quantity stabilization vs. price stabilization

Suppose the state organizes production and determines some individual consumption decisions. The most obvious liberalization policy is a big bang, which distributes state assets to consumers, allows firms to make production decisions by profitability, and sets prices according to supply and demand. Although the first welfare theorem implies that the resulting equilibrium will be Pareto optimal, the equilibrium will generally not be Pareto-improving relative to the planning allocation. Some consumers are primarily endowed with factors that are over-priced under planning relative to their market value. For instance, the market will force firms to eliminate obsolete technologies, diminishing the demand for some factors and leaving some factor owners worse off. In principle, losers under a big bang could be fully compensated via lump-sum transfers from winners. But policymakers may not know enough to calculate the transfers.

Quantity stabilizations therefore offer an appealing alternative. As we will see, the viability of quantity stabilization hinges on whether the state under planning dictates exact purchases of all goods and balances its budget. If individuals under planning make their own consumption decisions, perhaps subject to rationing constraints, the government will not know all of the quantities traded under planning, which can make quantity stabilization infeasible. Price stabilizations suffer from fewer existence difficulties and curiously have some efficiency advantages that quantity stabilizations lack.

There are N goods and finitely many consumers, each consumer i having the utility function u_i: $R_+^N \to R$ and an endowment $e_i \in R_+^N$. Each u_i is strictly quasiconcave and strictly increasing in each good. This strong increasingness assumption ensures that quantity-stabilized equilibria exist.[4]

[4] This condition replaces the standard assumption that each agent's endowment of each good is strictly positive. But agents' consumption bundles under planning here play the role of endowments, and it is not reasonable to assume these are strictly positive. One may establish the feasibility of

Under planning, the government sets a price vector $\bar{p} \geq 0$ by fiat and rations consumer demand. Letting $z_i = x_i - e_i$ denote i's net trades or excess demands, the state requires, for any consumer i, that i's trade in good k satisfies either an inequality constraint, $z_i(k) \leq \alpha_i(k)$, or an equality constraint, $z_i(k) = \alpha_i(k)$. In the former case, i is *partially rationed in good k*, while in the latter, i is *completely rationed in good k*. If i is completely rationed in every good, we say that i is completely rationed; otherwise we say i is partially rationed. Let $\alpha_i = (\ldots , \alpha_i(k), \ldots)$. We assume that the plan gives each consumer positive consumption of some good and prescribes deliveries that are not greater than endowments: $e_i + \alpha_i > 0$. We also assume that $\Sigma_i (e_i + \alpha_i) \gg 0$.

Partial rationing constraints need not bind. Indeed, it may be that no consumer is rationed with respect to some good k, as when the state cannot monitor individual purchases or when consumer budget constraints by themselves sufficiently limit demand. We continue to define the constraint levels $\alpha_i(k)$, however, since under quantity stabilization they will serve as consumer rights to buy goods at plan prices. Since the $\alpha_i(k)$ may be set arbitrarily high, no loss of generality is introduced.

We will say that complete rationing occurs for the economy as a whole if every consumer is completely rationed. Although it is common to assume that planned economies are completely rationed, such an extreme form of planning has rarely if ever been employed in large-scale economies. Consumers in real planned economies maintain at least some rights about how to spend their incomes; even when the state dictates factor deliveries, agents retain discretion over some of their consumption purchases. Moreover, prices in planned economies play an important role in curbing demand. Under complete rationing, in contrast, prices serve no such function: for each consumer i, the bundle α_i will either be unaffordable, in which case the model is not internally consistent, or i's purchase of α_i is dictated by the state independently of relative prices. The plan could therefore be implemented without prices or any exchange of money.

Also, for typical values of α_i, if i is completely rationed then i's budget constraint will not be

quantity stabilizations under conditions weaker than our increasingness assumption – see Grandmont and McFadden (1972) for a more general approach to existence that could be applied here.

satisfied with exact equality: only in knife-edge cases will α_i satisfy $\bar{p} \cdot \alpha_i = 0$. In the generic cases, if consumers can afford their rationed bundles ($\bar{p} \cdot \alpha_i \leq 0$), then they will accumulate money. As we will see, if consumers do accumulate money under planning, then quantity stabilizations may be impossible. (If some goods are partially rationed, consumers may accumulate money as well, but it is not a generic event.) To ensure the consistency of planning equilibria, we assume that if rationing is complete then $\bar{p} \cdot \alpha_i \leq 0$.

Each consumer i maximizes $u_i(x_i)$ subject to $x_i \geq 0$, the budget constraint $\bar{p} \cdot (x_i - e_i) \leq 0$, and the rationing constraints discussed above. For each consumer i, $z_i(\bar{p}, \alpha_i, e_i)$ denotes the excess demands, $x_i - e_i$, that solve i's maximization problem.

Production under planning is organized by a finite set of state-owned firms. Each firm j has a production set Y_j, which gives the production vectors that are feasible for j. We assume that each Y_j is convex, closed, contains the negative orthant, and intersects the positive orthant only at 0. Firm j's production under planning is γ_j. With small notational changes, we could include in our model the capital goods that firms under planning receive from the state.

A consumer i's rationing constraint $\alpha_i(k)$ implicitly represents a right or obligation to trade some quantity of k with some set of firms. We use $\beta_j(k)$ to denote the sum of these claims on firm j. When possible, we suppress further mention of the $\beta_j(k)$ but we assume that consumer and firm claims are mutually consistent: $\Sigma_i\, \alpha_i(k) = \Sigma_j\, \beta_j(k)$.

