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NET PRESENT VALUE MAXIMIZATION AND IMPERFECTIONS IN THE LOAN MARKET: A NOTE

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NET PRESENT VALUE MAXIMIZATION AND IMPERFECTIONS

IN THE LOAN MARKET: A NOTE*

by

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There seems to be some confusion in the literature whether or not imperfections in the lending-borrowing market, and in particular differences in lending and borrowing rates, would destroy shareholder unanimity.[1] The purpose of this note is to show that imperfections in this market are not really relevant to stockholder agreement concerning the optimality of the net present value rule. To do this, the paper uses a new criterion guaranteeing unanimity and which basically only requires that investors are sufficiently competitive, i.e. spanning or the notion of firm competition recently proposed by L. Makowiski [9] generally turn out to be unnecessary for shareholder agreement.

In Section 1 a version of the state preference model and some of its properties which are well known, are quickly reviewed. Section 2 deals with the unanimity issue and, as an illustration of the findings, the certainty case is considered in some detail in Section 3.

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[1]For example, H. DeAngelo states in [3] on page 24: "..., Hirshleifer and Fama-Miller have shown that certain types of transactions costs--differential borrowing-lending rates, borrowing costs and certain types of capital rationing--will be associated with unanimity failure."

1. The Model and Some of Its Properties

Consider the traditional state preference model, extending over two-time periods, indexed respectively by 0 and 1. I consumers ($I < \infty$) indexed by i and F firms, indexed by f or g, operate in the economy. At time 1 only one of a finite set E of possible states of the world may occur, but at time 0 it is unknown into which state the economy will move. At time 0 a finite supply of a single perishable commodity basket is available which, at that moment is unable for consumption and investment in firms; at time 1 companies produce an amount y_{1f} of this consumption basket which is determined by f's production technology, input at time 0, y_{0f}, and the state of nature e $(e = 1, \ldots, E)$.

Once the state of nature has been revealed, consumers receive output net of all charges, according to their holdings at time 1. At the outset, investors bring their initial resources of the consumption basket and initial holdings in firms to market. After announcement of the company plans, an exchange equilibrium, characterized by market clearing and consumers maximizing their utility subject to their budget constraints, is attained. Specifically, each individual solves the next problem:

$$\max \; U_i\Big(x_{i0},\, x_{i1}(1), \ldots, x_{i1}(F)\Big)$$

$$\text{S.T.} \quad x_{i0} + \sum_f s_{if} v_f + L_i \leqq \bar{x}_{i0} + \sum_f \bar{s}_{if} \cdot (v_f - \xi_f y_{0f}) + B_i$$

$$x_{i1}(e) \leqq \bar{x}_{i,1} + \sum_f s_{if} \cdot \Big(y_{1,f}(y_{0f}, e) - \xi_f (1 + b_F) y_{0f}\Big)$$

$$+ L_i(1+\ell) - B_i(1+b)$$

$$(x_{i0}, x_{i1}) \in \mathbb{R}_+^{E+1} \;;\quad s_i \in \mathbb{R}_+^F \;;\quad L_i \in \mathbb{R}_+ \;;\quad B_i \in \mathbb{R}_+ .$$

U_i and x_i denote respectively utility and planned consumption by individual i, $\bar{x}_{i0}$ and $\bar{x}_{i1}$ his initial endowment of the consumption basket at time 0 and 1, $\bar{s}_i$ and s_i his initial and equilibrium holdings.

Next to trading in shares (the market value is denoted by v_f), the market mechanism allows riskfree lending and borrowing to take place but at different rates. Costs associated with going short simply rule out short selling of risky assets and are responsible for the borrowing rate (b for individuals and b_F for firms) to be higher than the lending rate ℓ. L_i and B_i denote respectively lending and borrowing by individuals; ξ_f is the debt-input ratio of firms. W.l.o.g. it is supposed that firms do not lend.[1]

One severe objection that could be raised against the present model is its ad hoc formulation of the imperfections in the market, especially the loan market. However no real loss in generality is involved. All that is necessary for the imperfections to arise naturally in the model, is to assume that, next to the states of the world, outputs are also subject to company or project specwfic risks; similarly one could suppose that $\bar{x}_{i1}$ depends on an individual state factor. Then one could invoke adverse selection problems (see for example J. Stiglitz and A. Weiss [14]) in combination with costly risk assessing procedures,

