COWLES FOUNDATION FOR RESEARCH IN ECONOMICS

AT YALE UNIVERSITY

Box 2125, Yale Station
New Haven, Connecticut 06520

COWLES FOUNDATION DISCUSSION PAPER NO. 704

A COMPARISON OF THE MICHIGAN AND FAIR MODELS: FURTHER RESULTS

Ray C. Fair and Lewis S. Alexander

May 6, 1984

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by

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I. Introduction

This paper examines the equation-by-equation accuracy of the Michigan and Fair models using the method in Fair (1980). Emphasis is placed on examining the possible misspecification of the equations. In an earlier study, Fair and Alexander (1984), we used the method to examine the accuracy of the complete models. In the present study we are interested in the accuracy of the individual equations when considered in isolation from the rest of the model.

II. The Method

Although the main use of the method in Fair (1980) is to compare complete models, it can be used to examine individual equations. When the method is applied to complete models, it accounts for the four main sources of uncertainty of a forecast: uncertainty due to 1) the error terms, 2) the coefficient estimates, 3) the exogenous variables, and 4) the possible misspecification of the model. Because it accounts for these four sources, it can be used to make comparisons across models. For present purposes we are not interested in the uncertainty from the exogenous variables. All variables in the individual equations being examined, both exogenous and endogenous, except for the dependent variable and lagged values of the dependent variable, are assumed to be known with certainty.[1] The following is a brief outline of the method as it pertains to individual equations. See Fair (1980) or Chapter 8 in Fair (1984) for a complete discussion of the method.

Assume that the equation being examined has p coefficients to estimate. Let s^2 denote the variance of the error term, and let V denote the covariance matrix of the coefficient estimates. V is $p \times p$. An estimate of s^2, say $\hat{s}^2$, is $(1/T)uu'$, where u is a $1 \times T$ vector of estimated error terms. T is the number of observations. The estimate of V, say $\hat{V}$, depends on the estimation technique used. Let $\hat{\alpha}$ denote a p-component vector of the coefficient estimates, and let u_t denote the error term for period t.

Uncertainty from the error terms and coefficient estimates can be estimated in a straightforward way by means of stochastic simulation. Given assumptions about the distributions of the error terms and coefficient estimates, one can draw values of both error terms and coefficients. For each set of values the equation can be solved for the period of interest. Given, say, J trials, the estimated forecast mean and estimated variance of the forecast error for each period can be computed. Let $\tilde{\tilde{y}}_{itk}$ denote the estimated mean of the k-period-ahead forecast of variable i, where t is the first period of the forecast, and let $\tilde{\sigma}^2_{itk}$ denote the estimated variance of the forecast error. $\tilde{\tilde{y}}_{itk}$ is simply the average of the J predicted values from the J trials, and $\tilde{\sigma}^2_{itk}$ is the sum of squared deviations of the predicted values from the estimated mean divided by J.

It is usually assumed that the distributions of the error terms and coefficient estimates are normal, although the stochastic-simulation

[1] An exception to this is when groups of equations are examined. In these cases all the variables determined within the group are endogenous.

procedure does not require the normality assumption. The normality assumption has been used for the results in this paper. Let u_t^* be a particular draw of the error term for period t, and let α^* be a particular draw of the coefficients. The distribution of u_t^* is assumed to be $N(0, \hat{s}^2)$, and the distribution of α^* is assumed to be $N(\hat{\alpha},\hat{V})$.

Estimating the uncertainty from the possible misspecification of the equation is the most difficult and costly part of the method. It requires successive reestimation and stochastic simulation of the equation. It is based on a comparison of estimated variances computed by means of stochastic simulation with estimated variances computed from outside-sample (i.e., outside the estimation period) forecast errors. Assuming no stochastic-simulation error, the expected value of the difference between the two estimated variances for a given period is zero for a correctly specified equation. The expected value is not in general zero for a misspecified equation, and this fact is used to try to account for misspecification effects.

Without going into details, the basic procedure is to estimate the equation over a number of different estimation periods and for each set of estimates to compute the difference between the two estimated variances for each length ahead of the forecast. The average of these differences for each length ahead provides an estimate of the expected value. Let $\bar{d}_{ik}$ denote this average for variable i and length ahead k. Given $\bar{d}_{ik}$, the final step is to add it to $\tilde{\sigma}^2_{itk}$. This sum, which will be denoted $\hat{\sigma}^2_{itk}$, is the final estimated variance. Another way of looking at $\bar{d}_{ik}$ is that it is the part of the forecast-error variance not accounted for by the stochastic-simulation estimate. Some of the specifics of the above procedure will become apparent in the discussion of the computations in Section IV.

III. Some Features of the Models

Before considering the individual equations, it will be useful to give a brief discussion of some of the differences between the two models. The Michigan model has 61 stochastic equations and 50 identities, and the Fair model has 30 stochastic equations and 98 identities. Even though the Michigan model has more stochastic equations than does the Fair model, it is to some extent less structural. The Fair model accounts for all balance-sheet constraints and flows of funds among the sectors, which the Michigan model does not. This is an important difference. It means that a variable like corporate profits is determined by an identity in the Fair model (revenue minus costs) and by a stochastic equation in the Michigan model. There are a number of variables in the Michigan model that are determined by stochastic equations that would be determined by identities if all the flow-of-funds constraints were met.

The Michigan model is also less structural in its determination of the unemyloyment rate. In the Fair model there are three stochastic equations explaining the labor force (equations for prime age men, prime age women, and all others), a stochastic equation explaining the number of people holding two jobs, and a stochastic equation explaining the demand for jobs by the firm sector. The unemployment rate is determined by an identity. It is equal to one minus the ratio of total employment to the total labor force. Total employment is equal to the total number of jobs minus the number of people holding two jobs. In the Michigan model the unemployment rate is determined by a stochastic equation. It is a function of a dummy variable, a time trend, and one minus the employment rate of adult men. The employment rate of adult men is determined by a stochastic equation. It is a function, among other things, of real GNP.

