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CHEAP TALK AND CO-ORDINATION WITH PAYOFF UNCERTAINTY

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Cheap Talk and Co-ordination with Payoff Uncertainty*

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1. Introduction

Two firms must simultaneously make an investment decision. There are strategic complementarities, so if there was common knowledge of actual payoffs, there would be two Nash equilibria: both invest and both do not invest. But firm 2 is uncertain about firm 1's payoffs, and attaches some probability to firm 1 being unable to invest (i.e., having a dominant strategy to not invest). This probability is sufficiently high that, in the absence of communication, investment would not occur even when it was feasible for firm 1 and efficient for the two firms together. What happens if the firms are able to communicate?

The answer hinges on the exact preferences of the type of firm 1 who is unable to invest. This "committed" type has a dominant strategy to not invest. If there were no communication, this is all we would need to know about the committed type's preferences. But with communication, the committed type's preferences over firm 2's actions are also important. If this type prefers that firm 2 not invest, there will be an efficient equilibrium with communication where the non-committed type of firm 1 invests when he is able to do so, and truthfully announces whether he intends to invest or not. But suppose that the committed type would prefer that firm 2 invest, even though he is not going to invest (i.e.,

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there are "positive spillovers" in the language of Cooper and John [1988]). Then the committed type of firm 1 would have an incentive to claim that he was going to invest, even though he wasn't, and no cheap talk statements would be credible. Thus there would (inefficiently) be no communication and no investment in any equilibrium.

This observation about the credibility of cheap talk statements is related to a famous argument of Aumann [1990] in the context of complete information games. However, the relation is delicate and we postpone until the conclusion a detailed discussion of the relation. Our purpose in this paper is to derive general lessons about communication in an incomplete information context. In particular, the above example (analyzed formally in the section 2) suggests the following general results for two player games:

Full Communication. If there is common knowledge that each player wishes the other player to take the same action as himself, then it is possible to achieve efficient outcomes.

No Communication. If there is common knowledge that each player's preferences over the other's actions is independent of his own type, then that player's statements about his intended actions cannot change the equilibrium set of outcomes.

In this paper, we prove such results, i.e., identify sufficient conditions for, respectively, full and no communication along the above lines. The most striking feature of our formal sufficient conditions, however, is how strong they turn out to be. We provide examples demonstrating that they are not excessively strong.

Consider first our full communication result. A complete information game is *self-signalling* if a player does not have an incentive to deceive his opponent about which action he intends to take. In particular, conditional on a player being forced to choose action a, he would prefer that his opponent choose the best response to a to his choosing the best response to any other action a'. A Nash equilibrium is a *commitment* equilibrium if no player could do better by committing to a particular action and allowing his opponent to choose a best response. We show that in a two player, private values, incomplete information game with self-signalling satisfied and a commitment equilibrium existing for every type profile, there exists an equilibrium with communication where players truthfully announce their types and play according to the commitment equilibrium for the announced type profile (proposition 4.5). The self-signalling condition rules out the difficulty identified by Aumann [1990]. The commitment equilibria requirement rules out games like Battle of the Sexes. Therefore, our full communication result shows that these are the only two conflicts between players that impede the successful transmission of information.

In a private values incomplete information game, a type has some induced preferences over the behavior of his opponent (assuming that he is able to choose a best response). The incomplete information game satisfies common induced preferences if each type of each player has identical induced preferences over mixed strategies of his opponent. We show that in a two player, private values, incomplete information game with independent types, all equilibria (satisfying a weak refinement) have no communication in equilibrium. Requiring common induced preferences over mixed strategies amounts to requiring common *cardinal* preferences. Yet we show by example that the result breaks down with only common ordinal preferences. Similarly, with correlated types communication may be valuable despite common induced preferences (proposition 4.11). Intuitively, with independent types and private values, all communication is essentially equivalent to communicating about intended actions. But with correlated types, equilibrium messages have (and must have) more complex equilibrium interpretations.

The above results are described in section 4 (the model is described in section 3). In section 5, we describe an correlated type example not covered by our general results (cheap talk in the electronic mail game). We postpone until section 6 our discussion of the relation to the Aumann critique, to cheap talk refinements and existing work on cheap talk with incomplete information.

2. Leading Example

Two players must decide whether to invest (I) or not invest (N). There is a cost c of investing, but if both players invest, each receives a return of 100. In addition, there is a small, positive or negative, externality x that each player receives if the other chooses to invest. Thus payoffs are given by the following matrix:

Opponent's Action

		I	N
Own Action	I	$100 - c + x$	$-c$
	N	x	0

FIGURE 1

The cost of investing is either low ($c = 90$) or high ($c = 110$), giving the following matrices:

		Opponent's Action	
		I	N
Own Action	I	$10 + x$	-90
	N	x	0

Low Cost

		Opponent's Action	
		I	N
Own Action	I	$-10 + x$	-110
	N	x	0

High Cost

FIGURE 2

Thus a player with high costs has a dominant strategy to not invest. But if it was common knowledge that both firms had low costs, there would be an equilibrium where both invested.

