COWLES FOUNDATION FOR RESEARCH IN ECONOMICS

AT YALE UNIVERSITY

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New Haven, Connecticut 06520

COWLES FOUNDATION DISCUSSION PAPER NO. 694

CURRENT ACCOUNT, EXCHANGE RATE, AND MONETARY DYNAMICS IN A STOCHASTIC EQUILIBRIUM MODEL

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March 2, 1984

October 1983

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Abstract: We construct a simple stochastic open-economy macro-economic model from the decision rules of rational, optimizing agents, solving explicitly for the relationship between the model's deep parameters, and the variance-covariance matrix of equilibrium returns on domestic and foreign assets. We use the model to study the dynamic relationship between exchange rate changes and current account flows in response to domestic monetary shocks, to establish the conditions under which exchange rate depreciations accompany current account deficits, and to investigate the link between time preference, risk aversion, and the dynamic exchange rate and current account response to monetary disturbances. The model generates current account time series which exhibit persistent deviations from balance, even for the special in which domestic and foreign shocks are purely transitory.

In the early stages of this research I had the benefit of reading an unpublished paper by Carl Walsh (1981) in which he examines the influence of money supply rules on the portfolio decisions of agents. I would like to thank Ben Friedman and Tom Sargent for many useful discussions and Virginia Mellinger for excellent editorial advice. The detailed comments of an anonymous reference were invaluable in the revision of this paper. Conversations with Danny Quah, Paul Richardson, Olivier Blanchard and the comments of members of money workshops at Boston College and Harvard have also been of great help. The usual disclaimer applies.

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0: INTRODUCTION

Recent research in open economy macro-economics has highlighted the link between the saving and portfolio choices of a country's residents and fluctuations in that country's exchange rate and balance on current account. Contributions to this so-called portfolio balance approach to current account and exchange rate and determination include recent papers by Dornbusch and Fisher (1980), Rodriguez (1980), Obstfeld (1981), and Branson (1981), and Kouri (1982).

In this paper we construct a dynamic, stochastic rational expectations model which fills an important gap in the portfolio balance approach literature. Existing portfolio balance models are deterministic and, excepting the work by Obstfeld (1981), aggregative. As such, they cannot be used to study relationships between the preferences, endowments, and technologies of an economy, the consumption and portfolio decisions of its agents, and the resulting equilibrium time series behavior of its current account and exchange rate. To study these relationships, we use the consumption and portfolio decision rules of rational, optimizing agents to construct a simple stochastic macroeconomic model which generates current account and exchange rate time series that are consistent with a rational expectations monetary equilibrium. In particular, we use the restrictions implied by market clearing and rational expectations to identify the relationships between the "deep" parameters - those which chacterize the preferences, policies and stochastic processes endogenous to the model - and the coefficients of the macro-economic model. Once identified, the model is used to answer theoretical and policy questions such as:

(1) What is the dynamic relationship between exchange rate changes and current account flows in response to domestic monetary shocks?

(2) Under what conditions do exchange rate depreciations accompany current account deficits?

(3) How do the rate of time preference and attitude towards risk of an economy's agents influence the dynamic behavior of its current account and exchange rate in stochastic equilibrium?

The model we use to answer these questions is complementary to Obstfeld's (1981) in that we depart from the aggregative, static approach prevalent in the recent international finance literature by basing consumption, saving, and portfolio behavior on explicit, intertemporal optimization. However, unlike Obstfeld who studies a non-stochastic, infinite-horizon Sidrauski (1967) economy, we construct a stochastic, overlapping generations version of the small open economy introduced in the influential paper of Kouri (1976). Young agents can save by carrying domestic money and foreign currency bonds into old age. The supply of domestic money and the foreign price level are stochastic, so that the yields on the two assets are random. Agents are averse to variance in their second period consumption which leads them to hold both assets in their portfolios. Asset demands and the consumption and savings decisions depend upon agents' subjective distributions of asset returns. We close the model by assuming that expectations are fully rational, by which we mean that the first and second moments of the asset price vector implied by the model are identical to those moments anticipated by agents when formulating their optimal decision rules.

This paper exploits the the important feature of overlapping generations models that non-proportional monetary injection and money financed fiscal expenditures are, in general, not neutral. In our set-up this implies that changes in the stock of money, whether anticipated or not, can influence the real equilibrium saving and portfolio decisions of agents. Futhermore, we are able to show how the co-movement between the exchange rate and the current account in response to a given monetary shock depends upon the interaction between the individual's motive to tilt consumption away or towards the present and his hedging motive to reduce the variance of consumption when old.

Although there is a theoretical presumption that current account deficits should be accompanied by exchange rate deprecitations, the empirical evidence on this relationship is somewhat mixed. In one of the first studies on this subject Sachs (1982) finds that[1]

> (a)mong the large (OECD) countries, there is a clear pattern of nominal and real exchange rate appreciation among surplus countries . . . and depreciation among the rest. . . Interestingly, the pattern seems to break down for many of the smaller countries where we observe large deficits together with nominal and real exchange rate appreciations. . . There are as many observations of deficits with appreciation as with depreciation for the smaller countries.

Multivariate time series work by Branson (1981) and Loopesko (1982) confirms a dynamic relationship between exchange rate depreciations (appreciations) and current account deficits (surpluses) for Japan, Germany, and the U.S. While Branson identifies low-order stochastic difference equations for the detrended current accounts of these three countries, he finds for Britain that current account deviations from trend are white noise.

This paper represents a first attempt at studying the dynamic interrelationships among the current account, the exchange rate, and the money supply within an optimizing, dynamic stochastic equilibrium framework. As such it exploits particularly simple parameterizations of preferences and policies to obtain concrete answers to specific questions. The simplicity of the model is necessary since we wish to solve explicitly for the relationships between the model's deep parameters and the coefficients of the macro-economic model under the restriction, uncommon in the aggregative rational expectations literature, that the first and second moments generated by the reduced form are identical to those moments which are anticipated by agents in calculating their optimal rules. The plan of the paper is as follows. In section I, we set up the model. In section II, we solve for the model's rational expectations monetary equilibrium. We then use the restrictions implied by rationality to identify the coefficients of the macro-economic model studied in section III. In section III, we study the dynamic and steady-state behavior of the current account and exchange rate time series generated by the model. Section IV provides some concluding remarks.

