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COORDINATING COORDINATION FAILURES IN KEYNESIAN MODELS

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This paper focuses on the importance of strategic complementarity in agents' payoff functions as a basis for macroeconomic coordination failures. We first analyze an abstract game and find that inefficient equilibria and a multiplier process may arise in the presence of strategic complementarities (essentially positively sloped reaction curves). We then place additional economic content on the analysis of this game by considering strategic complementarities arising from: production functions, matching technologies and commodity demand functions in a multi-sector economy.

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I. INTRODUCTION

It is said that there are two types of people in the world: those who divide the world into two groups of people, and those who do not. By contrast, there are three types of papers in the macroeconomic literature on unemployment theory. First, those of the new classical macroeconomics have sought to argue that underemployment/unemployment arises from intertemporal substitution of leisure and/or misperceptions of prices due to an inability to distinguish perfectly between changes in relative prices and changes in the general level of prices. Second, articles in the Keynesian tradition suggest that unemployment arises from non-rational expectations and/or wage and price rigidities; many insights of these theories have been formalized in the fix-price literature. And, third, there is a group of papers which start with the observation that there are two theories of unemployment--new classical and Keynesian--and then offer an alternative model. This discussion here is concerned with papers in this third category.

More specifically, a number of authors have recently constructed examples of economies which exhibit underemployment equilibria, but where the results do not derive from the usual Keynesian assumptions; see, for example, Bryant (1983), Diamond (1982), and Weitzman (1983). The models in these papers instead generate their results from the inability of agents to coordinate their actions successfully in a many-person, decentralized economy. Unfortunately, these examples of coordination failure are superficially very dissimilar: for example, Diamond's model is grounded in search theory, Bryant emphasizes imperfect information, and Weitzman stresses increasing returns. As a result, the similarities of these models have been obscured and the crucial elements of the coordination failures have not been fully identified. One of the aims of this paper, therefore, is to provide a general framework which we can use to analyze these different models and explain how they relate to one another. Our more ambitious goal is to use this framework to yield further insights into Keynesian coordination failures and to relate this literature to other models in the Keynesian tradition, including those with fixed prices.

The feature highlighted throughout the paper is that of a positive externality in optimal strategies; following Bulow, Geanakoplos and Klemperer (1983), we term this strategic complementarity. That is, we consider the condition that the optimal level of "effort" of a particular agent depends positively on the levels of effort of the other agents in the economy. In such circumstances an all-around expansion of economic activity might be desirable, but an individual increase in effort may not be to the advantage of any given single agent. Our analysis of strategic complementarity has two components. First, we construct a simple game, essentially devoid of economic structure, and show that strategic complementarity is associated with the presence of "Keynesian features" such as underemployment equilibria and a multiplier. Second, we attempt to place additional economic content on the analysis of this game by means of a number of examples, drawing on the previous literature where appropriate.

In Section II, therefore, we develop a game in which players' optimal strategies depend upon the strategies of other agents. Because we wish to analyze situations where agents fail to coordinate their actions, the natural equilibrium concept to utilize is that of Nash equilibrium, which is by its nature non-cooperative. Given strategic complementarity, there may be multiple symmetric Nash equilibria; we examine the welfare properties of these and show that our game may exhibit inefficient (underemployment) equilibria. In addition, we show that strategic complementarity is necessary and sufficient for multiplier effects in our game.

In Section III, we go beyond the abstract game of Section II and consider the possible economic interpretations of strategic complementarity. The economic system can generate complementarities in agents' payoffs, both at the level of technology and preferences and through the market and non-market interactions of agents. First, we discuss complementarities arising from the production technology, using some simple examples, and present a market interpretation of the model proposed by Bryant (1983). Diamond's (1982) model is then used to illustrate how strategic complementarity can arise from the matching technology. Finally, we present an extended example where complementarities arise from agents' demands. In Section IV, we summarize our arguments and discuss possible extensions and questions for future research.

II. SYMMETRIC NASH EQUILIBRIA AND STRATEGIC COMPLEMENTARITY

To begin the analysis, we consider a relatively abstract game exhibiting a multiplicity of symmetric Nash equilibria. Assume there are I agents indexed $i = 1, 2, \ldots, I$ where agent i controls a variable $e_i \in R_+$. (For motivation, we sometimes call e_i the effort of agent i.) Denote the payoffs of agent i by $\sigma^i(e_i, e_{-i})$ where $e_{-i} \equiv (e_1, e_2, \ldots, e_{i-1}, e_{i+1}, \ldots, e_I)$ is the vector of efforts chosen by the other players. Assume that the payoff functions are continuously differentiable and define $\sigma^i_j \equiv \partial\sigma^i/\partial e_j$ and $\sigma^i_{jk} \equiv \partial^2\sigma^i/\partial e_j \partial e_k$. Finally, assume that $\sigma^i_{ii}(\cdot) < 0$.

In a Nash equilibrium, agent i takes the actions of the other agents, e_{-i}, as given and optimizes over e_i. The optimal choice of e_i,

$e_i^*(e_{-i})$, satisfies $\sigma_i^i(e_i^*(e_{-i}), e_{-i}) = 0$. We will assume that agents have identical payoff functions and hence focus on symmetric Nash equilibria (SNE). By a slight abuse of notation, we denote the payoff to agent i of action e_i when all other agents take effort $\bar{e}$ by $\sigma^1(e_i, \bar{e})$ and let $e_i^*(\bar{e})$ be the optimal effort of agent i when $e_j = \bar{e}$ for all $j \neq i$.[1] The SNE of the game are defined by

$$S = \{e \in R_+ | e_i^*(e) = e \text{ for all } i\} .$$

At a SNE, if all other agents are choosing effort e , it is in the interest of the remaining agent to choose effort e as well. To characterize the set S further, denote the derivative of agent i's payoff function with respect to agent i's effort when $e_j = e$ for all $j \neq i$ by $\phi^i(e)$; i.e., $\phi^i(e) = \sigma_i^i(e,e)$. Since agents are identical, we will drop the superscript i on this function (so that $\phi^i(e) = \phi(e)$ for all i).

To ensure that S is non-empty, we assume that $\lim_{e_i \to 0} \sigma_i^i(e_i, e_{-i}) \geq 0$ and $\lim_{e_i \to \infty} \sigma_i^i(e_i, e_{-i}) \leq 0$ for all e_{-i} . Hence $\lim_{e \to 0} \phi(e) \geq 0$ and $\lim_{e \to \infty} \phi(e) \leq 0$. Given our continuity assumptions on $\sigma^i(\cdot)$, $\phi(e)$ is continuous and so there will exist an $e^* \in S$ (see Figure 1).

As discussed in the Introduction, one aim of this paper is to develop a game-theoretic analysis of models with "Keynesian features," including conditions for multiple inefficient SNE. From Figure 2, we see that a necessary condition for multiple SNE is that $\phi'(e^*) \geq 0$ for some e . A sufficient condition is that $\phi'(e) \geq 0$ for $e \in S$.[2]

From our definition of $\phi(e)$, $\phi'(e) = \sum_{j=1}^{I} \sigma_{ij}(e,e)$. Given our assumption that $\sigma_{ii}(\cdot) < 0$, a necessary condition for multiple symmetric Nash