The Michigan model has more disaggregation with respect to the expenditure variables. The differences pertain to consumer durable expenditures and nonresidential fixed investment. In the Michigan model durable expenditures are disaggregated into four components: new autos, motor vehicles and parts less new autos, furniture and household equipment, and all other. There is one stochastic equation for each of these components. In the Fair model there is one stochastic equation explaining total durable expenditures. Nonresidential fixed investment is disaggregated into four components in the Michigan model: structures, producers' durable equipment in production, producers' durable equipment in agriculture, and producers' durable equipment except in agriculture and production. There is one stochastic equation for each of these components. In the Fair model there is one stochastic equation explaining total nonresidential fixed investment. There is also a separate equation in the Michigan model explaining the number of new car sales, which is used as an explanatory variable in the automobile expenditure equation. Considerable work has gone into the Michigan model in explaining automobile expenditures.

There is a heavy use of dummy variables in the Michigan model, and many of the dummy variables are subjective in nature. Two of the more subjective variables are a dummy variable to reflect increased awareness of gas mileage in the cost of running a new car and a dummy variable to reflect auto rebates and reaction to higher auto prices. Of the 345 estimated coefficients in the Michigan model, 70 are coefficients of dummy variables or variables that are a function of dummy variables, which is 20.3 percent of the total. These coefficients appears in 29 different stochastic equations.

Dummy variables play a much less important role in the Fair model. There are eleven dummy variables, six of which account for the effects of the dock strikes in the import equation. The other five dummy variables appear in four different stochastic equations. Of the 169 estimated coefficients in the Fair model, 13 are coefficients of dummy variables or variables that are a function of dummy variables, which is 7.7 percent of the total.

The heavy use of dummy variables in the Michigan model poses a problem for the comparison method. One problem, which is not of concern here, is that it is difficult to account for exogenous-variable uncertainty with respect to dummy variables, since it is difficult to know how to estimate the uncertainty of these variables. Another problem is simply that it is difficult to know what to make of an equation that is heavily tied to dummy variables. Even though an equation like this may do well in tests, the use of dummy variables may have biased the results in favor of the equation. In some sense, the more is an equation based on dummy variables, the less has it explained.

An example of this problem is the following. As noted above, a dummy variable links the employment rate of adult men to the overall unemployment rate. The former is much easier to explain than the latter because the labor force of adult men fluctuates much less than does the labor force of other groups. The Michigan model thus links a relatively easy-to-explain variable to a relatively hard-to-explain variable by the use of a time trend and a dummy variable. It is not clear how much of the unemployment rate is actually explained by this procedure, even though the equation fits well and (as will be seen) does well in tests.

Another example of the dummy variable problem concerns the key price equation in the Michigan model, equation A2, which determines PPNF the private nonfarm deflator. There are two dummy variables in the equation that pertain to the price freeze, and there is a productivity trend variable that is a function of three other dummy variables. One of the latter three variables takes on a value of 1 between 1954 I and 1967 IV and 0 otherwise; one takes on a value of 1 between 1968 I and 1973 IV and 0 otherwise; and one takes on a value of 1 between 1979 I and 1979 IV and 0 otherwise. The specification of this equation may mean that a fairly large part of the fluctuations in the price deflator is explained by the dummy variables, and if this is true, the method will underestimate the uncertainty from the price equation.

The Michigan model has also used what seem to be some questionable explanatory variables. For example, the discount rate is used as an explanatory variable in the bill rate equation. It is by far the most significant variable in the equation. On a quarterly basis the two variables are highly correlated, but this is because the discount rate generally follows the bill rate with a lag of a few weeks. The discount rate is not generally the policy instrument used by the Fed to influence short term rates. It is simply a passive instrument. Another example of this type is the use of the minimum wage in the wage rate equation. It seems more likely that the aggregate wage rate affects the minimum wage rate rather than vice versa. Both the discount rate and the minimum wage are exogenous in the model.

IV. The Equations Analyzed

The equations that have been analyzed are listed in Table 1. In some cases a group of equations has been analyzed rather than a single equation, and the groups are also presented in the table. The groups for the Michigan model are the following. Five equations are used to explain consumer expenditures on durables: an equation explaining new car sales and four equations for four categories of durable goods. The new car sales variable is an explanatory variable in the first two expenditure equations. Four equations are used to explain nonresidential fixed investment, one equation for each of four categories of investment. Inventory investment is determined by a stochastic equation and an identity. The identity determines the stock of inventories. The stock of inventories appears as an explanatory variable in the inventory investment equation with a lag of one quarter. The unemployment rate for the Michigan model is determined by three stochastic equations and an identity. This specification was discussed above. The output per manhour variable is an explanatory variable in the equation determining the employment rate of males 20 and over. Finally, the money supply is determined by two stochastic equations.

The groups of equations for the Fair model are the following. Housing investment is determined by a stochastic equation and an identity. The identity determines the stock of housing, which appears in the housing investment equation with a lag of one quarter. Nonresidential fixed investment is also determined by a stochastic equation and an identity. The identity determines the stock of capital, which appears in the investment equation with a lag of one quarter. Inventory investment is determined by a stochastic equation, which explains the level of production,

TABLE 1. The Equations Analyzed

Michigan

Individual Equations

Variable	Equation
Consumer expenditures, services (CS72)	C7
Consumer expenditures, nondurables (CN72)	C6
Housing investment (IRC72)	C13
Imports (M72)	C16
Private nonfarm deflator (PPNF)	A2
Wage rate (JCMH)	A1
Bill rate (RTB)	E2

