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A GENERAL EQUILIBRIUM EXPRESSION OF THE PARADOX OF THRIFT

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ABSTRACT

A model is presented which is derived from some observations of Keynes on the nature of capital. The allocation of investment is analyzed in two economies with random demand shocks which are identical except for the types of markets. In the first, the combination of an asset and forward markets realizes the complete set of markets. In the second, the forward markets are replaced by spot markets. Consumers and entrepreneurs are rational and markets clear. A clear definition of the paradox of thrift is proposed and its existence is proven. The substitution of spot markets for forward markets generates fluctuations of the aggregate variables. The equilibrium with fluctuations is not always a constrained Pareto optimum.

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I. Introduction

The central issue in macroeconomics is the extent to which the economy regulates itself through the markets. A well-known example of such a problem is the "coordination" of saving and investment in an economy subjected to various shocks. This is a major point in Keynes' analysis of effective demand. In a celebrated chapter he describes the effect of an increase of the propensity to save:[1]

> An act of individual saving means--so to speak--a decision not to have dinner today. But it does *not* necessitate a decision to have dinner or to buy a pair of boots a week hence or a year hence or to consume any specified thing at any specified date. Thus it depresses the business of preparing to-day's dinner without stimulating the business of making ready for some future act of consumption. It is not a substitution of future consumption-demand for present consumption-demand,--it is a net diminution of such demand. Moreover, the expectation of future consumption is so largely based on current experience of present consumption that a reduction in the latter is likely to depress the former, with the result that the act of saving will not merely depress the price of consumption-goods and leave the marginal efficiency of existing capital unaffected, but may actually tend to depress the latter also. In this event it may reduce present investment-demand as well as present consumption-demand.
>
> If saving consisted not merely in abstaining from present consumption but in placing simultaneously a specific order for future consumption, the effect might indeed be different. For in that case the expectation of some future yield from investment would be improved, and the resources released from preparing for present consumption could be turned over to preparing for the future consumption. Not that they necessarily would be, even in this case, on a scale equal to the amount of resources releated; since the desired interval of delay might require a method of production so inconveniently "roundabout" as to have an efficiency well below

[1] Keynes (1936), pp. 210-211. (italics in the text, emphasis added).

> the current rate of interest, with the result that the favourable effect on employment of the forward order for consumption would eventuate not at once but at some subsequent date, so that the *immediate* effect of the saving would still be adverse to employment. In any case, however, an individual decision to save does not, in actual fact, involve the placing of any specific forward order for consumption, but merely the cancellation of a present order. Thus, since the expectation of consumption is the only *raison d'être* of employment, there should be nothing paradoxical in the conclusion that a diminished propensity to consume has *cet. par.* a depressing effect on employment.

These ideas are important from a theoretical point of view because they provide the elements for an explanation of slack by "insufficient" demand, and employment fluctuations, which is not based on price rigidities. They are also at the origin of the well-known paradox of thrift.[2]

The insights presented in the quotation have been pathbreaking, and various formalization have been proposed since then. However in the contemporary language of economic theory, the above description is only a "story," and the paradox of thrift a conjecture.

The key aspect of the problem is the absence of complete Arrow-Debreu markets (as it was already clear to Keynes), a fact which is well recognized empirically. No coordination problem arises in an economy with complete contingent markets. This information failure between future consumption plans and investment decisions has sometimes been analyzed with the concepts of saving ex ante and ex post or desired and actual savings. A different interpretation is proposed here. I consider two stylized economies which differ only in the types of markets.

In the first, the set of complete markets is realized by one asset which provides a means of savings, and a set of forward markets between

[2] Samuelson (1980), pp. 225-227.

this asset and the future consumption goods. The purpose of this model with complete markets is to provide a tool for a thought experiment: it is an ideal situation with perfect coordination. The consumption intentions of consumers can be conveyed exactly to entrepreneurs. As in the standard "corn" economy, the level of intended savings is equal to investment.

In the second economy, spot markets (between the asset, the produced goods and labor), replace the forward markets. All other characteristics are unchanged. This is a rather fundamental empirical fact in the structure of exchanges, but casual empiricism may be insufficient in the present context. It is best to justify here the assumption of spot markets by considering an economy where consumers are randomly allocated between markets (similar to the islands of Phelps, 1968), in future periods. They are then prevented from contracting in the present, with dealers of different locations.

In this economy, consumers save, but do not place specific order for future consumption. The (unique) asset price is insufficient to convey all information to entrepreneurs. The responses of investment to the *same* variations of taste is compared between the two economies with full information and with asymmetric information, respectively.

This construction presents some advantages over other *ad hoc* formulations. The role of the market for savings and of the interest rate, which is somewhat obscure in the previous quotation, is emphasized in this model of general equilibrium. The paradox of thrift has a clear definition, and is proven in a model with rational expectations and market clearing.

The problem of the coordination of saving and investment is obviously related to an interpretation of aggregate fluctuations. In this sense, the present model provides an explanation for aggregate fluctuations which is based on the absence of forward markets. To highlight the issue, it is possible to choose parametric values of the model such that aggregate fluctuations are infinitesimal in the economy with complete markets and are large in the spot market economy.

The present model also shows that the intertemporal demand fluctuations are described by Keynes, are insufficient to explain the paradox of thrift. An essential element here is a set of alternative technologies with investment inputs at different dates, which produce the same good for consumption at a specific date. The technologies with a lower production period are less efficient and are not utilized in the economy with complete markets. However, in the spot market economy, it may be valuable from a social point of view to reduce investment until the spot markets reveal more information about the state of consumers' preferences, and eventually use some of the technologies with a lower production period. This is exactly what happens in the model of general equilibrium with rational expectations described here. It is noteworthy to remark that this outcome does not depend on the investment timing decision by individual firms;[3] in the model firms have only one opportunity for investment.

The paradox of thrift is discussed in the next section. The model is introduced in Section 3. The equilibria of the two economies with complete and with spot markets respectively, are compared in Section 4. Finally, it is interesting to analyze the role of fiscal policy in this model where "demand failures" occur because of the absence of forward markets. The concluding remarks are presented in the last section.

[3]For a synthesis of the literature on the timing problem of investment under uncertainty in partial equilibrium, see Jones and Ostroy (1984).

There is an obvious relation between this typical macroeconomic model of general equilibrium and some theoretical studies on incomplete markets and asymmetric information.[4] The same types of problems about the existence of equilibrium or its uniqueness are encountered here. These results are now well known. The non-existence or the multiplicity of (Pareto comparable) equilibria are illustrated in the Appendix by two special cases of the model.

II. The Paradox of Thrift

The paradox of thrift can be found in the literature under different forms. One version goes vaguely like this: a shift occurs in preferences such that individuals given their opportunity set (prices, quantity constraints if any, etc., ...), want to increase their savings. But when they alter their behavior, the mutual interactions of all agents generate in the end, a lower level of investment. This distinction between the individual intentions (presumably in partial equilibrium), and the general equilibrium outcome may also be the intuition that the terms saving "ex ante" and "ex post" try to capture, although they may not clarify the problem.

There is more than one explanation for this difference between individual intentions and aggregate result. For the sake of clarify, it is important to focus only on one of them. As Keynes was well aware of,[5] an increase of the taste for future consumption may reduce temporarily aggre-

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[4]The example of non-existence is similar to that of Kreps (1977). For multiple equilibria with incomplete markets, see Hart (1975). The model with complete markets used here has a structure which is very similar to that of Grossman (1981).

[5]See the above quotation in the previous section, second paragraph.

gate investment even in an economy with complete markets. However, this is not very interesting and certainly not a paradox.