I: THE MODEL

Preliminaries

We construct an overlapping generations version of the model introduced in Kouri (1976). The economy is small in that its citizens take as given the stochastic process which governs $\bar{p}_t$, the goods price of foreign currency. It is open in the sense that its residents can freely import (export) units of the one good by issuing (purchasing) claims denominated in the foreign currency on future domestic (foreign) output. The nominal rate of interest on these traded securities, i, is assumed constant. Agents live two periods. Agents born in period t are "young" in t and "old" in t+1. Agents receive a perishable endowment w in each period. For simplicity, we consider a stationary economy in which there is neither population nor endowment growth, and we set the population of each generation equal to one. Domestic money is issued by the government to finance fiscal deficits, and is held entirely by domestic residents. Since there is only one - costlessly traded - good, it is natural to require that the goods market arbitrage relationship $p_t = e_t\bar{p}_t$ holds in each period, where p_t is the goods price of domestic money and e_t is the foreign currency price of domestic currency, the exchange rate. Note that a depreciation of the exchange rate implies a fall in e_t. Since we assume that foreigners do not hold domestic money, the price of domestic money equalizes the supply of money and the demand for it by domestic residents.

Agents

We assume the young act so as to maximize a time-separable function of life- time consumption,

$$u(c_t^y;t) + \alpha u(c_{t+1}^o;t+1); \tag{1}$$

subject to the following budget constraints,

$$c_t^y = w - p_t m_t - \bar{p}_t \bar{d}_t; \tag{2}$$

$$c_{t+1}^o = w + p_{t+1} m_t + \bar{p}_{t+1} r \bar{d}_t. \tag{3}$$

c_t^y is consumption by the young in time t, c_{t+1}^o is consumption by the old in time t+1, $\alpha \equiv 1/(1+\delta)$ where δ is the positive rate of time preference, $r \equiv 1+i$ is the gross nominal rate of interest on traded securites $\bar{d}_t$, and m_t is the stock of domestic money held at time t.

To obtain tractable decision rules, we shall assume the u( ) to be linear in the mean and variance of consumption conditional on information available when young. This yields an objective function of the form:

$$c_t^y + \alpha_t E(c_{t+1}^o) - \eta_t \mathrm{var}(c_{t+1}^o); \tag{4}$$

where, ${}_tE(c_{t+1}^o)$ and ${}_t\mathrm{var}(c_{t+1}^o)$ are the mathematical mean and variance, respectively, of time t+1 consumption by generation t conditional on all information available at time t, and η is the marginal disutility when young of variance in second period consumption.

Formally, the agent's problem may be expressed as

$$[P] \qquad \max_{\{c_t^y,\, c_{t+1}^o,\, m_t,\, \bar{d}_t\}} \quad c_t^y + \alpha_t E(c_{t+1}^o) - \eta_t \text{var}(c_{t+1}^o) \tag{4}$$

s.t.

$$c_t^y = w - p_t m_t - \bar{p}_t \bar{d}_t, \tag{2}$$

$$c_{t+1}^o = w + p_{t+1} m_t + \bar{p}_{t+1} r \bar{d}_t, \tag{3}$$

$$c_t^y \geq 0, \tag{5}$$

$$c_{t+1}^o \geq 0. \tag{6}$$

We begin by solving the <u>unconstrained</u> version of [P] in which the restrictions $c_t^y \geq 0$ and $c_{t+1}^o \geq 0$ are not imposed. Substituting (2) and (3) into (4) and differentiating with respect to m_t and $\bar{d}_t$, the first-order conditions for the unconstrained problem yield the following optimal portfolio rules:

$$\Psi_t^{\bar{d}} \equiv \bar{d}_t = \chi(\alpha r_t \bar{p}_{t+1} - \bar{p}_t)/\kappa \;-\; \beta(\alpha_t p_{t+1} - p_t)/\kappa, \tag{7}$$

$$\Psi_t^m \equiv m_t = (\alpha_t p_{t+1} - p_t)/\kappa \;-\; \beta(\alpha r_t \bar{p}_{t+1} - \bar{p}_t)/\kappa, \tag{8}$$

$$\text{where} \quad \chi \equiv {}_t\sigma^2_p / {}_t\sigma^2_{\bar{p}} r^2; \quad \beta \equiv {}_t\sigma_{p\bar{p}} / {}_t\sigma^2_{\bar{p}} r \;; \quad \kappa \equiv \eta({}_t\sigma^2_p - r^2\beta^2\, {}_t\sigma^2_{\bar{p}}),$$
$$\text{and} \quad {}_tx_{t+1} \equiv {}_tEx_{t+1} \;;\; \sigma^2_x \equiv {}_t\text{var}(x_{t+1}). \tag{9}$$

In an appendix we derive the conditions under which κ is positive - it is non-negative by the Cauchy-Schwartz inequality - and in what follows we assume this to be the case. Noting that the expected return on foreign assets is $r_t\bar{p}_{t+1}/\bar{p}_t$ and that the expected return on domestic money is ${}_tp_{t+1}/p_t$, we see the elasticity of demand for either asset with respect that asset's own return is positive. The cross-return elasticities may be either positive of negative, depending upon the young's anticipation of the covariance between the returns on domestic and foreign assets. When β is positive, the young expect the returns on the two assets to covary positively. A rise in the expected return on money raises the demand for that asset, and because its return is positively correlated with that of the foreign asset, holdings of the latter are reduced to hedge against the increased variance in second period consumption brought about by the higher money holdings. If the young expect the returns to covary negatively, the hedging of second period consumption variance is accomplished by channeling additional savings into foreign bonds. Thus, asset demands depend not only on asset prices and expected returns, but also upon the anticipated variance and covariance of next period's asset prices. Note that for the mean-variance preferences we have assumed, the agent's demands for these risky assets do not depend on his lifetime endowments, a feature which [P] shares with more general savings-portfolio choice problems in which the u( ) exhibit constant absolute risk aversion.[2]

