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LIQUIDITY FLOWS AND FRAGILITY OF BUSINESS ENTERPRISES

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Abstract. This paper considers the efficiency of financial intermediation and the propagation of business cycle shocks in a model of long-term relationships between entrepreneurs and lenders, where lenders may be constrained in their short-run access to liquidity. When liquidity is low, relationships are subject to breakups that lead to loss of joint surplus. Liquidity outflows cause damage to financial structure by breaking up relationships, and damage persists due to frictions in the formation of new relationships. Feedbacks between aggregate investment and the structure of intermediation greatly magnify the effects of shocks. For large shocks, financial collapse may become inescapable in the absence of external intervention.

1. Introduction

This paper develops a new approach to modelling frictions in credit markets, built on two key hypotheses. First, we assume that financial intermediaries channel funds to profitable investment opportunities through *long-term relationships* established with the personnel who operate the investment. These relationships convey benefits to their members, and

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losses may be suffered when relationships are severed. Second, the flow of funds within a relationship is constrained by the lender's ability to acquire funds, i.e. the relationship itself may be liquidity constrained in the short run. We refer to our approach as the *liquidity flows* model of financial intermediation.[1]

In the liquidity flows model, restricted access to liquidity can lead to severance of relationships, even though the borrower and lender would obtain positive joint surplus from continuing the relationship. In this way, business enterprises are *fragile* with respect to variations in liquidity flow. An outflow of liquidity in the aggregate induces a spike in severances and consequent destruction of valuable enterprises that are essential to the channelling of liquidity. In addition to the losses of surplus borne by the members of the severed relationships, the overall structure of financial intermediation is altered in a manner that impairs the efficiency with which liquidity is utilized.

We explore these ideas using a matching and contracting framework that builds on the labor market models of Mortensen and Pissarides (1994) and Ramey and Watson (1997a). Business enterprises are modelled as long-term *liquidity relationships* between an entrepreneur, who exerts effort to operate the firm, and a lender, who channels needed liquidity into the firm. The entrepreneur and lender can freely contract with one another, subject to two restrictions: contractual payments cannot be enforced in the event that the relationship is severed, i.e. there is limited liability; and the lender has access to a limited amount of liquidity.[2] We show that the entrepreneur and lender will sever their relationship when liquidity is low, even though it would be mutually beneficial for the agents to continue the relationship. Such breakups occur despite the fact that agents are free to renegotiate their contracts, i.e. inefficient severance does not result from any suppression of renegotiation.

Liquidity relationships are formed through a matching process, whereby unmatched lenders seek to identify successful new projects from pools of proposals submitted by prospec-

[1] Empirical evidence supporting these two hypotheses is discussed in the following section.

[2] In our setting, lenders are illiquid to the extent that they may be unable to draw on future-period returns to make needed current-period payments to the entrepreneur.

tive entrepreneurs.. The efficiency of financial intermediation is positively related to the number of active liquidity relationships. An outflow of aggregate liquidity reduces the prospective liquidity flow obtaining to each lender, and thereby induces a spike in the rate at which relationships break up. Output is persistently lower, as relationships must be rebuilt gradually through the matching process. In the long run, high levels of aggregate liquidity are shown to correspond with lower rates of business failures, greater ease in locating profitable investment opportunities, and a larger number of business enterprises.

We endogenize the determination of aggregate liquidity by incorporating an investment decision, giving rise to investment feedbacks that greatly reinforce the harmful effects of fragility. Investment feedbacks are significant because financial intermediation and investment are complementary: a larger number of liquidity relationships increases the return to investment, while greater investment supports a larger number of relationships. As a consequence, the improvements in financial structure that flow from higher liquidity generate increasing returns to investment. This can lead to multiple steady-state equilibria associated with different levels of aggregate liquidity, including a financial collapse equilibrium, in which all credit market activity ceases.

Investment feedbacks also serve as a mechanism for propagating business cycle shocks. Upon impact, a negative shock reduces the number of investment relationships; this leads to a fall in investment, which induces a larger and more persistent decline in the number of relationships. The effect of the shock is thereby magnified and made more persistent. We show that, if the shock is sufficiently large, capital market collapse becomes the *unique* equilibrium outcome: agents' attempts to rebuild financial structure through rematching are swamped by the ongoing investment feedbacks that destroy relationships, so that the market cannot escape collapse. Here sustained policy intervention to restore liquidity becomes essential to reviving the capital market.

We extend the model by allowing existing liquidity relationships to seek out a second lender, in order to build financial depth and thereby to reduce the probability of surplus-reducing breakup. Despite their advantages, two-lender relationships do not necessarily emerge in equilibrium, as the rent extracted by second lenders from one-lender relationships may exceed their added value. Further, even when two-lender relationships are formed, aggregate output may be reduced relative to one-lender equilibria, due to diminishing returns within relationships. Overall, the model shows how the structure of financial relationships, as well as the efficiency of intermediation, are determined by agents' responses to the problem of limited liquidity access.

Our framework for modelling capital-market imperfections offers an alternative to the "internal equity" approach that has been used to study the effects of financial frictions on aggregate fluctuations; see Bernanke and Gertler (1989), Kiyotaki and Moore (1997) and Carlstrom and Fuerst (1997). In the internal equity approach, the severity of contracting problems depends on the size of equity stakes owned by the firms' managers. Shocks are propagated through their effects on the equity stakes. The liquidity flows approach, in contrast, abstracts from the allocation of ownership within the firm, and instead focuses on the effects of limited access to liquidity. In essence, our emphasis is on the balance sheet of the intermediary, as opposed to that of the borrower.[3] Further, in restricting access to liquidity, our approach is reminiscent of "limited participation" models of monetary propagation, including Lucas (1990), Fuerst (1992) and Christiano and Eichenbaum (1995). Our innovation involves tying participation to long-term relationships that are subject to matching frictions.

Long-term borrower-lender relationships have been previously considered by Sharpe (1990), Gertler (1992) and Rajan (1993); these papers abstract from market interaction and focus on problems created by asymmetric information between borrowers and lenders. Connections between financial intermediation and the efficiency of investment have been analyzed by Greenwood and Jovanovic (1990) and King and Levine (1993); in these papers, greater liquidity raises the steady-state quality of investment projects. Within the context of employ-

[3] By focussing on access to liquidity as opposed to ownership, the liquidity flows model avoids the criticism of internal equity models that managers must hold unrealistically large equity stakes in order for propagation effects to be important.

ment relationships, den Haan, Ramey and Watson (1997) show how shocks are propagated by feedbacks between aggregate savings and the number or relationships, mediated by the rental rate of capital.

Recently, Cooper and Corbae (1997) have developed a model of financial collapse based on coordination failure in financial intermediation. In their paper, households must simultaneously commit to payments in order to finance the fixed costs of intermediation, and collapse occurs when households believe that other households will not contribute. Periodic collapse outcomes are tied to a sunspot process. The current paper instead considers coordination failure resulting from complementarity between intermediation and investment, and it links dynamics to a matching process that governs the evolution of financial structure. We show that collapse can occur as the unique equilibrium following a large shock, rather than as one of several equilibria that are conditioned on a sunspot.[4]

Section 2 presents empirical motivation for our approach, Section 3 lays out the basic model of liquidity relationships, and Section 4 incorporates capital-market matching and the liquidity allocation rule. Results for exogenous paths of aggregate liquidity are given in Section 5, while Section 6 incorporates investment feedbacks, and Section 7 extends the model to allow for two-lender relationships. Section 8 concludes.

2. Empirical Motivation

In the U.S. market for loanable funds, firms' liquidity needs are largely met through long-term relationships with lenders. Petersen and Rajan (1994), for example, report that firms tend to concentrate their borrowing to a single source: the smallest 10 percent of firms who have a bank as their largest single lender secure, on average, 95 per cent of their loans, by value, from that bank. This figure drops to a still-significant 76 percent of loans for the largest 10 percent of firms. Geographic proximity plays a role in borrower/lender relationships. For

[4] Dell'Ariccia and Garibaldi (1998) have recently developed a matching model of bank lending, focussing on how matching frictions and breakup costs affect the dynamic responses to short-term interest rate shocks. Their model abstracts from contracting problems between banks and borrowers, as well as aggregate investment flows.

their sample, Petersen and Rajan (1995) find that 50 percent of firms are located within two miles of their primary lending institution, while 90 percent of firms are within 15 miles. Brewster Stearns and Mizruchi (1993) present evidence that social ties within relationships are also important.

The significance of borrower/lender relationships can be inferred from their economic ramifications. Thus, Hoshi, Kashyap and Scharfstein (1990,1993) argue that Japanese firms' connections to *Keiretsu*, which are large industrial groups within which member banks supply liquidity, help shield investment from firm-specific liquidity fluctuations. Evidence from Berger and Udell (1995) and Gibson (1995) for the U.S. and Japan, respectively, indicates that the financial health of banks relates positively with the amount of investment by their borrowers. Here the channeling of liquidity through relationships shows up clearly in the effects on investment.

Further evidence of the value of relationships comes from Petersen and Rajan (1994), who find that capital availability increases with the age of the relationship, and Berger and Udell (1995), who show that borrowers pay significantly lower borrowing costs when they are members of more exclusive or more long-lived relationships. From this we may infer that relationships convey joint benefits that are shared with borrowers via lower costs. Events surrounding Continental Illinois in the mid 1980's, documented by Slovin, Sushka and Polonchek (1993), provide striking corroboration: stock prices of Continental Illinois' customers first fell, and then rose, with news of the bank's impending bankruptcy and the subsequent bailout plan. More recently, financial regulators in Japan have taken care to preserve firms' access to liquidity as part of a financial reform plan entailing bank failures.[5] The following assessment comes from Haines, Riding and Thomas (1991): "Finding a new bank is not a decision that most businesses take lightly. Significant costs and inconveniences are typically involved when a business decides to switch banks."

[5] As summarized by the New York Times (July 3 1998, p. C2), Japanese Finance Ministry officials stress that "(t)he advantage of the bridge bank approach is that it insures that corporate customers will continue to have access to loans, reducing the shock of a bank failure."

A second important feature of capital markets is that the allocation of liquidity to lenders, based on idiosyncratic factors, affects the availability of loanable funds. A wealth of evidence has established that banks' capital positions influence loanable funds and the response of lending to fluctuations in market conditions; see Keeley (1988), Sharpe (1995) and Kashyap and Stein (1997). Thus, banks experiencing a series of negative shocks affecting their capital have fewer funds to lend. According to Furlong (1992) and Kashyap and Stein (1997), bank size affects sensitivity of lending to market conditions, while Amos (1992) and Rose (1993) identify sectoral overspecialization of bank loan portfolios as a key factor in explaining bank failures. This evidence indicates that bank-specific factors are of great importance in determining how much liquidity is available to the bank's customers, as well as the degree to which liquidity varies over time.[6]

The allocation of liquidity is also subject to geographic segmentation. Mathis and Ulrich (1982), Neuberger and Zimmerman (1990) and Drabenstott and Meeker (1997) argue that liquidity is segmented within local and rural areas, with important segmentation effects remaining even after the recent expansion of interstate banking. The regional incidence of credit crunches gives further evidence of geographic segmentation: Amos (1992) identifies overspecialization in energy loans as the impetus to a financial crisis in the mid 1980's largely restricted to Kansas, Oklahoma and Texas, while Bernanke and Lown (1991) discuss capital inadequacies as the source of an early 1990's New England credit crunch.

Overall, it seems inappropriate, in view of this evidence, to regard the market for loanable funds as an anonymous spot market where aggregate demand meets aggregate supply. Instead there emerges a picture of borrowers and lenders forming relationships that have joint value, but are at the same time subject to idiosyncratic variations availability of funds. In the remainder of the paper, we attempt to formalize this picture and draw out its implications

[6] Availability of funds is also a key constraint in the fast-growing private equity market. As stressed by Prowse (1998), the process of raising private equity funds is time-consuming and costly, and the desirability of placing funds with private equity intermediaries depends on stochastic events like the intermediaries' recent track records.

for credit-market activity.

3. Liquidity Relationships

3.1. Contracting and Production. Our model of productive enterprises involves two types of agents: entrepreneurs who provide effort as an input to production; and lenders who supply a capital input. Effort and capital are combined to produce a single good, which may be consumed or accumulated as capital. Entrepreneurs and lenders go through a matching process, involving evaluation of proposed projects, to form long-term *liquidity relationships* that operate across time, through which capital input is channelled to the entrepreneurs. An active liquidity relationship consists of one entrepreneur and one lender, and production takes place across discrete periods $t = 1, 2, \dots$.[7] In this section we will describe the activities of a single liquidity relationship in isolation, deferring discussion of the broader credit market to Section 4.

The timing of events within a period for a given entrepreneur/lender pair is illustrated in Figure 1. In phase A, the lender obtains a strictly positive supply of previously-produced goods that may be used as capital input for period t, denoted h_t. The entrepreneur is assumed to begin the period with zero supplies of goods. Both agents observe h_t at this time. In phase B, the agents negotiate a contract governing the division of surplus. Contract negotiation may lead to dissolution of the productive enterprise in phase B, which entails severance of the entrepreneur/lender relationship. Further details of the contracting phase are discussed below.