A firm j with $\bar{p} \cdot \gamma_j < 0$ loses money under planning. The state must therefore transfer to j a sum of money $t_j > 0$ to ensure that j does not go bankrupt. When $t_j < 0$, j returns its profits to the state.

A *planning equilibrium* is a price vector $\bar{p}$, a α_i for each consumer i, and a (γ_j, t_j) for each firm j such that markets clear, and each j produces feasibly and makes nonnegative profits:

(7.1) $$\Sigma_i\, z_i(\bar{p}, \alpha_i, e_i) = \Sigma_j\, \gamma_j,$$

(7.2) $$\gamma_j \in Y_j \text{ and } \bar{p} \cdot \gamma_j + t_j \geq 0, \text{ for each firm } j.$$

Since consumers can accumulate money balances, the state may need to run a deficit, that is, issue positive net credits to firms, $\Sigma_j\, t_j > 0$. When consumers accumulate money, their withdrawal of purchasing power leads aggregate firm profits to be negative. So, to keep the economy's firms afloat, the government must in sum pay out positive subsidies. To see this, suppose that at least one consumer, say h, has excess money balances: $\bar{p} \cdot z_h(\bar{p}, \alpha_h, e_h) < 0$. Since $\bar{p} \cdot z_i(\bar{p}, \alpha_i, e_i) \leq 0$ for each consumer i, summing across consumers yields $\bar{p} \cdot \sum_i z_i(\bar{p}, \alpha_i, e_i) < 0$. Since condition (7.1) implies $\bar{p} \cdot \sum_i z_i(\bar{p}, \alpha_i, e_i) = \bar{p} \cdot \sum_j \gamma_j$, it must be that $\bar{p} \cdot \sum_j \gamma_j < 0$. Hence, by condition (7.2), $\sum_j t_j > 0$. As we will see, deficits raise difficulties for quantity stabilizations. Call the *government budget balanced* if $\sum_j t_j = 0$.[5]

The government's knowledge of the trades that occur under planning will be important. We assume that the government can gather information about trades solely from the rationing parameters α_i (the α_i may not be immediately accessible, but the government can as necessary verify claims about the α_i). The government in contrast can*not* ascertain actual purchase data for partially rationed goods. It would be more realistic to give the government other sources of information, but for our purposes all that matters is that we include some trades about which the government is not fully informed.

Although the current model is laid out as a description of planning, it potentially covers many inefficiencies. We have not specified how production decisions are made; they might therefore be made by profit-maximizing producers facing a distortion – say from externalities in production or commodity taxation. And since partial rationing constraints need not bind any of the agents, consumers can be traditional price takers.

Quantity stabilization

As in a big bang, quantity stabilizations distribute the ownership of firms to consumers and firms maximize profits. But unlike a big bang, agents retain certain rights and obligations to repeat their plan trades at the prices $\bar{p}$. In contrast to partial-equilibrium quantity stabilizations, we will see that consumers need not be obligated to repeat all of their prereform purchases. On the other hand, in

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[5] Lau, Qian, and Roland (1997) assume complete rationing obtains and there is no government sector. Consequently, in the generic case when consumers accumulate money, equilibria under planning will not exist. To see why, let firm profits and losses go to consumers, which is how Lau, Qian, and Roland (1997) model firms under planning. Letting Π denote the sum of firm profits, we have $\bar{p} \cdot \sum_i z_i(\bar{p}, \alpha_i, e_i) < \Pi$ if consumers accumulate money. But (7.1) implies $\bar{p} \cdot \sum_i z_i(\bar{p}, \alpha_i, e_i) = \bar{p} \cdot \sum_j \gamma_j = \Pi$.

general equilibrium the feasibility of quantity stabilizations is problematic. If we assume that rationing under planning is partial, then governments may not be able to institute a quantity stabilization. And even when rationing is complete, the government will face a revenue shortfall that also can make quantity stabilizations impossible.

When rationing is complete, the government will directly or indirectly know all of the agents' plan trades and can give agents the right to repeat those trades at the plan prices $\bar{p}$. This is the case considered by Lau, Qian, and Roland (1997, 2000). If rationing is partial, the state will not know all trades and cannot give agents the right to repeat their actual trades, only the right to buy or sell the quantities given by their plan rationing constraints, α_i for consumer i and β_j for firm j. If for good k all agents have these rights, whether for purchases or sales, then obligations for good k are *strong*. (Keep in mind that any consumer right to buy or sell a good entails a corresponding obligation of some firm to sell or buy that good.) In contrast, obligations for good k are *weak* if each consumer i has the right to buy k up to the constraint level $\alpha_i(k)$ at price $\bar{p}(k)$ but does not have any corresponding right to sell k. Obligations are *strong overall* (resp. *weak overall*) if obligations for each good k are strong (resp. weak). It might seem that weak obligations should lead to feasibility problems, but this turns out not to be the case: completeness of rationing (along with budget balance) guarantees that quantity stabilization is feasible (Theorem 3).

Consumers who are partially rationed under planning may well be able to purchase more goods at plan prices under a quantity stabilization than they actually purchased under planning. Although this possibility introduces feasibility problems, the state has little choice given its lack of information; if consumers do not retain rights to trade up to their rationing constraints, they may end up worse off.

A typical consumer i has three sources of income: endowment sales, a share of firm profits earned from stock ownership, and arbitrage profits or losses from purchases and sales at the plan prices $\bar{p}$. The ownership shares distributed by the government to i are given by $\theta_i = (..., \theta_{ij}, ...) \geq 0$. The profile of ownership shares, $\theta = (..., \theta_i, ...)$, must satisfy $\sum_i \theta_{ij} = 1$ for each firm j. Letting p denote the reform price vector and π the vector of firms' profits, the sum of endowment and profit income equals $p \cdot e_i + \theta_i \cdot \pi$. When obligations are strong overall, i's arbitrage profits equal $(p - \bar{p}) \cdot \alpha_i$ (cf. section 2).