We assume that there is incomplete information about costs. While player 2 is known to be low cost, player 1 is low cost with probability $\frac{4}{5}$ and high cost with probability $\frac{1}{5}$. Notice that we can analyze the equilibria of this incomplete information game without knowing the value of x: it is strategically irrelevant. If player 1 is high cost, he has a dominant strategy to not invest. Thus player 2 assigns probability at least $\frac{1}{5}$ to player 1 not investing. Thus the net gain to the (low cost) player 2 (from investing over not investing) is at most $\frac{4}{5}(10)+\frac{1}{5}(-90) = -10 < 0$; so there is no investment in any equilibrium.

This outcome is inefficient: both players would gain if they could co-ordinate on investment when both their costs are low (as long as $x > -10$). We will allow player 1 to make cheap talk statements before the players simultaneously choose actions, and see how this influences the outcome. It turns out that what is crucial is whether externalities are positive or negative: the *sign* of x is critical.

2.1. Adding Cheap Talk with Negative Externalities

Let $x = -1$, so that the above payoff matrices become:

		Opponent's Action	
		I	N
Own Action	I	9	-90
	N	-1	0

Low Cost

		Opponent's Action	
		I	N
Own Action	I	-11	-110
	N	-1	0

High Cost

FIGURE 3

The following is an equilibrium: player 1 truthfully announces his type. If he announces that he is low cost, both players invest. If he announces that he is high cost, both players don't invest.

2.2. Adding Cheap Talk with Positive Externalities

Let $x = 1$, so that the above payoff matrices become:

Opponent's Action

		I	N
Own Action	I	11	-90
	N	1	0

Low Cost

Opponent's Action

		I	N
Own Action	I	-9	-110
	N	1	0

High Cost

FIGURE 4

The above truth-telling behavior no longer constitutes as equilibrium. The problem is that now the high cost type of player 1 - who has a dominant strategy to not invest - would now strictly prefer that player 2 invests nonetheless. Thus the low cost type of player 1 can no longer credibly convey information. One can verify that every equilibrium of the game with cheap talk has no investment in equilibrium.

Thus by increasing x from -1 to 1 (i.e., making the both invest equilibrium more attractive for both players), we have paradoxically destroyed the possibility of efficient investment in equilibrium.[1]

3. The Model

There are two players, 1 and 2, and a finite action set of each player, A_1 and A_2 $(A = A_1 \times A_2)$. There is a finite number of types T_i $(i \in \{1, 2\})$ for player i $(T = T_1 \times T_2)$. The prior over the type space is $\pi \in \Delta(T)$. Notice that we allow correlation among players' types. Player i's utility function is $u_i : A \times T_i \to \mathbf{R}$. Notice that a player's utility depends on his own type but not on the type of the other player; i.e., this is a game with private values. Player i's conditional probability over the other player's types is denoted by $\pi_i(t_j \mid t_i)$. Therefore,we have defined an incomplete information game $[\{1, 2\}, A, T, \pi, \{u_i\}]$

Before the players play the game described above we allow them to engage in cheap talk. In the cheap talk stage, there are discrete message spaces for each

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[1] As V. Bhaskar has pointed out to us, all that actually matters is that x has increased for the committed (high cost) type. If x were positive for player 2 and the low cost type of player 1 but negative for the high cost type of player 1, we could still have communication in equilibrium.

player, M_1 and M_2 ($M = M_1 \times M_2$). Therefore, player i's strategy consists of a *talking strategy* $\mu_i : T_i \to \Delta(M_i)$ and an *action strategy* $\alpha_i : M \times T_i \to \Delta(A_i)$. Beliefs for player i are $\lambda_i : M \times T_i \to \Delta(T_j)$. We will be interested in weak *perfect Bayesian equilibria (wPBE)* of the game with cheap talk: $[(\mu_1, \alpha_1, \lambda_1), (\mu_2, \alpha_2, \lambda_2)]$ is a weak perfect Bayesian equilibrium [wPBE] if each player is playing optimally at all his information sets given the strategy of the other and beliefs are updated using Bayes' rule whenever possible. Extending each u_i to mixed strategies in the usual way[2], we have:

Definition 3.1. *$[(\mu_1, \alpha_1, \lambda_1), (\mu_2, \alpha_2, \lambda_2)]$ is a wPBE[3] if*

