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GAME THEORY, COMPLEXITY AND SIMPLICITY.
PART III: CRITIQUE AND PROSPECTIVE

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Contents

Abstract

A discussion of some of the problems in the utilization of game theoretic solution concepts is given. It is suggested that a considerable broadeninng of solution concepts is called for to take into account sufficient context. Mass agent simulations appear to offer promise for some economic and societal problems.

Rational Behavior: Greed, modified by sloth, constrained by formless fear and justified ex post by rationalization.

1 Game Theory, Economics and Physics

Before venturing further into a discussion of new approaches to solutions of games it is apposite to remind ourselves of the original observations of von Neumann and Morgenstern (1944)

> First let us be aware that there exists at present no universal system of economic theory....The reason for this is that economics is far too difficult a science to permit construction rapidly....Only those who fail to appreciate this condition are likely to attempt the construction of universal systems. Even in sciences which are far more advanced than economics, like physics, there is no universal system available at present.
>
> To continue the simile with physics: It happens occasionally that a particular physical theory appears to provide the basis for a universal system, but in all instances up to the present time this appearance has not lasted more than a decade at the best (p. 3)
>
> The next subject to be mentioned concerns the static or dynamic nature of our theory. We repeat most emphatically that our theory is thoroughly static. A dynamic theory would unquestionably be more complete and therefore preferable....A static theory deals with equilibria....For the real dynamics which investigates the precise motions, usually far away from equilibria, a much deeper knowledge of these dynamic phenomena is required (pp. 44, 45).
>
> The emphasis on mathematical methods seems to be shifted more towards combinatorics and set theory — and away from the algorithm of differential equations which dominate mathematical physics.
>
> It should be clear...that a theory of rational behavior — i.e. of the foundations of economics and of the main mechanisms of social organization.— requires a thorough study of the "games of strategy" (p. 46)...in the process of this analysis it will be technically advantageous to rely on pictures and examples which are rather remote from the field of economics proper and belong strictly to the field of games of the conventional variety (p. 47).

The theme of von Neumann and Morgenstern was to set up the apparatus to permit the careful study of statics. Their solution, other cooperative solutions and the noncooperative equilibrium solution of Nash were all devoted to this goal. It is a monument to their success that the time has arrived to move beyond this goal.

1.1 Structure, Intent and Behavior

The reader who has seen Parts I and II of this trilogy may wonder where game theory fits into complexity, mass particle behavior, chaotic systems and the other occupations of the Santa Fe Institute. The first part was a tutorial providing an exposition of the basic languages of game theory. It presented the basic concepts utilized to study many person conscious interactive optimization. The second part dealt with a large and diversified set of applications. These applications are proliferating. But successful application and an understanding of where are the weaknesses as well as the strengths of game theory, are not the same. *Homo Ludens* is the intellectual son of *homo oeconomicus*, who in turn is descended from the *Benthamite utilitarian man*. For those of us brought up in a reductionist tradition it is a source of wonder and satisfaction that so much insight into certain local optimization problems and some global principles could be gleaned from such simplifications. The next steps in furthering our understanding require a different approach.

The previous parts have shown how game theory provided a language to describe the structure of multi person decision making at three levels of detail. They are the *extensive form*, the *strategic form* and the *coalitional form*. It is suggested here that the coalitional form completely abstracts away from concerns with dynamics and behavior. It is implicitly assumed that all individuals know what they want, know the power of groups and can costlessly (and timelessly) form and dissolve all coalitions. The coalitional form is primarily used for normative investigations often associated with axioms involving symmetry, equity, efficiency and other "desirable features." The discussion here is limited to the strategic and in particular, the extensive forms of the game, where process and dynamics can be considered.

1.2 The Extensive Form, Information and Dynamics

The devising of the theory of games has radically transformed our understanding of ways to view strategic choice and individual decision making. But its very power has served to illustrate the enormous gaps we have in our understanding of human behavior. The elegance and precision of the game theoretic formulation has enabled us to see more clearly the radical simplifications involved in portraying *homo ludens*. The microeconomist's ideal of rational man is for most purposes a weak first approximation of an individual. This approximation is valuable in answering some questions about economic behavior and grossly misleading when utilized elsewhere where context counts.

By providing us with a precise language to describe rational fully conscious decision making with individuals with unbounded abilities to compute, the game theoretic models of human behavior have given us a means to examine both the power and success of this approach along with its limitations and glaring weaknesses.

In Part I the extensive form of a game was described. It is easy to see the strength and elegance of the game theoretic notation when applied to a description of chess. Without ever having to draw the extensive form for chess, by observing that it is a finite game with perfect information (i.e., each player is completely informed about everything that has transpired), in principle one can employ a backward induction. This can be used to show that chess is an *inessential game*. If two superbrains were to play chess the game should be over as soon as they had chosen who is to play Black. Each, by working backwards from every terminal node of the game tree would be able to calculate his optimal strategy and a win or draw would be declared without bothering to play.

The extensive form provides a language to describe an interactive conscious decision-making process in detail, identifying every move, all information conditions and indicating when each individual is required to make a local choice.[1] A formal definition of *strategy* has been developed. But the care and precision with which this definition was developed tells us immediately that humans do not utilize strategies in that sense. The idea of individuals looking over the complete set of strategies of size larger than $10^{10,000}$ where each strategy is an enormous book of instructions tells us that neither people nor machines play chess in this manner. They have algorithms, rules of thumb or other models of behavior which enable them to prune the tree and simplify search. They use rules to evaluate positions; expertise counts. Simon and Schaeffer (1992) note that chess masters probably carry more than 50,000 "chunks"or patterns which they recognize in play. They distinguish the *substantive rationality* which is dealt with in most game theory discussions, from procedural rationality which is directed to procedures and how to play rather than to existence proofs. They note that chess is the *Drosophila of artificial intelligence and cognitive science research*. It illustrates the reality of computational difficulty.

Game theory, for the most part, has ignored the role of expertise. It applies the modeling rule of *external symmetry*, i.e., all players are assumed to be alike in all aspects which are not otherwise specified in the description of the game. The atoms and cells of game theory and economics are, in general, undifferentiated. There is one type of actor, *homo ludens*.

