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NUCLEAR WARFARE, C^3I AND FIRST AND SECOND STRIKE SCENARIOS

(A Sensitivity Analysis)

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1. ZERO AND NONZERO SUM MODELING

This essay is one in a projected series on the study of certain aspects of conflict in general and command and control in particular. The methodology of game theory is utilized in our attempts to formulate well-defined models and to obtain certain numerical estimates. However we believe that careful analysis of many mathematical models in application to military problems may show a considerable sensitivity to what at first glance may appear to be slight changes in the initial assumptions.

Our basic theme is that the solution and analysis of a specific model without an explicit *conceptual sensitivity analysis* (especially when studying strategic problems), may introduce a mindset and a potentially dangerous bias. In particular the current most dangerous manifestation comes in the extremely different mindsets caused by using zero sum or nonzero sum

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models of nuclear interchange.

We suggest that for certain purposes (in particular worst case analysis and weapons evaluation) the zero sum model is attractive and worthwhile. For other situations it is misleading. In the remainder of this article we consider the same physical system modeled with zero sum and with non-constant sum payoffs.

2. A SIMPLE ZERO SUM MODEL FOR THE INTERACTION OF STRATEGIC FORCES

In this section we consider a simple strategic model under different zero sum scenarios, such as sequential and simultaneous games, perfect weapons versus imperfect weapons; systems with no warning mechanisms and those with perfect warning systems. We analyze how the value of weapons and targets and the accuracy of the weapons influence the strategies of the two adversaries and look at usefulness of warning systems. As our models are purposely extremely simple we view them more as part of a parable to show how sensitive conclusions are to variations in basic assumptions. We view this as a first step in promoting a discussion of conceptual sensitivity analysis on nuclear warfare.

2.1. The Basic Model

In our first model we consider two sets of military targets, which represent the agents of two (super) players of a zero sum game. The first player, Player 1, has one agent, called a, which consists of one missile with a single warhead. The second player, Player 2, has three similar agents, called x, y, and z, which are located in three different sites and are completely connected by two-way communication links. As we are not concerned with the issues of protracted nuclear war in this paper, we do not consider the longer term problems of command and control.* Each of the agents can be in one of the following four states:

1. missile site with its weapon
2. missile site having launched its weapon
3. missile site destroyed after having launched its weapon
4. missile site and weapon destroyed

We denoted by $a(i)$ if agent a is in state i and by $v_a(i)$ its associated value. A similar notation is adopted for any of Player 2's agents. Hence, there are 32 final states of the form $[a(i), x(j), y(k), z(m)]$, as can be seen from the game tree in Figure 1.

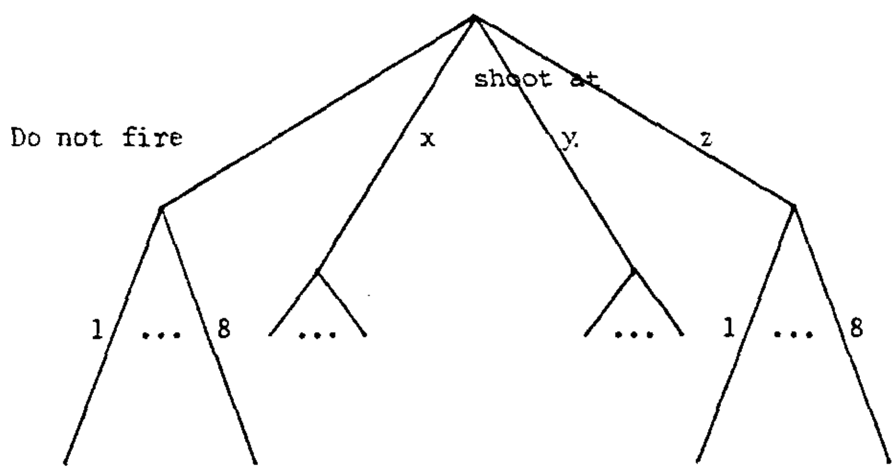

FIGURE 1

It is natural to assume that

$$v_t(1) > v_t(2) > v_t(3) > v_t(4) \ , \quad t = a, x, y, z \ . \tag{1}$$

The simplest case involves assuming that the pay-off functions are additive.

*We stress however (see 7.3) that C^3I for "grand tactics," i.e. nuclear war decisions to be made in less than twenty minutes pose qualitatively different problems than much of tactical C^3I.

So if the final state is $[a(i), x(j), y(k), z(m)]$ the pay-off of Player 1 is

$$P_1[a(i), x(j), y(k), z(m)] = v_a(i) - v_x(j) - v_y(k) - v_z(m) .$$

As we consider a zero-sum game the pay-off of Player 2 is: $P_2 = -P_1$.

2.2. Case 1: The Simultaneous Game with Perfect Weapons

To start with, we analyze the simultaneous game resulting out of the configuration described above under the further assumption that the weapons are perfect, so a launched missile always hits its target. The first player has four different strategies namely, not to shoot, to shoot at x , at y , or at z . The second one has eight strategies, from not to shoot, over launching one of his missiles or two, to launching all three of them. Hence, we get the following pay-off table for the first player (see table 1, page 6):

Analysis of the Model

As we consider here the case of perfect weapons one can quickly check that Player 2's strategies of shooting more than one missile will be inefficient and indeed under the assumption that

$$(2) \qquad v_s(3) - v_s(4) < v_t(1) - v_t(s) \ , \quad s, t = x, y, z \ ,$$

it can be shown that the strategies which impose on Player 2 to launch more than one missile are dominated by some mixed strategies of launching only one missile. Condition (2) is natural, as both sides of the inequality represent the values of the weapon; the left one when its site is destroyed while the right one is when it is undestroyed. Hence, under this assumption, the game is represented by the left half of the pay-off matrix given in Table 1.