[1] $\mu_i(m_i | t_i) > 0 \Rightarrow$
$$m_i \in \arg\max_{m_i' \in M_i} \sum_{t_j \in T_j} \sum_{m_j \in M_j} \pi_i(t_j | t_i) \mu_j(m_j | t_j) u_i((\alpha_i(\cdot | (m_i', m_j), t_i), \alpha_j(\cdot | (m_i', m_j), t_j)), t_i).$$

[2] $$\sum_{t_j' \in T_j} \pi_i\left(t_j' | t_i\right) \mu_j\left(m_j | t_j'\right) > 0 \Rightarrow \lambda_i(t_j | (m_i, m_j), t_i) = \frac{\pi_i(t_j | t_i) \mu_j(m_j | t_j)}{\sum_{t_j' \in T_j} \pi_i\left(t_j' | t_i\right) \mu_j\left(m_j | t_j'\right)}.$$

[3] $\alpha_i(a_i | (m_i, m_j), t_i) > 0 \Rightarrow$
$$a_i \in \arg\max_{a_i' \in A_i} \sum_{t_j \in T_j} \lambda_i(t_j | (m_i, m_j), t_i) u_i((a_i', \alpha_j(\cdot | (m_i, m_j), t_j)), t_i).$$

4. Results

We will be interested in two, polar questions: First, what conditions guarantee that enough information transmission occurs in the cheap talk stage to allow agents to play efficient Nash equilibria in the subsequent game? Second, when is it impossible to communicate information effectively so the cheap talk stage does not alter the equilibrium payoff set in the subsequent game?

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[2] I.e.,

$$\begin{aligned} u_i((a_i, \alpha_j), t_i) &= \sum_{a_j \in A_j} \alpha_j(a_j) u_i((a_i, a_j), t_i) \\ \text{and } u_i((\alpha_i, \alpha_j), t_i) &= \sum_{a_i \in A_i} \alpha_i(a_i) u_i((a_i, \alpha_j), t_i). \end{aligned}$$

[3] For perfect Bayesian equilibrium, we would require in addition that $\lambda_i(\cdot | (m_i, m_j), t_i)$ be independent of m_i and t_i. Our results do not depend on this distinction.

4.1. When Does Cheap Talk Work?

Consider a pure mapping from the type space to actions, $f : T \to A$, so $f_i(t)$ is player i's action if the type profile is $t = (t_1, t_2)$. We say that f is *implementable by cheap talk* (in a given incomplete information game) if f is played in an equilibrium of the incomplete information game with cheap talk. A simple way of implementing a mapping f is by a fully separating equilibrium where players truthfully announce their types. A mapping f can be implemented by a fully revealing strategies if and only if, for both i and all $t_i, t_i' \in T_i$ and $\tilde{a}_i : T_j \to A_i$,

$$\sum_{t_j \in T_j} \pi(t_j | t_i) u_i(f(t), t_i) \geq \sum_{t_j \in T_j} \pi(t_j | t_i) u_i((\tilde{a}_i(t_j), f_j(t_i', t_j)), t_i).$$

Notice that this implies that $f(t)$ is a Nash equilibrium for all type profiles $t \in T$.

The following lemma provides a simple sufficient condition for truthful announcements and following f to be a wPBE:

Lemma 4.1. *Rule f is implementable by cheap talk if for both players $i \in \{1, 2\}$ and all $t \in T$, $t_i' \in T_i$ and $a_i \in A_i$,*

$$u_i(f(t), t_i) \geq u_i((a_i, f_j(t_i', t_j)), t_i).$$

The above condition refers to the type space and is, therefore, on the incomplete information game. We would like to provide a natural sufficient condition for f to be implementable that depends only on the underlying complete information games. A complete information game is a pair $g = (g_1, g_2)$, each $g_i : A \to \mathbf{R}$.

Definition 4.2. *Complete information game g is self-signalling if for both i, all $a_i, a_i' \in A_i$, $a_j \in \underset{\tilde{a}_j \in A_j}{\arg\max}\, g_j(a_i, \tilde{a}_j)$ and $a_j' \in \underset{\tilde{a}_j \in A_j}{\arg\max}\, g_j(a_i', \tilde{a}_j)$,*

$$g_i(a_i, a_j) \geq g_i\left(a_i, a_j'\right).$$

This property says that if player i were planning to play action a_i, he would like to convince player j that he was going to play action a_i. The terminology comes from Farrell and Rabin [1996] who may have intended a weaker property. But this clearly builds in the feature that saying "I am going to play a_i" does indeed signal that player i intends to choose a_i.