To analyze the problem it is best to consider an economy where there are two types of states of nature. The "normal" state, and the states where a shift has occurred, which affects the relative tastes for present and future consumption. All these "shocks" are such that they generate an increase of aggregate saving at the prices which prevail in the normal state (in partial equilibrium). Also the production technology and the endowment are such that when markets are complete, all shocks generate an increase of aggregate investment (in general equilibrium).

Now consider an identical economy except that some forward markets are replaced by spot markets. The paradox of thrift occurs in a <u>strong</u> form if <u>all</u> the same demand shifts generate a decrease of the level of aggregate investment.

The paradox can also be given a definition in a <u>weak</u> form. Assume that the shocks occur with some probability distribution; the weak form implies that the <u>mean</u> of aggregate investment (over all shocks), is lower than in the normal state for the economy without complete markets (whereas <u>all</u> shocks increase aggregate investment in the Arrow-Debreu economy).

This paper will only consider the strong form of the paradox of thrift.

III. The Model

III.1. Introduction--The Framework

The main features of the model are taken from the description of Keynes. There is a large number of individuals. There are two periods and all individuals live for two periods.[6] Each individual is born with one occupation, consumer or entrepreneur.

The main departure from Keynes' original argument is the introduction of a market to carry wealth between periods. An important aspect of the problem is the fluctuations of the demands for goods which require some precommitment (investment) in order to be produced. Also these goods are perishable[7] (like prepared "dinners"). Therefore, the "current experience" of entrepreneurs could only be based on the current price of produced goods. To simplify the problem, there is no consumption good in the first period, and an asset market is introduced. The role of the market for savings is important and is somewhat neglected in the above quotation.

Consumers are endowed with a given good in each period. It is best to consider this good as "time." The consumer's utility depends on the consumption of time in both periods, and on the levels of consumption of two goods produced in the second period with labor input in the first period (investment).

An essential feature is that the production processes which require investment are run by entrepreneurs. Following standard tradition

[6]As shown later, the multiperiod framework considered by Keynes can be reduced formally to two periods.

[7]The storage of produced goods represents another investment decision. To limit the size of the problem, it is best to assume no storage of produced goods.

in macroeconomics, entrepreneurs do not consume the goods which they produce. They trade these goods for some of the second period time endowment of consumers (as for the first period, this is only a shortcut for a model where one unit of period 2 labor produces one unit of a consumption good which is also consumed by entrepreneurs). Entrepreneurs are risk-neutral if there is uncertainty, and maximize their expected consumption.

The preferences of consumers are random. For investment decisions only aggregate tastes matter. It may be strange to consider a model where the only "shocks" are generated by aggregate variations of tastes which spread like contagious diseases in a stationary environment. In a more realistic representation, these changes could be triggered by an exogenous event (as a supply shock), which is known to all. The only uncertainty would be about the response of tastes to this event. For the sake of simplicity the environment is stationary here. Only the variations of preferences remain.

There are two types of uncertainty which may occur. In the first consumers know their utility function exactly. The realization of the random variables which determine tastes occur <u>before</u> the beginning of the first period. With complete markets, there is no uncertainty. Without complete markets, entrepreneurs may not know exactly the state of nature in the first period, and the uncertainty is resolved only in the second period. In this case, when consumers want to shift consumption through time, there is no market to convey information about the desire for future consumption on specific goods.

Another type of situation is found when consumers do not know exactly their preferences which are determined only between the two periods. There is a liquidity premium for the consumers. This case is beyond the scope of the paper and will be analyzed separately.

In the present framework, the set of markets is complete when there is one asset between the two periods and there are in the first period, forward contracts for the second period. The asset is chosen here to be a claim to one unit of time in the second period.

The next step is to take into account the fact that with the structure of markets in a contemporary economy, the number of assets and forward markets is small with respect to the number of goods in the economy. A stylized representation is that there is only one asset, and that the various goods are traded with this asset in spot markets, in each period. This representation is achieved here by considering the situation where there are no forward markets for the produced goods in the first period. These goods are traded only in spot markets which open in the second period. In this case there is some uncertainty in the first period about the structure of consumers' preferences.

In the present context, a useful (and non-unique) justification for this assumption is that consumers are allocated randomly between different locations in the second period. They arrive in their new location at the beginning of the second period with their savings and time endowment, and they trade on spot markets to buy the produced goods from "dealers." The same produced goods can be purchased in each location (all brands are represented in all cities).[8] An accurate modelization of this segmentation of the economy à la Phelps would be a little heavy and would not provide much insight, at least for the present problem. With

[8]Presumably there are also in each city firms manufacturing parts of the produced goods which are then assembled (in some other city), and distributed to dealers throughout the country.

this description in the background, let us return now to the stylized model with economy-wide markets.

III.2. The Agents

2a. Consumers

There is a large number of consumers indexed by the parameter i. Each consumer is negligible with respect to the aggregate and is born with the utility function:

$$(1)\quad U^i = (1-\alpha^i-\beta^i-\gamma)\log(1-\ell_1^i) + \alpha^i \log x_1^i + \beta^i \log x_2^i + \gamma \log(\omega - \ell_2^i) .$$

The "time" endowments in the two periods are exogenous and identical for all individuals. (One could also assume a distribution of endowments which is independent of the taste parameters.) The variables ℓ_1^i, ℓ_2^i, x_1^i and x_2^i, represent the individual labor supplies in both periods, and the consumption of both produced goods in the second period, respectively.

The consumers' preferences are determined by the realization of the random variables α_i and β_i before the beginning of the first period. These variables are the sum of two components, the aggregate and the individual, which are independently distributed. It could be useful, for further studies, to think that the individual components of α^i and β^i have a large variance with respect to the aggregate variables α and β, respectively, and that the individual i has imperfect information on the aggregate shifts of preferences by introspection, i.e., by using the correlation between the individual taste parameters and their aggregate values.

For the investment decisions of entrepreneurs, only the aggregate variables α and β matter. The states of nature are defined by the values of these variables. Following Keynes, there are two types of shifts increasing the taste for the goods one and two, respectively. To simplify as much as possible, there are three (aggregate) states of nature, described in Table 1:[9]

(8) $\alpha^* + \beta^* > \alpha_0 + \beta_0$

The probabilities of states 1 and 2 are equal.

States of Nature	α	β
0	α_0	β_0
1	$\alpha^* + \varepsilon$	$\beta^* + \varepsilon$
2	$\alpha^* - \varepsilon$	$\beta^* - \varepsilon$

TABLE 1. The States of Nature

The interpretation of the table is straightforward. The state zero is the "normal" state. "Shocks" occur in states one or two. Both shocks increase the taste for total future consumption (represented by $\alpha + \beta + \gamma$) by the same amount. However, the relative taste for good one with respect to good two is greater in state one than in state two. The values α^* and β^* simply represent averages between the two states.

The following assumption will be used some time later.

Assumption (H1): *The difference* $\alpha^* + \beta^* - (\alpha_0 + \beta_0)$ *is infinitesimal with respect to* ε , *and is positive.*

[9] Radner (1979) has shown that with a finite number of states of nature, rational expectation equilibria with incomplete markets reveal, generically, the exact state of nature. Since this is not the case here, the example is not generic. But the example is chosen only for its computational simplicity, and is only a tool of analysis for a (more complex) economy with a two dimensional continuum of states of nature and a single asset.

When this assumption is satisfied, the shifts of aggregate demand across periods are negligible. This situation will highlight the aggregate fluctuations which are specific to the absence of forward markets (following the second paragraph of Keynes' quotation).

2b. Entrepreneurs and Production

All entrepreneurs have the same utility function in the beginning of the first period which is equal to the expected value of the time they can purchase from consumers in the second period (or of a good perfectly substitutable with leisure through a linear technology). Entrepreneurs have no endowment in the first period. They may be endowed with some "time" in the second period, so that they are never bankrupt.