If we further assume that

$$(3) \qquad v_a(1) - v_a(2) < \max_{t=x,y,z} [v_t(1) - v_t(4)]$$

which means that the value of Player 1's weapon is smaller than or equal to the most valuable target of Player 2, then the strategy "not to shoot" for Player 1 is dominated by the strategy to attack this most valuable target. Similarly, the "not to shoot" strategy of Player 2 is dominated if

$$(4) \qquad v_a(2) - v_a(3) > \min_{t=x,y,z} [v_t(1) - v_t(2)] \ .$$

This means that the value of Player 1's target without its weapon is greater than or equal to the least valuable weapon of Player 2.

Hence, under assumptions (1) through (4), which we assume throughout the paper, the simultaneous game reduces to a game where the "not to shoot" strategies are dominated for both players and each of them remains with three pure strategies where the optimal minimax strategy is some mixture of them. In the case that all the agents of Player 2 are identical, the optimal mixed strategies, for both players will be (1/3, 1/3, 1/3). On the other extreme if for example

$$v_x(2) - v_x(3) > \max_{t=y,z} [v_t(1) - v_t(4)]$$

then both players have an optimal pure strategy: Player 1 will shoot at agent x and Player 2 launches this agent's missile.

Player I \ Player II	not shooting	launch x	launch y	launch z	launch x, y	launch x, z	launch y, z	launch x, y, z
not shooting	$v_a(1)-v_x(1)$ $-v_y(1)-v_z(1)$	$v_a(4)-v_x(2)$ $-v_y(1)-v_z(1)$	$v_a(4)-v_x(1)$ $-v_y(2)-v_z(1)$	$v_a(4)-v_x(1)$ $-v_y(1)-v_z(2)$	$v_a(4)-v_x(2)$ $-v_y(2)-v_z(1)$	$v_a(4)-v_x(2)$ $-v_y(1)-v_z(2)$	$v_a(4)-v_x(1)$ $-v_y(2)-v_z(2)$	$v_a(4)-v_x(2)$ $-v_y(2)-v_z(2)$
shoot at x	$v_a(2)-v_x(4)$ $-v_y(1)-v_z(1)$	$v_a(3)-v_x(3)$ $-v_y(1)-v_z(1)$	$v_a(3)-v_x(4)$ $-v_y(2)-v_z(1)$	$v_a(3)-v_x(4)$ $-v_y(1)-v_z(2)$	$v_a(3)-v_x(3)$ $-v_y(2)-v_z(1)$	$v_a(3)-v_x(3)$ $-v_y(1)-v_z(2)$	$v_a(3)-v_x(4)$ $-v_y(2)-v_z(2)$	$v_a(3)-v_x(3)$ $-v_y(2)-v_z(2)$
shoot at y	$v_a(2)-v_x(1)$ $-v_y(4)-v_z(1)$	$v_a(3)-v_x(2)$ $-v_y(4)-v_z(1)$	$v_a(3)-v_x(1)$ $-v_y(3)-v_z(1)$	$v_a(3)-v_x(1)$ $-v_y(4)-v_z(2)$	$v_a(3)-v_x(2)$ $-v_y(3)-v_z(1)$	$v_a(3)-v_x(2)$ $-v_y(4)-v_z(2)$	$v_a(3)-v_x(1)$ $-v_y(3)-v_z(2)$	$v_a(3)-v_x(2)$ $-v_y(3)-v_z(4)$
shoot at z	$v_a(2)-v_x(1)$ $-v_y(1)-v_z(4)$	$v_a(3)-v_x(2)$ $-v_y(1)-v_z(4)$	$v_a(3)-v_x(1)$ $-v_y(2)-v_z(4)$	$v_a(3)-v_x(1)$ $-v_y(1)-v_z(3)$	$v_a(3)-v_x(2)$ $-v_y(2)-v_z(4)$	$v_a(3)-v_x(2)$ $-v_y(1)-v_z(3)$	$v_a(4)-v_x(1)$ $-v_y(2)-v_z(3)$	$v_a(3)-v_x(2)$ $-v_y(2)-v_z(3)$

TABLE 1

Simultaneous Game with Perfect Weapons

2.3. Case 2: The Sequential Game with Perfect Weapons and Warning

In this case we assume that both players have a warning system. Each of the players will know if he is attacked, but Player 1 will not know which missile the other launched and similarly Player 2 will not know which of his targets is attacked. After the warning they will have enough time to react.

It turns out that under assumptions (1) through (4) there is no essential difference between this model and the simultaneous game. This is the case since in the simultaneous game each of the players knows that the other will attack him. The only difference is that now the first to attack would be the one whose pay-off from the previous game is larger than his pay-off when both players use the "not to shoot" strategy. Here we can conclude that with the zero sum assumptions and perfect weapons there is no advantage in having warning systems in order to prevent a war.

2.4. Case 3: The Assessment Case

In this sequential game we assume perfect information in case that one player attacks the other. It means that if Player 1 attacks, then Player 2 will know where the missile is aimed at and he will have enough time to react accordingly. A similar situation exists if Player 2 attacks Player 1. Clearly, in case that a war is inevitable, each player prefers that the other shoots first, because then he can choose his best response. If, for example, Player 1 decides to shoot and aims at agent x, then of course, under assumption (1), Player 2 will launch agent's x missile. Therefore, Player 1 will start shooting only if

$$v_a(1) - v_a(3) < \max_{t=x,y,z} [v_t(1) - v_t(3)]$$

which means that even in the worst case Player 1's loss in a battle is smaller than that of Player 2.

On the other hand, if Player 2 decides to shoot first and launches, say missile x, then Player 1 will shoot back at site x, y or z for which $[v_x(2) - v_x(3)]$, $[v_y(1) - v_y(3)]$ or $[v_2(1) - v_2(3)]$ is maximal. Hence, Player 2 will start shooting only if

$$v_a(1) - v_a(3) > \min_{t=x,y,z} \{[v_t(1) - v_t(2)] + \max[v_t(2) - v_t(3), \max_{s \neq t}(v_s(1) - v_s(3))]\} .$$

Since it is not necessary that one of the above conditions will be satisfied, the assessment systems may prevent a war.

