Faulty Communication:
Some Variations on the Electronic Mail Game

by

Stephen Morris

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COWLES FOUNDATION FOR RESEARCH IN ECONOMICS

YALE UNIVERSITY

Box 208281

New Haven, Connecticut 06520-8281

http://cowles.econ.yale.edu/

Faulty Communication: Some Variations on the Electronic Mail Game*

Stephen Morris
Cowles Foundation,
Yale University,
P.O.Box 208281,
New Haven CT 06520, U. S. A.
stephen.morris@yale.edu

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Abstract

The electronic mail game of Rubinstein (1989) showed that a lack of common knowledge generated by faulty communication can make coordinated action impossible. This paper shows how this conclusion is robust to having a more realistic timing structure of messages, more than two players who meet publicly but not as a plenary group, and may be robust to strategic decisions about whether to communicate.

JEL: C72, D8

1. Introduction

When a group of individuals must coordinate on a risky course of action, it is valuable to collect the group together and publicly agree on how to proceed. By meeting together, it is possible for the underlying facts motivating their joint

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action and the details of their plan of action to become *common knowledge* among the group. Because everyone was present in the room together, each person is confident not only of the underlying facts and plan of action, but is also confident that those facts and that plan are common knowledge among the group.

Risky coordinated action *requires* some degree of common knowledge.[1] Without a public meeting, it is hard to achieve the requisite degree of common knowledge. Unless there is a very tightly prescribed protocol specifying how information (the underlying facts, the details of the plan of action) is transmitted to the group members, common knowledge breaks down very easily. A famous example of Rubinstein (1989) suggests why this might be the case.

A pair of players must decide whether to make a costly investment. They must make their choices simultaneously, but they have the opportunity to communicate about their plans before hand. If a player's private state is bad, he will lose two million dollars if he chooses to invest. If his private state is good, he will make a net profit of a million dollars if he invests, but only if the other player invests: there are strategic links between their projects and if only one of the players invests, that player will lose two million dollars, regardless of his private state. If either player decides not to invest, his profits are zero.

Player 2's state is good for sure. The ex ante probability that player 1's private state is good is $\frac{1}{2}$. If player 1's state is good, he passes that information on - by electronic mail - to player 2. (If his state is bad, he doesn't send any message). If player 1's message arrives safely, player 2 then sends a confirmation to player 1, informing him that she has received his message. Player 1 in turn sends a confirmation back to player 2, and so on. At each stage, there is a probability ε that the message will get lost. How many messages are required before the players are prepared to invest?

Clearly, player 1 does not invest if his state is bad and player 2 is not prepared to invest if she has not received a message from player 1. But how many messages are enough? The surprising answer is that they should never be prepared to invest. Suppose the players followed the rule that as long as a player knows that $k \geq 1$ messages have been successfully sent and received, he should invest. But then consider a player who has successfully received the kth message and has sent off the $(k+1)$th message but has not received a confirmation. He will assign probability at least a half to the possibility that his reply was lost (the other possibility is that his message arrived, but the confirmation was lost). But if

[1] See Monderer and Samet (1989) for the appropriate formalization of the requisite degree of common knowledge.

his reply was lost, then the other player knows only that $k-1$ messages have been successfully sent and received, and will therefore not invest. But given the payoffs (a loss of two million if the other player invests, a gain of one million if he does), it does not pay player 1 in terms of expected revenue to invest in that circumstance. A version of this argument by contradiction can be used to show that no investment ever takes place, however many messages end up being sent (this result is more formally reviewed in section 2).

The result suggests why coordination might be tricky with faulty communication. There seems to be a general logic at work: as players communicate using a faulty technology, every communication sent generates new uncertainty about whether that communication arrived. In order to show that a strategy profile is not an equilibrium, it is enough to find one type of one player who is supposed to invest under the putative equilibrium but who is sufficiently uncertain about what information has reached the other player that he is not prepared to risk it. But however general the logic, the example is highly stylized. We would like to explore the robustness of the conclusions of this model by examining more realistic variations.[2]

Binmore and Samuelson (2001) have examined one such variation of the email game with strategic communication decisions. The players of the e-mail game had no choice but to keep sending out confirmation after confirmation until a message got lost: the communication process was exogenously given. They ask what would happen if the players could choose whether to send a message or not. If players could commit ex ante to a rule for sending messages, they could solve the coordination problem by agreeing on a protocol where player 1 sends out one message only about his private state and player 2 sends at most one confirmation (if the probability of message loss was very small, such a scenario would allow both players to invest with very high probability in equilibrium and thus be better off).

But what if players could not commit ex ante to a communication rule, but faced an extra strategic decision about sending a message or not at each stage of the game? Binmore and Samuelson note that the good equilibria - where one or two messages get sent and coordinated investment takes place with high

[2] Rubinstein (1989) suggested that even in the original e-mail game, boundedly rational players with a high number of levels of knowledge of payoffs might act as if there was common knowledge. Dulleck (2000) has formalized this intuition with bounded recall reasoning. Dimitri (2000a) shows that the conclusions of Rubinstein (1989) continue to hold if the probabilities of messages getting are different for the two players, but not too different. Dimitri (2000b) examines mechanisms for achieving efficient coordination in the email game setting using a third party mediator.

probability - would exist in this setting. But so would the original bad equilibrium of the email game where messages keep getting sent and no one ever invests. There will also exist another bad "tacit" equilibrium where no messages get sent and no one ever invests. In order to make a prediction in this game with strategic communication, it is necessary to perturb the game and/or refine the equilibria. Binmore and Samuelson consider a perturbation of the game where (1) players must incur a small "attention" cost for each message they observe (this must be paid whether or not the message gets sent); and (2) there is a small cost for each message they actually send. Clearly, with costs of attending to and sending messages, the bad equilibrium of the e-mail game, with infinite useless messages, is no longer an equilibrium. But they also show that the tacit equilibrium with no messages and no investment fail an evolutionary stability criterion. Thus the only evolutionarily stable equilibria of this perturbed involve a finite number of messages being sent and a positive probability of investment in equilibrium.

