DEFAULT AND PUNISHMENT IN GENERAL EQUILIBRIUM

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Default and Punishment in General Equilibrium

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Abstract

We extend the standard model of general equilibrium with incomplete markets to allow for default and punishment. The equilibrating variables include expected delivery rates, along with the usual prices of assets and commodities. By reinterpreting the variables, our model encompasses a broad range of adverse selection, and signalling phenomena (including the Akerlof lemons model and Rothschild–Stiglitz insurance model) and some moral hazard problems in a general equilibrium framework.

Despite earlier claims about the nonexistence of equilibrium with adverse selection, we show that equilibrium always exists.

We show that more lenient punishment which encourages default may be Pareto improving because it allows for better risk spreading.

We define an equilibrium refinement that requires expected delivery rates for untraded assets to be reasonably optimistic. Default, in conjunction with this refinement, opens the door to a theory of endogenous assets. The market can choose default penalties and quantity constraints on sellers.

Keywords: default, incomplete markets, adverse selection, moral hazard, equilibrium refinement, endogenous assets

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1 Introduction

There is a substantial amount of default in the American economy. At first glance this would seem to be a sign of disequilibrium, and to call for economic models that radically depart from the orthodox paradigm of general equilibrium and market clearing.

Indeed, general equilibrium theory has for the most part not made room for default. In the Arrow–Debreu model of general equilibrium with complete contingent markets (GE), and likewise in the general equilibrium model with incomplete markets (GEI), agents keep all their promises by assumption. More specifically, in the GE model, agents never promise to deliver more goods than they personally own. In the GEI model, the definition of equilibrium (that has been developed in a rapidly growing literature) allows agents to promise more of some goods than they themselves have, provided they are sure to get the difference elsewhere. Agents there too must honor their commitments, though no longer exclusively out of their own endowments. Each agent can keep his promises because other agents keep their promises to him.

We build a model that explicitly allows for default, but is broad enough to incorporate conventional general equilibrium theory as a special case. We call the model $GE(R, \lambda, Q)$ because each asset j is defined by its promise R_j, the penalty rate λ_j which determines the punishment for default on the promise, and the quantity restriction Q_j attendant on those who sell it.

Fixing exogenously the set $\mathcal{A}$ of tradeable assets,

$$\mathcal{A} = \{(R_j, \lambda_j, Q_j) : (R_j, \lambda_j, Q_j) \text{ is tradeable}\},$$

we solve for equilibrium $E(\mathcal{A})$. The equilibrating variables include anticipated delivery rates on assets, along with the usual prices of assets and commodities.

One of the central features of our model is that assets are thought of as pools. Different sellers of the same asset will typically default in different events, and in different proportions. The buyers of the asset receive a pro rata share of all the different sellers' deliveries, just as an investor does today in the securitized mortgage market, or in the securitized credit card market. Pooling drastically reduces the information processing and transactions costs of trading assets, which explains its increasing prevalence in modern economies.

Just as buyers of commodities are assumed in perfect competition to regard prices as fixed, so we assume that buyers of assets regard default rates as fixed. Our general equilibrium model thus stands in contrast to models in which a single lender and a single borrower negotiate with each other. We have avoided a finite-player, game-theoretic treatment of default because, for the massive anonymous financial markets on which we focus attention, perfect competition is a better approximation to reality, and much more analytically tractable.

We have also avoided a (perfectly competitive but) partial equilibrium treatment of our subject because we wanted to evaluate the system-wide consequences of default. In a world in which promises can exceed physical endowments, each default can begin a chain reaction. A creditor in one market where payment does not occur is deprived of the means of delivery in another market where he is the debtor, thereby causing a further default in some other market, etc. The indirect effects of default might be as important as the direct effects, but they are missed in partial equilibrium models. We emphasize that these chain reactions occur exclusively in economies with intermediate levels of financial development, such as the system now in place in the United States. Once the asset markets become complete, the system of interlocking debts will be broken, as in the GE model, and no chain reactions will occur.

Another central feature of our model is that the subset $\mathcal{A}^* \equiv \mathcal{A}^*(E(\mathcal{A})) \subset \mathcal{A}$ of actively traded assets

$$\mathcal{A}^* = \{(R_j, \lambda_j, Q_j) \in \mathcal{A} : (R_j, \lambda_j, Q_j) \text{ is positively traded in } E(\mathcal{A})\}$$

also emerges in equilibrium. The promises, penalties, and sales limitations corresponding to actively traded assets can thus themselves be regarded as endogenous.

A crucial role in the endogenous determination of asset trade is played by the expectations agents have over the deliveries of assets that are not positively traded. (In game theoretic terms, this is analogous to beliefs off the equilibrium path.) We fix these expectations for non-traded assets at reasonable levels by a straightforward equilibrium refinement. The idea is to introduce an external ε-agent who sells ε units of each asset and fully delivers, and to take the limit as $\varepsilon \to 0$. This rules out irrational pessimism on expected deliveries from untraded assets. The simplicity of the refinement is due to our hypothesis of perfect competition consistently applied.[1]

Endogenous default necessarily involves adverse selection and moral hazard. Indeed we note that the adverse selection and signalling phenomena described by Akerlof (1972), Spence (1973), and Rothschild and Stiglitz (1976) can all be captured in our perfectly competitive framework.

Our first goal is to show that if agents have the mental powers to anticipate future rates of default (contingent on future events), just as they are presumed by conventional equilibrium theory to have the mental powers to anticipate future prices (contingent on future events), then default is consistent with the orderly function of markets. In Section 5 we prove the existence of equilibrium with default under exactly the same conditions necessary to prove the existence of equilibrium in the GEI model (where default is ruled out by assumption.) More precisely, we show that our refined equilibrium $E(\mathcal{A})$ exists for every collection $\mathcal{A}$ of assets (R, λ, Q) for which $Q < \infty$, or for which $Q = \infty$ but the promises R are all paid in the same numeraire.

Our second goal is to give a purely economic explanation for lenient default penalties. When markets are incomplete, default allows agents to tailor-make promises into deliveries that suit them best. In effect they can replace the given assets by more appropriate assets. Second, the span of the asset deliveries can be made much larger than the span of the asset promises, since a single given asset can be made into as many different assets as there are sellers, if different sellers default differently on the same promises.

[1] To the best of our knowledge, this appears to be the first analogue of the "trembling hand" refinement of game theory in perfectly competitive equilibrium.

Our final goal is to show that even in a world without trading costs, assets are endogenous. Whereas in GEI the selection of assets is usually regarded as outside the model, here we can resolve the asset selection problem by focusing on the endogenous determination of positively traded assets $\mathcal{A}^*$. Typically in GEI, every (nonredundant) asset is actively traded, so $\mathcal{A} = \mathcal{A}^*$. However, in equilibrium with default, there will typically be many assets in $\mathcal{A}\backslash\mathcal{A}^*$ which are priced by the market, but neither bought nor sold.[2] The reason is that with default, the sale of an asset is not the negative of its purchase. The buyer receives only what is delivered, but the seller gives up in addition penalties for what is not delivered. The marginal utility of buying may thus be strictly less than the marginal disutility of selling, leaving room for a price in between at which no agent will want to buy or sell.

Recall that each asset (R_j, λ_j, Q_j) is characterized by three dimensions. If the set $\mathcal{A}$ of available assets is comprehensive (i.e., all conceivable levels and combinations of the three asset dimensions are present in $\mathcal{A}$), then we prove in Section 9 that $\mathcal{A}^*$ will in effect select the Arrowian levels: completely spanning promises, with infinite penalties, and nonbinding quantity constraints. On the other hand, if two of the dimensions in $\mathcal{A}$ are *exogenously* restricted away from their Arrowian levels, then the forces of supply and demand will *endogenously* select the levels in the remaining dimensions in $\mathcal{A}^*$ to be far from Arrowian, as we show in Sections 10 and 11.

For example, suppose promises and quantity constraints are fixed exogenously as in Section 7, where we showed that optimal penalties *should* be intermediate. We can ask how harsh the penalties *will* be that endogenously emerge in $\mathcal{A}^*$. We find that the forces of supply and demand do select a unique penalty, which in the example turns out to be optimal.

We show that if promises and penalties are fixed exogenously in a particular way, our model includes the insurance contracts of Akerlof (1972) and Rothschild–Stiglitz (1976). In that case $\mathcal{A}^*$ endogenously selects quantity limits Q_j. This enables us to show how the phenomenon of signalling can be treated in perfect competition, moreover without jeopardizing the existence of equilibrium.

2 Default in Equilibrium: The $GE(R, \lambda, Q)$ Model

2.1 The Economy

As in the canonical model of general equilibrium with incomplete markets (GEI), we consider a two-period economy, where agents know the present but face an uncertain future. In period 0 (the present) there is just one state of nature (called state 0), in which H agents trade in L commodities and J assets. Then chance moves and selects one of S states which occur in period 1 (the future). Commodity trades take place again, and assets pay off. The difference from GEI is that in our $GE(R, \lambda, Q)$ model, assets pay off in accordance with what agents opt to deliver. Our notation for the exogenous variables is:

[2] In some applications we might choose to limit $\mathcal{A}$ exogenously; the point is that even if $\mathcal{A}$ is inclusive, $\mathcal{A}^*$ will still be limited.

$\ell \in L = \{1, ..., L\}$ = set of commodities
$s \in S = \{1, ..., S\}$ = set of states in period 1
$S^* = \{0\} \cup S$ = set of all states
$h \in H = \{1, ..., H\}$ = set of agents
$e^h \in \mathbb{R}_+^{S^* \times L}$ = initial endowment of agent h
$j \in J = \{1, ..., J\}$ = set of assets
$R_j \in \mathbb{R}_+^{S \times L}$ = promises per unit of asset j of each commodity $\ell \in L$ in each state $s \in S$
$u^h : \mathbb{R}_+^{S^* \times L} \to \mathbb{R}$ = utility function of agent h
$\lambda_{sj}^h \in \overline{\mathbb{R}}_+ \equiv \mathbb{R}_+ \cup \{\infty\}$ = real default penalty on agent h for asset j in state s
$Q_j^h \in \mathbb{R}_+$ = bound on sale of asset j by agent h

We assume that no agent has the null endowment, and that all named commodities are present in the aggregate, i.e.,

$$e_s^h = (e_{s1}^h, ..., e_{sL}^h) \neq 0$$

for all $h \in H$ and $s \in S^*$, and

$$e_{s\ell} = \sum_{h \in H} e_{s\ell}^h > 0$$

for all $s\ell \in S^* \times L$. Also each u^h is continuous, concave and strictly increasing in each of its $S^* \times L$ variables. Having assumed strict monotonicity and concavity, there is no further loss of generality in assuming that $u^h(x) \to \infty$ whenever $||x||_\infty \to \infty$.[3]

We can visualize the state space as a simple tree:

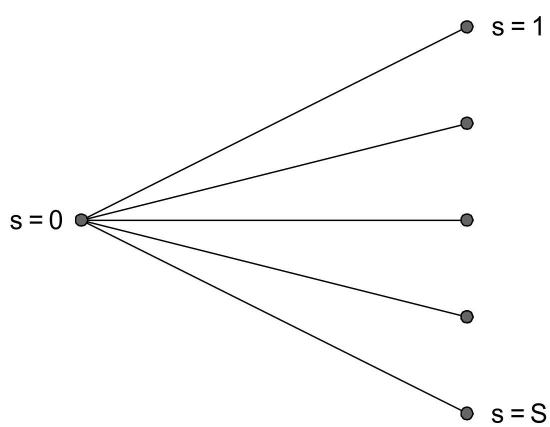

Figure 1

Agents h have heterogeneous, state-dependent endowments $e_s^h \in \mathbb{R}_+^L$ and disutilities of default λ_{sj}^h.

[3] Let $\Box = \{x \in \mathbb{R}_+^{S^* L} : ||x||_\infty \leq 2||\sum_h e^h||_\infty\}$. Let $\mathcal{L}$ be the set of affine functions $L : \mathbb{R}_+^{S^* L} \to \mathbb{R}$ such that $L(x) \geq u^h(x)$ for all $x \in \Box$. Define $\tilde{u}^h(x) \equiv \inf_{L \in \mathcal{L}} L(x)$. Then equilibrium with u^h and $\tilde{u}^h$ coincide, and $\tilde{u}^h$ has the desired properties.