For an additional case, see the appendix for the calculation of arbitrage profits when obligations are weak overall. Let $I_i(p, \theta_i)$ denote consumer i's income. So, if obligations are strong overall, $I_i(p, \theta_i) = p \cdot e_i + \theta_i \cdot \pi + (p - \bar{p}) \cdot \alpha_i$. Consumer i maximizes $u_i(x_i)$ subject to $p \cdot x_i \leq I_i(p, \theta_i)$ and $x_i \geq 0$. We represent the optimal $x_i - e_i$ by the excess demand function $z_i(p, I_i(p, \theta_i))$.

Firms must continue to receive from the state the money transfer t_j they received under planning; otherwise they may go bankrupt. A typical firm j's profits, π_j, therefore equals the sum of its operating profits $p \cdot y_j$, its transfer t_j, and its arbitrage profits or losses. For obligations that are strong overall, j's arbitrage profits equal $-(p - \bar{p}) \cdot \beta_j$ (see the appendix for the weak-overall case). Since j's arbitrage profits are lump sum (constant as a function of y_j), maximization of π_j reduces to maximization of $p \cdot y_j$.

A *quantity-stabilized equilibrium* is a p, a $y_j \in Y_j$ for each firm j, and a distribution of shares θ, such that

(7.3) $\Sigma_i \, z_i(p, I_i(p, \theta_i)) = \Sigma_j \, y_j$, and

(7.4) for each firm j: $\pi_j \geq 0$, and $y_j' \in Y_j \Rightarrow p \cdot y_j \geq p \cdot y_j'$.

A quantity stabilized equilibrium must be Pareto optimal; any variation in any agent's consumption comes at the cost of the same price vector p, and so the standard proof of the first welfare theorem applies. Furthermore, as argued in section 2 or in Lau, Qian, and Roland (1997), each consumer i is at least as well off at a quantity-stabilized equilibrium where obligations are strong overall compared to a planning equilibrium in which i is completely rationed. The same conclusion holds when the goods consumers purchase under planning are only partially rationed, as long as consumers are completely rationed in the goods they sell and obligations for the latter are strong under reform. Complete rationing of the goods consumers sell is more plausible than of goods consumers buy since consumers mainly sell factors and the state under planning may well dictate factor deliveries. Placing factors under strong obligations during a reform should then be feasible, since the state would have the necessary information about prereform trades. We state the Pareto improvement property as a theorem, but omit the (trivial) proof. (Below, consumer *i sells k* if $z_i(k) < 0$ at $(\bar{p}, \alpha_i, e_i)$.)

Theorem 2. Suppose consumer i is completely rationed in any good that i sells and and those goods are subject to strong obligations under reform. Then i cannot be worse off under a quantity stabilization.

Theorem 3 below reports that quantity-stabilized equilibria exist when consumers under planning are completely rationed and the government budget is balanced. Lau, Qian, and Roland (1997) argue that equilibria exist when in addition obligations are strong overall. Weak obligations turn out to be only a little more complicated; we need only ensure that $p \geq \bar{p}$ in equilibrium. Furthermore, even though obligations can be weak, there are quantity stabilizations that harm no agent.

Theorem 3. If under planning all consumers are completely rationed and the government budget is balanced, then, whether obligations for any good are weak or strong, there exist Pareto-improving quantity-stabilized equilibria.

A general-equilibrium quantity stabilization would presumably be implemented via repetition of trades rather than through state-issued transfers – the latter simply presupposes too massive an accumulation of information for a single authority (see section 2). But recall that the government or courts must be able to retrieve information about trades under planning as necessary; otherwise the obligation to repeat trades would be unenforceable.

Theorem 3's assumption that agents under planning are completely rationed is demanding and does not describe planned economies. And as we will see momentarily, existence of Pareto-improving quantity stabilizations can fail if rationing is only partial. But even if we suspend doubt about the plausibility of complete rationing, it is generically inconsistent with another assumption of Theorem 3, the requirement that the government budget under planning is balanced. As we saw earlier, if consumers are completely rationed and can afford their plan consumption bundle, then, except in some fluke cases, they accumulate money balances and hence the government must run a deficit. But government deficits cannot continue under a quantity stabilization; once rationing constraints are removed, aggregate demand for goods would outstrip supply. To see this, note that since profits are distributed to consumers and since the sum of the quantity stabilization transfers equals 0, net expenditures by consumers equals the sum of firm profits plus the sum of government transfers: $p \cdot \Sigma_i\, z_i(p, I_i) = \Sigma_j\, p \cdot y_j + \Sigma_j\, t_j$. Consequently, if the government budget is not balanced ($\Sigma_j\, t_j \neq 0$), then $p \cdot \Sigma_i\, z_i(p, I_i)$ will not equal $\Sigma_j\, p \cdot y_j$, contradicting (7.3).

The government must therefore levy enough taxes to cover its deficit under planning, $\Sigma_j\, t_j$. Unfortunately, the only revenue source that preserves the Pareto improvement conclusion without utilizing detailed information about individual agents is the distribution of firm shares. If eliminating the distribution of shares altogether does not generate $\Sigma_j\, t_j$ in revenue, the nonexistence problem persists. The state could instead impose the lump-sum tax $-\bar{p} \cdot \alpha_i$ on each individual i. Consumers can certainly afford these levies, but the tax bills would utilize information (the α_i) that government presumably would not have immediate access to. As we argued, the government's lack of direct access to the α_i does not by itself make quantity stabilizations impossible; but the indirect effect of this informational gap on the deficit may be fatal.

