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THE CONVENTIONALLY STABLE SETS IN NONCOOPERATIVE GAMES WITH LIMITED OBSERVATIONS: THE APPLICATION TO MONOPOLY AND OLIGOPOLY

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THE CONVENTIONALLY STABLE SETS IN NONCOOPERATIVE GAMES WITH LIMITED OBSERVATIONS: THE APPLICATION TO MONOPOLY AND OLIGOPOLY*

by

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Abstract: This paper applies the theory of the conventionally stable set to monopolistic and oligopolistic markets. A market model with a finite number of producers and a continuum of buyers is presented and then is formulated as a strategic game in which the producers' strategies are prices and the buyers' strategies are demands for commodities. It is shown that a conventionally stable set in this game corresponds to a conventionally stable one in a game where the producers are only players but the buyers are treated as a certain kind of demand function. Furthermore, it is shown that the theory of the conventionally stable set is compatible with the classical monopoly solution, the kinked-demand-curve solution and the leader-follower solution. This new theory makes their structures much more transparent.

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1. Introduction and Definitions

1. Kaneko [7] proposed a new noncooperative solution concept called "the conventionally stable set" for situations in which games in normal (strategic) form are repeated and players' observations about outcomes are limited. In [7], conventionally stable sets for several simple games are considered and the results are very strong and satisfactory. One result (Proposition 7 in [7]) states that the Nash equilibrium concept makes sense only in a very restricted situation which is very similar to a classical "perfectly competitive market."[1] However the Nash equilibrium concept has been traditionally employed as a solution concept for oligopolistic (duopolistic or monopolistic) markets. Obviously the above result contradicts this tradition. Therefore we have to consider the problem of what the new theory predicts if it is applied to oligopolistic markets. The purpose of this paper is to apply the new theory to a price-quantity oligopolistic market model.

The application of the new theory to the oligopolistic market gives solutions for several unsolved questions in economic and game theoretic literature in certain manners. The following list of the questions may help the reader to capture the meanings of the results of this paper:

(Q-1) In the standard oligopoly theories, the demand sides are treated just as demand functions. This may be equivalent to assume that buyers behave as price takers. However there is no formal theory in which buyers are treated as players who behave strategically and at the same time which can explain fully the assumption of demand functions.[2] The question is to explain this assumption from the game theoretical view point.

(Q-2) Consider a simple monopolistic market which consists of a monopolist (a seller) and a large number of buyers. In this market, the classical monopolistic solution which maximizes his profits defined by the demand function would be plausible. However there is no game theoretical explanation of it.[3] The question is to give a satisfactory explanation of it.

(Q-3) In the economic and game theoretic literature, there are several theories which do not have any fully theoretical foundations but have been accepted by many empiricists, e.g., the kinked-demand-curve solution and the leader-follower solution. The question is to give the theoretical foundations of them.

These three questions are mutually related and it is the most desirable to solve these questions simultaneously from one theoretical view point. In fact, the theory of conventionally stable set gives solutions to these questions simultaneously. The purpose of this paper is to show it.

The rest of this paper is written as follows. This section gives the definition of the conventionally stable set again. In Section 2, we provide a price-quantity market model which consists of a finite number of oligopolists (a monopolist) and a continuum of buyers and in which oligopolists' decision variables are prices and buyers' decision variables are quantities of commodities. It is shown that a conventionally stable set in this market corresponds to a conventionally stable set in the adjoint oligopolistic market which is generated from the original market by fixing each buyer's demand function. This result is really a foundation of our oligopoly theory and the answer to the question (Q-1). Therefore this result enables us to restrict our consideration to the adjoint oligopolistic market, which is the standard treatment of an oligopolistic market. Next we will show that the new solution concept can explain the classical monopoly solution in a certain sense in a monopolistic market. In fact, this explanation has a much deeper meaning than just an explanation. In oligopolistic markets, we show the much more surprising result that a kinked-demand-curve solution, a leader-follower solution in a certain sense and a hybrid of them appear as conventionally stable sets.

2. This subsection gives the definition of the conventionally stable set and explains the meaning of it briefly. For a fuller explanation of it, see Kaneko [7].

A game in normal form with limited observations is given as a quintuple $G = (N, \{S_i\}_{i \in N}, S, \{h_i\}_{i \in N}, \{a_i\}_{i \in N})$. The constituents are as follows: N is the set of all players, which may be finite or infinite. S_i is the set of all pure strategies of player $i \in N$.[4] S is the set of all outcomes $s = (s_i)_{i \in N}$ which are functions from N to $\bigcup_{i \in N} S_i$ with conditions

$$s_i \in S_i \text{ for all } i \in N , \tag{1}$$

$$\text{for all } i \in N , \ (s_{-i}, s_i') \in S \text{ for all } s_i' \in S_i , \tag{2}$$

where s_{-i} is the restriction of s on $N - \{i\}$. h_i is the payoff function of player $i \in N$, which is a function from S to R (the set of all real numbers). a_i is a partition of S which satisfies

$$\text{for any } s \in T \in a_i \text{ and } s' \in T' \in a_i , \text{ if } s_i \neq s_i' \text{ or } h_i(s) \neq h_i(s') , \text{ then } T \neq T' . \tag{3}$$

We call $\{a_i\}_{i\epsilon N}$ an <u>observation structure</u>. Let $a_i(s)$ be the function on S such that $s \epsilon a_i(s) \epsilon a_i$ for all $s \epsilon S$, which we call the <u>observation function</u> of player $i \epsilon N$.

It should be emphasized that we are considering a situation where the game G is played repeatedly. Due to the repetition of the game the observation structure $\{a_i\}_{i\epsilon N}$ can be easily interpreted as follows. When the game is played and an outcome s appears, player i observes the outcome s and obtains information $a_i(s)$. Then he will behave in the next period using this information. The most fundamental assumption made is that no player can directly know the other players' payoff functions but he can only know what he has observed in the other players' behavior.

Condition (3) is interpreted as follows. If player i plays two different strategies or if he receives different payoffs, then he can distinguish one outcome from the other. This is the minimum requirement for rational players. An important note is that we do not assume that player i can recognize $a_i(s)$ as a set but that player i can distinguish $a_i(s)$ from other $a_i(s')$. This implies that even if a_i were the finest partition, player i could not recognize each outcome itself but could only distinguish it from the others.

Now we are in a position to define the conventionally stable set. A <u>response function</u> of player $i \epsilon N$ is a function from S to S_i which satisfies

$$\text{if } a_i(s) = a_i(s'), \text{ then } r_i(s) = r_i(s'). \tag{4}$$

A <u>response configuration</u> r is a function from S to S such that for all $i \epsilon N$, r_i is a response function. r_{-i} denotes the restriction of r on $N-\{i\}$.

Since we are considering a situation where the game G is played repeatedly, it will be assumed that each player's temporal behavior depends on temporal states . This does not mean that each player's behavior is independent from his previous history. Rather, it is assumed that each player has fully learned (with respect to his observations) the society's responses to his actions. Although each player's temporal behavior depends upon temporal states. his entire behavior depends upon his knowledge on the society's responses to his actions, which has been acquired from the previous history. In particular, condition (4) means that his temporal behavior depends upon his temporal observation. This assumption will be generalized a little in Section 5.

In the following, let r be a fixed response configuration. We call the set $F(r) = \{s \in S : r(s) = s\}$ the <u>stationary set</u> of r and call each s in $F(r)$ a <u>stationary state</u>.

Let s and s' be two outcomes, which may be the same. We say that s <u>is absorbed by</u> s' iff

$$\text{for a finite } k \text{ , } \quad r^k(s) = \overbrace{r \cdot r \cdot \ldots \cdot r}^{k \text{ times}}(s) = s' \in F(r) \text{ .} \tag{5}$$

We call $\{s, r(s), \ldots, r^{k-1}(s), s'\}$ <u>an absorption of</u> s <u>by</u> s' .

Let $\{s^1, \ldots, s^k\}$ be an absorption of s^1 by s^k , i.e., $s^t = r^{t-1}(s^1)$ for $t = 1, \ldots, k$. After player i observed s^t $(1 \leq t \leq k)$, he gets information $a_i(s^t)$. If he notices at s^t that this absorption has started at s^1 , then he can use not only the observation $a_i(s^t)$ but also the accumulation of observations $a_i(s^1), \ldots, a_i(s^t)$. To represent accumulations of observations, we extend $a_i(s)$ as follows: for any absorption $\{s^1, \ldots, s^k\}$, we define $\hat{a}_i$ by

$$\hat{a}_i(\{s^1\}) = a_i(s^1) \tag{6}$$

$$\hat{a}_i(\{s^1, \ldots, s^t\}) = a_i(s^t) \cap r[\hat{a}_i(\{s^1, \ldots, s^{t-1}\})]$$

$$\text{for } t = 2, \ldots, k .$$

Note that if $\{s^1, \ldots, s^k\}$ is an absorption, then $\{s^t, \ldots, s^k\}$ and $\{s^t, \ldots, s^k, \ldots, s^k\}$ are also absorptions for every t $(1 \leq t \leq k)$.

Let s and s' be two outcomes. We say that s' _is connected to_ s by a _deviation_ $\{t_i^1, \ldots, t_i^k\}$ of player i from his response function r_i iff

$$t_i^m \text{ is a response function of player } i \text{ for } m = 1, \ldots, k , \tag{7}$$

$$(r_{-i}, t_i^k)\cdot(r_{-i}, t_i^{k-1}) \cdot \ldots \cdot (r_{-i}, t_i^1)[s] = s' . \tag{8}$$

We say that player i _is effective for_ $s \in S$ _in_ r iff

$$r_{-i}(s) = s_{-i} . \tag{9}$$

If player i is effective for s in r, then he can make s a stationary state, and, in fact, s is a stationary state unless he changes his strategy.

We say that player i _can improve upon_ an absorption $\{s^1, \ldots, s^k\}$ of s^1 by s^k at s^h $(1 \leq h \leq k)$ iff player i has a deviation $\{t_i^1, \ldots, t_i^m\}$ such that for any $\hat{s} \varepsilon \hat{a}_i(\{s^1, \ldots, s^h\})$, player i is effective for the outcome s'' which is connected to $\hat{s}$ by the deviation $\{t_i^1, \ldots, t_i^m\}$ and $h_i(s'') > h_i(s^k)$. (10)

Note the last inequality $h_i(s'') > h_i(s^k)$, i.e., player i compares the new stationary payoff $h_i(s'')$ with the original stationary payoff $h_i(s^k)$, and that s'' is uniquely determined by the deviation $\{t_i^1, \ldots, t_i^m\}$ and $\hat{s}$.