Adverse selection enters the picture because agents have different endowments out of which to keep their promises, and also different disutilities of default.

Promises must be of a limited kind $j \in J$ fixed a priori. A promise $j \in J$ specifies bundles of goods (or services) to be delivered in each state:

$$\text{Promise } R_j = \begin{pmatrix} \\ \\ \\ \\ \end{pmatrix} \begin{matrix} \} - \text{state 1 goods} \\ \} - \text{state 2 goods} \\ \\ \} - \text{state } S \text{ goods.} \end{matrix}$$

Agents h make promises by selling various quantities φ_j^h of each asset j. An agent's ability to keep a promise depends on how many promises he sells, both of the same kind j, and of other kinds $j' \neq j$. Moral hazard enters the picture, since a buyer of an asset (i.e., lender) does not know which other promises the seller (i.e., borrower) has made, and because borrowers have the option to default.

Each kind of asset prescribes a limit on its sale, $\varphi_j^h \leq Q_j^h$. Limits on sales of promises are necessary to any realistic model of credit.[4] If $Q_j^h = 0$, then agent h is essentially forbidden from selling asset j. If the limits Q_j^h are very large, they may be entirely irrelevant, as they mostly are in the examples of Sections 7 and 8.[5] But if they are small, then they may be used as a signal that the sellers are not making many promises, and hence that the promises are reliable. We explore signalling in example B1.

An economy is defined as a vector

$$\mathcal{E} = \left((u^h, e^h)_{h \in H}, \ \left(R_j, ((\lambda_{sj}^h)_{s \in S}, Q_j^h)_{h \in H} \right)_{j \in J} \right).$$

Note again that an asset consists of promises, penalties for default, and limits on sales.

2.2 Equilibrium

2.2.1 Macro Variables and Individual Choice Variables

Let us define

$p \in \mathbb{R}_{++}^{S^* \times L}$ = commodity prices
$\pi \in \mathbb{R}_{+}^{J}$ = asset prices
$K \in [0,1]^{S \times J}$ = expected delivery *rates* on assets
$x^h \in \mathbb{R}_{+}^{S^* \times L}$ = consumption of h
$\theta^h \in \mathbb{R}_{+}^{J}$ = asset purchases of h
$\varphi^h \in \mathbb{R}_{+}^{J}$ = asset sales of h
$D^h \in \mathbb{R}_{+}^{(S \times L) \times J}$ = deliveries by agent h on asset $j \in J$

[4] Evidence abounds that finite bounds are always imposed in the extension of credit. Even the best "name" among borrowers has a limited credit line.

[5] In Section 5 we are able to prove the existence of equilibrium even when $Q_j^h = \infty$, provided $\lambda \gg 0$ and the R_j all deliver in the same good.

In conventional general equilibrium theory, market prices (p, π) convey all relevant information (trade is anonymous).

The possibility of default forces us add delivery rates K as macro variables

In our model, an agent h buys θ_j^h shares of asset pool j and sells φ_j^h promises into the same pool. Different sellers h and $\tilde{h}$ may deliver differently, $D_j^h \neq D_j^{\tilde{h}}$, but their promises cannot be distinguished. We suppose that buyers and sellers do not trade bilaterally, but through the anonymous pool. The buyers (shareholders) of pool j receive a pro rata share of all its different sellers' deliveries. Each share of pool j delivers the fraction

$$K_{sj} = \frac{\sum_{h \in H} p_s \cdot D_{sj}^h}{\sum_{h \in H} p_s \cdot R_{sj} \varphi_j^h}$$

of its promises $p_s \cdot R_{sj}$ in state s. The shareholder of pool j does not know, or need to know, the identities of the sellers or the quantities of their sales. All that matters to him is the price π_j of the share and the delivery rate K_{sj}.

Pooling dramatically reduces the information needed to buy a diversified portfolio of risks: instead of forecasting individual deliveries K_{sj}^h for many different individuals h, a buyer need only concern himself with a single average delivery K_{sj}. Figuring out K_{sj}^h for one individual is typically no less difficult than estimating K_{sj} for a pool with a large population. Thus pooling overcomes the costly information processing problems inherent in multiple bilateral negotiations, and is one reason why it is becoming so prevalent in modern economies.

The pooling also leads to *adverse selection*, since a buyer must worry that unreliable sellers with a proclivity for lower deliveries will tend to sell more promises into the pool, worsening the anticipated rate K_{sj}. *Signalling*, by publicly committing oneself to a small quantity of sales, therefore has an important role to play, because it suggests to the buyer that deliveries may be more reliable. To incorporate it in our model, we suppose that there are many pools j, each with its own quantity limit Q_j imposed on sales into the pool. This opens up the opportunity for agents to signal their restraint by selling into a pool with low Q_j.

By enabling each agent to trade anonymously as part of a large aggregate, pooling already takes us part of the way toward perfect competition. We fully get there by postulating that all agents view (p, π, K) as fixed. Perfect competition thus further reduces the information a buyer requires: there is no need for him to forecast how the delivery rate at any pool would vary if the price were changed, since he can't change the price.

The terms $(R_j, ((\lambda_{sj}^h)_{s \in S}, Q_j^h)_{h \in H})$ of pool j are set exogenously, just as the location, date, and quality of a commodity are in traditional general equilibrium theory. The prices π_j, the anticipated delivery rates K_{sj}, and the trades at each pool j are all determined endogenously at equilibrium by the market forces of supply and demand.

2.2.2 Household Budget and Payoff

The *budget set* $B^h(p, \pi, K)$ of agent h is given by:

$$B^h(p,\pi,K) = \Big\{ (x,\theta,\varphi,D) \in \mathbb{R}_+^{S^*\times L} \times \mathbb{R}_+^J \times \mathbb{R}_+^J \times \mathbb{R}_+^{J\times S\times L} :$$
$$p_0 \cdot (x_0 - e_0^h) + \pi \cdot (\theta - \varphi) \leq 0;\ \varphi_j \leq Q_j^h \text{ for } j \in J;\ \text{and, } \forall s \in S,$$
$$p_s \cdot (x_s - e_s^h) + \sum_{j\in J} p_s \cdot D_{sj} \leq \sum_{j\in J} \theta_j K_{sj} p_s \cdot R_{sj} \Big\}$$

The budget set allows agent h to deliver whatever he pleases. On the other hand, the agent expects to receive a fraction K_{sj} of the promises made to him on asset j in state s. The first constraint says that agent h cannot spend more on purchases of commodities x_0 and assets θ than the revenue he receives from the sale of commodities e_0^h and assets φ. Moreover he can never sell more than Q_j^h of any asset j. The second constraint applies separately in each state $s \in S$. It says that agent h cannot spend more on the purchase of commodities x_s and asset deliveries $\sum_j D_{sj}$ in state s than the revenue he gets in state s from commodity sales e_s^h and asset receipts $\sum_j \theta_j K_{sj} p_s R_{sj}$.

The only reason that agents deliver anything on their promises is that they feel a disutility λ_{sj}^h from defaulting. The payoff of (x, θ, φ, D) given prices p, to agent h is

$$w^h(x,\theta,\varphi,D,p) = u^h(x) - \sum_{j\in J} \sum_{s\in S} \frac{\lambda_{sj}^h [\varphi_j p_s \cdot R_{sj} - p_s \cdot D_{sj}]_+}{p_s \cdot v_s}.$$

where $v_s \in \mathbb{R}_+^L$ is exogenously specified with $v_s \neq 0$. Note that $[\varphi_j p_s \cdot R_{sj} - p_s \cdot D_{sj}]^+ \equiv \max\{0, \varphi_j p_s \cdot R_{sj} - p_s \cdot D_{sj}\}$ is exactly the money value of the default of h on his promise to deliver on asset j in state s.

3 Default Penalties

Once we allow for default it is evident that society has much to gain from punishing those agents who fail to keep their promises. In a multiperiod world, market forces themselves might provide some incentive to keep promises, since agents who acquired a bad reputation for previous defaults might find it more difficult to obtain new loans. Collateral is also a very important device for guaranteeing at least partial payment (see Geanakoplos, 1997); but here we ignore it. For reasons of simplicity and tractability, we confine attention to a two period model with exogenously specified default penalties which are increasing in the size of the default. These penalties might be interpreted as the sum of third party punishment such as prison terms, pangs of conscience, (unmodeled) reputation losses, and (unmodeled) garnishing of future income.

Default in our model can either be strategic or due to ill fortune. Penalties are imposed on agents who fail to deliver, whatever the cause. Debtors choose whether to repay or to bear the penalty for defaulting; creditors cannot observe why default occurs. Agents who have no resources to repay will be punished as severely as they would if they had the resources but chose not to repay.[6] The consequences of default penalties are therefore two-fold: they tend to induce agents to keep promises when they are able, and they tend to discourage agents from making promises that they know in advance they will not always be able to keep.

Although in practice the severity of the penalty (e.g., a felony vs. a misdemeanor) depends on the nominal amount, and that is only adjusted slowly in the face of inflation, we suppose the adjustment is instantaneous, so that the penalties depend on the "real" default. Accordingly, we divide nominal defaults by the market price in state s of a fixed basket of goods v_s.

Notice that the budget set is convex, and the payoff function w^h is concave, in the household choice variables (x, θ, φ, D). Had we expressed these choices with other (apparently natural) variables, such as $\delta^h_{sj} \equiv$ delivery per unit promised, the budget set would no longer be convex, nor would w^h be concave.

It is worth noting a *scaling property* of the budget set (which is immediate from its definition and the fact that $e^h_s \neq 0$ and $p_s \gg 0$ for all $s \in S^*$): $(x, \theta, \varphi, D) \in B^h(p, \pi, K)$ and $0 < \alpha < 1 \Rightarrow (\alpha x, \alpha\theta, \alpha\varphi, \alpha D) \in B^h(p', \pi', K')$ for all (p', π', K') sufficiently close to (p, π, K). This property will often be useful to us.[7]

For simplicity (and for the facility of doing comparative statics) we have taken the default penalty to be linear and separable in the amount of default.[8] But we can easily accommodate more general payoffs w^h which allow for the marginal rate of substitution between goods to depend on the level of default. All that is needed for Theorem 1 is the continuity and concavity of w^h. For Theorem 2 we need to assume, in addition, that given any x, $w^h(x, \theta, \varphi, D, p) < u^h(e^h)$ if the default in any state, on any asset, is sufficiently large.

One could easily imagine a legal system that imposes penalties that are discontinuous in the size of the default, for example trigger penalties that jump to a minimum level at the first infinitesimal default. Our model does not explicitly allow for these possibilities. But as we show in our working paper [—], with a continuum of households, such modifications to the default penalties do not destroy the existence of equilibrium.

We shall also analyze default with netting, transactions costs, and confiscation, which make for nonconvex budget sets, or nonconcave payoffs. Then it becomes necessary to introduce a continuum of households. We defer this discussion to Sections 6, 8, and 12.

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[6] In our model default penalties do not distinguish fraud from ill fortune. In reality they are hard to separate, but ever since Las siete Partitas of Don Alfonso X "the wise," bankruptcy law has sought to distinguish them.

[7] An alternative scaling property, also satisfied by the budget set, is obtained if we replace $(\alpha x, \alpha\theta, \alpha\varphi, \alpha D)$ with $(\alpha x, \alpha\theta, \varphi, \alpha D)$. Our entire analysis remains intact with this version of scaling.

[8] See Shubik–Wilson (1977).

3.0.3 Market Clearing

We are now in a position to define a $GE(R, \lambda, Q)$ equilibrium. It is a list $\langle p, \pi, K, (x^h, \theta^h, \varphi^h, D^h)_{h \in H}\rangle$ such that (1) to (4) below hold.

(1) For $h \in H$, $(x^h, \theta^h, \varphi^h, D^h) \in \arg\max w^h(x, \theta, \varphi, D, p)$ over $B^h(p, \pi, K)$

(2) $\sum_{h \in H}(x^h - e^h) = 0$

(3) $\sum_{h \in H}(\theta^h - \varphi^h) = 0$

(4) $K_{sj} = \begin{cases} \sum_{h \in H} p_s \cdot D^h_{sj} / \sum_{h \in H} p_s \cdot R_{sj}\varphi^h_j, & \text{if} \quad \sum_{h \in H} p_s \cdot R_{sj}\varphi^h_j > 0 \\ \text{arbitrary}, & \text{if} \quad \sum_{h \in H} p_s \cdot R_{sj}\varphi^h_j = 0 \end{cases}$

Condition (1) says that all agents optimize; (2) and (3) require commodity and asset markets to clear. Condition (4), together with the definition of the budget set, says that each potential lender (i.e., buyer) of an asset is correct in his expectation about the fraction of promises that do in fact get delivered. Moreover, his expectation $K^h_{sj} = K_{sj}$ of the rate of delivery does not depend on anything he does himself; in particular, it does not depend on the amount θ^h_j he loans (i.e., purchases) of the asset. Every lender gets the same rate of delivery.