Putting aside the deficit problem, why is it that a government's ignorance of agent purchases under planning can by itself threaten the feasibility of quantity stabilizations? Absent this knowledge, a government aiming for a Pareto improvement must allow consumers to repeat purchases at the plan prices $\bar{p}$ up to their rationing constraints. Following Theorem 2, assume that consumers are completely rationed in the goods they sell under planning, and that obligations for these goods are strong under the reform. The existence difficulty is that if the reform prices of consumption goods (goods k where $\alpha_i(k) > 0$) are high relative to their plan prices, then consumers will buy as much of these goods as their rationing constraints allow, thus bankrupting firms that sell those goods. But if the reform prices of consumptions goods are low, then consumers will buy none of them at the plan prices, which can bankrupt firms with obligations to buy factors at plan prices. An example in the appendix illustrates the problem.

To sum up, quantity stabilizations for planned economies face formidable obstacles. To ensure the feasibility of quantity stabilizations, the government must directly or indirectly know the exact trades, agent by agent, that occur under planning. In effect, this requires that agents under planning are completely rationed. But this assumption, although strong, is not enough. Quantity stabilization requires that the government budget under reform be balanced. Consequently, in the generic case where the government under planning runs a deficit, the government must introduce additional taxes that match its deficit. The only way to raise enough revenue may be to introduce taxes that require a central authority to marshal detailed information about individual trades.

Price stabilization

When policymakers do not know the exact level of ex ante trades, consumers must be allowed to trade arbitrary quantities at $\bar{p}$ to ensure that they are not worse off under reform. Price stabilization is based on just this principle and does not rely on knowledge of ex ante trades. Correspondingly, there is no need to suppose that consumers under planning are completely rationed – indeed price stabilization is easiest to model when consumers under planning are entirely unrationed.

As in a big bang or a quantity stabilization, some or all of the state's ownership shares are distributed to consumers and firms maximize profits. But unlike a quantity stabilization, the monetary subsidies to firms are withdrawn – as are any handouts of capital goods to firms. As we will see, withdrawal of subsidies can be important for production efficiency.

The state distributes to consumer i the ownership shares $\theta_i = (..., \theta_{ij}, ...) \geq 0$, where the profile of ownership shares θ cannot exceed one, $\sum_i \theta_{ij} \leq 1$. We assume that if all of the ownership shares are distributed to consumers then demand at the planning prices $\bar{p}$ is infeasible. (We could instead introduce a money good, as in section 6, or model explicitly the material resources the state controls under planning and distributes under reform.)

Firms face the producer prices q and each firm j maximizes profits $q \cdot y_j$ subject to $y_j \in Y_j$. Letting $\pi = (..., \pi_j, ...)$ denote firms' profits, each consumer i maximizes $u_i(x_i)$ subject to $\bar{p} \cdot x_i \leq \bar{p} \cdot e_i + \theta_i \cdot \pi$ and $x_i \geq 0$. Letting $I_i(\theta_i) = \bar{p} \cdot e_i + \theta_i \cdot \pi$, we use $z_i(\bar{p}, I_i(\theta_i))$ to denote the excess demands, $x_i - e_i$, that solve consumer i's maximization problem.

A *price-stabilized equilibrium* is a $q > 0$, θ, and a $y_j \in Y_j$ for each firm j such that (1) demand equals supply, $\sum_i z_i(\bar{p}, I_i(\theta_i)) = \sum_j y_j$, and (2) each j maximizes profits, $q \cdot y_j \geq q \cdot y_j'$ for all $y_j' \in Y_j$.

For a planning setting, Theorem 1 can be restated as follows. *If no rationing constraint under* *planning binds and the plan is production inefficient, there exists a price-stabilized equilibrium that is production and partially allocatively efficient and that Pareto improves on the planning equilibrium.*

Quantity and price stabilization compared

(a) *Information*. One way to compare the informational demands of price stabilization and quantity stabilization is to count the pieces of information required. If the policymaker knows how much consumer wealth to distribute and assuming the adjustment process for producer prices works as described in section 2, then instituting a price stabilization requires knowledge only of the N prices at which goods exchanged under planning. If more types of goods exist under reform that under planning, the additional consumer prices need not be regulated: to secure a Pareto improvement, agents need only have the opportunity to buy or sell the economy's preexisting goods at plan prices. A quantity stabilization, on the other hand, assuming both that agents are completed rationed and that the government budget is balanced ex ante, requires knowledge of current prices and the exchanges previously made by each agent. So, $N + NI$ pieces of information are needed, where I is the number of agents. (We here count the information the government needs only indirect access to.)

(b) *Existence*. Our conclusions about the feasibility of price and quantity stabilization reveal a complementarity underlying the two approaches to reform: our existence result for price stabilization assumes that agents under planning face no binding rationing constraints, while our existence result for quantity stabilization requires that agents under planning are completely constrained. As is, therefore, neither model covers partially rationed agents, which is the most realistic case.

The existence problem for price stabilization is less severe, however. Since consumers under a price stabilization are entirely unconstrained in their market trades, consumer demand can exhibit a discontinuity when rationing constraints are relaxed: even with no distribution from the state and with consumer prices fixed at their planning levels, the discrete removal of rationing constraints can cause consumer demand to jump. If, following the elimination of the constraints, aggregate consumer demand remains feasible, then price stabilization is still possible. But even if demand becomes infeasible, an alternative version of price stabilization can be achieved by leaving the rationing constraints in place. Production efficiency will continue to obtain since profit-maximizing firms still organize production. Of course, since marginal rates of substitution no longer equalize, partial allocative efficiency is sacrificed.

