COWLES FOUNDATION FOR RESEARCH IN ECONOMICS
AT YALE UNIVERSITY

Box 2125, Yale Station
New Haven, Connecticut 06520

COWLES FOUNDATION DISCUSSION PAPER NO. 780

Note: Cowles Foundation Discussion Papers are preliminary materials circulated to stimulate discussion and critical comment. Requests for single copies of a Paper will be filled by the Cowles Foundation within the limits of the supply. References in publications to Discussion Papers (other than acknowledgment that a writer had access to such unpublished material) should be cleared with the author to protect the tentative character of these papers.

THE MONETARY-FISCAL MIX: LONG-RUN IMPLICATIONS

James Tobin

January 10, 1986
American Economic Association  
New York, December 28, 1985

THE MONETARY-FISCAL MIX: LONG-RUN IMPLICATIONS

James Tobin∗

Since 1981 the United States has faced, for the first time in history, the prospect that the federal debt would grow faster than the national product indefinitely. Economists have been prominent among the Cassandras deploring runaway public debt, but they have not been very specific about its hazards to the health of the nation. The usual story is "crowding out." The citizens’ savings are limited. The more that the federal government borrows, the less are available for capital investments, the sources of productivity advances on which the living standards of the future depend. This is an unexciting story of slowdown in growth, and we are usually imprecise about magnitudes and speeds, and especially about whether, how, and when government borrowing leads to a catastrophic crisis.

The purpose of this paper is to present explicitly and precisely the crowding-out story, in a way that exposes the roles of the parameters of fiscal and monetary policies and the macroeconomic structure. The model is first presented algebraically, and then illustrated numerically by simulations assuming arbitrary but, it is hoped, plausible parameter values.

For many sets of parameter values, these simulations do end in catastrophes, which can be precisely described and dated. I want to

∗Cowles Foundation for Research in Economics, Yale University, New Haven, CT 06520. I thank Daphne Butler and Willem Thorbecke for valuable help in computations.
be clear at the outset that these are illustrative exercises, warnings not
predictions. They are, it is true, motivated by recent trends in the United
States. But my prediction is that tax and/or expenditure policies will
sooner or later be changed enough to stabilize the debt/GNP ratio. Indeed
those changes appear to be on the way already.

I. The Model: General Structure

Since I am here concerned with long-run trends, the structure of the
model is borrowed from neoclassical growth theory. Full employment of an
exogenously growing labor force is assumed. Gross output is produced by
labor and fixed capital; it is divided between consumption and investment by
the saving decisions of households, who are both workers and capitalists.
The production function allows for variable proportions of the two inputs.
The marginal productivity of capital determines the short-term interest
rate.

Government debt is private wealth in this model. That is why it crowds
out the alternative store of value, productive fixed capital. Ricardo-Barro
equivalence effects, whatever their general validity, are not appropriate
here. No one expects deficit-reducing tax increases or spending cuts when
none have been put in place and the government denies or ignores their
necessity. The message of this exercise is that they do need to be put in
place so that people have credible reason to expect them.

Transactions with the rest of the world are not modeled in this
exercise. In practice, as shown by recent experience, crowding out of net
foreign investments (current account surpluses) mitigates the impact of
budget deficits on domestic capital formation. But borrowing abroad cannot
in general spare the economy the consequences of allowing government debt to absorb ever increasing shares of private saving. It could do so only if foreigners were willing to lend to us indefinitely at real interest rates below our economy's trend rate of real growth.

There is nothing new in studying long-run crowding out by use of neoclassical growth models. That is commonly done by comparative static analysis of steady states. Reduction in the fraction of national product saved and invested, resulting in the case at hand from governmental dissaving, moves the economy slowly from one steady-state path to another. The second path has a smaller capital/labor intensity than the first, therefore lower real wages and lower per capita consumption. Neither the transitional dynamics nor the differences between equilibrium paths are very dramatic. Students to whom this scenario of crowding out is exhibited yawn and wonder what all the shouting is about.

What is different in my simulations is attention to the possibility that, for quite realistic values of parameters, no steady states exist, or that the only one close to the initial conditions is an unstable equilibrium. In these cases an unstable vicious circle can lead fairly quickly to a dramatic crisis.

The key departure from the usual projections of deficits and debt growth and their effects is to make interest rates endogenous. Interest costs contribute to deficits and the growth of public debt. Increases in rates are the mechanism by which government borrowing squeezes capital investment.

Monetary policy enters the model via the fraction of public debt monetized by the central bank. Here I assume that the Federal Reserve holds
the inflation rate constant and determines the degree of monetization accordingly. A higher inflation target slows down the pace of crowding out, because greater "seignorage" lowers the interest cost on total debt. In this full employment model, there are no direct monetary effects on capital formation, which is governed wholly by saving. Given the fiscal parameters, the only way the central bank can alter policy is to change its inflation target.