Given that severance has not occurred, production takes place in phase C, where the entrepreneur makes his effort choice. The entrepreneur may select either high or low effort. A choice of high effort generates output of $f(h_t)$, where we assume $f(0) = 0 < f'(h_t)$ and $\lim_{h_t \to \infty} f(h_t) = \infty$. In this case, the relationship between the entrepreneur and lender continues into the following period. If the entrepreneur chooses low effort, then output

[7] In Section 7 we expand the model to allow for liquidity relationships consisting of one entrepreneur and two lenders.

is zero; instead, the entrepreneur receives a strictly positive private benefit of x, and the relationship is severed following phase C.[8] Finally, in phase D any transfers agreed to in the contracting stage are carried out by an external enforcement authority, as long as the relationship has not been severed in phases B or C.

We now spell out details of the phase B contract negotiation. Output produced in period t, together with the lender's supply of liquid assets, constitutes a liquidity pool that is verifiable for purposes of contracting. Thus, a contract specifies transfers from this liquidity pool to each agent, to be enforced in phase D in the event that the entrepreneur chooses high effort. Bargaining determines the transfers according to exogenously-specified bargaining weights, subject to the constraints that the entrepreneur must have an incentive to choose high effort and total transfers cannot exceed the available liquidity. The disagreement point is dissolution of the productive enterprise and severance of the liquidity relationship. Importantly, we assume that no transfers can be enforced in the event of severance, i.e. there is *limited liability.* When severance occurs, the lender receives a payoff of w_t for severance in phase B, and $\beta E_t[w_{t+1}]$ for severance in phase C, where w_t indicates the value of future returns from entering the pool of unmatched lenders in period t, and β is a discount factor. Thus, by delaying severance from phase B to phase C, the lender forgoes his opportunity in the current period to locate an entrepreneur having an attractive new project. The entrepreneur obtains a payment of zero in the event of severance, and his value from entering the pool of prospective entrepreneurs also has value zero, as a consequence of free entry.

To simplify the analysis, we assume throughout that the entrepreneur has zero bargaining

[8] There are two basic motivations for why low entrepreneurial effort should lead to severance of the relationship. First, low effort may induce rapid decay of enterprise value, to the point where returns from continuation fall short of operating costs. Low effort might also be directly tied to liquidation, e.g. the entrepreneur may abscond with essential assets. Second, contractual enforcement mechanisms used by the partners to sustain cooperation may entail a costly and time-consuming dispute resolution process in the event that the entrepreneur chooses low effort; see Ramey and Watson (1997b) for a detailed discussion of such mechanisms. When dispute resolution costs are sufficiently high, the entrepreneur and lender will opt to sever their relationship instead.

weight in the phase B contract negotiation, which can be interpreted as a situation in which the lender acquires 100% equity in the enterprise. Implications of weakening this assumption, as well as the assumption that the entrepreneur receives zero liquidity at the start of each period, are discussed in the Conclusion.

3.2. Severance Decision. We now derive conditions under which the agents choose to sever their relationship. Let p_t denote the transfer to the entrepreneur specified by the phase B contract. The entrepreneur has an incentive to choose high effort if and only if

$$p_t + g_t^e \geq x, \tag{1}$$

where g_t^e indicates the present value of the entrepreneur's expected future returns from continuing the existing liquidity relationship. Observe that the right-hand side of (1) reflects only the entrepreneur's current-period private benefit of low effort, as he obtains a future value of zero when the relationship is severed.

The bargaining outcome will specify the smallest nonnegative value of p_t that satisfies (1), subject to the constraint that the entrepreneur has zero liquidity:

$$p_t = \max\{x - g_t^e, 0\}. \tag{2}$$

The lender will sever the relationship in phase B unless the contract gives him at least the value of his outside option.[9] Joint returns net of h_t may be written $f(h_t) + g_t$, where g_t indicates the present value of joint expected future returns from continuing the relationship, net of liquidity supplied in future periods. Thus, the relationship gives the entrepreneur and lender a joint surplus of $f(h_t) + g_t - w_t$, which measures the value of the enterprise. Using (2), it follows that the lender's share exceeds the outside option value if and only if

[9]Note that the lender will not agree in phase B to an arrangement that allows the entrepreneur to choose low effort in the ensuing phase C: severance following phase C would give the lender a payoff of $\beta E_t[w_{t+1}]$, which is strictly less than the payoff w_t that the lender obtains by inducing severance in phase B. This is because under the latter policy, the lender can form a new relationship in the current period, giving a payoff no lower than the next-period severance value w_{t+1}, and strictly higher with positive probability.

$$\begin{aligned} & f(h_t) + g_t - (p_t + g_t^e) \\ = \quad & f(h_t) + g_t - \max\{x, g_t^e\} \geq w_t. \end{aligned} \tag{3}$$

Further, high effort can be sustained in period t only if the available liquidity pool is sufficiently large to finance this level of p_t. A necessary and sufficient condition for this is

$$f(h_t) + h_t \geq p_t. \tag{4}$$

It follows that the relationship will be severed in phase B if either (3) or (4) are violated. Conversely, if both conditions hold, then the agents will choose to continue their relationship: (3) implies $f(h_t) + g_t - w_t > 0$, so that continuation generates positive joint surplus; and p_t can be chosen to satisfy both the effort choice and liquidity constraints while giving positive surplus to the lender. This completes the proof of the following proposition.

Proposition 1. *The liquidity relationship is severed in period t if and only if either (3) or (4) fail to hold.*

According to Proposition 1, credit market relationships are *fragile*, in the sense that they are subject to breakdown even when they yield positive joint surplus to the entrepreneur and lender. Such breakdowns of relationships may occur for two reasons. First, returns may be too low for cooperation between the entrepreneur and lender to remain feasible: as soon as (3) is violated, either the entrepreneur chooses low effort, or the lender induces severance, for *any* p_t that the agents might consider. From (3) it follows that breakdown of cooperation leads to a loss of joint surplus of up to $\max\{x, g_t^e\}$. Maintaining cooperation may also require transfers to be made to the entrepreneur that exceed the liquidity available to the lender, leading to violation of (4) and severance of the relationship. Note that p_t represents a current-period claim on the lender, which must be settled using current-period consumption goods. Thus, illiquidity arises because the lender cannot draw on future-period returns to pay the entrepreneur in the current period.[10]

It is important to note that failure of either (3) or (4) will lead to breakdown of a mutually beneficial relationship, despite the agents' complete freedom to renegotiate their contract and equilibrium selection in a joint-surplus-maximizing manner. Constraints imposed by limited liability and limited liquidity, rather than inability to renegotiate, account for the fragility of liquidity relationships.[11]

The next proposition links severance to the quantity of available liquidity. The proofs of this and subsequent propositions are given in the Appendix A.

Proposition 2. *Assume that both the lender's and entrepreneur's future surpluses from continuing the liquidity relationship are nondecreasing in h_t. Then there exists a breakup margin $\underline{h}_t > 0$ such that the relationship is severed in period t if and only if $h_t < \underline{h}_t$.*

Observe that the breakup margin is necessarily positive, reflecting the fact that a minimal amount of liquidity is needed in order to make contractual payments that sustain the relationship.[12]

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[10] Severance due to violation of (3) reflects the contractual fragility effect considered by Ramey and Watson (1997a) in the context of employment relationships. The current formulation adds the possibility that separations occur due to $g_t^e > x$, which results from entrepreneurial liquidity constraints. It is shown in Appendix A however that $g_t^e < x$ must hold under the assumption that the entrepreneur has zero bargaining weight. Condition (4) is related to Farmer's (1988a,b) analysis of liquidity-constrained labor contracting, where low liquidity leads to inefficiently low labor supply. See also Sappington (1983), Kahn and Scheinkman (1985) and Nosal (1998).

[11] As another possibility for agreement, agents might seek to temporarily suspend their relationship when (3) or (4) are violated, in order to preserve match capital. Such suspensions will be infeasible, however, if the enterprise would experience rapid deterioration in the absence of managerial effort. For example, key personnel may leave during the suspension, or market dominance may be permanently lost. Further, contracts that support temporary suspension will be infeasible if a third party enforcement authority is unable to tell whether or not suspension resulted from a breach of contract by one of the partners.

[12] The latter property follows from our assumption that the entrepreneur has zero bargaining weight, which implies $p_t = x - g_t^e > 0$; see Appendix A.

4. Market Interaction

4.1. Formation of Liquidity Relationships. We now describe how liquidity relationships are formed. Assume that the credit market contains a unit mass of lenders, each of whom begins a period either matched with an entrepreneur in a liquidity relationship, or else unmatched and seeking to form a new relationship. There is a potentially infinite mass of potential entrepreneurs, from whom unmatched lenders solicit project proposals. At the beginning of each period, unmatched entrepreneurs can prepare a proposal at an effort cost of $c > 0$ and present it to an unmatched lender. Each unmatched lender then evaluates the submitted proposals, and a new liquidity relationship is begun if the submission pool contains at least one successful project. Otherwise, the lender continues to be unmatched in the following period. For simplicity, we restrict attention to the case where unmatched lenders can accept only one successful project, i.e. liquidity relationships may involve only one entrepreneur.

Let the total flow of new liquidity relationships in a period be given by $m(U_t, V_t)$, where U_t indicates the mass of unmatched lenders and V_t gives the mass of entrepreneurs who submit proposals. The function $m(U_t, V_t)$ is nonnegative, strictly increasing in each of its arguments, and satisfies constant returns to scale and $m(U_t, V_t) \leq \min\{U_t, V_t\}$. This matching function may be interpreted in terms of the probability that an unmatched lender locates a successful project from the pool of applicants. Each unmatched lender receives V_t/U_t proposals, and we may let $\lambda^l(V_t/U_t)$ denote the probability that at least one successful project is obtained. Then the matching function may be defined as $m(U_t, V_t) = U_t\lambda^l(V_t/U_t)$, and the assumed properties of $m(U_t, V_t)$ are derivable from natural restrictions on $\lambda^l(V_t/U_t)$.

The proposal evaluation process takes place in phase C, at the same time as production occurs in active liquidity relationships. Thus, lenders whose relationships are severed in the contracting phase can seek to form new relationships in the current period. Lenders who locate successful projects begin the next period in active liquidity relationships with the proposing entrepreneur.

4.2. Allocation of Aggregate Liquidity. To complete the specification of the model, we must indicate how the liquidity supplies h_t are determined. An individual lender's liquidity is related to the amount of liquidity that is supplied to the credit market in the aggregate, but it is also influenced by random factors associated with variations in the lender's individual circumstances. Let aggregate liquidity be denoted by H_t. The *liquidity allocation rule* for each lender is given by the distribution function $\nu(h_t \mid H_t)$, which is assumed to be atomless, except possibly for an atom at $h_t = 0$, and increasing in H_t according to first order stochastic dominance. Thus, greater aggregate liquidity makes it more likely that a lender will obtain a larger liquidity supply, but random variations remain. The support of $\nu(h_t \mid H_t)$ is bounded above by $h^u(H_t)$, where we assume $\lim_{H_t \to 0} h^u(H_t) = 0$. Further, to economize on special cases, we assume that $h_t = 0$ is contained in the support. This specification makes the simplifying assumption that, conditional on aggregate liquidity, idiosyncratic liquidity fluctuations are independent across lenders and over time. For example, the liquidity allocation rule may be given by $h_t = \varepsilon_t H_t$, where ε_t is an i.i.d. idiosyncratic liquidity shock satisfying $E[\varepsilon_t \mid H_t] = 1$.[13]

The liquidity allocation rule captures our idea that liquidity relationships serve as the channels through which financial intermediation is carried out. The rule incorporates idiosyncratic variations in the ability of lenders to obtain loanable funds; for example, a negative shock to a bank's capital will constrain its ability to make loans. Further, the liquidity allocation rule can incorporate unexpected fluctuations in entrepreneurs' costs that absorb liquidity. Central to our approach is the assumption that liquidity cannot freely flow between relationships in the short run, i.e. a lender cannot offset a shortage of liquidity by drawing on some centralized liquidity pool, nor can a lender with excess liquidity locate more profitable investments in the short run. In this sense, there is "limited participation" of the members of a liquidity relationship in the capital market.

[13] While it is more realistic to allow for persistence in the relationship-specific liquidity shocks, in this paper we focus on the time-independent case in the interest of simplicity.

4.3. Equilibrium. Credit market equilibrium for an exogenous path of aggregate liquidity may now be spelled out.[14] First, conditions (3) and (4) may be combined to determine the breakup margin:

$$f(\underline{h}_t) + \min\{g_t - \max\{x, g_t^e\} - w_t, \underline{h}_t - \max\{x - g_t^e, 0\}\} = 0. \tag{5}$$

Equation (5) determines the breakup margin as long as the solution satisfies $\underline{h}_t \in [0, h^u(H_t)]$. If the left-hand side of (5) is negative for all $\underline{h}_t \in [0, h^u(H_t)]$, then $\underline{h}_t = h^u(H_t)$; in this case, breakup occurs with probability one.