Each entrepreneur owns one firm and behaves competitively. There are three different technologies (T1), (T2) and (T3) respectively and two types of firms.

To simplify the notation, the population of consumers is normalized to one, and there are two representative firms, each controlled by one representative entrepreneur.[10] All aggregate quantities will be expressed per capita.

The first firm uses the technologies (T1) and (T2) in any combination. Its main characteristic is that inputs are made in the first period for production in the second period.

The level of the first produced good x_1, is determined by the first technology (T1), which transforms an input ℓ_{11} in the first period,

[10] There is no loss of generality, since this consumer could be defined as an arbitrary fraction of a caput. Also, we will use the words representative consumer (entrepreneur), and consumers (entrepreneurs), indifferently.

i.e., an investment, into an output

(9) $$x_1 = \ell_{11}^a \ , \quad 0 < a < 1 \ .$$

The second good can be produced by two technologies (T2) and (T3). The technology (T2), in the firm of the first type, requires an input ℓ_{12} in the first period for an output level x_{21} in the second period, with

(10) $$x_{21} = \ell_{12}^a \ .$$

The second type of firm is characterized by the technology (T3) which requires an input ℓ_2 of labor in the second period for an output x_{22}, with

(11) $$\ell_2 = Ax_{22} \ , \quad A > 1 \ .$$ [11]

There are two (related) interpretations for this production possibility set. The good two can be produced by precommitting resources in the first period or by waiting until the second period. The first type of technology (T2), has a higher degree of roundaboutness and is more efficient.

The model has also a three period interpretation which is closer to Keynes' original description. In this case, the second good represents the "dinners" for the day after tomorrow. They can be produced by committing resources for production today or tomorrow. The technology which has a larger period of production is more efficient. Obviously, this is

[11] We will need later a stronger condition for A .

only the production efficiency. The choice of technology will depend on the interest rate and on other factors.

III.3. The Markets

Since the random variables are realized before the first period, the set of markets is complete when there is an asset to carry wealth between the two periods, and there are forward markets in the first period, between this asset and the second period produced goods. This asset can be chosen arbitrarily. Here this "bond" is defined as the claim to one unit of time in the second period (or one unit of a good produced by labor with a linear technology), which will be the numeraire. This choice has also some similarity with the wage units in Keynes' work.

In the second economy analyzed here, the forward markets are replaced by spot markets which open only in the beginning of the second period after decisions have been made in the first period.

IV. The Equilibria

IV.1. The Demand for Bonds and the Supply of Labor

The characterizations of the equilibria are simplified as the assumption of the addi-log utility function. In general, the demand for bonds by consumers depends on the future prices when the forward markets exist, or on the anticipations of future spot prices between the two produced goods and leisure in the second period. The present case is simpler because with the functions (1) the marginal utility of leisure in the second period is independent of the relative prices in that period. The extension of the framework to more general functional forms would complicate significantly the algebra and would not at this stage, bring additional insights.[12]

The price of the bond in terms of units of time in the first period is equal to q .[13] Therefore the interest rate between units of time[14] in the two periods is equal to

$$r = \frac{1}{q} - 1$$

If the consumer i purchases z^i bonds in the first period, his marginal utility of income (measured in units of time), in the second period is equal to $(\alpha^i + \beta^i + \gamma)/(z^i + \omega)$. It is independent of the state of nature. The consumer sells in the first period an amount ℓ of labor and demands an amount $z^i = \ell^i/q$ of bonds. The first order condition between periods is

$$\frac{(1 - \alpha^i - \beta^i - \gamma)}{1 - \ell_1^i} = \frac{\alpha^i + \beta^i + \gamma}{\ell_1^i + \omega q} . \tag{12}$$

Whether there are forward markets or not, the supply of labor and the demand for bonds by consumer i are equal to

$$\begin{cases} \ell_1^i = (\alpha^i + \beta^i + \gamma)(1+\omega q) - \omega q \\ z^i = \ell_1^i/q \end{cases} \tag{13}$$

The aggregate labor supply is equal to the average of all individual supplies,

[12] Comments will be added later whenever results obtained are specific to the addi-log case.

[13] The implicit numeraire is the time in the second period.

[14] Or between two goods produced by labor through linear technologies in the two periods, respectively.

$$\ell_1^S = (\alpha+\beta+\gamma)(1+\omega q) - \omega q \ , \tag{14}$$

and the aggregate demand for bonds per capita is equal to

$$z^D = (\alpha+\beta+\gamma)\left(\frac{1}{q} + \omega\right) - \omega \ . \tag{15}$$

IV.2. The Markets for Produced Goods

The prices of the two produced goods in terms of the second period time are equal to p_1 and p_2, respectively. Whether these markets open in the first period (forward), or in the second (spot), the levels of demand for the produced goods can be expressed by the same functions which depend on the prices and on the total resources in the second period (time endowments and savings). These functions are defined by:

$$x_1^i = \frac{\alpha^i}{\alpha^i + \beta^i + \gamma}\,\frac{(\omega + z^i)}{p_1}, \quad x_2^i = \frac{\beta^i}{\alpha^i + \beta^i + \gamma}\,\frac{(\omega + z^i)}{p_2} \ . \tag{16}$$

By substitution of $\omega + z^i$, and using (13), these equations are equivalent to

$$p_1 x_1^i = \alpha^i\left(\frac{1}{q} + \omega\right) \ , \quad p_2 x_2^i = \beta^i\left(\frac{1}{q} + \omega\right) \ . \tag{17}$$

By aggregation over all individuals, the market demand functions are equal to

$$\begin{cases} p_1 = \dfrac{\alpha}{x_1}\left(\dfrac{1}{q} + \omega\right) \\[2ex] p_2 = \dfrac{\beta}{x_2}\left(\dfrac{1}{q} + \omega\right) \ , \end{cases} \tag{18}$$

where x_1 and x_2 represent the per capita levels of the two produced goods, respectively.

IV.3. The Equilibrium with Forward Markets

In the first period, entrepreneurs choose a level of investment ℓ_{11} and ℓ_{12} in the technologies (T1) and (T2) respectively, which are financed by issuing z_1^S bonds, in order to maximize expected profits (measured in period two time). Consider first the investment in (T1). When forward markets exist, the equilibrium is fully revealing in the first period. Since there is no uncertainty, the introduction of spot markets in the second period would not create new transactions. The price of output p_1 is perfectly defined in the first period. Since $\ell_{11} = qz_1^S$, the level of profit is equal to $p_1\ell_{11}^a - (\ell_{11}/q)$. The demand for labor is therefore equal to

(19) $$\ell_{11} = (aqp_1)^{1/1-a} \quad \text{if} \quad a < 1 .$$

The price p_1 depends on the total output x_1 in (18). Substituting in (19), one finds the demand for labor by the representative of all entrepreneurs in the first period, which is equal to (in per capita terms as usual),

(20) $$\ell_{11} = \left[\frac{a}{x_1}(1+\omega q)\right]^{1/1-a} .$$

Since x_1 is equal to ℓ_{11}^a, the demand can be expressed as a function of q and α :[15]

(21) $$\ell_{11} = a\alpha(1+\omega q) .$$

[15] The model can easily be extended to the case of linear technologies ($a = 1$). The demand for labor used in (T1) is perfectly elastic with a bond price q equal to $1/p_1$. The level of investment is then determined by the orders for x_1 on the forward market. The same remark applies for the technology (T2). (The case of incomplete markets is more delicate because some quantity mechanism is required.)