There are two parameters of fiscal policy. The main fiscal parameter is the ratio of the primary deficit — the deficit exclusive of after-tax interest outlays — to GNP. Given this parameter, changes in the income tax rate make a difference in two familiar ways. Private saving depends on after-tax income, and both saving and money demand may depend on after-tax interest rates.

II. The Model in Detail

The non-federal sector of the economy holds at all times a stock of wealth equal to a multiple \( \mu \) of the GNP. Life cycle theory suggests that non-human wealth is a multiple, \( w \), of after-tax labor income, which here is a constant fraction \( 1 - \alpha \) of GNP, where \( \alpha \) is the elasticity of gross output with respect to capital in a standard two-factor Cobb-Douglas constant-returns-of-scale production function. Then, writing the federal proportional income tax rate as \( \tau \) and letting \( w = 1 - \tau \), we have:

\[
(1) \quad \mu_0 = wu(1 - \alpha)
\]

Wealth demand may also be related to the after-tax interest rate \( uR \). Here this relation is assumed linear; a non-negative coefficient \( p \) represents the response of savings to interest.
\[ \mu = \mu_0 + \mu u(R - R_0) = \mu_0' + \mu uR \]

(\(R_0\) is introduced simply for calibration of the initial conditions of the simulations. Other variables similarly subscripted below play the same role.)

The wealth/GNP ratio \(\mu\) is composed of the capital/GNP ratio \(k\) and the government debt/GNP ratio \(d\):

\[ k + d = \mu \]

Debt takes two forms, nonmonetary, costing the government an after-tax real rate of interest \(r\), and monetary, costing the government zero nominal interest, thus a real rate the negative of the inflation rate \(\pi\).

The stock of base money is a fraction \(h\) of GNP. The demand for base money, relative to GNP, depends on the nominal interest rate \(r + \pi\). For a given inflation target \(\pi\), the central bank sets \(h\) to meet the demand:

\[ h = h_0 - y(r - R_0) - (\pi - \pi_0) = h_0' - y(r + \pi) \]

The non-negative coefficient \(y\) is higher the more sensitive are demands for base money to nominal after-tax interest rates.

The fundamental dynamic equation for \(d\) is:

\[ d = x + d(r - g_y) - h(r + \pi) \]

Here \(x\) is the "primary deficit" in ratio to GNP. The growth rate of real GNP is \(g_y\). Equation (5) says that the debt grows by the primary deficit if there is no outstanding debt at all (first term); that given an initial debt, it grows further by its net interest cost to the Treasury but declines relative to GNP by the economy's growth rate (second term); and that it declines by the amount of seigniorage (third term).

In a steady state real output can grow at its natural rate \(g\). The gross marginal productivity of capital is \(\omega/k\), and \(\delta\) is the constant rate
of capital depreciation:

(6) \( R = \alpha / k - \delta \)

The government's net interest cost of borrowing is lower than \( R \) for two reasons. One is that its creditors return part of their interest receipts in taxes on the interest, -- indeed on the nominal interest, a fact that saves the Treasury \( \tau \). The second is that the government can borrow with a credit-risk discount \( v \) from the after-tax return to capital equity. Thus:

(7) \( r = uR - v - \tau \)

The actual growth of GNP is:

(8) \( g_y = g + (\alpha / (1 - u))(k/k) \)

Combining (5) and (3) yields a differential equation in \( k \):

(9) \( k = \gamma - x - (\gamma - k)(r - g_y) + h(r + \tau) \)

This is the fundamental dynamic equation of the model. The strategy for its solution is to express all the variables in (9) in terms of \( k \) and of the policy parameters \( (x, u, \tau) \) and structural parameters \( (w, \alpha, \beta, \gamma, \delta, g, v) \). The other equations above enable this to be done.

Tedious algebra leads to the differential equation:

(10) \( k = Q(k)/V(k) \)

where \( Q(k) \) is a cubic polynomial and \( V(k) \) is a quadratic, with coefficients that depend on the parameters. If \( \beta \) and \( \gamma \) are both zero, the degrees of the two polynomials are reduced by one. This is a convenient reference case.

The more important of these two conditions is the assumed zero value of \( \beta \). This implies that crowding out is unrelieved by any increase in saving induced by rise in interest rates. A positive value of \( \gamma \), on the other
hand, tends to make matters slightly worse than in the reference case. As interest rates rise and the demand for money falls, the central bank has to monetize less debt in order to meet its inflation target. In any case, for realistic values of $h$, seignorage is quantitatively small.