But do the good equilibria with a finite number of messages sent and a positive probability of investment make sense? In such equilibria, there is a player who receives the last possible message and will only invest if he receives that last possible message. Would not the last player to receive a message be tempted to send an additional, unanticipated, confirmation? He knows that the other player is unsure whether he will invest and might think that the more confident the other player is, the more likely he is to invest. But unanticipated confirmations cannot occur in equilibrium, so if this argument in favor of sending unanticipated confirmations worked, the good equilibria would break down.

The player receiving the last message in Binmore and Samuelson's game has a *strict* incentive to *not* send an unanticipated confirmation since (1) he is sure that other player is investing anyway, so an unanticipated confirmation cannot increase that probability; and (2) there are strictly positive costs to sending messages. However, there are equally compelling ways of perturbing the strategic communication game so that the strict incentive goes the other way. In section 3, each player is assumed to observe an idiosyncratic payoff shock that ensures that there is always a very small probability that he will choose to not invest. This perturbation generates the very natural property that the probability that a player invests is always *strictly* increasing in the probability he assigns to his opponent investing. In this perturbed game, it is shown that all equilibria satisfying a natural refinement (investment occurs ex post whenever feasible in the post-communication game) have no investment in equilibrium.

This paper examines two more variations on the electronic mail game which

- while theoretically straightforward - highlight the features of communication structures that lead to a breakdown in common knowledge.

While it is natural to assume that there is a positive probability that any message gets lost, it was also assumed that there is no upper bound on the number of messages that could conceivably be sent. A more satisfying assumption might be that there is a finite amount of time available for communication before a decision must be made, and each message takes a random length of time to arrive. In section 4, such a timing game with real time messages is analyzed. Conditions on the message arrival technology that imply no coordinated action are identified, and a natural example satisfying them is provided.

The e-mail game involves two players who never meet face to face: if the two players ever met face to face, they could immediately generate the requisite common knowledge to co-ordinate their behavior. A perhaps more relevant scenario is when a large number of players must co-ordinate their behavior, but while subgroups receive information publicly within the group, they never receive information as a plenary group. In the locally public communication game of section 5, N players meet together publicly and often in groups of m players. In order to co-ordinate their actions, each player must be confident that at least n of the N will invest. If $m \geq n$, one meeting is enough to coordinate their behavior. If $m < n$ and there is significant uncertainty about what meetings will take place in the future, then under weak conditions it is possible to rule out coordinated actions, however many meetings take place.

2. The Electronic Mail Game

If a player's private state is good (G), he has an incentive to invest if his opponent invests. His payoffs are given by the following matrix (the row represents a player's action and the column represents his opponent's action):

G	Invest	Not Invest
Invest	1	$-c$
Not Invest	0	0

where $c > 0$. If a player's private state is bad (B), he has a dominant strategy to not invest, with payoffs given by the following matrix.

B	Invest	Not Invest
Invest	$-c$	$-c$
Not Invest	0	0

Player 2's state is good for sure, but the ex ante probability that player 1's state is good is $\frac{1}{2}$. If the state is good, player 1 sends a message to player 2, informing her of this fact.[3] The message gets lost with exogenous probability $0 < \varepsilon < \frac{1}{2}$. If player 2 receives the message, she sends a confirmation to player 1, stating that she received the message. This message also gets lost with probability ε. If player 1 receives the confirmation, he sends a re-confirmation, stating that he received the confirmation. This message also gets lost with probability ε. And so on.[4]

After all the messages have been sent (eventually a message will get lost) each player chooses an action (invest or not invest) simultaneously. Observe that from the communication stage, there are an infinite number of possible states of the system:

Player 1's State	Total Messages Sent	Messages Received by Player 1	Messages Received by Player 2	Probability
bad	0	0	0	$\frac{1}{2}$
good	1	0	0	$\frac{1}{2}\varepsilon$
good	2	0	1	$\frac{1}{2}(1-\varepsilon)\varepsilon$
good	3	1	1	$\frac{1}{2}(1-\varepsilon)^2\varepsilon$
good	4	1	2	$\frac{1}{2}(1-\varepsilon)^3\varepsilon$
good	5	2	2	$\frac{1}{2}(1-\varepsilon)^4\varepsilon$
.	.	.	.	.
good	$2n$	$n-1$	n	$\frac{1}{2}(1-\varepsilon)^{2n-1}\varepsilon$
good	$2n+1$	n	n	$\frac{1}{2}(1-\varepsilon)^{2n}\varepsilon$
.	.	.	.	.

Proposition 2.1. *If $c > 1-\varepsilon$, this electronic mail game has a unique equilibrium: both players never invest.*

Note that for small ε, this reduces to the requirement that $c \geq 1$, i.e., no investment is the risk dominant equilibrium of the game.

[3] For simplicity, it is assumed that no message is sent if the state is bad. However, there is no qualitative change in the results if player 1 communicates whatever he knows.

[4] This differs from the game analyzed by Rubinstein (1989). First, this version is "private values," so each player is certain of his own payoffs. Second, this version gives player 1 a dominant strategy for one private state (bad), while in the original game players simply faced the problem of coordinating on an efficient Nash equilibrium that varied across states. Both changes simplify the presentation of later results, but do not effect the qualitative conclusions.

