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MULTIPERIOD INSURANCE CONTRACTS

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I. Introduction[1]

Insurance contracts typically provide less than perfect insurance and insurance rates are generally adjusted to reflect previous accidents. For example, automobile insurance includes deductibles and frequent automobile mishaps lead to costlier insurance. The usual explanation of deductibles and coinsurance is the necessity to overcome problems of moral hazard and self-selection. In single-period models, Arrow [1965], Pauly [1968, 1974] and Shavell [1979] have shown that incomplete insurance provides an incentive for agents to take actions which reduce accident probabilities. In fact, the entire agency literature (see for example, Ross [1973], Shavell [1979] or Harris-Raviv [1979]) rests on this tradeoff between risk sharing and incentives. Rothschild-Stiglitz [1976] and Stiglitz [1977] show that imperfect insurance also induces the sorting of agents with different accident probabilities.

While the issues of self-selection and moral hazard have received considerable attention, the dynamic behavior of insurance rates is not as well understood. Recent papers by Radner [1981], Rogerson [1984], Rubinstein-Yaari [1983], and Spatt-Palfrey [1983], have shown how multi-period contracts affect moral hazard issues in insurance contracts. Dionne [1981], Malueg [1981] and Townsend [1982] have shown that despite information asymmetries, infinite length contracts in adverse selection models can yield a Pareto optimal allocation of risks.[2] The key to these results is the use of information about past experience in the formulation of insurance premia. Faced with this threat of adverse adjustments to insurance terms, agents are induced to take "appropriate" measures to reduce risks or to disclose their true accident probabilities. In either case, perfect insurance is again implementable.

In this paper we consider experience rating in a multi-period self-selection model. Experience rating of insurance contracts is done in two ways. The contract can depend upon the outcome of previous insurance contracts, or, in the case of retrospective experience rating, it depends on the insured's experience during the policy period. In both cases, premia and indemnities change as a function of experience. A variety of insurance contracts offer experience rating. These include automobile insurance, group health insurance, group life insurance and workmen's compensation. For group health and group life insurance, moral hazard seems unimportant and adverse selection is important in all of these insurance contracts.

To provide an explanation for the existence of both imperfect insurance and the conditioning of insurance terms on past experience we investigate a model of finite duration in which an agent's accident probabilities are private information. Firms do not observe risk types and hence insurance contracts must satisfy certain self-selection constraints. By offering multi-period contracts, insurance companies can adjust the terms of agreements over time. This adjustment allows for a more efficient sorting of agents relative to the single-period contract. Accident probabilities will be exogenously determined so there will not be any moral hazard problems.

We first investigate the optimal multi-period insurance contract when there is a single firm. In an example with two periods and two risk classes, our results indicate that the contract for low-risk agents will reflect experience while that offered to the high-risk agents will not. Low-risk agents who have accidents are faced with an increased premium and a lower indemnity while low-risk agents who do not have accidents will receive more favorable terms. We also discuss extensions of this example to more risk classes and an arbitrary number of periods. With a finite time horizon, contract terms are monotone functions of low-risk agents' accident records.

We also consider the use of multi-period contracts in a competitive market in two cases. First, we find the optimal contract when consumers are legally bound to a single firm for a number of periods. This yields contracts where high risk consumers' rates do not depend on past history, while low risk consumers are punished or rewarded based on their accident records. This contract is similar to the monopoly contract in terms of experience rating. The alternative situation is where consumers cannot be legally bound to a multiperiod contract. Thus, in a two period model consumers who have accidents (or don't) cannot be "punished" in a way that would result in second period expected utility that is lower than that which an entering firm could afford to offer (which, in our model, is the standard one period separating contract.) This additional constraint changes the optimal contract. The low risk consumers receive a contract which has first period utility that is lower than utility in the standard one period contract, but expected utility in the second period is greater than the standard one period contract if the consumer didn't have an accident and is exactly equal to the standard one period contract if he did have an accident. Firm profits from the first period contract will be positive and profits will be negative from the second period contract. These results are analagous to those in Harris and Holmstrom [1982] in that a form of "bonding" occurs. This analysis is also an example in which competition in the second period of a contract actually <u>reduces</u> social welfare.

One other issue we consider is the question of existence of a competitive equilibrium in a multiperiod model. Because two or more periods result in a better selection and increased utility, pooling contracts which break the competitive equilibrium in a one period model will be less likely to exist.

We also discuss the multiperiod pooling contract which will depend on accident history if possible. Results here are incomplete.

Section II of the paper outlines the basic model and investigates the optimal contract for a single firm. Section III discusses the competitive multi-period contract and section IV has results on pooling and existence of competitive equilibria.

II. Monopoly

We begin with a rather simple model to illustrate the role of multi-period insurance contracting. Consumers are identical except for their accident probabilities. They possess von Neumann-Morgenstern utility functions, $U(\cdot)$, which are strictly increasing and strictly concave. Their wealth is W if no accident occurs and $W-D$ if an accident takes place where $W > D$. We denote by π_H and π_L the probability of an accident for high-risk and low-risk agents respectively and assume $\pi_H > \pi_L$. These probabilities are out of the agent's control so that no moral hazard problem arises. There are N_H high-risk and N_L low-risk agents in the economy.

If agents do not purchase insurance, they obtain expected utility, $\bar{U}_i$. For $i = H, L$,

$$\bar{U}_i = \pi_i U(W-D) + (1 - \pi_i)U(W).$$

We call $\bar{U}_i$ the reservation utility of type i and note that $\bar{U}_H < \bar{U}_L$.

In this section of the paper we assume there is a single, risk neutral, insurance company. We begin by reviewing the analysis by Stiglitz [1977] on single-period insurance contracts. The monopoly offers to the agents an insurance contract which specifies for each date a premium P to be paid to the firm if no accident occurs and an indemnity I paid to the consumer if an accident takes place. We use the notation $\delta_i = \{P_i, I_i\}$ to denote the single-period contract offered to type $i = H,L$ agents.

First we analyze the optimal contract when agent's accident probabilities are known by the firm. We then investigate the case of asymmetric information.

A. Full-Information Solution

In the full-information solution, agent's accident probabilities are publicly known. Therefore the monopolist chooses δ_H and δ_L to

$$(2.1) \qquad \text{Maximize} \sum_{i=H,L} N_i((1 - \pi_i)P_i - \pi_i I_i)$$

$$(2.2) \qquad \text{subject to } V(\delta_i | \pi_i) > \bar{U}_i \text{ for } i = H,L.$$

The objective function is simply the sum of the expected profits from the contracts offered to each of the risk types. Constraint (2.2) is an individual rationality constraint where

$$(2.3) \qquad V(\delta_i | \pi_i) \equiv \pi_i U(W - D + I_i) + (1 - \pi_i)U(W - P_i)$$

is the expected utility of contract δ_i for agents with accident probability π_i.