Substituting the asset demands (7) and (8) into the budget constraints (2) and (3), we obtain the optimal consumption functions for the unconstrained problem:

$$c_t^y = w - p_t \gamma_t^m - \bar{p}_t \gamma_t^{\bar{d}}; \tag{10}$$

$$c_{t+1}^o = w + p_{t+1}\gamma_t^m + \bar{p}_{t+1}\gamma_t^{\bar{d}}. \tag{11}$$

Now for arbitrary w and stochastic processes p_t and $\overline{p}_t$, optimal consumption in the unconstrained problem is not necessarily non-negative. However, if we restrict our attention to stationary processes for home and foreign prices[3], then the fact that the asset demands are independent of the endowment sequence implies that, for w large enough, the optimal consumption functions (10) and (11) for the unconstrained problem also satisfy the non-negativity constraints $c_t^y \geq 0$ and $c_{t+1}^o \geq 0$. In what follows, we shall asume agents to be sufficiently well endowed so that we may work with the asset demands (7) and (8) and the consumption functions (10) and (11).

The other agent in the economy is the government. Governmment consumption c_t^g is just the sum of its endowment w_g and the goods purchased from (sold to) the young in exchange for money,

$$c_t^g = w_g + p_t(m_t^s - m_{t-1}^s). \tag{12}$$

II: SOLVING THE MODEL

Stochastic Specification

To solve the model it is necessary to specify the stochastic processes which govern the price of foreign money and the supply of domestic money. For simplicity, we shall assume both to evolve according to a first-order autoregression:

$$\overline{p}_t = \mu_{\overline{p}}(1-\omega) + \omega\overline{p}_{t-1} + o_t; \tag{13}$$

$$m_t^s = \mu_m(1-\phi) + \phi m_{t-1}^s + \zeta_t; \tag{14}$$

where we shall assume $-1 \leq \omega \leq 1$ and $-1 \leq \phi \leq 1$. Here o and ζ are white noises with variances σ_o^2 and σ_ζ^2 respectively, and with covariance $\sigma_{o\zeta}$.

Calculating a Rational Expectations Monetary Equilibrium

In an equilibrium, the demand for domestic money balances must equal the given supply. Since foreigners do not hold domestic balances, the price of domestic money adjusts in each period to equalize the supply of money and the demand for it by domestic residents. Using the derived demand for domestic balances - equation (8) - and the equilibrium condition $m_t^s = m_t$, we can express the equilibrium price of money as

$$p_t = \alpha_t p_{t+1} - \kappa m_t^s + \beta(\bar{p}_t - \alpha r_t \bar{p}_{t+1}). \tag{15}$$

We see that the equilibrium price of money - the inverse of the equilibrium price level - depends upon the supply of money and the price of the foreign bond, the expected prices of domestic and foreign assets next period, and the variances of and covariance between asset prices next period conditional on current period information. In a rational expectations equilibrium, the conditional first and second moments of the stochastic process which governs the equilibium price of domestic balances must be equal to those moments anticipated by agents when formulating their optimal portfolio, savings, and consumption policies. We can use these second moment restrictions to identify the relationships between the model's deep parameters and coefficients of the macromodel which describes the stochastic equilibrium of the economy.

Our strategy is as follows. We consider the class of solutions for which the rationally anticipated second moments ${}_t\sigma^2_p$, ${}_t\sigma^2_{\bar{p}}$, and ${}_t\sigma_{p\bar{p}}$ are time invariant. Following McCafferty and Driskill (1980), we then solve (15) by treating the second moment terms - and thus κ and β - as parameters. We then impose the restrictions that the conditional second moments of the "first moment rational" solution for p_t be consistent with κ and β, which are functions of these second moments as defined by (9).

We begin by studying the "first moment rational" solution to (15). Treating κ and β as parameters and using (13) and (14), it is straightforward to solve the first-order difference equation (15) forward. We obtain

$$p_t = \mu_p + \theta(\overline{p}_t - \mu_{\overline{p}}) - \Gamma(m_t^s - \mu_m); \tag{16}$$

where,

$$\mu_p \equiv [\beta\mu_{\overline{p}}(1-\alpha r)-\kappa\mu_m]/1-\alpha; \quad \theta \equiv \beta(1-\alpha r\omega)/1-\alpha\omega; \quad \Gamma \equiv \kappa/1-\alpha\phi \ . \tag{17}$$

The price of domestic money evolves according to a stationary stochastic process with mean μ_p. Using (16) and the goods market arbitrage relationship $p = e\overline{p}$, we obtain the folowing expression for the equilibrium exchange rate:

$$e_t = \theta + T/\overline{p}_t - \Gamma m_t^s/\overline{p}_t \tag{18}$$

where,

$$T \equiv [\mu_p + \Gamma\mu_m - \theta\mu_{\overline{p}}].$$

Positive mean deviations in the supply of domestic money require negative mean deviations in the price of domestic balances and exchange rate depreciation to equalize supply and the demand for money by the domestic young. The influence of mean deviations in the price of the foreign asset depends upon the anticipated covariance between the returns on the two assets, as well as rate of time preference α, the rate of interest r, and the rate at which such deviations are expected to decay, ω.

Up to this point, we have implicitly assumed that the equilibrium price of money is always positive. Under what conditions is this warranted? From inspection of equation (16), it is clear that a necessary condition for such a monetary equilibrium is that μ_p be positive. Using the definition of β and the fact that $1 - \alpha r > (<) \ 0$ as $i < (>) \ \delta$, we can state the following

> Result: A necessary condition for a monetary equilibrium is that ${}_t\sigma_{p\bar{p}}(i-\delta) < 0$.