The technologies (T2) and (T3) produce the same good. For the parametric values considered here, the interest rate will be sufficiently small and the input-output ratio A in (T3) sufficiently large, such that under perfect information, the technology (T3) is not used.[16] The investment decision for (T2) is then similar to that for (T1). The labor input ℓ_{12} can be expressed as a function of β and q :

$$\ell_{12} = a\beta(1+\omega q) \; . \tag{22}$$

The bond price is determined by the equilibrium of the market where labor and bonds are exchanged in the first period,

$$\ell_1^S = \ell_1^D = \ell_{11} + \ell_{12} \; . \tag{23}$$

Using (15), (21) and (22) after a straight manipulation, the equilibrium bond price is equal to:

$$q_F = \frac{(1-a)(\alpha+\beta) + \gamma}{\omega(1 - (1-a)(\alpha+\beta) - \gamma)} \; , \tag{24}$$

and the level of investment (or labor input) in the first period is

$$\ell_F = \frac{a(\alpha+\beta)}{1 - (1-a)(\alpha+\beta) - \gamma} \; . \tag{25}$$

A geometric representation of the equilibrium is given in Figure 1, with the demand and supplies of bonds, valued in units of time in the first period.

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[16] As shown in (37) below, the necessary and sufficient condition that the discounted value of the input output ratio $q_F A$ (where q_F is the equilibrium value), is greater than one.

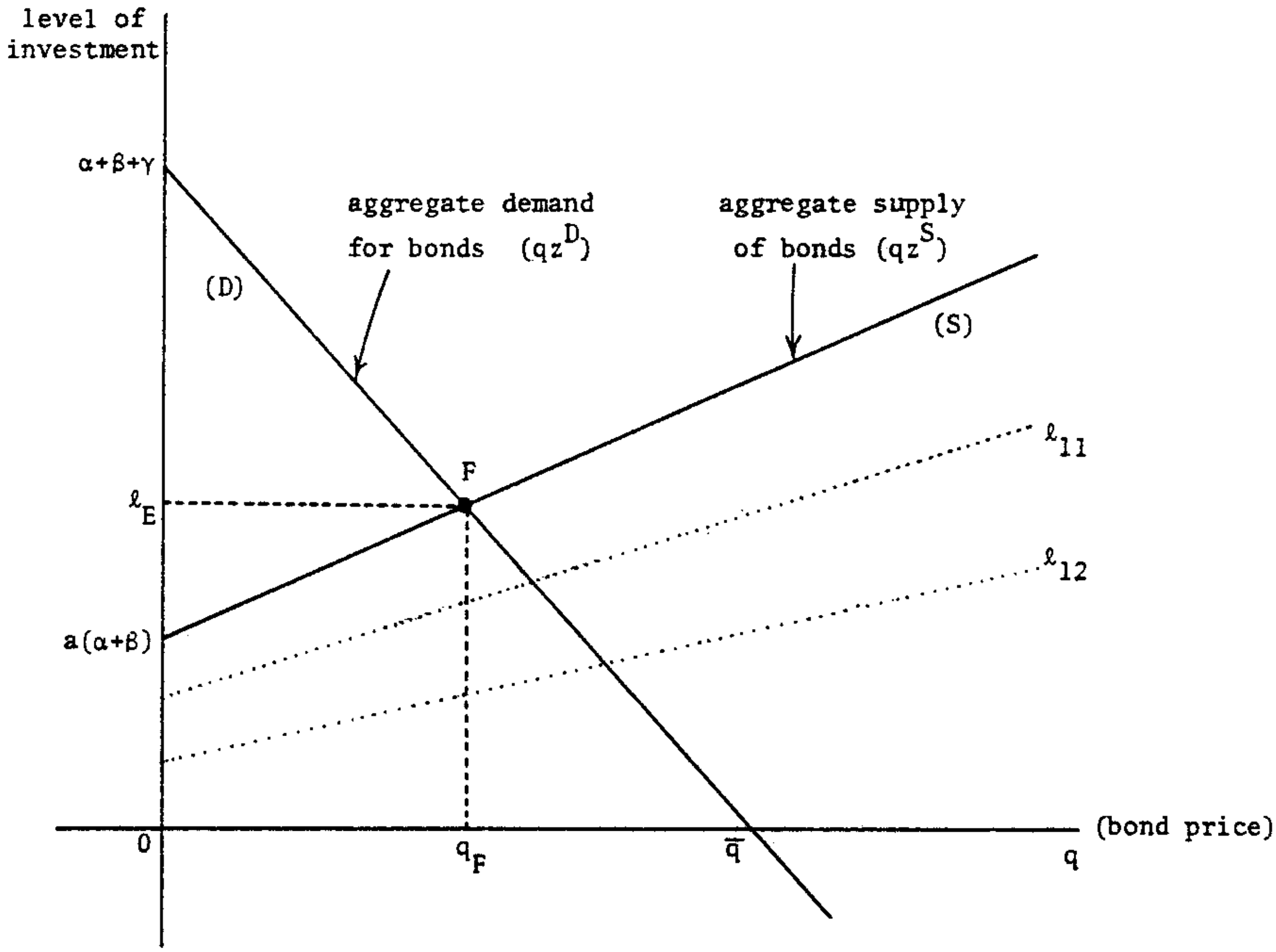

FIGURE 1

The Equilibrium with Forward Markets in the First Period

$$\bar{q} = \frac{\alpha+\beta+\gamma}{\omega(1-\alpha-\beta-\gamma)}$$

$$\begin{cases} qz^D = \ell_1^S \\ qz^S = \ell_1^D = \ell_{11} + \ell_{12} \end{cases}$$

The values of the forward market prices are equal to

$$(26)\qquad \begin{cases} p_1 = \dfrac{[\alpha(1+\omega q)]^{1-a}}{aq} \\[2ex] p_2 = \dfrac{[\beta(1+\omega q)]^{1-a}}{aq} \end{cases}$$

Three remarks can be made at this point. First, the equilibrium prices reveal the state of nature in the first period when entrepreneurs know the description of the states of nature in Table 1, but they do not need it since the price information is sufficient.

Second, the taste for total consumption in the second period increase with the sum $\alpha+\beta$. It is higher in states 1 and 2 than in the reference state 0. An important characteristic of this general equilibrium with complete markets is that <u>both demand shifts (in states 1 and 2) generate an increase of aggregate investment</u>.[17]

Finally, if assumption (H1) is satisfied the <u>aggregate</u> fluctuations of output and investment between the three states (measured in (25)) are small or even infinitesimal. The variations of economic activity occur mostly between the two technologies of production.

IV.4. The Equilibrium without Forward Markets

It is now assumed that the markets for the produced goods open only in the <u>second</u> period. As mentioned in Section IV.2, the prices on these spot markets can be expressed as functions of the bond price q and the quantities x_1 and x_2 , which are identical to those found in (18). The representative entrepreneur uses now the expected values of the output

[17] They, of course, generate an increase of saving in partial equilibrium with a fixed interest rate since the demand for bonds shifts upwards when $\alpha+\beta$ increases.

prices. A straightforward adaptation of the previous derivations shows that the levels of input demand for the technology (T1) is equal to

$$(27) \qquad \ell^D_{11} = aE(\alpha)(1+\omega q) \ ,$$

where $E(\alpha)$ is the expected value of α conditional on the entrepreneur's information in the first period.