The solution to (1) has full insurance offered to <u>both</u> risk classes and (2) binding for $i = H,L$. That is, the agents shed all of their risk to the insurance company at the expense of the consumers' surplus. Hence the full-information solution (δ_H^*, δ_L^*) satisfies

$$(2.4) \qquad U(W - D + I_i^*) = U(W - P_i^*) = \bar{U}_i \quad \text{for } i = H,L.$$

This solution is shown in Figure 1 where the horizontal axis is wealth in the "no accident" state. Point E is the initial endowment and points H^* and L^* are the wealth levels for the agents under δ_H^* and δ_L^* respectively. These indifference curves are convex due to the concavity of $U(\cdot)$. As discussed by Townsend, with full information, there are no gains to multi-period contracting.

B. Imperfect Information Solution

Once we relax the assumption that accident probabilities are public information, we need to constrain (2.1) further to ensure that agents have an incentive to reveal their types. These self-selection constraints, following the notation in (2.3), are

$$V(\delta_H|\pi_H) \geq V(\delta_L|\pi_H) \tag{2.5}$$

and

$$V(\delta_L|\pi_L) \geq V(\delta_H|\pi_L). \tag{2.6}$$

Expression (2.5) guarantees that high-risk agents prefer δ_H to δ_L. Expression (2.6) does the same for low risk agents.

From Figure 1, it is easy to see that the full-information solution will violate (2.5) since δ_L^* is preferred to δ_H^* by high-risk agents. Solving (2.1) subject to (2.2), (2.5) and (2.6) yields a solution in which:

(i) High-risk agents obtain full insurance,

(ii) Low-risk agents bear some risks, i.e., $I_L < D - P_L$,

(iii) $V(\delta_H|\pi_H) = V(\delta_L|\pi_H)$

$V(\delta_L|\pi_L) > V(\delta_H|\pi_L)$,

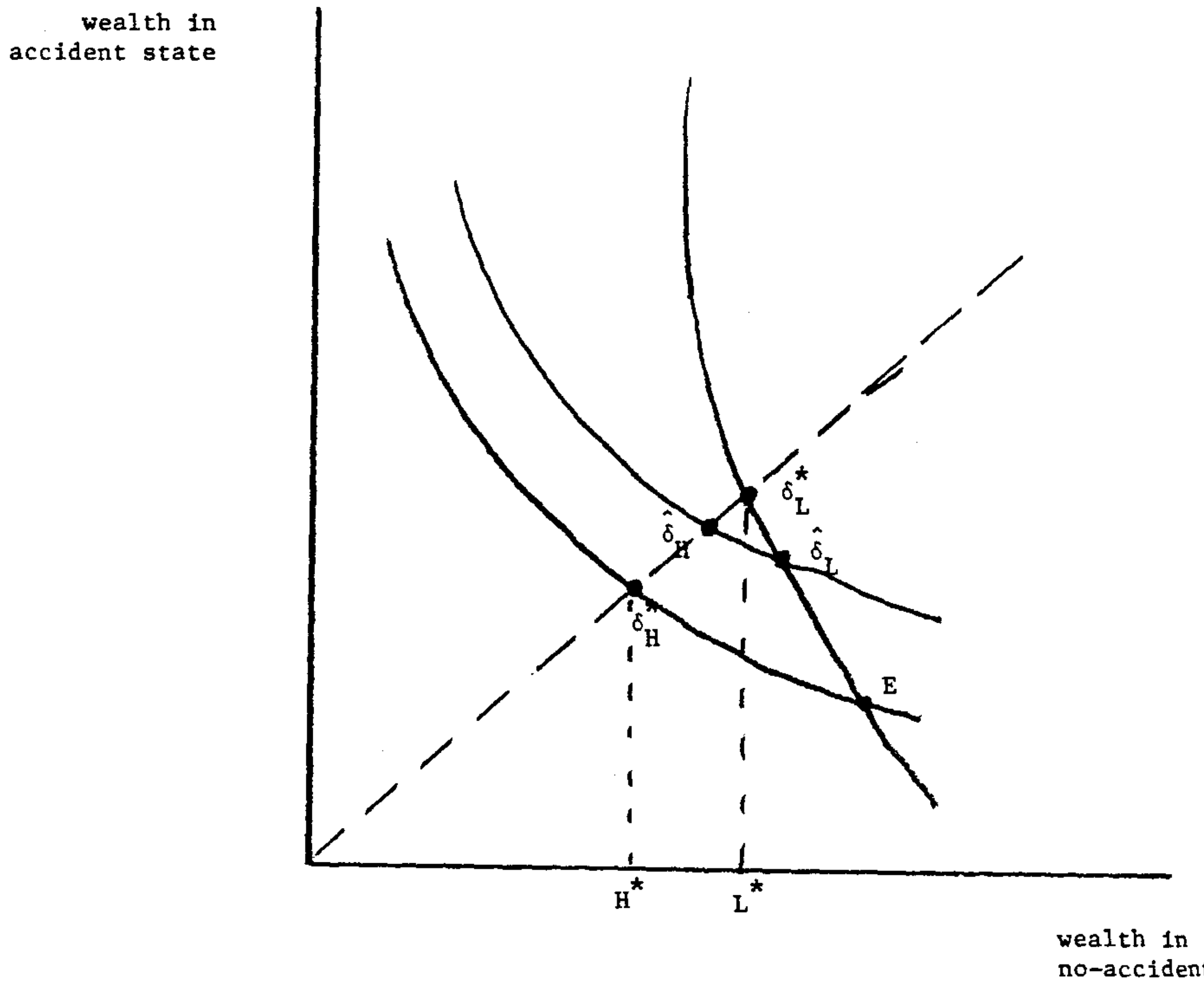
wealth in
accident state
δ_L^*
$\hat{\delta}_H$
$\hat{\delta}_L$
δ_H^*
E
H^*
L^*
wealth in
no-accident

FIGURE 1

and

$$\text{(iv)} \qquad V(\delta_H|\pi_H) > \bar{U}_H$$
$$V(\delta_L|\pi_L) = \bar{U}_L.$$

These results are shown formally in Stiglitz [1977].

This solution is depicted in Figure 1 as points $\hat{\delta}_L$ and $\hat{\delta}_H$. Essentially, the monopolist extracts all surplus subject to the self-selection constraints. The high-risk agents strictly prefer $\hat{\delta}_H$ to E and hence receive some consumers' surplus which the monopolist chooses not to extract. Due to the imperfect information, risks are not shared efficiently.