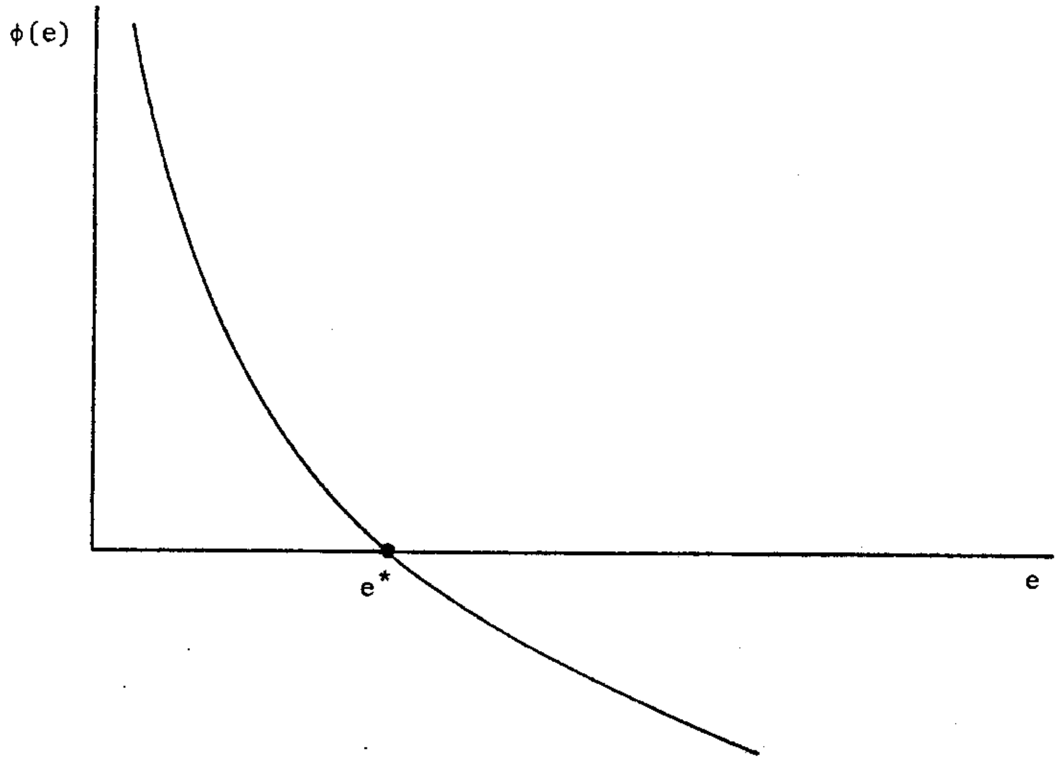

FIGURE 1

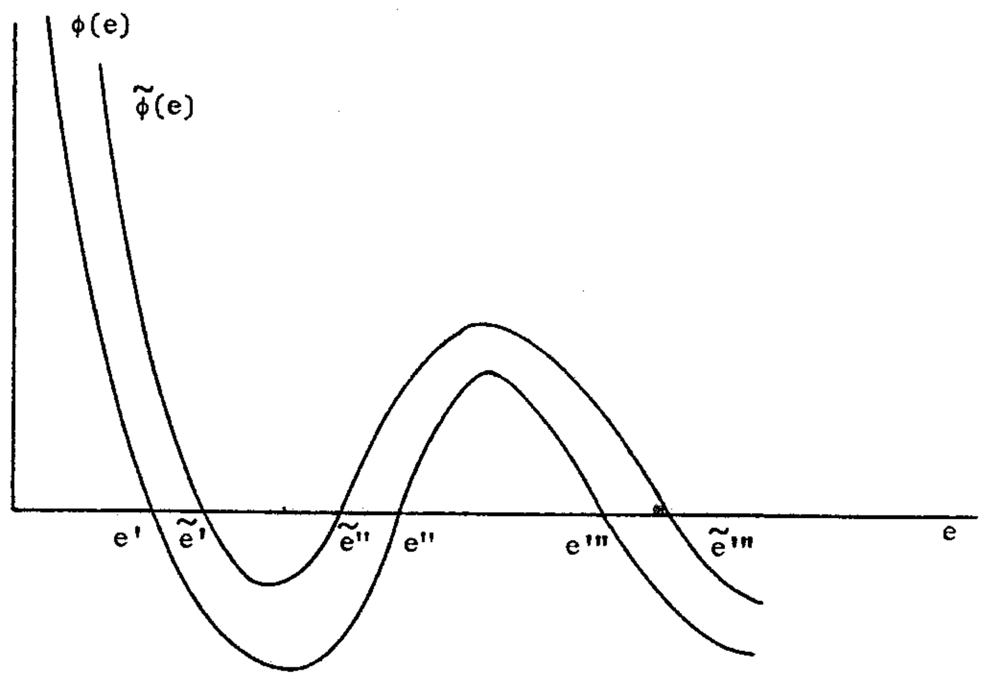

FIGURE 2

equilibria is that $\sigma_{ij}(\cdot) > 0$ for some j, which, following Bulow, Geanakoplos and Klemperer, we term strategic complementarity.

The role of strategic complementarity in generating multiple symmetric Nash equilibria is intuitively simple. Suppose that there are 2 players in the game $(I = 2)$. If the payoff functions exhibit strategic complementarity, then player 1's optimal effort level will be an increasing function of player 2's effort level (and vice-versa given the symmetry of the players). Hence the players' reaction functions will be upward sloping and multiple symmetric equilibria become a possibility.

Another of our interests is the welfare properties of the set of equilibria, S. Many leading examples of economies with Keynesian features make use of strategic complementarity (as we discuss below) and argue that Pareto-inferior allocations arise due to the inability of agents to coordinate their effort levels. In most of these examples, the equilibria are ordered by the levels of effort undertaken by the agents.

With regard to the welfare properties of allocations in S, there are two types of inefficiencies worth investigating. First, it may be that none of the $e \in S$ is efficient relative to the set of feasible allocations. This type of inefficiency is quite familiar and is generally due to externalities in the payoff functions. Agents, in choosing their effort levels, do not take into account their effect on others' payoffs. A second type of inefficiency arises when there are multiple SNE which can be Pareto-ranked. In many of the examples we explore below, strategic complementarities generate multiple equilibria.

To be more precise, suppose that e_i for $i = 1, 2, \ldots, I$ could be cooperatively set to

$$\text{maximize} \quad \sum_{i=1}^{I} \sigma^i(e_i, e_{-i}) \ .$$

The solution to this problem satisfies

$$\sum_{i=1}^{I} \sigma_j^i(e_i, e_{-i}) = 0 \quad \text{for all} \quad j \ .$$

Given the symmetry of the payoff functions, we focus on cooperative solutions in which all agents take the same level of effort. Define $\tilde{\phi}(e) = \sum_{i=1}^{I} \sigma_j^i(e,e)$ and denote the set of symmetric solutions to the first-order conditions for the planner's problem by

$$\tilde{S} = \{e | \tilde{\phi}(e) = 0\} \ .$$

To explore the relationship between the sets S and $\tilde{S}$, it is useful to note that

$$\tilde{\phi}(e) = \phi(e) + \sum_{i \neq j} \sigma_j^i(e,e) \ .$$

As one would expect, a sufficient condition (when the payoff functions are identical) for $e \in S$ but $e \notin \tilde{S}$ is that there be externalities present in the payoff functions, i.e. $\sigma_j^i(e,e) \neq 0$ for $j \neq i$.

In many of the cases we discuss, $\sigma_j^i(e,e) > 0$ for arbitrary e , $j \neq i$. If so, the $\tilde{\phi}(e)$ function lies uniformly above the $\phi(e)$ function as shown in Figure 2. (The $\tilde{\phi}(e)$ function is drawn with the same number of crossings as the $\phi(e)$ function which is not necessary.) In this case, it is quite clear that for each $e_i \in S$, there will exist an $\tilde{e}_i \in \tilde{S}$ with $\tilde{e}_i > e_i$ such that $\tilde{e}_i$ is preferred by all agents to e_i .

A second and more interesting type of inefficiency arises when there are multiple symmetric Nash equilibria (as shown in Figure 2). If $\sum_{i \neq j} \sigma^i_j(e,e) > 0$, then any equilibrium, say e'', with $\phi'(e'') > 0$ will be dominated by another, $e''' \in S$ with $e''' > e''$. Since $\tilde{\phi}(e) > 0$ for $e \in [e'', e''']$, moving from e'' to e''' increases the welfare of each agent. (Figure 2 shows an example of this.) Hence, at e'', agents are stuck at an inefficient equilibrium with a low-level of economic activity. This is a coordination failure. (At this level of generality, we cannot rank the other allocations in S.)

A final feature of Keynesian models that we wish to investigate is the multiplier. Generally, one thinks of multiplier effects arising when aggregates fluctuate more than individual variables. We show that strategic complementarity in the payoff functions is necessary and sufficient to generate multiplier effects. Intuitively, this is the case because changes in agent i's effort will induce changes in the actions of all other agents in the <u>same</u> direction, and this will in turn induce a further change in agent i's action, much as in the standard multiplier story.