Definition 4.3. *Strategy profile a^* is a Pareto-dominant Nash equilibrium of g if a^* is a Nash equilibrium and, for every other Nash equilibrium $a \in A$, $g_1(a^*) \geq g_1(a)$ and $g_2(a^*) \geq g_2(a)$.*

Definition 4.4. *Strategy profile a^* is a commitment equilibrium of g if a^* is a Nash equilibrium and, for both i, all $a_i \in A_i$ and $a_j \in \underset{\widetilde{a}_j \in A_j}{\arg\max}\, g_j(a_i, \widetilde{a}_j)$,*

$$g_i\left(a_i^*, a_j^*\right) \geq g_i(a_i, a_j).$$

A commitment equilibrium exists if no player could do better by (publicly) committing to an alternative action. A necessary condition for the existence of a commitment equilibrium is the existence of a Pareto-dominant pure strategy Nash equilibrium, i.e., player 1's "best" pure strategy Nash equilibrium corresponds to player 2's "best" one. Therefore, it exists in pure coordination games. A commitment equilibrium does not exist when player 1's favorite equilibrium does not coincide with player 2's. Therefore, it does not exist in the game Battle of the Sexes.

Proposition 4.5. *If, for all $t \in T$, [1] $g = u(\cdot, t)$ is self-signalling and [2] $f(t)$ is a commitment equilibrium of g, then f is implementable by cheap talk.*

Proof. As $f(t)$ is a commitment equilibrium of $g = u(\cdot, t)$ for all $t = (t_i, t_j) \in T$ we obtain: for all $t = (t_i, t_j) \in T$ and for both players:

$$u_i(f(t), t_i) \geq \ u_i((a_i, a_j), t_i) \text{ for all } a_j \in \ \underset{\widetilde{a}_j \in A_j}{\arg\max}\, u_j((a_i, \widetilde{a}_j), t_j). \tag{4.1}$$

Also, as $g = u(\cdot, t)$ is self-signalling for all $t \in T$ we obtain: for both i, $a_i \in A_i$, $a_j \in \underset{\widetilde{a}_j \in A_j}{\arg\max}\, u_j((a_i, \widetilde{a}_j), t_j)$, $t_i' \in T_i$, $a_j' \in \underset{\widetilde{a}_j \in A_j}{\arg\max}\, u_j((f_i(t_i', t_j), \widetilde{a}_j), t_j)$ and $t \in T$,

$$u_i((a_i, a_j), t_i) \geq u_i\left(\left(a_i, a_j'\right), t_i\right). \tag{4.2}$$

Finally, notice that $f_j(t_i', t_j) \in \underset{\widetilde{a}_j \in A_j}{\arg\max}\, u_j((f_i(t_i', t_j), \widetilde{a}_j), t_j)$ $j \in \{1, 2\}$. Therefore, combining (4.1) and (4.2), we obtain: for both players $i \in \{1, 2\}$ and all $t \in T$, $t_i' \in T_i$ and $a_i \in A_i$,

$$u_i(f(t), t_i) \geq \ u_i((a_i, f_j(t_i', t_j)), t_i).$$

Therefore, by Lemma 1, f is implementable by cheap talk. ■

Definition 4.6. *Complete information game g is a binary co-ordination game if [1] (Binary Actions) $\#A_1 = \#A_2 = \{I, N\}$; [2] (Strategic Complementarities): $g_i(I, I) - g_i(N, I) > g_i(I, N) - g_i(N, N)$ for $i = 1, 2$; [3] (Self-Signalling) $g_i(a, a) \geq g_i(a, a')$ for each $i = 1, 2$ and all $a' \neq a$.*

Corollary 4.7. *If, for all $t \in T$, [1] $u(\cdot, t)$ is a binary co-ordination game and [2] $f(t)$ is a Pareto-dominant Nash equilibrium of $u(\cdot, t)$, then f is implementable by cheap talk.*

Proof. By the proposition, it suffices to show that a Pareto-dominant equilibrium is a commitment equilibrium in binary co-ordination games. As there are strategic complementarities, there are three possible cases: action I is a dominant strategy for player 1, say; action N is a dominant strategy for player 1, say; there are two Nash equilibria, (I, I) and (N, N) with the first being Pareto dominant. We show that is all three cases, the Pareto dominant Nash equilibrium is a commitment Nash equilibrium.

Case 1: From the self-signalling property of binary coordination games, (I, I) is the unique Nash equilibrium. For player 2, this equilibrium displays the properties required of a commitment Nash equilibrium. We must check that it also does so for player 1. As I is the dominant action for player 1, $u_1(I, I) \geq u_1(N, I)$. As the game is self-signalling and as I is a dominant strategy for player $1, u_1(I, I) \geq u_1(I, N) \geq u_1(N, N)$. Therefore, $f(t) = (I, I)$ is a commitment Nash equilibrium.

Case 2: From the self-signalling property of binary coordination games, (N, N) is the unique Nash equilibrium. For player 2, this equilibrium displays the properties required of a commitment Nash equilibrium. We must check that it also does so for player 1. As N is the dominant action for player 1, $u_1(N, N) \geq u_1(I, N)$. As the game is self-signalling and as N is a dominant strategy for player 1, $u_1(N, N) \geq u_1(N, I) \geq u_1(I, I)$. Therefore, $f(t) = (N, N)$ is a commitment Nash equilibrium.