C. Multi-period Contracts

Relative to the full-information solution, the monopolist's profits are lower in the imperfect information case due to the self-selection constraints. In order to increase profits, the monopolist must relax the binding constraints. One means of doing so in a multi-period setting is to tie the terms of the insurance contract to past experience. This can increase profits by providing an alternative means for the sorting of agents.

The contracts now offered by the insurance company are somewhat more complicated since they reflect past experience. For the present, we consider a two-period extension of the basic model. An insurance contract will now include premia and indemnities for each of the periods contingent on past experience. We denote by $P_i(A)$ ($P_i(N)$) the second period premium of a type i agent experiencing an accident (no accident) in period 1. A similar definition holds for $I_i(A)$ and $I_i(N)$. With this notation,

$$\delta_H^2 = \{P_H,\ I_H,\ P_H(A),\ I_H(A),\ P_H(N),\ I_H(N)\}$$

and $\delta_L^2 = \{P_L,\ I_L,\ P_L(A),\ I_L(A),\ P_L(N),\ I_L(N)\}.$

The firm chooses δ_H^2 and δ_L^2 to

$$(2.7)\qquad \text{Maximize } N_H\{[(1-\pi_H)P_H - \pi_H I_H] + \pi_H[(1-\pi_H)P_H(A) - \pi_H I_H(A)] + (1-\pi_H)[(1-\pi_H)P_H(N) - \pi_H I_H(N)]\} + N_L\{[(1-\pi_L)P_L - \pi_L I_L] + \pi_L[(1-\pi_L)P_L(A) - \pi_L I_L(A)] + (1-\pi_L)[(1-\pi_L)P_L(N) - \pi_L I_L(N)]\}$$

subject to

$$(2.8)\qquad V(\delta_i^2|\pi_i) > 2\bar{U}_i \quad \text{for } i=H,L$$

$$(2.9)\qquad V(\delta_H^2|\pi_H) > V(\delta_L^2|\pi_H)$$

and

$$(2.10)\qquad V(\delta_L^2|\pi_L > V(\delta_H^2|\pi_L).$$

In this problem, the objective function is the sum of expected profits from the two risk types. Consider the N_H high-risk agents. They have a premium and an indemnity of $\{P_H, I_H\}$ in the first period. In the second period, those who had an accident in the first period face $\{P_H(A),\ I_H(A)\}$. Those who did not have an accident receive terms of $\{P_H(N), I_H(N)\}$. Expression (2.8) is simply a 2-period individual rationality constraint since the agents agree to <u>binding</u> two period contracts. Constraints (2.9) and (2.10) ensure the self-selection of agents. Following (2.3),

$$(2.11) \qquad V(\delta_i^2|\pi_j) = \pi_j U(W - D + I_i) + (1 - \pi_j)U(W - P_i) +$$
$$\pi_j[\pi_j U(W - D + I_i(A)) + (1 - \pi_j)U(W - P_i(A))] +$$
$$(1 - \pi_j)[\pi_j U(W - D + I_i(N)) + (1 - \pi_j)U(W - P_i(N))].$$

From this expression, it is clear that we are not permitting borrowing and lending by consumers. Furthermore, we have assumed that neither the firm nor customers discount future utility. Hence, in the absence of the incentive constraints, the optimal insurance arrangement would generate constant consumption for customers across both time and states of nature. As in the one-period problem this full-information solution will not be implementable when risk-types are private information. This will affect the variability of income both over time and states of nature. Our interest, indicated in Proposition 1, is mainly in the differences in contract terms across agents with different accident histories at a given point in time. Coupled with these differences across histories are, undoubtedly, variations in expected income across time since the insurance company is also acting as a "banker." These adjustments are not highlighted in our analysis and have been investigated, in a related setting, by Rogerson [1984].

Proposition 1. In the solution to (2.7), $V(\delta_L^2|\pi_L) = 2\bar{U}_L$ and $V(\delta_H^2|\pi_H) = V(\delta_L^2|\pi_H)$. Furthermore, in the optimal contract,

(i) The high-risk agents obtain perfect insurance and
$P_H(A) = P_H(N) = P_H$; $I_H(A) = I_H(N) = I_H$.

and (ii) the low-risk agents do not obtain perfect insurance and
$P_L(A) > P_L > P_L(N)$; $I_L(A) < I_L < I_L(N)$.

Proof. See Appendix A.

From this proposition we see that the monopolist uses insurance contracts contingent on past experience to increase expected profits. As in the single period case, high-risk agents continue to receive full insurance and the elements of δ_H^2 are not contingent on experience. However, the premia paid by a low-risk agent increase if an accident occurs in the first period and decrease if no accident occurs. Indemnities, on the other hand, adjust in the opposite direction.

This adjustment of terms across histories in the second period helps in the sorting of agents. High-risk agents who purchase δ_L^2 have a greater chance of facing costlier insurance in the second period than do low-risk agents. That is, δ_L^2 stipulates a lottery over period 2 contracts which high- and low-risk agents evaluate differently. With $\pi_H > \pi_L$, $P_L(A) > P_L(N)$ and $I_L(A) < I_L(N)$, high-risk agents assign a higher probability to the bad outcome of the lottery than do low-risk agents. By exploiting this difference, the monopolist can relax the single-period self-selection constraint and increase expected profits. As in the single-period case, low-risk agents have zero surplus.

It is possible to extend this analysis in a number of directions: Adding more risk classes and increasing the number of time periods. We conjecture that adding more risk classes would yield a solution in which all agents, except the high-risk class, faced premia and indemnities which are adjusted over time.

A more interesting extension concerns the addition of more time periods. To see the impact of this, we first require some additional notation. Let $h_{i,j}^t$ be the history at time t of agent j in risk class i = H,L. The relevant history is simply whether or not an agent had an accident in each of the t-1 periods. In terms of the optimal contract, the insurance company cares only about the number of accidents in the past t-1 periods.[3] Hence, $h^t_{i,j}$ is simply the number of accidents reported in the t-1 periods. We also define $\alpha^t_{i,j}$ as the probability that an agent in risk class $i = H,L$ will have j accidents through t-1 periods. Hence agents are identified by their risk class and number of accidents. These probabilities are given by the simple binomial distribution since we are concerned with repeated trials which are independent over time. Therefore,

$$\alpha^t_{i,j} = (\pi_i)^j (1 - \pi_i)^{t-1-j} \binom{t-1}{j}. \tag{2.12}$$

Finally, as in the two period case, δ^T_H and δ^T_L will refer to T-period contracts for high- and low-risk agents respectively. These contracts will specify both premia and indemnities as functions of agents' histories. Using the obvious generalization of (2.11) for $V(\delta^T_i | \pi_j)$, the monopolist chooses δ^T_H and δ^T_L to