Since heterogeneous borrowers may be selling the same asset, the realized rate of delivery K_{sj} is an average of the rates of delivery of each of the borrowers, weighted by the quantity of their sales. It might well happen that those borrowers with the highest rates of default are selling most of the asset, and this is the adverse selection and moral hazard that rational lenders must forecast.

We believe that our definition of $GE(R, \lambda, Q)$ equilibrium embodies the spirit of perfect, anonymous competition, and represents a significant fraction of the mass asset markets of a modern enterprise economy.

In the next sections we investigate the properties of equilibrium.

3.1 Untraded Assets

It is a curious fact that many of the large asset markets that our model seeks to describe have been initiated not by entrepreneurs but by government intervention. The government, for example, began the GNMA mortgage program by guaranteeing delivery on the promises of all borrowers eligible for the program (but not the timing[9] of delivery). It is likely, however, that these mortgage markets would function smoothly even without government guarantees. Private companies indeed do sell insurance on non-GNMA mortgages. A reasonable question to ask is why the pass through mortgage market did not begin on its own?

One possible explanation is provided by our model. When assets are traded, expected deliveries K_{sj} must be equal to actual deliveries. Expectations cannot therefore be unduly pessimistic. But for assets that are not traded, our model makes no

[9] A default induces the government to prepay the loan immediately, even if the lender would prefer the scheduled payments.

assumption about expectations of delivery (see (4)). In the real world, investors with no experience in observing default rates might tend to overestimate their probability. This can create serious problems, in practice as in our model. In the model, so far, there is nothing to stop the expectations from being absurdly pessimistic, which in turn will support trivial equilibria with no trade in the asset. The point is easily seen by a simple example. Consider an equilibrium of an economy in which certain assets are missing. Introduce these new assets j but choose their prices π_j close to zero. Then no agent will be willing to sell them, for he gets very little in exchange, but undertakes a relatively large obligation either to deliver commodities or to pay default penalties. Also choose the K_{sj} to be positive but even smaller. Then in spite of their low price, no agent will be willing to buy the assets since he expects them to deliver virtually nothing. Thus we have obtained trivial equilibria in which there is no trade of the new assets on account of arbitrarily pessimistic expectations regarding their deliveries.[10]

We believe that unreasonable pessimism prevents many real world markets from opening, and provides an important role for government intervention. But it is interesting to study equilibrium in which expectations are always reasonably optimistic. It is of central importance for us to understand which markets are open and which are not, and we do not want our answer to depend on the agents' whimsical pessimism.

3.2 Refined Equilibrium

Expectations for deliveries by assets that are not traded are analogous to beliefs in game theory "off the equilibrium path." Selten (1975) dealt with the game theory problem by forcing every agent to tremble and play all his strategies with probability at least $\varepsilon > 0$, and then letting $\varepsilon \to 0$. We shall also invoke a tremble, but in quite a different spirit. Our tremble will be "on the market" and not on households' (players') strategies. Indeed, no household could tremble the way we want: we introduce an external player who delivers more per unit than any of the real households.[11] This extraordinary delivery is what banishes whimsical pessimism.

Consider external ε-agent who sells and buys $\varepsilon = (\varepsilon_j)_{j \in J} \gg 0$ of every asset, and fully delivers on his promises. (One might interpret this agent as a government which guarantees delivery on the first infinitesimal promises.)

An equilibrium $E(\varepsilon)$ obtained with the ε-agent is called an ε-trembling hand equilibrium. Thus any such $E(\varepsilon) = \langle p(\varepsilon), \pi(\varepsilon), K(\varepsilon), (x^h(\varepsilon), \theta^h(\varepsilon), \varphi^h(\varepsilon), D^h(\varepsilon))_{h \in H} \rangle$ must satisfy:

[10] In the strategic market games literature it has been observed that markets can be arbitrarily shut because each agent expects that no other agent will go there, and hence does not go himself. But those markets are truly activated simply by announcing any price there: a low price will attract buyers and a high price will attract sellers. Except for degenerate cases, no price will eliminate all buying and selling. But as we saw in our thought example, it is *always* possible to announce (π, K) in such a way that eliminates all buying and selling. The purpose of our refinement is to pin down appropriate levels of (π, K). In strategic market games an external agent was introduced whose sole purpose was to trade. In this paper, the external agent must trade and deliver *fully*.

[11] Were we to invoke a tremble on strategies, e.g., forcing each household t to contribute $\varepsilon > 0$ to every pool, this would *not* meet our needs.

(1*) $(x^h(\varepsilon), \theta^h(\varepsilon), \varphi^h(\varepsilon), D^h(\varepsilon)) \in \arg\max w^h(x, \theta, \varphi, D, p(\varepsilon))$ over $B^h(p(\varepsilon), \pi(\varepsilon), K(\varepsilon))$

(2*) $$\sum_{h\in H}(x_s^h(\varepsilon) - e_s^h) = \begin{cases} 0 & \text{if } s = 0 \\ \sum_{j\in J} \varepsilon_j (1 - K_{sj}(\varepsilon)) R_{sj} & \text{if } s \in S \end{cases}$$

(3*) $$\sum_{h\in H}(\theta^h(\varepsilon) - \varphi^h(\varepsilon)) = 0$$

(4*) $$K_{sj}(\varepsilon) = \begin{cases} \dfrac{p_s(\varepsilon)\cdot R_{sj}\varepsilon_j + \sum_{h\in H} p_s(\varepsilon)\cdot D_{sj}^h(\varepsilon)}{p_s(\varepsilon)\cdot R_{sj}\varepsilon_j + \sum_{h\in H} p_s(\varepsilon)\cdot R_{sj}\varphi_j^h(\varepsilon)} & \text{if denominator} > 0\ (R_{sj} \neq 0) \\ 1 & \text{otherwise } (R_{sj} = 0) \end{cases}$$

Since the ε-agent buys and sells ε_j units of each asset j, asset market clearing (3*) is as before. But since he delivers fully $\varepsilon_j R_{sj}$ on his promises, and gets delivered only $\varepsilon_j K_{sj}(\varepsilon) R_{sj}$, on net he injects the vector of commodities $\sum_{j\in J} \varepsilon_j(1-K_{sj}(\varepsilon))R_{sj}$ into the economy in each state $s \in S$. This explains (2*). Finally, condition (4*) says that delivery rates must recognize the external agent. (The condition when denominator $= 0$ is now reduced to the trivial case when promises $p_s(\varepsilon)\cdot R_{sj} = 0$, hence when $K_{sj}(\varepsilon)$ is irrelevant.)

An $E = \langle p, \pi, K, (x^h, \theta^h, \varphi, D)_{h\in H}\rangle$ is called a trembling hand equilibrium if there exists a sequence of ε-trembling hand equilibria $E(\varepsilon)$ with $\varepsilon \to 0$ and $E(\varepsilon) \to E$.

Notice that the external agent boosts the delivery rate $K_{sj}(\varepsilon)$ above the level achieved by the real agents $h \in H$ in the ε-economy (unless they too are fully delivering, or not selling). As $\varepsilon \to 0$, this boost disappears for assets that are positively traded in the limit. But if $\varepsilon_j / \sum_{h\in H} \varphi_j^h(\varepsilon)$ does not go to zero and $\sum_{h\in H} \varphi_j^h(\varepsilon) > 0$ for all ε, the limiting rates K_{sj} will be boosted (unless there is no default by the real agents).

We could have imagined an external agent who delivers only 70% of his promises, instead of 100%. It is clear that any "100% refined equilibrium allocation" is a "70% refined equilibrium allocation," thus explaining why our choice of 100% deliveries gives the sharpest refinement.

In our definition of equilibrium, there is one price for each asset, including those assets that are not traded. There is therefore no possibility for price taking agents to offer different prices with the hope of luring a better selection of sellers.[12] We

[12] Putting the matter still differently, we regard an asset or contract as setting out the obligations of the seller, including the penalties if he fails to deliver, and the quantity limitations on his other sales. The price of the contract is set by competition between sellers and buyers, that is, by the market. Agents need only think about one prevailing price for each contract. In the Rothschild–Stiglitz view, the price is one of the terms of the contract. In this view, there is no such thing as a single contract; there are as many contracts as there are prices. Notice also that the Rothschild–Stiglitz view must regard market clearing as one of rationing. At most prices, the contract will not be traded, because *either* supply or demand is zero, and the other side of the market is rationed. This point of view has been admirably expressed by Gale. In our view competitive equilibrium should be defined by a single price at which both supply and demand are equal (possibly both zero), as long as expectations at that price are set at rational levels.

can interpret this situation in several ways. Suppose that buyers are aware of the composition of sales at the market prices, and perhaps of the composition of sales at prices a penny off from market prices. But they lack the knowledge or computing power to infer what the composition would be at prices far from market. They might therefore presume they will get the same selection of sellers no matter what price they quote, giving them no incentive to deviate from market prices. Alternatively, a buyer might understand full well the composition of sales, but he should make the cautious assumption that he alone cannot serve all the potential sellers, and that he is likely to be reached first by the sellers who are most anxious to sell, that is by the sellers who have the lowest reservation price for the asset. Thus again the buyer expects the same selection of sellers as elicited by the market price, and is left with no incentive to deviate from the market price.

In Sections 10 and 11, on the endogeneity of the asset structure, and in our sequel paper, we show that the equilibrium refinement plays a crucial role in determining whether an asset j is positively traded ($j \in \mathcal{A}^*$) or not ($j \in \mathcal{A} \backslash \mathcal{A}^*$).

3.3 Agents on the Verge of Trading

Suppose utilities u^h are differentiable, and suppose at a refined equilibrium $(p, \pi, K, (x^h, \theta^h, \varphi^h, D^h)_{h \in H})$ $p_s \cdot x_s^h > 0$ for all $h \in H$ and $s \geq 1$. We can define the marginal utility of money in state s to each agent h by $\mu_s^h = [\partial u^h(x^h)/\partial x_{s\ell}]/p_{s\ell}$ for any ℓ with $x_{s\ell}^h > 0$. The marginal utility of h of purchasing any asset j is then

$$MU_j^h = \sum_{s=1}^{S} \mu_s^h K_{sj} p_s \cdot R_{sj}$$

and the marginal disutility of selling asset j is

$$MDU_j^h = \sum_{s=1}^{S} p_{sj} \cdot R_{sj} \min \left\{ \frac{\lambda_{sj}^h}{p_s \cdot v_s}, \mu_s^h \right\}$$

An agent is said to be on the verge of buying (selling) asset j if he is not buying (selling) it, but would do so if the price π_j were ever so slightly lowered (raised):

$$\begin{array}{ll} \text{verge of buying:} & \pi_j = MU_j^h/\mu_0^h \\ \text{verge of selling:} & \pi_j = MDU_j^h/\mu_0^h \end{array}$$

If, in the refined equilibrium, $p_s \cdot R_{sj} > 0$ and $K_{sj} < 1$, then we know that in the perturbation some agent h was actually selling j and not fully delivering in state s (otherwise $K_{sj} = 1$ on account of the external agent). It also follows that some (other) agent was buying j (since markets clear in the perturbation and the external agent buys and sells the same amount of asset j). Passing to the limit, we conclude that at a refined equilibrium

$$\pi_j = \max_h \{MU_j^h/\mu_0^h\} = \min_h \{MDU_j^h/\mu_0^j\}$$

with all j with $0 < K_{sj} p_s \cdot R_{sj} < p_s \cdot R_{sj}$ for some $s \geq 1$.

In particular, it follows that our refinement uniquely specifies π_j (from the verge of selling condition) once we know all the p_s and x_s^h. Furthermore, if in addition we knew a priori the ratios $K_{sj}/K_{s'j}$ (for example $K_{sj} = K_{s'j}$) for all $s \geq 1$, then from the verge of buying equality we could deduce all the levels K_{sj} as well.