The concept "can improve upon" has the following meaning. Let an absorption $\{s^1, \ldots, s^k\}$ be occurring. If a player i notices this absorption with respect to his observations, and if he thinks that it is possible to make this absorption change and converge to another stationary state with a higher payoff than $h_i(s^k)$ by deviating from his response function r_i, then he wants to do so. If some player has such a possibility, then this absorption is not necessarily "stable." Hence a "stable" absorption should be free from such strategic behavior. We call an absorption $\{s^1, \ldots, s^k\}$ of s^1 by s^k stable iff any player can not improve upon this absorption at s^h for any $h = 1, \ldots, k$.

Now it is possible to define the conventionally stable set.

Definition. A response configuration r is said to be conventionally stable iff

(Stability): every absorption is stable, (11)

(Absorptivity): there is a finite k such that for any $s \varepsilon S$, there is an absorption $\{s^1, \ldots, s^h\}$ with $s = s^1$ and $h \leq k$. (12)

(Acyclicity): for any $i \in N$, there is a finite h such that (13)

for any $s \in S$ and $s_i' \in S_i$, $(r_{-i}, e_i)^h[(s_{-i}, s_i')]$ $= (r_{-i}, e_i)^{h-1}[(s_{-i}, s_i')]$, where e_i is the identity response function, i.e., $e_i(s) = s_i$ for all $s \in S$.

We call a subset V of S a conventionally stable set iff there is a conventionally stable response configuration r whose stationary set $F(r)$ coincides with V .

Note that condition (12) implies the nonemptiness of the stationary set.

The stability condition and the absorptivity condition require that any player have no incentive to deviate from his response function and the society reach a stationary state even if the society is purturbed by some reason. That is, every player obeys his response function as far as the others obey theirs, and conversely, since all the players obey their response functions, each must obey it. In this sense, a conventionally stable response configuration is a socially accepted standard of behavior, which is very similar to the concept of von Neumann and Morgenstern's "standard of behavior" [16]. Note that each player's knowledge about the response configuration is based on his observations and is limited in general. The acyclicity condition has the following meaning. In the past each player made trials and errors many times. The simplest trial of a player is to fix a strategy and then to observe the society's reactions to his fixing the strategy. In this case the acyclicity condition requires that the social reactions eventually reach a stationary state.

2. The Price-Quantity Oligopoly Game and the Adjoint Price-Oligopoly Games

This section presents a price-quantity oligopoly game Γ in which not only oligopolists but also buyers are treated as players, and shows that it is sufficient to consider conventionally stable sets in adjoint price-oligopoly games which are derived from the original game by fixing the buyers' response functions (i.e., observable demand functions).

3. Let $(N, \mathcal{B}, \mu)$ be a measure space of players, where $N = [-1,0] \cup \{1, 2, \ldots, n\}$ is the set of all players, $\mathcal{B}$ the minimal σ-algebra including every Lebesgue measurable set of $[-1,0]$ and every $\{i\}$ for $i = 1, \ldots, n$ and μ a finite measure on $\mathcal{B}$ with $\mu([-1,0]) > 0$, $\mu(\{j\}) = 0$ for all $j \in [-1,0]$ and $\mu(\{i\}) = 1$ for all $i = 1, \ldots, n$. A player in $[-1,0]$ is called a buyer and a player i $(i = 1, \ldots, n)$ is called an oligopolist (a monopolist if $n = 1$). We denote $[-1,0]$ and $\{1, 2, \ldots, n\}$ by T and N^*, respectively.

Each oligopolist produces and sells one commodity which is differentiated from the commodities produced by the other oligopolists. Oligopolist i's cost function is given as

$$C_i(q_i) = q_i \quad \text{for all} \quad q_i \geqq 0 . \tag{14}$$

Each oligopolist's cost function is assumed to have no capacity limit and he has a sufficiently large stock capacity. The cost functions are measured in terms of the composite commodity which is called "money." Every buyer $j \in T$ has a utility function $u_j(x) = u_j(x_1, \ldots, x_n, x_{n+1})$ on R_+^{n+1}, where x_i $(i = 1, \ldots, n)$ is a consumption level of the commodity produced by oligopolist i and x_{n+1} is a consumption level of money. Here R_+^{n+1} is the nonnegative orthant of the $n+1$ dimensional Euclidean space. We make the following assumptions:

(A) for all $j \in T$, $u_j(x)$ is a strictly quasi-concave and continuous function;

(B) for fixed $x \in R_+^{n+1}$, $u_j(x)$ is a measurable function of j on T .

Every buyer $j \in T$ is initially endowed with income $I_j > 0$, and I_j is a measurable and integrable function of j on T .

4. The Price-Quantity Oligopoly Game Γ : Consider the following rules of game: Every buyer reveals his demand vector and every oligopolist does his price simultaneously, and then the trades are determined according to these demand vectors and prices. Formally, the game Γ is defined as follows. Each buyer j's strategy space S_j is R_+^n and oligopolist i's strategy space S_i ($\equiv P_i$) is $[1,+\infty)$. The set of all outcomes S is the set of all pairs $(s_T, p) = ((s_j)_{j\in T}, p_1, \ldots, p_n)$ such that s_j is a measurable function on T with $s_j \in S_j$ for all $j \in T$ and $p_i \in P_i$ for all $i = 1, \ldots, n$. We define buyer j's trade function $g_j(s_T, p)$ on S by

$$g_j(s_T, p) = \begin{cases} s_j & \text{if } \sum_{i=1}^{n} p_i s_{ji} \leq I_j \\ s_j \cdot \nu_j & \text{otherwise} \end{cases} \qquad (15)$$

where $\nu_j = I_j / \sum_{i=1}^{n} p_i s_{ji}$. Note that $s_j = (s_{j1}, \ldots, s_{jn})$. We define payoff functions on S by

$$h_j(s_T,\ p) = u_j(g_j(s_T,\ p),\ I_j - \sum_{i=1}^{n} p_i g_{ji}(s_T,\ p)) \quad \text{for all} \quad j \in T^{5} \qquad (16)$$

$$h_i(s_T,\ p) = (p_i - 1)\int_T g_{ji}(s_T,\ p)d\mu(j) \quad \text{for all} \quad i \in N^* . \qquad (17)$$

The meaning of (15) is as follows: suppose that the buyers reveal their demand vectors and the oligopolists reveal their prices. If buyer j's demand vector s_j and the oligopolists' prices p satisfy j's budget constraint, then the trades are determined by them exactly, but otherwise, j's demand vector is reduced to the vector, $s_j \cdot \nu_j$, which has the same proportion as the original one and satisfies the budget constraint in equation.

In this paper, it is assumed that the game Γ has the following observation structure:

$$\text{for all} \quad j \in T , \quad a_j(s_T,\ p) = a_j(s'_T,\ p') \iff s_j = s'_j \ \&\ p_i = p'_i \quad \text{for all} \quad i \in N^* , \qquad (18)$$

$$\text{for all} \quad i \in N^* , \quad a_i(s_T,\ p) = a_i(s'_T,\ p') \iff \int_T g_{ji}(s_T,\ p)d\mu(j) = \int_T g_{ji}(s'_T,\ p')d\mu(j) \ \&\ p_t = p'_t \quad \text{for all} \quad t \in N^* . \qquad (19)$$

Condition (18) means that each buyer can observe only his demand and the oligopolists' prices revealed at the previous stage. Condition (19) means that each oligopolist can observe the amount which is sold to the buyer and the prices set by the oligopolists at the previous stage. Note that no oligopolist can observe buyers' individual demands.

5. The Ordinary Price-Oligopoly Game: To investigate conventionally stable sets in the game Γ, it is necessary to define another class of games. First, we define the ordinary price-oligopoly game G^*, which is a special case of the games which we will define. In the game G^*, it is assumed that every buyer behaves as a price-taker. That is, when the oligopolists set prices $p = (p_1, \ldots, p_n)$, each buyer solves the following maximization problem:

$$\max u_j(x_1^j, \ldots, x_n^j, x_{n+1}^j) \quad \text{subject to} \quad \sum_{i=1}^{n} p_i x_i^j + x_{n+1}^j \leqq I_j \tag{20}$$

Under assumptions (A) and (B) the individual competitive demand function $D^{*j}(p)$ is well-defined, i.e., $D^{*j}(p)$ solves (20). It is well-known that $D^{*j}(p)$ is continuous with respect to p and measurable with respect to j for fixed p. Since I_j is integrable and $p \geqq (1, \ldots, 1)$, $D^{*j}(p)$ is also integrable. We define $D^*(p)$ by

$$D^*(p) = \int_T D^{*j}(p) d\mu(j) . \tag{21}$$

We call $D^*(p)$ the competitive demand function.

The oligopolists' payoff functions f_i^* of the game G^* are given as

$$f_i^*(p) = (p_i - 1) D_i^*(p) \quad \text{for all} \quad p \geqq (1, \ldots, 1) . \tag{22}$$

In the game G^*, the buyers are not players but form only the environment the oligopolists face.

It is assumed that the game G^* has the finest observation structure, i.e.,

$$\text{for all} \quad i \in N^* , \quad a_i^*(p) = a_i^*(p') \iff p = p' . \tag{23}$$

That is, every oligopolist can observe the prices precisely.

We make the following assumption:

$$\text{(C)} \quad \text{for all } j \in T , \quad \sum_{i=1}^{n} p_i D_i^{*j}(p) < I_j \quad \text{for all } p \geqq (1, \ldots, 1) .$$

This assumption says that any buyer does not spend all his income to purchasing the commodities from the oligopolists when he behaves as a price taker.

Using the competitive demand function, we can characterize the nonexistence of a Nash equilibrium in the game Γ .[6]

Proposition 1. (i) An outcome (s_T, p) is a Nash equilibrium in the game Γ if and only if $s_j = D^{*j}(p_1, \ldots, p_n)$ for all $j \in T$ and $s_j = 0$ for almost all $j \in T$. (ii) There exists no Nash equilibrium in the game Γ if and only if for any p , $D_i^*(p) > 0$ for some $i \in N^*$.

It is easy to prove this proposition. From this proposition we can see that even if the game Γ has a Nash equilibrium, it is a pathological one.