To see this formally, consider a version of our model with $I = 2$. Let the payoff function for agent 2 be as described above, but let agent 1's payoff depend also on a random variable, $\tilde{\theta}$. That is, assume that $\sigma^1 = \sigma^1(e_1, e_2; \tilde{\theta})$, with $E\theta = 1$. We assume that $\sigma^1_3 > 0$ and $\sigma^1_{13} > 0$ (in an obvious notation). The game between the 2 players occurs after θ is determined. When $\tilde{\theta} = 1$, the payoff functions are assumed to be <u>identical</u> and one can characterize the Nash equilibria as the solutions $e^*_1 = e^*_2 = e^*$ to $\sigma^1_1(e^*, e^*; 1) = \sigma^2_1(e^*, e^*) = 0$.

As $\tilde{\theta}$ varies, we observe variations in the equilibrium values of e_1 and e_2. From the two first-order conditions, one can determine the partial effect of $\tilde{\theta}$ on the optimal choice of e_1 for a given e_2 as well as the equilibrium effect: $\partial e_1^*/\partial\theta$ and $de_1^*/d\theta$ respectively. First, one can show that $de_1^*/d\theta > \partial e_1^*/\partial\theta$ as long as $\sigma_{12}^1 \neq 0$.[3] That is, we get a type of multiplier effect associated with agent 1's effort whenever we have strategic complements $(\sigma_{12}^1 > 0)$ or strategic substitutes $(\sigma_{12}^1 < 0)$ in the payoffs.

More specifically, using the conditions for a Nash equilibrium at $\tilde{\theta} = 1$, we find that $de_1^*/d\theta = (\sigma_{11}^1\sigma_{22}^2/K)\cdot(\partial e_1^*/\partial\theta)$, where $K \equiv \sigma_{11}^1\sigma_{22}^2 - \sigma_{12}^1\sigma_{21}^2 > 0$ if we confine our attention to stable Nash equilibria. Therefore

$$\frac{de_1^*}{d\theta} = \frac{1}{1 - (\sigma_{12}^1/\sigma_{11}^1)^2}\,\frac{\partial e_1^*}{\partial\theta},$$

where we have made use of the fact that the payoff functions are identical when $\tilde{\theta} = 1$. Hence a multiplier effect is present from agent 1's viewpoint.

At an aggregate level, consider the effect on the aggregate effort level $(e_1 + e_2)$ of a change in θ . From the conditions for a Nash equilibrium, one can calculate that

$$\frac{d(e_1^* + e_2^*)}{d\theta} = \frac{1}{1 + (\sigma_{12}^1/\sigma_{11}^1)}\,\frac{\partial e_1^*}{\partial\theta}.$$

As $\sigma_{11}^1 < 0$, we see we have an aggregate multiplier effect when the payoff functions exhibit strategic complementarity. Since $de_2^*/d\theta$ is positive if $\sigma_{21}^2 > 0$, the aggregate change in equilibrium effort levels exceeds the partial equilibrium response of agent i . Finally, the magnitude of the multiplier depends on $-\sigma_{12}^1/\sigma_{11}^1 = -\sigma_{21}^2/\sigma_{22}^2$ which are the terms for the slopes of the agents' reaction functions. Hence steeper reaction functions generate larger multiplier effects.

As a final note, many of the examples we explore exhibit a continuum of equilibria. These have the virtue of providing clear cases of welfare orderings dependent on the level of economic activity. When there is a continuum of symmetric Nash equilibria, it follows by definition that $\phi(e) = 0$ over an interval, and hence $\phi'(e) = 0$ over that interval also. Provided that $\sigma^i_j(e,e)$ $(j \neq i)$ is positive, welfare will be increasing in e over the interval. In such examples the multiplier is infinite.[4]

III. SOME ECONOMIC EXAMPLES

In this section of the paper, we bring some economic life to the game discussed in Section II. We discuss both market and non-market games and focus on a number of sources of strategic complementarity. Think of an economy where agents' utility depends on their own efforts (e_i) and their ultimate consumption (c_i). If c_i depends on the effort levels of other agents (e_{-i}), then the coordination problems discussed in Section II can arise. What are the ways in which one can model the dependence of c_i on e_{-i}? Following Scitovsky (1954), we focus on externalities in technologies as well as externalities in the manner in which agents' transact. Drawing on models in the literature, we discuss how these sources of externalities can play a role in generating multiple, inefficient equilibria and, for some examples, a multiplier.

A. Input Games

To begin our study of the role of strategic complementarity in generating Keynesian results, we consider the problem of coordination among input suppliers to a shared production process.[5] As in our general model, we let e_i be the effort (input) of player i into the production of a single consumption good, c. The *per capita* production function is

$c = f(e_1, e_2, \ldots, e_I)$ with $\partial f/\partial e_i > 0$. Agents have identical utility functions defined over consumption and effort, $U(c,e)$, with $U_c > 0$, $U_e < 0$ and $U(\cdot)$ quasi-concave. These preferences, combined with the per capita production function, generate agent i's payoff function

$$\sigma^i(e_i, e_{-i}) \equiv U(f(e_1, e_2, \ldots, e_I), e_i)) .$$

Hence the properties of $\sigma^i(\cdot)$ are derived from the utility function and the production function. We have assumed for now that each of the players receives an equal allocation of the consumption good. Later, we analyze a market version of the game which endogenizes the distribution of the consumption good.

With our assumptions on $U(c,e)$, a necessary condition for strategic complementarity (and hence multiple symmetric Nash equilibria) is that $\partial^2 f/\partial e_i \partial e_j > 0$, i.e., inputs e_i and e_j are complements in the production process. This technological complementarity must override the "substitutability" between e_i and e_j induced by the (potential) concavity of $U(c,e)$ with respect to consumption.

There are a number of potentially interesting ways to generate complementarity of inputs within the production process. One could, for example, specify $f(e_1, e_2, \ldots, e_I) = g(\sum_{j=1}^{I} e_j)$ and assume that $g(\cdot)$ was globally increasing and convex over some interval. (It should be noted that while increasing returns of this variety can generate multiple symmetric Nash equilibria, it is quite different from the argument advanced by Weitzman (1982). As we discuss later, Weitzman made use of complementarities through demands rather than the strategic complementarity induced by increasing returns to scale.)

Alternatively, suppose that we have two agents with payoffs given by $U^i(c,e) = c - e^2 + \gamma_i e$ where $\gamma_1 = \gamma_2 = \gamma > 0$. Suppose that each agent has an endowment of effort (time) equal to $\bar{e} > \gamma$. Finally, let the per capita production function, $f(e_1, e_2)$ be $c = e_1 e_2$. It is straightforward to demonstrate that the reaction functions for the agents have the form $e_i = \frac{1}{2}(e_j + \gamma)$ as shown in Figure 3. The unique Nash equilibrium is $e_1^* = e_2^* = \gamma$. A social planner would want each agent to exert effort $\bar{e}$. In a decentralized system, the agents ignore the external benefits of their action and produce $\gamma < \bar{e}$.

This provides a useful example of the multiplier effect. Suppose that the parameter γ_1 varies. One can show that $\partial e_1^*/\partial \gamma_1 = 1/2$, $de_1^*/d\gamma_1 = 2/3$ and $de_2^*/d\gamma_1 = 1/3$. Hence equilibrium aggregates fluctuate more than individuals' effort levels. For this example, the multiplier, $1/(1 + \sigma_{12}^1/\sigma_{11}^1)$ equals 2.

Bryant (1983) assumed that $f(e_1, e_2, \ldots, e_I) = \min(e_1, e_2, \ldots, e_I)$. To understand the input game with this technology, we consider the input choice of agent i given the input decisions of other agents, e_{-i} . Let e_{-i}^{min} be the smallest effort level in the vector e_{-i} . The input choice of agent i can be seen in Figure 4. If $e_{-i}^{min} \leq \bar{e}$ where $\bar{e}$ solves $U_c(\bar{e},\bar{e}) = -U_e(\bar{e},\bar{e})$, then the optimal response of agent i is to set $e_i = e_{-i}^{min}$. Figure 4 displays agent i's choice set and the optimal choice of effort. Since agents are identical, there will be a continuum of equilibrium in the interval $[0,\bar{e}]$. These equilibria are Pareto-ranked (welfare increases in the effort level) with $\bar{e}$ being Pareto optimal. Due to coordination problems, the economy can get stuck at a low level of output.