This is an intuitive condition which highlights the interaction between the agent's motive to tilt his consumption profile and his hedging motive to reduce the variance of consumption when old. The condition is most easily understood for the case in which the price of foreign balances follows a random walk, $\omega = 1$. When $i > \delta$, the young want to tilt consumption towards the next period, which they can do by purchasing interest earning claims on next period's foreign output. However, if the return on domestic balances covaries negatively with the real return on the foreign bond, the young will also purchase domestic money at a positive price to reduce the variance of second period consumption:

$$ {}_t\mathrm{var}(c^o_{t+1}) = {}_t\sigma^2_p m^2_t + {}_t\sigma^2_{\bar{p}}(rd_t)^2 + 2{}_t\sigma_{p\bar{p}}(rm_t\bar{d}_t). \tag{19} $$

But this is just the condition stated above. When $i < \delta$, the young want to tilt consumption towards the present, and will do so by issuing claims denominated in the foreign currency on next period's endowment w. However, if the returns on domestic and foreign assets covary positively, the young will be willing to purchase domestic money at a positive price - partially or even completely offsetting the purchase by additional foreign borrowing - to reduce the variance of consumption when old.

Now the conditional second moments of p_t and $\bar{p}_t$ depend upon the β and κ, which themselves depend upon agents' assessments of these second moments as defined by (9). In a rational expectations equilibrium, agents' assessments must be equal to the actual conditional first and second moments of p_t and $\bar{p}_t$ generated by the model. In an appendix, we show that in a rational expectations monetary equilibrium, it must be the case that:

$$\kappa = \frac{(1-\alpha\phi)^2}{\eta\sigma^2_\zeta(1-\rho^2)}; \tag{20}$$

$$\beta = \frac{(1-\alpha\phi)(1-\alpha\omega)\sigma_{o\zeta}}{\eta\sigma^2_o\sigma^2_\zeta(1-\rho^2)(1-r)}. \tag{21}$$

Here $\rho \equiv \sigma_{o\zeta}/\sigma_o\sigma_\zeta$ is the correlation between shocks to domestic money and the goods price of foreign currency. Equations (20) and (21) use the restrictions implied by rationality and equilibrium to identify the (non-linear) relationships between β and κ and the model's deep parameters. This identification is important because it enables us to study the influence of time preference and the aversion to consumption variance on the dynamic exchange rate and current account responses to domestic monetary shocks.

Using (21) and the definition of β, we see that

$$_t\sigma_{p\bar{p}} = \frac{r(1-\alpha\phi)(1-\alpha\omega)\sigma_{o\zeta}}{\eta\sigma^2_\zeta(1-\rho^2)(1-r)}; \tag{22}$$

where recall that $\sigma_{o\zeta}$ is the covariance between unanticipated disturbances in domestic money and the price of foreign currency. Equation (22) shows that the domestic government can directly influence the sign and magnitude of $_t\sigma_{p\bar{p}}$ through a policy in which the supply of domestic money responds to unanticipated shocks in the price of foreign currency. For example, when $\sigma_{o\zeta} < 0$, an unanticipated fall in the price of foreign currency is met by an increase in the supply of domestic money. Such a policy induces a positive covariance between the returns on domestic and foreign assets, a covariance which is necessary for a monetary equilibrium in economies in which $i < \delta$. Conversely, for economies in which $i > \delta$, a monetary equilibrium requires the government to meet an unanticipated fall in the price of foreign currency with a contraction of the domestic money supply. Such a $\sigma_{o\zeta} > 0$ policy induces a negative covariance between the returns on the two assets, a covariance which encourage the young to hold domestic money in their portfolio to reduce the variance of second period consumption. In order to distinguish between that part of a given money shock which occurs in response to an innovation in $\overline{p}_t$ and that part which is independent of the o_t, we shall sometimes find it useful to work with the following simple specifications:

$$\zeta_t = \varepsilon_t + o_t, \qquad \text{when } i > \delta \ ; \tag{23}$$

and

$$\zeta_t = \varepsilon_t - o_t, \qquad \text{when } i < \delta \ ; \tag{24}$$

where ε_t is a white noise which is independent of the o_t. Note that (23) and (24) imply that $\sigma_{o\zeta} = \sigma^2_o$ and that $\rho^2 = \sigma^2_o/(\sigma^2_o + \sigma^2_\varepsilon)$

<u>The Equilibrium Current Account</u>

Since there is no investment in a perishable endowment economy, real domestic absorbtion a_t is just the sum of private and government consumption. Using (10), (11), and (12) we see

$$a_t = y + p_t[(m^s_t - m_t) - (m^s_{t-1} - m_{t-1})] - \overline{p}_t(\overline{d}_t - r\overline{d}_{t-1}); \tag{26}$$

where $y = 2w + w_g$ is gross domestic product. The equilibrium current account x_t is the difference between national income and equilibrium absorbtion. National income for this economy at time t is $y + i\overline{p}_t\overline{d}_{t-1}$ - gross domestic product plus the service income (expense) earned on foreign claims. Using the fact that in equilibrium $m_t^s = m_t$, we can express the equilibrium current account as

$$x_t = \bar{p}_t(\bar{d}_t - \bar{d}_{t-1}). \tag{27}$$

In what follows, we shall find it useful to work with the current account in nominal terms, as expressed in units of the foreign currency

$$\bar{x}_t \equiv x_t/\bar{p}_t = \bar{d}_t - \bar{d}_{t-1}. \tag{28}$$

Since investment is zero by assumption, the current account is equal to domestic saving. In this stationary, overlapping generations model, domestic saving is just equal to the difference between the purchases (issues) of foreign claims by the current young and the stock of such claims acquired (issued) by those young last period.