[1] Hence, as the transformation of "deposits" into loans consumers resources, clearing of the lending-borrowing market presumes

$$\sum_i L_i = \sum_i B_i + \sum_i y_{0f}\xi_f + T$$

with T the funds that will be consumed by the market system in the transformation of "deposits" into "loans" when current plans are executed.

to find differences in lending and borrowing rates and/or credit rationing and absence of short selling of shares. Such changes, together with borrowing subject to default risk, do not alter the content of any of the results; they only make the model (and for the present purposes unnecessarily) burdensome to handle.

Recall some well known properties of the precedingly mentioned exchange equilibrium.

First, whenever consumers act as if they cannot affect market prices, utility functions are differentiable and for each individual i at least one optimal consumption pattern (x_{0i}, x_{1i}) over time and states is strictly positive, the following relationship holds:

$$(1) \qquad \sum_e \pi_i\Big((y_{01}, \xi_1), \ldots, (y_{0F}, \xi_F)\Big)\Big[y_{1f}(y_{0f}, e) - y_{0f}\xi_f(1 + b_F)\Big] + \eta_{if} = v_f$$ [1]

with η_{if} a nonnegative Lagrange multiplier associated with the nonnegativity constraint on s_{if} and $\pi_i \in \mathbb{R}^E_{++}$, the vector of individual i's implicit prices, i.e.

$$(2) \qquad \pi_i = \frac{\partial U_i/\partial x_{1i}}{\partial U_i/\partial x_{0i}} .$$

For a particular individual, this vector is the same across all optimal consumption patterns. Because of the η_i-term, pricing in the market need not be additive, i.e. the equilibrium price of a security which time 1 payoff can be represented as a convex linear combination of other

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[1] Since only changes in the policy of firm f will be considered, reference to the plans of all companies other than f will henceforth be dropped from the notation. When no confusion is possible, also reference to the plan of firm f is deleted.

securities' payoffs, may be strictly below the amount one obtains when combining the prices of these other securities in the same way.

In addition, implicit prices also satisfy the condition:

$$(3) \qquad \frac{1}{\sum_e \pi_i(e)} = \kappa_i(1+\ell) + (1-\kappa_i)(1+b)$$

with $0 \leqq \kappa_i \leqq 1$.

κ_i is a ratio of the individual's Lagrange multipliers w.r.t. the nonnegativity conditions on lending and borrowing and has the following interpretation: $-\kappa_i = 1$: the individual is a lender;

$-\kappa_i = 0$: he is a borrower;

$-0 < \kappa_i < 1$: the investor neither lends nor borrows.

Hence κ_i determines the individual's certainty equivalent or marginal rate of substitution of certain consumption between time 0 and 1. As $_i$ is a ratio of Lagrange multipliers, (3) can also be shown to imply that at the optimum simultaneous lending and borrowing by the same individual never occurs.

More details on these issues can be found for example in M. Brennan [1], P. Cheng [2], N. C. Nielsen [11], G. Pye [12], L. Makowski [9].

2. Shareholder Competition and Unanimity

Given a set of firm proposals and the associated exchange equilibrium, a change in company f's plans from $(y_{0f}^{4}, \xi_f^{\ell_1})$ to $(y_{0f}^{2}, \xi_f^{\ell_2})$ is said not to offer a direct consumption benefit to investor i whenever the next condition is satisfied:

$$(4) \quad v_f^{\ell_2} - v_f^{\ell_1} \geqq \sum_e \pi_1(y_{0f}^{\ell_1}, \xi_f^{\ell_1}, e)\cdot\left[y_{1f}^{\ell_2} - y_{0f}^{\ell_2}\xi_f^{\ell_2}(1+b_F) - \left(y_{1f}^{\ell_1} - y_{0f}^{\ell_1}\xi_f^{\ell_1}(1+b_F)\right)\right]$$

It has been shown elsewhere (see [16]) that whenever (4) holds and the assumptions [1], [2], [3] of the subsequent theorem are satisfied--apart from the wealth effect and/or the fact that the proposal may destroy the feasibility of the investor's previously chosen optimal consumption pattern--it does not pay investor i to move away from his prechange income pattern. In other words, the L.H.S. of (4) gives the minimal impact on market value of the switch in plans such that the possible change in the set of available distributions and/or prices associated with this switch, can never make investor i better off.