$$\text{Maximize } N_H \{ \sum_{t=1}^{T} \sum_{j=0}^{t-1} \alpha^t_{H,j} [(1 - \pi_H) P_H(h^t_{H,j}) - \pi_H I_H(h^t_{H,j})] \} \tag{2.13}$$

$$N_L \{ \sum_{t=1}^{T} \sum_{j=0}^{t-1} \alpha^t_{L,j} [(1 - \pi_L) P_L(h^t_{L,j}) - \pi_L I_L(h^t_{L,j})] \}$$

subject to:

$$\frac{1}{T} V(\delta^T_i | \pi_i) > \bar{U}_i \quad \text{for } i = H,L \tag{2.14}$$

$$V(\delta^T_H | \pi_H) > V(\delta^T_L | \pi_H) \tag{2.15}$$

and

(2.16) $$V(\delta_L^T|\pi_L) \geq V(\delta_H^T|\pi_L).$$

As in the two-period solution described in Proposition 1, high-risk agents will continue to receive full insurance, (2.14) is binding for i=L and (2.15) is binding in the optimal T-period contract. Denoting the multiplier for the binding constraint in (2.14) by ϕ and the multiplier for (2.15) by λ,

(2.17) $$N_L = [\phi - \lambda(\frac{\alpha_{H,j}^t}{\alpha_{L,j}^t})(\frac{1 - \pi_H}{1 - \pi_L})] \; U'(W - P_L(h_{L,j}^t)) \text{ and}$$

(2.18) $$N_L = [\phi - \lambda(\frac{\alpha_{H,j}^t}{\alpha_{L,j}^t})(\frac{\pi_H}{\pi_L})] \quad U'(W - D + I_L(h_{L,j}^t)) \qquad \forall\ j,t.$$

These are the first-order conditions to (2.13)-(2.16) for an agent who had j accidents over t-1 periods and announced he was a low-risk consumer. So, in period t, given this history <u>and</u> his announcement, the agent will be charged a premium of $P_L(h_{L,j}^t)$ and receive an indemnity of $I_L(h_{L,j}^t)$.

We can use (2.17) and (2.18) to investigate the adjustments in the contract terms over histories and time. First, keeping t fixed, the adjustments in P_L and I_L will depend on the ratio of probabilities of agents having a given history. From (2.12),

(2.19) $$\frac{\alpha_{H,j}^t}{\alpha_{L,j}^t} = \frac{(\pi_H)^j(1 - \pi_H)^{t-1-j}}{(\pi_L)^j(1 - \pi_L)^{t-1-j}}$$

Therefore, as j, the number of accidents in the t-1 previous periods increases, from (2.19) we see that the ratio of probabilities will increase a well since $\pi_H > \pi_L$. From (2.17) this implies that $P_L(h_{L,j}^t)$ will increase wit j while $I_L(h_{L,j}^t)$ falls with j as seen from (2.18).

The monotonicity of these contract terms with respect to the histories should not be surprising. From (2.19), we see that the ratio of probabilities will satisfy the monotone likelihood ratio condition (see Milgrom [1981] or Grossman-Hart [1983]).

We also see from (2.17) and (2.18) that agents whose records strongly indicate that they are in the low-risk class will receive closer to full insurance. That is, if you have an accident record for which $\alpha^t_{H,j}$ is close to zero, then your wealth will be almost completely stabilized. Alternatively, if your history is more likely to be held by a high-risk agent, your wealth becomes more variable. Again, these adjustments are made to profitably sort the agents.

We can also consider the impact the changing the contract length, T. As $T \to \infty$, following the argument in Townsend [1982] we can show that the first-best contract is obtainable. One obtains this by choosing a feasible contract (not necessarily the one satisfying 2.17 and 2.18) and, using the law of large numbers to show that, as $T \to \infty$, average expected utilities converge to the full-information solution. The optimal T-period contract must therefore converge to the full-information solution as well. As T increases but remains finite we might expect that the adjustments in δ^T_L across histories would become less severe since the monopolist has more information to gather. We conjecture that λ falls as T increases which will reduce these adjustments.

One other interesting issue concerns the reporting of accidents if their occurrence is not public information, e.g., a fender-bender. If the terms of the contract are adverse enough following an accident, then agents may not inform the companies about bad realizations. This obviously can occur only in a multi-period insurance setting. Once insurance companies realize that agents may not report accidents, they will take that information into account when formulating the optimal contract. This issue merits further investigation.

III. Competitive Contracts

The model in this section differs from the model presented in section II because firms will be constrained to earn zero expected profits because of competition. The equilibrium concept will be a Nash equilibrium, as opposed to the reactive equilibrium proposed by Wilson. In the one period competitive model presented in Rothschild-Stiglitz [1976] it is shown that the equilibrium must be a separating equilibrium where different risk-type consumers receive different insurance. This equilibrium may not exist because contracts that pool all risk classes could yield higher utility for all consumers. However, a pooling contract cannot be an equilibrium. This section will consider the separating contracts. Section IV discusses the question of existence and pooling contracts.

A. Full Information Solution

With full information, the firm knows the risk type of each consumer. For each type the insurance contract would solve:

(3.1) $$\text{Maximize} \quad \pi_i\, U(W-D+I_i) \; + \; (1-\pi_i)\, U(W-P_i)$$
$$\text{subject to} \quad (1_i-\pi)P_i \; - \; \pi_i I_i = 0.$$

This maximizes the expected utility of the type i consumer along the zero profit locus of the firm. The solution yields full insurance for each type of consumer. No firm could offer a contract that two or more types of consumers would prefer (pooling two types) to the contract that solves (3.1) for his own type and that yields zero profits for the firm. The reason is that all risk types prefer their own contract to any contract for a higher risk class. A multi-period competitive contract where the risk classes of consumers are observable consists of the one period contract repeated in each period for each risk class. The contracts are δ_L^* and δ_H^* in figure 2. (Note that these contracts differ from the full-information monopoly solution despite the