2.5. Case 4: First Strike Forbidden

In the sequential game with perfect weapons at most one of the players has an incentive to shoot first. Therefore, if this player adopts the rule of "first strike forbidden" then peace will be guaranteed. A statement of no first strike would be only of political nature when it is made by a player who has no incentive for a first strike anyhow. On the other hand, as we pointed out before both players would like the other to shoot first if he has enough time to react to the attack with his best response. This balance could be upset if the time between launch and arrival at its target of a missile is so reduced that the attacked player would have no time to react. A preemptive attack would be much more likely.

3. NON PERFECT WEAPONS

In the first part of this paper we considered only perfect weapons. In order to study the sensitivity of this assumption on the results, we assume in the following part that a launched missile destroys its target only with probability p, and that all missiles have the same accuracy. As we are more interested in qualitative results, we are looking now for conditions under which the "non shooting" strategies are dominated or dominate the other strategies of the players.

3.1. Case 5: The Simultaneous Game

Similar to the simultaneous game with perfect weapons and non shooting strategy for Player 1 is dominated if one of Player 2's targets, say x, satisfies:

$$v_a(1) - v_a(2) < p\ [v_x(1) - v_x(4)] \tag{5}$$

and for $n = 1, 2, 3$

$$(1 - (1-p)^n)\ [v_a(4) - v_a(3)] + (1-p)^n\ [v_a(1) - v_a(2)] < p\ [v_x(2) - v_x(3)]\ . \tag{6}$$

The left-hand and right-hand sides of these two inequalities represent the expected loss of Player 1 and Player 2 respectively in case of a war: when Player 2 does not shoot in (5) and when he launches n of his weapons in (6). Of course, for $p = 1$, condition (5) is exactly condition (3) and (6) is then satisfied, since by (1), $[v_a(4) - v_a(3)] < 0$.

Under assumption (3), a straightforward application of (5) and (6) is that a sufficient condition for the "non shooting" strategy of Player 1 to be dominated by one of his other three strategies will be that the accuracy p is greater than or equal to $\overline{p}$ where

$$\overline{p} = \max\left[\frac{v_a(1) - v_a(2)}{v_x(1) - v_x(4)}, \frac{v_a(1) - v_a(2)}{v_a(1) - v_a(2) + v_a(3) - v_a(4) + v_x(2) - v_x(3)}\right] .$$

Similarly, the "not to shoot" strategy of Player 2 is dominated if also

(7) $$p\ [v_a(2) - v_a(3)] > \min_{t=x,y,z} [v_t(1) - v_t(2)]$$

which again is satisfied for p greater than or equal to p_2 , where

$$p_2 = \max\left[\overline{p}, \frac{\min\limits_{t=x,y,z} [v_t(1) - v_t(2)]}{[v_a(2) - v_a(3)]}\right] .$$

Hence, we can say (under the reasonable assumptions (3) and (4) on the values of the targets) that we are in the same situation as in the case of perfect weapons as long as the accuracy is good enough, see the conditions (5) to (7). If we interpret $v_t(1) - v_t(2)$ as the value of the missile at site t , $t = a, x, y, z$ we can say that p_2 is nondecreasing with respect to the value of the missiles. This is reasonable since if the value of the missile is high (e.g., it is expensive) it will be launched only if it has a reasonable accuracy.

<u>Non Shooting Dominates</u>

On the other extreme, if inequality (5) holds in the reversed sense for all targets of Player 2 and

$$(1 - (1-p)^n)\ [v_a(4) - v_a(3)] + (1-p)^n\ [v_a(1) - v_a(3)] > p\ [v_t(1) - v_t(4)]$$

for $n = 1, 2, 3$ and $t = x, y, z$, then the "non shooting" strategy of Player 1 dominates all his other strategies. These conditions are satisfied when the accuracy of the weapons is small enough. A sufficient condition will be that the accuracy p is smaller or equal to p^*, where

$$p^* = \frac{v_a(1) - v_a(2)}{3^*[v_a(1) - v_a(2) + v_a(3) - v_a(4)] + \max\limits_{t=x,y,z} [v_t(1) - v_t(4)]} .$$

Furthermore, if the accuracy p is small enough such that also

$$p\,[v_a(1) - v_a(4)] < \min_{t=x,y,z} [v_t(1) - v_t(2)]$$

then the strategy "not to shoot" dominates the other of Player 2's strategies.

Finally we can say that the non shooting strategies dominate the shooting strategies for both players if the accuracy of the weapons is smaller than or equal to p_1, where

$$p_1 = \min\left[p^*, \frac{\min\limits_{t=x,y,z} [v_t(1) - v_t(2)]}{v_a(1) - v_a(4)}\right] .$$

It is therefore more likely that non shooting dominates the greater the value of the missiles, $v_t(1) - v_t(2)$, or the smaller the value of the sites with missile, $v_t(1) - v_t(4)$. If the accuracy p lies between p_1 and p_2, then both players will adopt some mixed strategy which might include the non shooting strategy.

<u>Example</u>

Let the values for all targets $t = a, x, y, z$ be:

$$v_t(1) = 6 ,\quad v_t(2) = 4 ,\quad v_t(3) = 1 ,\quad v_t(4) = 0 .$$

Non shooting for Player 1 is dominated whenever the accuracy p of the weapons lies between 1/3 and 1. For Player 2 a shooting strategy dominates when p is greater than or equal to 1/2.

On the other hand the non shooting strategy for both players dominates all shooting strategies when the accuracy is smaller than or equal to 2/15.

3.2. Case 6: The Sequential Game with Warning

The warning system that we deal with here is the same as the one in the perfect weapon case. We also assume here that this system fails to tell each of the players the results of the shooting. We discuss two versions.