As is clear from Figure 4 as well, the reaction function of agent i with respect to e_{-i}^{min} coincides with the 45° line. Hence reaction curves

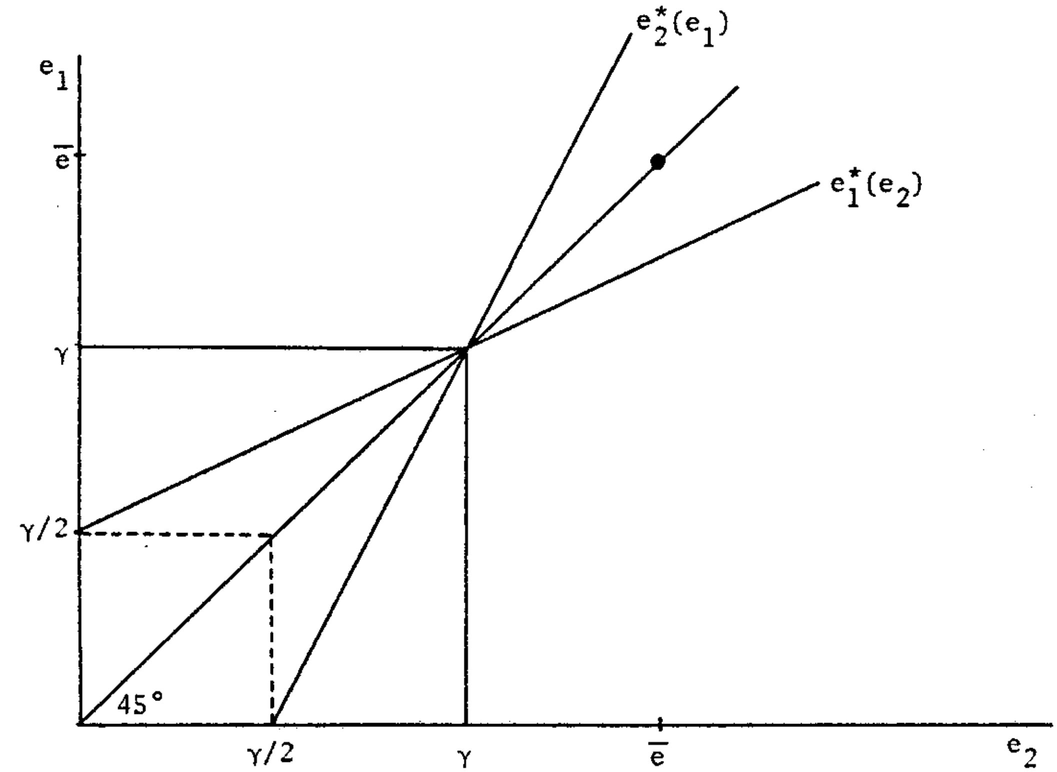

FIGURE 3

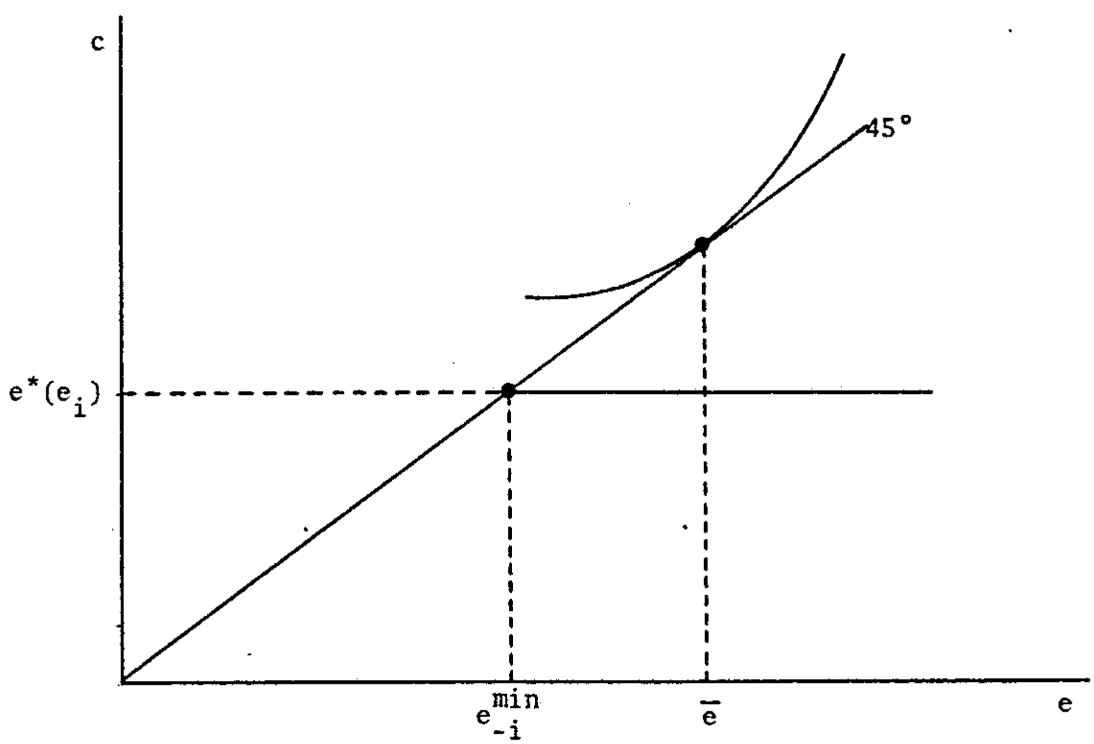

FIGURE 4

are positively sloped which is the essential feature of strategic complementarity. The multiplier is infinite here.

These examples serve to illustrate the importance of strategic complementarities in the production process in generating models of coordination failures. Since we assumed (as did Bryant) that agents simply shared output equally, the analysis leaves open the role of market structure. It would be useful to know whether these coordination problems arise in a competitive setting or whether some form of imperfect competition is important.

It is straightforward to consider a market version of the game outlined by Bryant.[6] Suppose, for simplicity, that $I = 2$ so there are two factor markets and a market for the consumption good. The intermediate goods producers sell their output to the consumption goods producers at the relative prices w_i (where the consumption good is the numeraire).

We will consider two configurations of market power. First, if there are a large number of intermediate and consumption goods producers, then we can investigate the competitive equilibria of this economy. For the economy described above, there exists only a single equilibrium with zero profits and market clearing. In this equilibrium, $w_1 = w_2 = 1$ and the efficient level of output, $\bar{e}$, is produced. Hence we immediately see that strategic complementarity in the technology is <u>not</u> sufficient to produce underemployment equilibria; clearly imperfect competition of some form must be important.

In considering imperfectly competitive economies, we can envisage a variety of possibilities distinguished by the source of market power. Assume for the moment that there is free entry (hence zero profits) into the production of the consumption good while the intermediate goods producers maintain monopoly power. Sellers of the intermediate good take the demand curves for their products as given. These demand curves, of course, depend on the decisions of the other suppliers. As before, we consider a Nash equilibrium.

We consider a two-stage game. In the first stage, input suppliers produce output levels, k_i, for $i = 1, 2$. Then, suppliers choose prices w_i for their input to maximize utility of $U(w_i e_i, k_i)$ where e_i is the amount of the input actually sold. We let z_i be the demand for input i by final goods producers and define

$$e_i \equiv \min(z_i, k_i) .$$

Here we have invoked the principle of no involuntary trades as used in the rationing literature.

To describe the determination of e_i (and hence w_i and k_i), we need to characterize z_i. We assume that there is a single final goods producer and allow entry only if profits for this activity are positive. This producer takes (w_j, k_j) for $j \neq i$ as given and determines z_i from

$$\max_{z_1, z_2} \left(\min(z_1, z_2) - \frac{w_1 z_1 - w_2 z_2}{2}\right)$$

$$\text{subject to} \quad z_j \leq k_j \quad \text{for} \quad j \neq i .$$

In this problem, the choice of z_i is constrained by the capacity constraints faced by the firm on the <u>other</u> input markets. This view of quantity-constrained equilibrium follows Benassy (1975) and is motivated by the view that firms determine input demands one market at a time.