With active default, $K_{sj} < 1$ and so we may well have that $K_{sj}\mu_s^h < \lambda_{sj}^h / p_s \cdot v_s < \mu_s^h$ and so $MU_j^h < MDU_j^h$, for all $h \in H$. Thus there may be room in between for a price π_j at which no agent would want to buy or sell asset j.

4 The Orderly Function of Markets with Default

Our first goal in this paper is to establish that default is completely consistent with the orderly function of markets. To that end we prove that under fairly general conditions, refined equilibrium always exists in our model.

The universal existence of equilibrium is somewhat surprising because of the historical tendency to associate default with disequilibrium (or more accurately, to make full delivery part of the definition of equilibrium), as we have already remarked. Furthermore, endogeneity of the asset payoff structure is known to complicate the existence of equilibrium with incomplete markets. But we show that no new existence problems arise from the endogeneity of the asset payoffs due to default.

The universal existence of equilibrium with default is also surprising because the pioneering papers placing adverse selection in a model of competition, by Akerlof (1972) on the market for lemons, and Rothschild and Stiglitz (1976) on insurance markets, purportedly showed that adverse selection is quite commonly inconsistent with equilibrium. We discuss Rothschild–Stiglitz in example B1.

We are now ready to state our main theorem, which is that $GE(R, \lambda, Q)$ equilibrium always exists, even if we insist on the equilibrium refinement discussed in Section 2.4.

Theorem 1 *For any $\lambda \in \overline{\mathbb{R}}_+^{HSJ}$ and $Q \in \mathbb{R}_+^{HJ}$, a trembling hand $GE(R, \lambda, Q)$ equilibrium exists, where $\overline{\mathbb{R}}_+ = \mathbb{R}_+ \cup \infty$.*

Proof Suppose first that penalties are finite, $\lambda \in \mathbb{R}_+^{HSJ}$. Fix a tremble $\varepsilon = (\varepsilon_j)_{j \in J} \gg 0$. For any small lower bound $b > 0$, define

$$\Delta_b = \Bigg\{ (p, \pi) \in \mathbb{R}_+^{S^* \times L} \times \mathbb{R}_+^J : \sum_{\ell=1}^{L} p_{s\ell} = 1 \; \forall s \in S^*,$$

$$b \leq p_{s\ell} \; \forall s\ell \in S^* \times L, \text{ and } 0 \leq \pi_j \leq \tfrac{1}{b} \; \forall j \in J \Bigg\}.$$

Choose M large enough to ensure that: $||x||_\infty > M \Rightarrow u^h(x) > u^h(2 \sum_{h' \in H} e^{h'})$ for all $h \in H$. (By assumption, $u^h(x) \to \infty$ as $||x|| \to \infty$, so such an M exists.) Now define, for each $h \in H$, $\Box^h = \{(x, \theta, \varphi, D) \in \mathbb{R}_+^{S^* \times L} \times \mathbb{R}_+^J \times \mathbb{R}_+^J \times \mathbb{R}_+^{SLJ} : \|x\|_\infty \leq M,\ \theta_j \leq 2 \sum_{h' \in H} Q_j^{h'},\ \varphi_j^h \leq Q_j^h$, and $||D||_\infty \leq ||Q||_\infty ||R||_\infty\}$. Let $\Box^H \equiv \times_{h \in H} \Box^h$.

Denote $\eta \equiv (p, \pi, K, (x^h, \theta^h, \varphi^h, D^h)_{h \in H}) \in \Delta_b \times [0,1]^{S \times J} \times \Box^H \equiv \Omega_b$.

Consider the map $\bar{K}_b : \Omega_b \to [0,1]^{S \times J}$ defined by

$$\bar{K}_{bsj}(\eta) = \begin{cases} \min \left\{ \dfrac{p_s \cdot R_{sj} \varepsilon_j + \sum_{h \in H} p_s \cdot D_{sj}^h}{p_s \cdot R_{sj} \varepsilon_j + \sum_{h \in H} p_s \cdot R_{sj} \varphi_j^h}, 1 \right\} & , \text{ if } R_{sj} \neq 0 \\ 1 & , \text{ if } R_{sj} = 0 \end{cases}$$

for each $s \in S$, $j \in J$. Clearly $\bar{K}_{bsj}$ is a continuous function.

Next, consider the correspondence $\psi_b^0 : \Omega_b \Rightarrow \Delta_b$ defined by

$$\begin{aligned} \psi_b^0(\eta) \quad = \quad & \underset{(p,\pi) \in \Delta_b}{\arg\max} \left\{ p_0 \cdot \sum_{h \in H} (x_0^h - e_0^h) + \pi \cdot \sum_{h \in H} (\theta^h - \varphi^h) \right. \\ & \left. + \sum_{s \in S} \sum_{h \in H} p_s \cdot \left[x_s^h - e_s^h - \sum_{j \in J} (1 - \bar{K}_{bsj}(\eta)) R_{sj} \varepsilon_j \right] \right\}. \end{aligned}$$

Clearly this map is non-empty and convex-valued, and USC.

Finally for each $h \in H$, define the correspondence $\psi_b^h : \Omega_b \Rightarrow \Box^h$ by

$$\psi_b^h(\eta) = \underset{x, \theta, \varphi, D}{\arg\max} \{ w^h(x, \theta, \varphi, D, p) : (x, \theta, \varphi, D) \in B^h(p, \pi, K) \cap \Box^h \}.$$

Notice that ψ_b^h is non-empty valued and convex-valued, thanks to the continuity and concavity of w^h, for all $h \in H$. To check that $B^h(p, \pi, K) \cap \Box^h$ is LSC, let p^n, π^n, $K^n \xrightarrow{n} \bar{p}$, $\bar{\pi}$, $\bar{K}$ with $\bar{p} \gg 0$. Let $(\bar{x}, \bar{\theta}, \bar{\varphi}, \bar{D}) \in B^h(\bar{p}, \bar{\pi}, \bar{K})$. Fix $0 < \alpha < 1$. Then $(\alpha\bar{x}, \alpha\bar{\theta}, \alpha\bar{\varphi}, \alpha\bar{D}) \in B^h(p^n, \pi^n, K^n) \cap \Box^h$ for sufficiently large n by the scaling property of the budget set, because $\bar{p}_s \cdot e_s^h > 0\ \forall s \in S^*$. Since α was arbitrary, this shows that $B^h(p, \pi, K) \cap \Box^h$ is LSC in (p, π, K) whenever $p \gg 0$. Since $B^h(p, \pi, K) \cap \Box^h$ is clearly USC, ψ_b^h is USC by the maximum principle.

Let $\psi_b : \Omega_b \Rightarrow \Omega_b$ be the correspondence defined by

$$\psi_b(\eta) = \psi_b^0(\eta) \times \{\bar{K}_b(\eta)\} \times \underset{h \in H}{\times} \psi_b^h(\eta).$$

By Kakutani's theorem ψ_b has a fixed point $\eta^b \equiv (p^b, \pi^b, K^b, (x^h(b), \theta^h(b), \varphi^h(b), D^h(b))_{h \in H})$. To avoid notational clutter, we suppress the b.

Note that in state 0, $p_0 \cdot (\sum_h (x_0^h - e_0^h)) + \pi \cdot (\sum_h (\theta^h - \varphi^h)) = 0$ (since, given the monotonicity of each u^h, this equality holds for each h individually in his budget-set). It follows that the "price player" could not make the value of excess demand (across commodities and assets) positive in period 0. Suppose for some $j \in J$, $\sum_{h \in H} (\theta_j^h - \varphi_j^h) > 0$. By taking $\tilde{\pi}_j = 1/b$ and $\tilde{\pi}_i = 0$ for $i \neq j$, it follows that

$$\frac{1}{b} \sum_h (\theta_j^h - \varphi_j^h) + \sum_{\ell \in L} \tilde{p}_{0\ell} \sum_{h \in H} (x_{0\ell}^h - e_{0\ell}^h) \leq 0,$$

for all $\tilde{p} \in \mathbb{P}_b \equiv \{q \in \mathbb{R}^L_+ : q_\ell \geq b \ \forall \ell \in L, \sum_{\ell=1}^L q_\ell = 1\}$. Hence

$$\sum_h (\theta_j^h - \varphi_j^h) \leq b||e_0||_\infty.$$

Similarly, if $\sum_{h\in H}(x_{0\ell}^h - e_{0\ell}^h) > 0$ for some ℓ, then by taking all $\tilde{\pi}_j = 0$ and $\tilde{p}_{0\ell} = 1-(L-1)b$ and $\tilde{p}_{0k} = b$ for all $k \neq \ell$, we get

$$\sum_{h\in H}(x_{0\ell}^h - e_{0\ell}^h) \leq \frac{(L-1)b||e_0||_\infty}{1-(L-1)b}.$$

From the fact that $\bar{K}_b$ fixed K, and from the fact that agents have optimized so that $p_s \cdot D_{sj}^h \leq p_s \cdot R_{sj}\varphi_j^h$, whenever $R_{sj} \neq 0$ we get

$$K_{sj} = \frac{p_s \cdot R_{sj}\varepsilon_j + \sum_{h\in H} p_s \cdot D_{sj}^h}{p_s \cdot R_{sj}\varepsilon_j + \sum_{h\in H} p_s \cdot R_{sj}\varphi_j^h} \leq 1.$$

Hence

$$\sum_{h\in H} p_s \cdot D_{sj}^h = \sum_{h\in H} K_{sj} p_s \cdot R_{sj}\varphi_j^h - (1-K_{sj})p_s \cdot R_{sj}\varepsilon_j.$$

From optimization of monotonic utilities in the budget set, we get

$$p_s \cdot (x_s^h - e_s^h) = \sum_{j\in J} K_{sj} p_s \cdot R_{sj}\theta_j^h - \sum_{j\in J} p_s \cdot D_{sj}^h.$$

Adding over agents $h \in H$, and substituting the above expression for $\sum_{h\in H} p_s \cdot D_{sj}^h$ we get

$$\begin{aligned} p_s \cdot \sum_{h\in H}(x_s^h - e_s^h) &= \sum_{j\in J}(1-K_{sj})p_s \cdot R_{sj}\varepsilon_j + \sum_{j\in J}\sum_{h\in H} K_{sj}p_s \cdot R_{sj}(\theta_j^h - \varphi_j^h) \\ &\leq \sum_{j\in J}(1-K_{sj})p_s \cdot R_{sj}\varepsilon_j + J||R||_\infty b||e_0||_\infty. \end{aligned}$$

Suppose $\sum_{h\in H}(x_{s\ell}^h - e_{s\ell}^h) - \sum_{j\in J}(1-K_{sj})R_{s\ell j}\varepsilon_j > 0$ for some $s \in S$. Since we are at a fixed point, the price player cannot increase the value of excess demand in state s by taking $\tilde{p}_{s\ell} = 1-(L-1)b$, and $\tilde{p}_{sk} = b$ for all $k \neq \ell$. Hence

$$\begin{aligned} &\sum_{h\in H}(x_{s\ell}^h - e_{s\ell}^h) - \sum_{j\in J}(1-K_{sj})R_{s\ell j}\varepsilon_j \\ &\leq \frac{1}{1-(L-1)b}\left\{(L-1)b\left[||e_0||_\infty + ||R||_\infty \sum_{j\in J}\varepsilon_j\right] + J||R||_\infty b||e_0||_\infty\right\}. \end{aligned}$$

We now let $b \to 0$. We argue that π_j must remain bounded as $b \to 0$. If $Q_j^h = 0$ $\forall h$, then replace π_j with 1. Otherwise, if $\pi_j \to \infty$, any agent h with $Q_j^h > 0$ could replace his entire action by selling Δ units of j, buying M ($\leq \Delta\pi_j/L$) units of each period 0 good, and delivering fully. Since $e_s^h \neq 0$ for all s, and commodity price ratios are bounded in each state, agent h can do this without incurring any default. But this gives him utility that exceeds $u^h(2\sum_{h'} e^{h'})$, which is more than he can possibly be getting at the fixed point (with all excess demands close to zero for small b), a contradiction. Thus all asset prices are bounded.