(c) *Production efficiency*. Under planning, a firm's production decisions are at least partly mandated by the state. But even subject to these constraints, firms under planning do not maximize profits. They are run by political functionaries and are moved by political imperatives. State firms may also be managed by corrupt administrators who siphon off profits; such cases can fit into our model is we suppose that firms purchase factors, such as certain managerial services, that serve no productive purpose. Under reform, firms are not likely to transform themselves immediately into full-fledged profit-maximizers that serve only the interests of their shareholders.

Non-profit-maximizing firms have significant implications for reform policy. Under a quantity stabilization, the state must continue to pay the t_j subsidies to firms that previously lost money under planning. If the state under planning also distributed capital goods to firms, then those distributions must also continue (for the same reason that the t_j must continue – without them, firms might go bankrupt). But if firms with either monetary or material subsidies are nonmaximizers, they can produce inefficiently and use their subsidies to stay afloat. In a price stabilization, in contrast, subsidies are entirely eliminated. Inefficient producers can therefore be driven from the market by the efficient producers.

Besides the absence of the subsidies, two conditions are crucial for production efficiency in the presence of nonmaximizers. First, the profit-maximizers must have access to technology that is at least as advanced as the nonmaximizers; if the nonmaximizers have superior technology, they could obviously produce inefficiently and still survive market competition. The other prerequisite is constant returns to scale. With decreasing returns to scale, firms earn positive rents due to nonpurchasable firm-specific inputs; nonmaximizers can use those rents as a buffer to subsidize inefficient production. But constant returns is a relatively mild assumption; it requires in effect that all inputs are marketed commodities.

Given prices q, each profit-maximizer j chooses a $y_j \in Y_j$ such that $q \cdot y_j \geq q \cdot y_j'$ for all $y_j' \in Y_j$.

Each non-profit-maximizer h simply chooses a $y_h \in Y_h$ such that $q \cdot y_h + t_h \geq 0$: a non-profit-maximizer may take any action it wishes as long as the sum of its operating profits and its subsidies is nonnegative. A *production equilibrium* is a q and a y_k for each firm k such that each firm's actions obey these restrictions.

Partition the economy's finite set of firms into a set P of profit-maximizing firms and a set NP of non-profit-maximizers. We assume that $|P| \geq 1$. Define $Y_P = \sum_{j \in P} Y_j$ and $Y_{NP} = \sum_{j \in NP} Y_j$.

Theorem 4. If Y_P exhibits constant returns to scale, $q >> 0$, $t_j \leq 0$ for each firm j, and Y_P contains Y_{NP}, then any production equilibrium is production efficient.

This result highlights an underappreciated feature of the standard general equilibrium model: profit-maximization is a sufficient condition for production efficiency under perfect competition but it is not necessary. As long as the maximizing firms have access to the same technology as the nonmaximizers and there is constant returns to scale, the maximizers will drive the nonmaximizers out of business. This explanation of market efficiency is in the spirit of Friedman's (1953) evolutionary theory. Efficiency does not rely on the rationality of each and every agent but on the capacity of the market to eliminate inefficient agents.

8. Conclusion: enforcement costs, and the interpretation of Pareto improvements

We have already emphasized the informational requirements of general-equilibrium quantity stabilizations; price stabilizations face enforcement challenges as well. A price stabilization relies on excise taxes to achieve production efficiency; although they are traditional policy tools, taxes create arbitrage opportunities and hence invite evasion. It is difficult to lay down a general principle that could say when quantity or price regulation is cheaper. Glaeser and Schleifer (2001) present cases where regulating quantities is likely to prove cheaper than regulating prices (e.g., a ban on Sunday liquor sales should cost less to enforce than a tax on Sunday sales). But in general equilibrium at least, where a quantity stabilization gives each consumer and firm the right to repeat a long list of agent-specific trades, quantity stabilization would surely end up the costlier alternative.

We have argued that a policymaker can gather the information needed for a quantity or price stabilization simply by observing market data. To complete this interpretation, suppose two economies that are near or exact replicas operate at two successive dates: the policymaker observes the date 1 economy and uses this information to set the date 2 economy's policies. The Pareto improvements we model therefore involve comparing the welfare of the date 2 agents with the welfare of the date 1 agents (as opposed to comparing the effect of a change in policies on the welfare of agents at a single date).

Assuming the date 2 agents are simply date 1 agents at a later point in time, two questions arise. First, are the Pareto improvements we describe somehow undermined if the date 1 agents anticipate the influence of their actions on the date 2 policy decisions? Agent *i*'s date 1 actions certainly affect a date 2 quantity stabilization since *i*'s date 2 transfer, $(p - \bar{p}) \cdot \bar{z}_i$, is in part determined by the date 1 demand $\bar{z}_i$. (The effect of an individual agent's date 1 demands on $\bar{p}$, on the other hand, is presumably small in a large economy.) But the influence of demands on transfers does not alter the conclusion that the date 2 allocation Pareto-improves on the date 1 allocation; it simply means that the date 1 allocation is now an endogenous variable. In the case of price stabilization, if each agent's date 1 influence on $\bar{p}$ is negligible, then even this endogeneity qualification is unnecessary.