Assume now, that the value of the input-output ratio A, in the technology (T3), is very large and that this technology is never used. The demand for labor in (T2) is of the same form as the one for (T1):

$$(28) \qquad \ell^D_{12} = aE(\alpha)(1+\omega q) \ .$$

The information of the entrepreneur is obtained only through the market price of bonds. Therefore this information is generated only by shifts of the demand for bonds (schedule (D) in Figure 1). Since this schedule is the same in states 1 and 2, the entrepreneur cannot discriminate between the two states. In general the bond price varies when demand shock occurs, with respect to its value in the state zero, and the entrepreneur can discriminate between the states (1,2) and the state 0. There exist , however, parametric values of the model such that no equilibrium exists. Such a case is presented in the Appendix, which is analogous to an example given by Kreps (1977), in a different context.[18]

The equilibrium values of the bond price and the level of aggregate investment are now equal to

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[18]There is also a difficulty if the technologies (T1) and (T2) are linear, for the determination of the scales of production since there is no quantity signal. In the economy with forward markets the quantity signals are provided by the forward orders.

$$(29) \qquad q = \frac{(1-a)E(\alpha+\beta) + \gamma}{1 - [(1-a)E(\alpha+\beta) - \gamma]}$$

$$(30) \qquad \ell = \frac{aE(\alpha+\beta)}{1 - (1-a)E(\alpha+\beta) - \gamma},$$

where $E(\alpha+\beta) = \alpha_0 + \beta_0$ if state 0 occurs

$E(\alpha+\beta) = \alpha^* + \beta^*$ if state 1 or 2 occurs.

Note that the aggregate values of investment in the three states of nature are the same as in the case of complete markets. In particular under Assumption (H1), the fluctuations of aggregate variables are infinitesimal. *In general, a shift of future demand does not reduce the level of aggregate investment, even when entrepreneurs do not know exactly which future goods consumers prefer.* The description of Keynes is not sufficient to explain the drop of investment when a preference shift occurs. The prices of produced goods may be low in some states of nature, but they are also high in other states. Since entrepreneurs are risk neutral, this uncertainty may not affect their decision. In the states 1 or 2, the spot prices of produced goods are equal to

$$(31) \qquad p_1 = \frac{\alpha}{\alpha^*} \frac{[\alpha^*(1+\omega q)]^{1-a}}{aq}, \quad \text{and}$$

$$(32) \qquad p_2 = \frac{\beta}{\beta^*} \frac{[\beta^*(1+\omega q)]^{1-a}}{aq}.$$

By comparison with the values found for the fully revealing equilibrium in (26), the prices have a higher variance, and the same mean. The higher price variance is explained by the fact that with spot markets, the levels of output for the two produced goods are the same in states 1 and 2, and in the second period, all the market adjustment to preference variations is made through prices. If state 2 occurs (strong taste for good 2), the price p_2 is higher in the spot market economy than in the economy with forward markets because the supply is smaller.

Assume now that the input-output ratio A of technology (T3) is not too large and that it provides an effective upper-bound on the price p_2 in the spot market: if this price increases above A, the entrepreneurs owning the technology (T3) start to produce. They impose an upper bound on the price p_2, which affects the optimization calculus of the entrepreneurs in the first period.

The technology (T3) is used in state 2 if the value of spot price found in (32) is greater than A, i.e., when

$$(33) \qquad \frac{\beta^* + \varepsilon}{\beta^*} \frac{[\beta^*(1+\omega q)]^{1-a}}{aq} > A .$$

The first order condition for the investment level in (T2) now takes the form:

$$(34) \qquad \frac{(p_2 + A)}{2} a\ell_2^{a-1} = \frac{1}{q} ,$$

with[19] $$p_2 = \frac{\beta^* - \varepsilon}{x_2}\left(\frac{1}{q} + \omega\right) , \quad \text{and} \quad x_2 = \ell_2^a .$$

The investment level ℓ_2 is the solution of the equation:

$$(35) \qquad \ell_2\left(2 - \frac{aAq}{\ell_2^{1-a}}\right) = a(\beta-\varepsilon)(1+\omega q) .$$

[19] As mentioned before, the parameters of the models are such that (T3) is not used under perfect information. This implies that it is not used in the state one, where the demand for the good 2 is low.

The algebra and the geometric exposition are simplified by the introduction of an assumption (which is not essential for the properties of the equilibrium):

Assumption (H2): *The term* 1-a *is strictly positive and its value is arbitrarily close to zero.*

In this case, the value of ℓ_2 is approximated by:

$$\ell_2 = \frac{(\beta^* - \varepsilon)(1+\omega q)}{2 - Aq} \quad \text{if} \quad q < \frac{1}{A}\left(1 + \frac{\varepsilon}{\beta^*}\right) . \tag{36}$$

The graphs of ℓ_1 and ℓ_2 as functions of q, are represented in Figure 2. All curves are represented with the values $\alpha = \alpha_0$, $\beta = \beta_0$ and $\varepsilon = \beta/2$. Under Assumptions (H1) and (H2) they are approximations of the curves found in states 1 and 2. The sum of ℓ_1 and ℓ_2 is equal to the aggregate investment which is equal to the total value of bonds supplied by firms. Its graph in states 1 or 2 is represented by (S_{12}). The schedule (S_0) applies in the state zero. The lines (D) and (S_0) are identical to those obtained under complete information (see Figure 1).

Under assumptions (H1) and (H2) a sufficient condition for the existence of the case represented by the figure is

$$\frac{1}{A} < q_F < \frac{1}{A}\left(1 + \frac{\varepsilon}{\beta^*}\right) . \tag{37}$$ [20]

The first inequality implies that the technology (T3) is not used when information about the state of nature is perfect. The second inequality

[20] The assumptions (H1) and (H2) simplify the sufficient conditions. Obviously (H1) and (H2) are not required for the main properties of the equilibria, and other sufficient conditions could be presented.