III. The Solution

Figure 1 depicts $Q$, $V$, and $k = Q/V$ for the reference case, all as functions of $k$. Since $d = \mu - k$ and $d = \mu - k$, it also depicts $d$ as a function of $d$. The quadratic $Q$ has a positive intercept. Its roots are its intersections, if any, with the horizontal axis. In Figure 1 there are none. The denominator $V$ is a negatively sloped line, which crosses the $k$ axis at $aq$. The vertical line at that point separates two quite different behaviors of $(k, k)$. To its left, $k$ is always positive, rising asymptotically to the dividing line. This part of the solution has no practical interest. The relevant region is to the right of the dividing line, where $Q/V$ will have roots if and only if $Q$ does. If these exist, they are steady state values of $k$ and thus also of $d$. The higher of the two is stable, the smaller unstable. If, as in Figure 1, $Q$ has no roots, $k$ is always negative in the relevant region; there is no steady state. Clearly whether or not $Q$ has real roots depends on the parameters. The general case, with $p$ and $\gamma$ non-zero, is qualitatively like the reference case but more complicated.

The economy is in trouble if there are no positive real roots of $Q$ and thus no steady states. From whatever initial condition in the right region, $k$ will steadily dwindle, at an ever increasing rate. The same trouble occurs even if roots and steady states exist, if the initial values
of \( k \) and \( d \) are to the left of the lower, unstable root. The question is what happens as \( k \) declines along the curve \( Q/V \), according to which \( k \) goes to minus infinity. Gross investment cannot be less than zero; the capital stock cannot decline faster than the depreciation rate \( \delta \). Accordingly the capital/output ratio \( k \) cannot decline faster than:

\[(12) \quad k = - (\delta + g)(1 - a)k \]

Equation (12), a line graphed in Figure 1, replaces (9) when \( Q/V \) is smaller than this number. The intersection of the two functions is the point of switching from Regime I to Regime II.

Although the rate of capital consumption is limited, the growth of debt continues. In regime II, \( k + d \) exceeds \( \mu \). What gives? Assuming the structural and policy parameters remain the same, the natural adjustment is the valuation of the capital stock. In regime I, the value of \( q \) was implicitly 1. But in regime II, when no gross investment is taking place, \( q < 1 \). Indeed such valuation is the neoclassical signal to agents that investment is an uneconomic use of output. The wealth constraint becomes:

\[(13) \quad qk + d = \mu \]

\[(14) \quad \dot{d} = - \dot{q}k - \dot{q}k + \mu \]

The equation for the interest rate \( r \) is also different. Holders of public debt compare its return not with \( uR \) but with \( uR/q + q/q \), what they can earn on equity. Thus (6) becomes:

\[(15) \quad r = uR/qk - u\dot{q}/q + uq/q - (v + \pi) \]

The dynamics of regime II can be found by substituting (12) - (15) into (5), the same strategy used above for regime I. The result is an expression giving \( q \) as a function of \( q \) and \( k \):

\[(16) \quad \dot{q} = (q, k)/V(q, k) \]
where, given \( k, Q \) and \( V \) are respectively quadratic and linear in \( q \).

In regime II the growth of debt "crowds out" \( q \), pushing it down to zero, to make room for \( d \) within the wealth/income ratio.

IV. Simulation Results

Table 1 reports for the reference case the values of parameters and initial conditions, and some of the results of its simulation. Figure 1 shows the simulation graphically. The initial condition for \( (k, k) \) is the point marked \( 0 \) on the curve \( Q/V \). From that point \( k \) declines, \( d \) increases, as shown on the curve for subsequent dates 5, 10, 11.6. At time 11.6 there is a regime switch. The regime II path is just a line from that point to the origin.

Figure 2 shows what happens in Regime II. The upper left panel shows the decline in \( q \) and \( qk \) as \( d \) continues to increase, until when \( q \) is zero at time 23 the debt absorbs all private wealth. The upper right panel tracks the amount by which debt plus capital stock valued at par exceeds the demand for wealth. The lower panel shows \( q \) over the same period.

Thus the debacle is the cessation of gross investment, followed by a decline in the stock market until the surviving capital stock is valueless.

Table 1 includes the simulated values of capital stock/GNP, debt/GNP, GNP itself, and national consumption (including government purchases) for the two critical dates, for regime shift and for \( q = 0 \). GNP and consumption are measured relative to what they would have been had they grown steadily at the economy's natural rate of growth \( q \). Note that before the regime change the shortfalls of those variables are trivial; indeed consumption has not suffered, because the decline in national saving has
offset the decline in capital intensity. As I remarked above, the visible penalties of gradual crowding out are undramatic. The crunch comes in Regime II, when losses of output and consumption become severe.