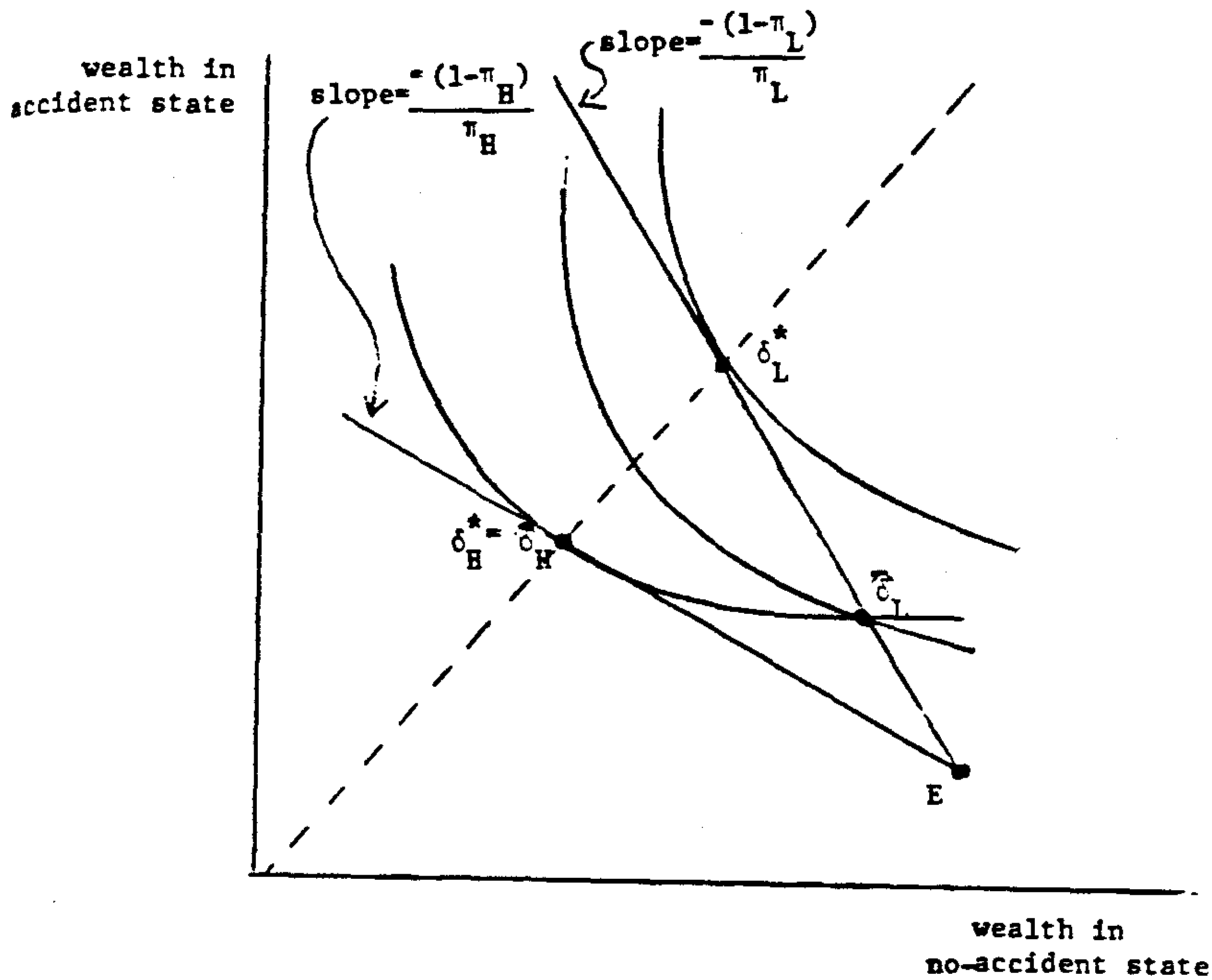
wealth in accident state
slope= $\frac{-(1-\pi_H)}{\pi_H}$
slope= $\frac{-(1-\pi_L)}{\pi_L}$
δ_L^*
$\delta_H^* = \bar{c}_H$
$\bar{c}_L$
E
wealth in no-accident state

FIGURE 2

duplication of the notation.)

B. Incomplete Information Solution

If consumer accident probabilities are not an observable characteristic, insurance contracts must be designed with the knowledge that consumers will misrepresent their risk class if this will increase expected utility. The full information solution cannot be implemented because high risk types have an incentive to claim that they are the lowest risk class as was the case in the monopoly section. If all consumers choose that contract, the firm will earn negative profits. Thus the solution to offer full insurance to all types cannot be a competitive equilibrium.

For a discussion of the one period information constrained competitive solution see Rothschild-Stiglitz (1975). The equilibrium contracts are $\bar{\delta}_H$, $\bar{\delta}_L$ pictured in figure 2. To summarize their results:

(i) The high risk types receive full insurance.

(ii) Low risk types receive less than full insurance.

(iii) The high risk type is indifferent between his contract and that for the low risk class.

(iv) Profits are zero on all contracts.

(v) If an equilibrium exists it will be a separating solution. No equilibrium may exist because of pooling possibilities.

Thus in a one period model where consumers know their accident probabilities and firms do not, if a competitive equilibrium exists, consumer types are revealed. In a model with multi-period contracts, a consumer's accident record may affect his insurance contract. We will first consider an example with two periods and solve for the contract when consumers can sign an agreement that (legally) binds them to the agreement for the two periods. Then we consider the two period contracts where the consumer can leave the contract after the first period. These two solutions are very different. We assume that the firm is always bound by any multi-period agreement.

C. Binding Two Period Contracts

If competitive firms can bind agents to two period agents, then it is straightforward to demonstrate that the separating solution will be qualitatively identical to the monopoly solution. That is, high risk agents will receive the contract $\bar{\delta}_H = \delta_H^*$ each period as in the one period problem. Given this contract and the self-selection constraint that $2V(\delta_H^*|\pi_H) > V(\delta_L^2|\pi_H)$, competitors will offer δ_L^2 to maximize $V(\delta_L^2|\pi_L)$ subject to zero expected profits. As shown formally in Appendix B, the optimal contract satisfies

$$(3.2) \qquad P_L(A) > P_L > P_L(N) \quad \text{and}$$
$$I_L(A) < I_L < I_L(N).$$

As in the monopoly solution, these adjustments represent a means of efficiently sorting agents. The main difference between the market structure is, of course, that the monopolist extracts the maximal consumers' surplus possible while, in the competitive case, firm's profits are driven to zero. Hence, when an equilibrium exists, it is characterized by an adjustment of the terms of δ_L^2 such that low-risk agents experiencing an accident are "punished" relative to those not having an accident.

D. Non-Binding Two-Period Contracts

We now relax our assumption that consumers sign a two-period binding contract and instead allow them to costlessly switch to another firm in the second period if they so desire. These second-period entrants are most likely to attract the low risk agents who are being "punished" for having an accident. Hence, we ask whether we can have adjustments in contract terms as a sorting mechanism in the face of entry in the second period.

To answer this, we first need to carefully specify the game played by the entrants in the second period. Then, taking entrants' decision rules as given, we can determine the optimal two-period contract for low-risk agents. In general, entrants know the two-period contract being offered (δ_H^2, δ_L^2) and may also have some information about individual agents such as their histories (in this case their first period realizations) and the type of contract they chose in period 1. Obviously, the optimal strategies of the entrants will critically depend on their knowledge about individual agents. In this paper we make the extreme assumption that entrants do not know either agents' accident histories or their choice of contract in the first period. Our future research plans are to relax this assumption and provide a welfare comparison of alternative information structures.