Case 3: In this case, $f(t) = (I, I)$ which is clearly a commitment Nash equilibrium.

Finally, as, for all $t \in T$, as binary coordination games are self-signalling and as $f(t)$ is a commitment Nash equilibrium, we can invoke the previous Proposition to claim that f is implementable by cheap talk. ■

We can illustrate by example the non-redundancy of the sufficient conditions of the proposition and corollary. First, consider the leading example of the section 2.2 with positive externalities (i.e., $x = 1$, see figure 3); can the players choose both invest when player 1's costs are low and both don't invest when player 1's costs are high? We saw that this was inconsistent with equilibrium. That rule does always select a commitment Nash equilibrium (one of the sufficient conditions of the proposition); it is also a binary co-ordination game, where a Pareto-dominant Nash equilibrium is always selected. But the complete information game when player 1 is high cost and player 2 is low cost fails the self-signalling condition.

The commitment Nash condition essentially requires that agents agree on which Nash equilibrium is best. The following Battle of the Sexes example illustrates the importance of the commitment Nash condition. Banks and Calvert [1992] explore more generally how much unmediated cheap talk can achieve in the Battle of the Sexes with incomplete information.

Suppose that player 1 is one of two types, t_1 and t_1'; player 2 has a unique type t_2. The players' payoffs are given by the following matrices:

Player 2's Action

Player 1's Action		L	R
	U	$1,2$	$0,0$
	D	$0,0$	$2,1$

Type Profile (t_1, t_2)

Player 2's Action

Player 1's Action		L	R
	U	$1,2$	$0,0$
	D	$2,0$	$4,1$

Type Profile (t_1', t_2)

FIGURE 5

Both complete information games are self-signalling. But consider the rule that players choose Nash equilibrium (U, L) if type profile is (t_1, t_2) and Nash equilibrium (D, R) if type profile is (t_1', t_2); the former is not a commitment Nash equilibrium. In fact, no information is revealed in any equilibrium.

4.2. When Does Cheap Talk Not Work?

We first prove that under three rather strong conditions, cheap talk does not work. Then we show by example why each of these conditions is required.

First, write $v_i(\alpha_j, t_i)$ for player i's indirect utility over the other player's actions, i.e., for any $\alpha_j \in \Delta(A_j)$,

$$v_i(\alpha_j, t_i) = \max_{a_i \in A_i} \sum_{a_j \in A_j} \alpha_j[a_j] u_i((a_i, a_j), t_i)$$

Definition 4.8. *There is common induced preferences if for all* $t_i, t_i' \in T_i$ *and all* $\alpha_j, \alpha_j' \in \Delta(A_j)$,

$$v_i(\alpha_j, t_i) \geq v_i\left(\alpha_j', t_i\right) \iff v_i(\alpha_j, t_i') \geq v_i\left(\alpha_j', t_i'\right)$$

Definition 4.9. *An equilibrium has degenerate messages if all types of each player are indifferent in equilibrium between all messages sent by any type of that player.*[4]

An equilibrium with degenerate messages has each player completely indifferent about which message is sent. As we will see, this does not rule out the possibility of information being conveyed.

Definition 4.10. *Types are independent if there exist* $\pi_1 \in \Delta(T_1)$ *and* $\pi_2 \in \Delta(T_2)$ *such that* $\pi(t_1, t_2) = \pi_1(t_1) . \pi_2(t_2)$ *for all* $(t_1, t_2) \in T_1 \times T_2$*.*

Proposition 4.11. *If there is common induced preferences and types are independent, then any equilibrium has degenerate messages.*

Proof. As types are independent, for all $t_i, t_i' \in T_i$, $i \in \{1,2\}$ $\pi_i(t_j \mid t_i) = \pi_i(t_j \mid t_i') = \pi_i(t_j)$. Let $((\mu_1, \alpha_1, \lambda_1), (\mu_2, \alpha_2, \lambda_2))$ be a PBE of the cheap talk game. Player 1 of type t_i sends messages to maximize

$$\arg\max_{a_1 \in A_1} \sum_{t_j \in T_j} \pi_i(t_j)\alpha_j\left((m_i, m_j(t_j)), t_j\right)[a_j] u_i(a_i, a_j, t_i) = v_i(\widehat{\alpha}_j(m_i), t_i)$$

where $\widehat{\alpha}_j(m_i)[a_j] = \sum_{t_j \in T_j} \pi_i(t_j)\alpha_j\left((m_i, m_j(t_j)), t_j\right)[a_j]$. Therefore, by common induced preferences, for all $t_i, t_i' \in T_i$

$$v_i\left(\widehat{\alpha}_j(m_i^*), t_i\right) \geq v_i\left(\widehat{\alpha}_j(m_i), t_i\right) \Longleftrightarrow v_i\left(\widehat{\alpha}_j(m_i^*), t_i'\right) \geq v_i\left(\widehat{\alpha}_j(m_i), t_i'\right).$$

Therefore, the set of messages that maximize the expected utility of player i of type t_i also maximize that of type t_i'. ■

The following examples illustrate why the proposition cannot be strengthened.