Second, do the Pareto improvements we have described necessarily leave each agent *i* better off following a policy change if we view *i*'s welfare as a function of his or her allocation on both dates taken together? In the case of planning at least, the answer is clear. If in the absence of reform at either date the government dictates that agent *i* will consume the same bundle $\bar{x}_i$ at both dates, then in the presence of reform at date 2, *i* will consume $\bar{x}_i$ at date 1 and a bundle preferred to $\bar{x}_i$ at date 2. So, assuming *i*'s overall welfare is an increasing function of date 2 utility, *i* will be better off in this expanded sense.

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Appendix

Proof of Theorem 1. Let $x_i(\lambda)$ = argmax $u_i(x_i)$ s.t. $\bar{p} \cdot x_i \leq \bar{p} \cdot e_i + m_i + \lambda$. Given that Y_T is closed and intersects the positive orthant only at 0, and given that the x_i functions are continuous, the set $\Lambda = \{\lambda \in R_+ :$ there exists $y \in Y_T$ such that $\Sigma_i\, x_i(\lambda) \leq \Sigma_i\, e_i + y\}$ is compact. Hence there exists a maximum $\lambda \in \Lambda$, say $\bar{\lambda}$. The assumption that y under autarky is in the interior of Y_T implies $\bar{\lambda} > 0$.

The vector $\Sigma_i\, x_i(\bar{\lambda})$ must be on the boundary of $\{x \in R_+^N : x \leq \Sigma_i\, e_i + y, y \in Y_T\}$, the set of feasible aggregate consumptions. Hence, letting $\bar{y} \in Y_T$ satisfy $\Sigma_i\, x_i(\bar{\lambda}) = \Sigma_i\, e_i + \bar{y}$, $\bar{y}$ is on the boundary of Y_T. Thus, production efficiency obtains for any such $\bar{y}$. Since Y_T is convex and contains

$-R_+^N$, there exists a $q > 0$ such that $q \cdot y' \leq q \cdot \bar{y}$ for any $y' \in Y_T$. This q and $\bar{y}$ satisfy the profit maximization requirements of a price-stabilized equilibrium. Finally, set each m_{di} equal to $\bar{\lambda}$. Since $\bar{\lambda} > 0$, each i has more income with m_{di} than at the autarky equilibrium and hence each i's utility increases.

The proof of allocative efficiency is standard. For any set of z_i' that, relative to $\hat{z}_i \equiv z_i(\bar{p}, p \cdot e_i + m_i + m_{di})$, improves the welfare of at least one agent and harms none, $\sum_i \bar{p} \cdot z_i' > \sum_i \bar{p} \cdot \hat{z}_i$. But since $\sum_i z_i' \leq \sum_i \hat{z}_i$, $\bar{p} \cdot \sum_i z_i' \leq \bar{p} \cdot \sum_i \hat{z}_i$. ■

Derivation of arbitrage profits

Suppose that obligations are weak overall (the strong overall case is discussed in the text). If $p(k) > \bar{p}(k)$ and $\alpha_i(k) > 0$, agent i will buy good k at price $\bar{p}(k)$ and resell at $p(k)$: i's arbitrage profit will then be $(p(k) - \bar{p}(k))\alpha_i(k)$. If $p(k) > \bar{p}(k)$ and $\alpha_i(k) < 0$, firms will exercise their option to buy from i leading i to have the return $\max[p(k) - \bar{p}(k), 0]\alpha_i(k) < 0$. If $p(k) < \bar{p}(k)$ and $\alpha_i(k) < 0$, firms will refuse to buy good k from i and if $p(k) < \bar{p}(k)$ and $\alpha_i(k) > 0$, i will refuse to buy k from firms. So i's total arbitrage profits equal $\sum_{k=1}^N \max[p(k) - \bar{p}(k), 0]\alpha_i(k)$.

As for a typical firm j, if $p(k) > \bar{p}(k)$ and $\beta_j(k) > 0$, consumers will invoke their right to buy from j, and j's arbitrage return will therefore be $-(p(k) - \bar{p}(k))\beta_j(k)$. When $\beta_j(k) < 0$ and $p(k) > \bar{p}(k)$, j will buy k at price $\bar{p}(k)$ and resell $p(k)$, leading to arbitrage profits of $-(p(k) - \bar{p}(k))\beta_j(k)$. The cases where $p(k) < \bar{p}(k)$ again induce no transactions. Summing, firm j receives total arbitrage profits of $-\sum_{k=1}^N \max[p(k) - \bar{p}(k), 0]\beta_j(k)$. ■

Proof of Theorem 3. We employ a standard technique, the social equilibrium existence argument of Debreu (1952) (surveyed by Debreu (1982)), that proceeds by setting (1) a truncated budget set for each consumer i that excludes only infeasible vectors, that is convex and compact for any $p \in \Delta_+^{N-1} = \{p \in R_+^N\colon \sum_{k=1}^N p_k = 1\}$, and that is continuous as a correspondence of p at any p such that $I_i(p, \theta_i) > 0$, and (2) a truncated production set for each firm j that excludes only infeasible vectors and that is convex and compact. Using a fixed point argument, the details of which we omit, it follows that there exists a (p, $\{z_i\}$, $\{y_j\}$) where $p \in \Delta_+^{N-1}$ such that (i) if $I_i(p, \theta_i) > 0$, then z_i gives consumer i at least as much utility, given prices p, as any other point in i's truncated budget set, and (ii) the supply vector y_j give firm j as least as much profit, given prices p, as any other point in j's truncated production set. Walras' law will then imply that $\Sigma_i z_i = \Sigma_j y_j$. It is sufficient to establish the continuity of i's budget correspondences only at p such that $I_i(p, \theta_i) > 0$ since we may assign i a set of pseudo excess demand vectors equal to i's entire truncated budget set whenever $I_i(p, \theta_i) = 0$, thereby preserving the upper hemicontinuity of the demand correspondence. As we will see, our assumptions imply that the (p, $\{z_i\}$, $\{y_j\}$) we find must satisfy $p >> 0$. Since $e_i + \alpha_i > 0$ for each i, $p >> 0$ implies that no i has $I_i(p, \theta_i) = 0$ at (p, $\{z_i\}$, $\{y_j\}$) and hence the pseudo excess demands are irrelevant. Also, since only infeasible points are truncated from the choice sets, the excess demands z_i and supplies y_j remain optimal when agents are free to choose from their original, nontruncated choice sets (we omit the details of this step too).