PROOF. If player 1's state is bad, he has a dominant strategy to not invest. If player 2 receives no message from player 1, she assigns probability

$$\frac{\frac{1}{2}}{\frac{1}{2}+\frac{1}{2}\varepsilon}=\frac{1}{1+\varepsilon}$$

to the possibility that player 1's state is bad. Thus her expected payoff to investing is at most

$$\frac{1}{1+\varepsilon}(-c)+\frac{\varepsilon}{1+\varepsilon}(1)=\frac{\varepsilon-c}{1+\varepsilon}<0,$$

so she cannot be investing in any equilibrium. If player 1 receives no confirmation from player 2, he assigns probability

$$\frac{\frac{1}{2}\varepsilon}{\frac{1}{2}\varepsilon+\frac{1}{2}(1-\varepsilon)\varepsilon}=\frac{1}{2-\varepsilon}$$

to the possibility that his message never arrived. Thus his expected payoff to investing is at most

$$\frac{1}{2-\varepsilon}(-c)+\frac{1-\varepsilon}{2-\varepsilon}(1)=\frac{1-\varepsilon-c}{2-\varepsilon}<0,$$

so he cannot be investing in equilibrium. Now let k be the smallest number greater than or equal to 1 such that a player who knows that k messages have been sent and received invests with positive probability. A player who receives the kth message but has not received a reply to the $(k+1)$th message assigns probability

$$\frac{\frac{1}{2}(1-\varepsilon)^{k-1}\varepsilon}{\frac{1}{2}(1-\varepsilon)^{k-1}\varepsilon+\frac{1}{2}(1-\varepsilon)^{k}\varepsilon}=\frac{1}{2-\varepsilon}$$

to the possibility that his message never arrived. If it did not arrive, then his opponent knows only that $k-1$ messages have been sent and received, and therefore will not be investing. Thus his expected payoff to investing is at most

$$\frac{1}{2-\varepsilon}(-c)+\frac{1-\varepsilon}{2-\varepsilon}(1)=\frac{1-\varepsilon-c}{2-\varepsilon}<0,$$

so he cannot be investing in equilibrium. This argument demonstrates by contradiction that there is no investment in equilibrium. ■

The result is tight in the following sense. If $\varepsilon \leq c \leq 1-\varepsilon$, this game has an equilibrium where player 1 always invests when his state is good and player 2 invests whenever she receives at least one message. If $c \leq \varepsilon$, this game has an equilibrium where player 1 always invests if his state is good and player 2 always invests.

3. Strategic Communication

The strategic communication game is identical to the electronic mail game of the previous section, with one change. When players in the electronic mail game were forced to send messages, players in the strategic communication game can choose whether to send the message or not. This gives rise to the somewhat elaborate extensive form game illustrated in figure 2.

insert figure 2 around here

Also fix $c > 1$ and $\varepsilon < \frac{1}{1+c}$.

We will analyze perfect Bayesian equilibria of this game. Thus each player has a strategy specifying behavior at each his information sets and each player has beliefs over the true node at each of his information sets. At each of a player's information sets, his strategy in the continuation game must be optimal given the strategies of other players and his beliefs at that information set. Each player's beliefs *on the equilibrium path* must be generated from equilibrium strategies by Bayes rule. *Off the equilibrium path*, there are two kinds of information sets for us to worry about. When players decide whether or not to send a message, they are at singleton information sets, so beliefs will be pinned down even off the equilibrium path. In additional, it will be assumed that when a player makes his investment decision at an unreached information set, he always believes his opponent is following his investment strategy (even though he must have deviated from his message sending strategy).

Behavioral strategies can be described by probabilities at each of the binary choices facing the players. The easiest way to describe them is to label the extensive form with these probabilities, as in figure 3.

insert figure 3 around here

Thus z is the probability that player 1 invests if his state is bad and x_0 is the probability that player 2 invests if she never receives a message. For all $k \geq 1$, the triple (π_k, x_k, y_k) describes the behavioral strategy of the player who has the option of the sending the kth message in the game: π_k is the probability that he chooses to send the message; x_k is the probability that he invests if he sends the kth message in the game but does not receive a reply; and y_k is the probability that he invests if he chose not to send the kth message. Thus $\left(z,(\pi_k,x_k,y_k)_{k=1,3,5,...}\right)$ describes the behavioral strategy of player 1 and $\left(x_0,(\pi_k,x_k,y_k)_{k=2,4,6,...}\right)$ describes the behavioral strategy of player 2. Since $z=0$ and $x_0=0$ in any equilibrium (because player 1 has a dominant strategy to not invest if investment conditions are bad, and player 2 assigns probability at least $\frac{1}{2}$ to player 1's state being bad), we focus attention on $(\pi_k,x_k,y_k)_{k=1,2,3,...}$ in the following analysis.

Write $f_0(\pi)$ for a player's set of mixed strategy best responses if his investment conditions are good and he expects his opponent to invest with probability π. Thus

$$f_0(\pi)=\begin{cases}\{0\}, \text{ if } \pi<\frac{c}{1+c}\\ [0,1], \text{ if } \pi=\frac{c}{1+c}\\ \{1\}, \text{ if } \pi>\frac{c}{1+c}\end{cases}$$

Figure 4 plots this correspondence.

insert figure 4 around here

We can use f_0 to state the equilibrium conditions on players' investment choices, taking as given their message sending strategies: for each $k=1,2,...$

$$\begin{aligned} y_k &\in f_0(x_{k-1}) \\ x_k &\in f_0\left(\frac{\varepsilon x_{k-1}+(1-\varepsilon)(1-\pi_{k+1})y_{k+1}+(1-\varepsilon)\pi_{k+1}\varepsilon x_{k+1}}{\varepsilon+(1-\varepsilon)(1-\pi_{k+1})+(1-\varepsilon)\pi_{k+1}\varepsilon}\right)\end{aligned} \tag{3.1}$$

Equilibrium conditions for message sending are more complicated to state in general but are simple enough to check in practise.