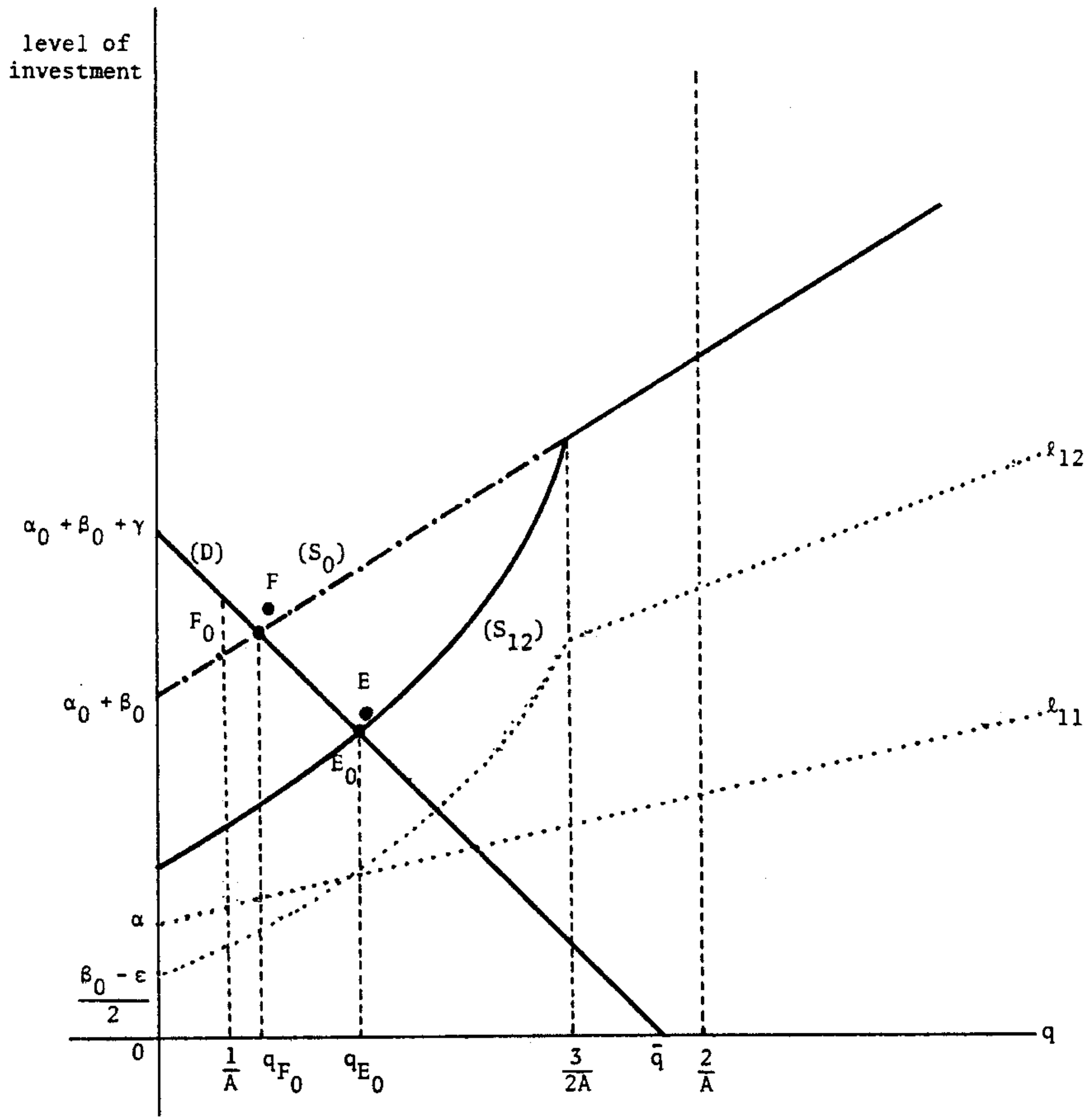

FIGURE 2

The First Period Equilibria with Forward Markets (F) and Spot Markets (S) implies that the technology (T3) is used in the spot market equilibrium for state 2 (when the level of the demand for the second good is high).

The equilibrium in the first period is represented by the points F_0 and E, in the "normal" state zero and when a shift occurs (state 1 or 2), respectively. The position of E is approximated by the point E_0, intersections of (D) and (S_{12}).[21] The point F represents the equilibrium with forward markets in states 1 or 2.[22]

As a reminder, the level of aggregate investment is greater at F than at F_0. All shifts of demand increase investment when there are forward markets. The geometry of Figure 2 implies immediately that the level of aggregate investment is smaller at the point S than at the point F. *All demand shifts induce a fall of investment when there are only spot markets.* This proves the existence of the paradox of thrift.

The explanation of the paradox of thrift should be clear from the previous description of the equilibrium. When the entrepreneurs make their investment decision in the first period they take a risk since they do not know if the demand for their product will be high or low in the second period. Because new entrepreneurs with the technology (T3) may enter the market when the demand is high, they put a ceiling on the price. This implies that the entrepreneurs are not fully compensated in the first period for the risk of a low price by a high price if demand is high. As a result, they reduce their investment.

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[21]This is the main use of Assumption (H2). With (41), it facilitates the placement of the points F and S in Figure 2, near the points F_0 and S_0, respectively, and the geometrical derivation of the properties of the equilibrium.

[22]The aggregate level ℓ is the same in both states, but ℓ_1 and ℓ_2 are different.

Because of the "back-up" technology (T3), the level of investment for the production of x_2 is reduced when a demand shift occurs (in state 1 or 2). In some sense, investment for x_2 is partly delayed to the second period until the exact state of demand is revealed to entrepreneurs. This is also the case in the three period interpretation of the model where the goods x_1 and x_2 represent consumption goods in the second and the third period, respectively.

It is important to note that *this process of waiting is not carried by individuals but is enforced by the invisible hand of the market.*

The entrepreneurs who make investment decisions for (T2) have only an investment opportunity in the first period, and by definition they cannot delay their decision since they do not have a second chance. However, the competition with the entrepreneurs of the technology (T3) in the second period, forces them to reduce investment when a demand shift occurs. From an aggregate point of view the result is the same as if an invisible hand had enforced a delay of investment. This delay is efficient, as we will see in the next section.

Aggregate Fluctuations

An important difference between the economies with forward and spot markets respectively, is that the levels of aggregate investment move in opposite directions under the same variations of consumers preferences.

In the present model output is equal to investment. This definition can be extended by assuming that there is also in the first period, a consumption good which is produced by labor in the same period. Assume for simplicity that the supply of this good is fixed. The level of total output varies obviously in the same direction as investment. A remarkable result is that in the economy with spot markets, the savings rate decreases when output decreases. This is in agreement with a stylized fact of the business cycle. Note that the "intended" and the "actual" savings rate, to use another terminology, which can be observed in the two economies, with and without forward markets, respectively, vary in opposite directions.

Also the correlation between investment and the interest rate is negative in the economy with complete markets, as can be expected when the supply savings fluctuates. This correlation has the opposite sign in the spot market economy.

When the Assumption (H1) is valid, the fluctuations of all aggregate variables and of the interest rate are infinitesimal in the economy with forward markets. In the spot market economy, the same demand shifts induce large variations of the same variables which can be interpreted as cycles. The introduction of forward markets would reduce significantly the amplitude of these cycles.

V. The Nonoptimality of the Equilibrium without Forward Markets and the Role of Policy

The equilibrium with spot markets (S-equilibrium), is inefficient with respect to the equilibrium with forward markets (F-equilibrium) in a first best sense. The important issue is the "constrained" inefficiency, i.e., whether there are policies with no more information than that revealed by the market, which improve the allocation of resources. This problem is solved by two propositions.

Proposition 1: *The allocation of resources in the* S-*equilibrium is efficient in the following sense: it maximizes the average level of utility over all consumers for a given value of expected profits, conditional on the information revealed by the equilibrium about the aggregate preferences.*

The result implies that macroeconomic policies which do not generate a gain of information cannot improve the allocation of resources in the economy with spot markets.

Proof: If state 1 or 2 occurs, the aggregate allocation of resources which is optimal in the Proposition's sense is the solution of the problem.

(P) Maximize $(1-\alpha-\beta-\gamma)\log(1-\ell_1) + E[\alpha \log x_1 + \beta \log x_2 + \gamma \log(1-\ell_2)]$,

subject to $E(\pi) = \pi_0$,

$$\begin{cases} x_1 = \ell_{11}^a , \\ x_2 = \ell_{12}^a + \frac{y}{A} , \\ \ell_{11} + \ell_{12} = \ell_1 , \\ \ell_2 = y + \pi , \quad y \geq 0 , \end{cases}$$

with the notation:

π : profit in the second period (measured in time),

y : labor input in the technology (T3).

The input decision y for the technology (T3) is taken only in the second period when the information is revealed.

The solution of the problem (P) determines the aggregate allocation. The individual efficient allocations can easily be deduced from it by an egalitarian distribution of income (for the additive welfare function), and market decentralization in each period.

It is a simple exercise to show that the first order conditions of the problem (P) are satisfied in the S-equilibrium. Because of the concavity of the objective function and of the production function, these conditions are also sufficient for a global optimum. The same argument applies in the state zero, where there is no uncertainty. Q.E.D.

The previous proof does not depend on the information structure used in the expectation operator E. Therefore, it also implies that the allocation of resources in the S-equilibrium is inefficient in the sense specified in the proposition, with respect to the allocation of resources in the F-equilibrium.[23]

Proposition 1 can be applied to the evaluation of macroeconomic policy. First note that in the present model, a lump-sum tax subsidy in the first period financed by a deficit is neutral when the bonds are repaid in the second period by lump-sum taxation. There is no effect on the allocation of resources because the budget of consumers is not altered. However other policies could stabilize the level of economic activity. But the stabilization of employment through linear taxes or subsidies is not desirable per se. More precisely, the above result shows that macroeconomic policies improve the allocation of resources only if they are able to enhance the information revealed by markets.

In the present model, a tax policy which applies only in the first period does not affect the signaling property of the interest rate and is not welfare improving (this property depends in an essential way on the additive separability of the utility functions of consumers).

