

COWLES FOUNDATION FOR RESEARCH IN ECONOMICS  
AT YALE UNIVERSITY

Box 2125, Yale Station  
New Haven, Connecticut 06520

COWLES FOUNDATION DISCUSSION PAPER NO. 586

Note: Cowles Foundation Discussion Papers are preliminary materials circulated to stimulate discussion and critical comment. Requests for single copies of a Paper will be filled by the Cowles Foundation within the limits of the supply. References in publications to Discussion Papers (other than mere acknowledgment by a writer that he has access to such unpublished material) should be cleared with the author to protect the tentative character of these papers.

MARGINAL COST PRICING, TAXATION AND SUBSIDIES IN URBAN TRANSPORT

John J. Beggs

March 31, 1981

# MARGINAL COST PRICING, TAXATION AND SUBSIDIES IN URBAN TRANSPORT

by

John J. Beggs

## Introduction

There is a well established body of knowledge in the classical tradition which carefully ties together the price, output and investment policies of entities both on the production and consumption sides of the market. In such a system, with perfect competition between production entities, and perfect knowledge and availability of alternatives for consumers, communication between producers and consumers is achieved through prices. Prices are related to costs in a systematic way, because if they are not, producers increase outputs which were more profitable, that is, for which price exceeded costs more than the case for other possible outputs. Indeed under sufficient assumptions a first-best solution will occur where price equates marginal costs. However, in practice, it is difficult to apply first-best principles of economic efficiency to the more general problems of resource allocation such as the problem of determining optimum city size. Though there is much discussion to the contrary, there is no standard city. Each location has particular advantages and disadvantages with respect to economies and diseconomies in providing services, including transport. Furthermore, the benefits from larger population concentrations, such as wider ranges of job and educational opportunities, recreation and entertainment, and variety

of environment, are not easily quantified. Even within a more partial equilibrium environment of the urban area there is the obvious interdependence between patterns of land-use and transport. Decisions about what are regarded as desirable land-use patterns cannot be made without information about the effects of alternative land-use patterns on transport costs and quality.

All this having been said, a strong tradition has established itself within the literature which takes the even more partial level of analysis as the urban transport sector itself. This lower level of analysis has been selected undoubtedly because of the apparent intractability of modeling the many involved aspects of land-use-transportation interaction (Engle et al. (1972), Ingram et al. (1972), Lowry (1964)). It is at the level of the urban transport sector that this paper takes focus. In particular, the paper is concerned with drawing out the two discernable approaches to the problem of finding second-best resource allocations with the urban transport sector. The distinctive difference, as will shortly be seen, is in the treatment of the resource and revenue implications of various pricing strategies.

The difficulty in any second-best world is to identify the constraints which are holding the system away from its first-best, or optimum, configuration. The notion of a resources constraint has its roots deeply embedded in traditional welfare economics. The "old" problem in economics was always seen to be one of allocating "scarce" resources so as to satisfy wants and needs, and to do so in such a way as to maximize the welfare of the society. The primary difficulty was not that of identifying the scarcity of resources, but rather of assessing the intensity of the wants and desires of individuals and groups in the society. The modern

economic theorist would explain that resources should be allocated according to the willingness of individuals or groups to pay for those resources, in short, according to some market clearing mechanism based on reservation prices. However, wide-spread market failure resulting from such phenomena as public goods, indivisibilities and externalities, substantially limits the role of the market in providing across-the-board solutions to the resource allocation problem. To contain the problem within tractable dimensions the task of the economist is then conceived of as a sub-optimization problem, confined to the sectoral level. Rather loosely defined criteria (political, social and economic) are used to paint in the broad picture, to determine the approximate size of the major sectors of the economy (e.g. defense, health, education, agriculture, etc.). The problem of the economist is to achieve the best resource allocation within each sector, given the constraint on the total amount of resources to be employed within each sector.