1. The two stage scenario: Here we assume that each of the players is taking decisions only once. This implies that Player 2 can launch his missiles only simultaneously.

In this case we have to distinguish between two possibilities. The first one is that the strategy "not to shoot" is dominated for both players. As in the perfect weapon case, there is no essential difference between this sequential game and the simultaneous one; the warning system does not prevent a war.

In the second possibility, when at least one of the players assigns a positive probability to the "non shooting" strategy in the simultaneous game, things are more complicated. Suppose in this case Player 1 shoots first then the situation now is as in the simultaneous game where the pure strategy "not to shoot" for Player 1 is eliminated. Of course, the value of this reduced game for Player 1 is smaller than or equal to the value of the original game. Hence, he will not shoot first if and only if he is better off by this new value than by the pay-off when both do not shoot.* Clearly, there is a possibility that this condition is not satisfied even if in the original game he assigns a positive probability for shooting. A similar argument is valid for Player 2. All this leads us to the conclusion that in the case of imperfect weapons with only two stages a warning system may prevent a war that otherwise could (with some probability) happen.

2. The more than two stages scenario: Here we assume that more than one action can be taken by each of the players. This basically implies that if Player 2 attacks first, he can shoot again after Player 1's response. The fact that a warning system can prevent a war is true also here but since the process may have three stages the number of pure strategies for Player 2 is larger** and so there is no easy comparison to the original game. We note that Player 2 has more pure strategies than he had in the previous scenario, therefore his willingness to start a war may increase.

3.3. Case 7: The Assessment Case

As in the case of perfect weapons, we assume that both players will have the information whether they are attacked or not and if attacked from where the missile has been launched and where it is aimed at. On the other hand, they will not know whether their own weapon hit the target or not. We make the assumption that there is only a limited window in which each of the players has to make his decisions and we do not consider the possibility of a "shoot-look-shoot" strategy.

In the case of perfect weapons it was clear by the assumptions on

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*Again, both players prefer that the other one will attack first if war is inevitable.

**We have to consider all the possibilities in both his decision's epochs.

the values that Player 2 would shoot back if for example Player 1 started shooting. This is not necessarily true when you have weapons which hit only with an accuracy of $p < 1$ because first of all Player 1 might miss and second the expected gain for Player 2 is also smaller. The latter is of course also true for Player 1 so that he has less incentive to shoot first. Nevertheless, he will start shooting if even in the worst case one of his shooting strategies has a higher pay-off than the pay-off when both players do not shoot. The same is the case for Player 2.

4. EXTENSIONS OF THE GAME WITH PERFECT WEAPONS

In the first part of this paper we considered the case where Player 1 had one missile site and Player 2 had three missile sites. In a more general setting we assume now that Player 1 has n sites, called $a(1), \ldots, a(n)$, and that Player 2 has m missile sites, called $x(1), \ldots, x(m)$. Let n be less than or equal to m.

4.1. The Simultaneous Game

First we are looking for sufficient conditions that the strategies of shooting twice at the same target are dominated. As Player 1 has less missiles than Player 2 targets and the weapons are assumed to always hit their target, it is clear that Player 1 will never shoot twice at the same target. The situation for Player 2 is different because he has more missiles than there are targets available. But under the assumption (2) for $s, t = x(1), \ldots, x(m)$ his strategies to shoot more than once at the same target are dominated.

Furthermore, Player 1's shooting strategies dominate the non-shooting strategy if

$$\min_i [v_{a(i)}(1) - v_{a(i)}(2)] < \max_j [v_{x(j)}(2) - v_{x(j)}(3)]$$

i.e., the less expensive weapon of Player 1 is cheaper than the most expensive target without weapon of Player 2. In a similar way the non-shooting strategy of Player 2 is dominated if

$$\min_j [v_{x(j)}(1) - v_{x(j)}(2)] < \max_i [v_{a(i)}(2) - v_{a(i)}(3)] .$$

Hence under similar conditions for the values of the targets as in the simple one versus three model the non-shooting strategies and shooting several times at the same target strategies are dominated for both players.

Some Remarks on Computational Complexity

We would like to note here, that without any assumptions on the values of the targets for both players, Player 1 originally has $(m+1)^n$ (m+1 to the n) strategies and Player 2 has $(n+1)^m$ strategies in the simultaneous game. After having eliminated shooting more than once at the same target both players are still left with

$$\sum_{i=1}^{n} n!m!/i!(n-i)!(m-i)! .$$

If we assume further (after renumbering of the missile sites), that

$$[v_{a(i)}(1) - v_{a(i)}(2)] < [v_{x(i)}(2) - v_{x(i)}(3)] \quad \text{for} \quad i = 1, \ldots, n$$

then Player 1 could clearly launch all his missiles. Hence, in this circumstance both players are left with $S = m!/n!(m-n)!$ strategies. Player 1 chooses the n out of Player 2's targets which he will attack and Player 2 chooses the n out of his m missiles which he will launch. This means that if you are to solve this simultaneous game you still have an S by S pay-off matrix, which can be fairly large. For example, for $m = 20$, $n = 10$, then $S = 20!/(10!)^2 = 19 \cdot 17 \cdot 13 \cdot 11 \cdot 4 = 184{,}866$ which is already unwieldly.

4.2. The Sequential Game with Warning

As we have seen in the simultaneous game above, non shooting is dominated for both players under very mild conditions. Therefore, once shooting dominates a warning system does not provide any new information. The player, whose pay-off out of the simultaneous game is bigger than the pay-off if both players adopt the non shooting strategy, will start shooting and the other player knows it.

Only if non shooting is part of an optimal mixed strategy for the simultaneous game does a warning system provide some new information. Against an attack the defender will be able to reoptimize and, hence, the attacker's pay-off may be reduced in comparison with the simultaneous fire game.