Since all choices and all macrovariables are uniformly bounded for small b, we can pass to convergent subsequences, obtaining $\bar{E} \equiv \langle \bar{p}, \bar{\pi}, \bar{K}, (\bar{x}^h, \bar{\theta}^h, \bar{\varphi}^h, \bar{D}^h)_{h\in H}\rangle$ as a limit point. Taking the limit of all inequalities derived above, we conclude that aggregate excess demand for commodities and assets is less than or equal to zero in E. It follows that the limit price ratios $\bar{p}_{s\ell}/\bar{p}_{sk}$ are bounded in each state $s \in S^*$. If $p_{s\ell}/p_{sk}$ became unbounded as $b \to 0$, some agent with $e_{s\ell}^h > 0$ could have consumed M units of commodity sk, obtaining more utility than $u^h(2\sum_{h'\in H} e^{h'})$, for all small b; but since excess demand goes to zero with b, $x^h < 2\sum_{h'\in H} e^{h'}$ for small enough b, contradicting that h has optimized. Thus the limiting $\bar{p} \gg 0$, and all agents have positive income in every state in $\bar{E}$. Thus individuals are optimizing in $\bar{E}$ on their untruncated budget sets. (This uses the concavity of ω^h and the fact that eventually the constraints on x and θ are not binding in his budget set.)

Note finally that if all commodity prices are positive, there cannot be excess supply in any commodity in $\bar{E}$, otherwise the price player would be making negative profits. For the same reason there cannot be excess supply of any asset j in $\bar{E}$, unless $\pi_j = 0$. But then no agent would sell j unless $\lambda_{sj}^h R_{sj} = 0$ for all $s \in S$. Without loss of generality we may in this case take $\theta_j^h = \varphi_j^h = 0$ for all h.

Thus we have shown that $\bar{E}$ is an ε-trembling hand equilibrium. Letting $\varepsilon \to 0$ and taking limits we obtain a trembling hand equilibrium. This proves the theorem for finite penalties λ.

If some penalties are infinite, we take limits of equilibria with increasing penalties. Since all actions must stay bounded along the sequence (because $Q_j^h < \infty$), any cluster point of these equilibria will serve as the desired trembling hand equilibrium.. ■

Our proof has used the fact that $\varphi_j^h \leq Q_j^h$ by assumption. Later the Q_j^h will play an important role as signals, but now the reader may wonder what would happen if they were eliminated, or taken to be enormously large. Recall that there is a pathology that occasionally occurs even when there is no default, for example in the GEI model. Sometimes two assets j and j' that promise different commodities nevertheless become nearly equivalent at some spot prices $(p_s)_{s\in S}$ because they then promise nearly the same money. At these prices the number of independent assets suddenly drops, and demand blows up as agents try to go infinitely long in asset j' and infinitely short in asset j (or vice versa). This destroys the existence of equilibrium. The bounds Q_j^h prevent this, as Radner (1972) long ago pointed out for the GEI model.

In the GEI model without short sale constraints like the Q_j^h, equilibrium can only be guaranteed if all the assets promise payoffs exclusively in the same good (say L) in each state $s \in S$. (See Geanakoplos–Polemarchakis (1986).) The asset matrix R

then is effectively reduced to $S \times J$ dimensions.

Default provides another reason why two assets that make different promises might, given certain macro variables (p, π, K), actually deliver the same money in every state. One should therefore wonder if default introduces additional difficulties in proving the existence of equilibrium. We have just seen that in the presence of the bounds Q_j^h it does not. Now we shall show that default also does not complicate the existence picture without the bounds Q_j^h.

Theorem 2 *Let all promises R_j be exclusively in good L for all $s \in S$ and let $R_j \neq 0$ for all $j \in J$. Define $GE(R, \lambda) = GE(R, \lambda, Q)$ with $Q_j^h = \infty$, $\forall h \in H$, $j \in J$. Then $GE(R, \lambda)$ exists for any vector $\lambda \in \overline{\mathbb{R}}_+^{HSJ}$ with $\sum_{s \in S} \lambda_{sj}^h R_{sLj} > 0$ for all $h \in H$ and $j \in J$.*

Proof Theorem 2 specializes the conditions of Theorem 1. Hence we have a $GE(R, \lambda, Q)$ equilibrium for all finite Q. Consider a sequence of equilibria, $\eta(Q) = (p(Q), \pi(Q), K(Q), (x^h(Q), \theta^h(Q), \varphi^h(Q), D^h(Q))_{h \in H})$, where $Q_j^h = Q \in \mathbb{N}$, for all $h \in H$, $j \in J$.

If there is a single Q with $\varphi_j^h(Q) < Q$, for all $h \in H$, $j \in J$, then by the concavity of each w^h, $\eta(Q)$ is a $GE(R, \lambda)$.

Passing to a convergent subsequence if necessary, we may suppose that for all $h \in H$ and $j \in J$,

$$\frac{\theta_j^h(Q)}{Q} \to \bar{\theta}_j^h, \ \frac{\varphi_j^h(Q)}{Q} \to \bar{\varphi}_j^h.$$

Moreover, we might as well assume that for at least one j and some h and h', $\bar{\theta}_j^h \neq 0$ and $\bar{\varphi}_j^{h'} = 1$.

For notational convenience, we shall write R_{sj} and D_{sj}, instead of the more accurate R_{sLj} and D_{sLj}, and we shall suppose that real default in each state $s \in S$ is measured in terms of the commodity bundle $v_s = 1_L$, which is one in the Lth coordinate, and zero elsewhere. Since all assets are exclusively delivering in the Lth good, no harm results from these simplifications. Finally, w.l.o.g. take $p_{sL} = 1$ for all $s \in S$.

Observe that for any $h \in H$, $s \in S$, $j \in J$, the level of default

$$d_{sj}^h(Q) \equiv [R_{sj}\varphi_j^h(Q) - D_{sj}^h(Q)]^+ \leq \frac{1}{\lambda_{sj}^h}[u^h(e) - u^h(e^h)],$$

whenever $\lambda_{sj}^h > 0$, for otherwise agent h would have done better not trading at all. (At any $GE(R, \lambda, Q)$, $x^h \leq \sum_{h'} e^{h'} \equiv e$.) Hence if $\varphi_j^h(Q) \to \infty$,

$$\frac{[R_{sj}\varphi_j^h(Q) - D_{sj}^h(Q)]}{\varphi_j^h(Q)} = \frac{[R_{sj}\varphi_j^h(Q) - D_{sj}^h(Q)]^+}{\varphi_j^h(Q)} = \frac{d_{sj}^h(Q)}{\varphi_j^h(Q)} \to 0.$$

It follows that $K_{sj}(Q) \to 1$ for all $s \in S$ with $R_{sj} > 0$, provided that $\sum_{h \in H} \varphi_j^h(Q) = \sum_{h \in H} \theta_j^h(Q) \to \infty$.

Furthermore, since relative prices $p_{s\ell}(Q)/p_{sk}(Q)$ stay bounded,

$$\sum_{j\in J} K_{sj}(Q)R_{sj}\theta_j^h(Q) - \sum_{j\in J} D_{sj}^h(Q)$$

must stay bounded. Otherwise agent h would eventually be consuming a negative quantity in state s, or a quantity exceeding the aggregate endowment e_s, contradicting commodity market clearing.

Putting these last statements together, we must have that

$$\lim_{Q\to\infty} \frac{\sum_{j\in J} K_{sj}(Q)R_{sj}\theta_j^h(Q) - \sum_{j\in J} D_{sj}^h(Q)}{Q} = R_s(\bar{\theta}^h - \bar{\varphi}^h) = 0,$$

for all $h \in H$, $s \in S$.

Consider any h with $\bar{\varphi}^h \neq 0$, and hence $\bar{\theta}^h \neq 0$. For any $Q \geq 1$,

$$\begin{aligned} \hat{\theta}^h &= \theta^h(Q) - \bar{\theta}^h \geq 0 \\ \hat{\varphi}^h &= \varphi^h(Q) - \bar{\varphi}^h \geq 0. \end{aligned}$$

At any large Q, the agent could feasibly have chosen

$$\hat{D}_{sj}^h = D_{sj}(Q) - R_{sj}\bar{\varphi}_j^h \geq 0 \text{ for all } j \in J.$$

With these choices he would pay exactly the same penalty as in the equilibrium $n(Q)$. He would receive exactly the same consumption at time 1 if $K_{sj}(Q) = 1$ for all j with $\bar{\theta}_j^h > 0$, and strictly more consumption otherwise. In order for him not to prefer this deviation, we must therefore have

$$\pi(Q)[\bar{\theta}^h - \bar{\varphi}^h] \leq 0 \text{ for all } h \in H.$$

But since $\bar{\theta}^h$ and $\bar{\varphi}^h$ are limits of $GE(R, \lambda, Q)$ equilibrium portfolios,

$$\sum_{h\in H} \bar{\theta}^h = \sum_{h\in H} \bar{\varphi}^h,$$

hence we must have

$$\pi(Q)[\bar{\theta}^h - \bar{\varphi}^h] = 0 \text{ for all } h \in H.$$

It now follows that household h would still prefer this deviation unless $\forall j \in J$, $\forall s \in S$,

$$[R_{sj} > 0, \text{ and } \bar{\theta}_j^h > 0 \text{ for any } h \in H] \Rightarrow [K_{sj}(Q) = 1].$$

Note finally that if $\bar{\varphi}_j^h > 0$, there must be some agent i with $\bar{\theta}_j^i > 0$, hence $K_{sj}(Q) = 1$ for all $s \in S$ with $R_{sj} > 0$ and either $\bar{\theta}_j^h > 0$ or $\bar{\varphi}_j^h > 0$.

Replacing $(p(Q), \pi(Q), K(Q), (x^h(Q), \theta^h(Q), \varphi^h(Q), D^h(Q))_{h\in H})$ with $(p(Q), \pi(Q), K(Q), (x^h(Q), \hat{\theta}^h, \hat{\varphi}^h, \hat{D}^h)_{h\in H})$ we get another $GE(R, \lambda, Q)$ with $\hat{\varphi}_j^h(Q) < Q$ for all h and j. (Notice that we are reducing sales and purchases only for assets with $K_{sj} = 1$, which therefore leaves the K unchanged.) ■

5 Chain Reactions, Netting, and Supernetting

5.1 Chain Reactions

In modern financial economies, agents often are long and short in many different assets. They rely on revenues from their loans to keep their own promises. But these revenues are only as reliable as the loans other agents have made to yet different parties, thus opening the possibility of a chain reaction of defaults. If α defaults against β, forcing β to default against γ, forcing γ to default against δ, then in our definition of equilibrium, α, β, and γ will pay default penalties, and the total utility loss from defaults will be large. Curiously this phenomenon is at its most dangerous when the financial system is at an intermediate level of development, with smoothly functioning markets that permit agents to go short, but without some finely tuned assets, forcing agents to hold complicated portfolios to achieve the risk spreading they desire.

Consider a world with four agents and three possible future events, each consisting of many different states of the world. Suppose β wants to consume in the first event, γ in the second event, and δ in the third event. Suppose agents β, γ, and δ have no endowment in the future states. Suppose α wants to consume in the present, but has a considerable endowment of goods in the future, except in one unlikely state ω in the third event.

If there were an advanced financial system of Arrow securities, agent α would in effect sell directly to each of the other three agents. For example, with just three Arrow securities, each one paying off exclusively in a different one of the three events, agent α would sell the first security to β, the second to γ, and the third to δ. Agent α by himself would default in state ω, and he alone would pay a default penalty.

Suppose, however, that in a less advanced financial system there are again three securities available. R_{123} promises 1 dollar in every state, R_{23} promises 1 dollar in (every state in) events 2 and 3, and R_3 promises 1 dollar in (every state in) event 3. Then in equilibrium we could expect α to sell R_{123}, β to buy R_{123} and to sell R_{23}, γ to buy R_{23} and to sell R_3, and δ to buy R_3. In the bad state ω in event three, the chain of defaults indicated above will take place. The penalty that α pays for starting the chain reaction may be very small compared to the total penalty incurred by the rest of the defaulters.

A diagram may make the situation clearer.

Figure 2

Notice that the asset span is exactly the same as with the three Arrow securities. What makes the chain of defaults possible is the interlocking asset trade, with investors receiving and delivering in a long chain, in some state. With Arrow securities this chain would never reach more than two links and one default.

One way around these chain reactions is to encourage market intermediation that nets payouts.

5.2 Netting

Consider the variation of our $GE(R, \lambda, Q)$ model in which an agent's purchases and sales of any given asset j are netted, so that he is deemed to have purchased $(\theta_j - \varphi_j)^+$ and sold $(\varphi_j - \theta_j)^+$.