The joint expected future return, g_t, is determined by

$$g_t = \beta E_t[(1 - \rho_{t+1}^x) \int_{\underline{h}_{t+1}}^{\infty} f(h_{t+1}) d\nu(h_{t+1} \mid H_{t+1}) \tag{6}$$

$$+ (1 - \rho_{t+1}^x)(1 - \rho_{t+1}^n) g_{t+1} + (1 - (1 - \rho_{t+1}^x)(1 - \rho_{t+1}^n)) w_{t+1}]$$

where ρ_t^n indicates the probability that $h_t < \underline{h}_t$, so that the relationship is severed in period t due to low liquidity:

$$\rho_t^n = \int_0^{\underline{h}_t} d\nu(h_t \mid H_t). \tag{7}$$

We refer to ρ_t^n as the *endogenous breakup probability.* Further, we assume that relationships may be severed for exogenous reasons, where ρ_t^x give the probability of exogenous breakup in period t. Any exogenous breakups are taken to occur between phases A and B.

The entrepreneur's expected return, g_t^e, satisfies

$$g_t^e = \beta E_t[(1 - \rho_{t+1}^x) \int_{\underline{h}_{t+1}}^{\infty} (p_{t+1} + g_{t+1}^e) d\nu(h_{t+1} \mid H_{t+1})]. \tag{8}$$

As for the lender's expected return from matching, w_t, we have

[14] Determination of aggregate liquidity will be endogenized in Section 6.

$$w_t = \lambda_t^l(g_t - g_t^e) + (1 - \lambda_t^l)\beta E_t[w_{t+1}], \tag{9}$$

where λ_t^l represents the lender's matching probability in period t:

$$\lambda_t^l = \lambda^l(V_t/U_t). \tag{10}$$

Imposing equilibrium in the matching market yields three final conditions. Let N_t denote the mass of liquidity relationships continuing into period t. The law of motion for N_t is given by

$$N_{t+1} = (1 - \rho_t^x)(1 - \rho_t^n)N_t + m(U_t, V_t), \tag{11}$$

where the first term captures liquidity relationships that continue from the preceding period, and the second term reflects newly-formed relationships. The pool of unmatched lenders in period t , U_t, may be expressed as

$$U_t = (1 - N_t) + (1 - (1 - \rho_t^x)(1 - \rho_t^n))N_t. \tag{12}$$

The first term in (12) indicates lenders who are unmatched at the start of the current period, while the second term captures lenders whose relationships have broken up in the contracting phase of the current period. Finally, the total mass of entrepreneurs submitting proposals, V_t, is determined by free entry. We assume that each proposal has the same chance of being accepted by some unmatched lender. As long as expected entrepreneurial returns exceed the proposal cost for some V_t, the free entry condition is given by

$$\frac{m(U_t, V_t)}{V_t} g_t^e = c, \tag{13}$$

Otherwise, entrepreneurs obtain zero expected returns for all V_t, and in this case $V_t = 0$.

To summarize, given paths of aggregate liquidity H_t and the exogenous breakup probability ρ_t^x, along with the initial number of relationships N_1, equations (5)-(13) jointly determine $\underline{h}_t$, g_t, ρ_t, g_t^e, w_t, λ_t, N_{t+1}, U_t and V_t for $t = 1, 2, ...$.

In steady-state equilibria of the model, aggregate liquidity and the exogenous separation probability satisfy $H_t = H$ and $\rho_t^x = \rho^x$ for all t, and the endogenous variables are constant across periods. We have

Proposition 3. *There exists a steady-state equilibrium.*

In the sequel, steady-state equilibrium values will be indicated by omitting the time subscripts.[15]

5. Aggregate Liquidity and Financial Structure

5.1. Persistent Effects of Liquidity Outflows. In the liquidity flows model, an outflow of aggregate liquidity induces a spike in the rate at which liquidity relationships break up, leading to losses of joint surplus by members of severed relationships. Breakup of relationships also causes damage to the structure of financial intermediation, in that there are fewer channels through which liquidity can flow to investment opportunities. Even after aggregate liquidity is restored, the efficiency of intermediation remains impaired since relationships can be rebuilt only gradually. Thus, the output effects of the liquidity outflow are made more persistent due to the adverse impact on financial structure.

We say that a *surprise liquidity outflow* occurs when agents anticipate that aggregate liquidity will be H in every period, but in period 1 the market is unexpectedly hit with $H' < H$. Liquidity is restored to H for all periods following period 1, which is expected by the agents. Thus, the liquidity path is $H_1 = H'$, $H_t = H$, $t = 2, 3, ...$, while the initial condition N_1 derives from the steady-state equilibrium for H. The implications of such a surprise outflow are derived in the following proposition.

Proposition 4. *Suppose the market begins in a steady-state equilibrium with* $\rho^n \in (0, 1)$. *If there is a surprise liquidity outflow in period* 1, *then there exists an equilibrium having the*

[15] Multiple steady-state equilibria are possible for given values of the parameters. The source of multiplicity is the fact that the value of $g - w$ determined by (6)-(9) may be decreasing in $\underline{h}$, so that (5) might hold for multiple values of $\underline{h}$.

following properties: (a) The endogenous breakup probability rises in period 1*, then returns to its steady-state value* ρ^n *in future periods; (b) The number of active liquidity relationships and aggregate output fall in period* 1*, then rise monotonically to their steady-state values; (c) All other variables remain at their steady-state values.*

An equilibrium of this form is illustrated in Figure 2. For convenience, the example uses a liquidity allocation rule having a two-point support, with one of the points being $h_t = 0$.[16] The surprise liquidity outflow amounts to a one percent decline in the long-run level H. Observe that the endogenous breakup probability ρ_t^n spikes upward in period 1, reflecting the fact that when aggregate liquidity is reduced, a greater proportion of relationships obtain a liquidity supply lying below the breakup margin. This leads to a drop in the number of relationships that survive into the following period. Other variables are not affected, since agents anticipate that liquidity will be restored to its previous level; correspondingly, the endogenous breakup probability returns to its steady-state level in period 2. The number of active matches, however, returns to the steady-state level only gradually, as severed relationships must be rebuilt through the matching process.

The output effects of the surprise liquidity outflow are depicted in Figure 3, where two cases are contrasted. First, the top line gives the path of output when $x = 0$ is specified; in this case, relationships never break up, and in the steady state all lenders are paired with entrepreneurs in every period. As the figure shows, the liquidity shock reduces output in period 1, reflecting the fact that a larger number of lenders obtain zero liquidity, but output is fully restored in period 2. The bottom line shows the normalized output level in the fragile economy for the example considered in Figure 2. In contrast to the $x = 0$ case, negative output effects persist past period 1 in the fragile economy, in line with the gradual

[16] For our examples, the support of $\nu(h_t \mid H)$ is taken to consist of 0 and $h^u(H)$, where $\nu(0 \mid H) = 1 - \gamma H^{0.2}$ and $h^u(H) = H(1 - \nu(0 \mid H))^{-1}$. Although this specification does not satisfy the earlier assumption that $\nu(h_t \mid H)$ should have no atoms at values $h_t > 0$, the discrepency is inessential, since we can regard the specification as approximating a distribution function that increases sharply at $h^u(H)$. Except for one case discussed in Section 7, the examples use the paramaters $v(h_t) = h_t^{0.33}$, $x = 3$, $\beta = 0.96$, $\rho^x = 0$, $c = 0.342$, $m(U_t, V_t) = 0.25U_t^{0.5}V^{0.5}$, and $\gamma = 0.401$.

rebuilding of severed relationships. Since aggregate liquidity returns to its steady state level, the output loss following period 1 results from reduced efficiency in financial intermediation. In this example, the cumulative loss of output following period 1 amounts to about one third of the period 1 reduction.

This result shows how a financial disturbance can exert output effects lasting many periods into the future, due to its effect on liquidity relationships. The key is that financial frictions are tied to the structure of intermediation, as reflected by the stock of liquidity relationships, and the latter can adjust only gradually. It should be noted that the rate at which relationships can be reformed following a disturbance is dictated by the lender matching probability, since this determines how quickly the liquidity flows can find their way back to entrepreneurs.

5.2. Steady-State Liquidity and Financial Activity. Our model offers predictions about the steady-state relationship between aggregate liquidity and measures of financial activity, including the number of liquidity relationships and the rates at which they form and break up. These effects are illustrated in Figure 4, which reports the sets of steady-state equilibrium values of N, λ^l and ρ^n as functions of H for the parameterization used in Figures 2 and 3. Note first that for a range of very low H, no liquidity relationships form in equilibrium; thus, financial intermediation cannot occur at all unless aggregate liquidity exceeds a minimum threshold.. Absence of relationships for H close to zero will hold generally, as condition (4) cannot be satisfied when liquidity is very low, so that the endogenous breakup probability is driven to unity. For higher values of H, there is a single positive-intermediation equilibrium, having positive values of N and λ^l and values of ρ^n below unity. In the latter equilibrium, an increase in H leads to a larger number of relationships and a higher lender matching probability, while the endogenous breakup probability falls. Further, for a middle range of H, the positive-intermediation equilibrium coexists with the zero-intermediation equilibrium, i.e. there are multiple steady-states. Overall, higher aggregate liquidity supports more efficient functioning of the financial system.

These results may be shown to hold more generally for the class of *low-breakup equilibria*, which are steady-state equilibria having the lowest values of ρ^n for given values of the exogenous parameters. A straightforward comparison of low-breakup equilibria across different levels of H is possible when feedbacks between the contracting and matching processes are small. We say that $\varepsilon > 0$ gives a *variability bound* on the lender matching probability if

$$m_V(1, V/U) < \varepsilon \text{ for all } V/U \in [0, \beta x/c]. \tag{14}$$

The variability bound restricts the degree to which changes in V lead to changes in λ^l, and thereby affect the contracting equations (5)-(9). We also impose an upper bound $\overline{H}$ on the possible aggregate liquidity levels, which is easily motivated as an implication of utility-maximizing investment behavior; see Section 6. With this, we have

Proposition 5. *Let aggregate liquidity be bounded above by $\overline{H}$, and the variability bound on the lender matching probability be sufficiently small. Then starting at any low-breakup equilibrium with $\lambda^l > 0$, a reduction in H leads to a low-breakup equilibrium with strictly higher ρ^n and strictly lower λ^l and N.*

Thus, plausible conditions exist under which the properties depicted in Figure 4 hold more generally for the class of low-breakup equilibria, and in particular, when steady-state equilibria are unique.[17]

6. Investment Feedbacks

6.1. Equilibrium with Endogenous Aggregate Liquidity. We now endogenize the determination of aggregate liquidity by tying liquidity supply to external investors who provide investment goods to the lenders. The passing of goods from investors to lenders is

[17] The variability bound is needed since under the equilibrium conditions, a fall in H lowers the lender matching probability at each $\underline{h}$, which tends to reduce w relative to g and strengthen the incentive to preserve the relationship. Interestingly, it can be shown that the comparative statics implications of Proposition 5 also hold when changes in vacancies have a very *large* effect on the lender matching probability, i.e. when (14) is altered to place a large uniform lower bound on $m_V(1, V/U)$. Anomolies can arise only in the intermediate case.

subject to idiosyncratic factors, as captured by the liquidity allocation rule, while lenders are assumed to remit the net proceeds of each period's operations, together with the liquidity supplied, directly back to the investors following the contract enforcement phase. Investors are taken to have utility that is linear in wealth, with common discount factor β. Further, for simplicity we assume that each investor obtains an average draw from the pool of lenders in each period, i.e. investors and lenders do not form long-term relationships. We add the assumption $\lim_{h_t \to \infty} f'(h_t) = 0$ to assure that investment is finite.

Let the aggregate net rate of return on investment be given by

$$R_t = \frac{N_t(1 - \rho_t^x) \int_{\underline{h}_t}^{\infty} (f(h_t) - p_t) d\nu(h_t \mid H_t)}{H_t}. \tag{15}$$

In equilibrium, aggregate liquidity must satisfy

$$R_t = \frac{1}{\beta} - 1. \tag{16}$$

Thus, for a given path of exogenous breakup probabilities ρ_t^x and initial condition N_1, an *investment equilibrium* obtains if (5)-(13) and (16) are satisfied. If $R_t < 1/\beta - 1$ for all H_t, then investors select $H_t = 0$.

6.2. Investment Feedbacks in Steady-State Equilibria. Structural features of the financial intermediation process, as reflected by the number of relationships and the exogenous breakup probability, enter into the average return, as seen in (15), and thus affect the flow of liquidity from investors. These investment responses feed back on the financial structure in a manner that can reinforce fragility effects in steady-state equilibria.

First of all, very strong feedbacks can suppress all economic activity in the capital market. As demonstrated in Section 5.2, low levels of aggregate liquidity will induce collapse of the credit market, and investment returns will be zero as a consequence of $N_t = 0$. Since R_t is zero, investors in fact choose $H_t = 0$, thereby inducing the collapse. This suffices to prove the following proposition:

Proposition 6. *There exists a steady-state investment equilibrium, in which* $\lambda^l = N = H = 0$.

Here the investment feedback is stark: absence of incentives to invest corresponds to absence of incentives to form liquidity relationships. Complementarity between investment and intermediation, reflected in the interaction between the average return to investment and the number of liquidity relationships, gives rise to an extreme form of coordination failure in the form of financial collapse.