Consider strong overall obligations first. To meet condition (2) above, let $\tilde{Y}_j$ denote the intersection of Y_j and a sufficiently large closed rectangle in R^N. For each $p \in \Delta_+^{N-1}$, define

$$\Pi_j(p) = \max_{y_j \in \tilde{Y}_j} p \cdot y_j - (p - \bar{p}) \cdot \beta_j + t_j.$$

Since $\beta_j = \gamma_j$ when consumers are completely rationed under planning, $\Pi_j(p)$ is j's maximum level of profits at prices p assuming obligations are strong overall. Setting $y_j = \gamma_j$,

$$p \cdot \gamma_j - (p - \bar{p})\beta_j + t_j = \bar{p} \cdot \beta_j + t_j \geq 0,$$

where the inequality follows from (7.2). Hence $\Pi(p) = (\ ..., \Pi_j(p), ...\ ) \geq 0$.

Fix an arbitrary distribution of shares θ. When consumer i is completely rationed under planning, $\alpha_i = z_i(\bar{p}, \alpha_i, e_i)$. Thus, when obligations are strong overall, i's budget constraint at prices p is $p \cdot z_i \leq (p - \bar{p}) \cdot \alpha_i + \theta_i \cdot \Pi(p)$, or equivalently,

(A.1) $$p \cdot x_i \leq p \cdot (e_i + \alpha_i) - \bar{p} \cdot \alpha_i + \theta_i \cdot \Pi(p) = I_i(p, \theta_i).$$

To meet condition (1), intersect i's budget set $\{x_i \in R_+^N : p \cdot x_i \leq I_i(p, \theta_i)\}$ with a large closed rectangle in R^N, thus generating a truncated budget set that is compact and a continuous correspondence of p whenever $I_i(p, \theta_i) > 0$.

The fixed point argument then establishes that there is a ($p > 0$, $\{z_i\}$, $\{y_j\}$) such that, for each firm j, y_j is optimal for j at p, and, for each consumer i with $I_i(p, \theta_i) > 0$, z_i is optimal for i at p. We show that $I_i(p, \theta_i) > 0$ for all i, implying the optimality of all the z_i at p. Given that (a) $p > 0$, (b) $\Sigma_i (e_i + \alpha_i) \gg 0$, (c) $\bar{p} \cdot \alpha_i \leq 0$ for all i, and (d) $\Pi(p) \geq 0$, at least one agent k must have $I_k(p, \theta_k) > 0$ – see (A.1). Since u_k is increasing in each good, it must be that $p \gg 0$; otherwise z_k would not be optimal at p. Our assumption that $e_i + \alpha_i > 0$ for each i then implies that each $I_i(p, \theta_i) > 0$.

Finally, to show that markets clear, we confirm that Walras law' holds at $(p, \{z_i\}, \{y_j\})$, i.e., $\Sigma_i\, p \cdot z_i - \Sigma_j\, p \cdot y_j = 0$. Using the agent budget constraints and the definition of firm profits,

$$\Sigma_i\, p \cdot z_i - \Sigma_j\, p \cdot y_j = \Sigma_i [(p - \bar{p}) \cdot \alpha_i + \theta_i \cdot \pi] - \Sigma_j [\pi_j + (p - \bar{p})\beta_j - t_j].$$

Since $\Sigma_i \alpha_i = \Sigma_j \beta_j$ and the government budget is balanced, $\Sigma_i\, p \cdot z_i - \Sigma_j\, p \cdot y_j = 0$, as desired. We conclude that $(p, \{y_j\}, \theta)$ is an equilibrium.

Next consider obligations that are weak for an arbitrary subset of goods. Again, fix the distribution of shares θ. Define the function $\bar{\lambda}: R_+^N \to R$ by $\bar{\lambda}(p) = \text{argmax}_\lambda\, \lambda \bar{p}$ s.t. $\lambda \bar{p} \leq p$ and $\lambda \leq 1$. If plan prices were to equal $\bar{\lambda}(p)\bar{p}$ and reform prices were to equal p, then agents would invoke all of their plan rights to buy goods and i's arbitrage profits would equal $(p - \bar{\lambda}(p)\bar{p}) \cdot \alpha_i$. Defining $\tilde{Y}_j$ as before, let $\tilde{\Pi}_j(p) = \max_{y_j \in \tilde{Y}_j} p \cdot y_j - (p - \bar{\lambda}(p)\bar{p})\beta_j + \bar{\lambda}(p) t_j$. Given that the function $\bar{\lambda}$ is continuous, i's truncated budget correspondence remains a continuous correspondence of p whenever the right hand side of the budget inequality

$$p \cdot x_i \leq p \cdot (e_i + \alpha_i) - \bar{\lambda}(p)\bar{p} \cdot \alpha_i + \theta_i \cdot \tilde{\Pi}(p)$$