6. Adjoint Price-Oligopoly Games: The first result is the following proposition.

Proposition 2. Let V be a conventionally stable set in the game Γ . Then every $(s_T, p) \in V$ satisfies

$$s_j = D^{*j}(p) \quad \text{for all } j \in T . \tag{24}$$

<u>Proof.</u> Let $V = F(r)$. Suppose that for some $(s_T, p) \in V$, there is a $j \in T$ such that $s_j \neq D^{*j}(p)$. Consider the absorption $\{(s_T,p),(s_T,p)\}$. By (18) and (19), if the buyer j changes his strategy s_j to $D^{*j}(p)$, no other player can observe this change, and so, no new response occurs. Hence the buyer j is effective for $(s_{T-j}, D^{*j}(p), p)$ and it gives him a higher payoff by (20). This argument is true for any $(s'_T, p') \in a_j(s_T, p)$ because $s'_j = s_j$ and $p' = p$ by (18). Therefore he can improve upon the absorption $\{(s_T,p),(s_T,p)\}$ by fixing his strategy to $D^{*j}(p)$. This is a contradiction. Q.E.D.

This proposition says that the buyers behave as price-takers in every stationary state in every conventionally stable set in Γ. The reason for this is that since each buyer's behavior does not influence the others through not only payoff functions but also direct observations, he can get a higher payoff by behaving as a price-taker in a stationary state.

Now we are going to show a stronger version of Proposition 2 in the following. Let $\tilde{N}$ be a subset of N. We say that $\{r_t\}_{t\in\tilde{N}}$ is <u>price-dependent</u> iff for all $t \in \tilde{N}$,

$$\text{if } p_i = p'_i \text{ for all } i \in N^*, \text{ then } r_t(s_T, p) = r_t(s'_T, p') . \quad (25)$$

If $\{r_t\}_{t\in\tilde{N}}$ is assumed to be price-dependent, then we sometimes regard each r_t as a function on $S_1 \times S_2 \times \ldots \times S_n$ instead of a function on S.

Let $r_T = \{r_j\}_{j\in T}$ be price-dependent. Then we define $D^{rj}(p)$ $(j \in T)$ and $D^r(p)$ by

$$D^{rj}(p) = g_j[r_j(p)] \quad \text{for all } p \geqq (1, \ldots, 1) , \quad (26)$$

$$D^r(p) = \int_T D^{rj}(p) d\mu(j) \quad \text{for all } p \geqq (1, \ldots, 1) . \quad (27)$$

We call $D^r(p)$ the <u>observed demand function derived from</u> r_T. Let G^r be the game which is obtained from the ordinary price-oligopoly game $G*$ by replacing the competitive demand function $D*(p)$ by $D^r(p)$. Of course, the players in the game G^r are the oligopolists, and we denote by $f_i^r(p)$ oligopolist i's payoff function, i.e., $f_i^r(p) = (p_i - 1)D_i^r(p)$. It is also assumed that the game G^r has the finest observation structure, i.e., it satisfies (23). The game G^r is called an <u>adjoint price-oligopoly game</u>. Of course, the ordinary price oligopoly game $G*$ is an adjoint price-oligopoly game, in other words, if r_j is the best response function for all $j \in T$, then G^r is the ordinary price-oligopoly game.

<u>Proposition 3</u>. Let $r_T = \{r_j\}_{j \in T}$ be price-dependent. Then r_{N*} is price-dependent and $r = (r_T, r_{N*})$ is conventionally stable in the game Γ if and only if r_{N*} is conventionally stable in the game G^r and for all $p \in F(r_{N*})$,

$$r_j(p) = D^{*j}(p) \quad \text{for all} \quad j \in T . \tag{28}$$

Before proving this proposition, let us consider its implication. A response configuration r is said to have the <u>price-taker property on the stationary set</u> iff (28) holds. Proposition 2 says that every conventionally stable response configuration in Γ has the price-taker property. Since each buyer can observe his demand and the oligopolists' prices revealed at the previous stage, his response function depends upon these observations in general. If the price-dependence property is assumed, his response function is independent from his previous strategy. This would be an innocuous requirement. Furthermore since each oligopolist can observe the prices and the total demand for his commodity, his response function depends upon these in general. The price-dependence property requires that his response function be independent from the total demand. However if the price-dependence property is assumed on the buyers, then the total demands for the commodities depend upon the prices as far as every player conforms his behavior to his response function and any disturbance does not occur. This implies that each oligopolist's response function depends upon the total demand indirectly. Therefore the requirement of price-dependence on the oligopolists would be also innocuous. Proposition 3 implies that if the price-dependence property and the price-taker property on the stationary set are assumed, it is sufficient to consider conventionally stable sets in the game G^r. As argued above, these assumptions are innocuous and so, for convenience sake, we restrict ourselves to the case of price-dependence in the following sections.

Propositions 2 and 3 are our answer to the question (Q-1). It is clear that this is not satisfactory as far as any good result in G^r is not reached. Therefore we will consider conventionally stable sets in G^r in the following sections.

Proof. Necessity: Since r_T is price-dependent, $D^{rj}(p)$ is well-defined by (26) and (28) is true by Proposition 2. It is easily verified that r_{N*} satisfies the absorptivity and acyclicity conditions in the game G^r. Let us prove the stability condition. Suppose that the stability condition does not hold. Then there is an oligopolist i_o who can improve upon some absorption $\{\hat{p}^1, \ldots, \hat{p}^k\}$ in the game G^r. That is, for some h $(1 \leq h \leq k)$, i_o has a deviation $\{\hat{t}^1_{i_o}, \ldots, \hat{t}^m_{i_o}\}$ such that

$$\hat{p}'' = (r_{N^*-i_o}, \hat{t}^m_{i_o}) \cdot \ldots \cdot (r_{N^*-i_o}, \hat{t}^1_{i_o})[\hat{p}^h] ,$$

$$f^r_{i_o}(\hat{p}'') > f^r_{i_o}(\hat{p}^k) , \tag{29}$$

and i_o is effective for $\hat{p}''$.

Note that G^r has the finest observation structure. It is sufficient to show that the game Γ also has an absorption which i_o can improve upon in Γ . We define the sequence $\{(s^1_T, p^1), \ldots, (s^k_T, p^k), (s^{k+1}_T, p^{k+1})\}$ in the game Γ by

$$p^\nu_i = \hat{p}^\nu_i \quad \text{for all} \quad i \in N^* \quad \text{and all} \quad \nu = 1, \ldots, k ,$$

$$s^1_j = r_j(p^1) \quad \text{for all} \quad j \in T ,$$

$$s^\nu_j = r_j(p^{\nu-1}) \quad \text{for all} \quad j \in T \quad \text{and all} \quad \nu = 2, \ldots, k+1 ,$$

$$\text{and} \quad p^{k+1}_i = \hat{p}^k_i \quad \text{for all} \quad i \in N^* .$$

It is easy to see that this sequence is an absorption in the game Γ . The oligopolist i_o can improve upon this absorption at (s^h_T, p^h) with a deviation $\{t^1_{i_o}, t^2_{i_o}, \ldots, t^m_{i_o}, t^{m+1}_{i_o}\}$ such that

$$t^\nu_{i_o}(s_T, p) = \hat{t}^\nu_{i_o}(p) \quad \text{for all} \quad (s_T, p) \in S \quad \text{and all} \quad \nu = 1, \ldots, m$$

$$\text{and} \quad t^{m+1}_{i_o} = e_{i_o}^{\hat{p}''_{i_o}} ,$$

where $e_{i_o}^{p''_{i_o}}$ is the constant response function with the value $\hat{p}''_{i_o}$.

Now we prove this assertion. Let $(\overline{s}_T, \overline{p})$ be an arbitrary outcome in $\hat{a}_{i_o}(\{(s^1_T, p^1), \ldots, (s^h_T, p^h)\})$. Then $(\overline{s}_T, \overline{p})$ satisfies $\overline{p} = \hat{p}^h$ by

(19). Therefore $(\bar{s}''_T, \bar{p}'') = (r_{-i_o}, t^{m+1}_{i_o}) \circ \ldots \circ (r_{-i_o}, t^1_{i_o})[(\bar{s}_T, \bar{p})]$ satisfies

$$\bar{p}'' = \hat{p}'' \quad \text{and} \quad s''_j = D^{rj}(\bar{p}'') \quad \text{for all} \quad j \in T .$$

Since $p^{k+1} = p^k$ and $s^{k+1}_j = D^{rj}(p^k)$ for all $j \in T$, we have

$$h_{i_o}(\bar{s}''_T, \bar{p}'') = f^r_{i_o}(\hat{p}'') > f^r_{i_o}(\hat{s}^k) = h_{i_o}(s^{k+1}_T, p^{k+1}) .$$

Clearly the oligopolist i_o is effective for $(\bar{s}''_T, \bar{p}'')$ in the game Γ. Therefore i_o can improve upon the above absorption. This is a contradiction to the supposition of Necessity.

<u>Sufficiency</u>: Let r_{N*} be conventionally stable in the game G^r. Then r_{N*} is clearly price-dependent, and $r = (r_T, r_{N*})$ satisfies the absorptivity condition and acyclicity condition in the game Γ. Since (28) holds and one buyer's behavior can not be observed by any oligopolist and any other buyer by (18) and (19), no buyer can improve upon any absorption in the game Γ. Furthermore it can be proved similarly to the proof of Necessity that if an oligopolist i_o can improve upon an absorption in the game Γ, then the oligopolist i_o can improve upon some absorption in the game G^r. Q.E.D.

3. The Monopolistic Market

7. This section considers the monopolistic market, i.e., $n = 1$. An adjoint price-oligopoly game G^r is a one-person game. Then it is easy to see the following proposition.

Proposition 4. In the adjoint price-oligopoly game G^r, r_1 is conventionally stable if and only if r_1 satisfies the absorptivity condition and

$$\text{for every } p_1 \in F(r_1),\ f_1^r(p_1) \geqq f_1^r(p_1') \text{ for all } p_1' \in [1,+\infty). \tag{30}$$

Propositions 3 and 4 imply

A price-dependent response configuration $r = (r_T, r_1)$ is conventionally stable in Γ if and only if r_1 satisfies the absorptivity condition in G^r, (30) and for all $(s_T, p) \in F(r)$, $s_j = D^{*j}(p_1)$ for all $j \in T$. (31)

The above result is interpreted as follows. The monopolist faces the observed demand function $D^r(p_1)$, and he maximizes his profits $(p_1 - 1)D^r(p_1)$. In particular, it should be noted that the buyers behave as price-takers in any stationary state but not in general. See Figure 1. The monopolist has tried many prices and has known the observed demand function which the monopolist has faced. Since each trial does not take long time and is transitory, every buyer might not behave as a price-taker in nonstationary states. Of course, if every buyer employs the best-response functions, then the observed demand function becomes the competitive demand function.