Average returns for the full range of H values are shown in Figure 5, which is based on the parameterization used to produce Figure 4. The region of low H for which $R = 0$ may be noted, and point A, at the origin, indicates the collapse equilibrium derived in Proposition 6. For a middle region of H, average returns rise with H, as the favorable effect of liquidity on financial structure outweighs the effect of diminishing returns within individual relationships. Complementarity between H and N thus generates increasing returns on this region. For high H, diminishing returns come to dominate. These conflicting effects give rise to a pair of equilibria with positive H, at points B and C. Returns for the $x = 0$ economy, in which lenders are always matched with entrepreneurs, are given in the upper curve. In contrast to the fragile economy, the $x = 0$ economy exhibits diminishing returns for all H, and the unique steady-state equilibrium lies at point D.

Because of complementarity, the response of H will tend to magnify the effects of a change in model parameters on steady-state financial activity, relative to the case where H does not adjust. To see this, consider how equilibrium values vary when the exogenous breakup probability, ρ^x, is increased. We focus here on *high-savings equilibria*, which are the steady-state savings equilibria having the highest level of H (for example, point C in Figure 5).

Proposition 7. *Let the variability bound on the lender matching probability be sufficiently small. Then starting at any high-savings equilibrium with* $H > 0$*, an increase in* ρ^x *leads to a high-savings equilibrium with strictly lower* H*. Further,* ρ^n *is increased, and* λ^l *and* N *are reduced, by a greater amount than in the case where* H *does not adjust.*

As the proposition shows, the investment response to a structural change that decreases the efficiency of intermediation serves to reinforce the adverse direct effects on endogenous breakup and matching probabilities and the number of relationships.

6.3. Investment Feedbacks and Propagation of Shocks. Feedbacks from investment to intermediation also help to propagate shocks to the capital market. To illustrate how shocks are propagated, we consider the market response to a *financial structure shock*, which is a surprise increase in the exogenous breakup probability, caused by changes in government regulations or consumer tastes, for example. In this case, agents expect the exogenous breakup probability to remain at ρ^x, but in period 1 proportion $\rho^{x\prime} > \rho^x$ of active relationships experience exogenous breakup. The path of exogenous breakup probabilities is thus given by $\rho_1^x = \rho^{x\prime}$, $\rho_t^x = \rho^x$, $t = 2, 3, ...$, with N_1 obtained from the steady-state equilibrium for ρ^x.[18]

Figure 6 presents numerically calculated equilibrium values associated with a financial structure shock.[19] Note the large and persistent decline in aggregate liquidity following the shock, which reflects the investment response to higher levels of ρ_t^n and lower levels of N_t. Correspondingly, ρ_t^n remains persistently above its steady-state level. This stands in contrast to the case of fixed H_t, depicted in Figure 2, where ρ_t^n returns to the steady-state immediately following the shock. The resulting effects on output are shown in Figure 7, which compares output in the investment equilibrium to the path that would emerge if H_t were held fixed. Observe that investment feedbacks serve to magnify the shock on impact, and overall they roughly double the output loss in this example.

The credit-market response to financial structure shocks involves two competing effects. On one hand, structure is repaired via the matching process, as liquidity relationships are reformed. On the other hand, adverse investment feedbacks raise the rate at which relationships break up along the adjustment path. The latter effect can dominate, so that the market

[18]Note that, according to (16) and (15), investment can respond in the current period to surprise movements in ρ_t^x. This reflects rapid movements of aggregate liquidity into and out of the capital market.

[19]In the example, we set $\rho^{x\prime} = 0.01$, while $\rho^x = 0$ as in the earlier examples.

becomes unable to escape the collapse outcome. The next proposition gives conditions under which this situation can arise.

Proposition 8. *Let $c/\beta x$ lie sufficiently close to $m(\infty, 1)$. For the path of exogenous breakup probabilities $\rho_1^x = \rho^{x\prime} > \rho^x = \rho_t^x$, $t = 2, 3, ...$ and initial condition $N_1 > 0$, if $\rho^{x\prime}$ is sufficiently large, then the unique investment equilibrium has $H_t = N_{t+1} = 0$ for all t.*

The key point here is that collapse of the credit market emerges as the *unique equilibrium* for a sufficiently large financial structure shock. In contrast to the existing literature, coordination failure in this case does not entail selection or sunspot arguments that serve to align agents' expectations. What happens instead is that a big shock does so much damage to financial structure that recovery becomes impossible. Proposition 8 identifies a condition on the vacancy posting cost, c, that is sufficient to ensure that the rate at which relationships are rematched is too slow to offset the ongoing increase in their destruction due to investment feedbacks.[20] The collapse outcome becomes an absorbing state, and only a sustained exogenous injection of liquidity can restore credit market activity.

7. Financial Depth

Our model shows how liquidity relationships face the prospect of mutually costly breakup when available liquidity is insufficient. Thus, there arises an incentive to acquire additional channels of liquidity, in order to reduce the probability of breakup. As financial depth is built up in this manner, enterprises grow larger, and their breakup probabilities decline.

To consider these issues, we extend the model to allow for liquidity relationships consisting of one entrepreneur and two lenders. Thus, relationships consisting of one entrepreneur and one lender may submit project proposals to unmatched lenders, along with unmatched

[20] It should be noted that steady-state equilibria with positive investment can exist under conditions supporting Proposition 8. In particular, assume that $\rho^x = 0$, and also that for given $\underline{h}$, the probability of $h \leq \underline{h}$ may be made arbitrarily small by taking H sufficiently large. Then positive-investment equilibria will exist under the conditions of the theorem as long as $f(h_t)$ is sufficiently large for h_t outside of a neighborhood of zero, i.e. for a sufficiently high level of productivity.

entrepreneurs who seek their first lender. One-lender relationships seeking a second lender do so at the same time as they produce. Assume that only one proposal may be submitted in a given period, i.e. entrepreneurs cannot match with two lenders in a given period. Once a second lender agrees to join a one-lender relationship, the relationship obtains two draws h_t^1 and h_t^2 from the distribution $\nu(h_t \mid H_t)$ in each period.

We add the following assumptions. First, the vacancy posting cost for one-lender relationships is zero. Second, at the point when the second lender is located, the second lender's bargaining weight is chosen in a manner that extracts all of his surplus from joining the relationship, i.e. his value from joining is equal to the value of remaining unmatched. Besides simplifying the model, each of these assumptions serve to increase the attractiveness to one-lender relationships of seeking a second lender.

Equilibrium conditions for the two-lender model are spelled out in Appendix B, and here we consider steady-state equilibria for the case of fixed aggregate liquidity. Figure 8 reports the equilibrium numbers of one- and two-lender relationships in comparison to the level of aggregate liquidity. We use the parameterization considered in the previous examples, with one important exception: in the earlier examples, the specification of $f(h_t)$ exhibits sharply diminishing returns, while here we consider a specification having nearly constant returns.[21] For a low range of H, one-lender relationships choose not to seek a second lender, as the rent extracted by the second lender (equal to the value of his outside option) exceeds the added benefits associated with a lower breakup probability. For a high range of H, however, the comparison goes in the opposite direction, and equilibria have both one- and two-lender relationships. Note that as H rises, the proportion of relationships having two lenders increases, as does the total number of relationships.

Figure 9 compares the endogenous breakup probabilities of one- and two-lender relationships for the upper range of H where two-lender equilibria exist. Observe that two-lender relationships have a significantly lower breakup probability, representing the key benefit of

[21] In particular, the earlier specification $f(h_t) = h_t^{0.33}$ is replaced by $f(h_t) = h_t^{0.9}$. Other aspects of the paramaterization remain unchanged. The figures report equilibrium values in low-breakup equilibria.

acquiring a second lender. The relationship between the latter benefit and aggregate liquidity explains why there are no two-lender equilibria on the lower range: when H is low, the breakup probability with two lenders is still high enough that it fails to outweigh the second lender's rent extraction. Only for larger H does the benefit of a second lender become sufficiently great.

It should be further noted that two-lender relationships will tend to be older than one-lender relationships, as the latter must pass through a phase of matching before they can become the former. Thus, these results are consistent with the observation that larger, older firms tend to have lower failure rates.[22] More broadly, the model rationalizes the process whereby enterprises build financial depth, through acquiring additional liquidity channels, in order to reduce their exposure to hazards associated with limited short-run liquidity access. Extremely large enterprises, having tiny breakup probabilities, would be possible if the model were extended further to allow for even more lenders. However, buildup of financial depth is not inevitable, as we have seen that the market does not move beyond one-lender relationships unless aggregate liquidity becomes sufficiently high.

Even when agents do have an incentive to form large enterprises, it is not necessarily true that economic efficiency is enhanced, in that returns to liquidity are subject to diminishing returns within relationships. This point is considered in Figure 10, where steady-state output in one- and two-lender equilibria is compared for the example in Figures 8 and 9. At around $H = 40$, two-lender relationships emerge in steady-state equilibrium, and output is increased by approximately 10 per cent relative to the equilibrium where only one lender is allowed. As H approaches 70, however, output becomes lower in the two-lender equilibria, even as agents continue to form two-lender relationships. In the latter case, the aggregate benefits of

[22] See Cooley and Quadrini (1998a,b) for discussions of empirical evidence relating firm size, age and failure rates. They also present an alternative theory that can account for the evidence, relating firm size to the gradual buildup of internal equity through accumulated profits.

having less fragile relationships are outweighed by diminishing returns within relationships.[23] The example demonstrates that private incentives to build financial depth need not align with social incentives, and that equilibrium financial structure may entail enterprises that are suboptimally large.

8. Conclusion

We have provided a new model of credit markets, based on the hypotheses that liquidity is channeled to borrowers through long-term relationships, and lenders may be constrained in their ability to obtain funds in the short run. The model shows how relationships may experience privately inefficient breakups when liquidity is scarce. Further, financial intermediation becomes less efficient when there are fewer relationships. Surprise liquidity outflows are propagated through the damage they inflict to relationships, while for steady-state equilibria, markets with low liquidity are characterized by relatively sluggish formation of relatively fragile enterprises. Complementarity between financial intermediation and investment supports the existence of multiple steady states, including a collapse outcome in which all financial activity ceases. Investment feedbacks serve to magnify business cycle shocks and make them more persistent. For a sufficiently large negative shock, the collapse outcome may be inescapable.

Our model demonstrates that the internal structure of financial market relationships can impact on the process of intermediation. We show, for example, that agents may profit from adding additional lenders to their relationship, and correspondingly financial intermediation is carried out through larger and more robust enterprises. While we have restricted attention to the simple case of either one or two lenders, our approach can be usefully extended to consider large firms or banks consisting of many entrepreneurs and lenders. A liquidity outflow from such a large enterprise may lead to inefficient severance of some of the component entrepreneurs, even as the enterprise as a whole survives; in this way, persistent damage to the structure of intermediation need not entail actual bankruptcy. Further, internal structure

[23]Further, for the specification $f(h_t) = h_t^{0.33}$, diminishing returns are so pronounced that two-lender equilibria do not appear for any value of H.

will vary with changes in the underlying production or contracting technology.

The paper makes a number of simplifying assumptions that may be relaxed to produce interesting new ramifications. Most significant is our assumption that aggregate liquidity is allocated to lenders according to an arbitrary distribution function. This assumption allows us to highlight the importance of search frictions for the efficiency of financial intermediation, but it also rules out alternative institutional arrangements that may increase the availability of funds in the short run. As one possibility, we have considered the building of financial depth internally by bringing in additional lenders. A wide variety of other arrangements, both internal and external, can be introduced into our basic model. For example, institutions analogous to commercial paper or loan sales markets might be modelled using a suitable respecification of the liquidity allocation rule for a subset of relationships. The important point is that the frictions we identify here give a key rationale for the existence such institutions. The rich mix of institutions observed in practice, as well as the emergence of new solutions, can be understood in terms of trade-offs between the benefits of greater liquidity access and the costs associated with setup and operation of the institutions.

We have also made a number of simplifying assumptions with respect to the nature of contracting within liquidity relationships. Our assumption that entrepreneurs have zero bargaining weight is easily modified to allow for any specification of bargaining weights; this would complicate the analysis, but the basic results would be unaltered. The assumption that entrepreneurs enter each period with zero liquidity is more fundamental, in that they are precluded from building up stocks of liquidity that can be used to make relationships less fragile, i.e. we have ruled out internal equity effects. Such "liquidity hoarding" may occur in a more complete model, where entrepreneurs are treated as households who allocate savings between their own enterprise and intermediaries who channel savings to other enterprises. Important new issues would arise in this setting, including the extent to which entrepreneurs trade off the easing of contracting problems within their own firms against the benefits of diversifying their investments across many firms.

More broadly, households who are not entrepreneurs or managers may also prefer to channel their savings to particular enterprises, as opposed to utilizing intermediaries. The importance of retained earnings as a source of finance is, after all, indicative of ongoing inflows of liquidity from existing equity holders. The formation of long-term relationships between investing households, as equity holders, and firms may be essential to understanding the structure of corporate finance. Outside finance may involve added costs in the form of search frictions, informational or contracting problems, and liquidity access that are lessened within long-term equity relationships. Extended versions of the liquidity flows model might be able to rationalize the preponderance of finance via retained earnings relative to reallocation of liquidity across firms by financial intermediaries.