One class of equilibria are "message threshold" equilibria. For some $\overline{k}\geq 1$, only $\overline{k}\geq 1$ messages get sent and each player is prepared to invest as long as he knows that at least $\overline{k}$ messages have been sent.[5] This equilibrium is illustrated in figure 5, for the case where $\overline{k}=2$ (the diagram includes the beliefs at each information set implied by equilibrium strategies).

insert figure 5 around here

[5] Thus $\pi_k=1$ for all $k\leq\overline{k}$ and $\pi_k=0$ for all $k>\overline{k}$; $x_k=0$ for all $k\leq\overline{k}-1$ and $x_k=1$ for all $k>\overline{k}-1$; $y_k=0$ for all $k\leq\overline{k}$ and $y_k=1$ for all $k>\overline{k}$.

The person who sends the $\overline{k}$th message is prepared to invest, even if he does not receive a confirmation, because under the equilibrium strategies he was not anticipating a confirmation. Because of this, he assigns probability $1-\varepsilon$ to his message arriving and thus has expected payoff $1-\varepsilon-\varepsilon c>0$.

This equilibrium relies on the fact the player who receives the $\overline{k}$th message chooses not to send a confirmation. Under the equilibrium strategies, he is indifferent between whether to send a confirmation or not. But notice that if he fails to send a confirmation, he knows that the other player believes that he will invest with probability $1-\varepsilon$. On the other hand, if he sent an unanticipated confirmation then (in this equilibrium) he knows that the other player will believe that he will invest with probability 1. This suggests that he should break his indifference in favor of sending a confirmation.

To model this intuition formally, consider a perturbed version of the game where a player does strictly prefer that his opponent is more confident that he will invest. In particular, suppose that payoffs to player i in the good state are:

G	Invest	Not Invest
Invest	$1-\theta_i$	$-c-\theta_i$
Not Invest	0	0

(3.2)

where θ_i is drawn according to a c.d.f. Ψ_δ with support $[-c,1]$ and is observed by player i only immediately before he makes his investment decision. Now if player i assigns probability π to his opponent investing, he will invest only if

$$-\theta_i+\pi(1)+(1-\pi)(-c)\geq 0.$$

The probability of this occurring is

$$f_\delta(\pi)=\Psi_\delta((1+c)\pi-c)$$

Thus each f_δ is a strictly increasing function with $f_\delta(0)=0$ and $f_\delta(1)=1$. We assume that as $\delta\to 0$, Ψ_δ concentrates mass around 0: $\Psi_\delta(-\delta)<\delta$ and $\Psi_\delta(\delta)>1-\delta$. This implies that as $\delta\to 0$, f_δ tends to the step correspondence, f_0, illustrated in figure 4. Figure 6 plots f_δ for small δ:

insert figure 6 around here

In this perturbed game, the equilibrium conditions for investment become:[6] for each $k = 1, 2, ...$

$$\begin{aligned} y_k &= f_\delta\left(x_{k-1}\right) \\ x_k &= f_\delta\left(\frac{\varepsilon x_{k-1} + (1-\varepsilon)\left(1-\pi_{k+1}\right) y_{k+1} + (1-\varepsilon)\pi_{k+1}\varepsilon x_{k+1}}{\varepsilon + (1-\varepsilon)\left(1-\pi_{k+1}\right) + (1-\varepsilon)\pi_{k+1}\varepsilon}\right) \end{aligned} \tag{3.3}$$

An equilibrium is *immune to strategic uncertainty* if there exists a sequence of equilibria in the perturbed game that converge to the candidate equilibrium as $\delta \to 0$.

The message threshold equilibria described above are not immune to strategic uncertainty. In any equilibrium of the δ-game close to the $\overline{k}$-equilibrium, we will have $x_{\overline{k}} \approx f_\delta(1-\varepsilon)$ while $y_{\overline{k}+1} \approx f_\delta(1)$. Thus the player receiving $\overline{k}$th message will have a strict incentive to send an unanticipated confirmation in the perturbed game.

Another class of equilibria are "punishment equilibria." For some $\overline{k} \geq 1$, only $\overline{k} \geq 1$ messages get sent and each player is prepared to invest as long as he knows that *exactly* $\overline{k}$ messages have been sent.[7] This equilibrium is illustrated in figure 7, for the case where $\overline{k} = 2$.

insert figure 7 around here

As in the message threshold equilibria, the person who sends the $\overline{k}$th message is prepared to invest, even if he does not receive a confirmation, because under the equilibrium strategies he was not anticipating a confirmation. But the person who receives the $\overline{k}$th message is no longer indifferent between sending an unanticipated confirmation or not. Under the punishment equilibrium strategy profile, he will be punished if he sends a $\left(\overline{k}+1\right)$th message: no player chooses to invest in equilibrium if he knows that a $\left(\overline{k}+1\right)$th message has been sent. Such punishment equilibria perhaps do capture why many confirmations are not sent in practise: over some threshold, players will become discouraged and not anticipate investment. However, the punishments are inefficient. Following an unanticipated

[6] We ignore the fact that player 2 will invest with positive probability even when she receives no message (i.e., $x_0 > 0$); this simplifying assumption does not change any qualitative conclusions.