Sets of Equations

Variable	Equation
Consumer expenditures, durables (CD72)	CD72 = CDAN72 + CDAO72 + CDFE72 + CDO72
Units of retail new car sales (AUTOS)	C1
Consumer expenditures, new autos (CDAN72)	C2
Consumer expenditures, motor vehicles and parts less new autos (CDAO72)	C3
Consumer expenditures, furniture and household equipment (CDFE72)	C4
Consumer expenditures, durable goods less CDAN72 + CDAO72 + CDFE72	C5
Nonresidential fixed investment (IBF72)	IBF72 = IBFNC72 + IPDQ72 + IPDO72 + IPDAG72
Structures (IBFNC72)	C8
Producers' durable equipment in production (IPDQ72)	C10
Producers' durable equipment except in agriculture and production (IPDO72)	C11
Producers' durable equipment in agriculture (IPDAG72)	C12
Inventory investment (IINV72)	C15
Stock of business inventories (SINV72)	$SINV72 = SINV72_{-1} + IINV72$
Global unemployment rate (RUG)	B3
Output per manhour (QMH77)	B1
Employment rate, males 20 and over (REM)	B2
Unemployment rate, males 20 and over (RUM)	RUM = 100 - REM
Money supply (M1BPLUS)	E11
M2 plus short term treasury securities (M2PLUS)	E1

TABLE 1 (continued)

Fair

Industrial Equations Variable	Equation
Consumer expenditures, services (CS)	1
Consumer expenditures, nondurables (CN)	2
Consumer expenditures, durables (CD)	3
Imports (IM)	27
Private nonfarm deflator (P_f)	10
Wage rate (W_f)	16
Bill rate (RS)	30

Sets of Equations Variable	Equation
Housing investment (IH_h)	4
Stock of housing (KH)	$KH = (1 - \delta_H)KH_{-1} + IH_h$
Nonresidential fixed investment (IK_f)	12
Stock of capital (KK)	$KK = (1 - \delta_K)KK_{-1} + IK_f$
Inventory investment (IV_f)	$IV_f = V - V_{-1}$
Production (Y)	11
Stock of inventories (V)	$V = V_{-1} + Y - X$
(Note: X = total sales. X is determined elsewhere in the model.)	
Civilian unemployment rate (UR)	$UR = U/(L1 + L2 + L3 - J_m)$
Labor force, males 25-54 (L1)	5
Labor force, females 25-54 (L2)	6
Labor force, all others (L3)	7
Number of moonlighters (LM)	8
Number of jobs in the firm sector (J_f)	13
Total employment, civilian and military (E)	$E = J_f + J_g + J_m + J_s - LM$
Number of people unemployed (U)	$U = L1 + L2 + L3 - E$
(Note: J_g = federal government civilian jobs, J_m = federal government military jobs, J_s = state and local government jobs. These variables are exogenous.)	
Money supply (M1)	$M1 = M_h + M_f + M_r + M_s$
Demand deposits and currency, household sector (M_h)	9
Demand deposits and currency, firm sector (M_f)	17
(Note: M_r = demand deposits and currency, foreign sector; M_s = demand deposits and currency, state and local government sector. These variables are exogenous.)	

Notes: All equation numbers are for stochastic equations.
See Belton, Hymans, and Lown (1981) for a description of the Michigan variables and equations.
See Fair (1984) for a description of the Fair variables and equations.

and two identities. Given the level of sales and the level of production, the identities determine the stock of inventories and inventory investment. The stock of inventories appears in the production equation with a lag of one quarter.

The unemployment rate is determined by five stochastic equations and three identities. The stochastic equations determine three labor force categories, the number of moonlighters (people holding two jobs), and the number of jobs in the firm sector. Total employment is equal to the total number of jobs less the number of moonlighters. Total unemployment is equal to the total labor force less total employment. The unemployment rate is the ratio of unemployment to the civilian labor force.

The money supply is determined by two stochastic equations and an identity. The two stochastic equations determine the demand deposits and currency of the household and firm sectors. The money supply variable is the sum of these two plus the demand deposits and currency of the foreign and state and local government sectors.

V. Calculations of the Results

Many steps were involved in obtaining the final results, and it is easiest to discuss the computation of the results in the order in which they are done. The results for the Michigan model will be discussed first.

Duplication of the Basic Estimates

Data for the Michigan model were taken from the TROLL version of the model that was current at the beginning of 1983.[2] The specification of this version of the model is in Belton, Hymans, and Lown (1981) (BHL). The first step was to duplicate the basic set of estimates, which we were able to do. For none of the 61 equations were the differences between our estimates and the BHL estimates large enough to call into question our duplication of the results.

Uncertainty with Respect to the Error Terms and Coefficient Estimates

Given the basic coefficient estimates, V and s^2 were estimated for each equation. When a group of equations was considered, the covariance matrix of the coefficient estimates was taken to be block diagonal, where the blocks are the V matrices for the individual equations. Similarly, the covariance matrix of the error terms for a group of equations was taken to be diagonal, where the diagonal elements are the s^2 estimates for the individual equations.

Table 2 contains the main result of this paper. The values in the a rows are stochastic-simulation estimates of the forecast standard errors based on draws of the error terms only. The values in the b rows are based on draws of both error terms and coefficients. The results are based on 250 trials for each of the two stochastic simulations. The simulation period is 1978 I - 1979 IV. In terms of the notation in Section II,

[2]We are indebted to Edwin Kuh and Steve Schwartz for providing us with a tape of the data. We are also indebted to Joane Crary for answering a number of questions about the model. None of these individuals are accountable for the results in this paper. We assume responsibility for all errors.

TABLE 2. Estimated Standard Errors of Forecasts for 1978 I - 1979 IV for Three Models

