# Social Security Investment in Equities I: Linear Case

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**Abstract**

This paper explores the general equilibrium impact of social security portfolio diversification into private securities, either through the trust fund or private accounts. The analysis depends critically on heterogeneities in saving, production, assets, and taxes. Limited diversification weakly increases interest rates, reduces the expected return on short-term investment (and the equity premium), decreases safe investment, increases risky investment and increases a suitably weighted social welfare function. However, the effects on aggregate investment, long-term capital values, and the utility of young savers hinges on assumptions about technology. Aggregate investment and long-term asset values can move in opposite directions.

*Keywords:* Social security, privatization, diversification

*JEL Classification:* H55

## Introduction

Policy discussions of social security portfolio diversification have concentrated on the consequences for retirement benefits and the budget viability of the system, ignoring general equilibrium repercussions.[1] In contrast, we analyze the general equilibrium ramifications for prices, for utility levels, and for investment. We show how critically they depend on heterogeneity in saving, in production, in assets, and in taxes.[2]

Among the elderly, Social Security income is distributed very differently than private pension and asset income.[3] For the bottom quintile of the income distribution, 81% of

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[1]See, e. g., Advisory Council on Social Security (1997).

[2]For examples of claims that make sense in a representative agent model but are not adequate once heterogeneity is recognized, see Financial Economists' Roundtable (1998), and Greenspan (1997). For a discussion of privatization in general, see Diamond (1999). For another analysis of portfolio diversification where general equilibrium effects matter, see Abel (2001). Our paper differs from that of Abel in that we attribute the lack of portfolio diversification of some workers to a lack of savings, while Abel focuses on a fixed cost of portfolio diversification. This difference implies a different response to social security portfolio diversification, with Abel finding an income effect lowering investment from consumers who stop paying the fixed cost and stop investing in the stock market because of the change in social security portfolio. Moreover, Abel assumes an aggregate production function, leaving no role for direct choice about the riskiness of aggregate production.

[3]See Mitchell and Moore (1997), Social Security Administration (1996).

income comes from Social Security, while only 6% is from pensions plus income from assets. For the top quintile, 23% comes from Social Security, while 46% is from pensions and assets — dramatically different percentages. Similarly, there are great differences in saving and investing among current workers. Among all those who were paying social security taxes in 1995, fully 59% held no stock, either directly or through pension plans. Even among those between 45 and 54 years of age, 50% held no stock, directly or indirectly.[4] These differences have important implications for diversification proposals.

We represent this heterogeneity in saving behavior by supposing that there are two types of representative agents, one of which does no saving (except through social security) and the other of which saves and selects a portfolio (and, for simplicity is assumed not to be covered by social security). We refer to the two types of agents as workers and savers.

Our analysis also recognizes the importance of heterogeneity in production. We suppose that there are two short-term technologies, which produce safe and risky output.[5] We also assume there are two long-term technologies, called safe land and risky land, which produce safe and risky output in perpetuity. Distinguishing between safe and risky output allows diversification to change equilibrium prices in such a way as to increase production in one technology sector and reduce it in the other. Considering long-term production allows changes in the equilibrium prices of land to redistribute wealth between generations. Wealth redistribution is interesting for its own sake, but also because of its effect on investment.

Social security diversification is likely to change the rate of interest, requiring higher income taxes to pay the higher coupons on government bonds. But the increased income tax burden may fall on households in different proportions than the social security taxes and benefits. This, in turn, will have feedback effects on the equilibrium interest rate.

In order to keep the analysis simple, social security is modeled as a combination of a pay-as-you-go system together with a defined contribution system, and not as a partially-funded defined benefits system. Social security diversification occurs when the asset mix in the defined contribution system is suddenly shifted from bonds toward equities, and then maintained at the higher equity level forever after. For example, if workers were suddenly given discretionary accounts, some of them with little or no outside saving would choose to invest part of their accounts in equities, and then our analysis would apply. Formally, we suppose there is a social security trust fund, which suddenly sells some of its bonds and invests the proceeds in stock, and passes through the difference in net returns between the stocks and bonds to the contemporaneous old.[6] The differences between defined benefit and defined contribution systems as distributors of rate-of-return risk have been explored in OLG models with a single representative agent.[7] This paper is meant to complement those studies. There is a brief discussion of how to extend the analysis to a defined benefit plan in Section 11.

The paper proceeds with a sequence of models that highlight different effects. In the first model (Section 5), technology determines the rates of return on both safe and risky assets. Diversification changes the level and riskiness of social security benefits. This raises expected utility of workers if they prefer some stocks to an all-bond portfolio, while having no effect on the utility of savers.[8] This effect persists in all of the models considered.

[4] Quoted in Geanakoplos, Mitchell, and Zeldes (1999). See Kennickell, Starr–McCluer, and Sunden (1997) and Ameriks and Zeldes (in progress).

[5] With two types of production, the aggregate uncertainty in production varies with the mix of the two types. This is a simple way to allow aggregate uncertainty to be a choice variable.

[6] Since we suppose that all workers have the same utility, it makes no difference whether social security accounts are personal or are managed by a trust fund, provided that they choose the same asset mix and benefits are adjusted in this way.

[7] See, for example, Bohn (1997, 1998, 1999), Diamond, (1997). We are not aware of other equilibrium studies considering portfolio diversification.

[8] The welfare gains come from the superior risk-sharing social security diversification permits when

Moreover, diversification in this model raises risky investment, lowers safe investment, leaves aggregate investment unchanged and so raises expected output. In the second model (Sections 6-8), the safe rate of return is not determined by technology. Diversification now raises the safe interest rate, requiring an increase in taxation. Thus there is a potentially redistributive term (from changing returns and changing taxes) in addition to the direct impact of a change in portfolio on the beneficiaries. Moreover, diversification raises risky investment, and so aggregate investment. Then, we add infinitely-lived assets to introduce redistribution across generations. A rise in the safe interest rate lowers the value of infinitely-lived assets, hurting the savers holding these assets and benefitting young savers in the future. This case seems the most realistic among the linear cases, and so we concentrate much of our attention there.

To show the dependence of the results on technological assumptions, we consider a model (Section 9) where there is only safe investment, holding the government interest rate fixed. In this case the price of infinitely-lived risky assets goes up, generating an intergenerational redistribution in the opposite direction. Moreover, this lowers safe investment, and so aggregate investment.

One conclusion which holds in all the linear models is that social security diversification creates the potential for welfare improvements. Diversification from a point of zero exposure to equities raises the sum total of weighted utility in the economy if household utilities are weighted so that the expected marginal utility of a dollar for sure is the same for every saver and every retired worker.[9] In addition to the welfare gains, social security diversification would also likely cause welfare redistributions, so that a Pareto improvement might require additional policy steps. Another conclusion that holds across the three linear models analyzed here is that diversification (weakly) decreases safe investment and (weakly) increases the safe rate of return, with the opposite movements for risky investment and expected risky return. But, aggregate investment and land prices may rise or fall, depending on details of the technology.

These models are easy to analyze since the technology leaves at most one endogenous rate of return. In a companion paper, we examine a more general technology where both rates of return are endogenous. A brief look at this model is in Section 10. The common conclusions of our linear models continue to hold there.

Of course, these results require assumptions. We suppose that the demands by savers for consumption when young, and for safe and risky consumption when old, are normal. (With expected utility, normality follows from decreasing absolute risk aversion and increasing relative risk aversion.) We suppose that increases in government bond interest payments raise the payments on government bonds held by social security more than they raise workers' income taxes (and thus raise savers' taxes more than they raise savers' income on the government bonds they hold). We also suppose that the level of risky investment does not affect the relative outputs across states (short-term risky production is along a ray in state space). Also, we suppose that the output from both short-term and long-term risky production is independently and identically distributed each period.

Proponents of social security diversification often say it would help young savers because stocks have traditionally earned a higher return than social security is projected to yield in the future. They have been rightly criticized for sometimes forgetting about

there are workers who do not have savings to invest on their own, and when, in the absence of social security diversification, social security benefits have a low correlation with stock returns. This point was made in Geanakoplos, Mitchell, and Zeldes (1999), who also tried to quantify the welfare gain in a special quadratic example.

[9] Since workers do not save, their marginal rates of substitution across time are not proportional to interest rates, and it is impossible to require marginal utilities for young workers to match up as well, for all time periods. But even if they do not match up in the welfare function, the welfare function must be increased by diversification.

the unfunded liability embodied in social security commitments to today's old, and for ignoring the riskiness of stock returns.[10] Naturally our model recognizes both of these considerations, and not surprisingly it shows that the equity premium would fall after diversification. Our analysis also makes clear that the welfare of young savers depends on at least two more considerations. First, their income taxes will rise (to pay the higher interest on government bonds). Second, the assets they buy and sell will change in value. Young savers, being net buyers of long-term assets, will lose utility if land prices rise. Unless long-term capital values go down substantially after diversification, young (and future) savers will be made worse off by social security diversification. On the other hand, today's old savers will be made better off if long-term capital values rise.

Proponents of social security diversification also sometimes argue that it will stimulate aggregate investment. We find that it does stimulate risky investment, but it also decreases safe investment. The effect on aggregate investment depends critically on technological assumptions. Investment is driven by the savings of the young. A rise in long term capital values, which reduces their welfare, tends to reduce their savings, and thus aggregate investments. Nevertheless, in the risky linear technology model, to which we devote the most attention, social security diversification *lowers* long term capital values and increases aggregate investment.

# 1 Technology

We analyze the equilibrium of a stochastic overlapping-generations economy, where each generation lives for two periods. There is one perishable consumption commodity in each date-event, which can either be eaten or invested using a productive technology. Young consumers have (nonstochastic) endowments, which can be interpreted as earnings from inelastically supplied labor with a technology that is linear and nonstochastic in labor.

At each date-event there are two short-term investment opportunities which transform the single perishable consumption good into (safe or stochastic amounts of) consumption goods in the next period. In Sections 1–9, we assume a linear short-term technology to avoid the complications from feedback of investment levels on rates of return to productive investments. The safe investment produces $R_0 k_0$ in the period following an investment of $k_0$, with no durability in the capital, where $R_0 > 1$ is a constant. (Thus we are assuming a positive safe rate of return.) The risky investment produces $Rk$ in the period following an investment of $k$, also with no durability in the capital, where $R > 0$ is a random variable. For convenience, we assume the risky returns to be independently and identically distributed each period. Each of these technologies may or may not be used in equilibrium, depending on rates of return.[11] Below, we add to the model two types of infinitely-lived assets, yielding safe and risky outputs each period and referred to as land.

# 2 Consumers

To bring out the difference between social security covered workers and wealth holders, we assume there are workers who do not save and savers who are not covered by social security; that is, two representative agents in each birth cohort.[12] We assume that each

[10] This line of criticism is developed in Geanakoplos, Mitchell, and Zeldes (1998, 1999).

[11] When a technology is not in use, we suppose that the marginal utility of beginning to use it is strictly less than the marginal cost of beginning to use it. For completeness, we mention that there are (knife-edge) regimes with a technology that is not in use but with the marginal benefit of beginning to use it exactly equal to the marginal cost of beginning to use it. Generically, these regimes will not be observed.

[12] Feldstein (1985) makes a two-types assumption in his classification of agents as rational and myopic. Having savers covered by social security would complicate the notation without changing the analysis.

worker receives $w$ in the first period, with each saver receiving $W$.