4.3. The Assessment Case

Suppose both players have perfect information about each other's actions. In the two stage scenario, where each player makes his decision in one epoch and cannot shoot, wait for the other's response and then shoot again, both players still prefer the other one to attack if war is inevitable. Furthermore, it is conceivable under this scenario that the aggressor has to shoot all of his missiles because as he has no possibility to shoot a second time all not launched missiles are unprotected targets for the other player. Hence, in this situation Player 1 would start only if there are n targets of Player 2 such that

$$\sum_{i=1}^{n} [v_{a(i)}(1) - v_{a(i)}(3)] < \sum_{j=1}^{n} [v_{x(j)}(1) - v_{x(j)}(3)] .$$

As Player 2 has more targets than Player 1 and, therefore, has more to lose when he attacks, he is much less likely to shoot first. This situation will change when we consider more than two stages in the sequential game. In this scenario the aggressor could react to the response of the attacked player. The latter could then shoot a second time as well and so on. As we do not consider a protracted war, the number of exchanges is certainly limited by the time the first missiles hit their targets and interfere with with or even interrupt the communication lines.

5. GAMES WITH INCOMPLETE INFORMATION

Until now we did not question how the values of the targets (in their four possibilities) are assigned. Suppose that there is more than one value a player might assign to his agent's missile sites, but only he knows the true value. For example, in our simple model if there are two possibilities for Player 1 and one for Player 2 then there are two possibilities as to what is the real game. For this more complicated issue with incomplete information, the players do not know the motivation of the opponent. In Harsanyi's treatment all that he knows *a priori* is the various possibilities and the probability distribution that "Nature" assigns over time (Harsanyi, 1967). Incomplete information models* of any size tend to be computationally different. An example of this situation is where Player 1 knows that Player 2 has some high value targets, but does not know which they are nor how valuable they may be.

*An exposition of incomplete information models is given in Shubik (1982, p. 276).

5.1. The Simultaneous Game

For simplicity, we go back to the example where Player 1 is to be chosen out of two possibilities (and so the values of his target) while Player 2 has only one. Suppose Player 1's possibilities are named a and b with the corresponding values $v_s(1), \ldots, v_s(4)$, $s = a, b$ with probabilities q and $(1-q)$. For Player 2 we use the same notation as before. As said above, any realization of Player 1 leads to a different scenario but since Player 2 does not know who his opponent is we have to model this as one (more complicated) game. For example, (any) Player 1 has to take into consideration the fact that Player 2 has only imperfect information. Here a pure strategy for the set of Player 1 has two components, each assigns a pure strategy to a different possible realization. The pay-offs now will be the expected pay-offs of the corresponding pay-offs of the two simpler games. Obviously, the pay-off matrix now will have 16 rows and 8 columns.

Under assumptions (1) and (2) with the further assumptions

$$(3') \qquad \max_{s=a,b} [v_s(1) - v_s(2)] < \max_{t=x,y,z} [v_t(1) - v_t(4)]$$

and

$$(4') \qquad \min_{s=a,b} [v_s(2) - v_s(3)] < \min_{t=x,y,z} [v_t(1) - v_t(2)]$$

we get that any strategy which includes "not shoot" for any of the players is dominated and so we are left with a 9 by 3 pay-off matrix which can be used in order to find the value and the optimal mixed strategies.

5.2. The Sequential Game

As in the original model, under assumptions (1), (2), (3') and (4'), a warning system cannot prevent a war. In the assessment case the approach is as in the original game even though the calculations are more complicated. We note that in a more general model (i.e., in the case of a larger number of agents on both sides), the fact that one shoots first changes the (*a priori*) probability distribution over the true state. The new distribution can be found using a Bayesian approach, and both players should take this fact into consideration. We add, however, that the use of Bayesian updating of the subjective probabilities implicitly implies that no information is interpreted as changing the cognitive map of the overall system.

6. NONCONSTANT SUM MODELS WITH THE SAME TECHNOLOGICAL STRUCTURE

The zero-sum game is based upon several attractive assumptions for those who like their models to be tidy, solutions concepts clear, human factors minimized and mathematics nontrivial, but capable of producing "hard numbers."

When we move into the never-never land of nonconstant sum game theory a host of new and by no means fully solved basic problems appear. In particular we must question at least three basic assumptions which could be answered easily in the zero-sum context. They are:

(1) Do we believe in external symmetry of the players?

(2) What are we going to accept as a solution concept?

(3) Can we describe the payoff functions?

A fourth question of importance that was ignored in the zero sum treatment and will be ignored here, but cannot be ignored in human affairs is that there are a set of strategies and outcomes which neither side has foreseen in their planning exercises. Unexpected innovation may play a decisive role.

6.1. External Symmetry of the Players

In many war games the antiseptic titles of "Red" and "Blue" are hung on the two major teams.* This somehow is meant to make the game more general than Americans (Red) and Russians (Blue) or Cowboys and Indians or Chinese and Japanese.

When two abstract players are considered, often implicit in the analysis is that personality, culture, training, morale and abilities are all equal.

In weapons evaluation it may be reasonable to regard the competence and morale of both sides as equal for some purposes. In the study of potential war between the Soviet Union and the United States the assumption of symmetry can be highly misleading.

6.2. What Is the Solution Concept?

For a two-person zero sum game it is easy to argue that the maximin solution is a reasonably sound extension of individual rationality. Not only does this happy state of affairs not go over to the nonconstant sum game, but an extremely dangerous "worst case" bias can appear when the computationally attractive but conceptually unsupported maximin solution is applied to a nonconstant sum game.

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*Why Red and Blue instead of Blue and Grey or Rouge et Noire is an interesting question in itself (see Shubik, 1975).