It is now possible to derive an expression for the equilibrium current account. Using equation (8), we see that in equilibrium

$$(\alpha_t p_{t+1} - p_t)/\kappa = m_t^s + \beta(\alpha r_t \bar{p}_{t+1} - \bar{p}_t)/\kappa. \tag{29}$$

Substituting (29) into (7), simplifying, and using (28), we obtain an expression for the equilibrium current account

$$\bar{x}_t = \Omega(\bar{p}_t - \bar{p}_{t-1}) - \beta(m_t^s - m_{t-1}^s); \tag{30}$$

where,

$$\Omega \equiv (\alpha r \omega - 1)/\eta \sigma_o^2 r^2. \tag{31}$$

The influence of money on the equilibrium current account depends critically on the equilibrium conditional covariance between the returns on domestic and foreign assets. If the young expect the returns to covary positively, an increase in money leads them to increase equilibrium foreign borrowings - thus shinking the economy's current account surplus - in an effort to reduce the variance of second period consumption. If β is negative, the hedging of consumption variance is accomplished by channeling additional savings into foreign assets, improving the the economy's current account position. In general, an increase in the price of the foreign asset reduces the lending - increase the borrowing - abroad by the domestic young, reducing the economy's current account surplus.

III: CURRENT ACCOUNT, EXCHANGE RATE, AND MONETARY DYNAMICS

The Macro-Economic Model

Collecting the stochastic processes (16), (18), and (30) which govern the equilibrium price of domestic money, exchange rate, and current account, and (13) and (14), the laws of motion for the price of foreign currency and domestic money, we construct the following simple stochastic macro-economic model:

$$p_t = \mu_p + \theta(\overline{p}_t - \mu_{\overline{p}}) - \Gamma(m_t^s - \mu_m); \tag{16}$$

$$e_t = \theta + T/\overline{p}_t - \Gamma m_t^s/\overline{p}_t; \tag{18}$$

$$\overline{x}_t = \Omega(\overline{p}_t - \overline{p}_{t-1}) - \beta(m_t^s - m_{t-1}^s); \tag{30}$$

$$\overline{p}_t = \mu_{\overline{p}}(1-\omega) + \omega\overline{p}_{t-1} + o_t; \tag{13}$$

$$m_t^s = \mu_m(1-\phi) + \phi m_{t-1}^s + \zeta_t; \tag{14}$$

The dynamic interactions among the time series for money, prices, the exchange rate, and the current account which are consistent with a rational expectations monetary equilibrium are conveniently studied using this system of stochastic difference equations.

Time Series Properties of the Current Account

We begin by investigating the properties of the current account time series generated by the model. Substituting (13) and (14) into (30), and using the specifications given in (23) and (24), we see that the equilibrium current account evolves according to zero-mean arma(2,2) process:

$$(1-\omega B)(1-\phi B)\overline{x}_t = F(B)o_t - \beta(1-\omega B)(1-B)\varepsilon_t; \tag{32}$$

where,

$$F(B) \equiv [\Omega(1-\phi B) + |\beta| (1-\omega B)](1-B). \tag{33}$$

Here B is the "backshift" operator $Bx_t \equiv x_{t-1}$. Consistent with the time series evidence reported in Branson (1981), deviations from current account balance do, in general, persist and evolve according to a low-order stochastic difference equation. Two special cases are worth noting. If the price of foreign balances follows a random walk , $\omega = 1$, and mean deviations in domestic money are white noise, $\phi = 0$, then deviations from current account balance still persist, but evolve according to an ma(1) of the form

$$\overline{x}_t = [\Omega + |\beta|(1-B)]o_t - \beta\varepsilon_t. \tag{34}$$

If both the price of foreign currency and domestic money follow random walks, the current account is white noise

$$\overline{x}_t = [\Omega + |\beta|]o_t - \beta\varepsilon_t. \tag{35}$$

Permanent shocks thus have no impact on the path of the current account expected to prevail in periods subsequent to the disturbances.

The time series properties of the exchange rate, a non-linear function of two stationary processes, are not conveniently summarized analytically. We now study the dynamic co-movement between the exchnage rate and the current account in response to an independent monetary disturbance.

Dynamic Response to an Independent Monetary Shock

We next study the dynamic relationship between exchange rate changes and current account flows in response to domestic monetary shocks. To do this, we calculate and compare the theoretical responses of the exchange rate and the current account to an impulse in ε, a disturbance to domestic money which is independent of shocks to the price of foreign currency assets. Suppose that a some time t=0, the economy is in steady state with

$$p_0 = \mu_p; \quad \bar{x}_0 = 0; \quad \bar{p}_0 = \mu_{\bar{p}}; \quad m_0^s = \mu_m. \tag{36}$$

The equilibrium exchange rate in this state is

$$e_0 = \mu_p/\mu_{\bar{p}}. \tag{37}$$

At time t=0, there is an unanticipated, independent increase in the domestic money supply, $\varepsilon_0 > 0$. Using (14), the autoregressive specification for the domestic money supply, we know that

$$m_t^s - \mu_m = \phi^t \varepsilon_0; \tag{38}$$

that is, mean deviations in the money supply decay at the rate $1-\phi$.

(41) and equation (16), the reduced form expresion for the price of domestic balances, imply the following exchange rate response to an impulse ε_0:

$$e_t - e_0 = -(\Gamma/\mu_{\bar{p}})[m_t^s - \mu_m]. \tag{39}$$

In response to an independent monetary disturbance at t=0, exchange rate deviations from e_0 are proportional to money supply deviations. In particular, an independent τ per-cent positive mean deviation in domestic money at t=0 requires a $\tau\Gamma\mu_m/\mu_p$ per-cent depreciation of the exchange rate at t=0. If there are no other shocks to the economy, then at t=1 the exchange rate begins to appreciate, the extent to which depending upon the parameter ϕ. When $\phi = 0$, money supply deviations are white noise and the exchange rate re-appreciates fully to e_0. When $0 < \phi \leq 1$, mean deviations in the money supply persist and thus do deviations of the echange rate from e_0. As the money supply returns to steady state at the rate $1-\phi$, the exchange rate appreciates, with $1-\phi^n$ per-cent of the appreciation completed after n periods.