With this assumption, the second-period behavior for entrants is really no different from that of firms in the one-period model. As we indicate below, high-risk agents will continue to receive δ_H^*. Hence competition will force entrants to offer either the separating contract to attract low-risk agents or the pooling contract from the one-period solution. Given our focus on separating solutions as a means of circumventing the existence problem, second period competition implies that entrants will offer $\bar{\delta}_L$, the optimal separating contract in the single-period model.

Given that entrants will provide $\bar{\delta}_L$, we can determine the optimal two-period contract. For notational purposes, define $\delta_L^2 = \{\delta_L^1, \delta_L^A, \delta_L^N\}$ where $\delta_L^1 = (P_L, I_L)$, $\delta_L^A = (P_L(A), I_L(A))$ and $\delta_L^N = (P_L(N), I_L(N))$. Competition ensures that δ_L^2

(3.3) maximizes $V(\delta_L^2|\pi_L)$

subject to

$$2V(\delta_H^*|\pi_H) \geq V(\delta_L^1|\pi_H) + \pi_H \max\{V(\delta_L^A|\pi_H),\ V(\bar{\delta}_L|\pi_H)\} + (1-\pi_H)\max\{V(\delta_L^N|\pi_H), V(\bar{\delta}_L|\pi_H)\} \tag{3.4}$$

$$V(\delta_L^2|\pi_L) \geq 2V(\delta_H^*|\pi_L) \tag{3.5}$$

$$V(\delta_L^A|\pi_L) \geq V(\bar{\delta}_L|\pi_L) \tag{3.6}$$

$$V(\delta_L^N|\pi_L) \geq V(\bar{\delta}_L|\pi_L) \tag{3.7}$$

$$(1-\pi_L)P_L - \pi_L I_L + \pi_L[(1-\pi_L)P_L(A) - \pi_L I_L(A)] + (1-\pi_L)[(1-\pi_L)P_L(N) - \pi_L I_L(N)] = 0. \tag{3.8}$$

Constraints (3.4) and (3.5) are similar to the two-period self-selection contraints that arose in the binding contracts problem. However, in this problem, high-risk agents who declare themselves low-risk agents have the option of taking the entrants' contract ($\bar{\delta}_L$) in the second period as well. Constraint (3.4) explicitly allows this option. Since entrants can attract low risk agents as well, we have added (3.6) and (3.7). Because of (3.6) and (3.7) we do not have to include the options of leaving the low risk contract in the sorting constraint for low risk agents. Finally, (3.8) is the expected zero profits constraint.

Before solving this problem, it is insightful to determine whether the optimal binding two-period contract is feasible. It is not surprising that the punishments described by (3.2) will be too severe--i.e., (3.6) will be violated. To see this assume that, to the contrary, $V(\delta_L^A|\pi_L) \geq V(\bar{\delta}_L|\pi_L)$ in the solution to the binding contracts problem. The two possible configurations that satisfy both (3.2) and (3.6) are shown in figure 3. The δ' contracts are ruled out as they are not incentive compatible—i.e., (3.4) is violated. The $\underline{\delta}$ contracts are also ruled out as expected profits are

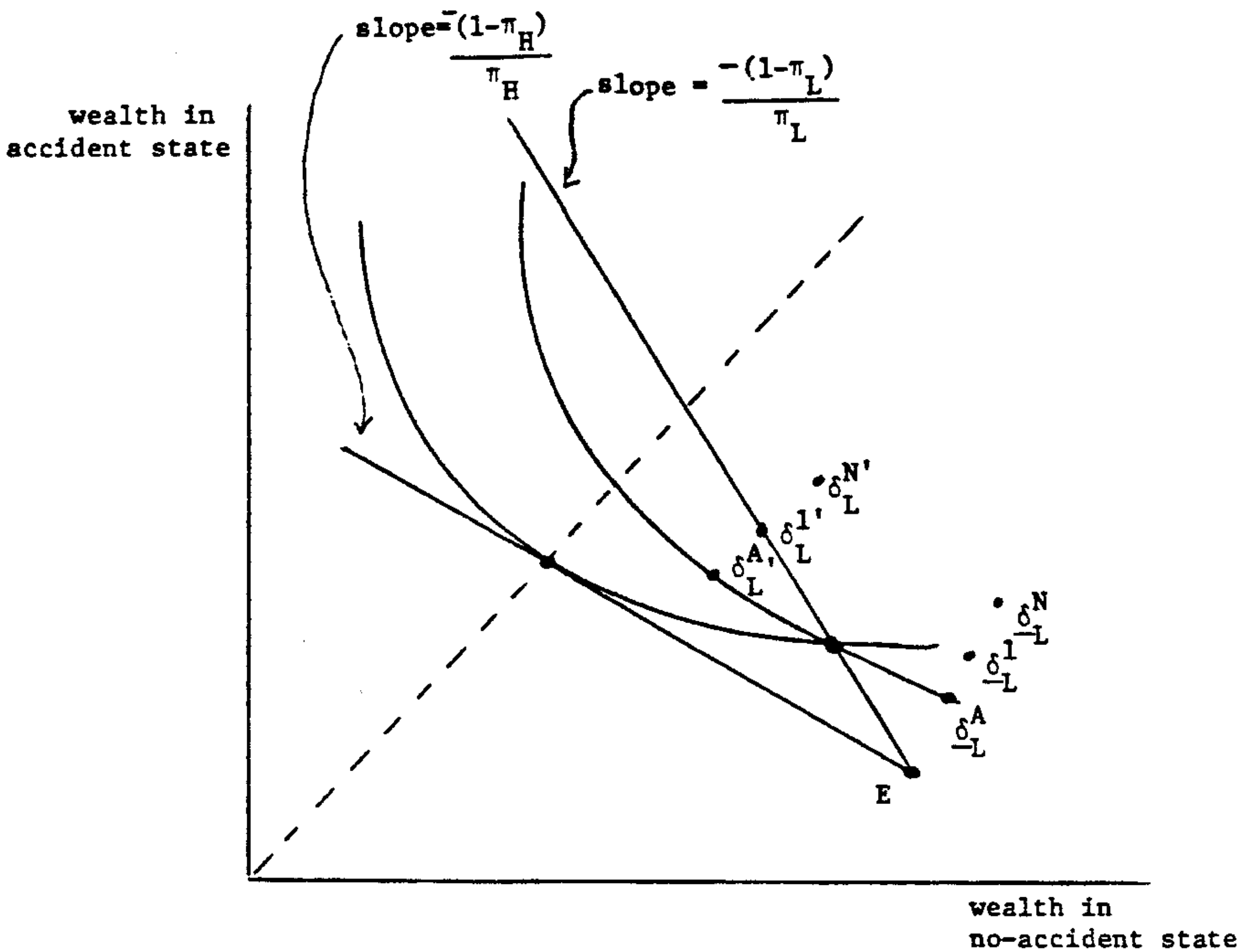
slope= $\frac{-(1-\pi_H)}{\pi_H}$
slope = $\frac{-(1-\pi_L)}{\pi_L}$
wealth in
accident state
$\delta_L^{N'}$
$\delta_L^{1'}$
$\delta_L^{A'}$
$\underline{\delta}_L^{N}$
$\underline{\delta}_L^{1}$
$\underline{\delta}_L^{A}$
E
wealth in
no-accident state