Dreyfus and Dreyfus (1986) in their critique of Artificial Intelligence suggest that the performance of individuals can be characterized by five levels

1. Novice
2. Advanced beginner
3. Competence
4. Proficiency
5. Expertise

The novice learns the rules and consciously manipulates in more or less a context free manner. The advanced beginner improves with considerable experience which

[1] However the extensive form representation of a game may not be unique and apparently insignificant differences may challenge the use of the various noncooperative equilibrium solutions. The deep and careful analysis of Kohlberg and Mertens (1986) illustrates this.

enable him to recognize situational elements. However the level of competence requires more than an inventory of rules and situational elements; the individual learns how to adopt an appropriate hierarchical procedure. They suggest that when cognitive scientists speak about "problem-solving" they have in mind the thought process characterizing competence involving plans, goals and strategies.

Dreyfus and Dreyfus (1986, pp. 27–35) argue that the next two stages "are characterized by a rapid, fluid, involved kind of behavior that bears no apparent similarity to the slow, detached reasoning of the problem-solving process." Words such as intuition and "know how" refer to the proficient agent's spotting and reacting to special features of a situation. When the level of expert is reached, the observation of Frank Lloyd Wright applies: "An expert is one who does not have to think. He knows."

Much of past game theory has been devoted to harvesting a rich crop of answers to questions which could be usefully asked while ignoring individual differences and limited abilities. The intellectual ground has now been cleared sufficiently that a new set of questions must be answered and the ability to answer them depends on enlarging the scope of the models.

1.3 Solutions Past and Solutions Future

The major cooperative solutions investigated have been the core, value, nucleolus, bargaining set and stable set of an n-person game. A discussion of these solutions has been given in the previous parts of this paper. The details are not important here. What is important is that they all have been essentially normative solutions. Given that we require that a solution satisfies various axioms concerning efficiency, symmetry, bargaining power, additivity and other conditions, then the solution tells us which set of outcomes have the required properties.

The Nash noncooperative equilibrium can also be considered axiomatically where efficiency is given up but mutual consistency of individual expectations is required. In spite of being able to regard the noncooperative equilibrium solution as static, the thrust of much of the theoretical work on modifying the noncooperative equilibrium (for a good coverage see Van Damme, 1996) has been devoted to considering the game in extensive form and a considerable body of experimentation is concerned with whether the behavior of human players leads to a noncooperative equilibrium.

It is my belief that the future directions in the development of game theory solutions may call for a different emphasis than the previous developments. In particular the modification of the concept of noncooperative equilibrium is only one of many approaches. The work instigated by Harsanyi (1967) on Bayesian players and the work of Aumann (1976) on games with incomplete information on the rules of the game and lack of common knowledge (see Geanakoplos, 1994 for a survey) are valuable philosophical extensions of the classical "one type of agent" model; but a vast expansion of more specific and context related models is more congenial to the development of testable game theoretic dynamic models. The work of Rubinstein (1986) on finite automata and Hammerstein and Selten (1994), Weibull (1996) and others on game theory models in evolutionary biology are indicative of the change taking place. Much more change is about to come.

In the remainder of this discussion a sketch is given of the sources of the needed distinctions which call for specialized models. I doubt that a "general theory of everything" is in the cards for Physics and I doubt even more so that it is in the cards for all forms of human behavior for all times. We are at the point in time where new models are feasible, given the basic developments in mathematics in general, computational methods, simulation, ways of observation, growth of data banks and the development of the social and biological sciences.

It is my belief that in the development of dynamic models *context is everything* in understanding the nature of decision making. Human dynamics is, at best, a multi person controlled stochastic process where history frequently counts and unique predictable outcomes are a rarity. Von Neumann and Morgenstern based their original model of the environment and the decision makers on analogies with formal parlor games, played by goal oriented optimizing individuals. These game are chess, Poker, Bridge and others where the relevance of the environment and the context is minimal. The players are abstract intelligent individual agents acting in pursuit of their own limited goals *in vitro*, not managers, generals, crooks, fools, relatives, bureaucrats or politicians in a complex fluid environment.

A striking example of the relevance of context is given by the number of proverbs which come in apparently contradictory pairs; for example: "He who hesitates is lost," or "Look before you leap." Another pair is "In the kingdom of the blind the one-eyed man is king" or "In the kingdom of the blind the one-eyed man is mad." The difference comes in the understanding of context.

A contrast between the game theory definition of strategy and the military definition of strategy illustrates the dangers and limitations in trying to misapply a beautifully precise mathematical construct with a far less precise but far more encompassing set of guidance rules.

A military strategy is an overall plan for the execution of a war, involving not merely general and specific geographical considerations and timing, but organizational delegation of decision-making and an explicit recognition of goals, terrain, technology, doctrine, force structure, chain of command and a recognition of the state of morale and other social and psychological factors influencing the forces involved.[2]

2 On Numbers

Quantitative differences often lead to qualitative differences. In the applications of the theory of games the number of agents and the context of their activity is critical. There are two interrelated ways in which numbers play an important role in understanding the strengths and limits of game theory and its relationship to other approaches. The first concerns how the theory is influenced by changes in the number

[2] A perceptive discussion of the development of the military view of strategy is provided by Paret (1986) covering the works of Sun Tzu, Clausewitz, Jomini, and many others. The themes of organization and delegation and the importance of control and the appropriate processing of information abound.

of agents[3] and the second concerns the ability of the individual agent to communicate and to process information.

Numbers count and they frequently appear in the limits to human perception. In a classical article in psychology George Miller (1956) discussed *The magic number seven plus or minus two* pointing out the seven deadly sins, the seven virtues, the five continents, the nine muses. He suggested that the existence of the numbers indicated some of the limits on human capacity for processing information.