One can go a step further and consider supernetting across different assets that an agent has traded in.[13] Now deliveries are no longer made separately on each asset, but there is one combined payment in every state $s \in S$.

Netting and supernetting curtail chain reactions, but they are difficult to implement in practice, though they are becoming more common. It is interesting to note that netting destroys the convexity of the budget set, since we must have

$$p_s \cdot (x_s^h - e_s^h) + \sum_{j \in J} p_s \cdot D_{sj}^h \leq \sum_{j \in J} K_{sj} p_s \cdot R_{sj} (\theta_j^h - \varphi_j^h)^+$$

for all $s \in S$. In our working paper [—] we show nevertheless that refined equilibrium exists with a continuum of agents.

6 The Economic Advantages of Intermediate Default Penalties with Incomplete Markets

There are four fundamental drawbacks to reducing the default penalties λ so far that some agents choose to default in at least some states in equilibrium: (1) creditors,

[13] Institutionally this may be regarded as a clearinghouse.

rationally anticipating (on account of direct and indirect reasons) that they might not be repaid, are less likely to lend; (2) borrowers may not repay even in contingencies that have been foreseen, and even though they are able; (3) imposing penalties is a deadweight loss; (4) the default of unreliable agents imposes an externality on reliable agents who, because they cannot distinguish themselves from the unreliable agents, are forced to borrow on less favorable terms.

Akerlof regarded the fourth (externality) cost of default as so important that for this reason alone he suggested it would always be worthwhile to reduce default by imposing penalties on defaulters. By analogy one could ask manufacturers of products to issue guarantees to replace any defective parts, and in addition to pay for all damages caused by defective parts.

Our second goal in this paper is to show that despite myriad reasons why default is socially costly, the benefits from permitting some default often outweigh all of these costs. The benefits from allowing default are basically twofold, and both stem from the fact that markets are incomplete to begin with. First, an agent who defaults on a promise is in effect tailoring the given security and substituting a new security that is closer to his own needs, at a cost of the default penalty. With incomplete markets one set of assets may lead to a socially more desirable outcome than another set. Second, since *each* agent may be tailoring the same given security to his special needs, one asset is in effect replaced by as many assets as there are agents, and so the dimension of the asset span is greatly enlarged. A larger asset span is likely to improve social welfare (although this gain must be weighed against the deadweight loss of the default penalties that are thereby incurred). In short, permitting default allows for a plethora of additional assets that do not have to be specified in advance. Each agent can tailor the simple standard contract to fit his idiosyncratic situation.

A third benefit from allowing default, which is closely related to the first two, is that when there is no netting, agents can go long and short in the same security, thereby doubling their asset span. We make use of this in the following example. (The examples could be presented with netting, or supernetting, but then we would need more assets and a more cumbersome analysis to make the same points.)

Let there be three agents and $S = 3$ states of nature, let there be one good in each state, $L = 1$, and suppose agents have no utility for consumption at $t = 0$. Each agent has the same utility

$$u(x_1, x_2, x_3) = \sum_{s=1}^{3} \log(x_s).$$

The endowments of the agents are

$$e^1 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}; \; e^2 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}; \; e^3 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}.$$

We take the collection of asset promises to be

$$R_0 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix};\ R_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \equiv 1^1;\ R_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \equiv 1^2;\ R_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \equiv 1^3.$$

We take default penalties to be one of three types:

$$\begin{aligned} \lambda_{sj}^h &= \infty,\ \forall h, s, j;\ \backslash \\ \lambda_{sj}^h &= \lambda > 0,\ \forall h, s, j; \text{ or} \\ \lambda_{sj}^h &= \begin{cases} \infty & \text{if } e_s^h = 1 \\ 0 & \text{if } e_s^h = 0 \end{cases}. \end{aligned}$$

We take

$$Q_j^h = \infty \quad \forall h, j.$$

Notice that the first two penalties are completely anonymous, since they are the same whatever the name of the defaulter, and whatever his circumstances. The last penalty type is infinite when agents have the resources to pay, and 0 otherwise. They do not depend on the name of the defaulter, but they do depend on his circumstances; they require more information to carry out. The information required is identical to the sort of information an insurance company must obtain to verify that an accident has occurred. Indeed in our sequel paper we shall use these penalties precisely in order to render insurance a special case of default.

It is very important to observe that all these examples satisfy the assumptions of Theorem 2.[14] Here we can be sure a refined equilibrium exists. In our calculations we shall often be able to show that there is a *unique* equilibrium satisfying the "on the verge" condition. Since refined equilibria must also be on the verge, we can conclude that the "on the verge" equilibrium must be a refined equilibrium, sparing us the tedious computations of the ε-trembles.

Version A0: Arrow Assets: Pure Promises, Infinite Penalties, and Infinite Quantity Constraints

The Arrow–Debreu equilibrium in our example can easily be calculated as $p = (1, 1, 1)$ and $x^h = (2/3, 2/3, 2/3)$, $\forall h \in H$. It can be implemented as a $GE(R, \lambda, Q)$ if $\mathcal{A}$ consists of the three Arrow assets $j = 1, 2, 3$. Let $R_j = 1^j$, where 1^j is the jth unit vector, be the pure promise for state j, and let the default penalties and sales constraints be set at infinity, $\lambda_{sj}^h = Q_j^h = \infty$, $\forall h \in H$, $s \in S$, $j \in J$. The equilibrium is given by $(p, \pi, K, (x^h, \theta^h, \varphi^h, D^h)_{h \in H})$ where $p = (1, 1, 1)$, $\pi = (1, 1, 1)$, $K_{sj} = 1$, $\forall sj$, $x^h = (2/3, 2/3, 2/3)$, $\theta^h = 2/3 \cdot 1^h$, $\varphi^h = 1/3 \cdot e^h$, and $D_{sj}^h = 1/3$ if $h \neq s = j$, and 0 otherwise.

[14] Actually $\log(x)$ is not continuous at 0, so by "$\log x$" we really mean $\log x = \begin{cases} \ln x \text{ if } x \geq \delta \\ \frac{1}{\delta} x + \ln \delta - 1 \text{ if } 0 \leq x \leq \delta \end{cases}$ where $\ln x$ is the conventional logarithm base e, for some very small $\delta > 0$.

In the $GE(R, \lambda, Q)$ equilibrium just described, the volume of trade is 2/3 in each of the three asset markets. Notice that there is some trivial multiplicity in the equilibria, since agents could engage in wash sales and buy and sell the same asset. We could instead have taken $\theta^h = (2/3, 2/3, 2/3)$, $\varphi^h = e^h$, which has volume of trade equal to 2 in each of three asset markets. However, with the tiniest of transactions costs, wash sales would be eliminated, and the volume of trade would fall to 2/3.

Version A1: The Optimal Default Penalty with Incomplete Markets

In Version A0 we found that setting $\lambda^h_{sj} = \infty$ gave a Pareto efficient outcome, because it eliminated default. Setting $\lambda^h_{sj} = \lambda < \infty$ would have led to a Pareto worse outcome. Nevertheless, we shall argue in this section that when markets are incomplete, it is often better to set intermediate default penalties. In Version A0, markets for risk sharing were effectively complete.

Consider the economy as in A0, but with only one asset $R_0 = (1, 1, 1)$. Suppose that the reason for default cannot be observed, so $\lambda^h_{sj} = \lambda$, $\forall h$, s, j. Agents who promise delivery but do not have the good will default and suffer the penalty. Anticipating this they will make fewer promises, and risk-sharing will be reduced.

We can calculate the equilibrium and agent utilities for any value of $\lambda \in (1, \infty)$. When $\lambda \leq 0$ buyers realize that sellers will not deliver anything, so demand will be zero and equilibrium will involve no trade. When $\lambda \to \infty$ buyers will anticipate full delivery, but sellers will realize that with probability 1/3 they will not be able to avoid a crushing penalty, and so again equilibrium trade goes to 0. By setting an intermediate level of default penalties we can make everybody better off. We graph the situation schematically in welfare space:

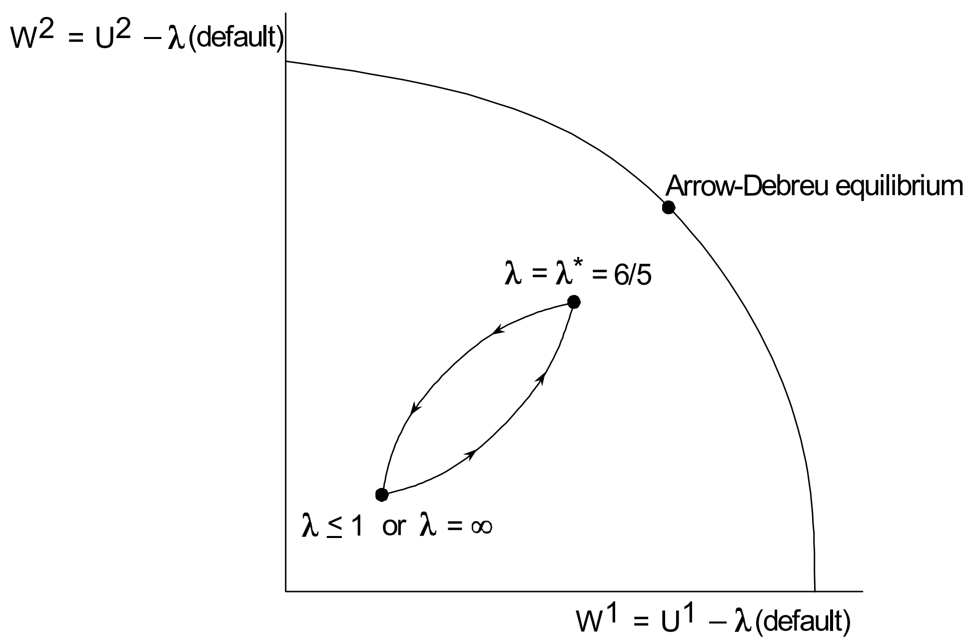

Figure 3

In our equilibrium different sellers default differently. The buyers of the asset receive the average deliveries of all the sellers. When $\lambda \geq \lambda^* = 6/5$, sellers in their good states deliver fully, and sellers in their bad state default completely. Thus our example illustrates the pooling aspect of assets, namely that investors buy shares of a pool of individually sold promises.

At $\lambda = 6/5$, $x^1 = (1/3, 5/6, 5/6)$, $x^2 = (5/6, 1/3, 5/6)$ and $x^3 = (5/6, 5/6, 1/3)$, $\theta^h = \varphi^h = \varphi = 1/2\ \forall h$ and $K_{sj} = K = 2/3\ \forall s$. By buying and selling 1/2 unit of the asset R_0, agent h gains $1/3 = 2/3(1/2) = K\theta^h$ when $s = h$ and loses $1/6 = 2/3 \cdot 1/2 - 1/2 = K\theta^h - \varphi^h$ in the two states $s \neq h$. Agent h delivers fully when $s \neq h$ because his marginal utility of consumption after delivery is $1/(5/6) = 6/5 = \lambda^*$. When $s = h$, agent h defaults completely since his marginal utility of consumption $1/(1/3) = 3 > \lambda^*$. Since for any $s \in S$ we have 2 agents with $h \neq s$, $K_{s0} = 2/3$. Thus the asset promise $R_0 = (1, 1, 1)$ actually delivers $(2/3, 2/3, 2/3)$ per unit promise. Agent $h = 1$ delivers $1/2 \cdot (0, 1, 1)$, agent $h = 2$ delivers $1/2 \cdot (1, 0, 1)$, and agent $h = 3$ delivers $1/2 \cdot (1, 1, 0)$. The reason each agent buys and sells only 1/2 a unit of asset R_0 instead of a full unit to get to the Arrow–Debreu allocation is that the sale of φ units of the asset is accompanied by the loss of $\varphi\lambda$ utiles for the inevitable default in state $s = h$. The marginal utility from buying the asset is $(2/3)(6/5)+(2/3)(6/5)+(2/3)\cdot(3) = 18/5$; the marginal disutility from selling is also $(6/5) + (6/5) + (6/5) = 18/5$. (It is therefore more convenient to take $\pi_0 = 18/5$.)

A consequence of pooling is that the volume of trade is high. In equilibrium (when $\lambda = 6/5$), each agent sells 1/2 unit of the asset, giving a total volume of trade equal to $3 \cdot 1/2 = 3/2$, much greater than the volume of trade per asset in the Arrow–Debreu equilibrium.