6.3. A Disclaimer on Nuclear War Models and Nonconstant Sum Games

All of the results sketched in Sections 2 to 5 were based upon the utilization of a zero sum pay-off matrix. Basic modeling considerations and preliminary results which will be described in a subsequent paper indicate that the stability of the system depends in an important manner on the structure of the nonconstant sum pay-off. Although we believe that some problems of interest can be posed and examined in the zero sum context we stress that although worst case analysis provides a pessimistic assessment of resource requirements it grossly distorts the understanding of threat structure and wipes out consideration of mutual accommodation.

7. A PROGRAM IN THE STUDY OF STRATEGIC SYSTEMS DEFENSE

7.1. First Strike Defense: The One Person Nonconstant Sum Game

Our basic theme is that the models and mathematics of game theory can be utilized to help to clarify concepts and to improve the specification of problems in strategic systems defense. When the problem is clear enough and sufficiently important then the actual calculation of a solution may be called for. Thus there are two distinctive uses to our approach. The first is to further the ongoing process of conceptualizing the important problems and the second involves the obtaining of solutions.

If we assume no first strike by Player 1 and we characterize Player 1 by a network of targets, then Shubik and Weber (1981) have suggested that we could represent the systems defense network of Blue (Player 1) by the *characteristic function** used in portraying an n-person game in coalitional

*The characteristic function $v(S)$ defined for all $S \subset N$ is a superadditive set function defined for 2^n values where $n = |N|$. Each value $v(S)$ indicates the worth that can be obtained by the coalition S acting alone.

form. Here however the reason for using the characteristic function is to be able to portray the different values which can be attributed to the survival of different configurations of the original defense system.

In essence there are three levels of complexity which merit consideration in evaluating systems defense. We can consider the pay-offs to be given as (1) a linear function of surviving targets, (2) a characteristic function of the surviving set of targets, or (3) a partition function of the surviving network.

The linear model is the easiest to compute and the least relevant to network defense studies. The Colonel Blotto game literature provides examples of this type of analysis.

The characteristic function representation leads to extended Colonel Blotto games which reflect the complementarity among surviving parts of a system. But the actual possibility of describing the outcomes goes up as 2^n instead of as n in the linear case, where n is the number of nodes in the network. Thus it becomes practically impossible to carry out an exhaustive combination for any system with more than 10 to 15 nodes using a characteristic function unless linear approximations or other simplifying assumptions can be made about many of the configurations.

A third representation is to use the partition function. This acknowledges that it is not merely the set S of targets which survive that counts, but precisely how they are connected. Mathematical intractability will in general rule out considering the partition function. But a key factor in making defense calculations is to understand the nature of the biases introduced by simplification in order to keep calculation manageable.

If we use the characteristic function to evaluate the value of the surviving system we implicitly aggregate target and communication net destruction and do not distinguish the possibility that S targets may survive with many different states to the communication net among them. If a system is represented by a graph with nodes being targets and arcs being communication links then the characteristic function can be used to consider attacks which by knocking out nodes destroy only the arcs emanating from that node. We might however wish to consider both the elimination of a node and the cutting of an arc as might be caused by an electromagnetic pulse, cutting of cables or jamming of transmission. Figure 2 illustrates a five target net where, if the center target were destroyed $v(\overline{2345})$ would have only one value, but if we considered the possibility of communication disruption the system could be in any of the four states indicated in Figures 2b, c, d, and e.

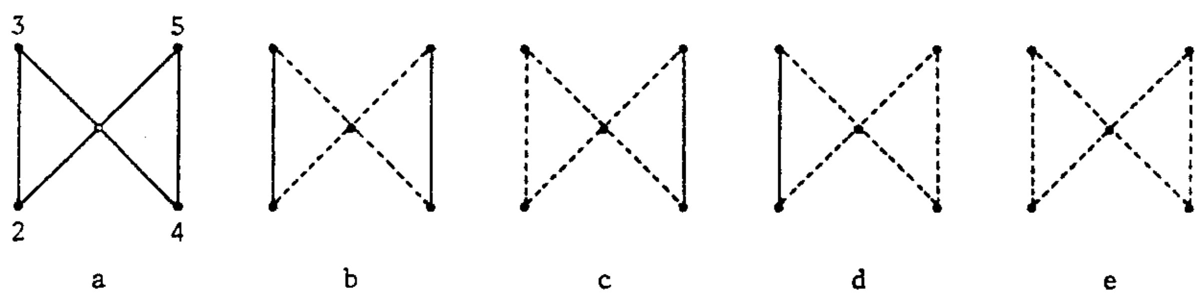


FIGURE 2

As was indicated by Shubik and Weber (1981) the one person first strike defense model can be used for the analysis of systems defense posture on the assumption that Blue is struck first and that both Blue and Red know the value of any surviving configurations to Blue. It implicitly calls for a maxmin or zero sum analysis but can offer some insights into the effects of hardening silos, varying C.E.P. error, launch error and other sources of technological variability.

The danger in mathematical analysis here is that the relatively clean game formulation can mislead the analyst into forgetting that most of the fundamental questions concerning nuclear war are at the strategic level and the answers to these are sensitive to assumptions concerning pay-off functions.

7.2. First and Second Strike Analysis: Two Person Sequential Zero Sum Games

Sections 2 to 4 have concentrated on zero-sum models. Zero-sum situations may arise naturally in games such as chess or two-person Poker; or they may provide a reasonable approximation to the fundamental opposition found in a duel or many other tactical situations. A completely different reason for modeling a game as zero-sum is the belief that the other side will attempt to maximize the damage done to you *regardless* of cost. This is a "worst case" analysis. A third reason for modeling a situation as a zero-sum game is when a damage exchange rate is deemed to be a useful measure. If the actual pay-off functions are given by $P_1(s_1, s_2)$ and $P_2(s_1, s_2)$ (where s_1 and s_2 are strategies of Players 1 and 2 respectively) then we may form a zero-sum game by considering a new pay-off to Player 1 of the form

$$\Pi_1 = P_1(s_1, s_2) - P_2(s_1, s_2) \; ; \quad \text{and} \quad \Pi_2 = -\Pi_1 \; .$$

In general nuclear war models do *not* fit the zero-sum scenarios. The only possible exceptions are where the modeler believes that a worst case analysis may provide useful benchmark and an easy place from which to start a sensitivity analysis. It is with this in mind that we have begun with a zero-sum analysis.

