COWLES FOUNDATION FOR RESEARCH IN ECONOMICS

AT YALE UNIVERSITY

Box 2125, Yale Station
New Haven, Connecticut 06520

COWLES FOUNDATION DISCUSSION PAPER NO. 727



# EXPANSIONARY GOVERNMENT POLICY IN AN ECONOMY WITH COMMODITY AND LABOR CONTRACTS

Russell Cooper

October 24, 1984

# EXPANSIONARY GOVERNMENT POLICY IN AN ECONOMY WITH COMMODITY AND LABOR CONTRACTS

Russell Cooper*

October, 1984

ABSTRACT

This paper considers a model in which all exchange is mediated by contracts. The analysis explores the indexation of labor and commodities contracts to observable variations in government spending financed by money creation. In one of the many equilibria, prices and nominal wages are shown to be independent of current money shocks. Except in the extreme equilibrium exhibiting full indexation, policy shocks will generate correlated movements in output and employment over time. The analysis thus suggests an inverse relationship between indexation of contracts and persistence of policy effects.

*Cowles Foundation for Research in Economics, Yale University.
I am grateful to Costas Azariadis for helpful discussions on this topic.

## I. Purpose

The goal of this paper is to investigate the effects of government macroeconomic policy in an economy in which transactions are mediated by contracts. While the fiction of the "Walrasian auctioneer" has certainly served a useful purpose in the study of general equilibrium, macroeconomics requires a more specific model of the wage-price formation process. To understand the responsiveness of wages and prices to anticipated and unanticipated government policies, it is necessary to have an explicit model of wage and price setting. Towards this end, this paper considers a macroeconomy in which labor services and commodities are traded through contracts. One may view this as an attempt to integrate labor contracting behavior with Okun's [1981] "customer markets." When exchanges are mediated entirely by contracts, it is quite natural to view informational asymmetries as well as spatial and intertemporal separation as a basis for the incomplete nature of agreements in terms of contingencies over time, space and states of the world.

There are a number of puzzles in macroeconomics which this approach may help to understand. First is the diverse patterns of indexation including contracts with nominal rigidities found in both labor markets and the financial sector. In previous work (Gray [1976], Fischer [1977], Cooper [1983]), it has been noted that in the presence of nominal shocks, some indexation of nominal wages to prices was desirable.[1] However, in those models, the prices of commodities were determined in auction markets and "automatically" responded to shocks. So indexing wages to prices substitutes for the (infeasible) indexation of wages to underlying shocks. Any potential interaction in the responsiveness of wages and prices to these shocks was effectively ignored. By modeling the joint determination of wages and prices in a contractual setting as in this paper, this interaction is highlighted. Since prices will not necessarily respond to nominal and real shocks, wages will not necessarily be indexed. In particular, the results of this paper indicate there may exist multiple equilibria in terms of the responsiveness of wages and prices to underlying, observable shocks.[2] One equilibria will have non-contingent wages and prices.

A second puzzle relates to the observed implementation of labor contracts relative to the predictions of contracting theory. Optimal labor contracts establish employment schedules which bear no specific relationship to a firm's labor demand schedule. The majority of actual contracts, to the contrary, appear to leave the employment decision at the discretion of the firm. This behavior was taken as an "institutional given" in the macro-contracts research by Fischer [1977] and Gray [1976]. As stressed in Barro's [1977] critique of the Fischer-Gray paradigm, the resulting employment-compensation pattern is inefficient.

The model investigated in this paper provides an explanation for contracts in which employment is determined by the firm. As described in the following sections of the paper, firms are parties to both labor and commodity contracts. _Ex post_, the firm is committed to provide a quantity of goods stipulated by the commodity agreements. Hence, firms _cannot_ sell as much as they wish at the "going price." This constraint on sales produces an "effective demand for labor" in the sense of Barro-Grossman [1971] and Malinvaud [1977]. To avoid a breach of their commodity contract, employment levels must be at the firm's discretion.[3]

The paper proceeds as follows. The model is explained in Section II. An equilibrium is defined and its properties characterized in Section III. The final section summarizes the results and discusses a number of open and challenging issues.

## II. Overview of the Model

As a means of generating a role for money, we consider an overlapping generations model. Commodities are not storeable and there are no other assets so that money is the sole store of value. As in other overlapping generations models, money serves as a means of undertaking trades between agents who are otherwise intertemporally separated.

The critical feature of the model is the market structure. All exchanges of labor, money and consumption goods will be mediated by contracts. That is, once the state of nature is known, commodity and labor contracts stipulate the trades which will be undertaken. Contracts, in turn, are determined by an *ex ante* game between firms to attract customers and workers. The specification of this game and a characterization of its outcomes is addressed in the following section.

The basis for contracting in this model will be risk sharing. There will be two types of agents in the model, risk averse workers and risk neutral firms. These differences in attitudes towards risk are taken to be exogenous though they may be rationalized through a selection process.[4]

We assume there are $N$ workers (or consumers) born each period who survive for two periods. Workers are endowed with a unit of leisure time which they supply to a firm. No worksharing is possible. These agents consume the single consumption good in old age. A generation $t$ employed worker consumes $c_{t+1}$ in period $t+1$ and has a utility function of $u(c_{t+1})$ with $u'(\cdot) > 0$ and $u''(\cdot) < 0$. An unemployed generation $t$ worker receives $k$ units of leisure yielding utility of $\hat{u} \equiv u(k)$ in period $t$ and dies. We assume all workers are identical.