Intuitively this result is not surprising. For condition (4) basically states that the change in market price is at least as big as the utility the investor would derive from consuming next to his current optimal time 1 income pattern also a small amount of $\left[y_{1f}^{\ell_2} - y_{0f}^{\ell_2}\xi_f^{\ell_2}\cdot(1+b_F) - \left(y_{1f}^{\ell_1} - y_{0f}^{\ell_1}\xi_f^{\ell_1}\cdot(1+b_F)\right)\right]$. In other words, the cost of the distribution $\left[y_{1f}^{\ell_2} - y_{0f}^{\ell_2}\xi_f^{\ell_2}\cdot(1+b_F) - \left(y_{1f}^{\ell_1} - y_{0f}^{\ell_1}\xi_f^{\ell_1}\cdot(1+b_F)\right)\right]$ is above (not necessarily strictly) the individual's utility for it.

Using this finding a similar version of the next theorem has been shown to hold (see [16]):

<u>Theorem</u>: Consider a firm f with L input-financing plans in its opportunity set Y_f, i.e. $Y_f = \left\{(y_{0f}^1, y_{1f}^1, \xi_f^1), \ldots, (y_{0f}^L, y_{1f}^L, \xi_f^L)\right\}$. Suppose that investors know that $(v_f^1 - \xi_f^1 y_{0f}^1) \geqq (v_f^2 - \xi_f^2 y_{0f}^2) \geqq \ldots$ $\ldots \geqq (v_f^L - \xi_f^L y_{0f}^L)$. Assume this pricing vector over projects satisfies the condition that no individual receives a direct consumption benefit even when the firm moves consecutively from project 1 to project 2,

from project 2 to project 3, ..., from project $L-1$ to project L .
Expressed mathematically:

$$(5)\begin{cases} v_f^2 - v_f^1 \geqq \sum\limits_e \pi_i(y_{0f}^1, \xi_f^1, e)\cdot\left[y_{1f}^2 - \xi_f^2\ (1+b_F)y_{0f}^2 - \left(y_{1f}^1 - \xi_f^1\cdot(1+b_F)y_{0f}^1\right)\right] \ ; \\ v_f^3 - v_f^2 \geqq \sum\limits_e \pi_i(y_{0f}^2, \xi_f^2, e)\cdot\left[y_{1f}^3 - \xi_f^3\cdot(1+b_F)y_{0f}^3 - \left(y_{1f}^2 - \xi_f^2\ (1+b_F)y_{0f}^2\right)\right] \ ; \\ \vdots \\ v_f^L - v_f^{L-1} \geqq \sum\limits_e \pi_i(y_{0f}^{L-1}, \xi_f^{L-1}, e)\cdot\left[y_{1f}^L - \xi_f^L\cdot(1+b_F)y_{0f}^L - \left(y_{1f}^{L-1} - \xi_f^{L-1}\cdot(1+b\ )y_{0f}^{L-1}\right)\right] \ . \end{cases}$$

Assume also that:

[1] individuals correctly perceive that the decisions of company f do not influence the value of the shares of the other firms;

[2] U_i is pseudo-concave and strictly increasing in its arguments for all $i \in I$;

[3] $(x_{0i}, x_{1i}) \in \mathbb{R}_{++}^{E+1}$ for all $i \in I$ (each individual consumes something in each period and state e).

Then all initial shareholders' preferences over plans coincide with the ordering according to net present value, while all other investors are indifferent w.r.t. the firm's choice of project. The proof is identical to that of Theorem 1 in [16] and is therefore not repeated here.

Thus all the theorem requires for the net present value rule to hold is that investors are sufficiently competitive in the sense that they bid up prices "high enough." Explicit assumptions on distributions or portfolios are absent and consumption effects are not ruled out.

In particular:

— when going down the sequence from project 1 to project 2, ..., from project $L-1$ to project L , indirect consumption effects (i.e., changes in utility due to the previously chosen optimal time 1 distribution being destroyed by the change in the company's plan) are not excluded;

— when moving upward along the sequence, next to indirect consumption effects, direct consumption benefits may occur also.