Proposition 1 does not imply however, that the equilibrium in the spot market economy is a constrained Pareto optimum.

Proposition 2. *In the spot markets economy, there exist policies which do not require more information than that provided by markets, and induce a more efficient allocation of resources, in the sense of Proposition 1.*

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[23]The F-equilibrium does not in all cases Pareto-dominate the S-equilibrium: the level x_1 is higher in the S-equilibrium, a situation which may be preferred by individuals with a strong consumption taste for the first produced good. It is however possible that for some particular parametric values, a Pareto ordering exists between the two equilibria.

Some remarks may be useful before the proof of the result. The fundamental cause for suboptimality of the S-equilibrium is the inability of the one bond price to provide information about the two markets for produced goods. In a general model with a continuum of states of nature, individuals would infer conditional probability distributions on α and β from the observation of q (and possibly also from observing their own α^i and β^i).

Fiscal policies deterministic or state contingent, affect in general the excess-demand of agents for given endowment and tastes. When the mapping from tastes (states of nature), to the bond price depends on policy, the signalling property of the bond price (the reverse function from the bond price to a probability distribution on the states of nature), may also be altered by a policy (which does not depend on an advantage in information). In the end, the nature of the equilibrium may be significantly modified.

This effect is illustrated in the proof of Proposition 2. The principle of the proof is to introduce a small disturbance in the economy (the policy), in order to induce a new equilibrium which is in the generic set of equilibria with complete information (Radner, 1979). The fundamental equation is the intertemporal first order condition (12), of the consumers.

Assume that a contingent tax policy is announced in the first period; in the second period the spot markets reveal to all agents the state of nature which has occurred before the first period. The fiscal policy is to implement linear taxes on produced goods and a uniform lump sum refund to all consumers in the second period only if state 1 is revealed as the state of nature. The tax revenues per capita T , are assumed to be small. Given the form of the indirect utility function of consumers (which is additively separable in income and prices), the consumers' marginal utility of income (in units of time), is now equal to $1/(z^i + \omega + T)$.

In the states one or two,[24] the first order condition (12) of individual i is now equal to:

$$(38)\qquad \frac{1-\alpha^i-\beta^i-\gamma}{1-\ell^i} = \frac{\alpha^i+\beta^i+\gamma}{2}\left(\frac{1+2\theta^i}{\ell^i+\omega q+T} + \frac{1-2\theta^i}{\ell^i+\omega q}\right) ,$$

where $\frac{1}{2}+\theta^i$ is the subjective probability for individual i that state one has occurred, conditional on his available information. The individual's labor supply is equal to

$$(39)\qquad \ell^i = (\alpha^i+\beta^i+\gamma)(1+T+\omega q) - (T-\omega q) - T\frac{1-\ell^i}{\ell^i+\omega q}(\alpha^i+\beta^i+\gamma)\theta^i .$$

When T is small, a first order approximation (in T) is equal to

$$(40)\qquad \ell^i = (\alpha^i+\beta^i+\gamma)(1+\omega q) - \omega q - T(1-\alpha^i-\beta^i-\gamma)(1+\theta^i) .$$

The subjective probability of state one, $\frac{1}{2}+\theta^i$, is determined by "introspection," i.e., by observing the individuals' values α^i and β^i . These values are the sum of aggregate and individual shocks. When individuals observe a positive difference $\alpha^i-\beta^i$, they do not know if this is due to an aggregate or an individual event. However, since by definition, the individual variations are independent of aggregate variations, the subjective probability of state one is greater than when the difference $\alpha^i-\beta^i$ is negative. Without loss of generality, and in order to simplify the arithmetics, θ^i is assumed to be a linear function of the dif-

[24] In state zero, there is perfect information, and the equation (12) is unchanged.

ference $\alpha^i - \beta^i$:[25]

(41) $$\theta^i = \lambda(\alpha^i - \beta^i) \ , \quad \lambda > 0 \ .$$

The average labor supply over individuals is obtained by summing (40). In the state two,

(42) $$\ell^S = (\alpha^* + \beta^* + \gamma)(1+\omega q) - \omega q - T(1 - \alpha^* - \beta^* - \gamma + \lambda(\tilde{\alpha}-\tilde{\beta})) - T\lambda(1-\gamma)\varepsilon \ ,$$

where $\tilde{\alpha}$ and $\tilde{\beta}$ represent the averages of the squares of α^i and β^i over individuals.

If the price q is the same in states one and two, the last term in the right hand side of (42) implies that the aggregate labor supply is greater in state two than in state one. But the labor demand depends only on the bond price and is the same in both states. Therefore, there cannot be an equilibrium with the same bond price in states one and two. Since the bond price is different in the two states, it is fully revealing and there is a perfect information. Also the value of T is small and the equilibrium solution is in the neighborhood of the F-equilibrium.

The utility levels of individuals are not as high as in the F-equilibrium because of the tax distortions in the second period. But since T is small, these distortions are only of the second order (with respect to T), and the new equilibrium with policy is more efficient than the S-equilibrium. Q.E.D.

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[25] This relation is exact if the state 1 is defined as follows: $\alpha^i = \alpha^* + \eta$, $\beta^i = \beta^* - \eta$ for a fraction μ of consumers, and η takes the opposite sign for the fraction $1-\mu$. The state 2 is symmetrical.

The key equation in the proof is the intertemporal first order conditions of consumers, (38). The policy has an effect because it has an impact on the average expected marginal utility of income in the second period, and through this channel, on the relation between the state of nature and the supply of labor (or the demand for bonds). This impact on the average expected marginal utility of income occurs because consumers have, on average, some (imperfect) information on state one or two.

The effect fails if consumers do not use introspection, i.e., if they (incorrectly) do not see any relation between their individual tastes and the aggregate. In this case the above policy is ineffective.

VI. Conclusion

The model presented here shows how an increase of the propensity to save can induce opposite responses of investment in economies with and without forward markets and how aggregate fluctuations can be explained by the absence of forward markets.

The model has some Keynesian properties but the role of policy is somewhat different from the traditional prescriptions of expenditures financed by debt. Although the property of constrained Pareto optimality is shown to be false by a counter-example, the effect of policy on the informational content of prices does not seem obvious for a more realistic framework, and the role of policy in the improvement of the allocation of resources may be rather limited.

However, the public debt could be analyzed with the present model in the second best situation. The standard purpose of the debt is then to remove the period specific budget constraints for the government and to equalize the excess-burden of taxation between period.[26]

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[26]See Barro (1979) for a quadratic welfare cost function, and Chamley (1983) for a general analysis in a utility maximizing framework.

There are two main issues at this point, which will be examined in future work. The future preferences of consumers may not be known to themselves, and this uncertainty may provide a liquidity motive. In this context, the menu of assets should be extended to analyze more specifically the role of money.

APPENDIX

1. The Non Existence of Equilibrium

In the present model, the bond price (or the interest rate), plays an essential role in providing information to entrepreneurs about the state of consumers' preferences. The value of the demand for bonds can be represented as a function of the bond price (as in Figure 1), in the next figure (schedule D). The value of the supply of bonds can be represented in the same way (schedule S). Its level depends on the anticipation of entrepreneurs which depend on the equilibrium bond price. When the desire for future consumption increases, the schedule D shifts upwards. For a given supply schedule, the bond price would rise. But this rise would reveal information about the increased taste for future consumption. Entrepreneurs then supply more bonds (for a given bond price): the supply schedule S shifts also upwards which generates a reduction of the bond price. For a particular choice of parametric values (described below), the effects of the two shifts on the bond price cancel each other. But in this case the bond price cannot reveal information. The perfect information does not exist.

An equilibrium with imperfect information brings also a contradiction: if entrepreneurs cannot discriminate between different states of nature the supply schedule S is the same over these states. But consumers have different demand schedules and the equilibrium price varies over the states of nature, providing an instrument to entrepreneurs to discriminate between the state of nature.