In fact, this result has a little paradoxical feature. Consider curve I in Figure 1. One might think that if the monopolist fixed his price to a little higher price, say, p_I' than p_I, then the buyers would change their demands to the competitive demands and so the monopolist would get a higher stationary payoff. If the monopolist knew the competitive demand function $D^*(p_1)$, then this reasoning would be true. But he can not know the competitive demand function. He knows only the observed demand function. He does not know if the stationary payoff is raised by setting his price as p_1' for long time. If the competitive demand function was drawn as curve II' , then he would lose. Thus the monopolist maximizes his stationary payoff based on his observations, but if he changed and fixed his price to a different one, then he might get a higher or lower stationary payoff. Therefore he can not try a different price from p_1 for long time. Therefore every price in a conventionally stable set is stable.

Although the classical monopolistic solution looks extremely simple and very plausible, there had been no fully game theoretical explanation of it. For example, Aumann [1] argued that the core theory can not explain the monopolistic solution and expects that the value theory would do it. Gardner [3] showed, however, that the value theory also can not explain it. See also Schotter and Schwödiauer [13, Sec. 5.1]. Therefore our success of the explanation of the monopolisitc solution has great significance. That is, if our theory could not explain the classical monopolistic solution, the application of the theory to more complicated economies, i.e. duopoly and oligopoly, has no significance because it was rejected in a simpler model.

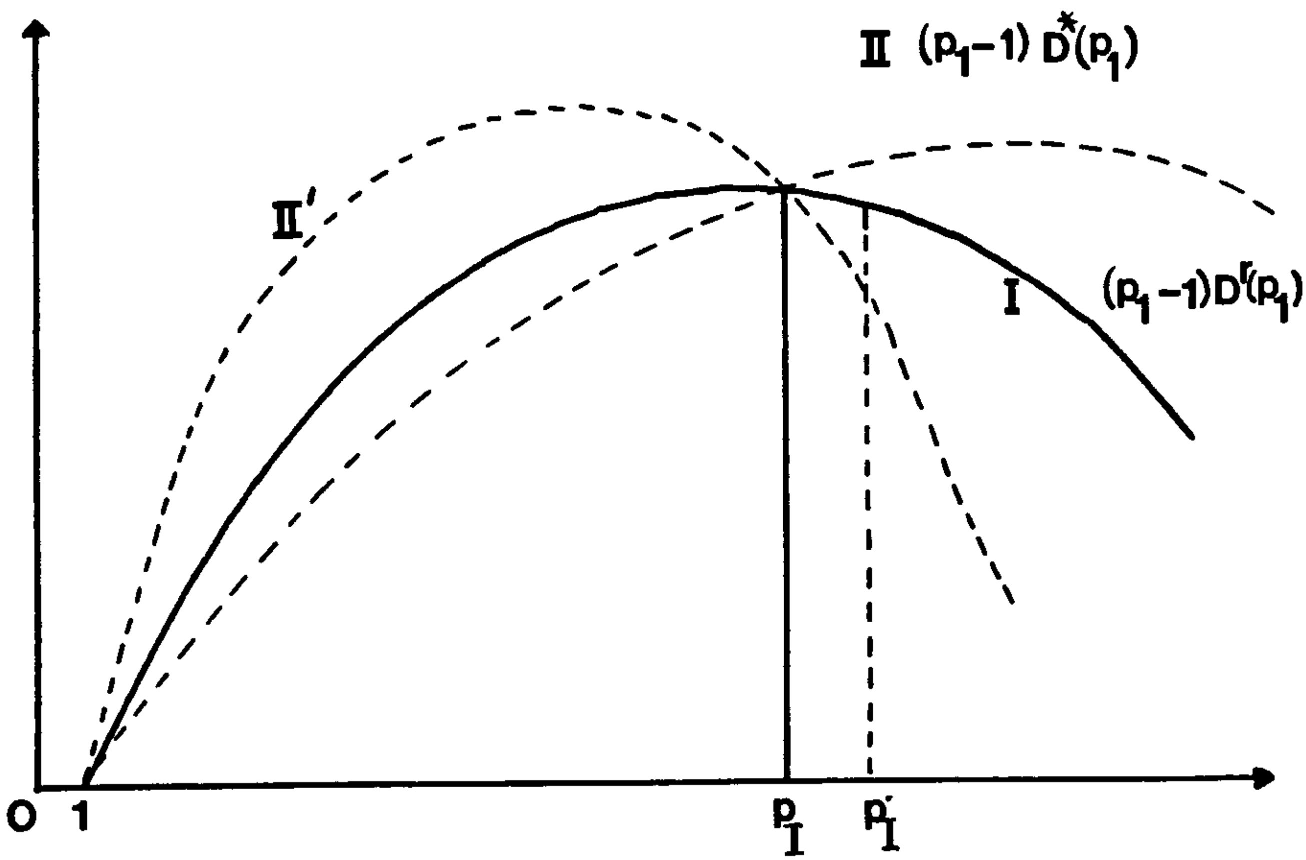

Figure 1.

4. Kinked-Demand Curve Solutions

8. In the present and succeeding sections, we provide concrete examples of conventionally stable response configurations in an adjoint price-oligopoly game G^r . Of course, we assume $n \geqq 2$. Two important classes of conventionally stable response configurations are presented. The response configurations in one class are kinked-demand-curve solutions which have been discussed in classical textbooks, or rather exhibit more explicit structures than the classical argument. The response configurations in the other classes are leader-follower solutions. The structures of them are, however, quite different from that of the classical leader-follower solution. They would be much more reasonable and intuitively appealing than the classical one. Furthermore there are an enormous number of hybrids of solutions in these two classes. The most important thing is the existence of a hybrid of these two kinds of solutions, i.e., the two arguments of the kinked-demand-curve solution and the leader-follower solution can be compatible in a price-oligopoly game. This paper does not attempt to list every conventionally stable configuration.

Consider an adjoint price-oligopoly game G^r . We make the following assumptions:

(D) The game G^r has a Nash equilibrium $p^N = (p_1^N, \ldots, p_n^N)$;

(E) There is a path $\pi = \{\overline{p}(\alpha) : \alpha \in [0,1] \ \& \ \overline{p}(0) = \overline{p}^N\}$ in $P_1 \times \ldots \times P_n$ such that for all $i \in N^*$, (i) $\overline{p}_i(\alpha)$ is an increasing function of α , (ii) for all $\alpha \in [0,1]$, $f_i^r(\overline{p}(\alpha)) \geqq f_i^r(\overline{p}_{-i}(\alpha), p_i)$ for all $p_i \geqq \overline{p}_i(\alpha)$ and (iii) $f_i^r(\overline{p}(\alpha))$ is a nondecreasing function of α .

Here $\overline{p}_{-i}(\alpha) = \overline{p}_{N^*-i}(\alpha)$.

It is well-known that the ordinary price-oligopoly game G^* does not necessarily satisfy assumptions (D) and (E). But these are not so strange conditions. The following example satisfies these assumptions.

Example 4.1. Let $n = 2$ and $\mu([-1,0]) = 6$. Assume that every buyer has the identical utility function $u(x_1, x_2, m) = -(x_1 - 15)^2 - (x_2 - 15)^2 - (x_1 + x_2 - 20)^2 + m$. Let $I_j = 1000$ for all $j \in T^*$. The individual competitive demand function $D^{*j}(p_1, p_2)$ is given as

$$D^{*j}(p_1, p_2) = \begin{cases} ((70 - 2p_1 + p_2)/6,\ (70 + p_1 - 2p_2)/6) & \text{if } D^{*j}(p_1, p_2) > 0 \\ ((70 - p_1)/4,\ 0) & \text{if } D_1^{*j}(p_1, p_2) > 0 \ \&\ D_2^{*j}(p_1, p_2) = 0 \\ (0,\ (70 - p_2)/4) & \text{if } D_1^{*j}(p_1, p_2) = 0 \ \&\ D_2^{*j}(p_1, p_2) > 0 \\ (0,0) & \text{otherwise} \end{cases}$$

The ordinary oligopoly game G^* has a unique Nash equilibrium, which is $(p_1^N, p_2^N) = (24, 24)$. The symmetric joint-profit maximization p^C, i.e.,

$$p_1^C = p_2^C \quad \text{and} \quad \sum_{i=1}^{2} (p_i^C - 1)D^*(p_i^C) \geqq \sum_{i=1}^{2} (p_i - 1)D_i^*(p) \quad \text{for all} \quad p \geqq (1,1) ,$$

is given as $p^C = (71/2, 71/2)$.[7] The segment from p^N to p^C, $\pi = \{t(1,1) : 24 \leqq t \leqq 71/2\}$, satisfies assumption (E).

The following proposition is a direct corollary of Proposition 1 of Kaneko [7].

Proposition 5. In the price-oligopoly game G^r, there is the conventionally stable set which consists of a Nash equilibrium p^N.

9. Sweezy [15] and Hall and Hitch [4] proposed a model as an explanation of rigid prices, which was taken as an empirical fact. Although this model was quickly accepted as the theory of oligopoly by many textbook writers, the theory has never been explained in a satisfactory form except Marschak and Selten's work [9, 10, 11]. It is a very surprising result that our theory is compatible with the classical version of the kinked-demand-curve solution, and that the solution which our theory provides is a purer version of it than Marschak and Selten's.

Let $\overline{p}(\alpha) = (\overline{p}_1(\alpha), \ldots, \overline{p}_n(\alpha))$ be the function determining the path π of assumption (E). Let $\beta_i(p_i)$ $(i \in N^*)$ be the inverse function of $\overline{p}_i(\alpha)$, i.e., $\overline{p}_i(\beta_i(p_i)) = p_i$ for all p_i with $\overline{p}_i(0) \leqq p_i \leqq \overline{p}_i(1)$. We define a response configuration $r_{N^*} = (r_1, \ldots, r_n)$ in the adjoint price-oligopoly game G^r by

$$r_i(p) = \begin{cases} \overline{p}_i(\min\{\beta_j(p_j) : p_j \leqq \overline{p}_j(1)\}) & \text{if } \overline{p}(0) \leqq p \ \& \ p \not\geq \overline{p}(1) \\ \overline{p}_i(0) & \text{if } \overline{p}(0) \not\leqq p \\ \overline{p}_i(1) & \text{if } p \geqq \overline{p}(1) \end{cases} \tag{32}$$

This response configuration has the kinked-demand-curve behavior and is conventionally stable. See Figure 2.

<u>Proposition 6</u>. The response configuration r_{N^*} defined by (32) is conventionally stable and $F(r_{N^*}) = \pi$.