From this problem, we can derive the demand curve facing firm i given the (w_j, k_j) chosen by the other firms, $z^i = D^i(w_i | w_j, k_j, j \neq i)$.

In the second stage of the game, input producer i chooses w_i to maximize utility subject to (w_i, e_i) lying on the demand curve of the final goods producer. Since this demand curve depends on (w_j, k_j), we obtain the strategic interaction between producers which will form the basis of the coordination problem. In the first stage of the game, input producers have conjectures about the capacity levels chosen by the other producers and also anticipate the resolution of the second stage game. We focus on the Nash equilibrium of this two-stage game.

Proposition 1: There is a continuum of equilibria for this two-stage game with $w_i = 1$ and $k_i = z_i = \hat{e}$ for all i with $\hat{e} \in [0,\bar{e}]$.

Proof: Suppose we solve the second stage game given $k_i = \hat{e}$ for all i. We need to show that $w_i = 1$ if $w_j = 1$ for all $j \neq i$. Given the assumed fixed coefficients technology, the demand function facing input producer i is shown in Figure 5. The utility maximizing point is clearly $w_i = 1$ and $e_i = z_i = \hat{e}$. If $w_i > 1$, the final goods producer is out of business and if $w_i < 1$ the input producer loses revenue and still only sells $\hat{e}$.

In the first stage game, if $k_j = \hat{e}$ for $j \neq i$, then agent i anticipates the second stage of the game just described and hence correctly conjectures the demand function shown in Figure 5. Therefore it is optimal to set $k_i = \hat{e}$. □

Hence, we see there is a market equilibrium corresponding to each of the equilibria in Bryant's model. In these equilibria with $\hat{e} < \bar{e}$, individual goods producers all wish to produce more at the input price of 1, but cannot do so because of the demand constraints they face. We again see inefficient equilibria due to coordination problems.

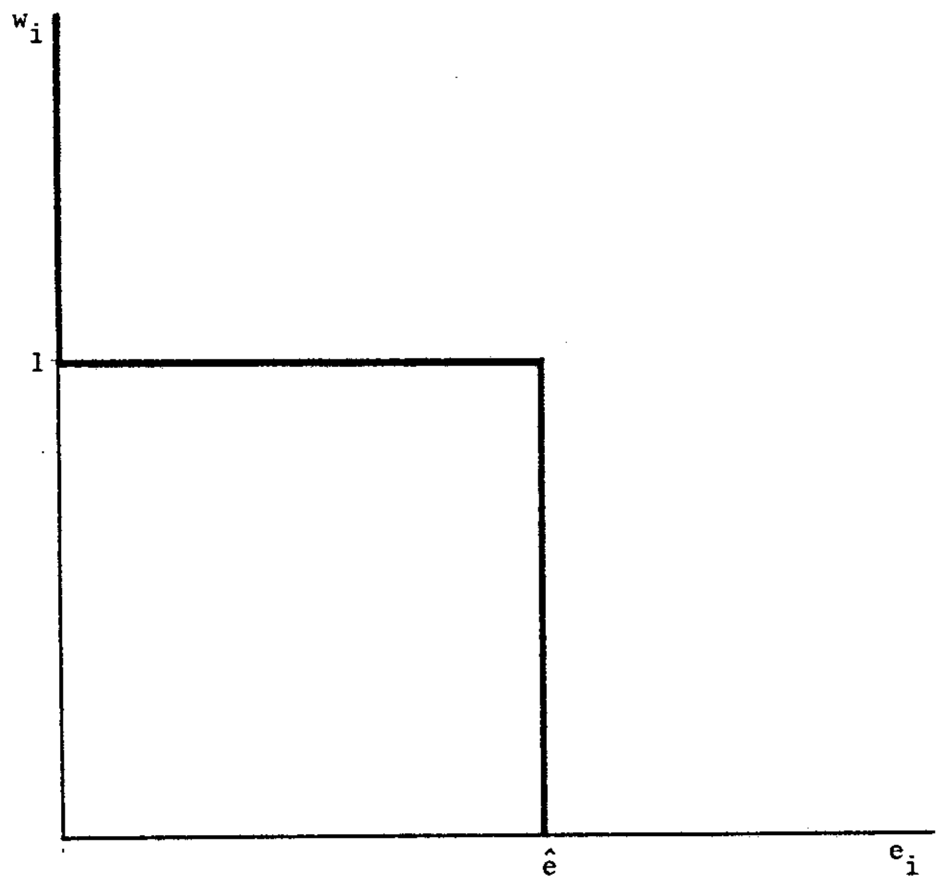

FIGURE 5

It should be noted that this result of a continuum of inefficient equilibria does not arise if the market power lies with final producers. In that case, the single producer could coordinate all activities and the inefficiency would be the usual one associated with imperfect competition.

As a final note, it is important to remember that there are two types of externalities arising in these input games. In the example shown in Figure 3, there is a unique, inefficient equilibrium due to the presence of a technological externality. The importance of strategic complementarity arises in the multiplier process. In the market and non-market versions of Bryant's model, the strategic complementarity generates a continuum of equilibria. A planner would not want to change any <u>one</u> of the input levels at an inefficient equilibrium since marginal products are zero. Hence the coordination of individual effort choices is the more important problem in this example.

B. Trading Externalities

Another important model which exhibits strategic complementarity is that of Diamond (1982). In this model, the externality arises from the trading technology. In Diamond's economy, individuals face production decisions which arrive stochastically and have varying costs; having made a decision to produce, agents then seek trading partners, who also arrive stochastically. Individuals trade on a one-for-one basis and consume the good so obtained; utility depends negatively upon the cost of production and positively upon consumption. While Diamond's model is set in continuous time, the essential point, for the purposes of our paper, can be illustrated in a static model where agents face a single production opportunity with an uncertain cost and then face a given probability of finding a trading partner. If an individual does not find a trading partner, the produced good perishes.

In terms of the game of Section II, an agent's payoff is an expected utility; if an individual elects to produce, then he or she faces a certain cost, and a return conditional on finding a trading partner. We define the payoff as that expected prior to the arrival of the production opportunity. An individual's strategic decision is therefore that of whether or not to accept a given production possibility, or--equivalently--it is the choice, *ex ante*, of a cut-off cost of production such that he or she will produce if and only if the cost of production is less than or equal to this cut-off value. This latter interpretation yields a continuous strategy variable, in keeping with the game in Section II. Letting $\tilde{e}_i$ be the realized cost of the production opportunity, agent i chooses a cut-off e_i such that he or she produces for $\tilde{e}_i \leq e_i$. When $\tilde{e}_i = e_i$, the agent is indifferent between producing and not producing. The assumption in Diamond's model that generates strategic complementarity is that the probability of finding a trading partner is an increasing function of the number of individuals seeking to trade. Hence the expected payoff to agent i from producing increases as more individuals produce--that is, as other agents raise their cut-off choices (e_{-i}). Since agent i's expected payoff increases, he or she will expect more production opportunities to be profitable, and will increase e_i . Consequently, an increase in e_{-i} generates an increase in e_i .

The presence of strategic complementarity can be shown even more clearly as follows. Let U be the utility obtained from consuming the produced good (with zero utility from zero consumption) and let $\tilde{e}_i$ be the (realized) cost of producing in utility terms. Assume that $p = p(e_{-i})$ is the probability that agent i finds a trading partner and let $G(\tilde{e})$ be the distribution of production costs. Then it follows that

$$\sigma^i(e_i, e_{-i}) = \int_0^{e_i} (pU - \tilde{e})g(\tilde{e})d\tilde{e} ,$$

where $g(\tilde{e})$ is the probability density function associated with $G(\tilde{e})$, and hence

$$\sigma^i_i(e,e) = \phi^i(e) = (p(e)U - e)g(e) .$$

So

$$\phi'(e) = [U(\sum_{j \neq i} \partial p/\partial e_j) - 1]g(e) ,$$

and

$$U(\partial p/\partial e_j)g(e) = \sigma^i_{ij}(\cdot) > 0 .$$

At a symmetric Nash equilibrium, $\phi(e) = 0$, which implies that $e^*_i = p(e_{-i})U$.