There may be other motivations for treating the sectoral resource constraint as the operative constraint on the system. These motivations stem from the desire of policy oriented economists to say "something" relevant about resource allocation at the micro level, but at the same time not tackle the question of the grand design of the collective economy. The obvious way to procede is to phrase the research effort as follows: suppose the sector under investigation remained the same size relative to other sectors of the economy, are there then ways in which resources can be reallocated within the sector so as to improve welfare? This leaves open, for later debate, the question of whether the sector should be reduced or expanded relative to other sectors.

The notion that the revenue constraint is the operative constraint

in resource allocation decision is of more recent vintage, and has its origins in modern public finance literature. The basic premise is that if everything is optimally organized throughout the economy, and lump-sum transfers are possible, prices in a sector such as urban transport ought to, under first-best conditions, equate marginal cost. The inability of the economy to achieve an optimum is due to the government's need to raise finance, and its inability to do this by means, such as lump-sum taxation, which will not impact on the efficiency of the economy. The urban transport pricing problem can be viewed as part of this overall government taxing and pricing problem where the government puts taxes on private sector outputs, and chooses public sector prices such that an optimum is reached. The appropriate prices should reflect the conditions under which the urban transport sector operates, and ideally should be determined as part of the overall government optimizing task. At the economy-wide level, however, the dimensionality and complexity of the problem remains so great that practical policy prescriptions are rarely ever available. In practice one finds that both policy and theory can be better advanced at the sectoral level, in this case at the level of the urban transport sector. Institutional and legal structures seem to be in good accord with this sectoral approach to the problem. Publicly owned and publicly regulated utilities are often required to operate with financial constraints, such as, that the utility break even, or that it achieve some target rate of profits.

A common feature of the resource constrained and revenue constrained second-best worlds to be considered in this paper is the operational assumption

of restricting the subject of investigation to an individual sector. The "general" resource allocation of the economy is beyond the scope of this paper. It will be seen that the optimum solution to the pricing problem, at the second-best level, is quite different depending on whether the sector is considered to be resource constrained or revenue constrained. The paper will proceed to illustrate the interesting consequences of these different views of the urban transport sector. Section 2 will discuss the resource constrained solutions and Section 3, the revenue constrained solutions. The final section of the paper draws the subject to some conclusions.

#### Resource Constrained Second-Best

One commonly used rule for second-best resource allocation argues that if all expenditure decisions are made efficiently then the rate of return on the marginal dollar will be the same for all projects (that is, the price-marginal cost ratio should be the same on all projects). If this is not the case then it will be to society's benefit to reallocate resources from those uses showing a lower rate of return to those offering a higher rate of return (Little (1957), Mishan (1972)). A model, in the style of Vickery (1963), Smeed (1964), Mohring (1965), Shipman (1971) and Sherman (1967, 1971, 1972), which formally generates this proposition, and a series of variations on this proposition, is presented below.

Throughout use is made of the line integral form of the objective function employed in Braeutigam (1979). Benefits are measured by integrating under commodity demand curves. Total surplus is measured as the difference between the area under the demand curve and total production

costs. In other words, it is the sum of producer and consumer surplus. It is assumed that Hotelling's (1938) conditions for the path independence of the line integral are satisfied. This requires equality of cross price derivatives between all goods whose prices change. This will hold strictly only if the income elasticities of the goods in question are equal (this follows straight forwardly from the Slutsky equation; see also Neuburger (1971)). Hotelling's result can be obtained approximately by assuming that the urban transport sector is sufficiently small that changes in consumer expenditure on urban transport will leave the marginal utility of income reasonably stable

Let us suppose that there are only two means of transportation, private and public transport. Denote the quantity of output per period of each sector as  $X_1$  and  $X_2$  respectively. The products will be substitutes and their inverse demand functions may be written:

$$(1) \quad \begin{aligned} P_1 &= P_1(X_1, X_2) \\ P_2 &= P_2(X_1, X_2) . \end{aligned}$$