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[4] Formally, fix weak perfect Bayesian equilibrium $[(\mu_1, \alpha_1, \lambda_1), (\mu_2, \alpha_2, \lambda_2)]$. Let

$$M_i^* \equiv \{m_i \in M_i : \mu_i(m_i | t_i) > 0 \text{ for some } t_i \in T_i\}$$

Let $\widetilde{v}_i(m_i | t_i)$ be the expected equilibrium utility of type t_i who sends message m_i, i.e.,

$$\widetilde{v}_i(m_i | t_i) \equiv \sum_{t_j \in T_j} \sum_{m_j \in M_j} \pi_i(t_j | t_i) \mu_j(m_j | t_j) u_i\left(\left(\alpha_i(\cdot | (m_i', m_j), t_i), \alpha_j(\cdot | (m_i', m_j), t_j)\right), t_i\right).$$

Now $[(\mu_1, \alpha_1, \lambda_1), (\mu_2, \alpha_2, \lambda_2)]$ has degenerate messages if $\widetilde{v}_i(m_i | t_i) = \widetilde{v}_i(m_i' | t_i)$, for each $i = 1, 2$, all $t_i \in T_i$ and all $m_i, m_i' \in M_i^*$.

Example 1. Allowing Degenerate Messages. It is hard to rule out the possibility that even with common induced preferences, player 1 conveys information to player 2, but player 2 uses that information is such a way that player 1 is indifferent about player 2's action. This is illustrated in the following example.

Let $T_1 = \{t_1, t_1'\}$ and $T_2 = \{t_2\}$. The probability of t_1 is $\frac{1}{2}$. Let $A_1 = \{U, D\}$ and $A_2 = \{L, C, R\}$. Let payoffs be given by:

Player 2's Action

Player 1's Action

	L	C	R
U	$1,0$	$2,1$	$3,0$
D	$0,1$	$1,0$	$2,1$

Type Profile (t_1, t_2)

Player 2's Action

Player 1's Action

	L	C	R
U	$0,0$	$1,1$	$2,0$
D	$1,1$	$2,0$	$3,1$

Type Profile (t_1', t_2)

FIGURE 6

There is an equilibrium where player 1 announces m if his type is t_1 and announces m' if his type is t_1'; he then chooses action U if type t_1 and D if type t_1'. Player 2 announces nothing, and then chooses action C if message/type profile is (m, t_2), and randomizes 50/50 between L and R if message/type profile is (m', t_2).

Example 2. Allowing Correlated Types As long as types are independent, and under our maintained assumption of private values, equilibrium announcements have (equilibrium) interpretations that they reveal information about the actions that the sender will take. But with correlated types, messages may have more complex interpretations, i.e., depending on the type of the receiver, they convey different information about the sender's intended actions. This allows information to be conveyed, even in equilibria with non-degenerate messages and under the common induced preferences assumption.

Let $T_1 = \{H, L, H', L'\}$ and $T_2 = \{t_2, t_2'\}$ and let the prior π be given by the following matrix:

Player 2's Type

Player 1's Type

	t_2	t_2'
H	$\frac{\alpha}{2(1+\alpha+\alpha^2+\alpha^3)}$	$\frac{\alpha^2}{2(1+\alpha+\alpha^2+\alpha^3)}$
L	$\frac{1}{2(1+\alpha+\alpha^2+\alpha^3)}$	$\frac{\alpha^3}{2(1+\alpha+\alpha^2+\alpha^3)}$
H'	$\frac{\alpha^2}{2(1+\alpha+\alpha^2+\alpha^3)}$	$\frac{\alpha}{2(1+\alpha+\alpha^2+\alpha^3)}$
L'	$\frac{\alpha^3}{2(1+\alpha+\alpha^2+\alpha^3)}$	$\frac{1}{2(1+\alpha+\alpha^2+\alpha^3)}$

FIGURE 7

where $\alpha < \frac{1}{9} < \frac{\alpha+\alpha^2}{1+\alpha^3}$. Both types of player 2 and types L and L' of player 1 have the low cost payoffs of the positive externality example of section 2.2 (see figure 3). Types H and H' of player 1 have the high cost payoffs.