In our version of Diamond's economy, it is clear that the existence or non-existence of multiple symmetric Nash equilibria depends on the form of the $p(e_{-i})$ function. More precisely, we should note that the probability of finding a trading partner is assumed to depend positively upon the number of individuals seeking to trade, which in turn depends positively on agents' effort levels via the $G(\tilde{e})$ function. Thus $p(e_{-i})$ in fact embodies two functions, both of which are increasing; hence $\partial p/\partial e_j > 0$, which is necessary but not sufficient for multiple symmetric Nash equilibria. Placing some restrictions on this relationship, $\sum(\partial p/\partial e_j) \geq 1/U$ is also necessary but not sufficient for multiple equilibria. A necessary and sufficient condition, from Section II, is that $\sum(\partial p/\partial e_j)\big|_{e^*} \geq 1/U$ for some e^* satisfying $e^* = pU$.

It is easy to show that a special case of this model will generate a continuum of equilibria, as in the Bryant model. Suppose that production costs, $\tilde{e}$, are uniformly distributed over $[0,U]$. Then

$$\sigma^i = \int_0^{e_i} (pU - \tilde{e})\left(\frac{1}{U}\right)d\tilde{e} = pe_i - \frac{e_i^2}{2U} .$$

Suppose further that each agent seeking a trading partner randomly meets one other person after trading decisions have been made, and is able to trade if and only if that other agent also elected to produce. From these assumptions it follows that

$$p = \left(\frac{1}{I-1}\right)\sum_{j\neq i} e_j/U = \frac{e_j}{U} .$$

So $\sum_{j\neq i} \partial p/\partial e_j = 1/U$, confirming the existence of strategic complementarity, and $e_i^* = pU = e_j$, implying that there is a continuum of symmetric Nash equilibria. (We can also confirm that $-\sum_{j\neq i} \sigma_{ij}^i/\sigma_{ii}^i = 1$.) The correspondence with Bryant's model should be evident. Since $\sum_{j\neq i} \sigma_j^i = e_i/U > 0$, the equilibria here are Pareto-ranked, with higher levels of e preferred and $e_i = U$ Pareto-optimal.

It is possible to argue that Diamond's externality can be interpreted as complementarity in production or demand. Noting that the trading technology is outside the control of the agents, one could argue that the trading technology is simply a part of the production process. Conversely, the number of potential traders can be taken as an index of demand for an individual's output, which calls to mind a demand externality. These similarities--or ambiguities--are not surprising, for they reflect the common intuition in the three cases. In the next subsection, we clarify this further by examining complementarities in demand.

C. Demand Externalities

As a final example, we consider complementarity between the demands expressed by agents in a multi-sector economy. This is perhaps the most compelling of our models since it captures the intuition that economies may get stuck at low levels of activity when agents are constrained in their sales. It is also perhaps the most "Keynesian" of our examples in that demand constraints play a crucial role. There is a coordination problem in such economies if low-level equilibria could be avoided by all agents' simultaneously increasing their demands. Since such coordination lies outside our non-cooperative equilibrium concept, a role for government is created in our model.

Coordination problems of this type are excluded by assumption in a Walrasian economy where agents can sell any amount at the given price. As with our discussion of Bryant's model, therefore, we would expect to find that this externality will only arise in a market structure where agents require information on both prices and quantities in making their decisions. For most of this section, we discuss a production economy where agents are imperfect competitors and hence do not face an infinitely elastic demand curve. At the end of the section we also discuss the possibility of generating similar results in a fix-price economy.

The key to our analysis is strategic complementarity in demands across various sectors of the economy. Because of the Keynesian flavor of this idea, it is perhaps not surprising that there are more examples in the literature of this type than of complementarities in the production or matching technology. (In particular, recent papers by Hart (1982), Heller

(1984), Roberts (1984) and Weitzman (1983) can be interpreted as examples where a positive demand externality plays an important role.) Consequently, rather than interpreting and extending particular models in terms of our framework (as with our discussion of Bryant and Diamond), we choose instead to construct a parsimonious model that captures those elements of the models in the literature which are of most importance to us.

The model we present is thus designed to fulfill a number of functions. First, it shows how strategic complementarities in demand may generate co-ordination failures; hence it also relates demand complementarities to our earlier examples. Second, it is intended to clarify some aspects of the other examples in the literature, highlight their important features, and illustrate the connections between these models. We discuss these models in more detail later. Third, we use our model to extend these previous examples in a number of ways. In particular, we clarify the nature of the unemployment generated by demand complementarities, the role of increasing returns and imperfect competition, and the relationship between these models and those in the fix-price literature. We do <u>not</u> seek great generality with our example, and hence we specify a very simple stylized structure of technology and preferences.

We consider a simple economy with three markets: a competitive labor market and two imperfectly competitive commodity markets indexed by $i = 1, 2$. There are m firms (indexed by $j = 1, \ldots, m$) in <u>each</u> of these latter markets. These firms can produce output according to the technology:

$$q_{ij} = \max[L_{ij} - \gamma, 0] ;$$

where q_{ij} is the output of the j^{th} firm in sector i, L_{ij} is labor employed by that firm, and $\gamma \geq 0$ is a parameter which can be interpreted as a fixed labor cost. This latter is included to eliminate certain low-level equilibria and to facilitate later comparisons with the model of Weitzman; we stress, however, that the increasing returns in this technology is not essential for our results, as can be confirmed by setting $\gamma = 0$ throughout. We normalize by setting the wage equal to unity. Hence the profits of a given firm (when $q_{ij} > 0$) are

$$\pi_{ij} = q_{ij}p_i - L_{ij} = q_{ij}(p_i - 1) - \gamma .$$

Workers are assumed to be (initially) homogeneous, and have preferences given by $c - nk$, where n is the number of hours worked, c is consumption of the (single) good that a given worker consumes, and $k < 1$ is the disutility of work. There are N workers, each of whom has a working-time endowment of T (hence $n \leq T$). Workers can be employed in either sector, but, once employed, are assumed to develop an immense and immediate distaste for the good they produce. (Ask someone who works in a chocolate factory how much he or she enjoys chocolate!) As a consequence, workers consume only the output of the sector in which they are not employed. This structure of employment and consumption (also found to varying degrees in most of the papers cited earlier) captures in a stylized manner the idea that workers specialize in production and generalize in consumption; it has the effect of excluding the possibility that firms can generate demand for their own output by increasing their employment. In a multi-sector economy, we could of course allow workers to consume more than one good.[7]

Similarly, we assume that (the owners of) firms also spend all their profits on the output of the other sector. By construction, therefore, the (circular) flow of income between the two sectors has no leakages, which (as will be seen below) gives rise to a continuum of equilibria. Allowing leakages by diversifying consumption bundles, introducing savings, etc., might generate locally unique equilibria, some of which could be inefficient. (See, for example, Roberts (1984).)

We assume that workers and firms treat the wage as given, and that firms behave as Cournot-Nash competitors within each sector. Assume in addition that firms recognize the potential influence of their choice of production plan on the willingness of workers to work in the other sector. That is, they recognize that if they choose too low a quantity, it may imply a price such that the real wage of workers in the other sector is below their reservation wage; in this case, demand for their product goes to zero. Since all revenue earned in one sector is spent in the other, the demand curve facing sector i is given by $q_i = \frac{p_{-i}q_{-i}}{p_i}$ (where $q_i = \sum_{j=1}^{m} q_{ij}$), provided that $\frac{1}{p_i} \geq k$ (i.e., the real wage in sector i is greater than or equal to the reservation wage). This demand curve is of course unit-elastic. Firm j in sector i takes as given $q_{i,-j}$, p_{-i} and q_{-i} , and hence solves the following problem:

$$\max_{q_{ij}} \pi_{ij}$$

$$\text{subject to } p_i = \frac{p_{-i}q_{-i}}{q_{ij} + q_{i,-j}} ,$$

$$\pi_{ij} = q_{ij}(p_i - 1) - \gamma , \quad p_i \leq 1/k \text{ and } q_{ij} > 0 .$$

$$\pi_{ij} = -(q_{ij} + \gamma) \qquad , \quad p_i > 1/k \text{ and } q_{ij} > 0$$

$$\pi_{ij} = 0 \qquad , \quad q_{ij} = 0 .$$

A Nash equilibrium for this economy is then described by $(p_i^*, q_{ij}^*,$ $i = 1, 2\ ;\ j = 1, \ldots, m)$ such that:

(i) q_{ij}^* solves the above maximization problem $\forall i, j$, given $p_{-i} = p_{-i}^*$, $q_{i,-j} = q_{i,-j}^*$ and $q_{-i,j} = q_{-i,j}^*$;

(ii) the total demand for labor does not exceed the total supply.