We assume no population growth and normalize the population so that there is a unit measure of (identical) savers and a measure of size $n$ of (identical) workers. The representative saver maximizes expected lifetime utility of consumption, taking prices as given. Expected lifetime utility, $V$, is equal to $U_1[C_1] + E\{U_2[C_2]\}$, where $C_1$ is consumption when young, and $C_2$ is consumption when old, and with $U_i$ increasing, concave and twice continuously differentiable. In the model without land, the savers divide exogenous first period wealth, $W$, among consumption and (up to) three tangible assets — government bonds, $B$, and two types of physical capital: $k_0$, which is the safe asset, and $k$, which is the risky asset. In addition, the savers pay income taxes, $T$, in the second period.[13] Thus, we denote expected utility maximization for the representative saver by:

$$\begin{aligned} V &= \max U_1[C_1] + E\{U_2[C_2]\} \\ &\quad \text{s.t. } W = C_1 + B + k_0 + k \\ &\qquad C_2 = (1+r)B + R_0 k_0 + Rk - T, \end{aligned} \tag{1}$$

where the rate of return, $R$, is random, but taxes are not, as of the time of first-period decisions. If the safe real asset is held in equilibrium, then $1 + r$ is equal to $R_0$, since the government bond and the safe real asset are perfect substitutes.

Consumer choice can also be viewed in terms of three (composite) consumer goods — first-period consumption and safe and risky second-period consumption, which we denote as $C_1$, $J$ and $K$.[14] It is therefore convenient to imagine that there is a safe financial asset promising one unit of safe consumption and also a risky financial asset promising one unit of risky consumption $R$, so that $J$ and $K$ can be bought directly.[15] With first-period consumption as numeraire, we denote the price of second-period risky consumption as $p_K$. The price of one unit of second-period safe consumption is denoted by $p_J$. When the risky investment is undertaken in equilibrium, $k > 0$, then we must have $p_K = 1$. When safe investment is undertaken in equilibrium, $k_0 > 0$, then we must have $p_J = 1/R_0$. Whether or not real safe investment is undertaken in equilibrium, $p_J$ is always equal to $1/(1+r)$, as long as the supply of government bonds to savers is positive. We now restate the consumer choice problem as:

$$\begin{aligned} V &= \max U_1[C_1] + E\{U_2[J + RK]\} = \max V^*(C_1, J, K) \\ &\quad \text{s.t. } C_1 + p_J J + p_K K = I \\ &\qquad I = W - p_J T. \end{aligned} \tag{2}$$

Demands for all three consumer goods are functions of the prices of second-period safe and risky consumer goods, and of net lifetime wealth. We denote them by $C^*[p_J, p_K, I] = C^*[p_J, p_K, W - p_J T], K^*[p_J, p_K, I] = K^*[p_J, p_K, W - p_J T]$ and $J^*[p_J, p_K, I] = J^*[p_J, p_K, W - p_J T]$.

[13] Taxes are used to pay interest on government bonds. By collecting taxes in the second period of life, they are paid back to the same cohort they are collected from. Collecting taxes in the first period instead would be equivalent to changing the level of government debt outstanding.

[14] Since all trading and production opportunities can be written in terms of these composite commodities, analysis of equilibrium can be done in these terms. Written in this form, the usual properties of compensated demands hold for the vector of consumptions. On the properties of compensated demands in the presence of uncertainty, see Diamond and Yaari (1972) and Fischer (1972). Moreover, analysis can be done in this form without the assumption of expected utility.

[15] When risky real investment is being undertaken, we can interpret the risky financial asset as shares in the output of a risky firm. When there is no real investment being undertaken, then this risky financial asset is like a contingent futures contract. An investor can acquire the right to future risky consumption by buying the risky financial asset (i.e., the shares or the futures contract) without having to actually undertake any risky investment, provided that he can find somebody who is willing to sell the asset.

We *assume* the function $V^*(C_1, J, K)$ is such that all three of first-period consumption, and safe and risky second-period consumptions are normal goods. The normality of the three goods in turn *guarantees* that all three goods are Hicksian substitutes (given the intertemporally additive structure of preferences described in (2)). A sufficient condition for normality of all three goods is that second period utility displays decreasing absolute risk aversion (DARA) and increasing relative risk aversion (IRRA). (For proofs of these assertions, see Aura, Diamond, Geanakoplos, (1999).)[16]

In contrast, we model workers, who also have two-period lifetimes, as nonsavers. Each worker earns a wage, $w$, in the first period (with inelastically supplied labor), pays payroll taxes $t_w$ in the first period, and consumes $w - t_w$. In the second period, workers consume social security benefits, $b$, which may be random, less income taxes $t$. We denote lifetime utility by $v$ and note that it satisfies:

$$v = u_1[c_1] + E\{u_2[c_2]\} = u_1[w - t_w] + E\{u_2[b - t]\}. \tag{3}$$

We distinguish two sources of taxes since the payroll tax will be used for social security, while the second period income tax will be used to pay part of the interest on the government debt outstanding.

The lack of randomness in income for young workers, $w$, guarantees a lack of randomness in the pay-as-you-go component of the financing of social security benefits for contemporaneous old workers, as we shall see in the next section.

# 3 Government and the Social Security System

It is assumed that each period the government rolls over one-period debt with a value of $G$. The interest payments on this debt are financed by taxes on older workers and older savers, with the principal rolled over to preserve the debt outstanding.

$$T_t + nt_t = Gr_{t-1}, \tag{4}$$

where taxes collected in period $t$ are used to pay interest at rate $r_{t-1}$ on debt issued in period $t-1$. We assume that taxes are divided in the proportions $a$ and $1-a$, $(0 < a < 1)$ giving the period and steady-state relations:

$$\begin{aligned} T_t &= aGr_{t-1}; \quad t_t = (1-a)Gr_{t-1}/n; \\ T &= T(r) = aGr = aG(1-p_J)/p_J = T(p_J); \\ t &= t(r) = (1-a)Gr/n = (1-a)G(1-p_J)/np_J = t(p_J). \end{aligned} \tag{5}$$

We use the same symbols $T$ and $t$ whether they depend on $r$ or $p_J$; this is not likely to confuse.

We model the social security system as a combination of a pay-as-you-go system together with a defined contribution system without worker choice of portfolio. Equivalently, we can think of the system as a partially-funded defined benefit system where the stochastic returns on stocks are fully used in determining that period's retirement benefits. The social security trust fund holds the value $F$ of government debt, and the value $\kappa$ of risky assets. It thus holds $K^f = \kappa/p_K$ of risky second period consumption (possibly equal to zero at the outset). Denoting the total value of the trust fund by $F_0$, and supposing the trust fund holds only short-term assets, the trust fund budget set in any period is:

$$F + \kappa = F + p_K K^f = F_0. \tag{6}$$

[16] For the reader interested in non-expected utility maximization, we must assume that $V^*$ is such that all three goods are normal, *and* that all pairs of goods are Hicksian substitutes.

Given the need to maintain the trust fund value allocations in bonds and equity, $F$ and $\kappa$, and given constant payroll taxes $t_w$, and given a stationary population, social security benefits satisfy the period and steady-state relations:

$$\begin{aligned} b_t &= t_w + ((1+r_{t-1})F - F + (RK^f_{t-1} - \kappa))/n \\ b &= t_w + (rF + (R - p_K)K^f)/n. \end{aligned} \tag{7}$$

Thus the expected utility of workers, $v$, satisfies the period and steady-state relations:

$$\begin{aligned} v &= u_1[w - t_w] + E\{u_2[t_w - t + ((1+r_{t-1})F - F + (RK^f_{t-1} - \kappa))/n]\} \\ &= u_1[w - t_w] + E\{u_2[t_w - t + (rF + (R - p_K)K^f)/n\} \\ &= u_1[w - t_w] + E\{u_2[t_w - t + (rF_0 + (R - (1+r)p_K)K^f)/n]\}. \end{aligned} \tag{8}$$

We have given (7) and (8) first as accounting identities (defining the benefits for workers), and second as relations that obtain if there is a steady state. Observe from equation (7) that all the variations in risky asset payoffs held by the trust fund are passed through directly to the current retirees. There is no risk sharing across generations, as there could be in a defined benefits plan, either by spreading return risks across several cohorts or by varying the payroll tax rate. The wage and the payroll tax rate are assumed to be constant over time; the retirement benefits, however are free to vary, and will do so if the rates of return earned on the trust fund holdings vary. Similarly, the second-period income tax will change if the interest rate on government debt changes.

A crucial part of our analysis is that if $K^f = 0$, young workers at time $t-1$ can look forward with certainty to the social security benefits they will receive when they are old at time $t$. The return $r_{t-1}$ they will get from the trust fund bond investment is already locked in. Furthermore, they can perfectly predict the pay-as-you-go portion of their benefits, since, in stationary equilibrium, wages of the young at time $t$ are nonrandom. In reality, of course, future real wages cannot be predicted with certainty. In our judgment, however, they are substantially less random than stock returns.

# 4 Stationary Equilibrium without Land

In stationary equilibrium, prices and young savers' consumption and asset holdings are constant through time and across states of nature. All that varies is output, consumption of the old savers and old workers, and social security benefits. With a single commodity, and stationary and independent productivity shocks, stationary equilibrium will exist. Since wages do not vary, a new steady state is reached starting with the generation born immediately after a permanent policy change.

When savers undertake risky investment, $p_K$ is equal to one. Stationary equilibrium in the model with short-term risky production, but without land, is then defined by prices and quantities $(r, C_1, C_2, B, k_0, k)$ such that given $r$ and taxes $T$ equal to $T(r)$, the choices $(C_1, C_2, B, k_0, k)$ solve the savers' optimization problem (1), and such that savers' demand for government bonds equals the supply available to savers:

$$B = G - F = G - F_0 + \kappa = G - F_0 + p_K K^f \tag{9}$$

If safe real investment is undertaken in equilibrium, then the interest rate on government bonds equals the return on safe investment. If not, then the government interest rate is determined by market clearance with no additional supply of safe assets.[17]

[17] Since the savers are both the demanders and the suppliers of real investment, the investment markets

Alternatively, we can write the market clearing conditions in terms of the variables $C^*$, $J^*$, $K^*$ introduced in budget set (2). The budget set (2) separates the consumption and savings decisions of the savers from the production decisions of firms. From now on we interpret $k_0$ to be the safe production chosen by the firms, and we interpret $k$ as the risky production chosen by the firms. Safe consumption market clearing becomes

$$J^*(p_J, p_K, W - p_J T(p_J)) = (G - F_0 + \kappa)/p_J + R_0 k_0 - T(p_J). \tag{10}$$

Savers' demand for safe second-period consumption must just meet the supply of safe second period consumption to savers, which is equal to the total principal and interest payments of government bonds, less what is held by the social security system, plus the safe production, less what is owed in taxes. Using the same variables $C^*$, $J^*$, and $K^*$, we can write market clearing in the risky good market as

$$K^*(p_J, p_K, W - p_J T(p_J)) = k - \kappa/p_K. \tag{11}$$

The supply of risky second period consumption to savers is equal to risky production, less what is held by the social security system.[18]

Market clearance could occur with or without each type of production, depending on rates of return. This gives

$$\begin{aligned} p_J = 1/(1+r) = 1/R_0 \quad &\text{if} \quad k_0 > 0 \\ \leq 1/R_0 \quad &\text{if} \quad k_0 = 0 \end{aligned} \tag{12}$$

$$\begin{aligned} p_k = 1 \quad &\text{if} \quad k > 0 \\ \leq 1 \quad &\text{if} \quad k = 0 \end{aligned} \tag{13}$$

Stationary equilibrium is now defined as a vector $(p_J, p_K, k_0, k)$ such that (10)–(13) hold. The condition defining the savers' holdings of government bonds $B$ given by (9) must still hold, and we continue to use $B$ as a convenient shorthand for the RHS of (9). But equation (9) will not be treated as an independent equation, since it follows from (10)–(13). Given the budget constraint in (2), if (10)–(13) hold, then the consumption good market clears as well.