However for a price system compatible with the constraints (5) to exist, some conditions have to be met.

[16] shows these conditions boil down to the following: if there is to be unanimous indifference between two plans with the <u>same net present value</u>, at least some of the final shareholders have to be able to choose another portfolio at at least one of these two corporate input-financing levels, which is equally optimal but which does not contain any shares of the company under consideration (i.e., for the plans $(y_{0f}^{\ell_1},\xi_f^{\ell_1})$ and $(y_{0f}^{\ell_2},\ \xi_f^{\ell_2})$ some final shareholders of f must be able to compose optimal portfolios with $s_{if}(y_{0f}^{\ell_1},\ \xi_f^{\ell_1}) = 0$ in the exchange equilibrium corresponding to f choosing plan $(y_{0f}^{\ell_1},\ \xi_f^{\ell_1})$ and/or $s_{if}(y_{0f}^{\ell_2},\ \xi_f^{\ell_2}) = 0$ in the exchange equilibrium corresponding to $(y_{0f}^{\ell_2},\ \xi_f^{\ell_2})$). It has been shown also in [16] that such a condition can be regarded as a weaker form of Ekern-Wilson spanning, but which is still sufficient to rule out consumption effects. Presently, however, due to the imperfections in the loan market, this spanning constraint is defined w.r.t. time 1 corporate returns <u>net</u> of financial changes.

When profits with different net present value are compared, no such condition is (implicitly) required anymore, at least not for those consumers who initially owned some of f's stock and hence are affected in their budgets by the firm's choice. This is not surprising, for all the theorem is about, is to guarantee that, whenever consumption effects with utility impact opposite to the budget effect are present, the latter one always dominates. This is the reason why, when a budget effect is missing, consumption effects necessarily have to be zero and why the earlier mentioned spanning condition w.r.t. any two projects with the same net present value, is implicitly present in the theorem.

The no direct consumption benefit condition in the downward direction of the theorem can even be weakened further, as illustrated in the next corollary:

Corollary 1: Recall the sequence of L feasible investment-financing plans from the preceding theorem and consider in particular the two consecutive plans ℓ (the current level) and $\ell+1$. Suppose that for investor i there exists an optimal portfolio associated with the plan $\ell+1$ with the property that none of its components offers him a direct consumption benefit if the firm would switch to the plan $\ell+1$. Assume this condition is met for all initial shareholders and every two consecutive proposals in the sequence. If in addition assumptions [1], [2] and [3] are met, the net present value rule holds.

Proof: Identical to the one of Theorem 1. □

The corollary thus implies, for example, that an initial shareholder i with η_{if} strictly positive for the plan ℓ (see equation (1))--and hence has sold off all of his holdings in f in the current exchange equilibrium--may find that, when the firm switches to $\ell+1$, condition (5) is violated. As long as the surplus in consumption value does not fully offset η_{if} however, he cannot derive a benefit from repurchasing shares in f in case the $(\ell+1)$ plan is implemented.

Thus, although he perceives a direct consumption benefit, this investor is not able to take advantage of it.

In summary, the theorem and corollary state that whenever they are sufficiently competitive, initial shareholders unanimously wish the firm to choose its production and financing plan in such a way that the net present value of the shares is maximized.

Remark

The preceding theorem and corollary presume that shareholders are able to correctly assess the net present value of every investment-financing plan of company f . One could rightly ask oneself whether this assumption can be really reconciled with the deficiencies in information acquisition-processing mentioned in Section 1 and justifying the imperfections in trading opportunities considered here. Even if one feels the rational expectations hypothesis w.r.t. pricing hard to swallow, the preceding findings may still remain valid. In particular, suppose shareholders have difficulty in estimating the effect of a change in corporate plans, but management is able to correctly pick out the project with the highest net present value, still there is no problem. In that case the theorem and corollary should be reinterpreted as stating that, if they are sufficiently competitive, all initial shareholders are actually made best off when managers maximize the firm's net present value. Simultaneously however the nature of the unanimity result has somewhat been altered: instead of being reduced to a passive implementation mechanism, management has an active role to play. To quote H. Leland from his paper on information and the role of managerial choice:

> In a world with no transactions costs, shareholders could be consulted with respect to each decision a firm might undertake. The managerial

> role would be limited to implementation only, with no decision-making powers. Yet it is manifestly clear that managers do have considerable responsibility in making investment, production, and financing decisions. An important reason for the active managerial role lies in the transactions costs associated with information. The manager typically knows a great deal about the firm's operations. This information would be useful to shareholders in making decisions for the firm. But it is costly to communicate. Rather than make decisions without information, or incur the costs of information communications, stockholders grant managers the authority to make decisions for them. But this independence of managerial decision making poses a fundamental question: Given their "inside" information, will managers make decisions which are approved by shareholders?[1]

The answer is yes, if shareholders are competitive and managers pursue a net present value maximizing policy.

It follows from the preceding discussion that the only effect of an imperfect loan market seems to be that the no direct consumption benefit conditions (5) are defined w.r.t. income streams after instead of those before financial charges. The reason for this change is not far to seek: with different lending and borrowing rates, the investors may not be able to change on their own account the financial leverage position of the firm by personal lending and borrowing, without affecting their portfolio's return treatment w.r.t. the riskfree opportunity. However, as long as the no direct consumption benefit conditions are met, this consumption effect is dominated by the impact on wealth. Therefore also, as long as it does not lead to a violation of the conditions (5), neither the individuals' borrowing rate nor their lending rate is relevant to unanimity.

Likewise rationing of or imperfect access to the loan market by

[1] H. Leland [8], p. 529.

shareholders doesn't look important either.

Also the company's borrowing rate doesn't seem to matter.

The two (related) questions the preceding theorem does not solve however, are whether or not in equilibrium the company financing decision is of any importance w.r.t. valuation and whether or not the production and corporate financing decision can still be separated. To some extent the answer will probably depend upon whether or not some firms are able to borrow at more "favorable" terms than others. However, to settle these issues some information on how the market values the firms' respective return distribution is needed, i.e. some empirical evidence is required.

3. The Certainty Case

To illustrate the preceding findings, suppose that the set of time 1 possible states of the world consists of a single element. The theorem thus predicts unanimity if direct consumption benefits are absent. However, in this special case even more can be said:

Corollary 2:

1. When, under conditions of certainty, company f does not affect the market price of other firms by its decisions, individuals receiving a direct consumption benefit as a consequence of a switch in plans is incompatible with equilibrium. Hence there is unanimity.
2. Then also, whenever correct price perceptions prevail and there is some lending in the exchange equilibrium, investors' estimation of the share value corresponding to a specific plan is equal to output net of all changes (including the financial ones), discounted at one plus the lending rate.

3. In addition, if the firm's borrowing rate is above the lending rate, the optimal corporate policy uses only equity.

Proof. The first two assertions follow from the fact that under certainty all future income streams may only differ in a scale factor, hence are perfect substitutes and hence have to offer the same return if there is to be some investment in all of them. Part 3 is a direct consequence of assertion 2. □

To see graphically why unanimity obtains, consider the following familiar picture (cf. J. Hirshleifer [6], [7], E. Fama and M. Miller [5]).