		Individual Equations or Groups								Complete Models							
		1978				1979				1978				1979			
		I	II	III	IV	I	II	III	IV	I	II	III	IV	I	II	III	IV
Consumer expenditures, services																	
Michigan:	a	.27	.38	.44	.50	.55	.60	.63	.67	.28	.39	.47	.54	.59	.65	.70	.74
	b	.29	.39	.48	.56	.61	.67	.71	.75	.30	.41	.52	.62	.67	.74	.80	.85
	d	.46	.70	.88	1.06	1.23	1.41	1.64	1.82	.48	.74	.93	1.15	1.37	1.63	1.94	2.21
	d/b	1.59	1.79	1.83	1.89	2.02	2.10	2.31	2.43	1.60	1.80	1.79	1.85	2.04	2.20	2.43	2.60
Fair:	a	.32	.42	.51	.59	.67	.71	.74	.81	.30	.40	.53	.61	.72	.81	.89	1.00
	b	.31	.43	.54	.62	.72	.79	.84	.92	.28	.41	.55	.67	.81	.93	.99	1.10
	d	.40	.46	.56	.67	.76	1.05	1.39	1.50	.35	.55	.87	1.19	1.41	1.68	1.94	2.21
	d/b	1.29	1.07	1.04	1.08	1.06	1.33	1.65	1.63	1.25	1.34	1.58	1.78	1.74	1.81	1.96	2.01
AR8:	a	.28	.40	.49	.60	.69	.72	.78	.81								
	b	.30	.44	.54	.64	.76	.84	.95	1.05								
	d	.51	.81	1.05	1.31	1.61	1.90	2.27	2.56								
	d/b	1.70	1.84	1.94	2.05	2.12	2.26	2.39	2.44								
Consumer expenditures, nondurables																	
Michigan:	a	.48	.59	.65	.65	.70	.70	.75	.71	.54	.70	.85	.98	1.14	1.18	1.27	1.38
	b	.46	.58	.69	.73	.78	.82	.84	.84	.52	.70	.88	1.02	1.14	1.25	1.39	1.45
	d	.77	1.03	1.13	1.21	1.36	1.50	1.58	1.65	.86	1.28	1.55	1.78	2.10	2.43	2.86	3.28
	d/b	1.67	1.78	1.64	1.66	1.74	1.83	1.88	1.96	1.65	1.83	1.76	1.75	1.84	1.94	2.06	2.26
Fair:	a	.57	.66	.69	.81	.75	.77	.70	.76	.58	.73	.82	.99	.99	1.07	1.07	1.16
	b	.68	.73	.82	.87	.89	.88	.84	.91	.70	.78	.96	1.07	1.11	1.17	1.23	1.33
	d	.85	.95	1.03	1.23	1.41	1.49	1.53	1.66	.84	.94	1.02	1.21	1.25	1.16	1.03	.98
	d/b	1.25	1.30	1.26	1.41	1.58	1.69	1.82	1.82	1.20	1.21	1.06	1.13	1.13	.99	.84	.74
AR8:	a	.57	.89	1.07	1.26	1.37	1.43	1.45	1.48								
	b	.61	.97	1.16	1.33	1.51	1.62	1.71	1.81								
	d	.88	1.35	1.37	1.12	1.28	1.50	1.73	1.88								
	d/b	1.44	1.39	1.18	.84	.85	.93	1.01	1.04								
Consumer expenditures, durables																	
Michigan:	a	1.12	1.34	1.42	1.47	1.57	1.76	1.79	1.73	1.15	1.53	1.82	2.20	2.36	2.73	2.98	3.18
	b	1.30	1.72	1.88	2.13	2.40	2.63	2.79	3.11	1.31	1.93	2.32	2.77	3.33	3.85	4.30	4.85
	d	3.28	2.89	2.82	.99	i	i	i	i	3.52	3.97	4.91	4.55	5.23	6.40	7.84	8.99
	d/b	2.52	1.68	1.50	.46	-	-	-	-	2.69	2.06	2.12	1.64	1.57	1.66	1.82	1.85
Fair:	a	2.15	2.29	2.16	2.13	2.30	2.22	2.25	2.34	2.17	2.44	2.79	3.26	3.72	3.69	4.09	4.25
	b	2.08	2.39	2.56	2.53	2.54	2.47	2.70	2.56	2.11	2.55	3.16	3.52	3.94	4.15	4.40	4.61
	d	3.44	4.44	5.16	5.38	4.74	3.84	4.30	3.64	3.09	3.58	5.09	6.40	7.95	8.97	10.20	11.66
	d/b	1.65	1.86	2.02	2.13	1.87	1.55	1.59	1.42	1.46	1.40	1.61	1.82	2.02	2.16	2.32	2.53
AR8:	a	1.91	2.60	3.15	3.36	3.60	3.92	4.18	4.19								
	b	2.13	2.82	3.39	3.83	3.80	4.45	4.69	5.12								
	d	4.32	5.54	6.39	6.64	6.93	7.97	9.09	9.92								
	d/b	2.03	1.96	1.88	1.73	1.82	1.79	1.94	1.94								

TABLE 2 (continued)

