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COMBINATORIAL OPTIMIZATION FOR ANALYZING ECONOMIC LEGISLATION
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ABSTRACT

Blockmodeling, a combinatorial technique for relational data analysis, is applied to studying texts of complex economic legislation. By making this area a subject for mathematical modeling, using methods related to combinatorics, logic, and discrete optimization, we describe a new type of frontier between law and economics.

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As a technique of mathematical modeling, blockmodeling provides a versatile way of identifying and analyzing complex relational patterns contained in social networks [see Arabie, Boorman, and Levitt, 1978 for review of the area]. We here propose that blockmodeling need not be limited to systems of persons or organizations, but may also be employed to analyze certain extremely complicated cultural products associated with legislative expression of economic policies (e.g., the Internal Revenue Code, Social Security Act, ERISA, bankruptcy laws, law and regulations of the Fed, etc.). Such complex statutes, often containing thousands of component rules interacting with each other, generate a distinct--fundamentally more abstract--level of social structure than markets, hierarchies, or social networks. These rule systems are sufficiently important as well as often independent in patterns of development [Clark, 1977; Ackerman, 1977] to justify their own special classes of models and methods of analysis. Aside from a handful of contributions to the law and economics literature [e.g., Landes and Posner, 1976 pursuing a legal rules/capital stocks analogy], approaches to the analysis of complex rule systems have remained an open modeling problem. In this paper we employ blockmodeling as a general mapping tool to chart statutory "legal landscapes" [Calabresi, 1982] on an operational basis.

1. Statutes as Data Structures: Approaches to Blockmodeling Systems of Ideas

We introduce blockmodels from a combinatorial optimization viewpoint which follows Boorman [1981; see also Arabie, Boorman, and Levitt for relations to other combinatorial problems]. The central data structure

in this field is a multigraph $\mathcal{A} = \langle S; \{A_m\}_{m=1}^k \rangle$ where S is a set of N entities and $\{A_m\}_{m=1}^k$ is a family of directed graphs each contained in $S \times S$, commonly identified with 0-1 incidence matrices $[a_{pq}^{(m)}]_{N \times N}$. The first goal of blockmodeling is to partition S into $c > 1$ blocs, $P = \{B_1, B_2, \dots, B_c\} \in \mathcal{L}(S)$ [the partition lattice on S], to separate as effectively as possible high-density from low-density regions of the A_m [i.e., submatrices of the permuted and blocked incidence matrices]. Here c reflects the degree of data aggregation $1 < c < N$ desired by the investigator [Boorman, 1970].

This "optimal separation" goal may be made concrete by seeking to maximize the following objective function over $\{P : P \in \mathcal{L}(S) \ \& \ |P| = c\}$

$$T = \sum_{m=1}^k \sum_{i,j=1}^c s(B_i, B_j) \left[d_{ij}^{(m)} - \mu_m \right]^2 \quad (1)$$

where

$$s(B_i, B_j) = |B_i| |B_j| - \delta_{ij} |B_i|, \quad (2)$$

$$d_{ij}^{(m)} = \frac{1}{s(B_i, B_j)} \sum_{p \in B_i} \sum_{q \in B_j} a_{pq}^{(m)}, \quad A_m = [a_{pq}^{(m)}]_{N \times N} \quad (3)$$

$$\mu_m = \frac{1}{N(N-1)} \sum_{p,q=1}^N a_{pq}^{(m)}, \quad (4)$$

δ_{ij} = Kronecker delta, $|X|$ = size of set X . Observe that this formulation excludes contributions from diagonal elements $a_{ii}^{(m)}$, which are to be coded as zeroes in the A_m ; see Arabie, Boorman, and Levitt (1978), also noting connections with structural equivalence ideas in model theory within mathematical logic [Shoenfield, 1967]. By

definition, $d_{ii}^{(m)} = 0$ if $|B_i| = 1$.

As in the Traveling Salesman Problem and many other problems in the NP-complete class [Garey and Johnson, 1979], it is possible to achieve (formally local) optima for \underline{T} through stepwise hillclimbing methods. The optimum reported in Fig. 1 was the best of 100 such local optima derived from random initial partitions using a "first improvement" move heuristic [Papadimitriou and Steiglitz, 1982, p. 469].