Depending on whether $k_0 > 0$ or $k_0 = 0$, and whether $k = 0$ or $k > 0$, equilibrium can be one of four different types, or regimes.[19] The effect of social security diversification depends crucially on which regime the original equilibrium is in.[20] In each case, we analyze the effect on equilibrium of a change in trust fund investment in risky assets: $d\kappa = -dF > 0$. Because the wages of both workers and savers are not influenced by past events, the economy achieves stationary equilibrium in a single period after a change in a parameter such as the portfolio allocation of the trust fund. If the unanticipated change comes at some date $t$, then generations born at date $t$ and after will consume as in the new

automatically clear if savers solve (1). The consumption market automatically clears as well, once the bond market clears. To check this, we can verify that supply of consumption equals demand,

$$W + nw + (R_0 - 1)k_0 + (R - 1)k + (R - 1)K^f = C_1 + C_2 + nc_1 + nc_2.$$

The reader can verify that after substituting for $C_1$ and $C_2$ from (1), $c_1$ and $c_2$ from (3), taxes from (5), and benefits from (7), this equation reduces to (9).

[18] To complete the picture we could explicitly model the production decisions of the firms to maximize profit:

$$\max[p_J R_0 k_0 - k_0] + \max[p_K k - k].$$

[19] As mentioned above, we shall restrict attention to economies that have no knife-edge stationary equilibria in which $k_0 = 0$ and $p_J = 1/R_0$, or in which $k = 0$ and $p_K = 1$.

[20] Furthermore, small changes in the trust fund create small changes in equilibrium. Equilibrium before and after social security diversification will therefore be of the same type.

steady state, and generations born at date $t-2$ and before will consume as in the original steady state. The generation born at date $t-1$ will consume as if it made consumption and asset choices when young in the original equilibrium, but was then forced to pay taxes and liquidate assets at date $t$ at the new steady state prices.

# 5 Social Security Diversification with Both Safe and Risky Investment

In this section, we assume the economy is such that in equilibrium both physical assets are held, $k_0 > 0$, $k > 0$. In equilibrium, the interest rate on government bonds must then be equal to the (exogenously fixed) rate of return on the safe asset, and the price of the risky consumption good must be 1, the cost of the risky physical asset. Since prices do not change when the trust fund alters its portfolio, savers are left unaffected. (Since the interest rate does not change, second-period taxes do not change, so the budget set of savers is indeed unaffected.) With unchanging prices, savers demand the same combination of all three consumption goods — first-period, second-period safe and second-period risky consumptions. Thus, if the trust fund sells some bonds to savers, the savers maintain the same lifetime consumption, and also finance the purchase simply by investing less in the safe technology. The trust fund uses the proceeds of the sale to undertake risky investment. Thus aggregate risky investment goes up, aggregate safe investment goes down and aggregate investment is unchanged. Since the expected return on risky investment must exceed the return on safe investment (for both to be held by risk-averse savers), expected aggregate output goes up. [21]

If the trust fund initially has only a small amount of the risky asset, this policy is a (weak) Pareto gain — savers are not affected and workers gain since the workers are not risk averse for the first bit of investment in risky assets. To see this, consider the change in worker expected utility (noting that interest rates $r$ and therefore taxes $t$ are unaffected) assuming that $\kappa = K^f$ is zero:

$$\begin{aligned} dv/d\kappa &= E\{u_2'[c_2]((R-1-r)/n - dt/d\kappa + F_0(dr/d\kappa)/n)\} \qquad (14) \\ &= E\{u_2'[c_2](R-1-r)\}/n \\ &= u_2'[c_2]E\{(R-1-r)\}/n > 0. \end{aligned}$$

The last equality is obtained by noting that second period consumption of workers is certain, hence so is second period marginal utility, so it may be brought outside of the expectation operator. The final inequality follows from the excess expected risky return (see footnote 21). Thus we have shown

**Proposition 1.** *Suppose both the safe and risky assets are held in stationary equilibrium (in positive quantities). Then, increased trust fund investment in risky assets will raise aggregate risky investment, lower aggregate safe investment, leave aggregate investment unchanged and increase expected output. If the trust fund initially held no risky assets, then the diversification will lead to a weak Pareto improvement, increasing the utility of the young workers and leaving the utility of the old workers and all savers unchanged.*

[21] To see this, note that with $p_K = 1$ and both assets held, the savers' first order condition is:

$$U_1'[C_1] = E\{U_2'[C_2](1+r)\} = E\{U_2'[C_2]R\};$$

Since $C_2$ and $R$ are perfectly correlated, $U_2'[C_2]$ and $R$ are negatively correlated. Hence the equality of expectations can only hold if $E\{R\} > 1 + r = R_0$.

The equity premium is defined as the difference between the expected return on the risky investment and the return on the safe investment, $E\{R\} - (1+r)$. Since the equity premium must be consistent with the portfolio choice of risk-averse savers (who hold a strictly positive quantity of risky assets by hypothesis), it must be positive in equilibrium. As long as the equity premium is positive, there is an expected utility gain to workers from diversification in a model where they bear no other risk.

The crucial step in this argument in favor of social security diversification is the paradoxical claim that workers are *more* risk tolerant *on the margin* than savers. One might suspect the contrary, that savers are more risk tolerant than workers, all else being equal. That is, it may well be that the worker utility $u$ is a concave transformation of the saver utility $U$, thereby displaying more risk aversion at any level of consumption. And workers have lower incomes on average than savers, which also makes them more risk averse, given an assumption of decreasing absolute risk aversion. But all else is not equal. The savers hold the entire risky capital stock of the nation, while the workers hold none (if $K^f$ is zero). Our proof that there are welfare benefits from social security diversification goes through no matter how much more risk averse $u$ is than $U$, and no matter how poor the workers are. It only needed that both $u$ and $U$ are differentiable, and that workers are not exposed to any stock market risk or other risks correlated with stock market risk.

In reality, workers' retirement income is not completely statistically independent of stock returns. Social security benefits are connected by an explicit formula to real wages. Over career horizons as long as forty years, there is considerable covariance between real wages and stock returns. The question then becomes, how big is worker exposure to stocks, how big is the equity premium, and how risk averse are workers? Addressing this question in detail is beyond the scope of this paper.[22] Our judgment is that after properly calibrating the stock exposure implicit in aggregate wages, one would come to the conclusion that the average worker is less exposed to stock returns than savers. At the point where the trust fund holds no stock, it seems very likely to us that the average worker would be better off by some investing in equity. The converse would hold only if it would be optimal for such a worker just starting to save, to hold a portfolio with no stocks at all.

However, what is best for the average worker may not be best for every worker. Though our model has assumed that all workers are identical, in reality some workers may be far more risk averse, so that for them any additional stock exposure may be bad, preventing social security diversification from being a Pareto gain (Pestieau and Possen, 1999). However, in considering a more general setting with heterogeneous workers, the reader should bear in mind that the lowest income workers would be protected by the safety net (SSI).

By the same logic used in the proof of Proposition 1, further increases in social security risky asset holdings would also be weak Pareto gains in a representative worker model until the optimal portfolio for workers was reached, unless the saver's holdings of the safe real investment reached zero first.[23] In considering the optimal level of social security diversification, we note that since social security benefits become more correlated with stock returns as diversification increases, the welfare benefits to further diversification decline. The proof of Proposition 1 is thus an argument for limited diversification.

The welfare gains from social security diversification described in Proposition 1 sound superficially similar to the popular argument repeatedly seen in the press about the excess return stocks have traditionally earned over bonds. However, if savers were also covered

[22] If the workers' and savers' utilities $u$ and $U$ display similar risk aversion, and both display increasing relative risk aversion, then the poorer workers should have a higher fraction of their wealth invested in stocks than the richer savers.

[23] The Social Security program is a response to the inadequacy of retirement saving by many workers. Just as mandating savings can raise utility for many workers, adjusting the portfolio can add to utility.

by social security, Proposition 1 would claim no (ex ante) gain from social security diversification for the typical saver, despite the equity premium (except for some savers who were 100% in stocks in their portfolio and wanted some of social security to be in stock as well). On the margin, savers should not expect any excess return to stocks in utility terms. For every dollar in the actual social security trust fund that is shifted to equity, the welfare gains described in Proposition 1 only apply to that fraction of each dollar that goes to support the benefits of workers with little financial wealth who do not borrow, and are therefore holding no stocks.[24] If there were no such constrained workers, as is the case in a representative agent model with only rational savers, a small enough change in trust fund portfolio policy would have no effects at all (Smetters, 1997 and Bohn, 1997, 1998).

Our judgment that nonsavers are less risk averse on the margin than savers gives rise to the same welfare benefits to diversification in all of our subsequent models. This effect is perhaps controversial. It is not controversial that a substantial fraction of American workers lack the resources to undo in their private portfolios what the social security trust fund might do on their behalf. If the technology is not perfectly elastic, then the presence of these nonsavers will force savers to alter their portfolio holdings after social security diversification. This implies that social security diversification must have real effects, which we investigate in the following models. These real effects, including additional welfare effects from changing asset prices and taxes, do not depend on our assessment of the marginal risk aversion of nonsavers. Indeed, precisely because they do not save, the effect of social security diversification on asset prices and taxes is independent of their (marginal) risk tolerance.

## 5.1 An Alternative Interpretation of the Model

In this bilinear model, a greater willingness by financial investors to hold risk (in aggregate) results in riskier investment (in aggregate) and so riskier output. This result is missing in the two linear models with just one type of investment analyzed below. It can also be missing in models without an investment choice other than the level of investment. Yet, firms do make choices that affect the riskiness of their output. One way to model a technology for such choices would be an activity analysis model. Assume that each firm has two activities with different risks and chooses the overall riskiness of its output by choosing the mix of the two activities. Assume that all firms are the same. By the Modigliani-Miller theorem, it would not matter which firm made which production choice as long as the aggregate of investment in each technology were the same. If one of these technologies is safe, while the other is risky, we have exactly the model above. It might be interesting to extend the analysis in the next section to have two risky technologies, instead of just one to explore the same issue.

[24] Some workers are unable to hold stocks because they have not saved enough. Others do not hold stocks even though they could. Some of the latter may not be optimizing and would also gain from the diversification, while some may not have been willing to bear the cost of learning about stocks and would also gain since they do not have to pay a cost if investment is done centrally. However, workers so risk averse that they should hold no stocks would lose from diversification, as noted above. Some workers may mistakenly be overinvested in stocks and would also lose from trust fund diversification if they do not reduce their stockholdings in response to trust fund investment.