I suggest that in the general study of game theory there are seven important divisions which depend on the number of agents. They are:

1. One person decision problems
2. Two person games (constant and nonconstant sum)
3. Three person games
4. "Few person" games (where few is from 4 to around 20)
5. "Many person" games (where many may be from around 20 to around a few hundred)
6. Large, but finite games
7. Games with a continuum of agents

We all would like to see a simple all encompassing "theory of everything" emerge from a few basic rules. I believe that there are undoubtedly several broad applications where the conventional apparatus of current game theory complete with the actors as true descendants of Benthamite man still promise high scientific payoffs. Unfortunately the general study of the nature and the number of human actors calls for a development of new game theoretic models capable of addressing problems in the behavioral sciences in specific, in contrast with trying to beat all problems in the behavioral sciences onto an early 20th century microeconomic utilitarian bed of Procrustes. The division of the behavioral sciences into biology, anthropology, economics, political science, sociology, social psychology and psychology is based on substantive considerations which are not reflected in merely aping the economist's methodology. Among humans with language, society and culture, the quantitative differences in numbers lead to qualitative differences in the nature of interaction. The best abstract model of the anonymous buyer of 100 shares in a mass market is not necessarily the same as that of the individual buying a rug in a face-to-face deal with a rug merchant. In each instance we must consider context and the questions we are trying to answer. Before the general theory of everything, there are many special questions to be answered. I believe that there are many interesting general properties of the mass behavior of individuals where the analogy between the individual in society and the particle in a physical system may be close enough to be worth considering. I also believe that there are other questions where this analogy may be of little help.

The one person decision problem deals with the single agent confronted with a nonrandom or a random optimization problem. It is here that the modeling choice of

[3] Rephrasing this more technically, holding the nature of the individual agents constant, is there any important form of convergent behavior encountered as numbers increase?

what constitutes the individual decision maker must be addressed. From the viewpoint of the microeconomist or the operations research practitioner the model is that of utilitarian man. The psychologist, social-psychologist, sociologist and psychiatrist will envision a different primitive unit. In the context of solving a cost minimization production problem the first model is probably the more productive. When the problem at hand is to find out why Bobbi is absent from school and lies about where he has been, a different primitive unit is called for. Context counts. In the first problem, to a good first approximation society and history are irrelevant. When dealing with the second, the psychiatrist's view of the individual cannot avoid the imprint of society (Brothers, 1997).

Those who utilize Bayesian updating in the study of the behavior of the individual under uncertainty, must remember that the original priors come from somewhere and that somewhere may involve the society to which the individual belongs.

Two person games (constant and nonconstant sum), preferably with two strategies for each agent have provided the rich source of metaphors and analogies in the use of conversational game theory. Even at this level of simplicity much of the force of the problems posed by multi person optimization appear. Students of social-psychology and international relations will continue to obtain valuable insights from the game theory developments. But hopefully it will be a two way street with the game theorist beginning to appreciate the relevance of socio-psychological insights such as observations that oriental females with no game theory experience play the repeated Prisoners' Dilemma game differently from Australian union members. The development of two person multistage games will benefit considerably both from experimental gaming and from joint work of game theorists with social psychologists.

Three person games call for a special study. Three is the first number for which, if information is aggregated, the individual is unable to attribute actions specifically to another agent. It is also the first number at which there is a choice in multi person coalition formation. Three is still a small enough number that face-to-face communication may be intense.

"Few person" games (where few is from 4 to around 20). It is here (and with the three person games) that we encounter what we might intuitively call combinatoric complexity. In any scientific investigation the dynamics of the three or four body problem is hardly comprehended, except in highly special cases. From the view point of game theory and its applications in economics, political science and elsewhere, I have selected the range for the few agent problem to be 4 to around 20. The numbers 1, 2 and 3 are all special cases requiring independent study, thus the lower bound on the general few agent problem begins at 4. However the size of the upper bound is not fixed as it is heavily determined parametrically and by context. In long lasting relationships it is feasible for twenty members of a small institution to know each other on a first name basis. In an industry with high turnover and regional dispersion it is unlikely that twenty firms maintain detailed knowledge of each other. Somewhere in this grey zone between 4 and around 20 the levels of face-to-face contact attenuate;

the degree of common knowledge goes down and the role of aggregation increases. Aggregation can come about in the formal processing of statistics, in the generation of stereotypes or in the production of social or professional norms. Oligopoly theory, international relations theory, small committee and club theory as well as pack and small herd behavior of some animals are relevant in this range.

A key question in many of the social sciences is "how many is many?" In the study of industrial organization this is often asked in the context of the regulation of industry to ensure competition. In the context of organization theory there are questions concerning optimal span of control. An adequate answer depends at least on numbers, duration of relationship, spacial distribution and nature of the communication net.

"Many person" games (where many may be from around 20 to around a few hundred) provide the twilight zone above which anonymity appears. But before we consider anonymity, an understanding of the nature of the communication network and the upper bounds on size of a group of agents where the atomic structure of recognition of individual agents by individual agents is still predominant. Small, low mobility villages, small firms, small tribes may have numbers in this range.

Large, but finite games provide the testing ground for the development of a science of mass behavior. It is at this level where the concentration is less on the strategic action among few individuals but on mass agent quasi-anonymous behavior. In the immediate future I believe that the highest payoffs lie here for the game theorist and mass behavior social-psychologist utilizing both simulation and machine computation methods.

Games with a continuum of agents are an idealization used primarily for mathematical convenience in studying situations with many agents where each has negligible strategic power. As with the distinction between continuous and finite time (discussed below), so it is with games with a countable number of agents and with a continuum. A game theory technique for considering the influence of increasing numbers is by replication. A game with n players is replaced by kn players where k can become arbitrarily large. The key question to consider when dealing with large, but finite games and games with a continuum of agents is whether or not, as the game with countable number of agents becomes indefinitely large, does the solution approach the solution to the game with a continuum of agents. It is frequently analytically easier to work with continuous approximations to large but discrete phenomena. If it can be shown that the difference in the behavior of the solutions becomes arbitrarily small then the tractability is bought at no cost. If the limiting behavior does not approach the behavior at the limit the explanation of the discrepancy may be of considerable importance in understanding the phenomenon at hand.

2.1 On Atoms, Molecules, Cells, People and Context

The atomic table currently has 112 elements; the human body has around 260 differentiated cells. In the universe around us there are subatomic particles, atoms, molecules and cells; in the animal kingdom there is of the order of 1.4 to 2 millions of species of insects, around 8,600 species of birds and around 4,630 species of mammals. There are around 6 billion individual humans with a biomass of less than that of all ants. The human biological, social, economic and political units involve families, extended families, clubs, tribes, voting districts, churches, political parties, nations and international organizations. The odds are that there are other planetary civilizations in other galaxies with other forms of intelligent life with whom we have no direct communication whatsoever. The message is diversity. There are many basic building blocks and there are many species in the animal kingdom. Does game theory or chemistry provide the better model for the study of decision making? The answer to this is probably context and question sensitive.