There are $F$ agents, called entrepreneurs who are active for three periods. They are endowed with $\bar{e}$ units of the consumption good in the middle period of their life. These agents have access to a linear technology, $Y = L$ , which converts labor (L) into output (Y) . The costs of acquiring this technology is the $\bar{e}$ units of endowment goods. An entrepreneur who makes use of this technology in their middle period becomes a firm. Profits from this activity are used to finance consumption $(z_{t+1})$ in old age. If an entrepreneur does not choose to become a firm, the $\bar{e}$ units of endowment are consumed in the second period and the agent dies. We will call a firm producing in period $t$ a generation $t$ firm.

The uncertainty in the model stems from government policy. The aggregate money supply in period $t$ , $M_t$ , follows the stochastic process

$$M_t = M_{t-1}\tilde{x}_t .$$

Here $\tilde{x}_t$ is a random variable with a cumulative distribution function $F(x)$ . We assume that $F(\cdot)$ is independent of time and that $\tilde{x} \in [1,\bar{x}]$ with $E\tilde{x} = \mu > 1$ . Hence we concentrate on expansionary periods in terms of government policy.[5]

In contrast to many papers on the role of monetary policy (e.g., Lucas [1972]), we do not assume that injections of new money are proportional transfers. The new money created by a realization of $\tilde{x}_t$ , $M_{t-1}(\tilde{x}_t - 1)$ , is used to finance government expenditures. Finally, we assume that realizations of $\tilde{x}_t$ are public information. Despite the fact that money injections are observable, they will have real effects in the period they occur since the transfers are non-proportional. Whether or not these real effects persist will be an issue for investigation.

Given this structure of endowments, preferences, technology and uncertainty, we turn attention to the structure of contracts and the trading opportunities of the agents. There are essentially two types of trades that agents wish to undertake. Workers sell labor services to firms in youth. Firms and workers both desire to consume in old age by purchasing the consumption goods from a firm in the next generation. Given the overlapping generations structure, money is the link between generations as no trades in real terms can be negotiated across time. Within a generation, firms and workers do co-exist for two periods. However, after the production period, any profits from the firms are returned to the single shareholder (i.e. the entrepreneur) and the firm is dissolved. Hence, we assume, that the worker and the firm cannot negotiate a labor contract in real terms since the firm is effectively a one period entity. As a result labor contracts must be specified in nominal terms. The point behind this distinction of shareholders and firms is to create a means of separating the exchange of labor from the acquisition of commodities. One could have approached this differently by introducing spatial separation to generate the lack of a double coincidence of wants between workers and firms necessary to support nominal contracting.[6]

With this need for nominal contracting in mind, we can specify the types of contracts we would observe. First, a period $t$ labor contract will specify wages $w_t$ and employment $L_t$ as functions of the monetary history and the current observable money shock. That is,

$$w_t = w(M_0, x_1, x_2, \ldots, x_t) \quad \text{and} \quad L_t = L(M_0, x_1, \ldots, x_t) \ .$$

We let $\delta_t^j = \{w(\cdot), L(\cdot)\}$ represent a labor contract offered by firm $j$ in period $t$.

Second, commodity agreements will specify a per unit price, $\pi_t = \pi(M_0, x_1, x_2, \ldots, x_t)$, for period $t$ consumption for a given monetary history and realization of the current $x_t$. An active generation t firm precommits to this schedule in period of t-1 prior to the realization of both $x_{t-1}$ and $x_t$ . We assume that non-linear pricing is infeasible as quantities cannot be monitored. We also make the strong assumption that firms do not ration customers at the ex post price stipulated in the commodity contract. Hence quantities are demand determined. This is taken as "a rule of the game" for the purposes of the analysis.

The interaction of these contracts of a generation t firm is depicted in Figure 1. Period t firms are involved in three types of contracts. They have a labor contract associated with current generation workers and a commodity contract for the exchange of goods with the past generation of workers and shareholders. Since these commodity contracts are structured so that output (and hence employment) are demand determined, the only negotiable aspect of the labor contract is the compensation schedule. Generation t firms will also consume next period from firms in generation t+1 announcing $\pi_{t+1}$ . It is important to remember that the commodity contracts for period t , $\pi_t(\cdot)$ , were announced in period t-1 before the determination of $x_{t-1}$ but given the history $(M_0, x_1, x_2, \ldots, x_{t-2})$ . This timing allows workers to "negotiate" their labor contracts given pricing schedules announced by firms. Hence workers are parties to two contracts both of which are in nominal terms.

With this set-up we can focus on the interaction of labor and commodity contracts as a means of insuring worker's real consumption. The point is simply one of matching the indexation of wages and prices to underlying random variables. Since many of our outlays--mortgage payments, taxes, etc.--are predetermined in nominal terms and known prior to the determination of labor contracts, the conjecture is that nominal wage and price rigidity may emerge as an equilibrium for this model.