Observe that the present version of blockmodeling carries over to a setting of nonmarket social structures a principal hallmark of general equilibrium models in neoclassical economics [Arrow and Hahn, 1971], namely, the characteristic simultaneous dependence of every part of the solution (here, bloc assignment of a given member of \underline{S}) on every other part (the prevailing bloc assignments of all other members of \underline{S}). Also as in general equilibrium, nonuniqueness issues may arise in blockmodeling, corresponding in the present setting to solutions with \underline{T} values close to the best one.

In sociometric and related applications of blockmodeling, \underline{S} is usually a set of specific persons and $\{A_m\}_{m=1}^k$ is a collection of (binary) social networks, e.g., recording passage of favors or of information [Boorman and Levitt, 1981]. In the present application, we let \underline{S} be a set of "rules," typically separate sections of some piece of complex (governmental) legislation but possibly also rules of private sector origin (e.g., a set of accounting standards). The $\{A_m\}_{m=1}^k$ are then chosen to be different types of citations connecting the sections, as developed from statutory texts, legislative histories, administrative interpretations, or judicial materials (cases).

This picture of a complex statute as a (multi)relational structure

is considerably closer to the ways lawyers tend to describe such material [e.g., Frankfurter, 1947; Stewart, 1975, p. 1813] than to the ways neo-classical economists are accustomed to embedding most problems in \mathbb{R}^n (e.g., the Landes-Posner approach to legal rules as capital stocks). Following the lead provided by sociometric uses of blockmodeling, the combinatorial optimization of (1) may be used to obtain, not merely a partition of the sections, but also a summary of the "legal topography" via the simplified relational structure $\mathcal{D} = \langle \{1, 2, \dots, c\}; \{\mathcal{D}_m\}_{m=1}^k \rangle$ whose nodes are the c blocs and whose relations $\mathcal{D}_m = [d_{ij}^{(m)}]_{c \times c}$ are valued graphs [from (3) above] defining the "connectivity" of the statute.

Where the connectivity of the statute may be seen as evolving (e.g., because groups of sections become linked through judicial opinions interpreting the law) one may also construct time-dependent blockmodels using case data to obtain a "moving picture" of the statutory evolution, as a more complete picture of connections among the rules emerges under litigation. Details of this approach, which may be especially useful in the spirit of an "X-ray" technology for detecting stress points or trouble spots in complex legislation [Gilmore, 1965, p. x], will be developed elsewhere.

2. Application: Law (UCC Article 9) Governing Secured Transactions

We illustrate statutory blockmodeling using an important body of 1950's-vintage state-level law particularly relevant to an economy with many bankruptcies [Scott, 1977]. With some local variations, Uniform Commercial Code (UCC) Article 9 governs security interests in personal property in 49 states and is renowned as a field of formidable legal complexity [see 1500-page treatise of Gilmore, 1965]. The Article 9

statute, containing 55 sections in its 1978 Official Text, will be analyzed as four networks $\{A_m\}_{m=1}^4$ (see Fig. caption), seeking optima of T for $c = 8$ blocs only (which by conventional legal analysis standards remains fairly "coarse"). Also to save space, we report in the Figure only the Boolean sum $\bigvee_{m=1}^4 A_m$, permuted and partitioned to correspond to the best 4-network optimum identified ($T = 170.17$).

Bloc I contains sections delineating the law's broadest outlines-- general policy decisions aimed at legal unification, transactions included and excluded, and the cornerstone Article 9 concept of "attachment" (defining the point when a security interest is created). Bloc II states the rules of "perfection," establishing Article 9 mechanisms for fortifying claims to a high priority when rival security interests of multiple creditors clash. Bloc III collects the basic priority rules themselves. While these three blocs are all small, they demarcate three of the most important technical unifications imposed on a jumble of earlier law by the enactment of Article 9, as well as the parts of Article 9 of most immediate interest to economic modelers and policy analysts [e.g., Jackson and Kronman, 1979 interpreting Bloc II and III rules as reducing transaction costs, cf. Williamson, 1979; see also Dauer and Stern, 1976 on Bloc III unanticipated interactions with the 1966 Federal Tax Lien Act, with microeconomic effects reducing capital availability to the construction industry].

Bloc IV may be characterized as ancillary, primarily technical provisions occupying a supporting role vis-a-vis Blocs I-III. Supporting this interpretation, note that $d_{4i} \geq d_{i4}$ ($i = 1, 2, 3$) and $d_{44} \ll 1$ (calculating densities from [3] applied directly to the Boolean union matrix reproduced in Fig. 1).