# 6 Social Security Diversification with only Risky Investment

While the riskiness of aggregate output is plausibly endogenous, as in the bilinear model above, it is unrealistic to suppose that technology fixes the returns on safe and risky assets, independent of preferences. We now consider an economy with the expected return on risky investment given by technology, but without any safe investments, implying that the return on government bonds is endogenous. That is, in equilibrium: $k_0 = 0$ and $k > 0$. Given the constant marginal returns to risky investments, equilibrium requires $p_K = 1$. The interest rate on government debt is determined by the supply of and demand for bonds, with market clearance given in equation (10). Restating (10), and substituting for $p_J(J^* + T)$ from the budget constraint in (2), we have a single equation in a single variable, $p_J$, which equals $1/(1+r)$.[25]

$$\begin{aligned} G - F_0 + K^f &= W - C^*[p_J, 1, W - p_J T(p_J)] - K^*[p_J, 1, W - p_J T(p_J)] \\ &= W - C^*[p_J, 1, W - (1 - p_J)aG] - K^*[p_J, 1, W - (1 - p_J)aG]. \end{aligned} \tag{15}$$

Note that with $G$, $F_0$, and $W$ all fixed, the response of aggregate investment, $K^* + K^f$, to portfolio policy is minus the response of the consumption of savers, $C^*$. To analyze the effect of diversification, we differentiate (15) with respect to $\kappa$ $(= K^f)$, giving the response of the interest rate to the increase in the supply of bonds available to savers. Any change in the interest rate (with no change in gross debt outstanding) requires a change in income taxes to cover interest costs. Thus, to sign the change in the interest rate, we need an assumption on the share of taxes paid by savers, as well as our assumptions on demand functions. Given the change in both the interest rate and taxes, we have a redistribution between savers and workers, along with the gain to workers from diversification, as in Section 5. We proceed by first relating the changes in utilities to the change in the interest rate and then arguing that the interest rate does indeed increase.

## 6.1 Income Taxes, the Interest Rate, and the Redistribution of Income

From (2), (10), (9), the definition of taxes given in (5), and the first order condition for bond demand, the change in utility to savers from a change in $p_J$ is given by

$$\begin{aligned} \partial V/\partial p_J &= -U_1'\{J^* + d[p_J T(p_J)]/dp_J\} \\ &= -U_1'\{[G - F_0 + \kappa - (1 - p_J)aG]/p_J + d[(1 - p_J)aG]/dp_J\} \\ &= -U_1'\{[B - (1 - p_J)aG]/p_J - aG\} \\ &= -U_1'[B - aG]/p_J \\ &= -E\{U_2'\}[B - aG]/p_J^2. \end{aligned} \tag{16}$$

A change in the interest rate causes a redistribution of income between taxpayers and interest recipients. Hence, if the shares of marginal second-period taxes paid by savers, $a$, and workers, $1 - a$, do not match their shares in the holding of government debt (directly by savers and indirectly through social security for workers), $B \neq aG$, a change in the interest rate affects utilities. Recognizing that $p_J$ is the only endogenous price, and relating the change in $p_J$ to the change in $r$, it follows from (16) that the response of expected utility to trust fund portfolio diversification satisfies:

$$\begin{aligned} dV/d\kappa &= -E\{U_2'\}(B - aG)/p_J^2\{dp_J/d\kappa\} \\ &= E\{U_2'\}(B - aG)\{dr/d\kappa\}. \end{aligned} \tag{17}$$

[25] Thus, $dp_J/dr = -p_J^2$.

Workers are affected by trust fund investment and by the impact of the interest rate on benefits and taxes. Differentiating expected lifetime utility of workers given by the last line of (8), and setting $p_k = 1$ and $\kappa = K^f$, and then using the definition of taxes from (5) and the bond clearing equation (9), we have:

$$\begin{aligned} dv/d\kappa &= E\{u_2'((R-1-r)/n - dt/d\kappa + (F_0-\kappa)(dr/d\kappa)/n)\} \\ &= E\{u_2'(R-1-r+(F-(1-a)G)(dr/d\kappa)\}/n \\ &= E\{u_2'(R-1-r)\}/n - E\{u_2'\}(B-aG)(dr/d\kappa)/n. \end{aligned} \tag{18}$$

Diversification affects workers through two channels. As was the case in the regime with both investments, the expected utility of workers increases from bearing some risk if they were bearing none before diversification. In addition, the effect on workers from the change in the interest rate has the opposite sign from its effect on savers, as can be seen by comparing (17) and (18). If $a$ is equal to $B/G$, then this effect is zero and workers only gain from improved risky investment. When $a = B/G$, we have a (weak) Pareto gain, as in the case with both investments.[26]

Denoting the social evaluation of the marginal utility of second-period safe consumption of a worker relative to that of a saver by $m$, the impact on a social welfare function for each cohort in steady state (SWF) equals the weighted sum of individual impacts:

$$\begin{aligned} dSWF/d\kappa &= m\{ndv/d\kappa\} + 1\{dV/d\kappa\} \\ &= mE\{u_2'(R-r-1)\} - E\{mu_2' - U_2'\}(B-aG)dr/d\kappa. \end{aligned} \tag{19}$$

Thus there is a direct utility gain from improved risk bearing and a redistributive term, which vanishes if $a$ equals $B/G$. If there is an income distribution change ($a \neq B/G$) , its effect depends on the direction of transfer and the sign of $E\{mu_2' - U_2'\}$. In particular, if other policy tools result in a level of $m$ so that one unit of second-period safe consumption gives the same expected marginal social welfare to every old agent, then the redistribution term drops out and total weighted utility is increased by diversification.

Relying on the rise in the interest rate, as will be shown in the next subsection, we can state the utility impacts as:

**Proposition 2a.** *Suppose there are no long-lived assets, and no safe real assets held in equilibrium. Suppose there are constant marginal returns to risky investment, and that risky investments are undertaken in equilibrium. Suppose that the share of taxes of savers is at least as large as their share of bond holdings, $B \leq aG$, and that all three demands are normal. Then, starting from a trust fund portfolio entirely in bonds, diversification raises the expected utility of workers in every generation (except the original old, who are unaffected). If in addition, $B = aG$, then trust fund diversification does not affect the utility of young savers in every generation (up to first order, increasing it up to second order). If instead, $B < aG$, then diversification (without any other policy changes) lowers the utility of young savers in every generation. Nevertheless, when $B \leq aG$, trust fund purchases of risky investment increase the weighted sum of utility of all workers and savers, weighted so that the marginal social utility of second-period consumption is the same for all.*

**Proof of Proposition 2a.** Without land, old savers and old workers at the time diversification is first implemented are not affected. Given a rise in the interest rate, as shown in Proposition 2b, equations (17), (18), and (19) demonstrate the utility gains. ■

[26] Actually, if $a = B/G$, then savers obtain a second order benefit from trust fund diversification, assuming $dr/d\kappa$ is not 0.

Whether $B < aG$ depends on the size of the trust fund and how tax policy responds to increased interest costs. If $F = 0$, then $B = G > aG$, and an increase in interest rates helps savers, because for every extra dollar in interest receipts received, they pay only $a < 1$ dollars extra in taxes. On the other hand, if the trust fund holds all the government bonds, then $B = 0 < aG$, and savers lose from an increase in interest rates.[27] In reality, the social security trust fund pays benefits to nonsavers and savers. But because of the redistributive nature of the social security benefit rules, nonsavers have a claim on benefits that exceeds their share of income. At the end of 2000, the trust fund was over $1 trillion (OASI plus DI), and increasing rapidly. If it is not the case now, surely in a few years $B < aG$ will be more plausible than $B > aG$. Once the trust fund is big relative to the outstanding stock of government bonds, interest rate increases can be expected to help workers and hurt savers.

## 6.2 Interest Rate and Aggregate Investment

Increasing the supply of government bonds available to the savers will raise the interest rate if demand slopes down. The proof requires confirming that the indirect effects of the interest rate on demand (through income taxation) do not offset the direct effect.

**Proposition 2b.** *Suppose there are no long-lived assets, and no safe real assets held in equilibrium. Suppose there are constant marginal returns to risky investment, and that risky investments are undertaken in equilibrium. Suppose that the share of taxes of savers is at least as large as their share of bond holdings, $B \leq aG$, and that all three demands are normal. Then, trust fund purchases of the risky investment increase the interest rate on government debt, and increase aggregate real investment, though by less than the quantity of the trust fund purchases.*

**Proof of Proposition 2b.** Suppose we are in equilibrium given by (15). If $K^f$ increases, equation (15) can be brought back into balance by decreasing $C^*[p_J, 1, W-(1-p_J)aG] + K^*[p_J, 1, W-(1-p_J)aG]$. With $B \leq aG$ , this can only happen by decreasing $p_J$, that is, by increasing $r$. To see this, differentiate (15), giving:

$$dp_J/d\kappa = -1/(d\{C^*[p_J, 1, W-(1-p_J)aG] + K^*[p_J, 1, W-(1-p_J)aG]\}/dp_J). \quad (20)$$

Letting $I = W - (1-p_J)aG$ be net wealth at the original equilibrium, and letting $V$ be utility at the original equilibrium, and using the standard Slutsky equation[28] relating demand to the sum of compensated demand and income effects, and using the size of the income effect given by (17), we have

$$dp_J/d\kappa = -1/(C^c_p[p_J, 1, V] + K^c_p[p_J, 1, V] + ((C^*_I[p_J, 1, I] + K^*_I[p_J, 1, I])(-[B-aG]/p_J))) < 0, \quad (21)$$

where subscripts $p$ and $I$ refer to partial derivatives with respect to $p_J$ and income $I$, and superscript $c$ means compensated demand. To see that this expression is less than zero, first note that compensated changes keep expected utility constant and marginal utilities

[27] To consider who in reality is a receiver of government bond interest payments net of the taxes levied to pay for them, we need to consider which taxes are raised if interest costs are higher. If it is just the income tax increased, then low income people are not taxed at all. However, if the earned income tax credit is altered along with the income tax (violating our assumption that it is taxes on older workers that are adjusted), then the impact is throughout the income distribution. A realistic case would indeed consider tax changes on young workers and savers as well as on the old. That would have additional effects, which we are not analyzing.

[28] $dC^*[p_J, 1, W-(1-p_J)aG]/dp_J$ equals $C^c_p + C^*_I dI$, where $dI = dV/U'_1[C_1]$ is the change in income that would give the same utility at the old prices as given by the new prices and the new income.

are proportional to prices, implying that $0 = C_p^c + p_K K_p^c + p_J J_p^c = C_p^c + K_p^c + p_J J_p^c$. Since compensated own effects are always negative, $J_p^c < 0$, it follows that $C_p^c + K_p^c > 0$. Since both $C$ and $K$ are normal goods, and $B - aG \leq 0$, the denominator of (21) is positive. Note that this analysis holds for any $\kappa \geq 0$ consistent with equilibrium with only risky investment.