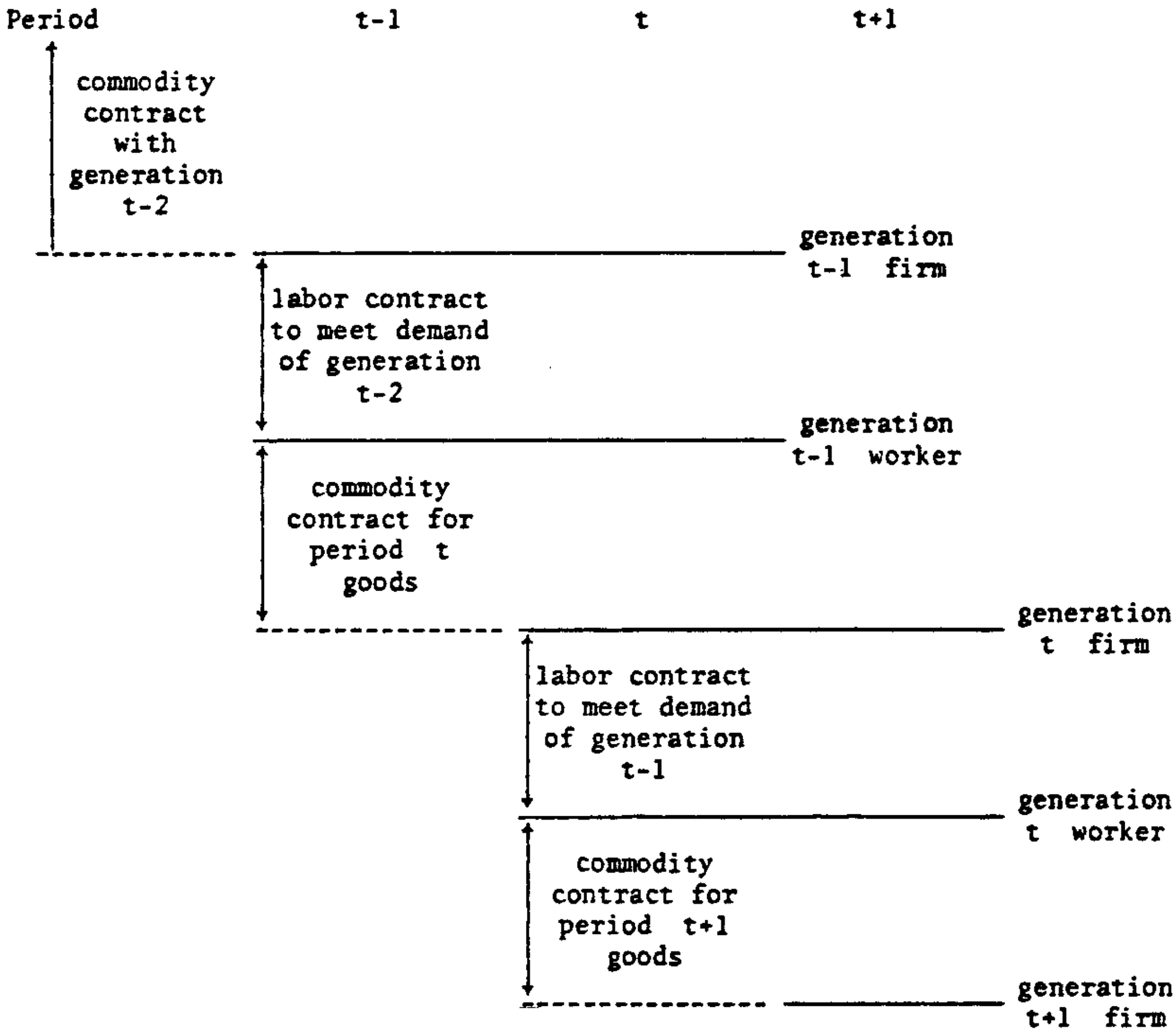


FIGURE 1

STRUCTURE OF CONTRACTS

The other aspect of this model worth stressing concerns the channel of government policy. If commodity contracts do not specify prices which are fully indexed to the current $x_t$, then government purchases will generate output movements since firms agree to meet all demand. With labor as the only productive input, to avoid breach of contract, these output commitments generate a derived demand for labor. So expansionary government policy will generate comparable movements in output and employment.

In sum, the model is structured to highlight nominal contracting and the interaction of labor and commodity agreements. With this overview in mind, we begin the formal analysis of an equilibrium.

## III. Characterization of an Equilibrium

We take a game theoretic approach to analyze the equilibria of the model described above. The strategic interaction is largely between firms of a given generation in their competition for workers and customers. We begin by outlining the strategies of firms, workers and consumers.

### a. Firm Behavior

As discussed in the previous section, a generation $t$ firm offers commodity contracts in period $t-1$ for the delivery of commodities in period $t$. We let $\pi_t^j(x_t, x_{t-1}, I_{t-1})$ be the per unit price offered by firm $j$ of generation $t$. This function is dependent on the known monetary history of the economy prior to the realization of $x_{t-1}$ in period $t-1$. We denote this history by $I_{t-1} = (M_0, x_1, x_2, \ldots, x_{t-2})$. This price function is also contingent upon realizations of $x_{t-1}$ and $x_t$ which occur after the equilibrium price functions are determined. By assumption, firms meet all demand forthcoming in period $t$ for given realizations of $x_{t-1}$ and $x_t$. The source of this demand is old workers, old entrepreneurs and the current government's nominal spending.