Equation (30), the reduced form expression for the current account, and (41) imply the following current account response to an impulse ε_0:

$$\bar{x}_t = \beta(1/\phi - 1)[m_t^s - \mu_m]; \quad t \geq 1,\ \phi \neq 0. \tag{40}$$

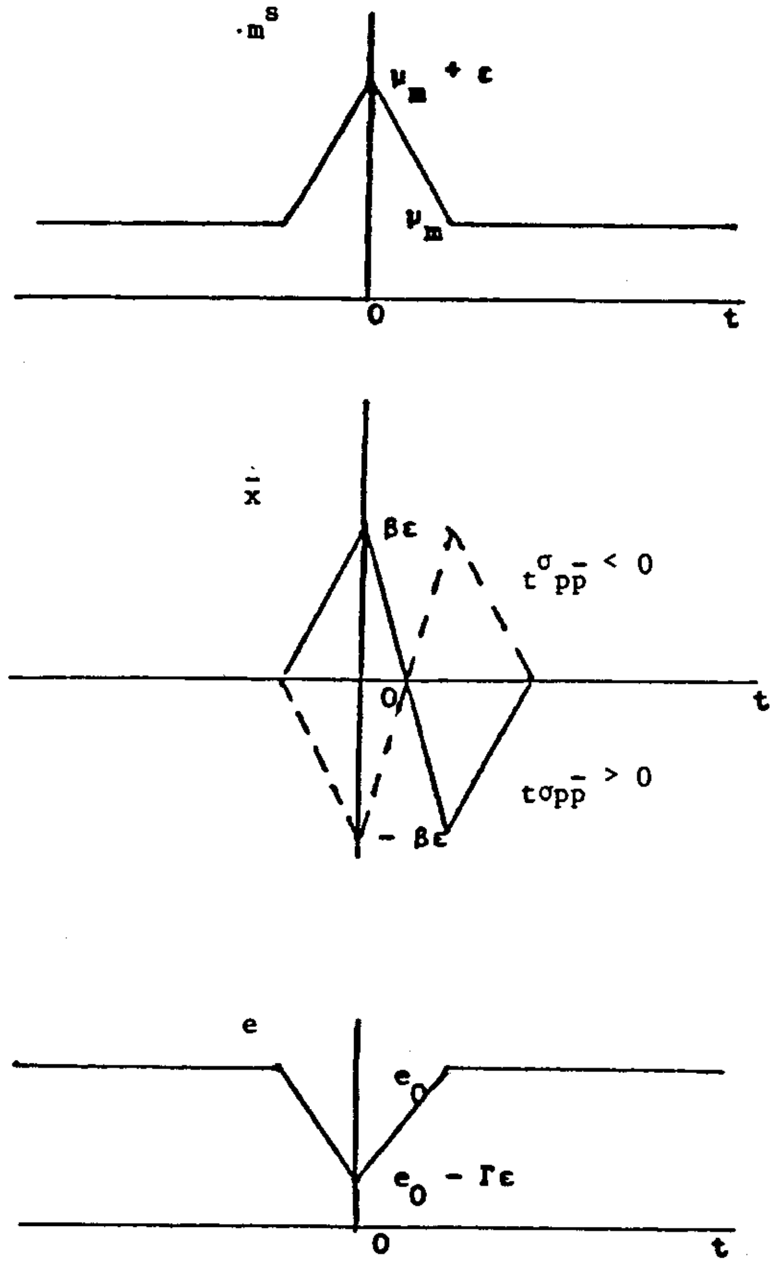
$$\bar{x}_0 = -\beta[m_t^s - \mu_m]. \tag{41}$$

In response to an independent monetary disturbance at t=0, deviations from current account balance are proportional to mean deviations in the supply of domestic money. However, the direction of influence on the equilibrium current account of independent mean deviations in the domestic money supply depends upon the interaction between the motive to tilt the consumption profile and the hedging motive to reduce the variance of consumption when old. As discussed in section II, when $i <$ (>) δ, the equilibrium conditional covariance between asset returns - and thus β - must positive (negative). We conclude that an independent positive monetary shock ε_0 causes a current account <u>deficit</u> in economies for which $i < \delta$ and a <u>surplus</u> in economies for which $i > \delta$. In the former (latter) case, agents reduce the ex-ante variance of second period consumption by offsetting (augmenting) their purchases of domestic balances with increased issues (purchases) of foreign currency denominated debt (assets) which are used to finance present (future) consumption. If there are no further shocks, then in period t=1 the current account is in surplus in economies for which $i < \delta$ and in deficit in $i > \delta$ economies. In the former (latter) case, the t=1 decline in domestic balances held by the young tends to reduce their issues (purchases) of foreign currency debt (assets) used to finance their present (future) consumption. When $0 < \phi \leq 1$, persistent mean deviations in the money supply which result from independent monetary disturbances induce persistent real and nominal current account surpluses (deficits) in $i < (>) \delta$ economies, with both money and the current account returning to the steady state at the rate $1-\phi$. For the case in which money supply deviations are white noise, the t=1 surplus (deficit) in $i < (>) \delta$ economies equals the t=0 deficit (surplus) which results from the positive ε_0 shock. Figures 1, 2, and 3 depict the response of the exchange rate and cxurrent account to a monetary shock for the respective cases in which the shock is transitory, persistent, and permanent.