To consider the impact of changing social security portfolio policy on aggregate investment, we need only determine its effect on the consumption of young savers, since the consumption of young workers does not change. From (15) we know (using Slutsky and the income effect term from (17)) that

$$\begin{aligned} dk/d\kappa &= d\{K^* + K^f\}/d\kappa = -(dC^*[p_J, 1, W - (1-p_J)aG]/dp_J)(dp_J/d\kappa) \qquad (22) \\ &= -\{C_p^c[p_J, 1, V] + C_I^*[p_J, 1, I](-[B - aG]/p_J)\}(dp_J/d\kappa) \\ &= \{C_p^c - C_I^*[B - aG]/p_J\}/\{C_p^c + K_p^c - (C_I^* + K_I^*)[B - aG]/p_J\} > 0. \end{aligned}$$

We already saw that the denominator is positive. Also $C_p^c[p_J, 1, V] > 0$, since $J$ and $C$ are Hicksian substitutes if $J$ is normal. Furthermore, $C_I^* > 0$, since $C$ is normal. Thus if $B \leq aG$, substitution and income effects go the same way. Hence trust fund diversification lowers $C^*$, thereby raising total risky investment $K^* + K^f$. Our analysis also shows that $dk/d\kappa = d\{K^* + K^f\}/d\kappa < 1$, when all the goods are normal, for then $K_p^c > 0$ and $K_I^* > 0$, and the denominator of (22) is larger than the numerator. ■

Complementing these propositions, we note that if an increase in the government bond interest rate redistributes wealth from workers to savers, $B > aG$, then the interest rate may rise or fall.[29]

## 7 Adding Infinitely-Lived Assets

In the models above, no assets last more than one period. Thus a change in the interest rate does not redistribute wealth across generations. To consider intergenerational redistribution, we now add two infinitely-lived assets. A change in social security policy that changes the prices of the long-lived assets will redistribute wealth between the old, who own the assets at the time the policy is implemented, and all future generations who buy them. We assume fixed quantities of both types of infinitely-lived assets, referred to as safe land and risky land. Each unit of safe land provides one unit of consumption, independent of the state of nature, in each period.[30] Each unit of risky land produces the same (realized) output as one unit of the (contemporaneous) risky investment.[31] We denote the supplies of the two assets by $L_0$ and $L$, and their prices by $p_0$ and $p$. Because of the stationary structure of the economy, in stationary equilibrium, these prices are constant over time. The effects of trust fund diversification are modified by the presence of land, but not drastically changed. As before, the analysis depends on which short-term investments are undertaken in the original stationary equilibrium. The case where both are undertaken, $k_0 > 0$ and $k > 0$, is exactly like the case without land since no prices change, and it is not repeated here. In the rest of Section 7, we reformulate the definition of equilibrium to include land.

[29] So far, we have considered two different models with perfectly elastic and perfectly inelastic supplies of safe assets. We could consider an intermediate model with a downward-sloping demand by foreigners for government debt. This would give a change in the equilibrium interest rate that was between the two cases analyzed. In this case, the increase in the interest rate on government debt would involve increased payments abroad as well as transfers from taxpayers to trust fund beneficiaries.

[30] This might be a fixed number of government consols, the interest on which is financed by taxation on successive generations.

[31] Since the return to risky investment is independent of the level of investment, there is no distinction between land that provides output and land that provides capital input.

In reality, the stock market is made up of both short-term and long-term investments, and so the effect of social security diversification on stock market prices involves both short-term and long-term asset price changes.

## 7.1 Land and the Dynamic Asset Span

Land (of either type) lasts forever and gives new output forever and is sold each period by the older generation to the younger one. Given a stationary economy, and the assumption that land output is independent and identically distributed each period, the price of land (just *after* the realization of output) is constant across time, and across realizations of output.[32] The one-period gross return from purchasing land is equal to its dividend that period, plus a constant capital value. The one-period returns on either type of infinitely-lived land are therefore (endogenous) convex combinations of the returns on risky short-term investments and the safe return on government bonds. Thus we can incorporate land into our model without introducing a new risk characteristic. We do not need to reformulate the choice problem of savers in terms of the three composite consumer goods.

If a young saver buys one unit of safe land, it costs $p_0$ and yields safe consumption in the second period (from output and resale) of $1+p_0$. Since this is a perfect substitute for buying $(1+p_0)/(1+r)$ units of the government bond, we have

$$p_0 = (1+p_0)/(1+r), \tag{23}$$

or

$$p_0 = 1/r = p_J/(1-p_J). \tag{24}$$

Similarly, by spending $p$ on risky land, the consumer gets the risky dividend that can be purchased at a price of $p_K$, (by investing in the short-lived risky asset) and the ability to sell the asset at price $p$, which has a current value of $p/(1+r)$. Thus, by arbitrage, we have the equilibrium price of risky land satisfying:

$$p = p_K + p/(1+r) = p_K + pp_J \tag{25}$$

or

$$p = (1+r)p_K/r = p_K/(1-p_J). \tag{26}$$

Thus the prices of both kinds of land are determined by the price of the short-term assets. Both land prices increase with the price of second-period safe consumption; equivalently, land prices decrease when the interest rate rises. The total value of land is

$$P = p_0 L_0 + pL = L_0 p_J/(1-p_J) + L p_K/(1-p_J). \tag{27}$$

Any equilibrium where the trust fund holds $L^f$ acres of risky land is equivalent to an equilibrium in which the trust fund holds $L^f$ units of the risky asset and $L^f pp_J$ bonds. (The dividends are the same, and by (26) the bond payoffs $pL^f$ can be used each period to repurchase the same portfolio.) In particular, starting from a portfolio exclusively in short-term assets, a trust fund purchase of $L^f$ acres of risky land (obtained by selling bonds) would have precisely the same effect as the purchase of $L^f$ units of the risky short-term asset (obtained by selling bonds). However, the effect of a further trust fund purchase of land will differ in the two cases because it will change asset prices, giving a different capital gain to the two portfolios. To keep the analysis simple, we shall continue to suppose that the trust fund holds only short-term assets.

[32] The iid assumption implies that there is never any "news" about future returns, so land values never change.

### 7.2 Stationary Equilibrium with Land

We now define equilibrium in terms of the three goods $C^*$, $J^*$, $K^*$, as we did earlier. The presence of land does not change the expected utility maximization problem of savers given in (2) nor their demand curves. Now, savers acquire safe future consumption by buying bonds, and both types of land. Safe consumption $J^*$ plus taxes for savers must equal the return from holding all the government bonds not held by the trust fund, plus the payoff from safe land, plus the resale value of all land, safe and risky, plus the payoff from short-term safe production (if any). Similarly, risky consumption for savers, $K^*$, is now the demand for the sum of risky short-term investments and the one-period payoffs of risky land, less what is held by the trust fund. Thus, (10) and (11) become

$$J^*(p_J, p_K, W - p_J T(p_J)) + T(p_J) = (G - F_0 + \kappa)/p_J + L_0 + P + R_0 k_0, \qquad (28)$$

$$K^*(p_J, p_K, W - p_J T(p_J)) + \kappa/p_K = L + k. \qquad (29)$$

## 8 Social Security Diversification with Risky Investment and Land

We suppose the economy with land is such that in stationary equilibrium there is only risky production, $k_0 = 0$ but $k > 0$. As in Section 6, equilibrium boils down to the market for safe consumption in equation (28). Using the budget constraint (2) of the savers, the equality $p_K = 1$, the definition of $T(p_J)$, and the formula (27) for the value of total land, we can write the market clearance for bonds as:

$$\begin{aligned} G - F_0 + k &= p_J J^*(p_J, p_K, W - p_J T(p_J)) + p_J T(p_J) - p_J(L_0 + P) \qquad (30) \\ G - F_0 + K^f &= W - C^*[p_J, 1, W - p_J T(p_J)] - K^*[p_J, 1, W - p_J T(p_J)] - p_J(L_0 + P) \\ &= W - C^*[p_J, 1, W - (1-p_J)aG] - K^*[p_J, 1, W - (1-p_J)aG] - p_J(L_0 + L)/(1-p_J). \end{aligned}$$

This differs from (15) in Section 6 by the addition of the last term on the RHS. As before, we have a single equation in a single variable, $p_J$. And, with the same assumptions as before, we will again find that $p_J$ goes down, equivalent to the interest rate going up.

From (29), aggregate investment in short-lived production, $k$, is trust fund demand for real investment, $K^f$, plus the demand of savers for risky consumption, $K^*$, minus the portion of that demand that is satisfied by purchasing risky land, $L$. Thus, using the second line of (30), and $p_K = 1$, and $P = p_K L + p_J(L_0 + P)$, aggregate investment in risky short-term assets, $K^f + K^* - L$, can be written:

$$K^f + K^* - L = p_K(K^f + K^* - L) = W - (G - F_0) - C^*[p_J, p_K, W - p_J T(p_J)] - P. \quad (31)$$

That is, short-term investment equals the wealth of savers less what they spend on consumption, on bonds and on land. Hence, with $W$, $G$, and $F_0$ all fixed, the response of aggregate investment in short-term assets to social security diversification is minus the sum of the response of consumption of savers, $C^*$, and the change in the value of total land, $P = p_0 L_0 + pL$. With the same assumptions as before, we will again find that aggregate investment goes up.

### 8.1 Expected Utility

In the previous models without land, old savers were not affected by social security diversification. But with the introduction of land they have something to sell, whose value might be affected by social security diversification. For example, if land prices go down in value (as the interest rate rises), the old savers at the time of the trust fund diversification lose, *ceteris paribus*. Young savers gain, as do savers in every succeeding generation.

We begin with old savers at the time of implementation of the policy change:

$$\begin{aligned} \partial V_{\text{old}}/\partial p_J &= U_2'[dP/dp_J] \\ dV_{\text{old}}/d\kappa &= (\partial V_{\text{old}}/\partial p_J)(dp_J/d\kappa) \\ &= U_2'[dP/dp_J](dp_J/d\kappa). \end{aligned} \tag{32}$$

Since the new stationary equilibrium is achieved immediately after the trust fund purchases, young savers at the time of the purchases are affected exactly the same way as all future savers, namely

$$\begin{aligned} \partial V/\partial p_J &= U_1'\{-J^* - d[p_J T(p_J)]/dp_J\} \\ &= U_1'\{-\{[B-(1-p_J)aG]/p_J + L_0 + P\} - d[(1-p_J)aG]/dp_J\} \\ &= -U_1'[(B-aG)/p_J + L_0 + P] \\ dV/dk &= (\partial V/\partial p_J)(dp_J/d\kappa) \\ &= \{-U_1'[(B-aG)/p_J + L_0 + P]\}(dp_J/d\kappa). \end{aligned} \tag{33}$$

This differs from (16) in Section 6 by the presence of $L_0 + P$. The first term in the last line of (33) reflects the within-cohort redistribution between savers and workers as a consequence of different shares in government bonds and in the taxes to pay the interest on the bonds. The second term reflects the across-cohort redistribution from changes in the price of safe consumption which is purchased from the previous generation by buying land. The formulas in (33) appear to depend on the value of land, not the change in this value, but the link between the two follows from the fact, shown below, that:

$$dP/dp_J = [P + L_0]/(1-p_J). \tag{34}$$

This implies that the change in expected utility, (33), can be written:

$$dV/d\kappa = -U_1'[(B-aG)/p_J + (1-p_J)dP/dp_J](dp_J/d\kappa). \tag{35}$$

Now it is clear that changing land values does affect young savers. If social security is diversified at time 1, and land prices fall, the young at time 1 do not gain by the whole drop in land prices, since the resale value of the land when they get old also falls.