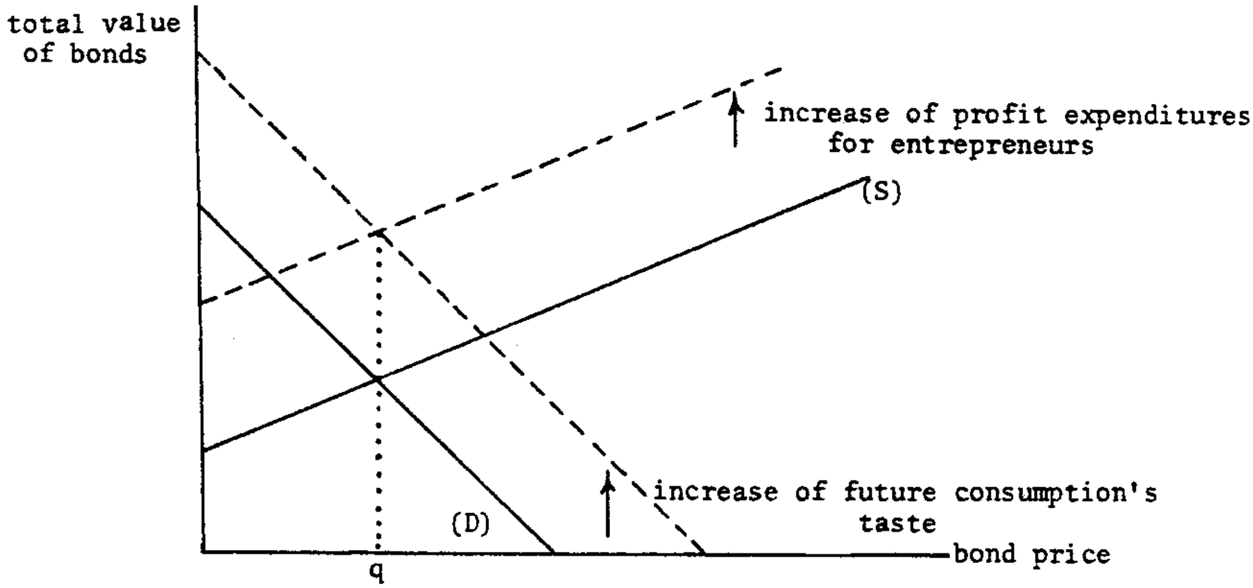

FIGURE 3

The non-existence of equilibrium is now proven for a special version of the model presented in the text. There is only one produced good x_1 , produced with the technology (T1). The consumers are represented by a single individual with the utility function:

$$U = \frac{1}{1-\sigma}(1-\alpha-\gamma)(1-\ell_1)^{1-\sigma} + \alpha \log x_1 + \gamma \log(1-\ell_2) \tag{43}$$

There are two possible values for the parameter α , α_1 and α_2 respectively $(\alpha_1 < \alpha_2)$, which determine the state of nature. The price of the produced good p_1 , in the second period, is equal to

$$p_1 = \frac{\alpha}{\alpha+\gamma}\left(\frac{1+z}{x_1}\right) , \tag{44}$$

where z is equal to ℓ_1/q (see expression (16) in the text).

The value of the supply of bonds by the entrepreneur is equal to the labor demand:

(45) $$\ell_1^D = qa(1+z)E\left(\frac{\alpha}{\alpha+\gamma}\right) ,$$

where $E\left(\frac{\alpha}{\alpha+\gamma}\right) = \frac{\alpha}{\alpha+\gamma}$ if there is perfect information. Since $z = \ell/q$, the entrepreneur is on its demand schedule when

(46) $$\begin{cases} \ell = \dfrac{qaE\left(\frac{\alpha}{\alpha+\gamma}\right)}{1 - aE\left(\frac{\alpha}{\alpha+\gamma}\right)} \\ \ell = \dfrac{qa\alpha}{(1-a)\alpha + \gamma} , \quad \text{if there is perfect information.} \end{cases}$$

The graph of this function is represented by the schedule (S) in Figure 3. It shifts upwards with the value of α.

The labor supply is determined by the first order condition of the consumer (similar to condition (12))

(47) $$\frac{1-\alpha-\gamma}{(1-\ell)^\sigma} = \frac{\alpha+\gamma}{\ell+q} .$$

The graph of ℓ as a function of q is represented by the schedule (D). For a proper choice of the parametric values there exist two values α_1 and α_2, such that the equilibrium price is the same when entrepreneurs have perfect information, which proves the existence of the case of Figure 3. The non-existence of equilibrium follows from the previous discussion. The values of α_1 and α_2 are found by substituting for ℓ in (47) the expression (46) and solving the equation in α for the values: $a = .9$, $\gamma = .51$, $q = 1$, $\sigma = 2$.

2. Multiple Equilibria and Pareto Comparison

The model presented in the text generates two equilibria when its parametric values are modified. The first equilibrium reveals exactly the state of nature. The second equilibrium is of the same type as the one considered before, and does not discriminate between states 1 and 2, in the first period.

The technologies (T1) and (T2) are now represented by the functions

$$y_1 = \ell_{11}^{a} \qquad \text{(T1)}$$
$$y_2 = \ell_{12}^{b}, \quad b < a \qquad \text{(T2)}$$

The technology (T3) is unchanged. For simplicity there is only one consumer, behaving competitively, with the utility function.

$$U = (1-\alpha-\beta-\gamma)\log(1-\ell_1) + \alpha \log x_1 + \beta \log x_2 + \gamma \log(1-\ell_2) .$$

Also, only states 1 and 2 of Table 1 are retained since information is perfect in state zero.

The labor supply schedule is given in (14),

$$\ell^S = \alpha^* + \beta^* + \gamma - q(1-\alpha^*-\beta^*-\gamma) ,$$

and is the same in states 1 and 2. The labor demand is obtained from (21) and (27), and is equal to

$$\ell^D = (a\alpha^* + b\beta^* \pm (a-b)\varepsilon)(1+q) , \quad \text{under perfect information,}$$
$$= (a\alpha^* + b\beta^*)(1+q) , \quad \text{if entrepreneurs cannot discriminate between states 1 and 2.}$$

There are two equilibria. With perfect information, the bond price is equal to

$$q = \frac{(1-a)\alpha^* + (1-b)\beta^* + \gamma \pm (a-b)}{1 - [(1-a)\alpha^* + (1-b)\beta^* + \gamma \pm (a-b)\varepsilon]}$$

It is higher in state 1 than state 2, and therefore reveals the state of nature to entrepreneurs. There is also an equilibrium where the bond price is the same in the two states, and equal to

$$q = \frac{(1-a)\alpha^* + (1-b)\beta^* + \gamma}{1 - [(1-a)\alpha^* + (1-b)\beta^* + \gamma]}$$

This equilibrium is the extension of the equilibrium considered in the text to the case $a \neq b$. Note that it is more "natural" than the equilibrium with perfect information since the labor supply function is the same in both states and there is no mechanism, in the present framework, to induce different bond prices in the two states of nature.

The Pareto comparison is especially simple when the difference between a and b is small. In this case the two equilibria are approximated by those described in the text. Also there is here a single (representative) consumer. It follows immediately from Proposition 1 that the fully revealing equilibrium dominates the equilibrium with imperfect information, in the sense of Pareto.

The property of multiple equilibria seems to be more robust then the property of non-existence considered above. More specifically, the present framework of a discrete set of states of nature is only a simplification of a model with one price and a continuum of states of nature with a dimension two. It would be possible to find parametric values in this more general model such that the property of multiple equilibria holds under small perturbations of the parameters.

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