Generation $t$ firms are also parties to labor contracts. These contracts are negotiated with period $t$ workers prior to the realization of $x_t$ but with full knowledge of $I_t = (M_0, x_1, x_2, \ldots, x_{t-1})$ . So firms have more information when they sign labor contracts than at the time commodity agreements are made. Since labor is the only productive input and firms, by assumption, are not allowed to ration customers, they do not have the flexibility to negotiate employment rules in their contracts. Hence firms compete for workers by offering compensation schedules. We let $w_t^j(x_t, I_t)$ denote the wage paid to an employed worker at firm $j$ of generation $t$ when $x_t$ occurs given a monetary history $I_t$ .

We let $w_t = (w_t^1, w_t^2, \ldots, w_t^F)$ and $\pi_t = (\pi_t^1, \pi_t^2, \ldots, \pi_t^F)$ summarize the strategic choices of generation $t$ firms. Finally, we let

$$w_t^{-i} = (w_t^1, w_t^2, \ldots, w_t^{i-1}, w_t^{i+1}, \ldots, w_t^F) \quad \text{and}$$
$$\pi_t^{-i} = (\pi_t^1, \pi_t^2, \ldots, \pi_t^{i-1}, \pi_t^{i+1}, \ldots, \pi_t^F)$$

be the schedules offered by firms other than firm $i$ .

b. Consumer Behavior

In period $t-1$ , generation $t$ firms offer $\pi_t$ to generation $t-1$ agents who will be consuming in period $t$ . These consumers include generation $t-1$ workers and entrepreneurs who are also parties to labor contracts $(w_{t-1})$ and (in the case of firms) commodity agreements $(\pi_{t-1})$ . Workers in generation $t-1$ evaluate the price schedule of firm $j$ of generation $t$ by

$$V_{t-1}(\pi_t^j | w_{t-1}^k, \rho_{t-1}^k) = E_{x_{t-1}, x_t} u\left(\frac{w_{t-1}^k}{\pi_t^j}\right)\rho_{t-1}^k(x_{t-1}, I_{t-2}) + (1 - \rho_{t-1}^k(\cdot))\hat{u}$$

Here $u(\cdot)$ is the strictly increasing, strictly concave utility function for workers. Note that the expectation is taken over both $x_{t-1}$ and $x_t$ since the competition for consumers occurs prior to the realization of those random variables. The functions $w^k_{t-1}$ and $\rho^k_{t-1}$, represent this worker's compensation schedule and employment probabilities with firm k respectively and are explained below. Workers, taking their labor contract as given, choose to buy from the firm offering the largest $V_{t-1}$.

Generation t-1 firms also choose a generation t firm from which to purchase consumption goods. Since firms are risk neutral, they select the producer for which $E(1/\pi_t)$ is the largest.

Finally, the government is a consumer in each period--since the new money it creates is used to purchase commodities. We assume that the government purchases from the same firm as do the old entrepreneurs.

Since we will focus on symmetric equilibria, "tie-breaking" rules will be important. When firms offer identical pricing schedules, consumers are split evenly across them.

Finally, we need to make some statements about the enforceability of these contracts. We assume that firms precommit to a pricing schedule in period t-1 which is binding. We assume that consumers can costlessly search across firms to find the best supplier according to $V(\pi^j_t|\cdot)$ but are bound to the firm they decide to consume from. In a symmetric equilibrium, we could have also allowed costless search by consumers *ex post*.

c. Worker Behavior

As in the case of consumers, workers choose the best labor contract offered by the firms of its generation. To evaluate the compensation schedule of firm j, generation t $(w^j_t(x_t, I_t))$, the worker needs to know the price schedule of that firm, $\pi^j_t$, and the number of workers committed to firm j, $N^j_t$. The worker uses $\pi^j_t$ to infer the labor requirements of firm j for each realization of $x_t$ and then uses $N_t^j$ to determine the probability that a worker will be employed in a given state, $\rho_t^j(x_t, I_t) = \frac{L_t^j(x_t, I_t)}{N_t^j}$ where $L_t^j$ is the state-contingent emyloyment level for firm j, generation t. This employment level is determined by the firm's price function $(\pi_t^j)$ and the nominal money balances of its customers (including the government). In a symmetric equilibrium, each firm will have a nominal demand for its product of $M_t/F$ in period t. Hence, workers have sufficient information to calculate employment probabilities and evaluate the contract offered by a given firm. We let $V_t^j(w_t^j|\pi_t^j, \pi_{t+1}^*, N_t^j)$ be the workers' valuation of the wage offer by firm j, generation t. That is,

$$V_t^j(w_t^j|\pi_t^j, \pi_{t+1}^*, N_t^j) = E_{x_t,x_{t-1}} u\left(\frac{w_t^j}{\pi_{t+1}^*}\right)\rho_t^j(x_t, I_t) + (1-\rho_t^j)\hat{u} .$$