When $1 < \lambda < 6/5$, agents default in every state, delivering nothing in their bad state and delivering $D(\lambda)$ only up to the point where the marginal utility of consumption equals λ in their good state. The reader can verify that $K(\lambda) = (6\lambda - 6)/(4\lambda - 3)$, $D(\lambda) = 3 - (3/\lambda)$, $\varphi(\lambda) = (4/3) - (1/\lambda)$, and $x'(\lambda) = (2(1 - (1/\lambda)), 1/\lambda, 1/\lambda)$, etc. Clearly as[15] $\lambda \to 1$, $x'(\lambda) \to (0, 1, 1)$, $D(\lambda) \to 0$, and $K(\lambda) \to 0$. (Asset trade $\varphi(\lambda)$ does not go to 0 as $\lambda \to 1$ because the log utility is $-\infty$ at zero consumption.) as $\lambda \uparrow 6/5$, $\varphi(\lambda)$, $D(\lambda)$, and $K(\lambda)$ are monotonically increasing, as is the utility of final consumption.

For $\lambda \geq 6/5$, the agents always deliver if they have the goods on hand. Thus K_{s0} is maintained at 2/3, but asset trade again begins to drop because the inevitable punishment makes selling less attractive. The formulas are messy and we do not bother to present them here. An increase in the penalty rate beyond $\lambda = 6/5$ does not improve risk bearing (since φ begins to drop), and it also increases the deadweight loss from punishing agents who cannot deliver anyway. It thus strictly lowers welfare.

Furthermore, observe that as λ rises from 1 to $\lambda = 6/5$, the deadweight utility loss from default

$$\lambda\varphi + 2\lambda(\varphi - D) = \frac{4}{3}\lambda - 1 - \frac{10}{3}\lambda + 4 = 3 - 2\lambda$$

[15] Recalling that $\log x = \ln x$ only for $x \geq \delta$, we really require $2(1-(1/\lambda)) \geq \delta$, that is, $\lambda \geq 2/(2-\delta)$. By taking δ small, $2/(2-\delta)$ is just about 1.

actually falls, to 3/5. Since the allocation is improving, and the default penalty is falling, we deduce that $\lambda^* = 6/5$ leads to the Pareto best outcome among all economies with $\lambda^h_{sj} = \lambda$.

Example A1 illustrates that the optimal default penalty might be low enough to encourage some real default, and the attendant deadweight loss, when markets are incomplete. It also illustrates that the possibility of default makes the asset payoffs endogenous, since we do not know before an equilibrium is calculated what the default rates will turn out to be. If we change the utilities or endowments of the agents, or the default penalties, the equilibrium will change, the default rates will change, and the asset payoffs will be different.

7 Transactions Costs and the Advantages of Pooling

Even if assets and penalties could be chosen simultaneously, there is good reason to suppose that not every Arrow security would be actively traded. In practice, that would be much too costly.

If every agent tried to market a personalized asset, tailor-made to his needs, buyers would be confronted with a bewildering array of choices. The information processing and evaluation costs would be prohibitive, forcing each buyer to consider only a few of the assets. Trading costs would also be high because every market would be thinly traded, with only a few buyers and just one seller.

In our working paper [—] we formalized some of these costs. We supposed that there are liquidity costs of selling or buying which decline in the volume of *total* trade $\theta = \varphi = \sum_h \varphi^h = \sum \theta^h$, but increase in the amount θ^h or φ^h of individual trade. We also supposed that there is a further fixed "information-evaluation" cost of buying an asset that increases in the variance of K. Whatever the precise formulation of these costs, it is evident that there can be tremendous efficiency gain to pooling. Buyers are spared evaluation costs and sellers reap the benefits of liquid markets.

In the real world, promises are indeed standardized, enabling liquid markets, even though deliveries are idiosyncratic. Thus two agents take out the same insurance policy, under the same terms, even though it is perfectly understood that payments on each will come in different states of the world. We present a concrete example to illustrate ideal pooling, and to show how our model of default encompasses insurance. The example shows that in some cases the Arrow–Debreu equilibrium can be achieved with just one liquid asset, instead of many thinly traded Arrow assets.

Version A2: The Advantages of Standardized Pooled Assets

Consider the economy described in Section 7 with $H = \{1, 2, 3\}$ and with just one asset $R_0 = (1, 1, 1)$. Suppose that the default penalties are

$$\lambda^h_{sj} = \begin{cases} \infty & \text{if} \quad s \neq h \text{ (i.e., if } e^h_s = 1) \\ 0 & \text{if} \quad s = h \text{ (i.e., if } e^h_s = 0) \end{cases}$$

that is, default penalties are infinite when agents have the resources to pay, and 0 otherwise. We might interpret state s as the state in which a bad accident happens to agent $h = s$.

Let $\pi_0 = 3$, $p = (1, 1, 1)$. Agent h buys *and* sells 1 unit of the asset, delivering fully when his endowment is 1, and defaulting completely when his endowment is 0. The upshot is that on net, agent h has effectively bought an insurance contract. Indeed every agent has formally obtained the same insurance contract (by virtue of making identical asset trades) but each has insured his own idiosyncratic risk.

Since in every state two agent types deliver and the other type defaults, $K_{s0} = 2/3$, $\forall s \in S$. Consumption by h in the state $s = h$ where he has no endowment is thus $K_{s0}\theta^h_{s0}R_{s0} = (2/3)(1)(1) = 2/3$. Consumption in the other states where he delivers is $e^h_s + K_{s0}\theta^h_{s0}R_{s0} - D^h_{s0} = 1 + 2/3 \cdot (1)(1) - 1 = 2/3$. We verify that this is a $GE(R, \lambda, \infty)$ equilibrium by noting that the marginal utility of owning an extra unit of the asset is $\sum_{s=1}^{S}(\partial u/\partial x_s)K_{s0}R_{s0} = \frac{3}{2}\left(\frac{2}{3}\right) + \frac{3}{2}\left(\frac{2}{3}\right) + \frac{3}{2}\left(\frac{2}{3}\right)$, which is equal to the marginal disutility of selling the asset $\sum_{s=1}^{S} R_{s0} \min\left[\frac{\partial u}{\partial x_s}, \lambda^h_{s0}\right] = \frac{3}{2}(1) + \frac{3}{2}(1) + 0$, where $3/2 = [d\log(2/3)]/dx$ is the marginal utility of consumption in each state.

Version A2 seems at first glance like an artificial example,because the penalties themselves are idiosyncratic. But, as we said earlier, they are no more idiosyncratic than insurance contracts.

8 Endogenous Asset Structures without Transactions Costs

We have argued that with transactions costs it is efficient for trade to be conducted via a relatively small number of liquid assets. We now show that, even *without* transactions costs, the market forces of supply and demand will force all trade into a small subset of the available assets. This is so even though we confine attention to refined equilibria in which optimistic expectations tend to boost trade in every market.

In some contexts it has become customary to think of endogenizing the asset structure by allowing atomic agents to invent new assets (often one at a time) to upset a prevailing equilibrium. These asset-creating agents are hypothesized to be motivated by payoffs that might depend on the perceived volume of trade which would take place in their new asset if no other prices changed (or in the new trading equilibrium after all prices equilibrated), or in some other way on their perceived profits from introducing the new asset. When the status quo assets are chosen so that none of these agents has an incentive to introduce a new asset, the asset structure is said to have been endogenously determined. This approach to endogenizing the asset structure inevitably involves a combination of price taking behavior and oligopolistic-Nash thinking on the part of the asset-creating agents.

By contrast we follow a relentlessly competitive approach to the problem of endogenous assets. Every agent is a price taker. An asset is endogenously missing in our approach if it is not in $\mathcal{A}^*$, i.e., if there is a price at which no agent wants to sell or buy it.

Recall that an asset is specified not just by its vector R_j of promises across states, but also by the associated default penalties λ^h_{sj}, and quantity constraints Q^h_j. If the government could simultaneously and without limitations choose these parameters, it would set them at the Arrowian levels: promises with full span, infinite penalties, and nonbinding quantity constraints. Now we show the market would do the same.

Version A3: Arrow Securities Emerge When Default Penalties Are Infinite

Consider our standard example with $H = \{1, 2, 3\}$, but with four assets $R_j = 1^j$, $j = 1, 2, 3$, and $R_0 = (1, 1, 1)$. Let the penalties be $\lambda^h_{sj} = \infty$ if $j = 1, 2, 3$, and $\lambda^h_{s0} = \lambda^* = 6/5$ for all h and s. Despite the fact that the default penalty for asset 0 has been chosen "optimally," the unique equilibrium (ignoring redundant trades) is the Arrow–Debreu equilibrium of Version A0, so that asset 0 is not traded at all. The forces of supply and demand determine that the Arrow securities are traded and other assets are not.

We elevate this example to a theorem:

Theorem 3 *Let $\mathcal{E} = ((u^h, e^h)_{h\in H}, (R_j, ((\lambda^h_{sj})_{s\in S}, Q^h_j)_{j\in J}))$ be an economy which includes all the Arrow securities: for each $s \in S$, there is an asset $i = i(s)$ such that $R_{sLi} = 1$ and $R_{s'\ell i} = 0$ otherwise, with $Q^h_i = \infty$ $\forall h$ and $\lambda^h_{si} = \infty$ $\forall h$ and $\forall s$. Then for any $GE(R, \lambda, Q)$ equilibrium $\eta = ((p, \pi, K), (x^h, \theta^h, \varphi^h, D^h)_{h\in H})$, we can find prices $q \in \mathbb{R}^{(1+S)L}_{++}$ such that $(q, (x^h)_{h\in H})$ is an Arrow–Debreu equilibrium. Moreover, if $\lambda \gg 0$, no agent defaults on any actively traded asset in η, even if there are assets $j \in J$ with low λ^h_{sj}. Finally, there is an equilibrium η', possibly η itself,with the same $((p, \pi, K), (x^h)_{h\in H})$ such that the only actively traded assets in η' are Arrow securities.*

Proof Let η be given. Let $q_0 = p_0$ and let $q_s = \pi_{i(s)}(p_s/p_{sL})$, $\forall s \in S$. Let

$$v^h(q) \equiv \max\{u^h(x) : q \cdot x \leq q \cdot e^h,\ x \in \mathbb{R}^{(1+S)L}_{+}\}.$$

Observe that $K_{sj} = 1$ for each asset j with $\lambda^h_{sj} = \infty$ $\forall h, s$, if $R_s \neq 0$, since no agent will default in the refinement and the external agent will be fully delivering. It follows that by never defaulting, each agent h could, by selling and buying the Arrow securities, achieve at least $v^h(q)$, that is,

$$u^h(x^h) \geq u^h(x^h) - \text{default penalty} \geq v^h(q).$$

It follows that $q \cdot x^h \geq q \cdot e^h$ $\forall h \in H$. Since η is an equilibrium $\sum_{h\in H} x^h = \sum_{h\in H} e^h$. Hence $q \cdot x^h = q \cdot e^h$ $\forall h \in H$, and $(q, (x^h)_{h\in H})$ is an Arrow–Debreu equilibrium, and the default penalty actually borne by each agent $h \in H$ is zero.

Clearly each agent is indifferent to achieving x^h via the actively traded assets in η, or via Arrow securities. If every agent trades exclusively via Arrow securities, then supply will equal demand, and we achieve the desired equilibrium η'. ■

Theorem 3 shows that the market selects the Arrow promises in $\mathcal{A}^*$ if default penalties are infinite (for these promises). We have seen that with no penalties, $\mathcal{A}^*$ will be empty. As penalties are made harsher, $\mathcal{A}^*$ will tend to increase, provided that all promises are available in $\mathcal{A}$. The reason is that for every state $s \in S$, there is likely to be some agent who knows his punishment will be relatively low in that state. He will have incentive to sell the corresponding Arrow promise j and debase its K_{sj}, and therefore its price π_j. This will effectively prevent agents intending to deliver in state s from selling j. By raising the general level of default penalties, this phenomenon is discouraged.