[7] Thus $\pi_k = 1$ for all $k \leq \overline{k}$ and $\pi_k = 0$ for all $k > \overline{k}$; $x_{\overline{k}} = 1$ and $x_k = 0$ for all $k \neq \overline{k}$; $y_{\overline{k}+1} = 1$ and $y_k = 0$ for all $k \neq \overline{k}+1$.

$(\overline{k}+1)$th message, there is an efficient equilibrium where all players always invest from that point on. But if players were expected to behave in that way, the original equilibrium would break down. In what follows, we will focus on *ex post efficient* equilibria, where, contingent on strategies in the message sending phase, equilibrium strategies in the investment game maximize the amount of investment (there is such a "maximal investment" equilibrium because of the strategic complementarities in the game). The punishment equilibria are not ex post efficient. What equilibria are left?

There is the "infinite messages - no investment" equilibrium, where players always send messages and never invest. There is also a tacit equilibrium, where player 1 does not send his first message if his state is good, but both players do always send confirmations if (out of equilibrium) they receive messages. These are essentially the only equilibria that are ex post efficient and immune to strategic uncertainty:

Proposition 3.1. *All ex post efficient perfect Bayesian equilibria immune to strategic uncertainty in the strategic communication game have both players not investing after every history; every player sends a confirmation with probability close to 1, whenever they receive a message.*[8]

Formal definitions of the refinements and a proof of the proposition are presented in the appendix.

The proposition says that once a message has been received, each player must have an incentive to send a confirmation with high probability in any equilibrium. This implies no investment in equilibrium. One equilibrium has an infinite numbers of costless messages sent, and no investment, while another equilibrium has the first player never sending a message about his private state (since it will not do any good anyway). Since there presumably is a small cost of sending messages, we might imagine that this equilibrium would be played in a richer model with small costs.

The refinements ("ex post efficiency" and "immunity to strategic uncertainty") are ad hoc. One can imagine other formal routes to reach similar conclusions. The implications of the ex post efficiency requirement would also be delivered by a forward induction requirement that no unanticipated message is sent unless the sender expects it to increase the probability of the other player investing.

[8] More precisely, $\pi_k > \frac{1-\varepsilon(1+c)}{(1-\varepsilon)^2}$ for all $k = 2, 3, ...$; and $x_k = 0$ and $y_k = 0$ for all k. For small ε, this implies π_k close to 1.

However, the modest purpose here is to suggest that making message sending a strategic choice need not remove the coordination problem from the e-mail game and reasonable views of rational play in the game are consistent with that view.

The above analysis formalized an intuition based on the Pareto-improving nature of unanticipated confirmations. One can also imagine quite different arguments in favor of the view that strategic message sending choices will not reduce message sending. If we look at either message threshold equilibria or punishment equilibria, the most preferred equilibria for player 2 (from an ex ante perspective) are one message equilibria, where player 1 invests whenever his state is good and player 2 invests whenever he receives one message from player 1. The most preferred equilibria for player 1 (from an ex ante perspective) are one message equilibria, where player 2 invests whenever he receives a message from player 1 and player 1 invests whenever he receives a confirmation from player 2. In the former case, player 1 bears the ex ante risk of misco-ordination (with probability $\frac{1}{2}\varepsilon$, player 1's message gets lost, player 1 invests and player 2 does not). In the latter case, player 2 bears the risk (with probability $\frac{1}{2}(1-\varepsilon)\varepsilon$, player 2's confirmation gets lost, player 1 invests and player 2 does not). Thus each player would like (ex ante) to be the last one to receive a confirmation. One can easily imagine stories where this lead to an ex post proliferation of messages. For example, if a player could credibly make statements like "I know you were not planning to send a confirmation; however, if I do not receive a confirmation, I will not invest," he would have an incentive to do so.

The strategic communication version of the e-mail game has a rich set of equilibria. The results presented in Binmore and Samuelson (2001) and in this section suggest that one could obtain many different conclusions by suitable perturbations of the game and choices of equilibrium refinement. Fine details of the strategic communication environment may be important in determining the likelihood of co-ordinated behavior using faulty communication channels.

4. Timing

The timing game is identical to the electronic mail game of the section 2, with one change. Instead of a message getting lost with exogenous probability ε, assume instead that it may take some time to arrive. Let $F(\tau)$ be the probability that a message will take less than or equal to τ minutes to arrive (and write $f(\cdot)$ for the density corresponding to $F(\cdot)$). At time 0, player 1 learns whether his state is good; if it is good, he sends a message to player 2; if the message arrives, she sends a confirmation; and so on. At time T, however many messages have been sent and received, the players must make their investment decisions.

Suppose that a player receives his last message at time t. What probability does he assign to his message having been the last? If he has just received a message at date t, the probability that his message never arrives is

$$N(t) \equiv 1 - F[T-t];$$

the probability that his message arrives but he never receives a reply is

$$A(t) \equiv \int_{\tau=0}^{T-t} f(\tau)(1 - F(T-t-\tau))\, d\tau.$$

The likelihood ratio of these two probabilities is

$$L(t) \equiv \frac{A(t)}{N(t)} = \frac{\int_{\tau=0}^{T-t} f(\tau)(1 - F(T-t-\tau))\, d\tau}{1 - F[T-t]}.$$

Thus the probability that his message was the last sent, conditional on not receiving a confirmation is

$$\frac{N(t)}{N(t) + A(t)} = \frac{1}{1 + L(t)}.$$

Proposition 4.1. *If $c > 1 - F(T)$ and $c > L(t)$ for all $t \in [0, T]$, the timing game has a unique equilibrium: both players never invest.*

This result is tight in the following sense. If $1 - F(T) \leq c \leq L(0)$, the timing game has an equilibrium where player 1 always invests when his state is good and player 2 invests exactly if she receives at least one message. If $c \leq 1 - F(T)$, the timing game has an equilibrium where player 1 always invests when his state is good and player 2 always invests.