In this expression $\pi_{t+1}^*$ is the best contract for the delivery of period t+1 consumption available to the worker. The probability of employment depends on both $N_t^j$ and $\pi_t^j$ as discussed earlier. The _ex ante_ mobility of workers across firms sets $N_t^j$ so that $V_t^j$ is equal across active firms. Note that workers have a positive utility value for leisure $(\hat{u})$ and severance pay is prohibited by assumption. We comment on the importance of this latter restriction later.

d. Feasibility

Before defining an equilibrium, we need to discuss feasibility conditions and present the payoffs of a firm. There are two conditions for feasibility of contracts. First, firms must be able to meet the nominal wage promises it makes to workers. That is, $\pi_t^j(x_t, x_{t-1}, I_{t-1}) \geq w_t^j(x_t, I_t)$

for all $x_t$ . Since wage contracts for firm $j$ , generation $t$ are <u>after</u> $x_{t-1}$ is known, the comparison of per unit revenues and costs can be easily made on a state-by-state basis. A contract satisfying this inequality generates non-negative consumption for firms. The second feasibility requirement concerns a firm's ability to "meet the demand forthcoming" at its announced price.

We define the period $t+1$ consumption of firm $j$ , generation $t$ by $z_t^j$ for a given $(x_{t-1}, x_t, x_{t+1})$ by

$$z_t^j(\pi_t^j, w_t^j | \pi_t^{-j}, w_t^{-j}, \pi_{t+1}) = \frac{L_t^j(\pi_t^j - w_t^j)}{\pi_{t+1}^*} .$$

Here $\pi_{t+1}^*$ is simply the preferred commodity contract offered by generation $t+1$ firms. Note that payoffs of a firm depend on strategies of other firms in its generation and the next generation's commodity contract. We see from this expression that $z_t^j \geq 0$ iff $\pi_t^j - w_t^j \geq 0$ .

e. <u>Equilibrium Defined</u>

Since firms are identical within generations we focus on symmetric Nash equilibria. We define an equilibrium by labor and commodity contracts $(w_t^*, \pi_t^*)$ for each firm of generation $t = 1, 2, \ldots$ . The equilibrium is sequential in that in period $t$ , prior to the realization of $x_t$ , <u>only</u> contract markets for workers in period $t$ and commodities for period $t+1$ delivery are active. In making offers in these markets firms take as given the compensation and price schedules to be offered by firms in future generations since payoffs depend on these schedules. The equilibrium will be perfect foresight in terms of these functions.

One way to view an equilibrium is to define $V_t^*$ as the expected utility of a generation $t$ worker for given $(w_t^*, \pi_t^*)$ , $t = 1, 2, \ldots$ . The equilibrium utility level is jointly determined by $(w_t^*, \pi_t^*, \pi_{t+1}^*)$ since each of these functions enters into $V_t^*$ . A firm contemplating a deviation from a candidate equilibrium path views itself as constrained by the sequences $(V_1^*, V_2^*, \ldots)$ . That is, to attract workers and/or customers, the firm must meet the appropriate expected utility constraint. This view of equilibrium is used in the definition which follows.

A sequence of wage and price functions $(w_t^*, \pi_t^*)$ for $t = 1, 2, \ldots$ constitutes an equilibrium if:

(i) $E_{x_{t-1},x_t,x_{t+1}} z_t^j(\pi_t^*, w_t^* | \pi_t^*, w_t^*, \pi_{t+1}^*) \geq E_{x_{t-1},x_t,x_{t+1}} z_t^j(\pi_t^j, w_t^j | \pi_t^*, w_t^*, \pi_{t+1}^*)$ for all feasible $(\pi_t^j, w_t^j)$ where $V_{t-1}(\pi_t^j, w_{t-1}^*, \pi_{t-1}^*) \geq V_{t-1}^*$ and $V_t(w_t^j, \pi_t^j, \pi_{t+1}^*, N_t^j) \geq V_t^*$ .

(ii) $E_{x_t,x_{t+1}} z_t^j(\pi_t^*, w_t^* | \pi_t^*, w_t^*, \pi_{t+1}^*) \geq E_{x_t,x_{t+1}} z_t^j(\pi_t^*, w_t^j | \pi_t^*, w_t^*, \pi_{t+1}^*)$ for all feasible $w_t^j$ where $V_t(w_t^j, \pi_t^*, \pi_{t+1}^*, N_t^j) \geq V_t^*$ .

(iii) $z_t^j(\pi_t^*, w_t^* \; \pi_t^*, w_t^*, \pi_{t+1}^*) \geq 0 \quad \forall \; (x_{t-1}, x_t, x_{t+1})$ .

(iv) $E_{x_{t-1},x_t,x_{t+1}} z_t^j(\cdot) \geq \bar{e}$ for all $j$ , $t$ .

(v) $M_t/F = \pi_t^*(x_t, M_{t-1}) L_t^*(x_t, M_{t-1})$ for all $(M_{t-1}, x_t)$ .

(vi) $N/F \geq L_t^*$ for all $(M_{t-1}, x_t)$ .