Before proving this proposition, we consider "personal faced demand function." Let $p^* \in F(r_{N^*})$ be given. We define oligopolist i's <u>personal faced demand function</u> $D_i^r(p_i/p^*)$ <u>at the current price vector</u> p^* by

$$D_i^r(p_i/p^*) = D_i^r(r_{-i}(p^*_{-i},\ p_i),\ p_i) \quad \text{for all} \quad p_i \geqq 1\ , \tag{33}$$

where $r_{-i} = r_{N^*-i}$ and $D_i^r(p)$ is oligopolist i's observed deamnd function in the adjoint price-oligopoly game G^r. In fact, this personal faced demand function is the kinked-demand function which the oligopolist i is facing at the given current price vector p^*.[8] If he raises his price, then the other oligopolists do not change their prices, but if he lowers it, then the others change theirs to the levels determined by (32). Therefore his faced demand function declines by a unit-increment in his price more than it rises by a unit-decrement in his price. That is, his faced demand function is kinked at the current price p_i^*. This is the same as the standard argument of the kinked-demand-curve-solution.

Example 4.2. Consider the ordinary oligopoly game G^* in Example 4.1. Let $\pi = \{t(1,1) : 24 \leqq t \leqq 71/2\} = \{\alpha(24,\ 24) + (1-\alpha)(71/2,\ 71/2) : \alpha \in [0,1]\}$. In this economy, (32) is written as

$$r_i(p) = \begin{cases} \min(p_1,\ p_2) & \text{if} \quad (24,\ 24) \leqq p \ \&\ p \not\geq (71/2,\ 71/2) \\ \bar{p}_i(0) & \text{if} \quad (24,\ 24) \nleqq p \\ \bar{p}_i(1) & \text{if} \quad p \geqq (71/2,\ 71/2) \end{cases} \tag{34}$$

and oligopolist 1's personal faced demand function $D_1^r(p_1/p^*)$ at the current price vector $p^* \in \pi$ is given as

$$D_1^r(p_1/p^*) = \begin{cases} \max(0,\ 70 - 2p_1 + p_2^*) & \text{if} \quad p_1 \geqq p_1^* \\ 70 - p_1 & \text{if} \quad 24 \leqq p_1 \leqq p_1^* \\ 70 - 2p_1 + p_2^N & \text{if} \quad 1 \leqq p_1 \leqq 24 \end{cases} \ . \tag{35}$$

The personal faced demand function $D_1^r(p_1/p^*)$ at the current price vector p^* is drawn as Figure 3. Of course, at each $p^* \varepsilon \pi$, each oligopolist has the personal faced demand function like Figure 3.

In this case, $\overline{p}(1)$ is a joint-profit maximization solution. Hence it might appear as a stationary state in this case. But we can replace π by $\pi' = \{\overline{p}(\alpha) \varepsilon \pi : 0 \leqq \alpha \leqq \alpha_0\}$ $(0 \leqq \alpha_0 \leqq 1)$ and Proposition 6 is still true for the new π'. Thus, mathematically, it is a rare case that $F(r)$ contains a joint-profit maximization solution.

The final note is on Marschak and Selten's very similar result to Proposition 6. They provide a kind of kinked-demand-curve solution as a candidate for their solution concept "convolution." But their response function is different from (32) and more complicated. They call the response configuration defined by (32) a "purer version of the kinked-demand behavior," but it can not be a convolution.[9] This fact would make the difference between these two theories clearer.

<u>Proof of Proposition 6</u>. It is easy to see that r_{N*} defined by (32) satisfies the absorptivity condition and the acyclicity condition. We show only that r_{N*} satisfies the stability condition. Consider an arbitrary point p. By (32), $p^* = r_{N*}(p)$ is a stationary state and so, $\{p, p^*\}$ is an absorption. Let $p^* = \overline{p}(\alpha^*)$. In this case, it follows from (32) that an oligopolist i can yield a new stationary state p^0 by a deviation such that (1) for some α^0 with $0 \leqq \alpha^0 \leqq \alpha^*$, $p_j^0 = \overline{p}_j(\alpha^0)$ for all $j \neq i$ and $p_i^0 \geqq \overline{p}_i(\alpha^0)$, or (2) $p_j^0 = \overline{p}_j(0)$ for all $j \neq i$ and $p_i^0 \leqq \overline{p}_i(0)$. First let us consider Case (1). By assumption (E), $f_i^r(p^0) \leqq f_i^r(p_{-i}^0, \overline{p}_i(\alpha^0)) \leqq f_i^r(\overline{p}(\alpha^*)) = f_i^r(p^*)$. Hence he can not improve upon the absorption $\{p, p^*\}$. Second, in Case (2), by assumptions (D) and (E), we have $f_i^r(p^0) \leqq f_i^r(p_{-i}^0, \overline{p}_i(0)) \leqq f_i^r(p^*)$. Hence

he can not improve upon the absorption. This game has the finest observation structure, and so, there is no accumulation of observations. Therefore it is sufficient to prove that $\{p, r_{N*}(p)\}$ is stable for all p . Q.E.D.

5. Leader-Follower Solutions and Hybrids

10. Although von Stackelberg [17] argued that the leader-follower behavior would lead disequilibrium and provide no definite result,[10] many economists still believe the leader-follower behavior and call the resulting equilibrium the Stackelberg equilibrium. Of course, there is no persuasive theory which can explain the Stackelberg equilibrium.[11] This section provides conventionally stable response configurations in which an oligopolist plays a role of a leader and the others do those of followers. The meanings of "leader" and "follower" are, however, quite different from those in the standard argument. The reader will find that our version of leader-follower behavior is purer than the standard one.

For the above purpose, the definition of the conventional stability given in Section 1 is a little restrictive, and is needed to be generalized. That is, we allow a response function to depend upon two previous outcomes.[12]

Consider the game of Subsection 2. A response function of player i is a function from $S \times S$ to S_i which satisfies

$$\begin{aligned} &\text{if } a_i(s^1) = a_i(\hat{s}^1) \text{ and } a_i(s^2) = a_i(\hat{s}^2) , \\ &\text{then } r_i(s^1, s^2) = r_i(\hat{s}^1, \hat{s}^2) . \end{aligned} \tag{36}$$

A response configuration r is a function from $S \times S$ to S such that for all $i \in N$, r_i is a response function. The set $F(r) = \{s \in S : r(s,s) = s\}$ is called the stationary set of r . Player

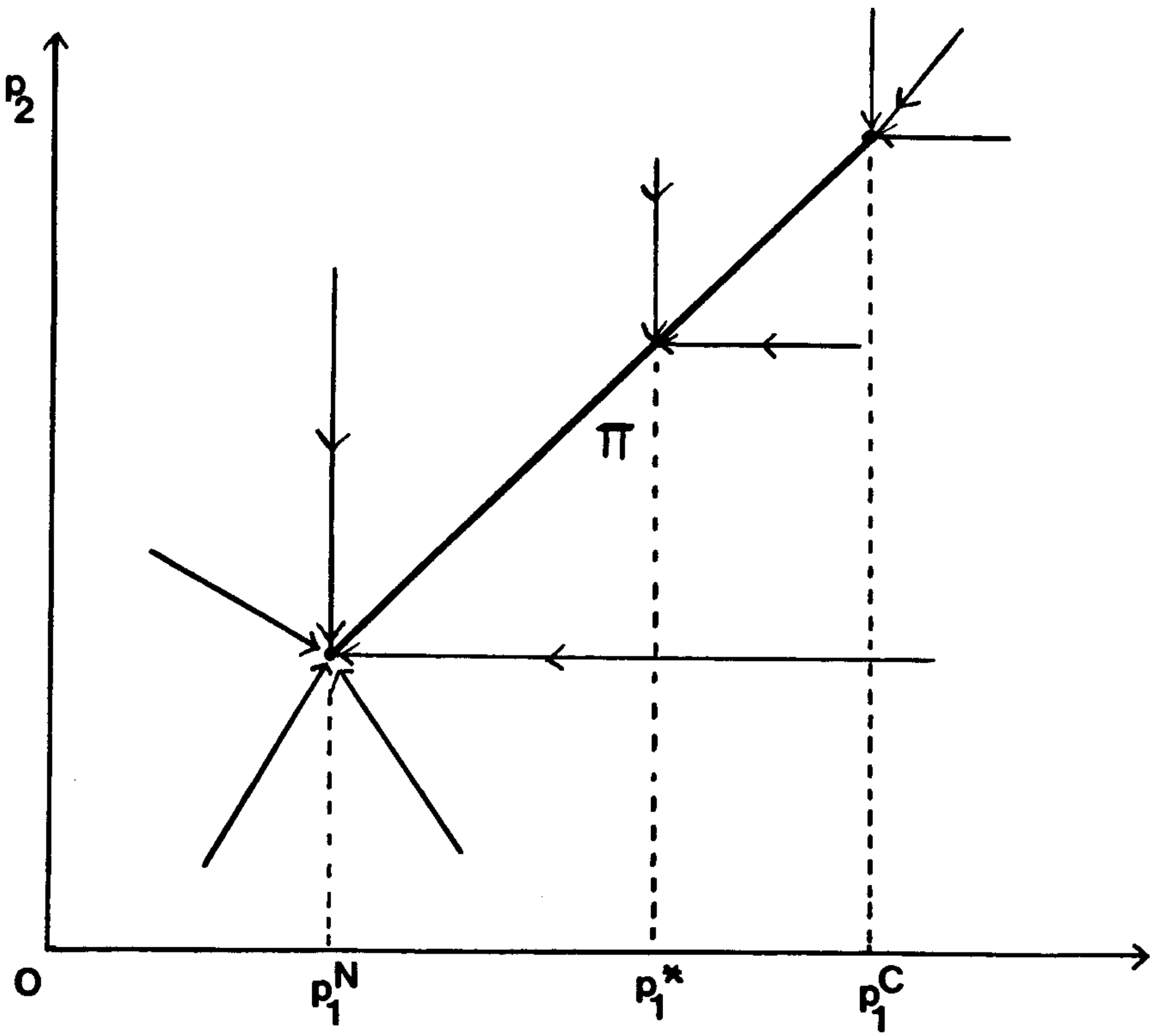

Figure 2.