Are all living things decision makers? Are planets decision makers? Can the interaction between the Earth and its moon be regarded as a two person game? Is the battle between a Pitcher plant and a fly a two person game?

Von Neumann (1966, p. 81) suggested that a self reproducing mechanism requires at least eight kinds of part components. We do not really understand what is the magic which differentiates the animate from the inanimate, but it appears that it is not only merely numbers but heterogeneous elements and connections and connections imply some form of information bond among the units.

Marvin Minsky (1985) has used the phrase "Society of the Mind" to suggest that a great variety of local optimizers may be organized to form human intelligence. For some purposes it makes sense to regard all humans as the undifferentiated atoms, decision makers or primitive units in the game. It is my belief that in the understanding of many aspects of the behavioral sciences it is worth differentiating the basic decision makers. Humans are highly adaptive organisms. Poets, painters and politicians, accountants, bureaucrats and scientists all began as babies of at least two sexes. But if one is trying to answer questions concerning society and its short term functioning, characterizing the population as being composed of consumer-voters, bureaucrats, entrepreneurs, financiers and politicians may be more fruitful than considering only bloodless homogeneous economic maximizers. For some purposes a single sex and single age population provides a justifiable simplification for other purposes it might even be helpful to observe that the population has an age distribution and sex differences.

2.2 Solutions and Prediction

> There is much which we must leave, whether we like it or not, to the un-"scientific" narrative method of the professional historian
>
> Norbert Weiner (1948, p. 191)

Von Neumann was dubious of game theoretic solutions which, in general, predicted a single point. In fact, in spite of the attempts to refine the noncooperative equilibrium solution to select a unique equilibrium point, uniqueness of prediction for any game theoretic solution is a rarity due to special properties of the problem at hand. Among the cooperative solutions which have been proposed, the Shapley value is unique for side payment games.[4] But it is incorrect to interpret this as a prediction. It is, in essence, a statistical average which awards the individual his expected marginal productivity on the assumption that all orders in which an individual enters any coalition are equiprobable.

In many highly practical operations research problems the estimates sought are statistical. One wants to know the number of items in stock which maximizes expected profit. No one tries to predict if Mrs Jones is going to buy a washing machine in the next six months. A serious fund manager or investment banker when buying microchip manufacturers will spend far less time trying to pick "the successful firm" than trying to buy a broad enough portfolio of well priced firms which will include enough of the stochastically determined winners. Brian Arthur's (1994a) example of growth following a Polya process tells us of the fundamental unpredictability of what happens to a single innovating firm. It does not argue against what happens to the mean performance of the universe of firms.

Per Bak's (1996) example of avalanches on a sand pile or of the Gutenberg–Richter law on the frequency (but not the timing) of the occurrence of earthquakes argue in favor of science with prediction of global but not individual properties. Ralph Gomory's (1995) observations on the unknowable are consistent with Bak's observations. At best we can hope to pick up statistical regularities. In many instances humans avoid the need for an intractable global prediction by substituting a solution based on local control. Rather than predict the weather for today's picnic, build a climate controlled sunroom and hold the picnic there.

In my opinion three person game theoretic dynamics is less tractable than the general three body problem. One will learn more about the dynamics of The British Raj, Afghanistan and Imperial Russia from the pen of a good historian than a mathematical political scientist, although a few game theory metaphors might aid in illuminating thought.

Arthur *et al.* (1997) in their study and simulation of stock market behavior and Arthur (1994b) in his observations of the changes in the predictions of the number of individuals attending an evening's music session at a local bar stress great changes in the predictions of individuals. A statistical regularity may still arise from the changes.

3 The Games Within the Game

One of the key questions in basic science is that concerning morphology. What are the mechanisms behind the creation of form? In the applications of game theory a basic question is what constitutes a player. In various conventional applications of the theory of games we study commitment; for example England may commit to preserving the independence of Belgium; individual A may commit to helping

[4] The value is not unique for nosidepayment games, but is generically a point set.

individual B if B starts a new business. A may also make a commitment to A in the form of a New Year's resolution to lose 20 pounds and give up smoking. In the first instance the decision maker is a compound construction consisting of a nation state acting on another nation state,[5], in the second it is a single individual acting on another individual, whereas in the last instance it is an individual dealing with himself as though the single individual consisted of a society or a game with many players. The question helps to determine the primitive concept of the player.

No one has yet done the appropriate analysis on "the portrait of the artist as a 2^{40} agent game," yet we might view the individual players to be the cells in the organism or institution known as a human being. At the next level of decisionmaking it might be a bunch of selfish genes manipulating the puppet cells who in turn are manipulating the human puppet.

There has been a considerable development in the application of game theory to evolutionary biology (for a good exposition see van Damme, 1996). A key feature to this application is that the individual agent is not modeled as an independent decisionmaker, but as a strategic dummy or a mechanism with a given fixed strategy. I suggest that there is a middle ground between the pure mechanism and *homo ludens*, this is the independent locally optimizing agent, where the influence of the overall behavior of all agents changes the environment in which they operate and thus drives a long term evolutionary process which is of little concern to the locally optimizing agents. Figure 1 shows a picture of an overlapping generations socio-political economy where the infinite tape represents the environment, physical, societal, political and economic institutions. The sequence of overlapping boxes below are the overlapping generations of individual humans. The figure as drawn indicates a finite nonstochastic length of life for the individual.[6]. Empirically, it is reasonable to view these boxes as representing a life currently having an operational upper bound of around 130 years. Among those who have neared the upper bound (say over 110) it does not appear too likely that they were concerned about decisionmaking in the same way as they were at 20 or 40.

[5] See Masters (1983) and Fessler (1988) on the state and Tulloch (1984) for insect societies.