9 Endogenous Default Penalties When Promises Are Incomplete

When all asset promises are available, the market should and will exclusively trade promises with infinite penalties. Let us suppose that the set $\mathcal{A}$ contains only a limited variety of promises, far short of a complete set of Arrow promises. Given these limitations on promises, in Section 7 we were able to ask how severe the default penalties *should* be to promote economic efficiency. Since our model allows for the possibility that different punishment regimes coexist at the same time, we can also ask how harsh the punishment scheme *will* be that endogenously emerges in equilibrium. For example, an agent could indicate his intention to perform a service, he could orally commit to performing the service, he could put in writing that he promised to perform a service, or he could draw up a contract with a lawyer announcing his promise to perform a service. If all four of these promises are treated equally by the courts, then there is no issue of selecting a punishment. But if the punishment in case of default is different for these different manners of making the same promise, then in effect the parties to the agreement are choosing the severity of default penalties attached to the promise. We shall now show that in our example, the forces of supply and demand select the optimal default penalties.

Version A4: Endogenous Default Penalties

Consider the model of version A1 with only one asset promise $R_0 = (1, 1, 1)$ and $\lambda^h_{s0} = \lambda^* = 6/5$, $\forall h \in H$ and $\forall s \in S$. It is natural to regard the penalty λ^* as imposed by a beneficent and knowledgeable government. But we may also regard λ^* as emerging from the equilibrium forces of supply and demand.

Now let there be a finite number of additional assets R_j, all making the same promises $R_j = (1, 1, 1)$, but with default penalties $\lambda_j = \lambda^h_{sj}$ for all $h \in H$, $s \in S$, ranging at intervals of $\lambda^*/100$ from 0 to $100\lambda^*$. The symmetry of the utilities, endowments, and penalties guarantees (by symmetrizing the proof of Theorem 2) that a symmetric, refined equilibrium must exist. (Symmetry means that $\varphi^h_j = \theta^h_j = \varphi_j$ for all $h \in H$ and $j \in J$.) We shall now show that despite the myriad of available assets, in every (symmetric) refined equilibrium, all trade will be conducted in the assets j for which $\lambda^h_{sj} = \lambda^*$. We begin by describing an equilibrium of this type, and then we show it is essentially the only (symmetric) equilibrium satisfying the "on the verge" condition.

The equilibrium will involve exactly the same prices, delivery rates, trades, and consumption as described in example A1 for the case $\lambda = \lambda^* = 6/5$. There we found that $x^1 = (1/3, 5/6, 5/6)$, $x^2 = (5/6, 1/3, 5/6)$, $x^3 = (5/6, 5/6, 1/3)$, and $\varphi^h_{j^*} = 1/2$ for all h, and $K_{sj^*} = 2/3$ for all s. We must now extend that equilibrium to define prices π_j and delivery rates K_{sj} for all the new assets. The "on the verge" condition uniquely specifies all these (π_j, K_j) for $j \neq j^*$. Set $\pi^* = 18/5$, and set $\pi_j = \min\{\lambda_j, 6/5\} + \min\{\lambda_j, 6/5\} + \min\{\lambda_j, 3\}$ for $j \neq j^*$, which is the marginal disutility of selling asset j. At these prices agents are just indifferent between selling j and j^*, so it is optimal to supply zero of j. Recall in example A1, $\pi_{j^*} = 18/5 =$ the marginal utility of buying or selling asset j^*.

The marginal utility of buying asset j must be equal to $(18/5)/\pi_{j^*} = 1$, i.e.,

$$\frac{\frac{6}{5}K_j + \frac{6}{5}K_j + 3K_j}{\pi_j} = 1.$$

Hence $K_j = (5/27)\pi_j$ for all $j \in J$.

By concavity, since the first-order conditions are satisfied, each agent is indeed maximizing by trading exclusively via asset j^*.

Having specified the prices π_j and delivery rates K_j for all the assets, we note that in equilibrium there is active trade only in asset j^* (aside from trivial wash sales in assets j with $\lambda_j > 0$ and $K_j = 1$). For $6/5 < \lambda_j < 3$, no agent will deliver anything on asset j in his bad state, since he consumed 1/3 and $3 > \lambda_j$. Hence if j is actively traded, $K_j \leq 2/3$, contradicting our formula $K_j = (5/27)\pi_j = (5/27)(6/5 + 6/5 + \lambda_j) > 2/3$. If $\lambda_j < 6/5$, and yet consumption in the good state is 5/6, then no agent who actively sells j will deliver anything on j in any state. Hence if j were active K_j would be zero, contradicting our formula for K_j.

We have thus displayed an equilibrium in which (almost) any default penalty is available, yet only a single one (namely the Pareto efficient penalty) is used in equilibrium.

We now argue that there can be no other refined (symmetric) equilibrium. In any (symmetric) equilibrium we have consumption $x^1 = (2x, 1-x, 1-x)$, and similarly $x^2 = (1-x, 2x, 1-x)$, and $x^3 = (1-x, 1-x, 2x)$. If $x = 1/6$, then all (π_j, K_j) are defined, as in the last paragraph, by the on the verge condition. If $x > 1/6$, then agent 1 has delivered up to a point in states 2 and 3 where his marginal utility of consumption $1/(1-x) > 6/5$. He would not have done that unless he was selling an asset with default penalty $\lambda_j \geq 1/(1-x) > 6/5$. If asset j delivers fully in every state, then it is irrelevant, since by symmetry each agent is buying and selling an equal amount of it. But from the argument in the proof of Theorem 2, if the asset did not fully deliver everywhere, then any agent buying and selling it would default completely in at least one state. Since by symmetry every agent buys and sells it, $K_j \leq 2/3$. The marginal utility to purchasing asset j is at most $\frac{2}{3}\left(\frac{1}{2x} + \frac{1}{1-x} + \frac{1}{1-x}\right) = \frac{2}{3}\frac{3x+1}{(1-x)2x} = \frac{1}{1-x} + \frac{1}{(1-x)3x} < \frac{3}{1-x}$ (if $x > 1/6$) of utility in period 1. The marginal disutility of selling asset j is at least $\frac{1}{1-x} + \frac{1}{1-x} + \frac{1}{1-x} = \frac{3}{1-x}$, a contradiction.

If $x < 1/6$, we shall show there can be no equilibrium price π^* for asset $j = j^*$. The marginal disutility of selling asset j^* is $\frac{1}{1-x} + \frac{1}{1-x} + \frac{6}{5}$, since $1/(1-x) < 6/5 = \lambda^*$. Hence, the marginal disutility of selling is less than $18/5$. It also follows that every agent would deliver in each of his two good states if he were selling asset j^*. Hence $K_{s0} \geq 2/3$, $\forall s \in S$, by our equilibrium refinement. The marginal utility of buying asset j^* is then at least $\frac{2}{3}\frac{1}{1-x} + \frac{2}{3}\frac{1}{1-x} + \frac{2}{3}\frac{1}{2x}$. For $x < 1/6$, the marginal utility of buying is always larger than $18/5$, hence larger than the marginal utility of selling, a contradiction.

10 Endogenous Quantity Constraints: Signalling

We saw in Section 8 that the forces of supply and demand could endogenously select unique default penalties that are active in equilibrium, out of an arbitrarily large array of possibilities. Here we show an analogous result holds for quantity constraints. We present a last example illustrating the theme from our sequel paper [—].

Version B

Consider our standard example, but now with 6 households whose endowments are $e^1 = (0,1,1)$, $e^2 = (1,0,1)$, $e^3 = (1,1,0)$, $e^4 = (1,0,0)$, $e^5 = (0,1,0)$, and $e^6 = (0,0,1)$. The utilities of all households are identical: $u(x) = \sum_{s=1}^{3} \log x_s$. Their default penalties are given by

$$\lambda^h_{sj} = \begin{cases} \infty & \text{if } e^h_s = 1 \\ 0 & \text{if } e^h_s = 0 \end{cases} \quad \text{for all } h \in H,\ s \in S,\ j \in J.$$

All assets $j \in J = \{1, 2, ..., 100\}$ entail the same promises $R_j = (1,1,1)$, but different quantity constraints $Q_j = j/30$.

We may think of households $h \in \{1,2,3\}$ as *reliable*, since they will deliver in 2 out of 3 states j and $h \in \{4,5,6\}$ as *unreliable*, since they deliver only in 1 state. The model now clearly displays the potential for adverse selection, since the unreliable have incentive to sell more of any asset, and therefore to be more than proportionately represented in the pool, thereby debasing the pool deliveries. Signalling one's reliability by selling an asset with a low quantity constraint, therefore has an important role to play. This is all the more so if an exclusivity constraint is imposed, prohibiting households from selling more than one asset.

Rothschild and Stiglitz argued (in an oligopolistic version of this example) that there might not be any equilibrium under these circumstances. But they showed that if an equilibrium exists, it must be separating: reliable and unreliable households will sell different assets.

The exclusivity constraint renders the budget set nonconvex, thus preventing this example from being covered by existence Theorems 1 and 2. Nevertheless, it can be shown that refined equilibrium *always exists*, and is always unique, in such insurance economies. (See Dubey–Geanakoplos [—].) In our numerical example, there is indeed an equilibrium in which all the reliable households buy and sell asset 9 up to its quantity limit $Q_9 = 9/30$, while all the unreliable households buy and sell 30 units of asset 100. Hence $K_9 = 2/3 = \pi_9$, and $K_{100} = 1/3 = \pi_{100}$. Assets $j \in \{10, 11, ..., 100\}$ are priced so that the *unreliable* households are on the verge of switching to them. Assets $j \in \{1, ..., 8\}$ are priced so that the reliable households are on the verge of selling them (unless Q_j is too low, in which case $K_j = \pi_j = 1$). For more details, see [—].

The universal existence and uniqueness of the insurance equilibrium is made possible by our perfectly competitive framework. In Rothschild–Stiglitz an entrant could upset a candidate equilibrium by introducing an asset with a new quantity constraint and a new price, (Q, π). The entrant could persuade buyers that this contract would yield a high K, thereby justifying its price and disrupting the old equilibrium. In our example, every Q is already marketed and priced at equilibrium (by the refinement). The "new" Q is a red herring. The real point of Rothschild–Stiglitz is that the entrant can quote a new price, which will attract a different clientele, and thus justify a different K. Perfect competition does not permit agents to quote new prices — they are all price takers.

We have in mind the huge, anonymous markets now becoming so common on Wall Street. Mortgages today are promises sold by homeowners to banks, who then sell them into mortgage pools (totalling around \$3 trillion). The bank plays a minor, administrative role, collecting payments and verifying the eligibility of the homeowners (according to criteria specified by the pool, not the bank). The bank receives a "servicing fee" for its efforts, and passes the default and prepayment risk on to the shareholders in the pool. The analysis therefore properly shifts from the one-on-one interaction of banker and homeowner to the pool level of perfectly competitive, anonymous, shareholders (lenders) and borrowers.

Imagine now that a little bank in Peoria decides to offer mortgages at 6%, instead of the prevailing pool rate of 7%. It figures to get a better clientele, attracting so many new, reliable homeowners (who were not until then borrowing at all) that the default rate would fall from 3% to 1%, thereby improving the net payoff from $4\% = 7\% - 3\%$ to $5\% = 6\% - 1\%$. In a massive market, we feel that the Peoria bank will be sadly disappointed. It seems likely that its first customers will be the old homeowners who will jump at the chance to refinance their 7% loans into 6% loans. The new homeowners, who found 7% too steep to borrow at all, will have much less incentive to run to Peoria. But the little Peoria banker will not have the trillions of dollars needed to give loans to everybody. He should cautiously assume that the customers most likely to reach him before he runs out of money are no better on average than those in the huge pool. This caution is embodied in our equilibrium refinement.

Our refinement is simple enough so that equilibrium always exists and is easy to compute. Yet it is strong enough to give a unique equilibrium, namely the separating equilibrium. We quote from Dubey–Geanakoplos [—]:

> Perfect competition not only simplifies the equilibrium, but also its refinement. In the contract theory literature, when two parties are in face-to-face meetings, an extensive form game is created, in which the refinements are vastly more complex. They require agents to engage in a long chain of

hypothetical reasoning about each other. For example, in the refinement of Cho and Kreps [3], h must think about what j thinks about what every other player k (including h himself) is thinking about, in order to deduce whether j will be able to deduce who he is dealing with. It presupposes common knowledge of private, individual characteristics; and calls upon each agent not only to think through many iterations, but to believe that others are doing likewise. Our refinement strains credulity less. There is no hypothetical reasoning and no chain. Agents think only about the observable macro aggregates K_j. The concrete, infinitesimal actions of the external agent are relevant only through their impact on the K_j; indeed their purpose is to render the K_j observable.

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