		Individual Equations or Groups								Complete Models							
		1978				1979				1978				1979			
		I	II	III	IV	I	II	III	IV	I	II	III	IV	I	II	III	IV
Housing investment																	
Michigan:	a	2.09	3.02	3.51	4.26	4.65	4.91	5.01	5.09	2.09	3.07	3.68	4.69	5.54	6.41	7.16	7.40
	b	2.23	3.19	3.86	4.62	5.20	5.76	6.22	6.89	2.24	3.22	3.98	4.86	5.93	7.20	8.61	10.37
	d	5.35	8.49	9.82	10.74	11.99	13.57	14.94	15.29	5.36	8.54	9.42	9.93	12.13	15.26	18.79	21.49
	d/b	2.40	2.66	2.54	2.32	2.31	2.36	2.40	2.22	2.39	2.65	2.36	2.04	2.05	2.12	2.18	2.07
Fair:	a	2.70	4.40	5.77	6.49	7.15	7.50	7.44	7.64	2.71	4.71	6.36	7.25	8.19	8.99	9.39	10.38
	b	2.82	4.82	6.41	7.57	8.39	8.80	9.15	9.68	2.80	4.83	6.83	8.46	9.58	10.77	11.88	13.35
	d	4.91	7.74	7.37	6.20	5.38	7.37	8.01	7.29	4.78	7.28	7.56	7.64	9.45	11.57	13.13	15.34
	d/b	1.74	1.61	1.15	.82	.84	.84	.88	.75	1.71	1.51	1.11	.90	.99	1.07	1.11	1.15
AR8:	a	2.61	4.22	5.92	7.06	7.66	8.02	8.21	8.23								
	b	2.78	4.68	6.20	7.42	8.53	8.96	9.34	9.42								
	d	6.43	11.25	12.24	10.89	9.58	10.35	12.46	12.77								
	d/b	2.31	2.40	1.97	1.47	1.12	1.16	1.33	1.36								
Nonresidential fixed investment																	
Michigan:	a	1.13	1.38	1.47	1.49	1.62	1.75	1.79	1.83	1.13	1.47	1.71	1.94	2.20	2.51	2.79	3.09
	b	1.08	1.51	1.65	1.79	2.00	2.11	2.16	2.09	1.08	1.57	1.85	2.16	2.56	2.88	3.22	3.40
	d	2.52	4.15	5.18	5.21	4.53	5.36	9.20	15.05	2.53	4.28	5.60	5.76	4.86	6.27	13.47	29.18
	d/b	2.33	2.75	3.14	2.91	2.27	2.54	4.26	7.20	2.34	2.73	3.03	2.67	1.90	2.18	4.18	8.58
Fair:	a	1.50	1.88	2.14	2.30	2.35	2.26	2.39	2.53	1.72	2.25	2.60	2.92	3.24	3.32	3.49	3.66
	b	1.50	1.99	2.21	2.33	2.56	2.74	2.85	2.98	1.81	2.56	2.97	3.05	3.48	3.77	4.00	4.09
	d	2.24	2.57	2.74	2.91	3.40	3.49	3.75	3.85	2.69	3.24	4.00	4.19	5.24	6.08	6.75	7.54
	d/b	1.49	1.29	1.24	1.25	1.33	1.27	1.32	1.29	1.49	1.27	1.35	1.37	1.51	1.61	1.69	1.84
AR8:	a	1.24	2.13	2.75	3.29	3.80	4.21	4.30	4.46								
	b	1.42	2.05	2.78	3.68	4.27	4.92	5.15	5.45								
	d	2.26	2.35	3.41	3.35	2.69	2.03	i	i								
	d/b	1.59	1.15	1.23	.91	.63	.41	-	-								
Inventory investment																	
Michigan:	a	3.71	3.87	3.95	4.47	4.25	3.81	4.32	4.44	3.86	4.20	4.38	5.18	5.06	4.81	5.17	5.33
	b	4.38	4.21	4.00	4.35	4.10	4.17	4.43	4.26	4.59	4.75	4.53	5.10	4.76	5.11	5.45	5.57
	d	6.36	6.83	7.05	7.06	6.82	6.74	7.04	7.09	6.28	6.93	7.37	7.87	7.12	6.10	6.47	7.33
	d/b	1.45	1.62	1.76	1.62	1.66	1.62	1.59	1.66	1.37	1.46	1.63	1.54	1.50	1.19	1.19	1.32
Fair:	a	4.70	5.35	4.88	6.55	5.92	5.47	5.21	5.32	4.66	5.22	5.10	5.52	6.12	5.54	5.38	5.77
	b	4.62	5.17	5.46	5.30	5.21	6.04	5.85	5.90	4.50	5.05	5.52	5.60	4.99	5.93	5.90	6.23
	d	4.76	5.40	5.69	5.51	5.42	6.18	6.00	6.10	4.64	5.28	5.80	5.88	5.35	6.23	6.22	6.58
	d/b	1.03	1.04	1.04	1.04	1.04	1.02	1.03	1.03	1.03	1.05	1.05	1.05	1.07	1.05	1.05	1.06
AR8:	a	5.22	5.45	5.81	5.72	5.69	6.41	6.02	6.38								
	b	5.33	6.18	6.45	6.75	6.64	6.67	6.75	7.32								
	d	6.67	7.66	7.92	7.51	7.78	8.12	8.18	8.61								
	d/b	1.25	1.24	1.23	1.11	1.17	1.22	1.21	1.18								

TABLE 2 (continued)

		Individual Equations or Groups								Complete Models							
		1978				1979				1978				1979			
		I	II	III	IV	I	II	III	IV	I	II	III	IV	I	II	III	IV
Imports																	
Michigan:	a	2.44	3.45	3.82	3.95	3.98	3.85	3.70	3.55	2.47	3.42	3.68	3.80	3.80	3.65	3.64	3.67
	b	2.70	3.45	3.75	3.90	4.23	4.17	4.33	4.48	2.65	3.49	3.64	3.90	4.42	4.47	4.63	4.78
	d	3.73	4.96	5.70	6.41	7.07	7.28	7.88	8.45	3.66	5.01	5.64	6.29	6.98	7.67	9.12	10.69
	d/b	1.38	1.44	1.52	1.64	1.67	1.75	1.82	1.89	1.38	1.44	1.55	1.61	1.58	1.72	1.97	2.24
Fair:	a	2.06	2.60	2.67	2.61	2.67	2.78	2.57	2.73	1.90	2.46	2.60	2.67	2.72	2.71	2.61	2.97
	b	2.15	2.61	2.90	3.03	3.10	3.26	3.39	3.32	2.22	2.44	2.66	2.76	2.81	3.28	3.56	3.70
	d	3.85	6.05	8.22	10.09	11.87	13.44	15.69	17.42	4.00	5.74	7.54	8.94	10.24	11.11	12.56	13.10
	d/b	1.79	2.32	2.83	3.33	3.83	4.12	4.63	5.25	1.80	2.35	2.83	3.24	3.64	3.39	3.53	3.54
AR8:	a	2.63	3.53	3.98	4.64	4.71	4.87	5.14	5.22								
	b	2.79	3.61	4.33	4.91	5.43	5.58	5.77	5.96								
	d	5.04	7.54	9.69	11.70	13.11	13.71	14.25	15.26								
	d/b	1.81	2.09	2.24	2.38	2.41	2.46	2.47	2.56								
Private nonfarm deflator																	
Michigan:	a	.27	.40	.50	.56	.64	.70	.77	.79	.27	.40	.51	.59	.70	.79	.90	.99
	b	.28	.41	.51	.62	.67	.77	.84	.92	.28	.40	.53	.66	.74	.88	1.01	1.17
	d	.34	.41	.43	.59	.63	.72	.90	1.01	.34	.40	.45	.62	.68	.79	.93	1.11
	d/b	1.21	1.00	.84	.95	.94	.94	1.07	1.10	1.21	1.00	.85	.94	.92	.90	.92	.95
Fair:	a	.39	.54	.67	.76	.83	.97	.89	.92	.38	.55	.68	.77	.84	.91	.92	.98
	b	.41	.56	.69	.81	.88	.93	.98	1.04	.41	.57	.70	.84	.91	.99	1.09	1.21
	d	.68	1.13	1.58	2.07	2.50	2.85	3.27	3.69	.66	1.12	1.55	2.05	2.44	2.80	3.17	3.49
	d/b	1.66	2.02	2.29	2.56	2.84	3.06	3.34	3.55	1.61	1.96	2.21	2.44	2.68	2.83	2.91	2.88
AR8:	a	.30	.48	.70	.91	1.11	1.27	1.39	1.50								
	b	.34	.53	.77	1.05	1.30	1.55	1.78	1.99								
	d	.70	1.18	1.72	2.57	3.24	3.75	3.98	3.74								
	d/b	2.06	2.23	2.23	2.45	2.49	2.42	2.24	1.88								
Unemployment rate																	
Michigan:	a	.21	.28	.34	.40	.43	.47	.50	.53	.23	.35	.44	.54	.58	.65	.72	.76
	b	.22	.30	.35	.41	.46	.50	.54	.59	.25	.36	.45	.54	.61	.66	.74	.84
	d	.26	.37	.43	.46	.45	.43	.41	.41	.34	.52	.67	.74	.71	.51	.62	1.07
	d/b	1.18	1.23	1.23	1.12	.98	.86	.76	.69	1.36	1.44	1.49	1.37	1.16	.77	.84	1.27
Fair:	a	.40	.54	.63	.65	.67	.71	.76	.77	.24	.38	.48	.54	.61	.65	.65	.68
	b	.40	.59	.67	.73	.77	.85	.86	.87	.26	.42	.52	.61	.70	.77	.78	.80
	d	.32	.56	.75	.90	1.05	1.22	1.35	1.45	.39	.61	.86	1.05	1.16	1.25	1.30	1.39
	d/b	.80	.95	1.12	1.23	1.36	1.44	1.57	1.67	1.50	1.45	1.65	1.72	1.66	1.62	1.67	1.74
AR8:	a	.29	.58	.81	.97	1.06	1.11	1.17	1.23								
	b	.29	.56	.83	1.04	1.19	1.30	1.37	1.41								
	d	.31	.37	.39	.29	i	i	i	i								
	d/b	1.11	.66	.47	.28	-	-	-	-								