[6] The reader might contemplate the differences that three modeling distinctions might make. The first, as is used in many economic models is a finite nonstochastic life span. No one is killed by wars, plague or accidents, but drops dead at the foretold end of the allotted hours. The second is the existence of a specific finite upper bound to human life. This comes reasonably close to what is. It could be that in centuries to come the active human lifespan will move to 200 years or more; but at this time those who live beyond 110 do not appear to be terribly well represented by *homo ludens* and do not appear to spend too much time devoted to working out lengthy backward inductions. There is no hard evidence that anyone has lived beyond 130. The third model is to assume that there is no fixed upper bound. Mel Brooks' 2,000 year old man, if he survives against the odds will survive as the same decision maker (whatever that is) as he was when he was 20.

Figure 1. The Games Within the Game

The key element in the construction in Figure 1 is that it offers the possibility of blending local optimization and evolution together. The individual of age T older than 80 with his or her essentially finite life worries about here and now (and possibly a big enough pension for up to age $T + 20$). The financier, bureaucrat, scientist or politician devote their active optimization to highly different goals in society. These different local actions feed into the ongoing environment. But, in game theoretic terms, the environment provides the "rules of the game." Yet on a longer horizon than the short sellers' stock play, or the politician's election campaign their actions modify the rules of the game and are fed back onto the local players at some time in the future. A more detailed model would show a complex set of "games within the game" or interlocking games all on different time scales. But most human decision makers do not devote their waking hours to all of these games. Global optimization by the individual is a myth perpetrated by those who have failed to contemplate the complexity of human life as a partially controlled stochastic process with interactive feedback.

The view that the behavior of local optimizers is consistent with a global evolutionary structure calls for a review of the relationship between rules and behavior and between form and function. The institutions, organizations and organisms which exist provide the rules for the current local games. The context they provide constrains behavior, but does not uniquely determine behavior. But behavior feeds back onto the context and changes the rules of the game. The relationship between the local behavior and the global evolution of the system leads us to regard with caution attempts to provide too simple a dichotomy between form and function. It is easy to confuse rationalization with causality.

The game theorist concentrating on noncooperative behavior tends to be reductionistic. The unit of interest is the individual. The approach of the cooperative theorist tends to be more holistic. Von Neumann's and Morgenstern's concern with the stable set solutions stressed complex properties emerging from the set of all players as a whole.

4 Where Some of the Problems Are

It has been suggested that quantitative differences may easily result in qualitative differences in phenomena. In this section several sources of the qualitative difference are considered.

4.1 Computational complexity

The advent of the computer has revolutionized the possibilities for reseach in the sciences in general and the social sciences in particlar. The growth of linear programming, convex programming, integer programming, dynamic programming together with the ability to compute has removed problems in economics and operations research from being intellectual toys to providing major applications Rust (1997), and Traub and Wozniakowski (1980) raise basic questions concerning computability. The understanding of dynamics in the behavioral sciences requires the investigation of complex adaptive systems. Do assemblies of individuals acting in parallel actually solve parallel stochastic dynamic programs? The evidence appears to be that this is not the way they behave. Could it be that the result of their behavior is that they act "as if" they solved these complex optimizations. An exploration of many of the high dimensional problems faced in economic life indicates that in many instances, unless special structure is present computational requirements increase as a power of the dimension.

The problem of computational limit is faced in its purest form by the individual operations research analyst given a well-defined one person optimization problem. When on top of the difficulties involved with high dimension, the dynamic game theoretic aspects of forming expectations about the moves of others and the acts of Nature are also included, it becomes reasonably evident that different models of individual behavior are called for.

4.2 On Fiduciary Choice

It might be viewed in sociobiology that the animal sacrificing itself to maximize "inclusive fitness" is acting in a fiduciary manner for its genes. Much of microeconomic theory is based on the individual with an egocentric utility function maximizing his or her own welfare. Yet even casual empiricism shows that the preponderant strategic action taken by individuals in a modern economy, polity or society is taken by fiduciary agents playing with other people's money or lives. Once more the *caveat* must stress that many of the problems in the application of game theory arise in the mapping from the world around us onto the abstractions of actor, strategy set and preferences.

4.3 Continuous and Finite Time

In many of the applications of mathematics to science in general, and the social sciences in particular, either continuous or discrete time models are used, but rarely are they intermixed. The reason why hybrid models are avoided appears to be primarily to preserve mathematical tractability. Differential-difference equations with stochastic elements are messy. Unfortunately life is messy. Life is lived continuously with stochastic or periodical events interspersed. Babies take approximately nine months to be born, taxes tend to be based on an annual basis, sleep tends to take place on a daily basis; marriages, the timing of births, murders and the granting of honorary degrees take place more or less in stochastic event time.

The modeling of time requires the selection of one of:

1. Continuous time,
2. fixed clock or periodic discrete time, and
3. stochastic discrete time,

or blends of all three selected on an *ad hoc* basis. The availability of high speed computers changes the nature of the modeling selection. Simulation and numerical approximation enable the modeler to avoid simplifications selected primarily to preserve analytical tractability.

The relationship between continuous time and finite time models is frequently highly worth while exploring. The critical question to be asked is does the limiting solution of the finite model, as the time increment becomes small approach the solution of the continuous time model. An illustration of the importance of this problem comes about in investigating the role of money in an economy where the velocity of its use in transactions may become arbitrarily fast.[7] It is well known that however it is measured, the velocity of money varies in an economy. If we permit it to have an infinite velocity then the amount required to run the economy approaches zero. But humans can only verify transactions in a finite amount of time. Thus although going to the limit may appeal to a sense of mathematical elegance, for the modeling of decision making it may be less appropriate than considering that there is a lower bound to human decision time.

4.4 Complexity and Context

The tendency in game theoretic modeling and analysis has been to consider highly sophisticated individual units capable of performing complex computations in environments of any level of complexity.[8]. It appears that at least one of the next steps should be to consider the interaction of less complex individual agents. I view the developments in behavioral science, not as an abandonment of game theory, but as a recognition that the conclusions from game theory forces us to the next steps beyond

[7] The amount of money needed in an economy which utilizes it for transactions appears to be analogous to the free energy in an open physical system.

[8] Slobodkin (1992, Ch. 3) gives an interesting discussion of games and context and Huizinga's (1950) classical study considers the play element in culture.

the standard model of *homo ludens*. The complexity may come from the interactions of simple agents.