FIGURE 3

negative. Hence the two-period binding contract is not implementable in the presence of second period entry. Therefore, the welfare of low-risk consumers (in terms of two-period expected utility) is <u>lower</u> due to the existence of second period entrants. Since all other agents (including firms) have the same expected utility as in the binding case, the possibility of second-period entry clearly <u>reduces</u> social welfare.

The qualitative characteristics of the solution are summarized below.

<u>Proposition 2</u>: In the solution to (3.3), constraints (3.4), (3.6) and (3.8) are binding. The relevant maxima in constraint (3.4) are $V(\delta_L^A | \pi_H)$ and $V(\delta_L^N | \pi_H)$. Furthermore:

(i) $I_L(N) > I_L$ and $P_L(N) < P_L$.

(ii) δ_L^1 yields positive expected profits to the firm.

(iii) $P_L(A) < P_L(N)$ and $I_L(A) < I_L(N)$.

And $V(\delta_L^N | \pi_L) > V(\delta_L^A | \pi_L)$.

(iv) $V(\delta_L^1 | \pi_L) < V(\hat{\delta}_L | \pi_L)$.

<u>Proof</u>: This proof proceeds in two steps. First, we need to determine which of the constraints, (3.4)-(3.8), are binding. Second, we need to determine which are the relevant maxima in (3.4). We begin by assuming that (3.7) is not binding but that the other constraints may be. With this conjecture (to be validated later), one can solve the programming problem for all the combinations of positive maxima in (3.4). We find, by generating contradictions, that the relevant maxima <u>must</u> be $V(\delta_L^A | \pi_H)$ and $V(\delta_L^N | \pi_H)$.

Given this, we solve (3.3) with all of the constraints other than (3.7). From this solution (which we leave to the reader to derive), one finds that $I_L(N) > I_L$ and $P_L(N) < P_L$ so that agents not having an accident in period 1 are rewarded. It is also straightforward to show that $V(\delta_L^N | \pi_L) > V(\bar{\delta}_L | \pi_L)$

so that (3.7) is not binding. This implies that $V(\delta_L^N|\pi_L) > V(\delta_L^A|\pi_L)$ so that accidents in the first period yield lower second period expected utility. From the first-order conditions one finds that the form of this punishment is $I_L(A) < I_L(N)$. In order to meet (3.4), (3.6) and (3.8), $P_L(A) < P_L(N) < P_L$.

Suppose δ_L^1 makes negative profits in contrast to (ii). Then δ_L^N does too, so that δ_L^A must make positive profits. This is inconsistent with $P_L > P_L(A)$. Finally, if δ_L^1 makes positive profits, $V(\delta_L^1|\pi_L) < V(\bar{\delta}_L|\pi_L)$ to meet (3.4).

The key aspect of this proposition is that entry will limit but <u>not</u> destroy the use of experience rating as a sorting device. From (i), we see that agents who avoid accidents are rewarded. Agents are also "punished" for having accidents as given by (iii). Since $P_L(A) < P_L(N)$, however, these punishments are different than in the case of binding contracts. Firms choose to "punish" accidents by lowering I since high-risk agents are more likely to be affected by this type of adjustment. As in the Harris-Holmstrom model, firms make profits in the first period as a means of "financing" the negative profits from the second period adjustments. In doing so, low risk agents are worse off in the first period (see (iv)) than in the single-period sorting equilibrium.

Denote the solution to (3.3) by $\hat{\delta}_L^2$. It remains to be shown that the contracts $\hat{\delta}_L^2$ and δ_H^* constitute a competitive equilibrium. Since $\hat{\delta}_L^2$ satisfies (3.4)-(3.8), it is incentive compatible and consumers accepting this contract have no incentive to leave the firm in the second period. Hence, given δ_H^*, it is the best contract that can be offered to low-risk agents. The contract δ_H^* is, as before, a zero-profit contract maximizing the utility of high-risk agents. Therefore, $\hat{\delta}_L^2$ and δ_H^* characterize a competitive equilibrium.

IV. Existence of Equilibria

In a one period competitive model, Rothschild-Stiglitz showed that pooling possibilities sometimes destroy the separating equilibria. A competitive equilibrium is always a separating equilibrium and a pooling solution is not a competitive equilibrium. A multi-period competitive contract may have a higher probability of existence than in a single period model. This result depends on the type of pooling. When pooling contracts simply means that all individuals are treated strictly the same, so the multi-period pooling contract is the one period pooling contract repeated each period, a competitive equilibrium is more likely to exist since a separating contract is more efficient in a multiperiod setting. However, in a multiperiod pooling contract firms will always choose to use experience ratings in the same way the separating contracts use experience. By basing second period insurance on accident history, expected utility for the low risk or for the low and high risk consumers may increase. If a pooling contract uses experience ratings, existence of a competitive equilibrium may or may not be more likely in multiperiod models than in a single period setting. We leave this as an open research question.

Conclusions

This paper provides a theoretical explanation for the common observation of both incomplete insurance and experience rating. Due to asymmetric information about accident probabilities, insurance contracts must provide incentives for consumers to reveal their true types to the firm. In a multiperiod setting, experience rating is a means of improving the sorting of consumers. The monopolist links the terms of future contracts to past experience in a manner which thwarts the incentive of high risk agents to claim that they are in the low risk class. In a competitive environment, experience rating can serve the same purpose though its effectiveness is tempered by the possible entry of other insurance companies once punishments for a bad accident record are imposed.

This model should be contrasted with the recent work of Harris-Holmstrom. In that model, information was imperfect and symmetric. In our framework, this would mean that neither the insurance company nor the consumer knew true accident probabilities.[4] Harris-Holmstrom show that in this setting the evolving terms of the contract would reflect the public information on consumer characteristics contained in past experience. In our model, consumers do know their true accident probabilities. Experience rating is still used as a means of extracting this information.

Our analysis is still incomplete in a number of important ways. We have restricted attention in the competitive case to separating solutions. We need to relax this assumption which may require the use of a Wilson equilibrium. Also, we have assumed that the choice of contract in the first period and histories are not public information. We plan to consider the optimal contract under these alternative information structures. Finally, the nature of multi-period pooling contracts and the existence of competitive equilibrium with multi-period contracts remains to be examined.