TABLE 2 (continued)

		Individual Equations or Groups								Complete Models							
		1978				1979				1978				1979			
		I	II	III	IV	I	II	III	IV	I	II	III	IV	I	II	III	IV
Wage rate																	
Michigan:	a	.31	.42	.58	.64	.71	.78	.82	.88	.31	.42	.59	.66	.78	.87	.97	1.10
	b	.35	.46	.56	.68	.79	.88	.98	1.07	.35	.47	.59	.72	.86	1.00	1.15	1.32
	d	.28	.42	.54	.67	.71	.80	.89	1.07	.28	.43	.59	.75	.83	.99	1.14	1.37
	d/b	.80	.91	.96	.99	.90	.91	.91	1.00	.80	.91	1.00	1.04	.97	.99	.99	1.04
Fair:	a	.56	.79	.92	.98	.99	1.05	1.10	1.19	.56	.86	1.00	1.13	1.20	1.30	1.32	1.38
	b	.59	.79	1.02	1.18	1.35	1.47	1.59	1.67	.59	.84	1.01	1.15	1.35	1.48	1.70	1.85
	d	.50	.60	.84	1.00	1.31	1.60	1.89	2.16	.37	.49	.60	.72	.98	1.07	1.44	1.76
	d/b	.85	.76	.82	.85	.97	1.09	1.19	1.29	.63	.58	.59	.63	.73	.72	.85	.95
AR8:	a	.21	.33	.43	.53	.60	.63	.68	.73								
	b	.26	.44	.58	.73	.86	.95	1.04	1.14								
	d	.43	.75	1.04	1.35	1.75	2.11	2.51	2.96								
	d/b	1.65	1.70	1.79	1.85	2.03	2.22	2.41	2.60								
Bill rate																	
Michigan:	a	.43	.50	.62	.77	.73	.72	.82	.92	.41	.48	.58	.70	.69	.71	.80	.91
	b	.43	.61	.81	1.11	1.28	1.37	2.29	3.57	.41	.57	.73	.96	1.03	1.06	1.26	1.49
	d	.67	.81	1.01	1.32	1.50	1.58	2.26	3.53	.64	.89	1.02	1.26	1.38	1.35	1.17	.97
	d/b	1.56	1.33	1.25	1.19	1.17	1.15	.99	.99	1.56	1.56	1.40	1.31	1.34	1.27	.93	.65
Fair:	a	.67	.96	1.10	1.15	1.27	1.34	1.31	1.35	.71	1.00	1.07	1.13	1.17	1.21	1.17	1.19
	b	.65	.94	1.12	1.19	1.30	1.42	1.44	1.52	.73	.94	1.04	1.03	1.15	1.26	1.31	1.45
	d	1.31	1.75	1.88	1.77	2.38	2.70	3.08	3.37	1.37	2.13	2.38	2.49	2.66	2.83	3.00	3.22
	d/b	2.02	1.86	1.68	1.49	1.83	1.90	2.14	2.22	1.88	2.27	2.29	2.42	2.31	2.26	2.29	2.22
AR8:	a	.52	.82	.92	.97	1.00	1.08	1.17	1.23								
	b	.54	.86	1.00	1.13	1.22	1.35	1.39	1.40								
	d	1.52	2.51	2.72	3.08	3.39	3.65	3.89	4.09								
	d/b	2.81	2.92	2.72	2.73	2.78	2.70	2.80	2.92								
Money supply																	
Michigan:	a	.74	1.23	1.69	1.96	2.32	2.73	3.01	3.29	.76	1.28	1.74	1.99	2.29	2.57	2.77	2.89
	b	.77	1.42	1.97	2.40	2.76	3.07	3.42	3.94	.83	1.51	2.10	2.67	3.11	3.42	3.87	4.58
	d	1.34	2.07	3.54	4.00	4.84	5.62	6.34	7.46	1.57	4.26	5.68	5.97	5.17	6.54	13.64	29.34
	d/b	1.74	1.46	1.80	1.67	1.75	1.83	1.85	1.89	1.89	2.82	2.70	2.24	1.66	1.91	3.52	6.41
Fair:	a	1.04	1.41	1.51	1.62	1.75	1.84	1.95	1.88	.98	1.35	1.49	1.66	1.82	2.00	2.03	1.98
	b	1.04	1.46	1.65	1.86	2.08	2.18	2.20	2.24	.95	1.37	1.51	1.77	2.11	2.32	2.38	2.54
	d	1.43	2.23	3.03	3.88	4.78	5.82	7.14	8.30	1.40	1.84	1.88	2.13	2.31	2.39	2.37	1.92
	d/b	1.36	1.53	1.84	2.09	2.30	2.67	3.25	3.71	1.47	1.34	1.25	1.20	1.09	1.03	1.00	.76
AR8:	a	.57	1.11	1.55	1.95	2.43	2.91	3.42	3.92								
	b	.57	1.17	1.68	2.33	3.08	3.89	4.83	5.77								
	d	2.10	3.50	4.26	5.27	5.91	7.05	8.85	10.39								
	d/b	3.68	2.99	2.54	2.26	1.92	1.81	1.83	1.80								