Bloc V, a singleton, has the high centrality one would expect of the main repository of statutory special definitions.

Bloc VI on first sight appears to be a medley of special rules, but on further scrutiny emerges heavily in a "liaison" role between Article 9 and diverse adjoining legal fields (in this connection, note the high sparsity of ties reflected in low densities d_{16} , d_{61}). By the same token, Bloc VI is a natural place to seek loopholes and other slippages resulting from draftsmen's specialist preoccupations. As instances of conceptual outreach, Bloc VI includes Article 9's remedies upon debtor's default--essentially collection problems presenting issues largely separable from Article 9's basic system [Davenport and Murray, 1978, p. 257; Leff, 1970; cf. also Green, 1974, p. 265]. Other examples include Bloc VI's linkages to laws establishing suppliers' lien priorities warping Article 9 rules (Section 9-310); to general property law (e.g., Section 9-202 cutting off possible interactions with traditional, and sometimes extremely muddy, "title" ideas); and to "information control" principles (Sections 9-208, 9-407) bearing on free access to information versus privacy choices [compare Gilmore, 1965, pp. 472-73].

Bloc VII wrenches one of the major priority rules (Section 9-313, covering priority of security interests in personal property affixed to realty vis-a-vis real estate interests) out of sequence and places it with two definitional provisions. This placement signals possible trouble, at any rate something unusual, and in fact Section 9-313's corner of the law has been associated with considerable legal instability even over Article 9's brief history. At the root of the difficulties is Anglo-American law's "island model," according to which "the law of real property and the law of personal property are separate and non-communicating compartments" [Gilmore, 1965, p. 54]. This distinction is essentially

artificial in a modern economy but is perpetuated through the shape of the law, including the fit of Section 9-313 in the larger Article 9 structure, and imposes real social costs as a byproduct of conceptual rigidities [compare also Green and Shoven, 1982].

Bloc VIII covers the main "Article 9 paperwork," also rearranging the published order of this statute to place in this bloc the closely related Section 9-103 addressing multiple jurisdiction (conflict-of-laws) problems. This bloc hence covers material which lawyers would tend to see as primarily administrative or practice-oriented.

Examining the blockmodel as a whole, note also the strikingly high "center" to "periphery" ratio (e.g., Bloc VI alone has 26 of 55 sections). This finding, which our experience in blockmodeling other statutes shows to be widespread, suggests that much complex legislation is actually far more primitive than first appears, with only a small fraction of the material being allocated to institutional innovation as economic policy-makers would be likely to understand it.

3. Discussion

Using concrete data from a significant area of economic legislation and policy, we have suggested how blockmodeling can provide a practical formal way of decomposing exceedingly complex rule systems into simpler components, with a far more perspicuous pattern also resulting as to the "division of labor" among parts of the statute [compare the traditions represented by Simon, 1981 and Futia, 1977 which, however, have not dealt extensively with actual data].¹ With limited exceptions (e.g., price control) economic analyses tend to adopt highly simplified representations of legal and other rules actually operating [e.g., see Hellwig,

1977 excluding security interests from a model of bankruptcy]. As regulatory schemes and other varieties of economic legislation continue to proliferate, however, and once established show themselves remarkably difficult to redesign and to control [Feldstein, 1976; Roberts et al., 1972], there is a need for new attention to the identification of patterns in these conceptual structures and their consequences for management of the economy.

Footnote

1. Discrete optimization methods may also be used to approach problems of optimal legislative design and drafting, exploiting Lenstra's (1974) observation that the optimal matrix seriation problem is reducible to a Traveling Salesman Problem (TSP). The optimal organization of new legislation (or reorganization of old) thus becomes a new class of applications for combinatorial optimization techniques.

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Figure 1. Article 9 blockmodel, shown imposed on union of four data

matrices $A_m = [a_{pq}^{(m)}]$:

$a_{pq}^{(1)} = 1$ if Section p cites Section q in Article 9 text,
0 otherwise;

$a_{pq}^{(2)} = 1$ if Official Comments to Section p cite Section q ,
0 otherwise;

$a_{pq}^{(3)} = 1$ if Cross-references listed followed Section p cite q ,
0 otherwise;

$a_{pq}^{(4)} = 1$ if Definitional cross-references so listed cite q ,
0 otherwise.

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