The first two conditions ensure that no firm can profitably deviate from the candidate equilibrium. These deviations can take the two forms presented in conditions (i) and (ii). A generation $t$ firm can, in period $t-1$ , alter its pricing policy for period $t$ to $\pi_t^j \neq \pi_t^*$ . In doing so, it may also be forced to adjust $w_t^j \neq w_t^*$ in order to attract a sufficient number of workers. If the firm is to attract consumers and workers by these deviations, it must meet the $V^*_{t-1}$ and $V^*_t$ constraints included in condition (i). The second condition is the usual one for the characterization of an optimal labor contract (see, e.g., Azariadis [1975]). There is an expected utility constraint on attracting workers, $V^*_t$ , which any contract to attract workers must meet. So condition (ii) says there are no deviations from the $w^*_t$ compensation schedule (given $\pi^*_t$ and $\pi^*_{t+1}$ ) which increase firms' expected profits without reducing workers' expected utility below $V^*_t$ .

Conditions (iii) and (iv) ensure that the firms have non-negative consumption and that entrepreneurs are willing to become firms rather than consume their $\overline{e} \geq 0$ units of the consumption good. When condition (iv) holds, we will assume that all F firms in each generation are active.

Condition (v) relates the output and hence employment of each firm to the money supply per firm in period t $(M_t/F)$ and the price schedule $\pi^*_t(x_t, M_{t-1})$ , for every realization of $(x_t, M_{t-1})$ . This condition is analogous to a "money market clearing" condition. In this model it arises from our assumption that firms agree to meet all demands which, in a symmetric equilibrium, is simply $M_t/F$ . The final condition for equilibrium is one of production feasibility. The level of employment in each state (defined in condition (v)) <u>cannot</u> exceed the number of workers per firm.

It may be useful to restate how one calculates $V^*_t$ for $t = 1, 2, \ldots$ in this equilibrium. Given $(w^*_t, \pi^*_{t+1})$ workers can calculate their consumption in each state $(x_t, x_{t+1})$ . Furthermore, given $\pi^*_t$ , workers can calculate the employment rule for the firm. From this, and our assumption that workers are distributed equally across firms in a symmetric equilibrium,

$$\rho^j_t(x_t, I_t) = \frac{L^*_t(x_t, M_{t-1})}{N/F} .$$

## f. Some Interesting Equilibria

Given that we are focusing on Nash equilibria, one should not expect there to be a unique equilibrium for this model economy. Moreover, since the role of contracts in this model is to insure the real consumption of workers, there is additional reason to suspect that many patterns of wage and price indexation to the money shocks can generate equal real wages. We focus here on two equilibria: one with wages and prices independent of money shocks and the other with full indexation to shocks occurring during the agent's youth. There are undoubtedly other equilibria displaying partial indexation. We focus on these two extremes since they correspond to the cases which have been of interest to macroeconomists.

We first consider an equilibrium with wages independent of contemporaneous money shocks. Coupled with this structure of labor contracts, will be price schedules which are independent of current and past money shocks. Hence workers obtain real wage insurance through rigid nominal wages.

Proposition 1: If $1/k \geq \bar{x}$ and $N \geq (\bar{x})^2/\bar{\beta}$, then there exists a non-contingent equilibrium with

$$w_t^* = \bar{\alpha} M_{t-1} \quad \text{for all } t$$

$$\text{and} \quad \pi_t^* = \bar{\beta} M_{t-2} \quad \text{for } t \geq 2 .$$ [7]

where $\bar{\alpha}/\bar{\beta} = k$ and

$$\bar{\beta} = \frac{\mu}{eF}(1 - k\mu) .$$

Proof: Recall that $k$ is the value of leisure of unemployed workers, $N$ is the number of workers in each generation, $\mu \equiv Ex$ and $\bar{x}$ is the upper support of the distribution of $\tilde{x}$. We need to examine whether the equilibrium specified in the proposition satisfies conditions (i) through (vi) defining an equilibrium.

(i) Here we consider the deviation of a generation $t$ firm from the wage-price strategy outlined above. Firms cannot profitably raise or lower its level of $\pi_t$ since the real wage paid to its workers is at a minimum. Given the labor contract offered to generation $t-1$ workers ($w^*_{t-1}$) any $\pi_t$ which is not independent of both $x_{t-1}$ and $x_t$ will not attract consumers. Any combination of these two deviations will not be profitable either.

(ii) Given $\pi^*_t$ for $t = 1, 2, \ldots,$ $w^*_t$ represents an optimal labor contract in that risks are shared efficiently. The average level of wages cannot be adjusted without reducing firm's profits.

(iii) If $\pi_t \geq w_t$ for all $(x_{t-1}, x_t)$, then condition (iii) will be met. $\pi_t \geq w_t$ iff $\bar{\beta}/\bar{\alpha} \geq \bar{x}$. Since $\bar{\alpha}/\bar{\beta} = k$ in this equilibrium, we need the condition that $1/k \geq \bar{x}$ as stated in the proposition.

(iv) This zero expected excess profits is met by construction of $\bar{\beta}$.

(v) This condition is used to define $L^*_t = x_t x_{t-1}/F\bar{\beta}$.