TABLE 2 (continued)

Notes: a = Uncertainty due to error terms.
b = Uncertainty due to error terms and coefficient estimates.
d = Uncertainty due to error terms, coefficient estimates, and the possible misspecification of the model.
i = The total estimated variance was negative.

1) 250 trials for each stochastic simulation.
2) Errors are in percentage points except for inventory investment, where the errors are in billions of 1972 dollars at an annual rate. Errors for all variables except the unemployment rate, the bill rate, and inventory investment are percents of the forecast means.
3) The exact variables tabled for each model are the following. See Belton, Hymans, and Lown (1981) for the Michigan notation, and see Fair (1984) for the Fair notation. The variables for the autoregressive model are the same as those for the Michigan model.

	Michigan	Fair
Consumer expenditures, services	CS72	CS
Consumer expenditures, nondurables	CN72	CN
Consumer expenditures, durables	C72-CS72-CN72	CD
Housing investment	IRC72	IHH
Nonresidential fixed investment	IBF72	IKF
Inventory investment	IINV72	IVF
Imports	M72	IM
Private nonfarm deflator	PPNF	PF
Unemployment rate	RUG	UR
Wage rate	JCMH	WF
Bill rate	RTB	RS
Money supply	M1BPLUS	M1

the b-row values are values of $\tilde{\sigma}_{itk}$.[3] The values in the left half of the table are for the individual equations or groups of equations. The values in the right half of the table are for the whole model.

Uncertainty from the Possible Misspecification of the Model

For the misspecification results the Michigan model was estimated and stochastically simulated 27 times. For the first set, the estimation period ended in 1974 IV and the simulation period began two quarters later in 1975 II. For the second set, the estimation periods ended in 1975 I and the simulation period began in 1975 III. For the final set, the estimation periods ended in 1981 II and the simulation period began in 1981 IV. The beginning quarters for the estimation periods remained unchanged from those for the basic period. The length of the first 20 simulation periods was eight quarters. Since the data ended in 1981 IV, the length of the 21st simulation period, which began in 1980 II, was only seven quarters. Similarly, the length of the 22nd period was six, and so on through the length of the 27th period, which was only one quarter. For each of the 27 sets of estimates, new estimates of the V matrices and the s^2 values were obtained. Each of the 27 stochastic simulations was based on 50 trials.

These results produced for the one-quarter-ahead forecast for each endogenous variable 27 values of the difference between the estimated forecast-error variance based on outside-sample errors and the estimated forecast-error variance based on stochastic simulation. The average of these 27 values was taken for each variable. In terms of the notation in

[3]As indicated in note 2) to Table 2, most of the errors are in units of percent of the forecast mean. See the discussion in Chapter 8 in Fair (1984) for the exact way in which the percentage errors are computed.

Section II, this average is $\bar{d}_{i1}$, where the i refers to variable i , and the 1 refers to the one-quarter-ahead forecast. The total variance of the one-quarter-ahead forecast of variable i is $\tilde{\sigma}^2_{it1} + \bar{d}_{i1}$, which in terms of the notation in Section II is $\hat{\sigma}^2_{it1}$. For the results in Table 2, t is 1978 I, and the d-row value for 1978 I for each variable is the square root of $\hat{\sigma}^2_{it1}$. The calculations for the two-quarter-ahead forecasts are the same except that there are only 26 values of the difference between the two estimated variances for each variable. Similarly, there are only 25 values for the three-quarter-ahead forecast, and so on.

Results for the Fair Model

A similar procedure was followed for the Fair model as was followed for the Michigan model. The version of the model used is the one in Fair (1984). The d-row values for the Fair model are also based on 27 sets of estimates of the equations. The same ending quarters were used here as were used for the Michigan results.

Results for the Autoregressive Model (AR8)

The Michigan data base was used for the autoregressive model. The estimation periods are the same as those for the Michigan model. The model consists of a set of eighth-order autoregressive equations with a constant term and time trend, one equation per each variable of interest. The equations are completely separate for each other. The same steps were followed for the autoregressive model as were followed for the Michigan model except that 100 rather than 50 trials were used for each of the 27 sets of stochastic simulations.

Results for the Complete Models

The results for the complete models are presented in the right half of Table 2. These results are taken from Fair and Alexander (1984). In this earlier study, uncertainty from the exogenous variables was also estimated and "c-row" values were presented. These values were estimated standard errors based on draws of the error terms, coefficient estimates, and exogenous-variable values. The d-row values were then computed using the c-row values as the base. In the present case there are no c-row values, and the d-row values are computed using the b-row values as the base. Otherwise, the present results are identical to those in the earlier study.