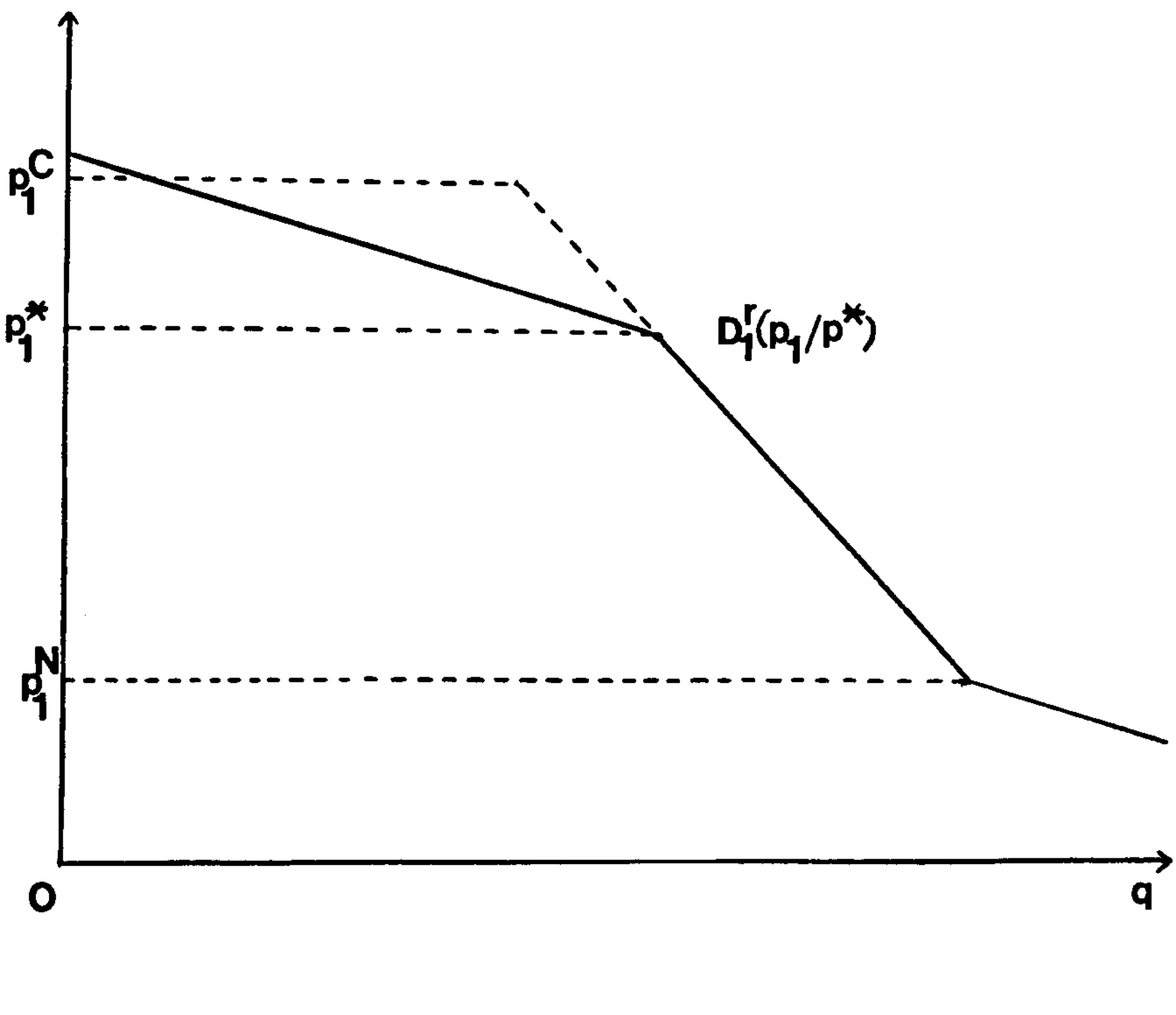

Figure 3.

i is said to be <u>effective for</u> $s \in S$ in r iff $r_{-i}(s,s) = s_{-i}$. A finite sequence $\{s^1, \ldots, s^k\}$ is called an <u>absorption</u> iff $r(s^{t-2}, s^{t-1}) = s^t$ for all $t = 3, \ldots, k$ and $s^{k-1} = s^k \in F(r)$. For any absorption $\{s^1, \ldots, s^k\}$, we define $\hat{a}_i$ by

$$\begin{aligned} &\hat{a}_i(\{s^1, s^2\}) = a_i(s^1) \times a_i(s^2) \\ &\hat{a}_i(\{s^1, s^2, s^3\}) = a_i(s^2) \times [a_i(s^3) \cap r[\hat{a}_i(\{s^1, s^2\})]] , \end{aligned} \tag{37}$$

and

$$\hat{a}_i(\{s^1, \ldots, s^t\}) = [a_i(s^{t-1}) \cap r[\hat{a}_i(\{s^1, \ldots, s^{t-2}\})]]$$

$$\times [a_i(s^t) \cap r[\hat{a}_i(\{s^1, \ldots, s^{t-1}\})]] \quad \text{for all } t = 4, \ldots, k .$$

Let s, s' and s'' be three outcomes. We say that s'' is <u>connected to</u> (s,s') by a <u>deviation</u> $\{t_i^1, \ldots, t_i^k\}$ of player i from his response function r_i iff (1) t_i^m is a response function of player i for $m = 1, \ldots, k$ and (2) $s^1 = (r_{-i}, t_i^1)[(s,s')]$ & $s^m = (r_{-i}, t_i^m)[(s^{m-2}, s^{m-1})]$ for $m = 2, \ldots, k$ and $s^k = s''$, where $s^0 = s'$. We say that player i <u>can improve upon</u> absorption $\{s^1, \ldots, s^k\}$ of (s^1, s^2) by s^k at s^h $(2 \leq h \leq k)$ iff

> player i has a deviation $\{t_i^1, \ldots, t_i^m\}$ such that for any $(\hat{s}^{h-1}, \hat{s}^h) \in \hat{a}_i(\{s^1, \ldots, s^h\})$ player i is effective for s'' which is connected to $(\hat{s}^{h-1}, \hat{s}^h)$ by the deviation $\{t_i^1, \ldots, t_i^k\}$ and $h_i(s'') > h_i(s^k)$.

We call an absorption $\{s^1, \ldots, s^k\}$ of (s^1, s^2) by s^k <u>stable</u> iff any player can not improve upon this absorption at any s^h for $h = 2, \ldots, k$.

<u>Definition</u>. A response configuration r is said to be <u>conventionally stable</u> iff

(<u>Stability</u>): every absorption is stable;

(<u>Absorptivity</u>): there is a finite k such that for any $(s^1, s^2) \in S \times S$, there is an absorption $\{s^1, s^2, \ldots, s^h\}$ with $h \leqq k$;

(<u>Acyclicity</u>): for any $i \in N$, there is a finite h such that for any $(s^1, s^2) \in S \times S$ and $s_i' \in S_i$,
$s^m = (r_{-i}, e_i^{s_i'})[(s^{m-2}, s^{m-1})]$ for $m = 3, \ldots, k$ and
$s^{h-2} = s^{h-1} = s^h$, where $e_i^{s_i'}$ is the constant response function with the value s_i', i.e., $e_i^{s_i'}(\bar{s}, \bar{\bar{s}}) = s_i'$ for all $(\bar{s}, \bar{\bar{s}}) \in S \times S$.

We call a subset V of S a <u>conventionally stable</u> set iff there is a conventionally stable response configuration r whose stationary set $F(r)$ coincides with V.

The essential idea of the above definition of the conventionally stable set is just the same as the original one. The difference from the original one is that the response functions depend upon two previous outcomes. It would not need any further comment.

<u>11</u>. This subsection provides a conventionally stable response configuration which has the structure of leader-follower behavior.

Let δ be positive number less than $\bar{p}_1(1) - \bar{p}_1(0)$. Consider the following response configuration $r_{N*} = (r_1, \ldots, r_n)$ in an adjoint price-oligopoly game G^r:

$$r_1(p^1, p^2) = \begin{cases} p_1^2 + \delta & \text{if } \beta_i(p_i^2) = \beta_1(p_1^1) \in (0,1] \text{ for} \\ & \text{all } i \neq 1 \ \& \ p_1^2 + \delta \leqq \bar{p}_1(1) \\ \bar{p}_1(1) & \text{if } \beta_i(p_i^2) = \beta_1(p_1^1) \in (0,1] \text{ for all} \\ & i \neq 1 \ \& \ p_1^2 + \delta > \bar{p}_1(1) \\ \bar{p}_1(0) & \text{otherwise} \end{cases} \quad (38)$$

and for all $j \neq 1$,

$$r_j(p^1, p^2) = \begin{cases} \bar{p}_j(\beta_1(p_1^2)) & \text{if } [p_1(0) + \delta < p_1^2 = p_1^1 + \delta \leqq \bar{p}_1(1) \\ & \text{or } p_1^2 = \bar{p}_1(1) < p_1^1 + \delta] \ \& \\ & p_i^2 \neq \bar{p}_i(0) \text{ for all } i \neq 1 \\ \bar{p}_j(0) & \text{otherwise} \end{cases} \quad (39)$$

Proposition 7. The response configuration defined by (38) and (39) is conventionally stable and $F(r_{N*}) = \{\bar{p}(0), \bar{p}(1)\}$.

Before proving this proposition, consider the response configuration r_{N*} in the following example.

Example 5.1. Consider the ordinary price-oligopoly game $G*$ in Example 4.1. In this game, $r_{N*} = (r_1, r_2)$ is written as follows:

$$r_1(p^1, p^2) = \begin{cases} p_1^2 + \delta & \text{if } 24 < p_2^2 = p_1^1 \leqq 71/2 \ \& \ p_1^2 + \delta \leqq 71/2 \\ 71/2 & \text{if } 24 < p_2^2 = p_1^1 \leqq 71/2 \ \& \ p_1^2 + \delta \geqq 71/2 \\ 24 & \text{otherwise} \end{cases} \quad (40)$$

$$r_2(p^1, p^2) = \begin{cases} p_1^2 & \text{if } \left[24 + \delta < p_1^2 = p_1^1 + \delta \leq 71/2 \right. \\ & \left. \text{or } p_1^2 = 71/2 < p_1^1 + \delta\right] \;\&\; p_2^2 \neq 24 \\ 24 & \text{otherwise.} \end{cases} \quad (41)$$

This response configuration r_{N*} is verbally explained as follows. Oligopolist 1 plays the role of a leader and 2 that of a follower, respectively. If Oligopolist 1 observed the two price vectors p^1 and p^2 and finds that oligopolist 2 followed him, i.e., $p_2^2 = p_1^1$ and the other additional conditions, then he raises his price to $p_1^2 + \delta$ or sets the highest price 71/2, and otherwise he lowers his price to the Nash price 24. The follower observes the leader's pricing, and if the leader obeys his response function r_1, then oligopolist 2 follows the leader's pricing, but otherwise lowers his price to the Nash price 24. Although the leader and follower play quite asymmetric roles, they have the same threats, i.e., if one of them finds that the opponent deviates from the response function, he uses the threat, the Nash price 24. This is the structure of a mutual deterrence similar to other conventionally stable ones (see Kaneko [7, Subsection 5.2]). Due to the mutual deterrence, the pair of the highest prices $(\bar{p}_1(1), \bar{p}_2(1))$ becomes stable. Of course then it is necessary that the pair of Nash prices is also stable. The conventionally stable response configuration defined by (32) has also a similar structure of a mutual deterrence. Thus this leader-follower behavior is quite different from the standard argument. The most important difference is that the leader-follower behavior emerges endogenously in our theory, while it is a priori assumed in the standard argument.