(vi) This feasibility condition holds if $N \geq (\bar{x})^2/\bar{\beta}$ as required in the proposition. Since $\bar{\beta}$ and $\bar{x}$ are independent of $N$, we can always increase $N$ to ensure the existence of a non-contingent equilibrium. □

There are a couple of important properties of this equilibrium. First, it may not exist if $(\bar{x})^2$ is too large relative to $N$. When $F \cdot L^*_t = N$, the economy reaches full-employment so that if variations in government demand generates a derived demand for labor (per firm) beyond

$N$ , we must have prices sensitive to current money shocks. So for a non-contingent equilibrium to exist, these states of "excess" demand must have a probability zero of occurring. Second, though the level of employment fluctuates with government demand there is no involuntary unemployment in this model. In this equilibrium, firms drive the real wage of workers down to their reservation wage, $k$ . This is a somewhat surprising result given the competitive structure of the problem. To see why $\overline{\alpha}/\overline{\beta} = k$ must hold, suppose that $\alpha/\beta > k$ . A generation $t$ firm could offer a commodity contract $\varepsilon$ below the candidate equilibrium $\pi_t^*$ . This would give this firm the entire goods market and hence monopsony power in the labor market next period. Hence unless $\alpha/\beta = k$ , this undercutting in prices will disrupt any potential equilibrium.

Finally, and perhaps most importantly, it should be noted that in this equilibrium, the effects of money show persistence. Given the nature of the money injections, there should be little surprise that $x_t$ and $L_t^*$ are correlated. However, in this equilibrium we see that $L_t^*$ is also correlated with $x_{t-1}$ . As in the work of Fischer [1977] and Taylor [1980], output movements are correlated due to intertemporal contracting arrangements.

As an alternative to this non-contingent equilibrium, there may also exist an equilibrium with indexed wages and prices. In this equilibrium, wages are indexed to current shocks and prices are proportional to inherited money balances. This solution does exhibit some price rigidity in that firms offer insurance to consumers by setting $\pi_t$ independent of $x_t$ . This corresponds to commodity price insurance explored in Azariadis-Cooper [1985]. We call this an indexed equilibrium.

Proposition 2: If $1/k \geq \bar{x}$ and $N \geq \bar{x}/\hat{\beta}$ then there will exist an equilibrium with

$$w_t^* = \hat{\alpha}M_t$$

$$\text{and} \quad \pi_t^* = \hat{\beta}M_{t-1} \quad \text{for all} \quad t$$

$$\text{where} \quad \hat{\alpha}/\hat{\beta} = k$$

$$\text{and} \quad \hat{\beta} = \frac{(1 - k\mu)}{\bar{e}F} .$$

Proof: Essentially follows the steps of the proof to Proposition 1. □

As was the case with the non-contingent equilibrium, an equilibrium with wages proportional to the money supply and prices proportional to the inherited money supply may not exist. This non-existence occurs if the money shock is too "variable" or if the population is not large enough. Since $\bar{x} > \mu > 1$ , it is easy to see that if a non-contingent equilibrium exists, an indexed equilibrium will exist as well. If the policy shock is too variable an indexed equilibrium may exist while a non-contingent equilibrium may not be feasible.

It is useful to compare the two equilibria characterized in Propositions 1 and 2. In both, workers have state independent real wages equal to $k$ . This is a result of the risk neutrality of firms and the capacity of the economy to absorb the aggregate shocks.

In both equilibria, competition between firms for workers and consumers drives expected excess profits to 0. With $\bar{e} > 0$ , this does not necessarily violate the ex post non-negativity constraint on firm's profits. If $\bar{e} = 0$ , then the condition $N \geq \bar{x}/\hat{\beta}$ will be violated since $\lim_{e \to 0} \hat{\beta} = \infty$ .

The main difference between the two equilibria is in the pattern of output (and employment) fluctuations. Since $\bar{\beta} = \mu\hat{\beta}$ , it is easy to see that

$$\bar{Y}_t = \frac{x_{t-1}}{\mu} \hat{Y}_t$$

when $\bar{Y}_t$ is the level of output in a non-contingent equilibrium and $\hat{Y}_t$ is the output level in an indexed equilibrium for a given $x_{t-1}$. (From Proposition 2, $\hat{Y}_t = x_t/\hat{\beta} \cdot F$.) Therefore, given $I_{t-1}$, $E[\bar{Y}_t | I_{t-1}] = E[\hat{Y}_t | I_{t-1}]$. However, output displays more variability in the non-contingent equilibrium.

It could, of course, be the case that neither of these equilibria exist. If $N < \bar{x}/\hat{\beta}$ (where $\hat{\beta}$ is defined in Proposition 2), then an indexed equilibrium will not exist since the economy does not have the aggregate resources to absorb all of the risk due to government policy. Hence $\pi_{t+1}$ cannot be independent of $x_{t+1}$ unless quantity rationing in the commodity markets is allowed. In either case, workers' consumption will no longer be perfectly insured.

g. Robustness

In calculating these equilibria, there were two restrictions placed on the form of contracts. First, we assumed that firms could not offer severance pay to laid-off workers. This is not a binding restriction in the equilibrium since workers are indifferent between being employed and unemployed.