For the present it will also be assumed that the average cost,  $C$ , of providing each service is independent of the cost of providing the other service, so that:

$$(2) \quad \begin{aligned} C_1 &= C_1(X_1) \\ C_2 &= C_2(X_2) . \end{aligned}$$

The assumption in the resource constrained second-best is that there is some fixed amount of resources,  $C^0$ , that society is prepared to spend on urban travel. These resources are to be allocated between

private and public transport in the most efficient manner. Implicitly the model requires that there be lower price elasticities for broad categories of goods and services (e.g. travel) than there are for sub-categories (e.g. bus or auto travel). In practice this may be an excellent working hypothesis, but it must be remembered that many long-run decisions rest on particular travel costs and opportunities that such a general position will not always be satisfactory. For example, subsidizing buses may in particular circumstances create transitional location problems. In applying the resource constrained optimization model one seeks to obtain such improvements in relative pricing among alternative modes that there will be an improvement in overall resource allocation. The resource constraint can then be formally written:

$$(3) \quad C_1(X_1) \cdot X_1 + C_2(X_2) \cdot X_2 = C^0$$

The constrained optimization problem can then be written formally as:

$$(4) \quad \text{Max: } S = \int_0^{X_1} P_1(\omega, 0) d\omega + \int_0^{X_2} P_2(X_1, \omega) d\omega - X_1 \cdot C_1 - X_2 \cdot C_2$$

$$\text{Subject to: } C_1 \cdot X_1 + C_2 \cdot X_2 = C^0 .$$

The Lagrangian function can then be formed as

$$(5) \quad \mathcal{L} = S + \lambda(C_1 X_1 + C_2 X_2 - C^0)$$

The optimum level<sup>†</sup> of private and public transport,  $X_1^*$  and  $X_2^*$  will

---

<sup>†</sup>A unique internal solution requires that the commodities be weak gross substitutes and that there be decreasing returns to scale in the cost functions. The presence of fixed costs raises the prospect of corner



satisfy the first order conditions:<sup>†</sup>

$$(6) \quad \frac{P_1}{MC_1} = \frac{P_2}{MC_2} = (1+\lambda) .$$

The MC term is the marginal social cost of an extra unit of output, and  $\lambda$ , the Lagrangian multiplier is the contribution to social welfare,  $S$ , of an additional dollar of resources. Clearly, when the resource constraint is not binding,  $\lambda = 0$ , (b) gives the best known of all the results on static allocative efficiency, namely, that price equal marginal social cost.

The solution offered in equation (6) could be characterized as a "centralized" solution to the welfare maximization problem. By this we mean that some central authority has the power to fix  $X_1$  and  $X_2$  at the appropriate welfare maximizing levels. In the urban transport sector decision making is a decentralized process whereby the marginal buyers of the transport services equate willingness to pay, price, and marginal private costs. To support the optimum allocation in equation

---

solutions, that is, an optimum that includes only private transport, or only private transport. Since such solutions are of only limited interest they will not be pursued here. The reader can proceed by interpreting the average cost functions as average variable cost functions, and bear in mind that the internal solutions which are presented in the paper could potentially be dominated by corner solutions.

<sup>†</sup>Path independence of the integral means that the objective function may be written

$$S = \int_0^{X_2} P_2(0, \omega) d\omega + \int_0^{X_1} P_1(\omega, X_2) d\omega - X_1 \cdot C_1 - X_2 \cdot C_2$$

hence simplifying the differentiation of the objective function.

(6) a system of taxes and subsidies will be required, that is, consumers will have to be coerced. In the case of private transport individual drivers will continue to use road space up to the point where price equates marginal private cost. Since all drivers are alike it follows that marginal private cost is also average private cost, or  $C_1$  in the notation above. The total cost of  $X_1$  users of the road system is given by,

$$TC = X_1 C_1$$

The marginal cost to society of an additional user is written,

$$(7) \quad MC_1 = C_1 + X_1 \frac{\partial C_1}{\partial X_1} = MPC_1 + E_1$$

which is decomposed into marginal private cost and the externality cost imposed by the marginal user. To coerce the road user to consume either more or less road space a tax or a subsidy (a negative tax) can be imposed on the road user so that two conditions are met:

$$(8) \quad P_1 = C_1 + \text{tax} \quad (\text{market clearing})$$

$$(9) \quad P_1 = (1+\lambda) \left( C_1 + X_1 \frac{\partial C_1}{\partial X_1} \right) \quad (\text{optimum allocation}).$$