Notice that the expected utility of young savers can increase or decrease, depending on the balance of redistributions between savers and workers, and redistributions between old savers and young savers. If the value of all land, $P$, exceeds the total of all government bond promises $G/p_J = G(1+r)$, then social security diversification must improve the welfare of young savers (assuming $dp_J/dk < 0$), even though it creates a redistribution from young savers to young workers if $B < aG$. Evidently young savers gain more from old savers than they lose to young workers. A similar conclusion holds if the value of safe land $p_0 L_0$ is greater than the value of government bonds $G$ outstanding, as can be seen from (33), and the identity $p_J(L_0 + P) = p_0(L_0 + L)$.

Equation (18) quantifying the effect of trust fund diversification on workers in the risky linear case without land applies without change in this risky linear case with land. As before, the increased exposure to risky stock and the rise in interest rates make workers better off, assuming $B - aG < 0$ and $dp_J/d\kappa < 0$.

**Proposition 3a.** *Suppose there is no safe short-term real investment, and that short-term risky investments display constant marginal returns, and are undertaken in equilibrium. Suppose there is both safe and risky land. Suppose also that the share of taxes of* *savers is at least as large as their share of bond holdings, $B \leq aG$, and that demands for all consumption goods are normal. Then, starting from a trust fund invested exclusively in bonds, trust fund purchases of risky investments increase the expected utility of all workers (except the old at the time the policy is implemented, who are unaffected). Old savers at the time the policy is implemented are hurt. If, in addition, $(aG - B) < p_0(L_0 + L)$, then all other savers are helped. If, however, $(aG - B) > p_0(L_0 + L)$, then all other savers are also hurt. Nevertheless, when $B \leq aG$, trust fund purchases of risky investment increase the weighted sum of utility of all workers and savers, weighted so that the marginal social utility of second-period consumption is the same for all.*

**Proof of Proposition 3a.** Given the rise in the interest rate, as proven below, the utility results follow from the equations above. We note that the SWF needed for the last conclusion has weights:

$$SWF = mnv^0_{\text{old}} + V^0_{\text{old}} + \sum_1^\infty \delta(t)[mnv^t + V^t]$$

The superscript $t$ refers to the generation of birth, and we suppose the diversification takes place at time $t = 1$. The weight $m$ is chosen, as before, so that starting from the original equilibrium, an additional dollar gives the same marginal social utility whether it is given to an old saver or an old worker from the same generation. Finally we suppose $\delta(t) = 1/(1+r)^{t-1}$, where $r$ is the interest rate prevailing in the original equilibrium. This also preserves the property that a simple redistribution has no impact on social welfare.

To calculate the effect of social security diversification on social welfare, the utility gains must be added across all generations. Using (32) and (35), and recognizing that $(1-p_J)$ is equal to $r/(1+r)$, the change in total land value does not affect social welfare. The sum of all savers' utility gains from the fall in land prices, discounted by the equilibrium interest rate, exactly balances the change in utility of the old from the generation in retirement at the time the policy was implemented. ■

## 8.2 Interest Rate, Investment and Land Values

From (24) and (26), the value of safe land and of risky land each move in the same direction as $p_J$, that is in the opposite direction of the change in interest rates. Thus trust fund purchases of risky short-term investments reduce the price of risky land (and also the price of safe land). It is a remarkable, and unanticipated, property of the current model that the increase in demand for risky land reduces its price! It is often claimed that if social security bought stocks, it would raise the value of the stock market. This breezy conclusion is seen to be more delicate than it sounds. Since the interest rate increases, it is not so surprising after all to find a tendency for stock prices to decline, for stock prices depend on discounting future returns. When technology fixes the return on short-term risky investments, the interest rate effect is the only one that affects stock prices. We pursue this question of land values further in the next sections.

It remains to show that trust fund purchases of risky investment increase the interest rate on government debt. An increase in the supply of government bonds available to the savers will lower the interest rate if demand slopes down. The proof requires confirming that the indirect effects of the interest rate on demand (through income taxation and the change in land values) do not offset the direct effect.

**Proposition 3b.** *Suppose there is no safe short-term real investment, and that short-term risky investments display constant marginal returns, and are undertaken in equilibrium. Suppose there is both safe and risky land. Suppose also that the share of taxes of* *savers is at least as large as their share of bond holdings, $B \leq aG$, and that demands for all consumption goods are normal. Then, trust fund purchases of risky investment increase the interest rate on government debt, and increase aggregate investment. The increase in aggregate investment may be larger or smaller than the trust fund purchase of risky investment. Moreover, the price of safe land falls, as does the price of risky land. The total value of land therefore falls, though $-dP/d\kappa < 1/p_J$.*

**Proof of Proposition 3b.** With $p_K$ unchanged at a value of one, we can compute the change in total land value by

$$\begin{aligned} dP/dp_J &= L_0[1/(1-p_J) + p_J/(1-p_J)^2] + Lp_K/(1-p_J)^2 \\ &= L_0/(1-p_J)^2 + Lp_K/(1-p_J)^2 \\ &= [L_0(1-p_J+p_J)/(1-p_J) + Lp_K/(1-p_J)]/(1-p_J) \\ &= [L_0p_J/(1-p_J) + Lp_K/(1-p_J) + L_0]/(1-p_J) \\ &= [P+L_0]/(1-p_J) \\ dP/d\kappa &= (dP/dp_J)(dp_J/d\kappa) \\ &= \{[P+L_0]/(1-p_J)\}(dp_J/d\kappa). \end{aligned} \tag{36}$$

We can get further information about the size of $dP/d\kappa$ by multiplying out the terms in (36). In the third line of (36), define $x$ by $-(C_I^*+K_I^*)(L_0+P)+dP/dp_J = p_J\{dP/dp_J\}+x$. Using (34), and the fact that $(C_I^* + K_I^*) < 1$, we know that $x > 0$. Now multiplying out the terms in (36), and using the fact that the rest of the terms in the denominator of (36) are positive and the fact that $dp_J/d\kappa < 0$, gives $-p_JdP/d\kappa < 1$.

Differentiating (31), using the Slutsky equation, using the impact of a price change on $V$, (33), and using the derivative of $P$ from (36), we have:

$$\begin{aligned} dp_J/d\kappa &= -1/d(C^* + K^* + P)/dp_J \\ &= -1/\{C_p^c + K_p^c + (C_I^* + K_I^*)(-(B-aG)/p_J - L_0 - P) + dP/dp_J\} \\ &= -1/\{\{C_p^c + K_p^c\} - \{(C_I^* + K_I^*)(B-aG)/p_J\} + \{-(C_I^* + K_I^*)(L_0+P) + dP/dp_J\}\} \\ &= -1/\{\{C_p^c + K_p^c\} - \{(C_I^* + K_I^*)(B-aG)/p_J\} + \{(L_0+P)(1/(1-p_J) - (C_I^* + K_I^*))\}\}. \end{aligned} \tag{37}$$

The first terms in the denominator of the last line of (37) are the compensated demands for first-period consumption and risky second-period consumption with respect to the price of safe second-period consumption and have a positive sum, as noted in the proof of Proposition 2. The next term reflects the redistribution between savers and workers and is positive if savers have a larger share in taxes than in bonds ($B \leq aG$) and have normal demands. The final term reflects the intergenerational redistribution between old savers when the policy is implemented and later cohorts. It is also positive when the demand for safe second-period consumption is normal (which implies that $C_I^* + K_I^* = 1 - p_J J_I^* < 1$) and the price of second period consumption, $p_J$, is between zero and one ($r > 0$), as we have assumed. Thus $dp_J/d\kappa < 0$, as was the case without land.

As noted in (31), the level of investment in short-term production possibilities, $K^f + K^* - L$, is equal to the endowment of young savers less their first-period consumption, less the amount spent on purchasing land, less the unified net debt of the government. Note that $L$ is constant. Hence the change in short-term risky investment is given by the change in the RHS of equation (31). Differentiating (31), using the Slutsky equation with the income effect from (33), and the derivative of $P$ from (36), gives

$$\begin{aligned} dk/d\kappa &= d\{K^* + K^f\}/d\kappa = -[d(C^* + P)/dp_J][dp_J/d\kappa] \\ &= -\{C_p^c + (C_I^*)(-(B-aG)/p_J + L_0 + P) + dP/dp_J\}[dp_J/d\kappa] \end{aligned} \tag{38}$$

$$= -\{\{C_p^c - (C_I^*)(B - aG)/p_J\} + \{-(C_I^*)(L_0 + P) + dP/dp_J\}\}[dp_J/d\kappa]$$
$$= -\{\{C_p^c\} - \{C_I^*(B - aG)/p_J\} + \{(L_0 + P)(1/(1 - p_J) - C_I^*)\}\}[dp_J/d\kappa] > 0.$$

The first term $C_p^c$ in the last line is the compensated cross elasticity of first-period consumption with respect to the price of second-period safe consumption. If the demand for riskless second-period consumption is normal, then $C^*$ and $J^*$ are Hicksian substitutes and this term is positive (Aura, Diamond, and Geanakoplos, 1999). With normality of demand for first-period consumption and redistribution from savers to workers ($B \leq aG$) the second term is positive. The third term is also positive when the demands for second-period safe and risky consumption are normal ($C_I^* < 1$) and the price of second-period consumption, $p_J$, is between zero and one ($r > 0$), as we have assumed. Multiplying by the minus sign in front and by $dp_J/d\kappa < 0$ gives a positive number.

Replacing $[dp_J/d\kappa]$ in the first line of (38) by the first line of (37), we get $dk/d\kappa = [d(C^* + P)/dp_J]/[d(C^* + K^* + P)/dp_J]$. From this we see that whether $dk/d\kappa$ is above or below one depends on the sign of $dK^*/dp_J$, which is the sign of $K_p^c - K_I^*\{[(B-aG)/p_J] + (L_0 + P)\}$. The compensated derivative, $K_p^c$, is positive since the two assets are Hicksian substitutes. Thus, we see that if $-(B - aG) > p_J(L_0 + P)$, then $d\{K^* + K^f\}/d\kappa < 1$. But if $-(B - aG) < p_J(L_0 + P)$, and $K_p^c$ is small, then $d\{K^* + K^f\}/d\kappa$ could be greater than 1. In such a case, the drop in the value of land gives such a big positive income boost to young savers, who are buyers of the land, that they increase their holdings of risky assets $K^*$ even while the competing return on safe assets has gone up. Thus a trust fund purchase of \$500 billion of risky assets could increase real investment by more than \$500 billion. ■

From (37) we note that the presence of long-lived assets decreases the sensitivity of interest rates to trust fund diversification, and thus decreases the size of the interest rate increase. We might interpret the quantitative part of 3b as follows. Given that a period in this model represents something like 30 years, and that the real interest rate has historically been about 2.3% per annum, a crude estimate of $p_J$ is about $\frac{1}{2}$. A \$500 billion transfer of trust fund assets from bonds into stock, maintained there forever, must lower land prices, but could not lower land prices by more than \$1 trillion.