Second, the rules of the game restricted firms to pure price strategies with quantities of goods determined by demand conditions. The question that remains is whether rationing would emerge as part of an equilibrium. Since firm's profits are linear in quantities, they do not have an incentive to ration customers at the equilibrium prices. Furthermore, a firm contemplating a deviation from either the non-contingent or indexed equilibrium would not make use of quantity constraints. Hence it appears that the prohibition against rationing is not restrictive. However, as discussed earlier, if N is not large enough to absorb all the government induced risk, rationing may arise as an equilibrium.

## IV. Summary and Extensions

The model investigated in this paper should be viewed as a starting point for a complete analysis of an equilibrium in which all trades are mediated by contracts. The main goal of the paper was to point out the intuitively appealing view that the indexation characteristics of labor contracts cannot be determined independently of the degree of indexation associated with commodity pricing. In this paper pricing policies for goods and labor are determined by risk sharing considerations. An indeterminacy arises with regards to the degree of indexation in the equilibria of the economy. The analysis does suggest a negative correlation between the degree of indexation to nominal shocks and the persistence of output movements generated by the shocks.

There appear to be a number of potentially interesting extensions of this analysis. One is to focus on the possibilities of involuntary unemployment. There are two possible ways to generate a result of this type. The first arises when $Eu(w_t/\pi_{t+1}) > u(k)$ so that laid-off workers prefer to be working. This is the type of equilibrium explored in Azariadis [1975] and requires restrictions on severance payments. An equilibrium with involuntary unemployment may emerge when demand variations are large relative to the labor force so that the real wage exceeds k in some states of nature. A second approach is to allow the level of demand in period t to be dependent on employment levels in the previous period. This would generate an additional coordination problem and the possibility of inefficient equilibria.

Another extension concerns the direct connection in this model between output and employment movements. If, for example, the firm also had inventory holdings, then employment fluctuations could be reduced and inventory variations used to meet demand fluctuations. One could then investigate the factors determining the responsiveness of these two inputs to aggregate shocks.

Finally, one might consider alternative applications of the general theme of this paper. For example, there should exist a positive relationship between the degree of indexation in financial instruments (such as mortgages) and the indexation in labor contracts.[8] Similarly, indexation of tax brackets and/or the movement to flexible exchange rates should also lead to more indexation in labor contracts.

FOOTNOTES

[1]The degree of indexation in these models was dependent on the relative variances of real and nominal shocks as well as the risk preferences of the parties to the contract.

[2]In a complementary paper on nominal rigidities, Farmer-Woodford [1984] focus on the multiplicity of rational expectations equilibria as opposed to the multiplicity of solutions to wage-price setting games explored here.

[3]An alternative formulation of the problem would give the labor contract procedence over the commodity contract. The present setting seems more natural.

[4]That is, less risk averse agents could choose to become firms and others choose to be workers.

[5]Contractionary policies require that the government be a seller and could be incorporated if we allow the government to hold inventories or have a production function of its own.

[6]This could be modeled in an "islands" setting where firms and workers are reallocated after the first period of life.

[7]For $t = 1$, $M_{t-2}$ is not defined. We assume $M_{t-2} = M_0$. So that $\pi_1 = \frac{(1-k\mu)}{\bar{e}F}$ will be the equilibrium price in period 1.

[8]In contrast to the work of Blinder [1977] indexation of wages and financial instruments may be complementary.

# REFERENCES


Azariadis, C. [1975]. "Implicit Contracts and Underemployment Equilibria," Journal of Political Economy 83, 1183-1202.

_________ and R. Cooper [1985]. "Predetermined Prices and the Allocation of Social Risks," forthcoming Quarterly Journal of Economics.

Barro, R. and H. Grossman [1971]. "A General Disequilibrium Theory of Income and Employment," American Economic Review 61, 82-93.

Barro, R. [1977]. "Long-Term Contracting, Sticky Prices and Monetary Policy," Journal of Monetary Economics 3, 305-316.

Blinder, A. [1977]. "Indexing the Economy through Financial Intermediation," Journal of Monetary Economics, Supplement, 69-106.

Cooper, R. [1984]. "Optimal Labor Contracts and the Role of Monetary Policy in an Overlapping Generations Model," Cowles Foundation Discussion Paper No. 656R.

Farmer, R. and M. Woodford [1984]. "Self-Fulfilling Prophecies and the Business Cycle," Caress Working Paper No. 84-12, University of Pennsylvania.

Fischer, S. [1977]. "Long-Term Contracts, Rational Expectations, and the Optimal Money Supply Rule," Journal of Political Economy 85:1, 191-205.

Gray, J. [1976]. "Wage Indexation: A Macroeconomic Approach," Journal of Monetary Economics 2, 221-235.

Iwai, K. [1981]. Disequilibrium Dynamics: A Theoretical Analysis of Inflation and Unemployment. New Haven: Yale University Press.

Malinvaud, E. [1977]. The Theory of Unemployment Reconsidered. Oxford: Blackwell.

Okun, A. [1981]. Prices and Quantities. Washington, D.C.: Brookings Institution.

Taylor, J. [1980]. "Aggregate Dynamics and Staggered Contracts," Journal of Political Economy 88, 1-23.