Finally, liquidity allocation may play a vital role in mediating the process whereby other capital and labor inputs are reallocated from enterprises having persistently low productivity to those with higher productivity.[24] For example, a surprise liquidity outflow might convey social benefits if it wipes out a low-productivity sector that has resisted reallocation. Financial intermediaries, however, might also accomplish this reallocation on an ongoing basis through their allocation of liquidity. There are deep connections between the way in which the market allocates liquidity in the short run and other inputs in the longer run, which represent a fascinating topic for future research.

[24] The implications of internal equity effects for reallocation of labor input have recently been considered by Caballero and Hammour (1998).

Appendix A: Proofs of Propositions

The following lemma is used throughout Appendix A.

Lemma A1. *If the entrepreneur has zero bargaining weight, then $x > g_t^e$ for every t.*

Proof. Given the entrepreneur's outside option of zero, it must be that $g_t^e \geq 0$. From (2), we see that the most an entrepreneur can obtain in any single period is x. Thus, $g_t^e \leq \sum_{k=1}^{\infty} x\beta^{k-1} = x/(1-\beta)$. Suppose there is a contingency (including realizations of h_t and other variables describing the economy) under which $g_t^e > \alpha$, for some $\alpha > x$. From (8) we know that there must be a contingency in this relationship occurring with positive probability, for which $g_{t+1}^e > \alpha/\beta$. Iterating this argument, observe that for each k there is a contingency occurring in period $t+k$ under which $g_{t+k}^e > \alpha/\beta^k$. This yields a contradiction for large k. *Q.E.D.*

Observe that Lemma A1 implies $\max\{x, g_t^e\} = x$ in our benchmark case of zero entrepreneurial bargaining weight.

Proof of Proposition 2. The lender's future surplus from continuation is given by $g_t - g_t^e - w_t$, which is assumed to be nondecreasing in h_t; since the entrepreneur's surplus g_t^e is nondecreasing, it follows that $g_t - w_t$ is also nondecreasing. Thus, if (3) or (4) hold for a given h_t, then they will hold for all larger values as well. Further, we have $\lim_{h_t \to \infty} f(h_t) = \infty$, so that both (3) and (4) are sure to be satisfied if h_t is sufficiently large. In view of Lemma A1, we have $p_t = x - g_t^e$; since $f(0) = 0$, it follows that (4) must fail to hold for h_t sufficiently small. We may define $\underline{h}_t$ as the smallest value of h_t that satisfies (3) and (4), and $\underline{h}_t$ will have the indicated properties in view of Proposition 1. *Q.E.D.*

We now list the conditions for steady-state equilibrium. To simplify the notation, define

$$Ef(\underline{h}) = (1-\rho^x)\int_{\underline{h}}^{\infty} f(h)d\nu(h \mid H),$$

Conditions (5)-(7) may be written

$$f(\underline{h}) + \min\{g - x - w, \underline{h} - x + g^e\} = 0, \tag{17}$$

$$g = \beta[Ef(\underline{h}) + (1-\rho)g + \rho w], \tag{18}$$

$$\rho = 1 - (1-\rho^x)(1 - \int_0^{\underline{h}} d\nu(h \mid H)). \tag{19}$$

Note that (19) defines ρ as the probability that breakup occurs for either endogenous or exogenous reasons. According to Lemma A1, $p + g^e = x$ for all h such that the relationship continues; thus, (8) becomes

$$g^e = \beta(1-\rho)x. \tag{20}$$

Equations (9)-(13) translate to

$$w = \lambda^l(g - g^e) + (1-\lambda^l)\beta w, \tag{21}$$

$$\lambda^l = \frac{m(U,V)}{U}, \tag{22}$$

$$N = (1-\rho)N + m(U,V), \tag{23}$$

$$U = (1-N) + \rho N, \tag{24}$$

$$\frac{m(U,V)}{V} g^e = c \text{ if } \frac{c}{g^e} < m(\infty, 1), V = 0 \text{ otherwise.} \tag{25}$$

Define

$$\varphi(\underline{h}, \rho, g^e, \lambda^l) = \frac{(1-\lambda^l)\beta Ef(\underline{h}) + \lambda^l g^e}{1-(1-\lambda^l)\beta(1-\rho)}, \tag{26}$$

Lemma A2. *If g and w satisfy (18) and (21), then $g - w = \varphi(\underline{h}, \rho, g^e, \lambda^l)$.Further, (18) and (21) are uniquely solved for g and w as functions of $\underline{h}, \rho, g^e,$ and λ^l.*

Proof. (18) and (21) may be combined to yield $g - w = \varphi(\underline{h}, \rho, g^e, \lambda^l)$. Further, as functions relating w to g, (18) and (21) are both linear. One can easily verify that $\beta < 1$ implies they have a unique intersection. *Q.E.D.*

Consider the following equation:

$$m(\frac{1}{\lambda^l}, \frac{g^e}{c}) = 1. \tag{27}$$

Lemma A3. *For $m(\infty, 1) > c/g^e$, (27) defines λ^l as an increasing and continuous function of g^e, with $\lambda^l \in [0, 1]$ and $\lambda^l \downarrow 0$ as $c/g^e \uparrow m(\infty, 1)$.*

Proof. Since $m(\infty, 1) > c/g^e$, we have $m(\infty, g^e/c) > 1$. Also, $m(1, g^e/c) \leq 1$ by assumption. That m is strictly increasing and homogeneous of degree one implies m is continuous. These facts imply there is a unique λ^l for which (27) holds, and $\lambda^l \in [0, 1]$. Further, λ^l is continuous as a function of g^e/c. Note finally that (27) is equivalent to

$$m(\frac{c}{g^e \lambda^l}, 1) = \frac{c}{g^e}.$$

As $c/g^e \uparrow m(\infty, 1)$, we must have $c/g^e\lambda^l \uparrow \infty$, whence $\lambda^l \downarrow 0$. *Q.E.D.*

The function defined by (27) may be extended continuously to values $c/g^e \geq m(\infty, 1)$ by setting $\lambda^l = 0$. Invoking (20) to substitute for g^e, we can write the function as $\lambda^l(\rho)$, strictly decreasing in ρ for $\rho < 1 - c/\beta x m(\infty, 1)$, and equal to zero for $\rho \geq 1 - c/\beta x m(\infty, 1)$.

Lemma A4. *(20) and (22)-(25) imply $\lambda^l = \lambda^l(\rho)$. Conversely, if $\lambda^l = \lambda^l(\rho)$ and (20) holds, then there exist values of N, U and V such that (22)-(25) are satisfied.*

Proof. First consider $c/g^e < m(\infty, 1)$. From (23) and (25), we have

$$V = \frac{\rho N g^e}{c}.$$

Plugging this and (24) into (25) gives

$$m(\frac{1 - N + \rho N}{\rho N}, \frac{g^e}{c}) = 1. \tag{28}$$

Finally, (22), (23) and (24) may be used to obtain

$$\lambda^l = \frac{\rho N}{1 - N + \rho N}.$$

Plugging this into (28) gives (27), and $\lambda^l = \lambda^l(\rho)$ follows using (20).

Next consider $c/g^e \geq m(\infty, 1)$. Suppose $\rho > 0$. If $N > 0$, then (24) implies $U > 0$, whence $\rho N = m(U, V) > 0$ from (23), contradicting $V = 0$. Thus, $N = 0$, which implies $U = 1$ and $\lambda^l = m(1, V/U) = 0$. Suppose instead $\rho = 0$. Then $m(U, V) = 0$, from (23), and we have $\lambda^l = 0$ for $U > 0$. For $U = V = 0$, λ^l is arbitrary, and we specify the limit as $U \downarrow 0$, which is $\lambda^l = 0$.

Conversely, suppose $\lambda^l(\rho) > 0$. Let V/U and N be defined by

$$\frac{V}{U} = \frac{\lambda^l(\rho) g^e}{c}, \tag{29}$$

$$N = \frac{m(1, V/U)}{m(1, V/U)(1 - \rho) + \rho}, \tag{30}$$

and let U be given by (24). We have $N \in [0, 1]$ as a consequence of $m(1, U/V) < 1$ and $\rho \in [0, 1]$. Plugging (29) into (27) gives (25); in turn, (25) and (29) imply (22), while (22) and (30) imply (23).

Finally, suppose $\lambda^l(\rho) = 0$. Then $m(\infty, 1) \leq c/g^e$, and setting $N = V = 0$, $U = 1$ satisfies (22)-(25). *Q.E.D.*

Proof of Proposition 3. We need to find $(\underline{h}, g, \rho, g^e, w, \lambda^l, N, U, V)$ satisfying (17)-(25), where ρ^n may be obtained from (19) using the parameter ρ^x. Let (19) be written $\rho = \rho(\underline{h})$ and define

$$\psi(\underline{h}) = f(\underline{h}) + \min\{\varphi(\underline{h}, \rho(\underline{h}), \beta(1 - \rho(\underline{h}))x, \lambda^l(\rho(\underline{h}))) - x, \tag{31}$$

$$\underline{h} - (1 - \beta(1 - \rho(\underline{h})))x\}.$$

Note that $\psi(\underline{h})$ is a continuous function.

Suppose first that we have $\psi(h^u) \leq 0$. Set $\underline{h} = h^u$, $\rho = \rho(h^u) = 1$, $g^e = 0$ and $\lambda^l = \lambda^l(1)$. Then g^e satisfies (20), and Lemmas A2 and A4 imply that we can define g, w, N, U and V such that (18) and (21)-(25) are satisfied. For these values, we have, using (31):

$$f(\underline{h}) + \min\{g - x - w, \underline{h} - x + g^e\} \leq 0.$$

This implies that the left-hand side of (17) is nonpositive for all $\underline{h} \leq h^u$, and thus $\underline{h} = h^u$ gives an equilibrium value.

The remaining possibility is $\psi(0) \leq 0 < \psi(h^u)$. By continuity, there exists $\underline{h} \in (0, h^u)$ with $\psi(\underline{h}) = 0$. Equilibrium values may be defined as above, and (17) holds at $\underline{h}$ for these values. *Q.E.D.*

Proof of Proposition 4. Let $(\underline{h}, g, \rho, g^e, w, \lambda^l, N, U, V)$ denote the initial steady-state values, which satisfy $\psi(\underline{h}) = 0$ and $\rho = \rho(\underline{h}) \in (0,1)$ at the aggregate liquidity level H. For $t = 2, 3, ,...,$ set $(\underline{h}_t, g_t, \rho_t, g^e_t, w_t, \lambda^l_t, V_t/U_t) = (\underline{h}, g, \rho, g^e, w, \lambda^l, V/U)$. Using the argument from the proof of Lemma A4, it follows that (17)-(22) and (25) are satisfied at these values, whence the corresponding equilibrium conditions (5)-(10) and (13) are satisfied for $t = 2, 3, ...$ Note that linear homogeneity of $m(U, V)$ is used here.

Now consider period 1. (6), (8) and (9) are satisfied by $g_1 = g$, $g^e_1 = g^e$ and $w_1 = w$, based on application of (18), (20) and (21). Using (17), it follows that $\underline{h}_1 = \underline{h}$ satisfies (5), and from (7) we have $\rho_1 > \rho$, since $\underline{h}_1 = \underline{h}$ and $H_1 < H$; correspondingly, $\rho^n_1 > \rho^n$. Defining $\lambda^l_1 = \lambda^l$ and $V_1/U_1 = V/U$ yields (10) and (13), using $g^e_1 = g^e$.

Next, for $t = 1, 2, ...$, let (N_{t+1}, U_t) be defined by

$$N_{t+1} = (1 - \rho_t)(1 - m(1, \frac{V_t}{U_t}))N_t + m(1, \frac{V_t}{U_t}). \tag{32}$$

with U_t given by (12). (12) and (32) may be combined to verify satisfaction of (11).

Finally, note that (23) and (24) imply that (32) holds at the steady-state values ρ, N and V/U. Then $\rho_1 > \rho$ and $V_1/U_1 = V/U$ imply $N_2 < N$, and the fact that $(1 - \rho)(1 - m(1, V/U)) < 1$ assures $N_t \uparrow N$. *Q.E.D.*

Proof of Proposition 5. Where $\lambda^l(\rho) > 0$, (20) and (27) may be combined to obtain

$$m(1, \frac{\lambda^l(\rho)\beta(1-\rho)x}{c}) = \lambda^l(\rho),$$

which implies

$$\frac{\partial\lambda^l}{\partial\rho} = \frac{-m_V(1, \frac{\lambda^l\beta(1-\rho)x}{c})\frac{\lambda^l\beta x}{c}}{1 - m_V(1, \frac{\lambda^l\beta(1-\rho)x}{c})\frac{\beta(1-\rho)x}{c}}.$$

It follows that for any $\xi > 0$, we can obtain $\partial\lambda^l/\partial\rho \in (-\xi, 0]$ for all $\rho \in [0,1]$ by choosing the variability bound sufficiently small.