# 9 Social Security Diversification with only Safe Investment and Land

To show alternative possibilities, we turn now to an economy with one-period safe real investments but no one-period risky investments undertaken in equilibrium. Many of our preceding results are now reversed. While not realistic, this case shows the importance of diminished risky investment opportunities. In this extreme case, the interest rate is determined by the return on safe real one-period assets, $p_J = 1/(1 + r) = 1/R_0$, and the price of safe land is technologically determined as well. With no change in the interest rate, taxes do not change. The price of risky consumption, and thus the price of risky land, are determined by market clearance, reflecting both the given interest rate and the evaluation of risky consumption by savers. With risky second-period consumption being a normal good, the demand is downward sloping and the price $p_K$ must rise to clear the market in response to an increase in trust fund demand for risky consumption.

We could envision someone supplying a short-term risky financial asset promising $R$ without any short-term real risky production. A seller of this asset could simply deliver out of land dividends, without producing any risky output. Thus we can define social security diversification exactly as before, namely as the sale of bonds and the purchase of short-term risky securities. Recall that starting from a position with no risky securities, trust fund purchases of short-term risky assets in exchange for bonds has exactly the same effect as the trust fund purchase of risky land in exchange for bonds.

In the absence of risky production, the only source of risky consumption is risky land, and each acre of risky land provides one unit of risky consumption. (Hence $K^*$ can be interpreted as the savers' demand for risky land.) Market clearing in the market for risky consumption, (29), now reduces to:

$$\begin{aligned} &K^*(p_J, p_K, W - p_J T(p_J)) + \kappa/p_K = L \\ &K^*(p_J, p_K, W - p_J T(p_J)) + K^f = L \\ &K^*(1/R_0, p_K, W - (1/R_0)aG(R_0 - 1)) + K^f = L. \end{aligned} \tag{39}$$

## 9.1 Expected Utility, the Price of Risky Consumption, the Price of Land, and Investment

With the interest rate fixed by the return on riskless investments, savers' utility changes only on account of a change in price of risky consumption. We have from the budget set (2), from the envelope theorem, and from (39) that:

$$\begin{aligned} \partial V/\partial p_K &= U_1'\{-K^*\} = -U_1'\{L - K^f\} \\ dV/d\kappa &= -U_1'\{L - K^f\} dp_K/d\kappa. \end{aligned} \tag{40}$$

Thus the expected utility of savers moves in the opposite direction from the price of risky investment. Since the trust fund adds to the demand for risky assets, we should not be surprised to find that $p_K$ rises after social security diversification. The result is that all savers, starting from the young at the time of social security diversification, lose.

This raises an interesting point for the current privatization debate. Many of today's young are clamoring for diversification on the grounds that stocks earn higher returns than bonds. But any rational young saver should already be investing so much of his wealth in stock that he is indifferent on the margin between further investments in stocks and bonds. Thus if prices did not change, the direct effect of social security diversification should be irrelevant to a young saver (even supposing he is covered by social security). However, if the extra demand for risky assets raises $p_K$ (equivalently, if it lowers the expected return savers can get over their lives), then equation (40) shows that it reduces their welfare, provided that the riskless rate does not also change.

**Proposition 4.** *Assuming the presence of safe real one-period investments, but no risky real one-period investments, and that demand for risky second-period consumption is normal, then, trust fund purchases of risky investment increase the price of risky consumption, and decrease aggregate real investment. Moreover, social security diversification raises the price of risky land, leaving the price of riskless land unchanged. The total value of land goes up.*

*Starting from a trust fund invested exclusively in bonds, trust fund purchases of risky investments increase the expected utility of all workers (except the old at the time the policy is implemented, who are unaffected). Old savers at the time the policy is implemented are also helped. All young and future savers lose utility as a result of the policy. Nevertheless, when $B \leq aG$, trust fund purchases of risky investment increase the weighted sum of utility of all workers and savers, weighted so that the marginal social utility of second-period consumption is the same for all.*

**Proof of Proposition 4.** Differentiating (39), and using the income effect from (40), we have:

$$\begin{aligned} dp_K/d\kappa &= -1/\{p_K\{dK^*[p_J, p_K, W - p_J T(p_J)]/dp_K - \kappa/(p_K)^2\}\} \\ &= -1/\{p_K K^c_{p_K} - p_K K^*_I[L - K^f]\} - \kappa/p_K\} > 0. \end{aligned} \quad (41)$$

Since compensated own price effects are always negative, and $K^*$ is a normal good, all the terms in the denominator are negative. Multiplied by the negative sign outside, we get the claimed result. From the connection between $p_K$ and the price of risky land, (26), we conclude that the price of risky land also rises. The price of safe land, (24), does not change.

$$\begin{aligned} &dp_0/d\kappa = (dp_0/dp_K)(dp_K/d\kappa) = 0 \\ &dp/d\kappa = (dp/dp_K)(dp_K/d\kappa) = [1/(1-p_J)](dp_K/d\kappa) \\ &dP/d\kappa = Ldp/d\kappa = [L/(1-p_J)](dp_K/d\kappa) = [LR_0/(R_0-1)](dp_K/d\kappa). \end{aligned} \quad (42)$$

With a rise in the price of land, the utility of old savers rises when the policy is implemented, since the value of the land they are holding rises. In turn, this lowers the expected utility of young savers and those in future cohorts. Starting from a trust fund invested exclusively in bonds, the expected utility of workers is increased by diversification in the same way as in Proposition 1.

It remains to prove that aggregate investment declines after trust fund diversification. Rearranging the equilibrium condition (28), using budget set (2), and market clearance for risky consumption (29) gives:

$$\begin{aligned} R_0 k_0 &= J^*(p_J, p_K, W - p_J T(p_J)) + T(p_J) - (L_0 + P) - R_0(G - F_0 + p_K K^f) \\ &= R_0(W - C^*(1/R_0, p_K, W - aG(R_0-1)/R_0) - p_K K^*) - (L_0 + P) - R_0(G - F_0 + p_K K^f) \\ &= R_0(W - C^*(1/R_0, p_K, W - aG(R_0-1)/R_0)) - (L_0 + P) - R_0(G - F_0 + p_K L). \end{aligned} \quad (43)$$

Dividing by $R_0$, differentiating with respect to $\kappa$, substituting for the derivative of $P$ from (42), and then using the Slutsky equation with the income effect derived in (40), we have:

$$\begin{aligned} dk_0/d\kappa &= -(dC^*/dp_K + L/(R_0-1) + L)(dp_K/d\kappa) \\ &= -(C^c_{p_K} - C^*_I[L - K^f] + LR_0/(R_0-1))(dp_K/d\kappa) \\ &= -(C^c_{p_K} + L[(R_0/(R_0-1)) - C^*_I] + C^*_I K^f)(dp_K/d\kappa) < 0. \end{aligned} \quad (44)$$

To see that the derivative $dk_0/d\kappa$ is negative, note first that since $K$ is normal, $C$ and $K$ are Hicksian substitutes, so $dC^c/dp_K > 0$. By normality of $K$ and $J$, $C^*_I < 1$. Since $R_0/(R_0-1) > 1$, the second term is positive. Finally, since $C^*$ is normal and the trust fund holdings of risky consumption are nonnegative, the last term is positive as well. Thus the sum in parenthesis is positive, and since $dp_K/d\kappa > 0$, safe investment declines. ■

# 10 Alternative technologies

The analysis so far was made simpler by the presence of at most one endogenous rate of return, the other rate of return and wages being given by technology independent of production decisions. If we assume that there are no short-term production possibilities or that both technologies are strictly concave, rather than linear, then we need to solve simultaneously two equilibrium equations in two unknown rates of return (wages remaining exogenous). This prevents the clear presentation of the underlying economic factors.

It also results in a more complex analysis. In a companion paper, we explore this model. Here we briefly summarize some of those results without proof.[33]

We suppose that the safe technology takes the form $f(k_0)$ and that the risky technology takes the form $g(k)R$, where $f$ and $g$ are twice differentiable and concave and $R$ is stochastic. We suppose the productive sectors of the economy are owned entirely by the savers.[34] Each saver receives a rent or profit from ownership of technology, in addition to his wage, as income.[35] This model includes the previous models as special cases.

By generalizing the model we can see what general qualitative properties persist across all the equilibrium regimes studied in Sections 1-9. We find in this section that, in general, social security diversification raises the riskless interest rate, and lowers the expected short-term risky return. It decreases safe investment and increases risky investment. Its effect on total investment could in general go either way.

To describe equilibrium in terms of budget set (2) and the variables $C^*$, $J^*$, $K^*$ requires recognition of the return from owning technologies as part of the definition of income, $I$. We begin with the productive sector, which is assumed to maximize profits, taking prices as given. Let

$$\Pi(p_J, p_K) = \max[p_J f(k_0) - k_0] + \max[p_K g(k) - k] \tag{45}$$

Income for the savers is now defined as

$$I(p_J, p_K) = W - p_J T(p_J) + \Pi(p_J, p_K). \tag{46}$$

With this definition of income, we can define savers' demands $C^*$, $J^*$, $K^*$ from budget set (2) as before. Stationary equilibrium is now described by a vector $(p_J, p_K, k, k_0)$ satisfying (45)–(51).

$$J^*(p_J, p_K, I(p_J, p_K)) + T(p_J) = (G - F_0 + \kappa)/p_J + L_0 + P + f(k_0) \tag{47}$$

$$K^*(p_J, p_K, I(p_J, p_K)) + \kappa/p_K = L + g(k) \tag{48}$$

$$\begin{aligned} p_J = 1/(1+r) &= 1/f'(k_0) \quad \text{if} \quad k_0 > 0 \\ &\leq 1/f'(k_0) \quad \text{if} \quad k_0 = 0 \end{aligned} \tag{49}$$

$$\begin{aligned} p_K &= 1/g'(k) \quad \text{if} \quad k > 0 \\ &\leq 1/g'(k) \quad \text{if} \quad k = 0 \end{aligned} \tag{50}$$

$$P = L_0 p_J/(1 - p_J) + L p_K/(1 - p_J). \tag{51}$$

We shall confine our attention to "regular economies" that satisfy two restrictions. The first is that if in any equilibrium, either safe or risky investment is not undertaken, then the corresponding price/marginal product condition in (49) or (50) is a strict inequality.

[33] A further generalization of the model would have been to introduce labor as a nonseparable input to production. If labor were applied at the same time as capital, for example, at planting time, before uncertainty is resolved, there would be little additional complication. But if labor is applied to production after uncertainty is resolved, for example, at harvest time (so that the capital of one generation combines with the labor of the next), then labor income becomes state dependent and there would be no steady state equilibrium (though perhaps a Markov equilibrium). One could also allow for distinct models of land, depending on whether ownership of land ensures a given level of (possibly stochastic) output each period, or whether the ownership of land provides a given level of capital input to production each period. When the marginal product of capital was given, the two approaches were the same.

[34] That is, each of the unit measure of savers owns access to these technologies in terms of own capital input. Since each saver will invest the same amount, we can do the analysis in terms of aggregates.

[35] This modeling approach differs from that with an externality that could result in the same aggregate output function, but without the separation of returns between the return on capital inputs and the return on ownership of technology. This alternative approach would give a larger return to trust fund investment in capital since there would not be an increase in the return to savers from owning technology.

The second restriction is that at every equilibrium, if we linearize the five equations (47)–(51), and then differentiate with respect to the five variables $(p_J, p_K, k, k_0, P)$, we get an invertible matrix. Since nearly every economy is regular, there is almost no loss of generality in looking only at regular economies.[36]

As in the linear model, there are four equilibrium regimes depending on whether risky or safe investment is undertaken. Nevertheless, since all four of these regimes are consistent with the hypothesis that the economy is regular, we can handle all the cases as part of the same analysis. Our first proposition shows that the effects of social security diversification on short-term prices and investment can be generalized from the special cases of the linear model to the more general concave model of this section.