PROOF. Clearly, player 1 will not invest if his state is bad. If player 2 receives no messages, she assigns probability $\frac{\frac{1}{2}}{\frac{1}{2}+\frac{1}{2}(1-F(T))}$ to player 1's state being bad and probability $\frac{\frac{1}{2}(1-F(T))}{\frac{1}{2}+\frac{1}{2}(1-F(T))}$ to player 1's state being good but player 1's first message never arriving. Thus her expected payoff is at most

$$\frac{\frac{1}{2}(1 - F(T))}{\frac{1}{2} + \frac{1}{2}(1 - F(T))}(1) - \frac{\frac{1}{2}}{\frac{1}{2} + \frac{1}{2}(1 - F(T))}(c) = \frac{1 - F(T) - c}{2 - F(T)} < 0.$$

Now let t^* be the earliest date for which a player who has sent a message at date t^* (but never received a confirmation) is prepared to invest with positive probability. He will assign probability $\frac{L(t^*)}{1+L(t^*)}$ to his message having arrived (but not the confirmation); and he assigns probability $\frac{1}{1+L(t^*)}$ to his message having never arrived. If his message never arrived, his opponent's last message (if any) must have been sent before date t, so his opponent cannot be investing. Thus his expected payoff at most

$$\frac{L(t^*)}{1+L(t^*)}(1) - \frac{1}{1+L(t^*)}(c) = \frac{L(t^*)-c}{1+L(t^*)}.$$

This is negative since $c > L(t^*)$. ■

To illustrate Proposition 4.1, suppose the density is exponential with $f(t) = \lambda e^{-\lambda t}$. Then

$$\begin{aligned} F(t) &= 1 - e^{-\lambda t} \\ N(t) &= e^{-\lambda(T-t)} \\ A(t) &= \int_{\tau=0}^{T-t} \lambda e^{-\lambda\tau} e^{-\lambda(T-t-\tau)} d\tau = \lambda e^{-\lambda(T-t)}(T-t) \\ L(t) &= \lambda(T-t) \end{aligned}$$

Thus if $c > \lambda T$ and $c > e^{-\lambda T}$, then there is no equilibrium where investment takes place. Since c is the ratio of the cost of investment to the potential benefit from investment and λT is the expected number of messages that can be sent and delivered, this is a rather restrictive condition. However, consider a density that is a sum of exponentials: $f(t) = \varepsilon\underline{\lambda}e^{-\underline{\lambda}t} + (1-\varepsilon)\overline{\lambda}e^{-\overline{\lambda}t}$. As $\underline{\lambda} \to 0$ and $\overline{\lambda} \to \infty$, this generates essentially the e-mail information system (with probability $1-\varepsilon$, the message arrives very quickly; with probability ε, it takes a very long time to arrive). In this case we have,

$$\begin{aligned} F(t) &= 1 - \varepsilon e^{-\underline{\lambda}t} - (1-\varepsilon)e^{-\overline{\lambda}t} \\ N(t) &= \varepsilon e^{-\underline{\lambda}(T-t)} + (1-\varepsilon)e^{-\overline{\lambda}(T-t)} \\ A(t) &= \varepsilon^2\underline{\lambda}e^{-\underline{\lambda}(T-t)} + (1-\varepsilon)^2\overline{\lambda}e^{-\overline{\lambda}(T-t)} + \varepsilon(1-\varepsilon)\left(\frac{\overline{\lambda}+\underline{\lambda}}{\overline{\lambda}-\underline{\lambda}}\right)\left(e^{-\underline{\lambda}(T-t)} - e^{-\overline{\lambda}(T-t)}\right) \\ L(t) &= \frac{\varepsilon^2\underline{\lambda}e^{-\underline{\lambda}(T-t)} + (1-\varepsilon)^2\overline{\lambda}e^{-\overline{\lambda}(T-t)} + \varepsilon(1-\varepsilon)\left(\frac{\overline{\lambda}+\underline{\lambda}}{\overline{\lambda}-\underline{\lambda}}\right)\left(e^{-\underline{\lambda}(T-t)} - e^{-\overline{\lambda}(T-t)}\right)}{\varepsilon e^{-\underline{\lambda}(T-t)} + (1-\varepsilon)e^{-\overline{\lambda}(T-t)}} \end{aligned}$$

Since $L(t)$ is decreasing, if $L(0) < c$ there is no investment ever. Figure 1 plots the values of $L(0)$ as $\underline{\lambda}$ varies, setting $T = 1$, $\varepsilon = \frac{1}{10}$ and $\overline{\lambda} = 100$.

insert figure 1 around here

5. Locally Public Communication

In the scenario of the electronic mail game, there is a lack of common knowledge because players never get to interact simultaneously. But technology that generates such simultaneous - and thus public - interactions (e.g., telephones) is readily available, so that particular two person scenario may not be of much practical importance. But in many contexts, there are strategic complementarities between the actions of a large population, there is uncertainty about the gains from or likelihood of success of a risky action, there is much information transmission within and between different groups in the population, but there is no information transmission that is public to the whole population. For example, there may be a lot of information transmission among a population on the edge of revolt against a repressive government, via both casual meetings of very small groups and clandestine meetings of larger groups, but there may not be information channels (such as the media) that transmit information publicly to large subsets of the population. Or consider media coverage of the economy. If all media channels are simultaneously reporting the same good news about the economy, and all report this fact, then the good news in common knowledge, and we may all be prepared to take risky actions on the understanding that others are doing so. On the other hand, if different news groups are reporting different things, then there is much information transmission of relevant information that occurs publicly within subsets of the population.[9] In each of these cases, different groups within the population either publicly (within the group) receive information, or may meet as a group and use the opportunity to communicate information, publicly within the group. If one such group receives positive information about the gains to a risky action, and are prepared to take the risky action independent of what people outside the group do, then the coordination problem is solved. But what if the groups are two small to make the risky action viable? How and when can common knowledge within the groups be translated into coordinated action?