No attempt is made here at an exegesis as to what is "complexity" I leave this exercise to those with a more philosophical bent. Nevertheless a few observations are called for. A search of the World Wide Web under "Definition of complexity" yields around 200 definitions. Claude Shannon (1948) in his development of information theory considers how to measure the degree of information contained in a message. In his attempt to do so he deals with messages without semantic context. Murray Gellmann (1994) observes a completely random generation of words is more complex than the works of Shakespeare in terms of Algorithmic Information Content (AKA Kolmogorov Complexity). Yet in terms of our intuitive feeling for the "interesting" both a sequence of random 0s and 1s and a sequence of 111111... are more or less equally uninteresting even though the latter can be coded more simply than the former, both.

In his book Gell–Mann (1994, p. 34) suggests the idea of crude complexity as being: "The length of the shortest message that will describe a system...employing language, knowledge, and understanding that both parties share." This suggested definition may be regarded as utilizing a shared understanding of context between the sender and recipient of the message. It is holistic, it depends on the whole set of communicators and their common history. The intrinsic idea is well illustrated by the old joke of the traveling salesmen telling jokes by numbers. One cries out "13" and all of the group except one very somber individual laugh. Someone asks him why he did not laugh. He replies "Not only have I heard that one before, he did not tell it well."

4.5 Complexity or Simplicity

> ...von Neumann's logical design of a self-reproducing cellular automaton provides a connecting link between natural organisms and digital computers. There is a striking analogy with the theory of games at this point. Economic systems are natural; games are artificial. The theory of games contains the mathematics common to both economic systems and games, just as automata theory contains the mathematics common to both natural and artificial automata. (A.W. Burke, 1966)

A lesson from the generation of fractals is how to obtain complexity from iterated simplicity. The highly intricate patterns which can be generated from the iteration of simple operators on elementrary structures illustrate how the apparently complex can be explained simply or coded parsimoniously.

The basic idea behind the ingenious answer to the banal and imprecise question of "how long is the coast line?" is that the answer is a function of the length of the ruler. The shorter the ruler, the more one can measure the nooks and crannies. Going to the limit by shrinking the ruler Mandelbrot (1983) proposed a nonintegral measure of dimension which he named a "fractal." Thus the jaggedness of the Norwegian coast produces a fractal dimension of 1.6. Mandelbrot observed that there are many phenomena, such as the ruggedness of landscapes and the changes in financial prices which show the same structure at any level of magnification.

An important breakthrough in the development of game theory was the switching of mathematical emphasis away from the differential calculus to combinatorics. But immediately the question arises "is combinatorics simpler or more complex than analysis of continuous functions?" For the very few agents it may be regarded as simple; for a middling number it appears to be complex and for extremely large numbers a simplicity may emerge in the form of mean or average properties of the system.

Stuart Kauffman (1993) has proposed a conjecture about life as an emergent property of the overall stochastic interaction of the elements in an open thermodynamic systems, i.e., a system with a net energy or food flow. In particular the methodological stress is on the importance of random graphs, where, as connections among initially isolated nodes are increased randomly and groupings of large components are created, the system dynamics based on the way neighbors influence each other is suggestive of phase transition where with low connectivity and low neighbor influence there is little dynamics, but as either the individual influence grows or the number of neighbors is increased the system activity increases until it reaches a threshold of chaotic behavior. Kauffman utilizes his $N\ K$ model where there are N nodes and K arcs to investigate the proposition that the greatest possibility for self-organization and the formation of life is at the boundary region beyond which the system becomes chaotic.[9]

The value of Kauffman's approach is not to be judged by the ingenuity of the abstraction, but by its fruitfulness and the evidence which can be mustered from biological investigation. But from the viewpoint of the game theorist, the modeling of agents, rules of the game and strategy sets appear to be appropriate.

In their book Epstein and Axtell (1996) argue for growing artificial societies "from the bottom up." They offer various simulations of sex, culture, conflict, disease and trade. In contrast with the work of Kauffman on the origin of life, or Arthur *et al.* on the stock market I find the basic modeling is somewhat less convincing. The aim of the simulations of Kauffman and Arthur *et al.* has been to answer specific questions where a case can be made that the representation of actors and context is adequate. It is possible that these simulations with a simple structure can be used to provide analogies in the study of the evolution of markets or culture, but this use is not unlike conversational game theory.

Arthur, Durlauf and Lane (1997) suggest six features of the economy which cannot be easily handled by prior mathematical economics. They are:

[9] Some time ago, prior to the development of simulation methods I proposed a quasi-cooperative solution to dynamic games called K-R stability (Shubik, 1959, Chs. 10,11) for application to games of economic survival whose equilibria were contingent on a random element and on the level of interaction among the agents. The static interpretation of the $K\ R$ relationship was as a code of behavior. The study of the emergence of a code requires an examination of a high dimensional model which is essentially not feasible without the use of the computer.

1. *Dispersed interaction* which comes with heterogenous agents each acting upon anticipation of the behavior of a limited number of others and on the aggregate state.

2. *No global controller* controls interaction, institutions provide the mediating devices for the intermix of competition and cooperation which typifies complex human behavior.

3. *Cross-cutting hierarchical organization:* there are many levels of organization, many of which serve as "building blocks" for more complex institutions, but the interconnections among them can be virtually any network.

4. *Continual adaptation:* Behaviors actions and strategies are under constant revision.

5. *Perpetual novelty:* Niches are constantly being created by new markets, institutions, technologies and behavior.

6. *The economy operates* far from any equilibrium. The incessant bombardment of change from technology, society, the polity.

The models of Kaufmann (1993), Bak (1996), Conway (see Sigmund, 1993) and others together with this perceptive list should be regarded as a challenge to the economist, biologist, other behavioral scientists and game theorists in particular. A central element is the study of coevolution in nonconservative, nonequilibrium systems. A new direction in game theory is called for to help in this task. For this purpose the stress must be on the study of masses of heterogenous agents with varied but limited abilities interacting in a stochastic environment. This need not cause unemployment to the devotees of *homo oeconomicus* in situations in or near equilibrium. There are still plenty of detail-dependent high information operations research oriented problems in the study of auctions, voting, marketing, assignment and agency which are worth examining. But the problems of emergent organizations and coevolution call for an expansion of the models of current conventional game theory.