**Proposition 5a.** *Suppose we have a regular economy with concave short-term production technology and land. Suppose that savers' demand is normal in all three goods, and that the savers' share of taxes is greater than or equal to their share of bonds. Then trust fund diversification (weakly) raises $p_K$ and $k$ and (weakly) lowers $p_J$ and $k_0$. In words, social security diversification raises the riskless interest rate, lowers the expected return on short-term risky securities, increases risky investment, and decreases safe investment.*

Proposition 5a includes the relevant portions of earlier Propositions as special cases. Furthermore, since in the risky linear case and in the safe linear case one of the prices $p_K$ or $p_J$ is fixed by the technology, Proposition 5a and the formulas for land prices yield the land price results above. On the other hand, the proof is indirect, and proceeds by finding a contradiction, yielding less insight. The proof is presented in the companion paper.

Proposition 5a is qualitative, so we cannot use it in general to sign the effect of social security diversification on aggregate investment, for that involves comparing the magnitudes of the effects on safe and on risky investment. But we can express the change in the total by the differentiation of the system of equations. Further assumptions then permit signing the expression, as well as that for the total value of land.

Since utilities can be written in terms of prices, we also have a general analysis of utility.

**Proposition 5b.** *Suppose we have a regular economy with concave short-term production technology and land. Suppose that savers' demand is normal in all three goods, and that the savers' share of taxes is greater than or equal to their share of bonds, and suppose that in the original equilibrium, $\kappa = 0$. Then trust fund diversification raises the utility of all young and future workers. If it also raises the price of land, then it helps old savers and hurts all young and future savers. If on the other hand, it lowers the total value of land, then it hurts all old savers and may help or hurt young and future savers. Nevertheless, when $B \leq aG$, trust fund purchases of risky investment increase the weighted sum of utility of all workers and savers, weighted so that the marginal social utility of second-period consumption is the same for all.*

# 11 Defined Benefits

We have restricted attention so far to a defined contribution social security system for analytical convenience, and to make the point that even there, social security diversification in moderation brings potential welfare gains. We show now that at least for our central

[36] That is, if we endow savers with a very small amount $s$ of safe consumption and a very small amount $r$ of risky consumption in their old age, then almost any choice of $s$ and $r$ (precisely, all except for a measure zero set of choices) will give a regular economy.

risky linear case, we can readily incorporate a defined benefit structure without changing the comparative statics conclusions.

In our defined contribution social security system, we supposed that the trust fund maintained a constant value $F$ invested in government bonds, and a constant value $\kappa$ invested in risky securities, distributing any surplus as changes in benefits to the contemporaneous old. If the fund acted only to maintain $F$, distributing a part of the surplus over $F$ as benefits to the contemporaneous old, and investing the rest in risky equities, over time the benefits and the size of the trust fund investment in risky equities would change.

Leaving tax rates fixed, the level of benefits would adapt to the level of the trust fund, thereby rising with the return on the portfolio, as does a defined contribution system, but not rising dollar-for-dollar, as the returns got spread over future cohorts. In this way the benefits of a cohort would depend on the realized returns over a longer period of time. With a sensible benefit rule, and a plausible stochastic process for the return on capital, this would raise expected utility for future generations, measured as of the time of implementation of the policy, since a diversified social security system could spread the return risk over many generations. Thus the gain from a diversified portfolio becomes larger with a good policy for determining benefits.

In any "defined benefits system" with risky investments, benefits (and or taxes) must be changed, depending on the returns of the risky investments. The point is to smooth benefits, while recognizing the need to satisfy a nonnegativity constraint should there be a prolonged period of low returns. In the presence of random returns on a nontrivial portion of the trust fund, it is necessary to recognize the probability that the portion of the trust fund invested in risky assets would become negative if both benefits and taxes were unchanged. Thus, every "defined benefit system" must be changed from time to time. The policies that determine such change need to be modeled in order to consider the value of smoothing that comes from defined benefits. In a model with randomness in other aspects of the economic and demographic environment as well, the change in uncertainty from a diversified portfolio would not be such a salient change in the system.[37]

In the risky linear case the variations described above in the trust fund holding of risky securities and in social security benefits have no effect on any equilibrium price. The extra money invested in risky securities is absorbed by an increase in risky production, with no effect on $p_K$. The environment of savers is thus exactly the same as it was in Section 8. The same comparative statics conclusions on prices and quantities for the defined contribution system of Section 8 would therefore apply to the defined benefits system described here, no matter what the precise benefit rule.

# 12 Concluding Remarks

We have considered the implications of changing trust fund portfolio policy away from 100% government debt in a general equilibrium model with heterogeneous households and heterogeneous production. We found that a small amount of diversification raises total welfare (suitably defined), and causes welfare redistributions among household types. Social security diversification changes relative prices, and the composition and amount of real investment. With the assumed structure of marginal taxes, social security diversification raises the interest rate, thereby increasing the payoffs of the social security trust fund and helping workers, but forcing an increase in income taxes that hurts savers. There is almost always a wealth transfer between young savers and old savers, with the direction

[37]If a defined contribution system is to fulfill its social purpose, it will also need periodic change in response to economic and demographic developments.

depending on the change in the value of land. Diversification reduces real safe investment, but increases risky investment. The effect on aggregate real investment and long-term asset prices is ambiguous. The common sense conclusion that trust fund diversification would (if it did anything at all) increase real investment and increase stock market value is thus seen to be questionable, as is shown in the risky linear case and the safe linear case. In both cases one or the other of the common sense predictions is reversed.

The paper assumed that the technology is iid. This leaves out the effect of news about future technologies on current asset prices. This would be an interesting extension. Presumably this would make asset returns riskier and add to the social value of sharing risks more widely and so strengthen the case for investment in equities. The paper assumed that labor is a separable input from capital. Allowing changes in investment to change wages would have created another interesting redistribution.

There are four points to make relative to the current policy debate.[38] First, contrary to some assertions, the heterogeneity of the population implies that trust fund portfolio choice does have real effects on the economy. Second, while it is appropriate to be concerned about the risk associated with a change in portfolio policy, it seems to us unlikely that workers are so risk averse that a portfolio completely invested in Treasury bonds is optimal. This point is reinforced by the ability of the government to spread risk over successive cohorts since Social Security is a defined benefit system. That is, if a defined benefit system is well-run, there is a stronger case for trust fund investment in private securities than in the models analyzed here which assumed a defined contribution system. Third, the marginal social benefit to diversification declines as the level of diversification increases (exposing workers to more risk), which puts a limit on the amount of socially desirable diversification. Fourthly, the models considered here have substituted equity investment for bond investment, holding constant the level of funding of Social Security. Many proposals for investment in stocks, whether through the trust funds or through individual accounts, use stock investments as a reason to increase or decrease the financing of Social Security (at least in the short run) relative to what might be proposed without such investment (e. g., see Smetters, 1997). Such a change involves intergenerational redistribution that is not incorporated in the analysis in this paper, though it could be accommodated by this model. Our analysis does apply, however, to proposals that would substitute a portfolio change for cuts in future benefits.

# References


[1] **Abel, Andrew B.** "The Effects of Investing Social Security Funds, in the Stock Market When Fixed Costs Prevent Some Households from Holding Stocks," *American Economic Review*, March 2001, *91*(1): 128-48.

[2] **Advisory Council on Social Security**. *Report of the 1994-1996 Advisory Council on Social Security.* 1997. Washington D. C.

[3] **Ameriks, John, and Zeldes, Stephen P.** "How Do Household Portfolio Shares Vary with Age?" In progress.

[4] **Aura, Saku, Diamond, Peter and Geanakoplos, John.** "Savings in a Two-Period Two-Asset Model," Unpublished manuscript, 1999.

[5] **Bohn, Henning.** "Risk Sharing in a Stochastic Overlapping Generations Economy," Unpublished, University of California at Santa Barbara, 1997.


[38] For more discussion of diversification, see Munnell and Balduzzi (1998) and Diamond (1999).

[6] ____. "Social Security Reform and Financial Markets." In *Federal Reserve Bank of Boston Conference on Social Security Reform*, 1998 (41).

[7] ____. "Should the Social Security Trust Fund Hold Equities? An Intergenerational Welfare Analysis," *Review of Economic Dynamics,* July 1999: *2*(3), 666-697.

[8] **Diamond, Peter.** "Macroeconomic Aspects of Social Security Reform," *Brookings Papers on Economic Activity.* 1997, (2), 1-66.

[9] ____ (ed.). *Issues in Privatizing Social Security.* Cambridge, MA: MIT Press. 1999.

[10] **Diamond, Peter and Yaari, Menahem.** "Implications of the Theory of Rationing for Consumer Choice under Uncertainty," *American Economic Review*, June 1972, *62*(3), 333-343.

[11] **Feldstein, Martin.** "The Optimal Level of Social Security Benefits," *Quarterly Journal of Economics*, May 1985, *100*(2), 303-20.

[12] **Financial Economists' Roundtable.** "Statement on Social Security," Unpublished manuscript, 1998.

[13] **Fischer, Stanley.** "Assets, Contingent Commodities, and the Slutsky Equations," *Econometrica.* March 1972, *40*(2), 371–385.

[14] **Geanakoplos, John, Mitchell, Olivia S. and Zeldes, Stephen P.** "Would a Privatized Social Security System Really Pay a Higher Rate of Return?" In R. Douglas Arnold, Michael Graetz, and Alicia H. Munnell (eds.), *Framing the Social Security Debate.* Washington, DC: National Academy of Social Insurance, Brookings Institution Press, 1998.

[15] ____. "Social Security Money's Worth." In Olivia S. Mitchell, Robert Myers, and Howard Young (eds.), *Prospects for Social Security Reform.* Philadelphia: University of Pennsylvania Press, 1999.

[16] **Greenspan, Alan.** Testimony before the Task Force on Social Security of the Committee on the Budget, U.S. Senate, November 20, 1997.

[17] **Kennickell, Arthur, Starr-McCluer, Martha and Sunden, Annika E.** "Family Finances in the U.S.: Recent Evidence from the Survey of Consumer Finances." *Federal Reserve Bulletin*, *83*(1), January 1997.

[18] **Mitchell, Olivia S. and Moore, James F.** "Retirement Wealth Accumulation and Decumulation: New Developments and Outstanding Opportunities," Working Paper 97-12, The Wharton School, University of Pennsylvania, 1997.

[19] **Pestieau, Pierre, and Possen, Uri M.** "Investing Social Security in the Equity Market. Does it Make a Difference?" Unpublished manuscript, University of Liege, 1999.

[20] **Munnell, Alicia H., and Balduzzi, Pierluigi.** "Investing the Trust Fund in Equities," Unpublished manuscript, Boston College, 1998.

[21] **Smetters, Kent.** "Investing the Social Security Trust Fund in Equity: An Options Pricing Approach," Technical Paper 1997-1, Congressional Budget Office, Macroeconomic Analysis and Tax Analysis Divisions, 1997.

[22] **Social Security Administration.** *Annual Statistical Supplement to the Social Security Bulletin*, 1996.