is strictly positive. Hence, just as in the strong overall obligations case, there exists a $(p^* \gg 0, \{y_j^*\}, \theta)$ such that if prices under planning equaled $\bar{\lambda}(p^*)\bar{p}$ and transfers to firms equaled $\bar{\lambda}(p^*) t_j$, then $(p^*, \{y_j^*\}, \theta)$ would be a quantity-stabilized equilibrium. Since $p^* \gg 0$, $\bar{\lambda}(p^*) > 0$. We therefore have $\Pi_j((1/\bar{\lambda}(p^*))p^*) = (1/\bar{\lambda}(p^*))\, \tilde{\Pi}_j(p^*)$. Each i's budget set at reform prices $(1/\bar{\lambda}(p^*))p^*$, plan prices $\bar{p}$, and firm subsidies t_j is therefore identical to the budget set that occurs with reform prices p^*, plan prices $\bar{\lambda}(p)\bar{p}$, and firm subsidies $\bar{\lambda}(p) t_j$. Since in addition each y_j^* is profit-maximizing at prices $(1/\bar{\lambda}(p^*))p^*$, $[(1/\bar{\lambda}(p^*))p^*, \{y_j^*\}, \theta]$ is a quantity-stabilized equilibrium.

The arguments in section 2 imply that the equilibria are Pareto improving. ■

Example of nonexistence of quantity stabilizations. Suppose that each possible quantity-stabilized equilibrium price vector is proportional to some $\hat{p}$.[6] There are two types of goods: *consumption goods*, of which no consumer is a net seller under planning, and *factors*, of which no consumer is a net buyer under planning. Suppose some firm j with a constant returns technology (1) produces one output, a consumption good k that is partially rationed, (2) purchases a single factor l, and (3) breaks even under planning ($t_j = 0$). Now consider a p proportional to $\hat{p}$ such that $p(k) = \bar{p}(k)$ and $\bar{p}(l) > p(l)$. Firm j will be forced to buy l and lose money on these transactions. Since good k induces no transfers, and given constant returns and (3), firm j loses money at p. If the parameter $\beta_j(k)$ is large enough, j will also lose money at any p such that $p(k) \geq \bar{p}(k)$.[7] In such a case, therefore, ensuring that j does not go bankrupt will require that equilibrium prices satisfy $p(k) < \bar{p}(k)$. But, to meet this restriction, it may be for some other consumption good k' that the inequality $p(k') < \bar{p}(k')$ is always satisfied. This occurs if $\bar{p}(k')/\bar{p}(k) > \hat{p}(k')/\hat{p}(k)$. We then have the opposite difficulty: consumers will not invoke their rights to buy k' at $\bar{p}(k')$ but firms that produce k' can still be forced to buy their plan factor inputs. Specifically suppose there is a firm j' that sells only k', purchases only the input l', and such that $t_{j'} = 0$. If at any p such that $p(k) < \bar{p}(k)$ it is also the case that $p(l') < \bar{p}(l')$, which occurs if $\bar{p}(l')/\bar{p}(k) > \hat{p}(l')/\hat{p}(k)$, then j' will make losses at such p and hence a quantity-stabilized equilibrium will not exist. ■

Proof of Theorem 4. If an equilibrium profile of the y_j were production inefficient, there would exist y_j' such that $\Sigma_{j \in J}\, y_j' > \Sigma_{j \in J}\, y_j$. Since $q \gg 0$, $q \cdot \Sigma_{j \in J}\, y_j' > q \cdot \Sigma_{j \in J}\, y_j$. But since the P firms are maximizing, $q \cdot \Sigma_{j \in P}\, y_j' \leq q \cdot \Sigma_{j \in P}\, y_j$. Hence $q \cdot \Sigma_{j \in NP}\, y_j' > q \cdot \Sigma_{j \in NP}\, y_j$. Since each $tj \leq 0$, $q \cdot \Sigma_{j \in NP}\, y_j \geq 0$. Hence $q \cdot \Sigma_{j \in NP}\, y_j' > 0$. Since $Y_P \supset Y_{NP}$, there exists $\hat{y}_j \in Y_j$, for each $j \in P$, such that

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[6] That is, given fixed preferences, endowments, firm technologies, and rationing constraints, for all θ, each quantity-stabilized equilibrium p is a multiple of $\hat{p}$; this would occur, for example, if each agent had the same homothetic preference relation.

[7] It is sufficient that $\beta_j(k) > \gamma_j(l)$. If there are consumers whose rationing constraints for k are slack at some planning equilibrium $(\bar{p}, \alpha_i, \gamma_j, t_j)$, then $\beta_j(k)$ and the $\alpha_i(k)$ can be increased and $(\bar{p}, \alpha_i, \gamma_j, t_j)$ remains a planning equilibrium. So the α_i and β_j parameters may be set to satisfy $\beta_j(k) > \gamma_j(l)$.

$\Sigma_{j \in P}\, \hat{y}_p = \Sigma_{j \in NP}\, y_j'$ and hence $q \cdot \Sigma_{j \in P}\, \hat{y}_j > 0$. Consequently, given that Y_P exhibits constant returns to scale, for any $\lambda > 0$ there exists a $(\tilde{y}_j)_{j \in P}$ such that $q \cdot \Sigma_{j \in P}\, \tilde{y}_j > \lambda$. Hence for any $\varphi > 0$, there exists a $j \in P$ such that some $\tilde{y}_j \in Y_j$ satisfies $q \cdot \tilde{y}_j > \varphi$, contradicting the assumption that the firms in P are maximizers. ■