Next, evaluating φ at $g^e = \beta(1-\rho)x$ and $\lambda^l = \lambda^l(\rho)$, we have

$$\frac{d}{d\rho}\varphi = \varphi_\rho - \varphi_{g^e}\beta x + \varphi_{\lambda^l}\frac{\partial\lambda^l}{\partial\rho}.$$

It can be verified using (26) that $\varphi_\rho \le 0 \le \varphi_{g^e}$, where at least one of the inequalities is strict for all ρ. Although we have $\varphi_{\lambda^l} \le 0$ in a neighborhood of equilibrium, $d\varphi/d\rho < 0$ can be assured to hold uniformly on $(\underline{h}, \rho) \in [0, \overline{H}] \times [0,1]$ by choosing ξ sufficiently small.

Now consider a low-breakup equilibrium with $\lambda^l > 0$, and let $\underline{h}'$ denote the value of $\underline{h}$ in this equilibrium. Proposition 2 implies $\rho > 0$. Further, according to Lemma A4, we have $\lambda^l = \lambda^l(\rho) > 0$, whence $\rho < 1$ and thus $\underline{h}' < h^u$. Lemmas A2 and A4 together imply $\psi(\underline{h}') = 0$, where ψ is defined in (31). Further, we must have $\psi(\underline{h}) < 0$ for every $\underline{h} < \underline{h}'$, else an equilibrium with smaller breakup probability could be constructed along the lines of the proof of Proposition 3. A reduction in H raises the value of $\rho(\underline{h})$ for every $\underline{h} \le \underline{h}'$, and so φ will be strictly lower; in turn, $\psi(\underline{h})$ will be strictly lower for all $\underline{h} \le \underline{h}'$, and the new low-breakup equilibrium must have $\underline{h} > \underline{h}'$. From this we conclude that ρ is strictly higher and λ^l is strictly lower in the new equilibrium. Finally, (22) and (23) imply

$$N = \frac{\lambda^l}{\lambda^l(1-\rho) + \rho}, \tag{33}$$

and it can be verified that N must fall when ρ rises and λ^l falls. *Q.E.D.*

Proof of Proposition 7. Note that

$$R \le \frac{Ef(0)}{H} \le \frac{f(H)}{H} \to 0$$

as $H \to \infty$. Thus, for sufficiently large $\overline{H}$, we must have $R < 1/\beta - 1$ for all $H > \overline{H}$, so that $H \le \overline{H}$ in any savings equilibrium.

Let the variability bound be small enough to obtain Proposition 5 for the value of $\overline{H}$ chosen above. For each H, let the other variables be determined by the low-breakup equilibrium associated with that H. For low-breakup equilibria with $\lambda^l = 0$, we have $N = 0$ according to (33), and so $R = 0$. Now consider low-breakup equilibria with $\lambda^l > 0$. As H rises, low-breakup equilibria have higher N and lower ρ, from Proposition 5, and the proof of Proposition 5 further establishes that $\underline{h}$ must fall. These effects lead R to rise, where we use the fact that $m = (1 - \beta(1 - \rho))x$ in steady-state equilibria. Thus, when low breakup equilibria are selected, any discontinuity in R as H increases must involve an upward jump. It follows that if H' is the savings level in the high-savings equilibrium, then $R < 1/\beta - 1$ in any low-breakup equilibrium for $H > H'$. Further, any other equilibrium associated with a given $H \ge H'$ must have greater ρ, whence greater $\underline{h}$ and smaller λ^l, and also smaller N, using (33). Thus, any high-savings equilibrium is a low-breakup equilibrium relative to the equilibrium savings level, and further, $R < 1/\beta - 1$ holds in any equilibrium associated with $H > H'$.

Now consider an increase in ρ^x. The proof of Proposition 6 is easily extended to show that, for each H, the low-breakup equilibrium must have higher ρ and lower λ^l and N, and further, $\underline{h}$ must fall; thus, we have $R < 1/\beta - 1$ for every $H \ge H'$. Conclude that the high-savings equilibrium must have a smaller level of H, leading to a further increase in ρ and further reductions in λ^l and N. *Q.E.D.*

Proof of Proposition 8. Using Lemma A1, we have $g^e_t = \beta(1 - \rho^x_{t+1})(1 - \rho^n_{t+1})x$. Thus, the following holds in any equilibrium:

$$\min\{g_t - x - w_t, \underline{h}_t - x + g^e_t\} \le \underline{h}_t - (1 - \beta)x.$$

It follows that $\underline{h}_t \geq \underline{h}'$ is implied, where $\underline{h}'$ is defined by:

$$f(\underline{h}') + \underline{h}' - (1-\beta)x = 0.$$

Now define $H' > 0$ by $h^u(H') = \underline{h}'$, and define $\rho^{x\prime}$ so that

$$\frac{N_1(1-\rho^{x\prime})f(h^u(H))}{H} < \frac{1}{\beta} - 1,$$

for all $H \geq H'$. Such a value of $\rho^{x'}$ exists due to the assumptions on f. If $\rho_1^x > \rho^{x\prime}$, then either $H_1 < H'$, implying $\rho_1^n = 1$, or $H \geq H'$, meaning that the return is less than $1/\beta - 1$ even in the absence of contracting constraints. Thus, when $\rho_1^x > \rho^{x'}$, the only value consistent with equilibrium is $H_1 = 0$.

Next, for $U_t = 1$ the entrepreneurial free entry condition is given by

$$m(\frac{1}{V_t}, 1) = \frac{c}{\beta(1-\rho_{t+1}^x)(1-\rho_{t+1}^n)x}.$$

As long as $c/\beta x$ is sufficiently close to $m(\infty, 1)$, we can be sure that V_t lies as close to zero as desired (regardless of ρ_{t+1}^x and ρ_{t+1}^n), so that $m(1, V_t) < N'$ holds, where $N' > 0$ is defined by $N'(1-\rho^x) = N_1(1-\rho^{x'})$. Note that $N_t < N'$ implies $H_t = 0$, by the same argument used above.

Now observe that $\rho_1^x < \rho^{x\prime}$ implies $\rho_1^n = 1$, and so $N_2 = m(1, V_2) < N'$. Thus, $\rho_2^n = 1$, and in fact $V_1 = 0$; then $N_2 = H_2 = 0$ is implied. By induction this result can be extended to all t, and these values give a savings equilibrium according to Proposition 7 and the fact that the right-hand side of (15) is nonpositive whenever $H_t < H'$. *Q.E.D.*

Appendix B: Equilibrium with Two-Lender Relationships

The breakup margin for a j-lender relationship, written $\underline{h}_t^j$, satisfies

$$f(\underline{h}_t^j) + \min\{g_t^j - \max\{x, g_t^{ej}\} - w_t, \underline{h}_t^j - p_t^j\} = 0,$$

where g_t^j, g_t^{ej}, and p_t^j denote the joint continuation value, the entrepreneur's continuation value, and the contracted payment to the entrepreneur, respectively, in a j-lender relationship.

Suppose a one-lender relationship successfully matches with a second lender in period t. We assume that the first and second lender can commit to long-term bargaining weights π^j, $j = 1, 2$, such that the expected future rent obtained by the second lender is equal to the value to the latter reentering the matching pool in period $t+1$, which is $\beta E_t[w_{t+1}]$. Thus, the net return to a one-lender relationship from locating a second lender is $g_t^2 - g_t^1 - \beta E_t[w_{t+1}]$, and the joint continuation value of a one-lender relationship may be written

$$g_t^1 = \beta E_t[(1 - \rho_{t+1}^x) \int_{\underline{h}_{t+1}^1}^{\infty} f(h_{t+1}^1) d\nu(h_{t+1}^1 \mid H_{t+1})$$

$$+(1 - \rho_{t+1}^x)(1 - \rho_{t+1}^{n1})(g_{t+1}^1 + \max\{\lambda_{t+1}^e(g_{t+1}^2 - g_{t+1}^1 - \beta E_{t+1}[w_{t+2}]), 0\})$$

$$+(1 - (1 - \rho_{t+1}^x)(1 - \rho_{t+1}^{n1}))w_{t+1}],$$

where ρ_t^{nj} denote the endogenous breakup probability of a j-lender relationship, and λ_t^e indicates the entrepreneurial matching probability:

$$\lambda_t^e = \frac{m(U_t, V_t)}{V_t}.$$

For two-lender relationships, we have

$$g_t^2 = \beta E_t[(1 - \rho_{t+1}^x) \int_{\underline{h}_{t+1}^2}^{\infty} f(h_{t+1}) d\nu^2(h_{t+1} \mid H_{t+1})$$

$$+(1 - \rho_{t+1}^x)(1 - \rho_{t+1}^{n2})g_{t+1}^2 + (1 - (1 - \rho_{t+1}^x)(1 - \rho_{t+1}^{n2}))w_{t+1}],$$

where $\nu^2(h_t \mid H_t)$ denotes the distribution of total liquidity $h_t = h_t^1 + h_t^2$ in two-lender relationships. The value obtained by an unmatched lender is given by

$$w_t = \lambda_t^l \frac{V_t - V_t^1}{V_t}(g_t^1 - g_t^{e1}) + (\lambda_t^l \frac{V_t^1}{V_t} + (1 - \lambda_t^l))\beta E[w_{t+1}],$$

where V_t^1 indicates the number of vacancies posted by one-lender relationships. Observe that the unmatched lender obtains $\beta E_t[w_{t+1}]$ when he is matched with a one-lender relationship.

Next consider stock-flow equations for the matching market. Let N^j_{t+1} indicate the total number of lenders in j-lender relationships. We have

$$N^1_{t+1} = (1-\rho^x_t)(1-\rho^{n1}_t)N^1_t + m(U_t,V_t)\frac{V_t - V^1_t}{V_t} - m(U_t,V_t)\frac{V^1_t}{V_t},$$

where the second term on the right-hand side indicates the inflow from new one-lender relationships, while the third term reflects the outflow from new two-lender relationships. Further:

$$N^2_{t+1} = (1-\rho^x_t)(1-\rho^{n2}_t)N^2_t + m(U_t,V_t)\frac{2V^1_t}{V_t},$$

where the second term on the right-hand side gives the inflow from new two-lender relationships. Note that each newly-formed two-lender relationship draws two lenders into the pool of lenders that are in two-lender relationships, one each from the pools of unmatched lenders and lenders in one-lender relationships. The pool of unmatched lenders is determined by

$$U_t = (1 - N^1_t - N^2_t) + (1-(1-\rho^x_t)(1-\rho^{n1}_t))N^1_t$$

$$+(1-(1-\rho^x_t)(1-\rho^{n2}_t))N^2_t.$$

Finally, free entry into the entrepreneurial matching pool determines V^1_t and V_t. As for the former, we have

$$V^1_t = N^1_t, \text{ when } g^2_t - g^1_t - \beta E_t[w_{t+1}] > 0,$$

$$V^1_t \in [0, N^1_t], \text{ when } g^2_t - g^1_t - \beta E_t[w_{t+1}] = 0,$$

$$V^1_t = 0, \text{ when } g^2_t - g^1_t - \beta E_t[w_{t+1}] < 0.$$

If $m(U_t/V_t, 1)g^{e1}_t > c$ for some $V_t > V^1_t$, then V_t satisfies

$$\frac{m(U_t,V_t)}{V_t}g^{e1}_t = c,$$

while otherwise $V_t = V^1_t$.

References

[1] Amos, O.M. Jr. (1992), "The Regional Distribution of Bank Closings in the United States from 1982 to 1988," *Southern Economic Journal* 58, 805-815.

[2] Berger, A.N., and G.F. Udell (1995), "Relationship Lending and Lines of Credit in Small Firm Finance," *Journal of Business* 68, 351-381.

[3] Bernanke, B. (1983), "Nonmonetary Effects of the Financial Crisis in the Propagation of the Great Depression," *American Economic Review* 73, 257-276.

[4] Bernanke, B., and M. Gertler (1989), "Agency Costs, Net Worth, and Business Fluctuations," *American Economic Review* 79, 14-31.

[5] Bernanke, B., and C. Lown (1991), "The Credit Crunch," *Brookings Papers on Economic Activity* 1991:2, 205-239.

[6] Brewster Stearns, L. and M. S. Mizruchi (1993), "Corporate Financing: Social and Economic Determinants," in R. Swedberg, ed., *Explorations in Economic Sociology*, New York: Russell Sage Foundation.

[7] Caballero, R.J., and M.L. Hammour (1998), "Improper Churn: Social Costs and Macroeconomic Consequences," draft, MIT, February.

[8] Carlstrom, C.T., and T.S. Fuerst (1997), "Agency Costs, Net Worth, and Business Fluctuations: A Computable General Equilibrium Analysis," *American Economic Review* 87, 893-910.

[9] Christiano,L.J., and M. Eichenbaum (1995), "Liquidity Effects, Monetary Policy, and the Business Cycle," *Journal of Money, Credit and Banking* 27, 1113-1136.

[10] Cooley, T.F., and V. Quadrini (1998a), "Monetary Policy and the Financial Decisions of Firms," draft, University of Rochester and Universitat Pompeu Fabra, February.

[11] Cooley, T.F., and V. Quadrini (1998b), "Financial Markets and Firm Dynamics," draft, University of Rochester and Universitat Pompeu Fabra, February.