5 Scale and Behavior

Subatomic physics and cosmology concern themselves with phenomena on vastly different scales. The unit of prime concern differs. In a similar manner the social scientist can select the level of reductionism beyond which the investigator chooses not to go. The agent selected by most game theorists is the individual and the basic assumptions concerning the decision structure were essentially given in the description of the game by von Neumann and Morgenstern. Probably due to our comfort with anthropomorphic concerns we are more comfortable with the individual as our basic unit. However depending on the scale at which we choose to work we may obtain highly different mappings from the elements in the physical world onto the players, the strategy sets, and the goals or payoffs. A game theoretic model to study the emergence of financial institutions may select economic individuals of varying expertise as the basic decision units. Theories devoted to the study of the emergence of life, of the minimal self reproducing systems (von Neumann, 1966, Kauffman, 1993) or evolutionary biology will employ different primitive units strategy sets and payoffs. Institutions as actors may provide the basic unit for the study of history and civilization.

5.1 Complexity and Redundancy

An organization that is elaborate and static may be less complex than one that has a simple structure but is designed to cope with dynamics.. In the elaborate static organization as long as the liturgy remains the same, once the neophyte has learned it, there is nothing more to learn.

A key concern in many organizations and organisms is survival and being able to survive in fluid environments with routines which are "good enough." A way of designing robust organizations is to ensure sufficient redundancy.[10] Von Neumann regarded error control (and hence redundancy) as one of the two key problems in the design of automata.

5.2 Game Theory Solutions, Rules of Thumb and the Passions

The great novel *Magister Ludi* by Hesse (1970) can be considered as an allegory illustrating the clash between the historical and analytical approaches to the study of society. The mathematical models of game theory, (in contrast with the "conversational" use of game theoretic analogies) has tended to reflect the bloodlessness of an abstract game. Strategy is considered without passion. A major project in the expansion of the uses of game theory is to accommodate the passions and those aspects of interaction among individuals whose function appears to provide the possibility for elaborate coding and decoding of messages which could not be otherwise handled by the capacity constrained individual. A broader game theory must encompass or at least be consistent with hope, fear, greed, envy, sloth, love, humor, anger, rage and all the other factors that a good "gamesman" knows by instinct are critical in human interaction.

Humans have passions. We tend to talk about bees or ants "being angry" when their nest is attacked. My guess is that insects do not have passions, nor do they carry out the calculations on mutually consistent expectations. Table 1 shows a 3×3 matrix game where I suspect that a pair of bugs confronted with playing, say $T = 300$ times would fail to do a backward induction and would be stupid enough to merely follow a best response cycle yielding an expected payoff of 1-1/2 per agent per period

[10] There is a literature in game theory on the structure of simple games which is related to the literature in electrical engineering on redundancy and the Chow factor measure. Dubey and Shapley (1979) discuss this. Shubik and Weber (1981) provide an example of a two person game where redundancy is critical for survival.

rather than play the sophisticated unique perfect equilibrium strategy of (1/3, 1/3, 1/3) yielding an expected payoff of 1 each.

Table 1.
A Simplex 3×3 Game

	1	2	3
1	2, 1	1, 2	0, 0
2	0, 0	2, 1	1, 2
3	1, 2	0, 0	2, 1

5.3 An Aside on an Eye-for-an-Eye and Tit-for-Tat

In recent years there has been a considerable interest in the emergence of cooperation. Much of the impetus to this work has derived from the imaginative work of Robert Axelrod (1984). Much has been made of the strategy of "tit-for-tat" and its success in a Prisoner's Dilemma game. I suggest that the emphasis on tit-for-tat and the randomly matched plays of the prisoner's dilemma are somewhat misplaced. The better generalization is "eye-for-eye and tooth-for-tooth" which is well defined for all games with symmetric outcome sets regardless of the underlying preference structure of the individuals. Furthermore it has the additional feature that by reacting with kind for kind, not merely are perceptions simplified but also the possibility for avoiding escalation of any variety is minimized. Tit-for-tat is not well defined on all games, even on all the 78 strictly ordinal preference 2×2 bimatrix games.[11]

6 Some Futures

6.1 Classical Game Theory: More of the Same

I suspect that both classical cooperative game theory and Bayesian noncooperative theory will continue to flourish. There is still much to be learned from parsimonious normative axiomatic models and in spite of the view of many laymen that the only active game theory is noncooperative theory, a perusal of the technical journals will show considerable activity in cooperative theory.

The activity in modifying the concepts of noncooperative play in stochastic games viewed in extensive form with incomplete information and lack of common knowledge provides employment for any number of highly intelligent and trained mathematically inclined game theorists, but the problem lies more in the way *homo ludens* has been modeled than it does in refinements of the definition of equilibrium.

[11] These points which might be regarded as game theoretic minor details require a detailed discussion to fully appreciate their importance. A tournament like that of Axelrod, but utilizing other of the 78 2×2 matrices is called for, possibly broken into several classes, such as those games with a unique pure strategy NEs, those which are symmetric and those with more than one NE.

6.2 Evolutionary Game Theory and Biology

I expect that evolutionary game theory will grow. Many a "pure game theorist" will feel that it is not "real" game theory because the actors really are preprogrammed mechanisms. But in one sense they can be regarded as reasonable simple models of agents with limited rationality Furthermore the possibilities are good for examining the experimental evidence for the predictions of evolutionary game theory. Evolutionary game theory methods may be used to examine the ramifications of goals such as "inclusive fitness" proposed by the socio-biologists (see, for example, Trivers, 1985). Basic biology and the study of animal behavior offer considerable scope for large scale simulations where masses of agents following relatively simple behavioral rules. At what point such models can or should not be regarded as "game theory" or relevant to game theory is a matter of taste and academic "turf war." My tastes run towards understanding problems rather than worrying too closely about the status of methodology.[12] The study of coevolution and innovation fit here.

6.3 Game Theory, Limited Machine Players and Artificial Intelligence

The theoretical work of Rubinstein (1986) has already been noted. I believe that along with the theory a considerable growth of experimentation with individuals playing artificial players in nonzero as well as zero sum games is called for (see for example, Hoggatt, 1969). This type of work provides for important comparisons and insights between human and machine performance. Shubik and Wolf (1971) while experimenting with a simple business game, out of sheer economy and ease in experimentation decided to employ an artificial player as the competitor. One of our students Scott Lockhart wrote the program for the artificial player. We glued in heuristics such as "cooperate unless you are double crossed." The artificial player was basically a follower. We asked the players to describe their competitor after they had played. This was essentially a Rorschak test We experimented with monopoly and duopoly games and found that although academic excellence was relevant to the scores on monopoly, the interpersonal view of competitiveness was far more important in determining success in duopoly games.