[9] See Chwe (2000) and Morris and Shin (2001), respectively, for more discussion of these applications.

In this section, a concrete scenario - generalizing the electronic mail game to many players - is described and analyzed. A population of N players will have to decide simultaneously whether to invest or not. A player's payoff from investing is 1 if a "public state" is good and at least n players (including himself) invest, where $1 < n < N$. If the state is bad, or if less than n players end up investing, then the payoff to investing is $-c$. The payoff to not investing is always 0.

If it became common knowledge among a group of players with at least n members that the state was good, there would be an equilibrium where they go ahead and invest (independent of what they expected others to do). This would trivially occur if a group with at least n members were publicly informed of the state. We want to focus on a scenario where players receive a lot of information publicly within groups of size smaller than n, both about the public state and about what others know. The other key feature of the scenario that will be maintained from the electronic mail game is after a player receives his last piece of information, he is uncertain whether further communication takes place. While very specific assumptions about the communication structure will be adopted to keep the analysis transparent, it will be clear that these two ingredients are driving results.

The ex ante probability that the state is good is $\frac{1}{2}$. The communication phase proceeds as follows. In each period, with probability $\varepsilon > 0$, communication ceases. With probability $1 - \varepsilon$, a group of $m < n$ players are randomly chosen (with uniform probability) to receive a communication. Thus the probability than any one player receives any given communication is always $\frac{m}{N}$. If a first communication takes place (this happens with ex ante probability $1-\varepsilon$), the players receiving that communication are informed whether the state is good or bad. All subsequent communications report on the state and the history of all previous communications (including who received them).[10] However, a player does not know if any communications occurred after the last one that he received.

Eventually, all the meetings are concluded. Each player must then decide whether to invest or not, based on the history of meetings attended.

[10] If $m > \frac{N}{2}$, this communication structure could be generated by having a random number of meetings consisting of m randomly chosen players. The state becomes known at the first meeting, and at each subsequent meeting, participants report everything that they know from previous meetings.

Proposition 5.1. *If $m < n$ and*

$$c > \max\left\{1, \frac{(1-\varepsilon)\frac{N-m}{N}}{1-(1-\varepsilon)\frac{N-m}{N}}\right\},$$

then the locally public communication game has a unique equilibrium: no player ever invests.

This result is tight in the following sense. If $c \leq 1$, there is an equilibrium where all players invest as long as they do not know that the state is bad. If $c \geq 1$ and *either* $m \geq n$ *or* $c \leq \frac{(1-\varepsilon)\frac{N-m}{N}}{1-(1-\varepsilon)\frac{N-m}{N}}$ and ε is sufficiently small, then there is an equilibrium where all players invest if they know that the state is good.

To interpret this result, recall that c is the cost of investing alone relative to the gains from successful coordinated investment (normalized to 1). As can be seen from the following proof, the expression $\frac{(1-\varepsilon)\left(\frac{N-m}{N}\right)}{1-(1-\varepsilon)\left(\frac{N-m}{N}\right)}$ represents the ratio of the probabilities a player assigns to another communication having occurred and having not occurred, after the last one that he received. For small ε, this expression tends to $\frac{N-m}{m}$. Thus - as one might expect - no investment occurs when the cost of investment is high and the number of players receiving each communication is low. A striking feature of the result is that as long as $n > m$ - i.e., no single communication is enough to generate coordination - then the number of players required to invest does not effect whether investment is possible in equilibrium.

PROOF. No player will invest if he knows that the state is bad. Consider a player who has never received a communication. The fact that he has never received a communication conveys no information about the state. Thus he assigns probability $\frac{1}{2}$ to the state being good. Thus his payoff to investing is at most

$$\frac{1}{2}(-c) + \left(\frac{1}{2}\right)(1) = \frac{1}{2}(1-c) < 0.$$

Now consider a player who knows that the state is good and who has just received a communication. With probability ε, no further meetings will take place. With probability $(1-\varepsilon)^k \left(\frac{N-m}{N}\right)^k \varepsilon$, exactly k more communications will take place, but he will not be in attendance. Thus the probability that there is at least one more communication, but none that he receives, is

$$\frac{(1-\varepsilon)\left(\frac{N-m}{N}\right)\varepsilon}{1-(1-\varepsilon)\left(\frac{N-m}{N}\right)}.$$

So the probability that any player assigns to the last communication he received being the last one will always be

$$\frac{\varepsilon}{\varepsilon + \frac{(1-\varepsilon)\left(\frac{N-m}{N}\right)\varepsilon}{1-(1-\varepsilon)\left(\frac{N-m}{N}\right)}} = 1 - (1-\varepsilon)\left(\frac{N-m}{N}\right).$$

Now let k^* be the smallest $k \geq 1$ such that a player who knows that the state is good and knows about exactly k communications invests with positive probability. If his last communication was the last communication, he knows that at most m players will choose to invest (since only m players received that last communication). Thus his payoff to investing is at most

$$\left(1 - (1-\varepsilon)\left(\frac{N-m}{N}\right)\right)(-c) + (1-\varepsilon)\left(\frac{N-m}{N}\right)(1) < 0. \quad \blacksquare$$

6. Conclusion

With faulty communication, fully rational players in a one-off strategic interaction may have great difficulty co-ordinating on an efficient outcome, even if they are able to communicate a lot. This problem arises not merely in the stylized example of the e-mail game, but in more realistic environments with many players, real time communication structures and strategic decisions about whether to communicate.