In the absence of cheap talk, the unique equilibrium has no investment. With cheap talk, there is an equilibrium where types H and L announce m; types H' and L' announce m'; player 2 announces nothing. Types L and L' of player 1 invest, types H and H' do not invest (independently of messages). Type t_2 invests only if message m is sent. Type t_2' invests only if message m' is sent.

Watson [1996] describes a setting where correlated types allows information to be conveyed between parties with conflicting interests.

Example 3: Weakening Cardinal to Ordinal Common Induced Preferences The common induced preferences condition is very strong: it requires that a player's types have the same *cardinal* preferences over the other players' actions. The following example shows that agreement on ordinal preferences over actions is not enough (even with independent signals).

Let $T_1 = \{t_1, t_1'\}$ and $T_2 = \{t_2, t_2'\}$. The independent probability of t_1 is $\frac{1}{2}$; the independent probability of t_2 is $\frac{2}{3}$. Let $A_1 = \{U, D\}$ and $A_2 = \{L, C, R\}$. Let payoffs of type t_1 of player 1 be:

		Player 2's Action		
		L	C	R
Player 1's Action	U	1	2	6
	D	0	1	5

FIGURE 8

Let payoffs of type t_1' of player 1 be:

		Player 2's Action		
		L	C	R
Player 1's Action	U	0	4	5
	D	1	5	6

FIGURE 9

Let payoffs of type t_2 of player 2 be:

	Player 2's Action			
		L	C	R
Player 1's Action	U	1	0	0
	D	0	1	0

FIGURE 10

Let payoffs of type t_2' of player 2 be:

	Player 2's Action			
		L	C	R
Player 1's Action	U	0	0	1
	D	0	1	0

FIGURE 11

There is an equilibrium where player 1 announces m if his type is t_1 and m' if his type is t_1'; he then chooses action U if type t_1 and D if type t_1'. Player 2 announces nothing, and then chooses action L if message/type profile is (m, t_2), C if message/type profile is (m', t_2) or (m', t_2'), R if message/type profile is (m, t_2').

Seidmann [1990] has demonstrated in sender-receiver games how ordinal conflicting interests does not lead to babbling.

5. Cheap Talk in the Electronic Mail Game

An incomplete information game literature has demonstrated how ex ante small probability events may have a major impact on equilibrium payoffs via higher order beliefs (see, e.g., Kajii and Morris [1995]). In particular, there may be arbitrarily high ex ante probability that payoffs are given by a certain complete information game, but nonetheless a strict and Pareto-dominant Nash equilibrium of that complete information game is never played in any equilibrium of the incomplete information game. Such conclusions require that types be highly correlated. Is this conclusion robust to allowing cheap talk?

Unfortunately, our “no communication” result relied on an independent types assumption. In this section, we show that nonetheless in a version of the electronic mail game of Rubinstein [1989], common induced preferences leads to no communication. The example can also be used to illustrate our full communication result.

Again, we have two players deciding whether to invest (I) or not invest (N) with payoffs given by the matrices in figure 2. Each player's type space is the set of non-negative integers, $T_1 = T_2 = \{0, 1, 2, ...\}$, with the following probability distribution over types:

Player 2's Type

		0	1	2	·	n
	0	ε	0	0	·	0
Player 1's Type	1	$\varepsilon(1-\varepsilon)$	$\varepsilon(1-\varepsilon)^2$	0	·	0
	2	0	$\varepsilon(1-\varepsilon)^3$	$\varepsilon(1-\varepsilon)^4$	·	0
	·	·	·	·	·	·
	n	0	0	0	·	$\varepsilon(1-\varepsilon)^{2n}$

FIGURE 12

Type 0 of player 1 is high cost (and thus has a dominant strategy to not invest). All other types of player 1, and all types of player 2, are low cost.[5]

A well known argument (see Rubinstein [1989]) shows that there is a unique equilibrium in this setting (without cheap talk): always play N. Type 0 of player 1 does not invest as it is a dominant strategy to not invest. Type 0 of player 2 attaches probability $\frac{1}{2-\varepsilon} > \frac{1}{2}$ to player 1 not investing, so she must not invest. The argument iterates.

Now suppose that cheap talk is allowed. That is, before choosing their actions, the players simultaneously send messages in some arbitrary message space. If $x < 0$, the conditions of proposition 4.5 are satisfied, and there exists an equilibrium where each player truthfully announces his type and then invests as long as player 1 is not of type 0.

But suppose that $x > 0$. In this case, a truth-telling equilibrium does not exist: type 0 of player 1 would have an incentive to claim to be some other type (in order to induce investment by player 2). In fact, *every* equilibrium has no investment by any type. To see why, fix an equilibrium and let i^* be the lowest type of player 1 who ever invests with positive probability (after any message). Since type 0 has a dominant strategy to not invest, we must have $i^* \geq 1$. Now all

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[5] The following story from Rubinstein [1989] may motivate the information structure. With probability $\varepsilon > 0$, player 1 is high cost and player 2 is low cost. With probability $1 - \varepsilon$, both firms are low cost. Firms know only their own costs. If firm 1 is high cost, he sends no message. If he is low cost, he sends a message to player 2, lost with probability ε. If received, 2 sends a confirmation, and so on. Now the type of each player corresponds to the number of messages sent.

types of player 2 less than $i^* - 1$ must attach zero probability to player 1 investing, and therefore must never invest in the equilibrium.