At a symmetric Nash equilibrium, $p_i^* = p_{-i}^*$ and $q_{ij}^* = q_i^*/m = q_{-i}^*/m$. Define $\underline{m} = 1/1-k$ and $\overline{m} = \sqrt{NT/2\gamma}$ and assume $\overline{m} > \underline{m}$.[8] We can then establish the following propositions about symmetric equilibria:

Proposition 2: When $\underline{m} \leq m < \overline{m}$, there exists a continuum of equilibria for this game with $p_1^* = p_2^* = \frac{m}{m-1}$, $q_{ij}^* \in \left[\gamma(m-1), \frac{NT}{2m} - \gamma\right]$ $\forall i,j$, and $L_{ij}^* \in \left[m\gamma, \frac{NT}{2m}\right]$ $\forall i,j$.

Proof: It is readily confirmed that, in general, at a symmetric Cournot-Nash equilibrium in sector i, $q_{ij}^* = \frac{m-1}{m^2} p_{-i}q_{-i}$ and $p_i^* = \frac{m}{m-1}$. The implied real wage in sector $-i$ is $\frac{m-1}{m}$, which is greater than or equal to k because $m \geq \underline{m}$. In order for q_{ij}^* to satisfy condition (i), it must also be the case that all firms are earning non-negative profits (for otherwise they would set $L_{ij} = 0$, produce zero output and earn zero profits). The profits of the j^{th} firm in sector i are $q_{ij}(p_i - 1) - \gamma = \frac{q_{ij}}{m-1} - \gamma$, and hence will be non-negative if $q_{ij} \geq \gamma(m-1)$. This places a lower bound of $\gamma(m-1)$ on the set of quantities (excluding zero) that satisfy condition (i). The total demand for labor is $\sum_i \sum_j (L_{ij}) = \sum_i \sum_j (q_{ij}^* + \gamma)$ $= 2m(q_{ij}^* + \gamma)$, and the total supply of labor is NT. Condition (ii) thus places an upper bound on q_{ij}^* equal to $\frac{NT}{2m} - \gamma$. Finally, the condition that $m < \overline{m}$ ensures that this upper bound on q_{ij}^* exceeds the lower bound, i.e., $\frac{NT}{2m} - \gamma > \gamma(m-1)$. (We note in passing that when $m = \overline{m}$, the continuum of equilibria collapses to a single point, with $q_{ij}^* = \frac{NT}{2m} - \gamma$ $\forall i,j$.) □

Proposition 3: When $m < \underline{m}$, there exists a continuum of equilibria for this game with $p_1^* = p_2^* = 1/k$, $q_{ij} \in \left[\frac{\gamma k}{1-k}, \frac{NT}{2m} - \gamma\right]$ $\forall i,j$, and $L_{ij}^* \in \left[\frac{\gamma}{1-k}, \frac{NT}{2m}\right]$ $\forall i,j$.

Proof: Since $m < \underline{m}$, firms cannot choose q_{ij} such that $p_i = \frac{m}{m-1}$, for this would drive employment in sector $-i$, and hence demand in sector i, to zero. Firms therefore maximize profits by setting $q_{ij}^* = kp_{-i}q_{-i} - q_{i,-j}$, which implies $p_i^* = 1/k$. At this price, the non-negativity of profits implies that $q_{ij}^* \geq \frac{\gamma k}{1-k}$, which places the lower bound on the set of (non-zero) quantities that satisfy condition (i). Condition (ii), as before, places an upper bound on q_{ij}^* of $\frac{NT}{2m} - \gamma$. This upper bound must exceed the lower bound as $m < \underline{m} < \overline{m}$. □

Proposition 4: When $m > \overline{m}$, the only equilibrium for this economy is autarky, i.e., $q_{ij}^* = 0$ $\forall i,j$, and $L_{ij}^* = 0$ $\forall i,j$.

Proof: Since $m > \underline{m}$, we know from Proposition 2 that firms will only earn non-negative profits at positive output levels if $q_{ij}^* \geq \gamma(m-1)$. Since $m > \overline{m}$, $m\gamma > \frac{NT}{2m}$, which implies $q_{ij}^* > \frac{NT}{2m} - \gamma$. But this violates condition (ii), and hence is not an equilibrium. □

Proposition 5: Whenever this economy has a continuum of equilibrium (i.e., $m < \overline{m}$), then, for any given m, these equilibria can be Pareto-ranked, with higher levels of output preferred and full employment $(q_{ij}^* = \frac{NT}{2m} - \gamma$ $\forall i,j)$ Pareto-optimal.

Proof: Firms earn profits equal to $q_{ij}^*(p_{ij} - 1) - \gamma$. Since this expression is increasing in q_{ij}^*, firms strictly prefer higher levels of output (holding m constant). If $m \leq \underline{m}$, workers are at their reservation wage, and hence are indifferent between working and not working. Since firms prefer greater output, the equilibria in this case are Pareto-ranked. If $m > \underline{m}$, then the real wage is strictly greater than the reservation wage, and workers strictly prefer to be employed than unemployed. Hence workers also prefer higher levels of output (employment) and, *a fortiori*, the equilibria are Pareto-ranked. □

We can make a number of observations about the solution to this game. First, at a symmetric Nash equilibrium, $q_{ij}^* = \left(\frac{m-1}{m^2}\right)p_{-i}q_{-i} = \frac{q_{-i}}{m} = q_{-i,j}$. Hence we can confirm that our example does indeed exhibit strategic complementarity across the two sectors of the economy. We can note also that firms' reaction functions have unit slope, which corresponds to our result in Section II for economies with continua of equilibria. The economic intuition for this is as follows. We know from Propositions 2 and 3 that the prices in each market depend *only* on the numbers of firms (when $m > \underline{m}$) or the reservation wage (when $m \leq \underline{m}$). Consequently, if producers in sector $-i$ increase production and hence demand for sector i output, producers in sector i respond with quantity changes that imply no changes in the price in sector i. A unit increase in output in one sector, therefore, induces a unit increase in the output of the other sector. We also know from our analysis in Section II that the multiplier in this economy is infinite--as one would expect, given the absence of leakages.

Second, we can note that our example displays involuntary unemployment whenever $m > \underline{m}$; unemployed workers would strictly prefer to earn the real wage $\frac{m-1}{m}$ rather than their reservation wage k, but cannot do so because the level of employment in this model is entirely demand-determined (except at full-employment). The usual thought experiment of an unemployed worker offering to work at a lower real wage does not seem to make sense in this game; an offer to work at a lower real wage in one sector is equivalent to an offer to raise the price level in the other sector, which contradicts the rules of the Cournot-Nash game that we have assumed. To put it another way, because the labor market is essentially passive in our model, the real wage is determined as a residual, and workers have no market power with which to influence it.

We can also emphasize again that increasing returns are not essential for our results, as can be readily confirmed by setting $\gamma = 0$ throughout. The important ingredients of our model are the strategic complementarity across sectors, and an element of imperfect competition arising from the fixed numbers of firms in each sector. (Under an assumption of constant returns $(\gamma = 0)$, the lower bounds of our continua of equilibria are zero for both $m > \underline{m}$ and $m < \underline{m}$, and there is no upper limit on the number of firms ($\overline{m}$ is infinite).) Increasing returns, in our model as in Weitzman's, can be understood as a means of endogenizing the number of firms when there is free entry. That is, for any given demand curve facing sector i, there will, under increasing returns, be a finite number of firms $\hat{m}$ (given by $\hat{m} = \sqrt{\frac{p_{-i}q_{-i}}{\gamma}}$) such that all firms are earning zero profits.[9] This is still consistent with a continuum of equilibria, where the number of firms varies with the level of output. Since we are assuming that firms earn zero profits, these equilibria will be Pareto-ranked whenever $m > \underline{m}$.