In fact, there are a lot of variations of the conventionally stable response configuration of (38) and (39). For example, the response configuration like Figure 5 could be conventionally stable. Since the response configuration is too complicated to write it down, it is omitted.

Proof of Proposition 7. It is easily verified that $F(r_{N*}) = \{\bar{p}(0), \bar{p}(1)\}$. It follows from (38) and (39) that any oligopolist i is effective only for $\bar{p}(1)$ and $(\bar{p}_{-i}(0), p_i)$ for all $p_i \in P_i$. It holds by assumptions (D) and (E)

$$f_i^r(\bar{p}_{-i}(0), p_i) \leq f_i^r(\bar{p}(0)) \leq f_i^r(\bar{p}(1)) \quad \text{for all} \quad p_i \in P_i . \tag{42}$$

First consider an absorption $\{p^1, \ldots, p^k\}$ with $p^k = \bar{p}(1)$. By (42), any oligopolist can not improve upon this absorption. Secondly, consider an absorption $\{p^1, \ldots, p^k\}$ with $p^k = \bar{p}(0)$. By (38) and (39), if an oligopolist i deviates from his response function, the resulting stationary state is $(\bar{p}_{-i}(0), p_i)$ for some p_i . Therefore, by (42), he can not improve upon this absorption. We have proved the stability property of r_{N*} . It is not difficult to see the absorptivity and acyclicity properties. Q.E.D.

12. This subsection notes that there are hybrids between the conventionally stable response configurations defined in Subsections 9 and 11.

Let α^0 be a number with $0 < \alpha^0 \leq 1$ and let δ be a positive number less than $\bar{p}_1(\alpha^0) - \bar{p}_1(0)$.[13] Consider the following response configuration $r_{N*} = (r_1, \ldots, r_n)$ in an adjoint price oligopoly game G^r :

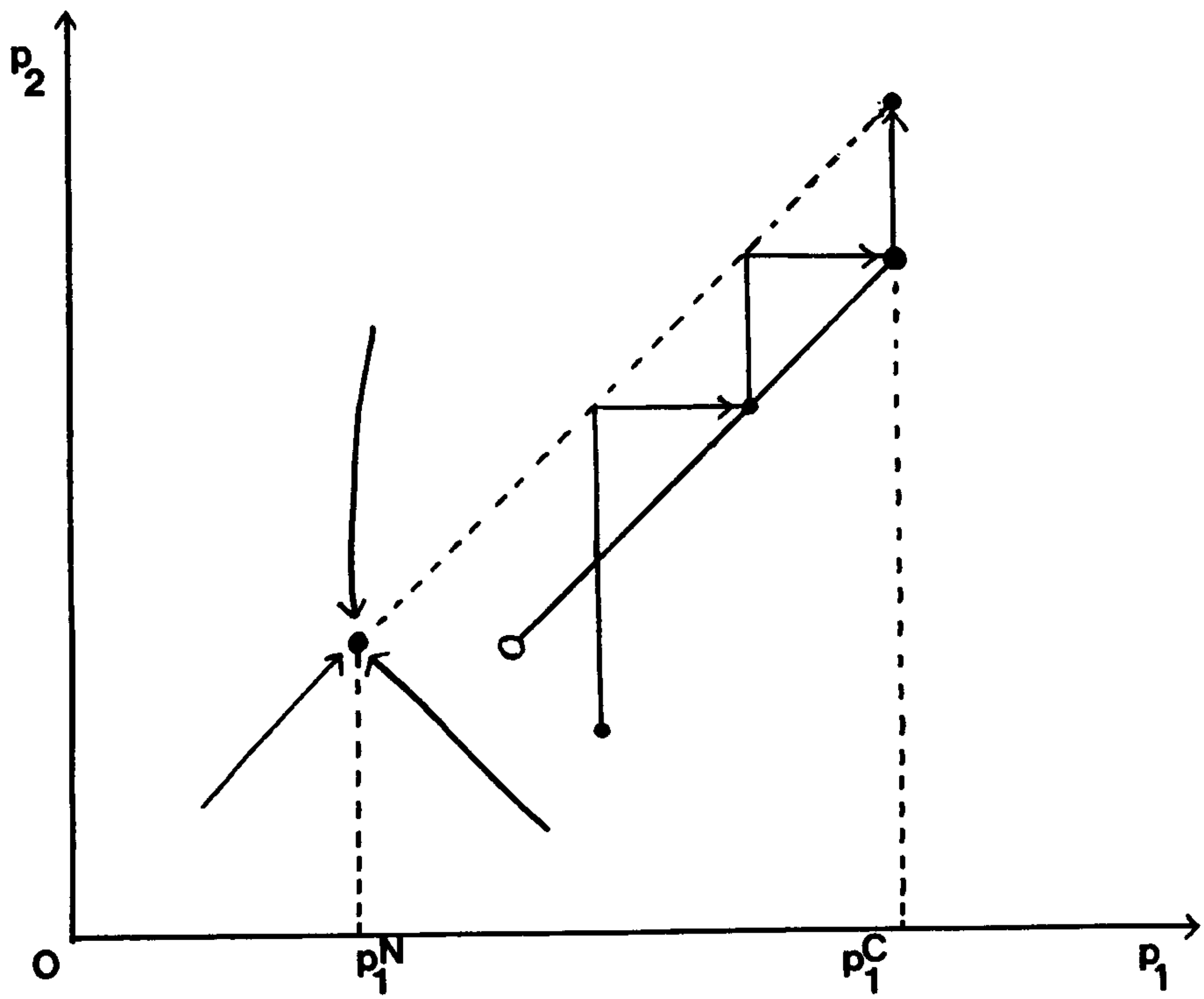

Figure 4.

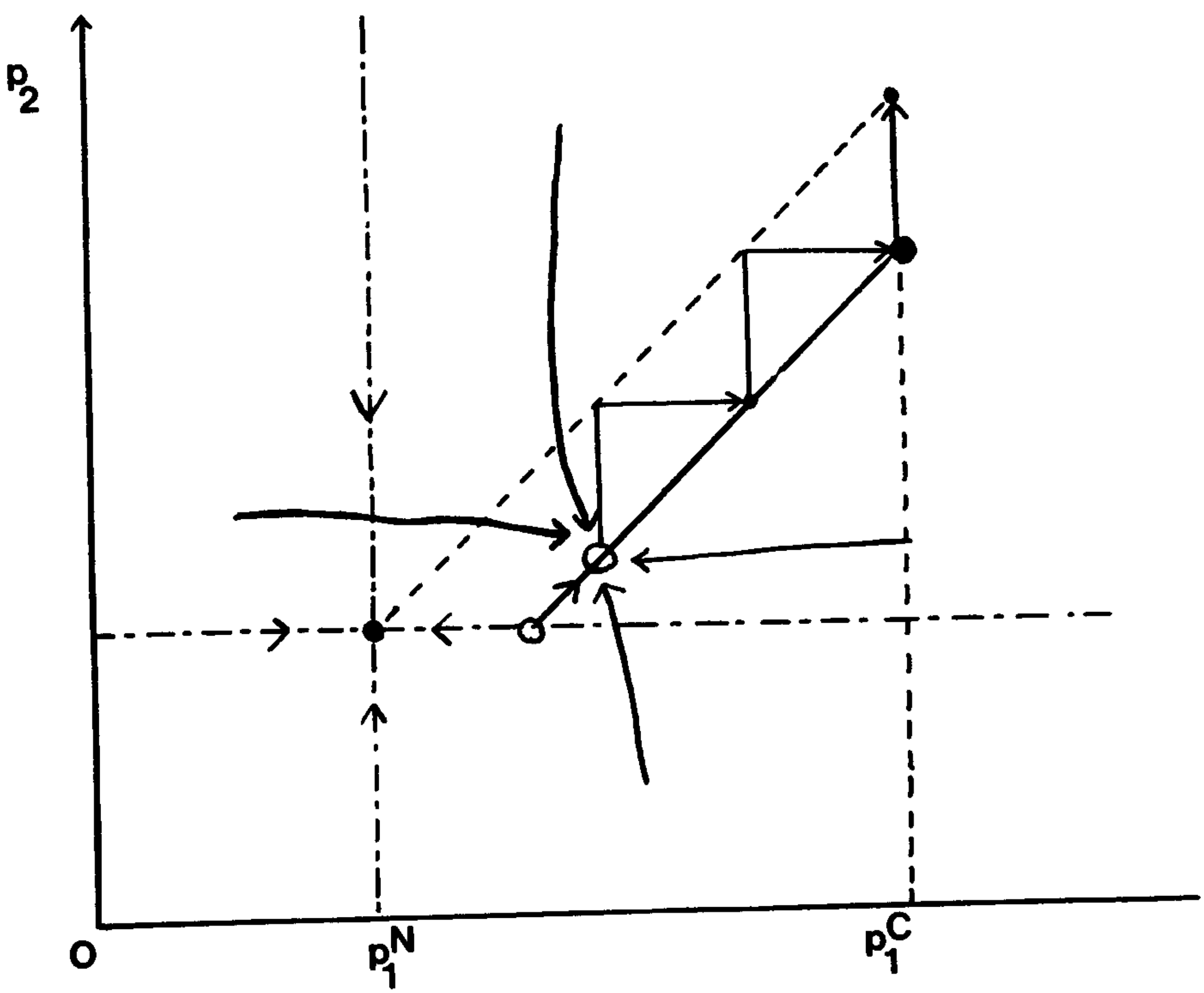

Figure 5.