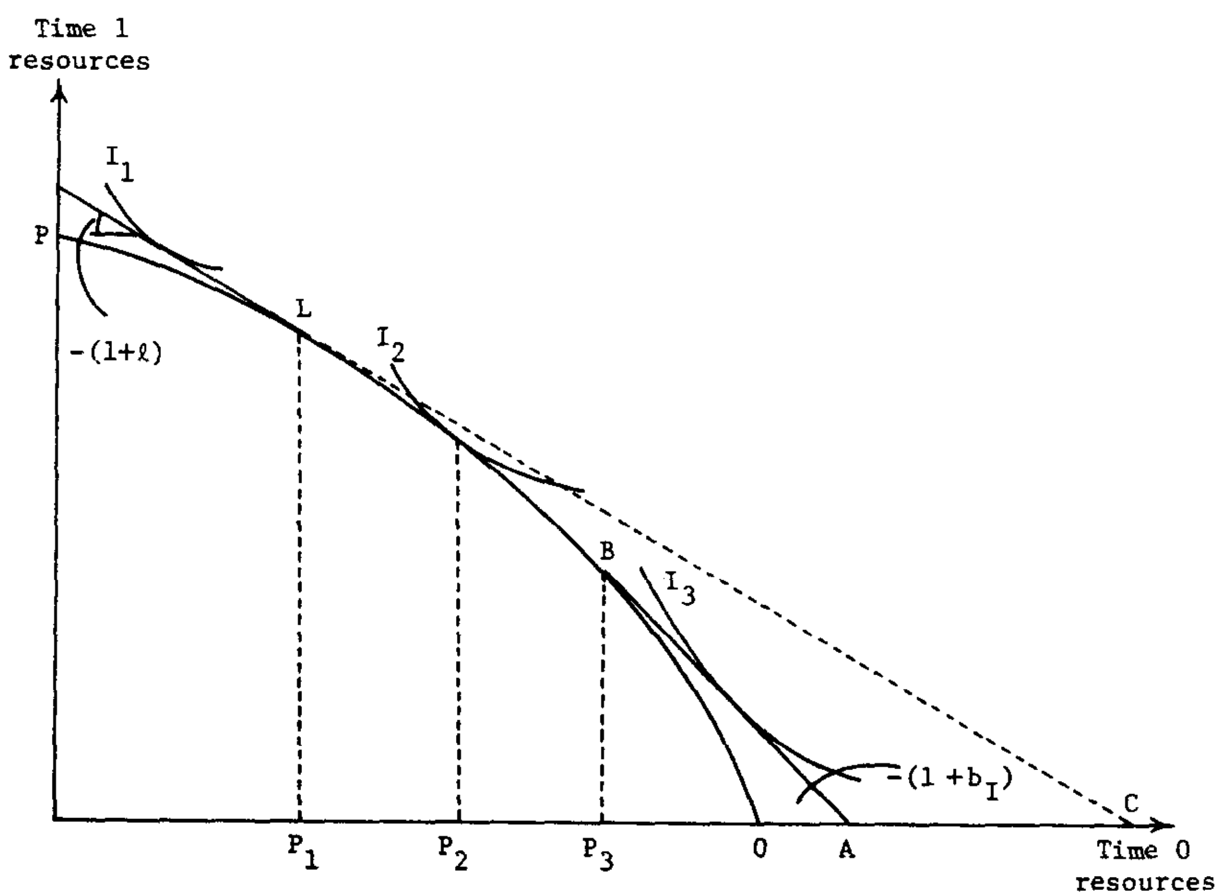

FIGURE 1

Consumption and Production Possibilities when Borrowing and Lending Rates Differ

In Figure 1 the curve AP represents the efficient productive opportunities and the straight lines through L and B respectively the lending and borrowing transformation curve. It seems obvious that the consumer with indifference curve I_1 would want the firm to invest AP_1, whereas the individuals having respectively the indifference curves I_2 and I_3 would prefer the company to invest only AP_2 and AP_3 (a more detailed discussion can be found in [5] and [7]). However by looking at matters in this way, one implicitly assumes no transactions in shares as possible. For if such trading is allowed, the price taking investor faces, next to the curve AP, also the straight market trading line with slope $(-(1+\ell))$, passing through the points L and C and everywhere dominating the curve AP (the existence of this line follows directly from the assumed trading possibilities and the equality of rate of return in similar investments). Therefore, if it is possible, the initial shareholders prefer to move along the trading line and not along the curve of productive opportunities. To approach the problem from still another point of view, remember that as long as $b > \ell$, simultaneous lending and borrowing by the same individual never occurs. Since lending and investment in shares are perfect substitutes in the present setting, it is strictly suboptimal for all non lenders to carry shares from time 0 to time 1. Therefore this class of individuals is not concerned with the fact that a (marginal) project may only return the input times one plus the lending rate. Only the price at which they can sell off their initial holdings matters to them. Obviously, as this price increases, their time 0 wealth increases also and they prefer the company to maximize its net present value.

Crucial to the preceding analysis is the availability of costless share trading possibilities. For if such trading is excluded, no dominant market trading line exists and the non lenders are forced to remain shareholders during period one. Consequently in that case the preceding conclusions would break down.

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