We noted earlier that we also wish to use our model to emphasize the connections between other models of coordination failure in the literature. Some brief comments are in order. In structure, our model most closely resembles that of Heller (1984), who also examines Cournot-Nash equilibria in a two-sector economy. Heller's model apparently has no leakages also, although he does not note the implications of this for the demand curves. We also model the labor market differently, which allows us to generate involuntary unemployment (which is not present in Heller's model).

Hart (1983) and Roberts (1984) present models which emphasize general equilibrium interactions in models with imperfect competition, and are not explicitly concerned with coordination failures. The inefficiencies in these models are generated by the imperfect competition and not by coordination failures; there is a unique equilibrium in these models due to the inclusion of a non-produced good. None the less, both models do incorpor demand externalities of the type considered here. In Roberts' model, this is apparent because of the structure of his model, which is very similar to that of Heller's, and indeed because of his brief discussion of coordination failures in an amended version of his model. In Hart's model, the connection is most apparent in his discussion of multiplier effects (see also footnote 7).

Weitzman (1983) uses a model with differentiated products; hence he assumes a very different structure of preferences and considers Nash equilibria in prices rather than quantities. He also did not explicitly model the labor market. Superficially, therefore, Weitzman's model is very different from ours. However, it can be verified that there is a correspondence between Weitzman's solution and our own, confirming that a demand externality is important in Weitzman's model also. As we noted earlier, we believe that it is this complementarity, not increasing returns, that is the essence of both our results and Weitzman's. Despite the apparent differences of all these models, therefore, our example allows us to argue that strategic complementarity in demands is of importance in them all.

Finally, we would like to relate our model to those in the fix-price literature. To do so, we consider a simple fix-price interpretation of our model under constant returns $(\gamma = 0)$. Suppose that, as before, there are m firms in each sector, but assume now that firms take all prices as given and face a quantity constraint on the amount of output they can sell. We assume a symmetric rationing scheme, so that each firm in sector i can sell $1/m$ of the total quantity demanded in sector i at the given price. We assume, following Benassy (1975, 1982) that agents express demands on each market, taking as given their constraints on other markets. (It will be recalled that we also made use of this interpretation of rationing models in our earlier discussion of Bryant's model.) Assume, finally, that $1/k > p_1 = p_2 = \overline{p} > 1$, so that prices in the two sectors are equal and above the competitive equilibrium price $(p_i = 1)$ but sufficiently low that the real wage exceeds the reservation wage.

A symmetric Nash equilibrium for this game is given by $(q^*_{ij}, \overline{q}_i \;\; \forall i,j)$, where $\overline{q}_i$ is the quantity ration facing sector i , such that:

(i) q^*_{ij} is optimal for each firm j in sector i , taking as given $p_i = \overline{p}$ and $\overline{q}_i$;

(ii) the quantities produced in sector i (q^*_{ij}) generate the quantity constraint $\overline{q}_{-i}$ in sector $-i$;

(iii) the total demand for labor does not exceed the total supply.

Proposition 6: For any given $\overline{p}$ (defined as above), there exists a continuum of equilibria for this game with $q^*_{ij} = L^*_{ij} \in \left[0, \frac{NT}{2m}\right] \;\; \forall i,j$ and $\overline{q}_i = mq^*_{ij} \;\; \forall i$.

Proof: Given $\overline{q}_i$, and the symmetric rationing scheme, each firm in sector i perceives a quantity constraint on its supply of $\overline{q}_i/m$. As profits are given by $q_{ij}(\overline{p}-1)$, which is positive and increasing in q_{ij} , firms will choose $q^*_{ij} = \overline{q}_i/m$ (implying $L_{ij} = \overline{q}_i/m$). Labor demand will not exceed labor supply if $\overline{q}_i \leq NT/2$. The sum of total payments to labor and total profits in sector i is thus $p_i\overline{q}_i = \overline{p}\,\overline{q}_i$. The implied quantity ration in sector -i , given $p_{-i} = \overline{p}$, is $\overline{q}_i$. By condition (ii) there is a Nash equilibrium for this game for any $\overline{q}_i = \overline{q}_{-i} \in \left[0, \frac{NT}{2}\right]$. □

As with our earlier interpretation of this game, the continuum of equilibria arises because there are no leakages in the system. While the proposition is of itself fairly trivial, it is of interest because it shows that fix-price economies may also exhibit strategic complementarity. It arises in this case because an increase in the output of firms in sector i leads to a relaxation of the quantity constraint facing firms in sector -i , and hence to an increase in the output of firms in sector -i . While the example here is special, this intuition is evidently more general --fix-price models with quantity rationing will usually exhibit strategic complementarity. This is striking, because it suggests that traditional fix-price Keynesian models are more closely linked to the models we have examined in this paper than one might at first expect; it is even more notable when one recalls that these recent models are often apparently motivated by the desire to generate Keynesian results without recourse to assumptions of price rigidity. This also perhaps provides strong circumstantial evidence for our intuition that strategic complementarity is a distinguishing element of models with Keynesian features.

CONCLUSION

Our principal finding in this paper concerns the importance of strategic complementarity in agents' payoff functions as a condition for model economies to display Keynesian features. We have argued that strategic complementarities can generate both multiple, inefficient equilibria and a multiplier process associated with changes in exogenous variables. The inefficiencies are driven by the presence of externalities in payoff functions while the multiplicity of equilibria and the multiplier derive explicitly from positive externalities at the level of strategic choices; i.e., positively sloped reaction curves.

We have placed our more general analysis into an economic context by drawing on a number of models in the literature displaying Keynesian features. Our analysis highlights the fact that these externalities can arise at the levels of preferences and technology (as in the Bryant example) or in the manner in which agents organize their transactions (as in the Diamond model and the models of demand externalities). We have also stressed the importance of imperfect competition in generating these inefficiencies.

One can view this approach as arguing for the importance of macroeconomic quantities in microeconomic choice functions. In many of our examples, an individual's choice of effort depended on a measure of the overall level of effort undertaken by others in the economy. This is the intuition behind the congestion problems found in search models and is extended to our examples to other settings.

We plan to extend our analysis in a number of directions. First, it would be useful to consider dynamic, stochastic versions of these examples to shed more light on intertemporal macroeconomic coordination problems.[10] This would allow us, for example, to focus more explicitly on the role of expectations in coordination failures. Given these failures, our second goal would be to understand the role of the government in coordinating economic activity, particularly in an intertemporal context.

FOOTNOTES

1. Unless indicated otherwise, the second argument of $\sigma^i(e_i, e)$ should be viewed as an I-1 vector with e in all components.

2. When $e \in S$ and $\phi'(e) > 0$, this will be a locally unstable equilibrium.

3. At $\tilde{\theta} = 1$, we assume $\sigma^1_{12}(\cdot) = \sigma^2_{21}(\cdot)$ and $\sigma^1_{11}(\cdot) = \sigma^2_{22}(\cdot)$ for arbitrary (e_1, e_2).

4. When $\phi'(e) = 0$, $\sigma^i_{ii} = -\sum_{j \neq i} \sigma^i_{ij}$ so that the denominator of the generalized multiplier $(1/(1 + \sum_{j \neq i} \sigma^i_{ij}/\sigma^i_{ii}))$ is zero.

5. Holmstrom (1982) investigates incentive problems that may arise in team production problems of a similar type. Here we focus on the set of non-cooperative equilibrium while Holmstrom focuses on schemes to support cooperative effort levels.

6. This discussion draws on Cooper (1983). Christophe Chamley, Allan Drazen and Jonathan Eaton provided useful comments on that earlier attempt to understand these issues.

7. One obvious extension would be to assume that there are r produced-good sectors in the economy, and that workers and firm-owners have homothetic utility functions over the $r-1$ goods they do not produce (with all goods weighted equally in the utility function). In that case, an increase in income in one sector would lead to proportionate increases in demand in all other sectors. We can note that this assumption on preferences resembles that in Hart (1982).

8. As will be evident from the proofs of the following propositions, the condition $\overline{m} > \underline{m}$ is necessary for the existence of non-autarkic equilibria.

9. In Weitzman's model, profits are a pure leakage from the system, so a zero-profit condition is also required for consistency.

10. See, for example, the recent work by Chamley (1984) on the paradox of thrift.

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