[12] Cooper, R., and D. Corbae (1997), "Financial Fragility and the Great Depression," NBER Working Paper No. 6094, July.

[13] Dell'Ariccia, G., and P. Garibaldi (1998), "Bank Lending and Interest Rate Changes in a Dynamic Matching Model," IMF Working Paper WP/98/93, June.

[14] Den Haan, W.J., G. Ramey, and J. Watson (1997), "Job Destruction and Propagation of Shocks," Discussion Paper No. 97-23, Department of Economics, UCSD, October.

[15] Drabenstott, M. and L. Meeker (1997), "Improving Rural Financing," *Independent Banker,* July, 33-35.

[16] Farmer, R. (1988a), "What is a Liquidity Crisis?" *Journal of Economic Theory* 46, 1-15.

[17] Farmer, R. (1988b), "Money and Contracts," *Review of Economic Studies* 55, 431-446.

[18] Fuerst, T.S. (1992), "Liquidity, Loanable Funds, and Real Activity," *Journal of Monetary Economics* 29, 3-24.

[19] Furlong, F.T. (1992), "Capital Regulation and Bank Lending," *Federal Reserve Bank of San Francisco Economic Review* 3, 23-33.

[20] Gertler, M. (1992), "Financial Capacity and Output Fluctuations in an Economy with Multiperiod Financial Relationships," *Review of Economic Studies* 59, 455-472.

[21] Gibson, M.S. (1995), "Can Bank Health Affect Investment? Evidence from Japan," *Journal of Business*, 68, 281-308.

[22] Greenwood, J., and B. Jovanovic (1990), "Financial Development, Growth, and the Distribution of Income," *Journal of Political Economy* 98, 1076-1107.

[23] Haines, G., A. Riding, and R. Thomas (1991), "Small Business Bank Shopping in Canada," *Journal of Banking and Finance* 15, 1041-1056.

[24] Hoshi, T., A.K. Kashyap, and D.S. Scharfstein (1990), "Bank Monitoring and Investment: Evidence from the Changing Structure of Japanese Corporate Banking Relationships," in R.G. Hubbard, ed., *Asymmetric Information, Corporate Finance, and Investment*, Chicago: University of Chicago Press.

[25] Hoshi, T., A.K. Kashyap, and D.S. Scharfstein (1993), "Corporate Structure, Liquidity and Investment: Evidence from Japanese Industrial Groups," *Quarterly Journal of Economics* 106, 33-60.

[26] Kahn, C., and J. Scheinkman (1985), "Optimal Employment Contracts with Bankruptcy Constraints," *Journal of Economic Theory* 35, 343-365.

[27] Kashyap, A.K., and J.C. Stein (1997), "What Do a Million Banks Have to Say about the Transmission of Monetary Policy?" NBER Working Paper No. 6056.

[28] Keeley, M.C. (1988), "Bank Capital Regulation in the 1980s: Effective or Ineffective?" *Federal Reserve Bank of San Francisco Economic Review,* Winter, 3-20.

[29] King, R.G., and R. Levine (1993), "Finance, Entrepreneurship, and Growth: Theory and Evidence," *Journal of Monetary Economics* 32, 513-542.

[30] Kiyotaki, N., and J. Moore (1997), "Credit Cycles," *Journal of Political Economy* 105, 211-248.

[31] Levine, R. (1997), "Financial Development and Economic Growth: Views and Agenda," *Journal of Economic Literature* 35, 688-726.

[32] Lucas, R.E. Jr. (1990), "Liquidity and Interest Rates," *Journal of Economic Theory* 50, 237-264.

[33] Mathis, S. and T. Ulrich (1982), "Small Business Credit: The Competitive Factor," *Bankers Magazine* 165, 41-45.

[34] Mortensen, D.T., and C.A. Pissarides (1994), "Job Creation and Job Destruction in the Theory of Unemployment," *Review of Economic Studies* 61, 397-415.

[35] Neuberger, J.A. and G.C. Zimmerman (1990), "Bank Pricing of Retail Deposit Accounts and The California Rate Mystery," F*ederal Reserve Bank of San Francisco Economic Review,* Spring, 3-16.

[36] New York Times (1998), July 3, p. C2.

[37] Nosal, E. (1998), "Financial Distress and Underemployment," *Review of Economic Studies* 65, 817-845.

[38] Petersen, M.A., and R.G. Rajan (1994), "The Benefits of Lending Relationships: Evidence from Small Business Data," *Journal of Finance* 49, 3-37.

[39] Petersen, M.A., and R.G. Rajan (1995), "The Effect of Credit Market Competition on Lending Relationships," *Quarterly Journal of Economics* 110, 407-443.

[40] Prowse, S.D. (1998), "The Economics of the Private Equity Market," *Economic Review*, Federal Reserve Bank of Dallas, Third Quarter, 21-34.

[41] Rajan, R.G. (1992), "Insiders and Outsiders: The Choice between Informed and Arm's-Length Debt," *Journal of Finance* 47, 1367-1400.

[42] Ramey, G., and J. Watson (1997a), "Contractual Fragility, Job Destruction, and Business Cycles," *Quarterly Journal of Economics* 112, 873-911.

[43] Ramey, G., and J. Watson (1997b), "Contractual Intermediaries," Discussion Paper No. 97-26, Department of Economics, UCSD, October.

[44] Rose, S. (1993), "Comment," in Michael Klausner and Lawrence J. White, eds., *Structural Change in Banking*, Homewood, Ill.: Business One Irwin.

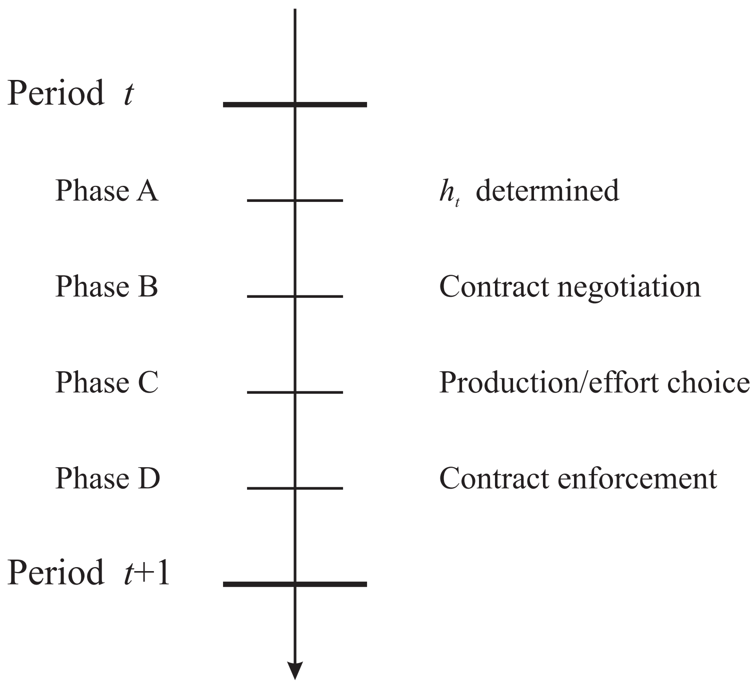
[45] Sappington, D. (1983), "Limited Liability Contracts between Principal and Agent," *Journal of Economic Theory* 29, 1-21.

[46] Sharpe, S.A. (1990), "Asymmetric Information, Bank Lending, and Implicit Contracts: A Stylized Model of Customer Relationships," *Journal of Finance* 45, 1069-1087.

[47] Sharpe, S.A. (1995), "Bank Capitalization, Regulation, and the Credit Crunch: A Critical Review of the Research Findings," Finance and Economics Discussion Series 95/20, May.

[48] Slovin, M.B., M.E. Sushka, and J.A. Polonchek (1993), "The Value of Bank Durability: Borrowers as Bank Stakeholders," *Journal of Finance* 48, 247-266.

Figure 1: Timing of events in investment relationships



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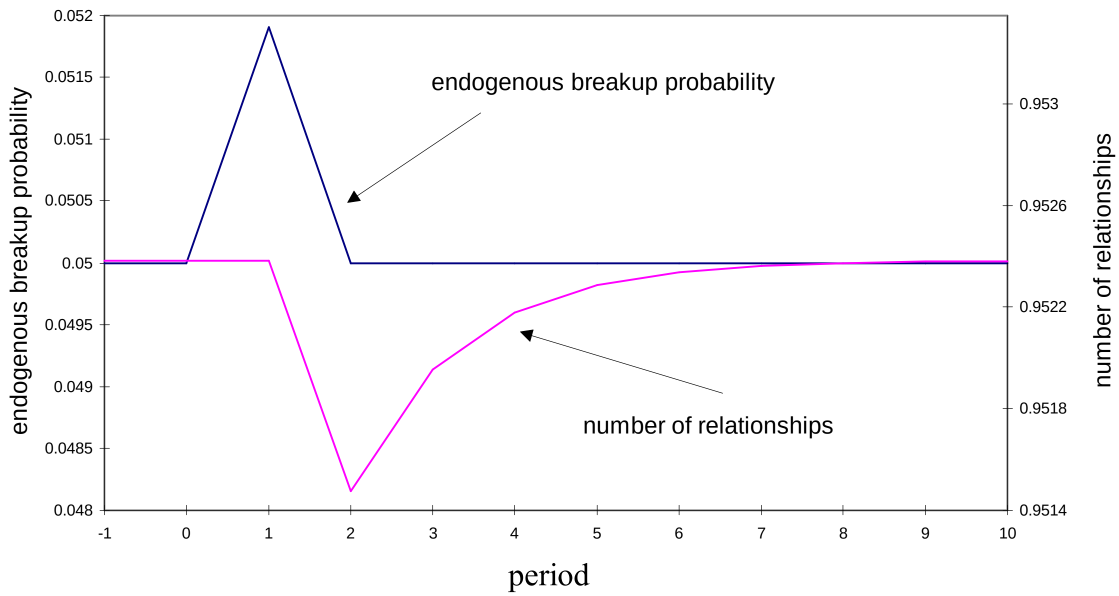


Figure : mplications for output

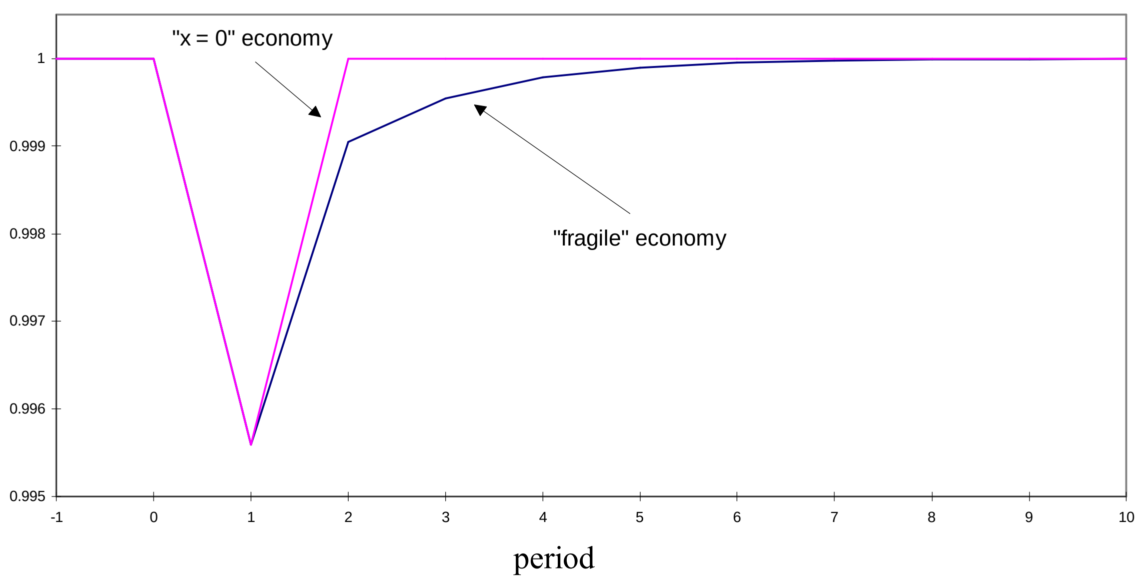
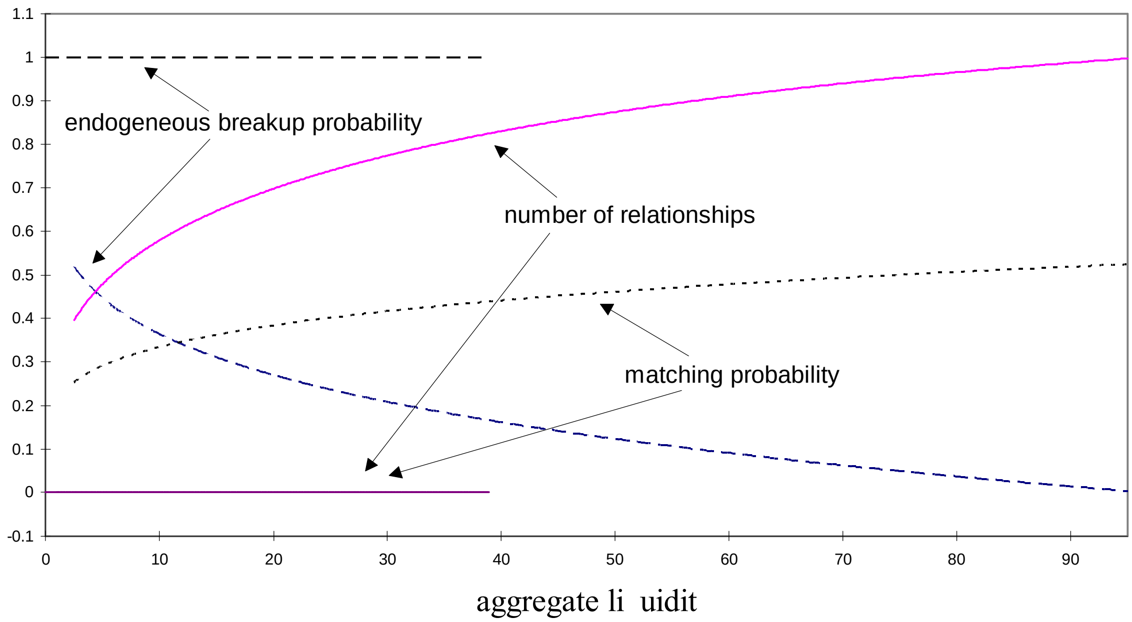