Of course, people carry out coordinated action without the benefit of public plenary meetings all the time. But they are most likely to be successful in doing so if they can commit ex ante to rules of communication that generate the approximate common knowledge required for coordination. Societies presumably evolve conventions for coordinating behavior that prevent inefficiently many risky communications. Similarly, armies have rules about who sends confirmations of what communications. The analysis in this paper suggests that such conventions and rules may not be entirely self-enforcing in each interaction, but may rely on outside constraints.

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APPENDIX

We first formally define the refinements described in the text.

Definition 6.1. *Perfect Bayesian Equilibrium* $(\pi_k, x_k, y_k)_{k=1,2,..}$ *is immune to strategic uncertainty if there exists* $\left(x_k^\delta, y_k^\delta\right)_{k=1,2,..} \to (x_k, y_k)_{k=1,2,..}$ *as* $\delta \to 0$ *such that given* $(\pi_k)_{k=1,2,..}$, $\left(x_k^\delta, y_k^\delta\right)_{k=1,2,..}$ *satisfy the equilibrium conditions (3.3) for each* δ.

Definition 6.2. *Perfect Bayesian Equilibrium* $(\pi_k, x_k, y_k)_{k=1,2,..}$ *is ex post efficient if, given* $(\pi_k)_{k=1,2,..}$, $(x_k, y_k)_{k=1,2,..}$ *is the largest solution to the equilibrium conditions (3.1).*

PROOF OF PROPOSITION 3.1. A player who sends the kth message but receives no confirmation believes that with probability

$$\frac{\varepsilon}{\varepsilon + (1-\varepsilon)(1-\pi_{k+1}) + (1-\varepsilon)\pi_{k+1}\varepsilon},$$

his message never arrived. Suppose that he expects that his opponent will not invest if he did not receive that message and will invest if he does receive that message. Then his expected payoff is

$$\frac{(1-\varepsilon)(1-\pi_{k+1})+(1-\varepsilon)\pi_{k+1}\varepsilon}{\varepsilon+(1-\varepsilon)(1-\pi_{k+1})+(1-\varepsilon)\pi_{k+1}\varepsilon}(1)-\frac{\varepsilon}{\varepsilon+(1-\varepsilon)(1-\pi_{k+1})+(1-\varepsilon)\pi_{k+1}\varepsilon}(c)$$

This expression is non-negative if

$$(1-\varepsilon)(1-\pi_{k+1})+(1-\varepsilon)\pi_{k+1}\varepsilon \geq \varepsilon c$$

or

$$\pi_{k+1} \leq \frac{1-\varepsilon(1+c)}{(1-\varepsilon)^2}.$$

Thus if $\pi_k > \frac{1-\varepsilon(1+c)}{(1-\varepsilon)^2}$ for $k = 2,..,\overline{k}$, we have by the usual inductive argument that no player invests unless he knows that at least $\overline{k}$ messages have been sent. Conversely, if $\pi_{\overline{k}+1} \leq \frac{1-\varepsilon(1+c)}{(1-\varepsilon)^2}$, there is an equilibrium where both players invest they know that at least $\overline{k}$ messages have been sent. So we have:

Observation: $(\pi_k, x_k, y_k)_{k=1,2,..}$ is an ex post efficient equilibrium if and only if there exists $\overline{k} \in \{1,2,3,...,\} \cup \{\infty\}$ such that $\pi_k > \frac{1-\varepsilon(1+c)}{(1-\varepsilon)^2}$ for $2 \leq k \leq \overline{k}$; if $\overline{k} < \infty$, $\pi_{\overline{k}+1} \leq \frac{1-\varepsilon(1+c)}{(1-\varepsilon)^2}$; $x_k = 0$ for all $k \leq \overline{k}-1$ and $x_k = 1$ for all $k > \overline{k}-1$; $y_k = 0$ for all $k \leq \overline{k}$ and $y_k = 1$ for all $k > \overline{k}$.

Now we check immunity to strategic uncertainty. Fix any ex post equilibrium $(\pi_k, x_k, y_k)_{k=1,2,..}$ of the above form with finite $\overline{k}$. Consider the rate of change of x_k^δ and y_k^δ with respect to δ, evaluated at $\delta = 0$. Since $f_\delta(0) = 0$ and $f_\delta(1) = 1$, this rate of change is equal to zero for all y_k^δ and all x_k^δ with $k \neq \overline{k}$ (since the limiting equations are of the form y_k or x_k equal to $f_0(0)$ or $f_1(1)$). But

$$x_{\overline{k}} = f_0(1-\varepsilon),$$

so $\left.\frac{dx_{\overline{k}}^\delta}{d\delta}\right|_{\delta=0} = \frac{d\Psi_\delta(1-\varepsilon(1+c))}{d\delta} < 0$. Thus for small δ, $x_{\overline{k}}^\delta$ will be strictly less than both x_k^δ and y_k^δ for all $k > \overline{k}$. But this implies that a strict incentive to send the $(\overline{k}+1)$th message, a contradiction.