An example of a question that this type of analysis can help to clarify and possibly answer is how important is retargeting for a second strike once the damage assessment from the first strike is known. Of particular interest is how sensitive the answer may be to the amount of error in the system. If weapons are inaccurate and information is poor then some of the finer combinatorics that may appear in game theoretic strategies could be washed out and a reasoned case might be made for fairly simple strategies.

7.3. First and Second Strike Analysis: Two Person Sequential Non Zero Sum Games

A basic feature of strategic analysis is that it invariably involves situations best modeled as nonconstant sum games. The concerns of the citizens of the Soviet Union and the United States are by no means reflected in a model of pure opposition. Unfortunately in even a model as simple as a two-person nonconstant sum game new basic phenomena which are not present in the two person zero sum game appear. In particular threats, ploys, counterploys, hot lines and the mixture of gamesmanship and game theory noted by Schelling (1960), Rapoport (1960) and others become relevant.

Language and contract have no role to play in the two person zero sum game. The actions speak for themselves and are the only items that matter. In nonconstant sum games there is usually room for bargaining, contract and threat.

As we go from two-person zero sum game formulations to the nonconstant sum versions we do not merely change mathematical techniques but mindsets as well. There is virtually a cultural and psychological change from two-person zero sum to nonconstant sum thinking.

Two-person zero sum game analysis is best suited to tactical problems, to worst case analysis and is oriented towards weapons evaluation, search or dueling problems. It is congenial with the mindsets of engineering "hard science" oriented operations research and a view that humans can be adequately accounted for by regarding them as imperfect machines. For many problems including much of tactical C^3I the assumptions of two-person zero sum game analysis are probably adequate.

Nonzero sum game analysis applied to nuclear war and to diplomacy is and has to be "squishy" for a host of reasons. Not the least of which are the human assessment problems of what is "victory" worth; how do we evaluate megadeaths and how are talk and promises weighed against deeds?

Strategic analysis cannot ignore the soft science component. Where the politicians, diplomats and bureaucrats fit into the command and control system can be ignored for much of the necessary systems design associated with tactical problems. But for an analysis of strategic nuclear war the C.E.P. estimates, launch errors, explosive force estimates are only a small part of the basic formulation. Herman Kahn coined the phrase "thinking the unthinkable" (1962). We suggest that this could be rephrased as forcing ourselves to "assess the meaning and value of the poorly quantifiable." A menu of hardware, probable deaths and destroyed cities does not provide a simple guide to the force of deterrence. The role of formal game theory analysis here is to force ourselves to join the technology of command, control, communications and information with the behavioral, bureaucratic and political soft factors which co-determine along with technology the basic structure of the nonconstant sum game.

Although threat solutions and noncooperative equilibria have been suggested as "solutions" to a noncooperative game there is as yet no clear consensus as to what constitutes an adequate solution concept beyond the paranoia of worst case analysis. Fred Iklé (1973) in a perceptive discussion on can nuclear deterrence last out the century? provides a balanced critique of how easy it is to throw the problem away by a premature mindset as to what constitutes a solution.

We suggest that the methodology of game theory directed towards models of nuclear warfare viewed as a two-person nonconstant sum game has much to offer in providing us a way to reconcile the many different mindsets which characterize the approaches to assessing nuclear warfare and its control.

8. CONCLUDING REMARKS

Jomini classified warfare into tactics, strategy and grand strategy. We suggest that the missile and atomic weapons have introduced a new category of grand tactics.

The scope of grand tactics is as broad as that of grand strategy; the stakes are as large or larger. But the time scale varies from a few minutes to a few days; a scale less than or equal to many tactical engagements. The implications of this new category are enormous. In particular the combination of brevity of time span together with the size of the stakes creates C^3I problems where the hard and soft factors must be considered together.

To some of us the miracle is not how bad matters appear to be; but how good they are in the sense that it is forty years since the advent of the atomic bomb and we have not yet blown ourselves to pieces. Hopefully in the next few years we can start to understand why and to improve the probabilities that no nuclear war is started.

APPENDIX

Summary of Calculations in Sections 2-4

Basic Assumptions

a) Two super players having n (resp. m) missile sites with one single missile.

b) There is perfect control and communication of the missile sites.

c) The values $v_t(i)$ of missile site t being in state $i = 1, 2, 3, 4$ satisfy:

i) $v_t(1) > v_t(2) > v_t(3) > v_t(4) \quad \forall t$

ii) $v_s(s) - v_s(n) \leq v_t(1) - v_t(2)$, $s, t = x, y, z$

iii) $v_a(1) - v_a(2) \leq \max\limits_{t=x,y,z} [v_t(1) - v_t(n)]$

iv) $v_a(2) - v_a(3) \geq \min\limits_{t=x,y,z} [v_t(1) - v_t(2)]$.