It is then seen that the optimum tax is:

$$(10) \quad (\text{tax})t = \lambda \cdot C_1 + (1+\lambda) X_1 \frac{\partial C_1}{\partial X_1} .$$

The tax in (10) has two components, a general tax on road users so as to make them conserve resources up to the point of satisfying the resource constraint, and a special tax to penalize road users for the externality costs which they generate. The argument for public transport runs along similar lines but the institutional setting is somewhat different. The public transport authority simply sets a fare schedule so that the condition (6) for optimum resource allocation is satisfied, and there are no problems of establishing how the tax might actually be levied.

To date little has been said about how the resource constraint  $C^0$  is established. As indicated in the Introduction to this paper there are two approaches. The first simply takes  $C$  as being given by some external planning authority. The second possibility is to take a sub-optimizing 'uncontrolled' urban transport sector and seek to improve resource allocation within the sector while still employing the same total amount of resources. This is possibly the most common scenario facing the applied economist in this field. While vested interest groups, urban travelers, may be prepared to accept some alteration of relative prices so as to improve their immediate welfare, they are less likely to accept a regime of taxes which transfers resources away from them as a group and towards improving the welfare of a much larger and less immediate group (namely the entire nation). An 'uncontrolled' market situation could be characterized as one where road users equate price and marginal private cost, and where the transit authority equates price and average cost.

Thus:

$$(11) \quad \begin{aligned} P_1 &= C_1 \\ P_2 &= C_2 . \end{aligned}$$

The market equilibrium levels of output will be those which satisfy the equations in (11) and they will be written  $(\hat{X}_1, \hat{X}_2)$ . The resources consumed at this equilibrium will be

$$(12) \quad \hat{X}_1 \cdot C_1(\hat{X}_1) + \hat{X}_2 \cdot C_2(\hat{X}_2) = \hat{C} .$$

The optimization problem is then to solve the objective function in (4) subject to the general constraint

$$(13) \quad X_1 C_1(X_1) + X_2 C_2(X_2) = \hat{C} .$$

The problem is illustrated in Figure 1.