How do variations in the policy parameters affect the outcomes? The key fiscal parameter is $x$, the ratio of the primary deficit to GNP. With the other parameters the same as in the reference case, $x$ must be below 0.00375 to make the system stable. But with $x$ at that value, the steady state values of $k$ and $d$ are respectively 2.37 and 0.13. The initial conditions (2.1, 0.4) are on the wrong side of that equilibrium. Hence $k$ dwindles, but so slowly that the economy is still well inside regime I after fifty years. To have a steady state equilibrium $k$ of 2.1 or lower, $x$ must be smaller than −0.001. In effect, a balanced primary budget is stable at or near the initial conditions. That is true in the reference case because the net real interest rate on federal debt, allowing for monetization, is very close to the natural growth rate. This choice of parameters and initial conditions was not accidental; it appeared to me that the United States debt/GNP ratio could be stabilized if the primary budget were balanced.

The inflation target makes a difference too. But in the reference case, with $x = 0.03$, it takes a 12% inflation to obtain a stable solution. Likewise, lowering the target from the reference value of 4% to 1% shortens the life of regime I by only one year.

I turn to the structural parameters. The most important one is $\phi$, the reponsiveness of private demand for wealth to the real interest rate. In current macroeconomic theory, the interest elasticities of saving and wealth demand are key parameters, and there is a lively debate about their
empirical magnitudes. In the reference case perfect inelasticity is assumed. At the other extreme, perfect elasticity at the initial interest rate, clearly there would be no crowding-out problem at all.

An informative summary measure of the effects of varying a parameter is the duration of regime 1, in years. In Figure 3, this measure is related to the value of $\phi$. To calibrate $\phi$, note that a value of 10 corresponds to an elasticity of about 1/4. The Figure assumes the reference case values of other parameters. Given so high an $\chi$, strict stability is not possible with any finite $\phi$. But positive values of $\phi$ do slow down the crowding-out dynamics and prolong regime 1. Values of 15 or higher make its duration longer than 25 years. The interest elasticity of money demand is less important. Introducing a $\gamma$ of 0.25, which corresponds at initial values to an interest elasticity of money demand of about 1/4, turns out to make very little difference.

I reiterate that these simulations are not predictions. They are designed only to illustrate why remedial policies should and will be adopted, and thus to make concrete the vague forebodings about runaway government debt.
<table>
<thead>
<tr>
<th><strong>Policy parameters</strong></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>x primary deficit/GNP</td>
<td>0.03</td>
</tr>
<tr>
<td>τ tax rate</td>
<td>0.20</td>
</tr>
<tr>
<td>π inflation target</td>
<td>0.04</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Structural parameters</strong></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>α capital share of GNP</td>
<td>0.30</td>
</tr>
<tr>
<td>δ depreciation rate</td>
<td>0.07</td>
</tr>
<tr>
<td>g natural growth rate</td>
<td>0.03</td>
</tr>
<tr>
<td>μ private wealth/GNP</td>
<td>2.50</td>
</tr>
<tr>
<td>h base money/GNP</td>
<td>0.05</td>
</tr>
<tr>
<td>η savings/interest coefficient</td>
<td>0</td>
</tr>
<tr>
<td>Ψ money demand/interest coefficient</td>
<td>0</td>
</tr>
<tr>
<td>ν interest premium</td>
<td>0.01</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Initial conditions</strong></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>d(0)</td>
<td>0.40</td>
</tr>
<tr>
<td>k(0)</td>
<td>2.10</td>
</tr>
<tr>
<td>R(0)</td>
<td>0.073</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Selected simulation results</strong></th>
<th>regime change</th>
<th>g = 0</th>
</tr>
</thead>
<tbody>
<tr>
<td>T year</td>
<td>11.6</td>
<td>23</td>
</tr>
<tr>
<td>k(T) capital/GNP</td>
<td>1.5</td>
<td>0.7</td>
</tr>
<tr>
<td>d(T) debt/GNP</td>
<td>1.0</td>
<td>1.8</td>
</tr>
<tr>
<td>Y(T)/Y*G(T) GNP/natural growth GNP</td>
<td>0.87</td>
<td>0.63</td>
</tr>
<tr>
<td>C(T)/C*(T) consumption/natural growth C</td>
<td>1.0</td>
<td>0.76</td>
</tr>
</tbody>
</table>
FIGURE 1: Simulation of the Model (Reference Case)

Numbers refer to years from "present." Regime I, years 0-11.6, Crowding Out. Regime II, years 11.6-, Capital Consumption.
FIGURE 2: SIMULATION OF REGIME II (Reference Case)
Upper left: decline of q. Upper right: excess supply of capital.

Lower panel: path of q
FIGURE 3: Effect of Interest Sensitivity of Savings on Duration of Regime I (Positive Gross Investment) for Two Inflation Targets, .04 and 0.

Note: β's higher than plotted points prolong regime I more than 25 years.