Suppose then that type $i^* - 1$ is the lowest type of player 2 who ever invests in the equilibrium. Let M_1^* be the set of messages that lead type $i^* - 1$ of player 2 to invest with positive probability. Type $i^* - 1$ of player 1 sends a message in that set with probability 1 (since he knows that type $i^* - 2$ is not investing, he chooses his message to maximize the probability that type $i^* - 1$ invests). Now recall that ex ante, type $i^* - 1$ of player 2 assigned probability $\frac{1}{2-\varepsilon}$ to player 1 being of type $i^* - 1$. Conditional only on observing a message in M_1^*, that probability must weakly go up (type $i^* - 1$ always sends a message in M_1^*, even though type i^* may not). Thus for at least one message in M_1^*, player 2 must assign probability at least $\frac{1}{2-\varepsilon}$ to player 1 being type $i^* - 1$, and therefore not investing. But then it is a best response for player 2 to not invest, a contradiction.

Thus we get a contradiction if $i^* - 1$ is the lowest type of player 2 who ever invests in the equilibrium. If i^* is the lowest type of player 2 who ever invests in the equilibrium, we can similarly construct a contradiction, reversing the roles of players 1 and 2 in the argument. If the lowest type of player 2 who ever invested in equilibrium were larger than i^*, then type i^* of player 1 would never invest in equilibrium, again a contradiction.

Unfortunately, this argument depends crucially on the special structure of types in this example and it is not clear how to generalize it.

6. Discussion

6.1. The Aumann Critique

Consider the case where there is common knowledge that both firm are low cost, so we have the following symmetric payoff matrix.

		Player 2's Action	
		I	N
Player 1's Action	I	$10 + x, 10 + x$	$-90, x$
	N	$x, -90$	$0, 0$

FIGURE 13

Suppose player 1 announces that he is going to invest. How credible is this statement? We may consider two kinds of informal tests (the terminology and discussion here follows Farrell and Rabin [1993]).

A statement is *self-committing* if a speaker who expected to be believed would have an incentive to make his announcement come true. This notion of credibility lies at the heart of cheap talk refinements building on Farrell's [1993] notion of neologism-proofness. The statement "I will invest" is self-committing in the above example, independent of the sign of x.

A statement is *self-signalling* if the speaker would want it to be believed only if it is true. The statement "I will invest" is self-signalling in the above example only if $x \leq 0$. If $x > 0$, player 1 would like to convince player 2 that he is going to invest independent of what player 1 is actually going to do. Aumann [1990] argued forcefully that 1's statements are surely not credible in this context. In particular, suppose player 1 thought that player 2 was likely to not invest but there was a slightly higher probability that 2 would invest if 1 claimed he was going to invest. Then it would be strictly optimal for player 1 to claim that he was going to invest, independent of what he was actually going to do.

It is an empirical question whether statements that are self-committing but not self-signalling are believed. Charness [1998] presents evidence that they are in some experimental settings. For those, like ourselves, for whom the Aumann critique is intuitively compelling, our incomplete information analysis offers an alternative line of defense. As a practical matter, there is rarely common knowledge of the underlying payoffs. By allowing for the ex ante possibility that player 1 is in fact committed to a particular action (perhaps with low probability), we force the inferences of the Aumann argument into play.

6.2. Cheap Talk Refinements

There is a large refinements literature for cheap talk games. In pure common interest games, these refinements typically pick out efficient outcomes (see, e.g., Matthews, Okuno-Fujiwara and Postlewaite [1991] and Conlon [1997]). Our full revelation result (proposition 4.5) simply demonstrated that efficient behavior was consistent with equilibrium (babbling equilibria also exist). Although we haven't pursued this line of research, our conditions are presumably sufficient for such refinements to generate full separation as a unique outcome.

6.3. Cheap Talk and Incomplete Information

Our paper belongs to a growing literature on cheap talk in incomplete information settings (both sender-receiver and more complex). Two themes that emerged in Crawford and Sobel [1982] have re-appeared repeatedly. First, information can be conveyed when the sender and receiver have common interests and not when they have exactly opposing interests. Second, when sender and receiver have mostly but not exactly opposing interests, it is often possible to find subtle dimensions along which information *can* be communicated. Our proposition 4.11 provides another example of a setting where only a very strong opposing interests condition is sufficient for babbling.

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