Figure : ffect of li uidit on stead state values



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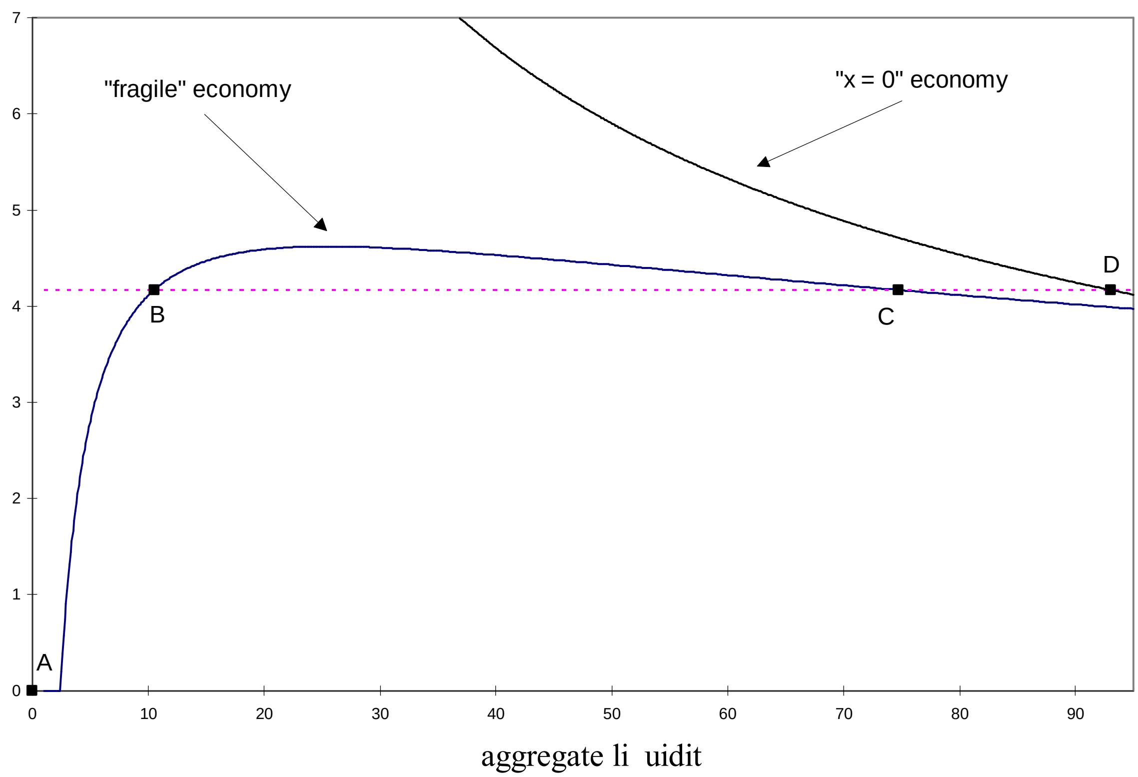

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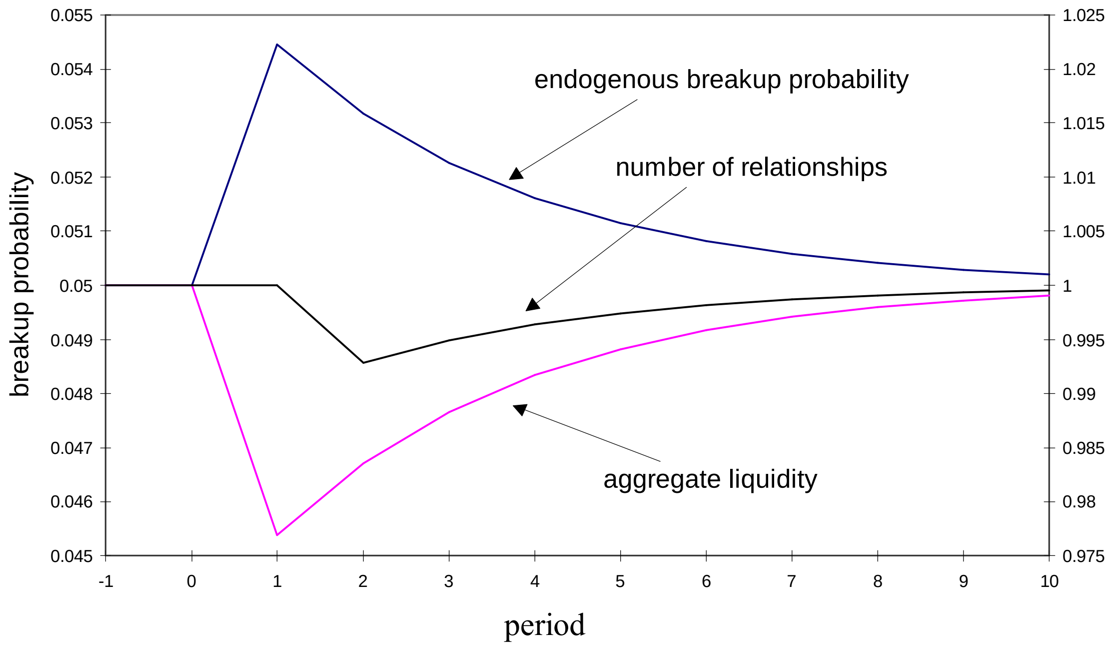

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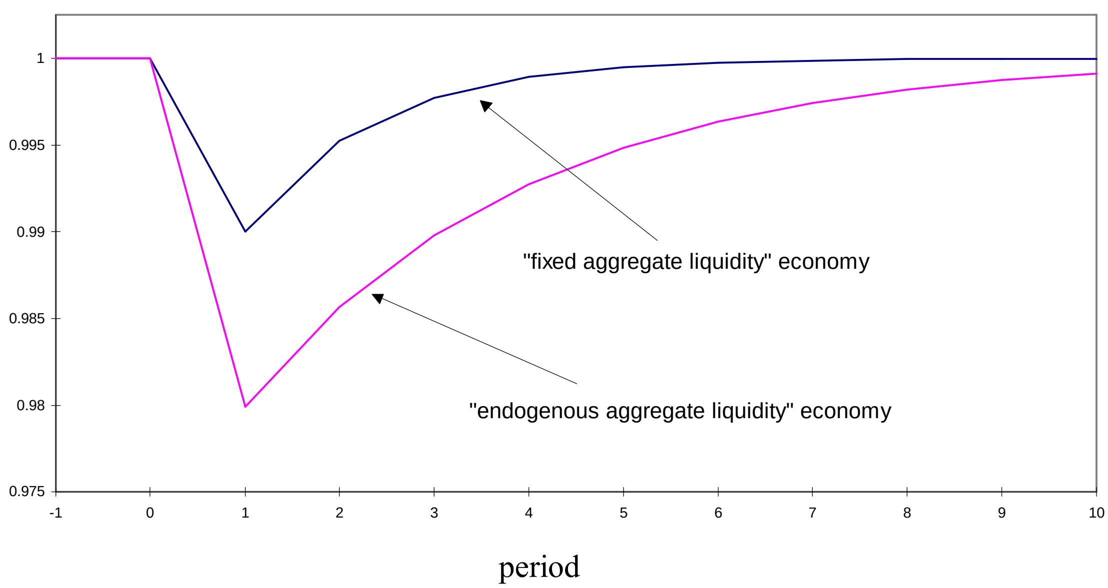

Figure : tead state values of num er of relationships

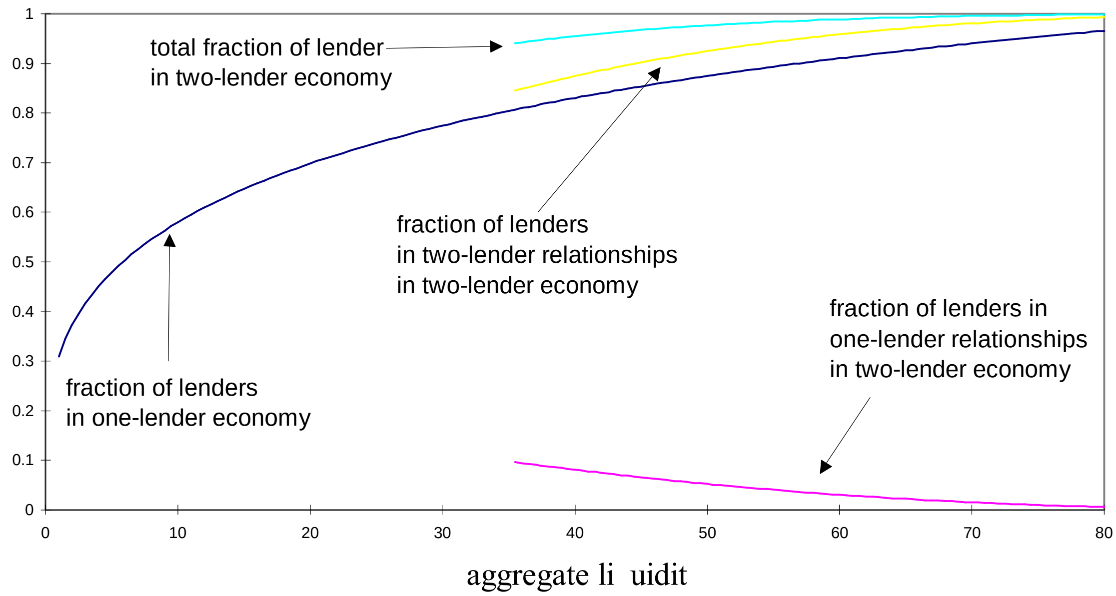

Figure : ndogenous rea up pro a ilities in t o lender economies

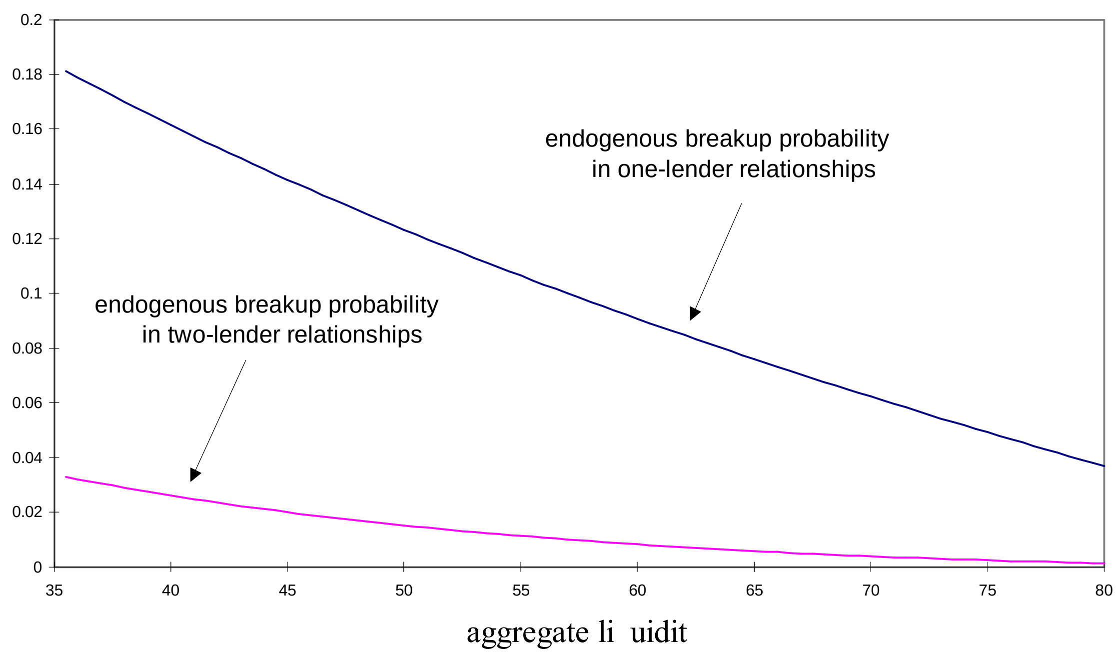

Figure 1 : tead state values of output