APPENDIX A

Proof of Proposition 1

Using ϕ_H and ϕ_L as the Lagrange multipliers for constraint (2.8), λ_H for (2.9) and λ_L for (2.10), it is necessary to first show that $\lambda_H > 0$ and $\lambda_L = 0$ in the solution to (2.7). This pattern of binding constraints is common in self-selection problems although this problem is more complicated due to the number of choice variables exceeding two.

First, one can demonstrate that <u>both</u> λ_H and λ_L cannot be positive. If the high-risk agents are indifferent between δ_L^2 and δ_H^2 then the low-risk agents will strictly prefer δ_L^2. Next, $\lambda_L > 0$ and $\lambda_H = 0$ leads to a violation of the individual rationality constraint for the low-risk agents.

Given that $\lambda_H > 0$, it is obvious from the first-order conditions that $\phi_L > 0$. Whether or not ϕ_H is positive remains an (unimportant) open question. The first-order conditions below assume $\phi_H = 0$ (which we conjecture is correct). Our main results reported in Proposition 1 do <u>not</u> depend on the sign of ϕ_H. It is straightforward to generalize these arguments to characterize the T-period contract as in (2.17) and (2.18).

With ϕ_L and λ_H positive, we can write the first-order conditions to (2.7) as:

(A.1) $\quad (P_L) \qquad N_L = \left(\phi_L - \lambda_H \dfrac{(1-\pi_H)}{(1-\pi_L)}\right) U'(W - P_L)$

(A.2) $\quad (I_L) \qquad N_L = \left(\phi_L - \lambda_H \dfrac{\pi_H}{\pi_L}\right) U'(W - D + I_L)$

(A.3) $\quad (P_H) \qquad N_H = \lambda_H U'(W - P_H)$

(A.4) $\quad (I_H) \quad N_H = \lambda_H U'(W - D + I_H)$

(A.5) $\quad (P_L(A)) \quad N_L = \left(\phi_L - \lambda_H \dfrac{\pi_H(1 - \pi_H)}{\pi_L(1 - \pi_L)}\right)U'(W - P_L(A))$

(A.6) $\quad (I_L(A)) \quad N_L = \left(\phi_L - \lambda_H \dfrac{\pi_H \pi_H}{\pi_L \pi_L}\right)U'(W - D + I_L(A))$

(A.7) $\quad (P_L(N)) \quad N_L = \left(\phi_L - \lambda_H \dfrac{(1 - \pi_H)(1 - \pi_H)}{(1 - \pi_L)(1 - \pi_L)}\right)U'(W - P_L(N))$

(A.8) $\quad (I_L(N)) \quad N_L = \left(\phi_L - \lambda_H \dfrac{\pi_H(1 - \pi_H)}{\pi_L(1 - \pi_L)}\right)U'(W - D + I_L(N))$.

Note that these do not include the conditions for the choice of $P_H(A)$, $P_H(N)$, $I_H(A)$ and $I_H(N)$. It is easy to show that (A.3) and (A.4) will characterize the choice of these variables. Hence, high-risk agents obtain perfect insurance.

Conditions (A.5) through (A.8) characterize the adjustments in the insurance of low risk agents. Since $\pi_H > \pi_L$, the remainder of proposition 1 is easy to show. Q.E.D.

APPENDIX B

First order conditions for the binding two period competitive contract are similar to those for the monopoly problem presented in appendix A. Let μ_i be the multiplier on the zero profits constraint for type i, and let λ_i be the multiplier on the incentive compatibility constraints. Then $\lambda_L = 0$ and $\lambda_H > 0$ from the discussion in section three, and $\mu_L > 0$ and $\mu_H > 0$. So first order conditions for the low risk types are:

$$\text{(B.1)} \qquad (P_L) \qquad \mu_L = \left(1 - \lambda_H \frac{(1-\pi_H)}{(1-\pi_L)}\right) U'(W-P_L)$$

$$\text{(B.2)} \qquad (I_L) \qquad \mu_L = \left(1 - \lambda_H \frac{\pi_H}{\pi_L}\right) U'(W-D+I_L)$$

$$\text{(B.3)} \qquad (P_L(A)) \quad \mu_L = \left(1 - \lambda_H \frac{(1-\pi_H)\ \pi_H}{(1-\pi_L)\ \pi_L}\right) U'(W-P_L(A))$$

$$\text{(B.4)} \qquad (I_L(A)) \quad \mu_L = \left(1 - \lambda_H \frac{\pi_H\ \pi_H}{\pi_L\ \pi_L}\right) U'(W-D+I_L(A))$$

$$\text{(B.5)} \qquad (P_L(N)) \quad \mu_L = \left(1 - \lambda_H \frac{(1-\pi_H)(1-\pi_H)}{(1-\pi_L)(1-\pi_L)}\right) U'(W-P_L(N))$$

$$\text{(B.6)} \qquad (I_L(N)) \quad \mu_L = \left(1 - \lambda_H \frac{(1-\pi_H)\ \pi_H}{(1-\pi_L)\ \pi_L}\right) U'(W-D+I_L(N))$$

For high risk types,

$$\text{(B.7)} \qquad (P_H) \qquad \mu_H = U'(W-P_H)(1+\lambda_H)$$

$$\text{(B.8)} \qquad (I_H) \qquad \mu_h = U'(W-D+I_H)(1+\lambda_H)$$

(B.9) $(P_H(A)) \quad \mu_H = U'(W-P_H(A))(1+\lambda_H)$

(B.10) $(I_H(A)) \quad \mu_H = U'(W-D+I_H(A))(1+\lambda_H)$

(B.11) $(P_H(N)) \quad \mu_H = U'(W-P_H(N))(1+\lambda_H)$

(B.12) $(I_H(N)) \quad \mu_H = U'(W-D+I_H(N))(1+\lambda_H)$

These conditions generate the adjustments in the contract for low-risk agents reported on page 16.

FOOTNOTES

[1]Beth Hayes was killed tragically on June 3, 1984. This revision of our February 1984 manuscript reflects comments of anonymous referees and is intended to clarify our earlier exposition.

[2]Rubinstein-Yaari deal with the problem of moral hazard while Dionne and Townsend concentrate on self-selection. Malueg combines the two.

[3]To see this formally, one must write down the T period version of (7). Since the probabilities associated with the history enter multiplicatively, their order does not matter. Hence, the contract terms will depend on the relative numbers of accident and no-accident probabilities preceding the firm's profits for that history. We leave the details to the interested reader.

[4]A similar situation of experience rating with imperfect and asymmetric information is considered by Bigelow [1983].

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