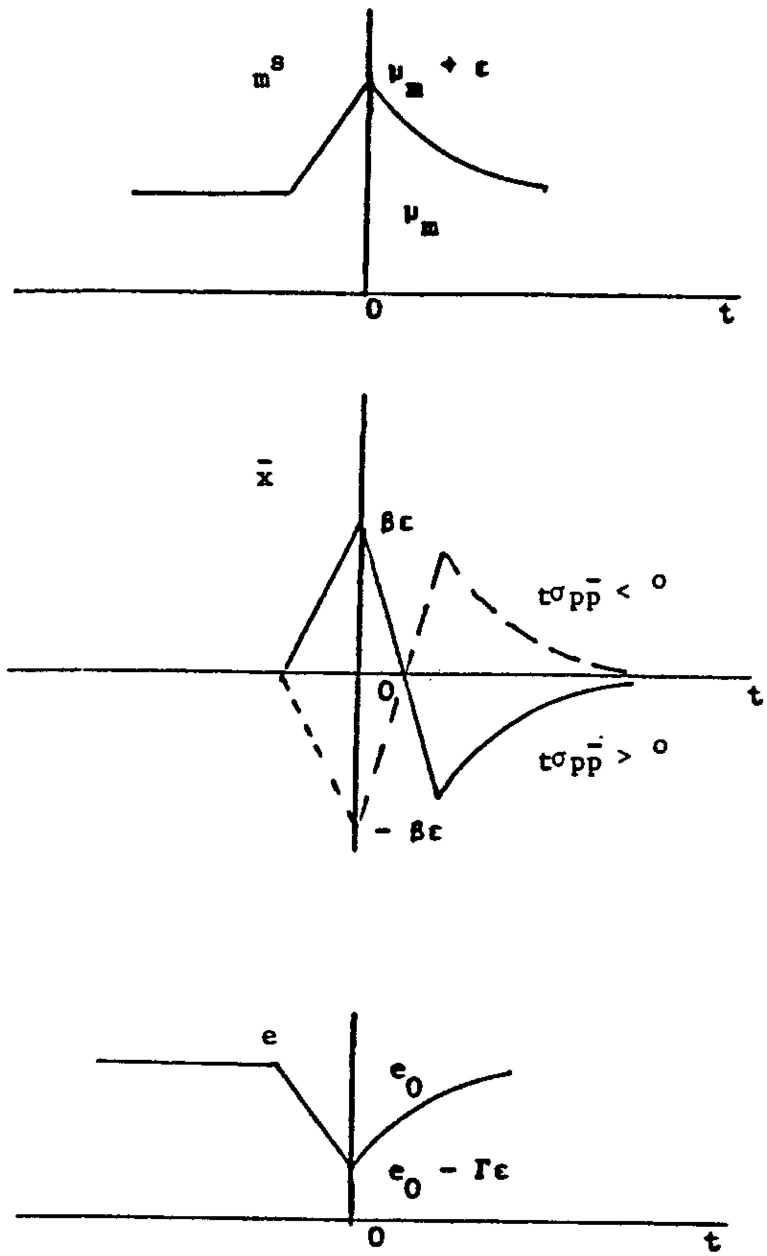
The conditions under which exchange rate depreciations accompany current account deficits in response to independent monetary disturbances are now apparent. Fundamentally, this association requires that the rate of time preference exceed the rate of interest on traded debt. If a monetary equilibrium exists for such an economy, it will exhibit a positive conditional covariance between the returns on domestic and foreign assets. This positive covariance induces a direct relationship between the desired purchases of domestic balances and issues of foreign currency denominated debt by the young to finance present consumption. In these economies then, there is a direct relationship between independent changes in nominal money balances and real domestic absorbtion.

Influence of Time Preference and Risk Aversion

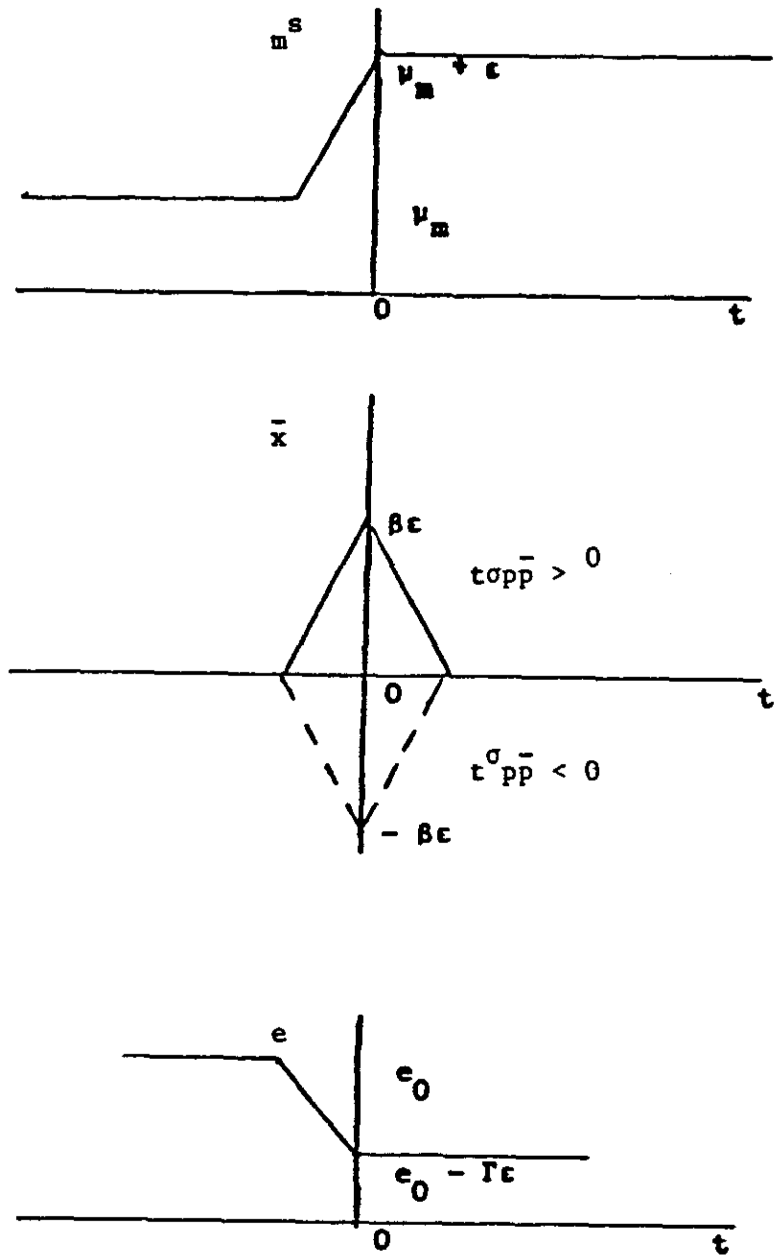
As we have seen, the dynamic exchange rate and current account responses to independent monetary disturbances are proportional to Γ and β, respectively.



<u>Figure One:</u> The dynamic response of the current account and exchange rate to a transitory monetary shock. Note that the current account response persists. The co-movement between the current account and exchange depends on the equilibrium conditional covariance between asset prices derived in the text.