The d/b Ratios

Table 2 also contains ratios of the d and b rows. The larger is this ratio for a given variable and length ahead of the forecast, the larger is the mean of the differences between the two estimated variances, $\bar{d}_{ik}$. In the following discussion these ratios will be referred to as measures of the misspecification of the equations or groups of equations. It should be noted, however, that this terminology is not quite right. Misspecification also affects the b-row values, and the ratios of the d and b rows measure the misspecification that is not already reflected in the b-row values. It should be noted that no tests of the hypothesis that the equations are correctly specified are made. It is merely assumed that the hypothesis is false. Given this, the objective is to estimate by how much the equations are misspecified. The estimates of misspecification are the $\bar{d}_{ik}$ values. Standard errors of these estimates are not available because the distribution of $\bar{d}_{ik}$ is not known.

A Note about Computer Work

The Fair-Parke program (1984) was used for all the computations in this paper. Once a model is set up in the program, all the estimation and stochastic simulation that are needed for the results in Table 2 can be done with a few commands. The program provides an easy way to debug the setting up of the model, and once this debugging has been done, few other errors are likely to arise.

VI. Discussion of the Results

It should first be noted that the present results cannot be used to compare the predictive accuracy of the three models because exogenous-variable uncertainty has not been taken into account for the Michigan and Fair models. In other words, the d-row values in Table 2 cannot be compared across models. The appropriate comparisons were made in Fair and Alexander (1984), which show, among other things, that the Michigan model is heavily tied to the use of exogenous variables. What we are interested in here is the size of the d/b values for a given equation and how sensitive these values are to the inclusion of the equation in the complete model.

Michigan Results

Michigan does extremely well with respect to the private nonfarm deflator, the unemployment rate, the wage rate, and the bill rate. The d/b values for these variables are close to one. One would thus conclude from these results that the equations are well specified. As noted earlier, however, the equations for the private nonfarm deflator and the unemployment rate are heavily tied to dummy variables and the equations for the wage rate and the bill rate have questionable explanatory variables. It is not clear how much the present results should be trusted with respect to these equations.

The worst variable for Michigan is nonresidential fixed investment. The d/b values average 3.43 separately and 3.45 when the entire model is used. This suggests that the estimated variances of the error terms and coefficient estimates substantially overestimate the predictive accuracy of the equations. Thus the determination of this variable appears to be highly misspecified.

The results for consumer durable expenditures are not sensible. It sometimes turns out in the successive reestimation and stochastic simulation of the model that some of the stochastic simulation estimates of the variances are much larger than the estimates based on outside-sample errors. This may result in large negative values of $\bar{d}_{ik}$, and these values when added to the square of the b-row values can yield negative values of the total variance. What this means is that the sample is not large enough to produce sensible results. This was the case for consumer durable expenditures for Michigan.

The remaining variables for Michigan have d/b values around two. From this group, the best results are for inventory investment. This variable has an average d/b value of 1.62 individually and 1.40 when it is included in the entire model. The worst results from this group are for housing investment. The average d/b values for this variable are 2.40 and 2.23.

In general, the results are not sensitive to the inclusion of the equations in the complete model. There is, however, one important exception. The average d/b value for the money supply increases by 65% when the equations are included in the complete model. In Fair and Alexander (1984) the Michigan model's ability to predict the money supply was shown to be much worse than that of either the Fair model or the AR8 model. The current results suggest that not all of the Michigan model's poor performance can be blamed on the specification of the two equations jointly determining the money supply. It appears that specification errors made in determining other endogenous variables substantially worsen the results for the money supply.

Results for the other variables, however, are not sensitive to their inclusion in the complete model. Excluding the money supply results, the average deviation of the individual equation d/b values from the d/b values for the complete model is only 9%.

Fair Results

Fair does well or reasonably well for consumer service expenditures, consumer nondurable expenditures, housing investment, nonresidential fixed investment, inventory investment, the unemployment rate, and the wage rate, where the ratios are generally closer to one than to two. The three worst equations are the equations for imports, the private nonfarm deflator, and the money supply. The remaining equations, for consumer durable expenditures and the bill rate, generally have ratios that are closer to two than to one.

The results for the Fair model are generally more sensitive to being included in the entire model than are those for the Michigan model. The extreme case is once again the money supply, although in this case the results are better when the equations are included in the complete model. The average d/b value is cut in half when the equations are included in the model. This suggests that errors in predicting other endogenous variables partly offset the errors in the equations determining the money supply.

Even excluding the money supply results, the Fair model results are more sensitive to being included in the model. For example, the average d/b value for consumer expenditures for services increases by 33% when that equation is included in the model. Also, the average d/b value for the wage rate decreases by 28% when it is included in the model. Excluding the money supply results, the average deviation of the d/b values for the individual equations from the d/b values for the complete model is 21%.

The difference in the sensitivity of the results for the two models to whether the equations are included in the complete model is consistent with results reported in Fair and Alexander (1984). These earlier results show that the Michigan model is closely tied to exogenous variables, i.e. the predictive accuracy of the model is sensitive to alternative corrections for exogenous variable uncertainty. These results and the present results suggest that the Fair model is more "endogenous" than the Michigan model.

Autoregressive Results

The autoregressive results are not sensible for nonresidential fixed investment and the unemployment rate (again, a small sample problem). For the remaining variables the autoregressive model does best for consumer nondurable expenditures and inventory investment, where the ratios are closer to one than to two. Otherwise, the ratios are closer to two. Excluding the results for nonresidential fixed investment and the unemployment rate, the average of the d/b values is 2.14. This suggests that any equation in a structural model that has d/b values above about two should be a cause for concern.

Conclusion

The method presented in Fair (1980) is capable of measuring the misspecification of equations. One can glean from the misspecification results in Table 2 the parts of the models that need the most improvement. One part for Michigan is clearly the equations for nonresidential fixed investment. Two parts for Fair are the equation for the private nonfarm deflator and the equation for imports. In general it would seem that any d/b value that is greater than two is cause for some concern and more work. There is also some concern for Michigan that the use of dummy variables has given a misleading impression of the quality of some of the equations.

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