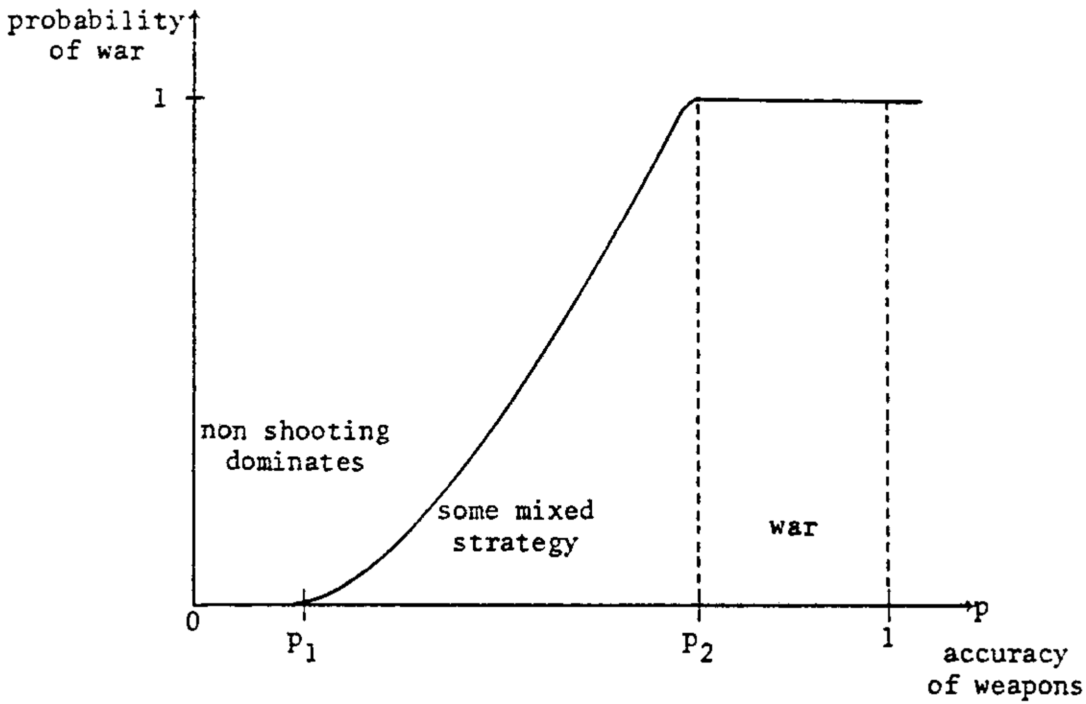
d) The utility functions for both players are additive, i.e.,

$P_I = \sum\limits_t v_t(i_t)$.

e) We only consider zero-sum games (i.e., antagonistic game).

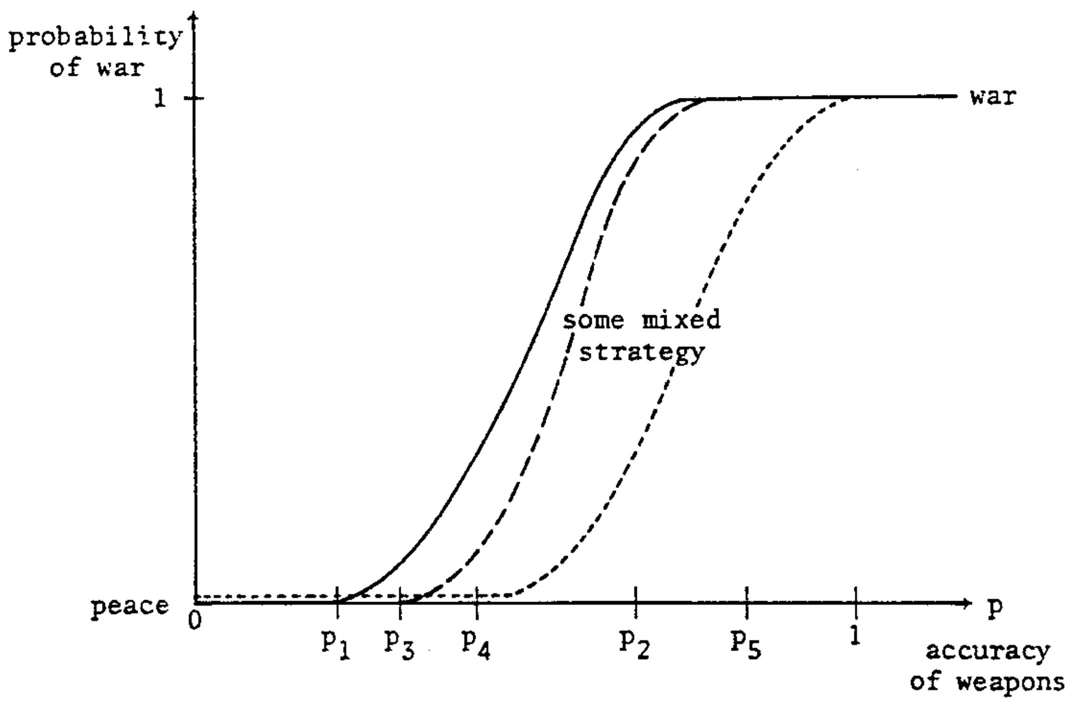
f) The accuracy of the weapons p is the same for all missiles and both players.

I. <u>The Simultaneous Game</u> (zero sum)



Where the p_j depend on the values of missile sites and are monotonic in $v_t(i)$.

II. The Sequential Game (zero sum)



——— no warning system at all

— — — — imperfect warning system tells you only whether you are under attack

------- (assessment) perfect warning system

Again the p_j are depending on the values $v_t(i)$ of the sites and they are monotone in the data.

BIBLIOGRAPHY

Harsanyi, J. C. (1967), "Games with incomplete information played by 'Bayesian' players, I: The basic model," Management Science, 14: 159-182.

Iklé, F. C. (1973), "Can nuclear deterrence last out the century?" Foreign Affairs (January), pp. 267-286.

Kahn, H. (1962), Thinking about the Unthinkable.

Rapoport, A. (1960), Fights, Games and Debates (Ann Arbor, MI: University of Michigan Press).

Schelling, T. C. (1960), The Strategy of Conflict (Cambridge, MA: Harvard University Press).

Shubik, M. (1975), Games for Society, Business and War (Amsterdam: Elsevier).

Shubik, M. (1982), Game Theory in the Social Sciences (Cambridge, MA: MIT Press).

Shubik, M. and R. Weber (1981), "Competitive valuation of cooperative games," Mathematics of Operations Research.