<u>Figure</u> <u>Two</u>: The dynamic current account and exchange rate response to a persistent monetary shock.



<u>Figure Three</u>: The dynamic response of the current account and exchange rate to a permanent monetary shock. Note that the current account response is transitory.

Using (20), (21), (23), (24), (39), and (40), it is easy to show that in a rational expectations monetary equilibrium, it must be the case that

$$\Gamma = \frac{(1-\alpha\phi);}{\eta\sigma^2_\varepsilon} \tag{42}$$

$$\iota\beta \,r = \frac{(1-\alpha\phi)(1-\alpha\omega)}{\eta\sigma^2_\varepsilon(r-1)}. \tag{43}$$

We see immediately that the exchange rate and current account responses to a given independent monetary disturbance are more pronounced the smaller the aversion to consumption variance or the higher rate of time preference of the economy's agents. The key to this current account result is that the absolute value of the equilibrium conditional covariance between p_{t+1} and p_{t+1} is inversely related to η and α. This implies that the positive relationship between desired purchases of domestic balances and issues (purchases) of traded debt (assets) used to finance present (future) consumption is stronger in economies with a smaller aversion to consumption variance or a higher rate of time preference. The inverse relationship between the exchange rate response to an independent monetary disturbance and the parameters η and α derives from the fact that κ, and in particular ${}_t\sigma^2_p$, are inversely related to these preference parameters. From equation (8), the derived demand for domestic balances, we see that

$$\partial m_t/\partial p_t = -1/\kappa. \tag{44}$$

The effect of a given price change on money demand is inversely related to κ, implying that a greater price and exchange rate adjustment are required to equalize the demand for domestic balances with a given change in money supply the larger is κ. κ captures the extent to which money balances are a good hedge against consumption variance. The higher is ${}_t\sigma^2_p$ relative to ${}_t\sigma_{p\bar{p}}$, the smaller is the reduction in consumption variance accomplished by saving an additional dollar.

IV: CONCLUDING REMARKS

In this paper, we construct a simple stochastic open-economy macro-economic model from the decision rules of rational, optimizing agents. We solve explicitly for the relationship between the model's deep parameters and the coefficients of the macro-economic model under the restriction that the first and second moments generated by the macro-economic are identical to those moments which are anticipated by agents in calculating their optimal rules. We use the model to study the dynamic relationship between exchange rate changes and current account flows in response to domestic monetary shocks, and established the conditions under which exchange rate depreciations accompany current account deficits. We show that that the exchange rate and current account responses to a given independent monetary disturbance are more pronounced the smaller the aversion to consumption variance and the higher rate of time preference of the economy's agents. Consistent with the time series evidence reported in Branson (1981), the model predicts that deviations from current account balance do, in general, persist and evolve according to a low-order stochastic difference equation, even for the special case in which domestic and foreign shocks are purely transitory.

In an important recent paper, Kareken and Wallace (1981) study a deterministic, two-country overlapping generations model of currency substitution. The key implication of their model, that equilibrium exchange rates are constant but indeterminant, does not carry over to this model because of our assumption of portfolio autarky which prohibits foreigners from holding domestic assets.

In particular, Nickelsburg (1980) has shown that introducing stochastic money supply processes to the original Kareken - Wallace model does not overturn the indeterminacy result.

APPENDIX

Under the assumption that the second moments are rationally anticipated, we obtain from (16)

$$ {}_t\sigma_{p\bar{p}} = \beta T_t\sigma_o^2 - \Gamma_t\sigma_{o\zeta} \tag{A1} $$

where

$$ T \equiv (1 - \alpha\omega r)/(1 - \alpha r). \tag{A2} $$

From the definition of β we obtain

$$ {}_t\sigma_{p\bar{\bar{p}}} = -\Gamma_t\sigma_{o\zeta} \;/\; (1 - T). \tag{A3} $$

We solve for Γ as follows. Rationality requires

$$ {}_t\sigma_p^2 = (\beta T)^2{}_t\sigma_o^2 + \Gamma^2{}_t\sigma_\zeta^2 - 2\beta T\Gamma_t\sigma_{o\zeta} \tag{A4} $$

Using the definition of κ and (A3), we can re-write (A4) as a quadratic in Γ:

$$ \Gamma^2{}_t\sigma_\zeta + \frac{2T\Gamma^2({}_t\sigma_{o\zeta})}{{}_t\sigma_o(1 - T)} - \frac{\Gamma^2({}_t\sigma_{o\zeta})^2(1-T^2)}{{}_t\sigma_o(1 - T)^2} - \frac{(1 - \alpha\phi)\Gamma}{\eta} = 0. \tag{A5} $$

This quadratric in Γ has two solutions, $\Gamma = 0$ and

$$ \Gamma = (1-\alpha\phi)/\eta_t\sigma_\zeta(1 - \rho^2). \tag{A6} $$

: : $\Gamma = 0$ implies that κ, β, and thus p_t are zero for all t. An equilibrium in which domestic money is without value is thus perfectly consistent with rationality. As our concern is with the co-movement between exchange rates and the current account, we study a stochastic equilibrium with the covariance structure implied by solution (A6).

NOTES

1. Sachs observes that most of the smaller OECD countries maintained some link between the value of their currencies and those of their major trading partners. He recognizes that theories and models of the relationship between nominal exchange rates and the current account assume a flexible exchange rate regime, and conjectures that the failure of this assumption to apply to the smaller OECD countries may explain why their experiences do not accord with the predictions of these theories and models.

2. Since a major objective of this paper is to solve the model under the restriction that the first and second moments it generates are rationally anticipated, it is important that we simplify the non-linearities in the objective function sufficiently to obtain a closed-form solution relating the model's endogenous variables to its "deep" parameters. In a recent article in this Journal, Anderson and Danthine (1981) employ, in their study of optimal hedging, an objective function which is linear in the mean and variance of consumption, as do Feldstein and Green (1980) in their recent model of corporate dividend behavior.

3. In the next section, we derive the conditions under which the equilibrium price of money evolves according to a stationary stochastic process.

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