$$r_1(p^1, p^2) = \begin{cases} \bar{p}_1(1) & \text{if } p^2 \geqq \bar{p}(1) \\ \bar{p}_1(\min\{\beta_j(p_j^2) : p_j^2 \leqq \bar{p}_j(1)\}) & \text{if } \bar{p}(\alpha^0) \leqq p^2 \ \& \ p^2 \ngeq \bar{p}(1) \\ p_1^2 + \delta & \text{if } \beta_i(p_i^2) = \beta_1(p_1^1) \in (0, \alpha^0] \\ & \text{for all } i \neq 1 \ \& \ p_1^2 + \delta \leqq \bar{p}_1(\alpha^0) \\ \bar{p}_1(\alpha^0) & \text{if } \beta_i(p_i^2) = \beta_1(p_1^1) \in (0, \alpha^0] \\ & \text{for all } i \neq 1 \ \& \ p_1^2 + \delta > \bar{p}_1(\alpha^0) \\ \bar{p}_1(0) & \text{otherwise} \end{cases} \tag{42}$$

and for all $j \neq 1$,

$$r_j(p^1, p^2) = \begin{cases} \bar{p}_j(1) & \text{if } p^2 \geqq \bar{p}(1) \\ \bar{p}_j(\min\{\beta_j(p_j^2) : p_j^2 \leqq \bar{p}_j(1)\}) & \text{if } \bar{p}(\alpha^0) \leqq p^2 \ \& \ p^2 \ngeq \bar{p}(1) \\ \bar{p}_j(\beta_1(p_1^2)) & \text{if } [\bar{p}_1(0) + \delta < p_1^2 = p_1^1 + \delta \leqq \bar{p}_1(\alpha^0) \\ & \text{or } p_1^2 = \bar{p}_1(1) < p_1^1 + \delta] \ \& \\ & p_i^2 \neq \bar{p}_i(0) \text{ - for all } i \neq 1 \\ \bar{p}_j(0) & \text{otherwise.} \end{cases} \tag{43}$$

The following proposition holds:

<u>Proposition 8</u>. The response configuration defined by (42) and (43) is conventionally stable and $F(r_{N*}) = \{\bar{p}(0)\} \cup \{\bar{p}(\alpha) : \alpha^0 \leqq \alpha \leqq 1\}$.

As this proposition can be proved similarly to the proofs of the previous propositions, we omit the proof.

In the region of price vectors greater than or equal to $\bar{p}(\alpha^0)$, this response configuration has the structure of the kinked-demand-curve behavior and in the other region it has the structure of the leader-follower

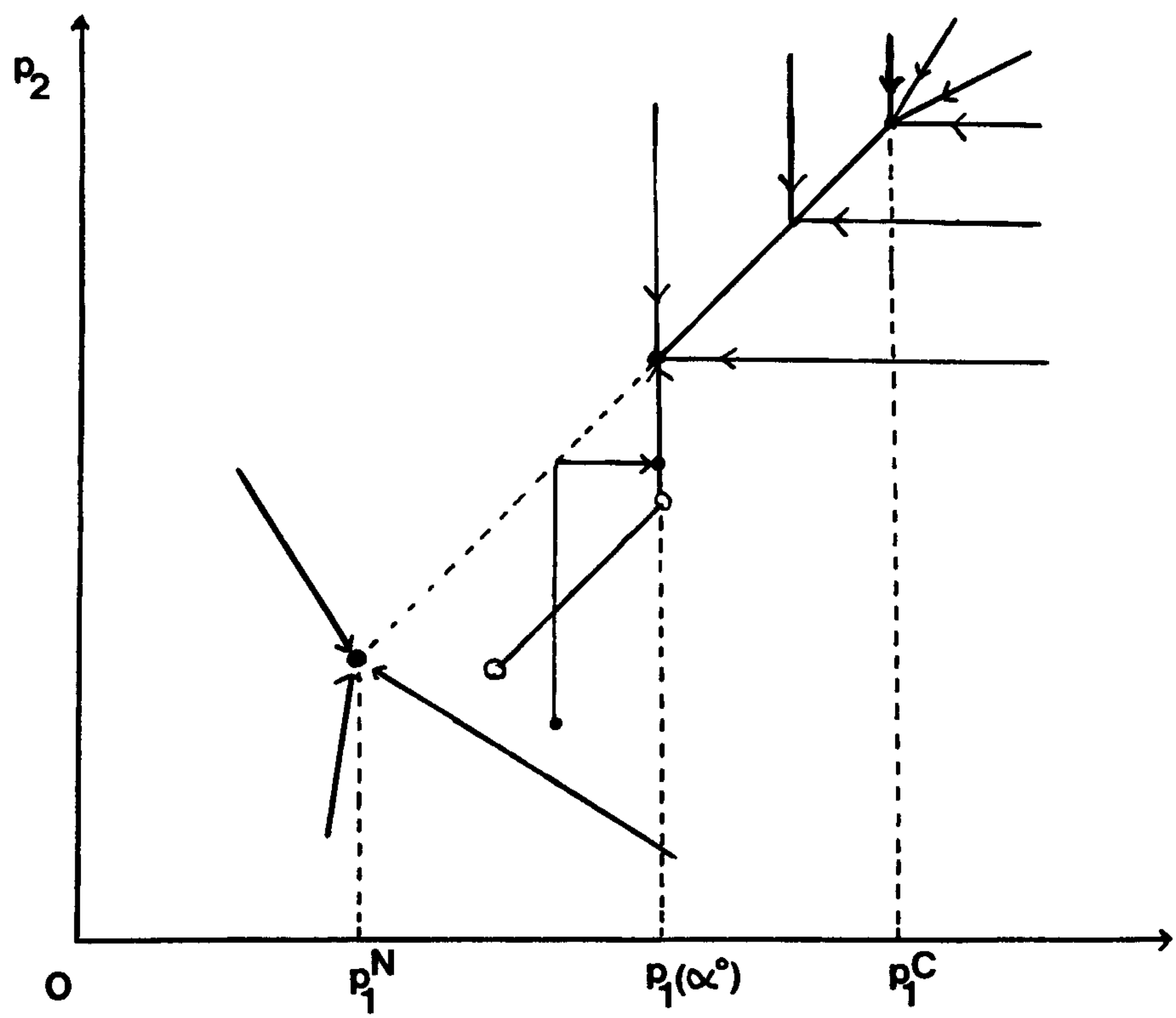

Figure 6.

behavior, i.e., oligopolist 1 behaves as a leader and others do as followers. See Figure 6. This is a hybrid between the conventionally stable response configurations defined in Subsections 9 and 11. Conversely, the response configurations of Subsections 9 and 11 are special cases of the above one, i.e., they correspond to the cases of $\alpha^0 = 0$ and $\alpha^0 = 1$ and there are a continuum of hybrids.

6. Concluding Remarks

13. This paper has applied the theory of the conventionally stable set to price-quantity oligopolistic markets. It has been shown that this application has solved the three unsolved questions in the oligopoly literature discussed in Section 1. The solution to the question (Q-1) given in this paper has no problem, but the solutions to the questions (Q-2) and (Q-3) deserve small comment. It is shown that monopolistic solutions, kinked-demand-curve solutions and leader-follower solutions are compatible with the theory of the conventionally stable set. Furthermore there are a lot of hybrids between kinked-demand-curve solutions and leader-follower solutions. Each conventionally stable set is a possible solution as an accepted standard of behavior. Then one might think that the above multiplicity or the indeterminacy would be a defect of the new theory, because the theory could not predict anything about the choice of a particular conventionally stable response configuration. Conversely, however, this is a point of the new theory. A standard of behavior or a convention has emerged from a long history, and it depends upon a history. A different history forms a different convention. Therefore the multiplicity is rather natural because the new theory takes a history into account. The new theory holds that the multiplicity of standard of behaviors is

inevitable and tells the importance of taking a history into account in oligopolistic situations.[14,15]

There are an important note and an open question which deserve comment. In the theory of this paper, the number of oligopolists is not restricted, but since the perfect observations about the oligopolists' prices are assumed in Γ and G^r, the number of oligopolists should be interpreted as a small number. To generalize our theory to be compatible with a market with a large number of oligopolists, it is necessary to introduce limited observations on the oligopolists' prices in some sense. Then it would be conjectured that if the number of oligopolists is very large, the multiplicity of conventionally stable sets would disappear and coincide with competitive equilibria.

Footnotes

[1]In fact, the author has postponed to hold this until satisfactory results are reached in the application of the new theory to monopoly and oligopoly.

[2]The subgame perfect equilibrium concept of Selten [14] can explain this assumption as follows: The game is formulated as a two stage model where the oligopolists decide and announce their prices at the first stage and the buyers decide their demands at the second stage under the information of the oligopolists' prices. In subgame perfect equilibria, the buyers behave as price takers. It is, however, easy to find certain defects in this argument. For example, this result is independent from the number of buyers, e.g., even if there is a unique buyer in the game, he behaves as a price taker. Obviously, this argument is not fully meaningful.

[3]See Aumann [1], Gardner [3] and also Schotter and Schwödiauer [13].

[4]This paper does not consider mixed strategies at all.

[5]For simplicity, this paper does not allow the redistribution of the oligopolists' profits among the buyers.

[6]An outcome s is called a <u>Nash equilibrium</u> iff for all $i \in N$, $h_i(s) \geqq h_i(s_{-i}, s_i')$ for all $s_i' \in S_i$.

[7]Any redistribution of profits among the oligopolists is not allowed.

[8]The personal faced demand function corresponds to Sweezy's imagined demand function. In our theory, it is not only imagined but also known by observations in the history.

[9]See Marschak and Selten [10, footnote 5].

[10]See Henderson and Quandt [5, pp. 229-230] and Fellner [2].

[11]Ito and Kaneko [6] consider the Stackelberg equilibrium (disequilibrium) in a quantity duopoly in terms of the subgame perfect equilibrium concept of Selten [14], formulating the market as a two stage game where the duopolists decide and announce to play a role of a leader or that of a follower and at the second stage they decide amounts of supply under the information of the first stage. It is shown, however, that the subgame perfect equilibrium implies a "dilemma," i.e., both the duopolists want to play as "leaders" and the resulting equilibrium is the same as the Nash (Cournot) equilibrium in the original market. See Ito and Kaneko [6].

[12] In fact, a similar result can be reached in the original definition of Section 1, but since it will be mathematically too complicated, we generalize the definition.

[13] If $\alpha^0 = 0$, then the following (42) and (43) are reduced to (32).

[14] Kaneko [7, Proposition 7] says that if the observation structure is coarse, roughly speaking, if it is similar to a "perfectly competitive market," then this multiplicity disappears, i.e., every conventionally stable set is included by the set of Nash equilibria.

[15] See also Kaneko [8].

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