Human knowledge is both of an individual and a social nature. Several bright undergraduates in game theory and I attempted to build a playable parlor game based on differentiated players endowed with know-how and know-who. A partial sketch is given to indicate the "flavor" of the modeling. There is a safe which has a combination consisting of many numbers in a particular order. In the safe there is a divisible prize. The coalition of individuals which succeeds in opening the safe may split the prize among its members. Each individual is given, by the rules a fixed finite memory size. Each individual can use the memory to store addresses of others or

[12] There may be a worthwhile cooperative game theoretic question in biology concerning the number of sexes. Why are there no species with three or more agents actively required in reproduction? Haploid and diploid relationships are the only ones which guarantee the existence of a core. Games with vital coalitions of size two appear to play a special role in many activities.

parts of the combination, but no individual has a large enough capacity to know the whole combination. Individuals may write over occupied memory. Communication and memory each have a cost. Each individual has a budget. Discussions between a pair of communicating individuals are regarded as costless or outside of the formal model. Any deal they make is enforceable only if it is still recorded by the participant who calls it into action. A deal is recorded by A if he has the name and address of B and the name of the deal. For example, in a three person game; between them B and C may each have the two needed strings of numbers of the safe's combination. But each does not have room for more than their piece of the combination and one address. Player A specializes in "know who." He makes a contract with both B and C for them to supply the numbers for the combination.

Games of this variety could be used as a hybrid AI device in the sense that one can impose some control on permitted usable memory size to see how humans organize trading off know who versus know how.

The popular book by Feigenbaum, McCorduck and Nii (1988) shows how far one can go and how useful it can be to construct elementary expert systems in the context of locating "useful" "doable" problems in the industrial or medical world. Once more I must return to my basic theme. The value of the answer depends on the understanding of the question. The understanding of the question depends heavily on the context of the problem. The executive working in Northrop or IBM or Toyota is not trying to unlock the secrets of the universe *in abstraco* but is willing to support an AI system if it applies to a problem at hand. The Dreyfus and Dreyfus (1986) critique of AI is only merited if one confuses the simple industrial applications of AI with the deep problems and the approaches outlined by Minsky (1985) or Gelernter (1991) and others. The work of the latter two is far closer to the directions which a dynamic game theory must take.

6.4 Crowd Behavior, Bubbles, Panics and Phase Change

In spite of the great book of Le Bon (1977) having been written over 100 years ago we still know surprisingly little about the behavior of mobs. When does a set of isolated individuals turn into a crowd? When does a crowd turn into a mob? What is the genesis of a financial bubble? What converts a bubble into a crash? What are the circumstances which leave mass anonymous behavior more or less uncorrelated? Are there fruitful analogies between these changes in crowd behavior and phase changes in physics?

The stock market and other mass anonymous economic markets may be viewed as mechanisms facilitating the coordination of millions of individuals who each regard themselves as a solitary player against an aggregate mechanism called the market. As long as their behavior is noncorrelated we expect to encounter no more than white noise around a noncooperative equilibrium. But it appears that in the way expectations about market behavior are formed that history matters. The sophisticated viewpoint of Arthur *et al.* (1997) with the simulation of individuals selecting from a host of heuristic rules as they "learn" from the market, offers a mass learning approach to the formation of expectations and price in the stock market; the approach of De Long *et al.* (1990) was more analytical; in contrast Bak, Pazuski and Shubik (1997) employed a simplistic model with fundamental analysts and chartists who were simpler than the Arthur agents, they, in essence learned nothing.

These two highly preliminary many agent simulations with both homogeneous and hetrogeneous agents are indicative of a new direction which, by the current purists would not be regarded as game theory but deal with the central issue of competition and collaboration in game theory in a different manner.

7 Concluding Remarks

In the overall view of the role of game theory in the behavioral sciences there has probably been a gross underestimation of the normative and applied value of cooperative theory of games and there has probably been a gross overestimation of the effectiveness of noncooperative theory.

The power of cooperative theory is present for two sets of reasons. First, it has provided a high level of abstraction which has enabled us to construct and analyze axiom systems designed to examine subtle concepts such as power, equity, fairness, decentralization and efficiency under many different conditions imposed on the measurability and comparability of preferences and the presence or absence of methods of side payment. The second set of reasons is that there are a host of directly applied problems where a normative analysis makes sense. These applications have been discussed in Part II.

The applications of noncooperative equilibrium theory to economics, biology, political science and law have been fruitful. The attempts to modify noncooperative equilibria for multistage stochastic games, especially without common knowledge may have some special applied value to problems such as agency relations in a modern corporation, but in general, I suspect that they are reaching the point of diminishing returns. Any reasonable examination of noncooperative equilibria in multistage games shows an enormous proliferation of equilibria.

The main lesson to be learned from surveying the work on solutions to multistage games is that there is no satisfactory general theory of strategic dynamics and it is unlikely that one will emerge from modifications of the definition of the noncooperative equilibrium preserving the *homo oeconomicus* model of the decision maker. It is important to remind ourselves that the model of rational fully informed man is a poor approximation of the individual and it is used in the study of many decision problems for mathematical convenience, not as the ideal to which human's strive. For many purposes it is a poor representation of mature, senile or immature humans. Furthermore the underlying bias in the model of *homo oeconomicus* is nonevolutionary.

The study of game theory gave us a powerful language which has helped us to examine the problems faced by consciously optimizing individuals in multiperson situations. The success of its application has shown its limits. The computational and knowledge requirements for analytical solutions tell us that this is not the way these problems are solved. The paradoxes involving the mismatch between individual and social rationality illustrate the difficulty in settling on any unique "right" n-person solution.

The growing body of evidence on how individuals behave in experimental games and the concern for understanding competition and cooperation among, genes, cells, insects, plants and other living organisms suggests the direction for the further development and applications of the theory of games.

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