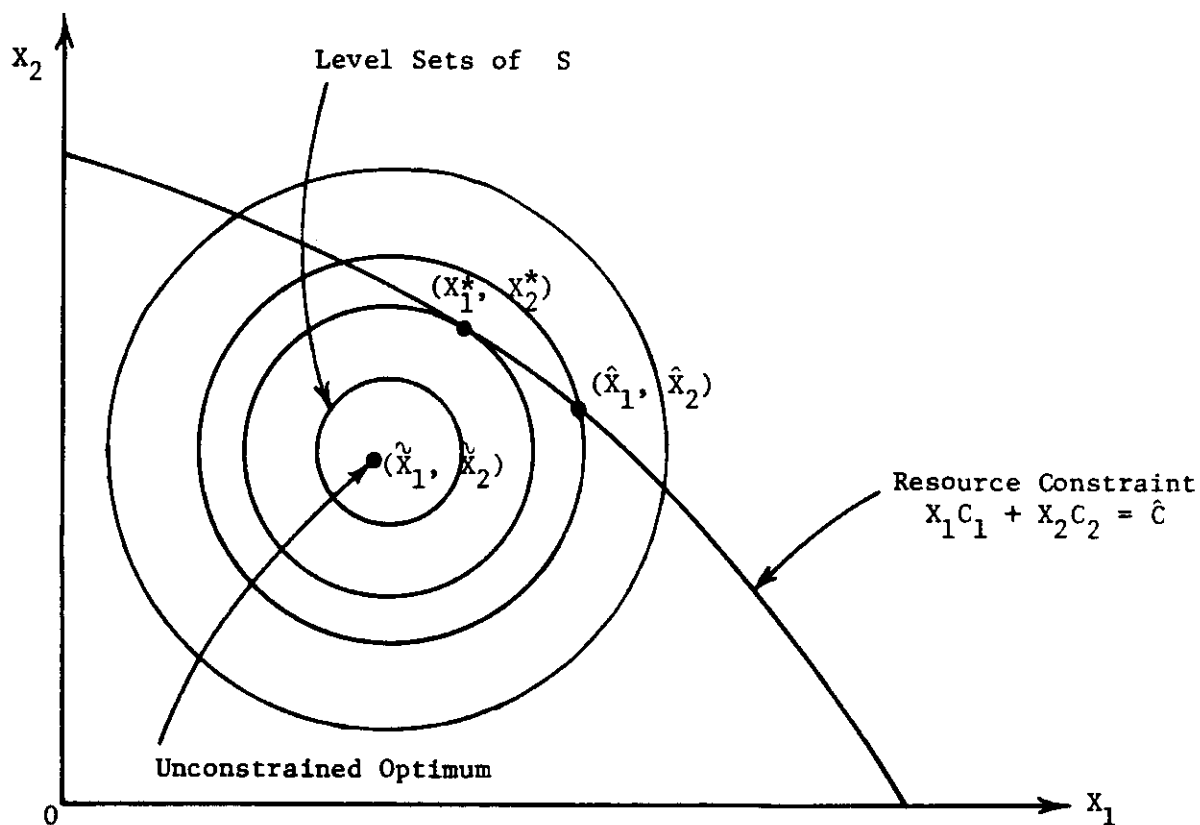


FIGURE 1. Resource Constrained Optimum

In this situation the free market solution  $(\hat{X}_1, \hat{X}_2)$  will consume more resources than the unconstrained optimum  $(\check{X}_1, \check{X}_2)$  simply because marginal users do not take account of the external cost which they impose on the system. Hence  $(\check{X}_1, \check{X}_2)$  will lie below the resource constraint line. The solution to the constrained optimization problem will then be one in which the solution to the Lagrangian multiplier is negative (additional marginal resources reduce total social surplus). The solution to the taxing problem is then of the form of equation (10) with the possibility of a negative tax (a subsidy). Each mode will receive a subsidy,  $\lambda C_1$  and  $\lambda C_2$ , and then will have part of that subsidy taxed away according to the extent of the externalities associated with the marginal units of consumption in that mode,  $(1+\lambda)X_1(\partial C_1/\partial X_1)$  and  $(1+\lambda)X_2(\partial C_2/\partial X_2)$ . In Figure 1, the diagram has been drawn in such a way that the new equilibrium involves fewer units of private transport and more units of public transport, reflecting the greater externalities associated with private transport. The welfare gain comes about as a result of the movement to a higher contour, or level set, of  $S$ .

The constrained optimization problem defined by the constraints in equations (12) and (13) is one in which prices are set less than marginal social costs. Indeed it is possible that prices could be set less than marginal private costs\* if the externality costs on a mode are small. This implies a net subsidy to that mode from general revenues.

The proposition that public transport be priced below marginal cost on the grounds that public transport contributes less to traffic congestion than private transport is a familiar one in the urban transport

---

\*This is true if  $|\lambda C_i| > (1+\lambda)X_i(\partial C_i/\partial X_i)$ .

literature (Shipman (1971), Sherman (1972), Braun (1972)). The argument is, however, usually advanced in a different fashion to that offered above. The operative constraint is taken to be the infeasibility of confronting road users with the costs of the delays they impose on other drivers. This is a technical constraint upon the central planner due to the absence of a suitable metering device that can be employed to identify which road users are travelling in congested areas at congested times of the day. The constraint may then be written as:

$$(14) \quad P_1(X_1, X_2) = C_1(X_1) \quad (\text{market clearing})$$

and this constraint must always be satisfied. The constraint (14) is shown in Figure 2 along with the constraint which satisfies the conditions

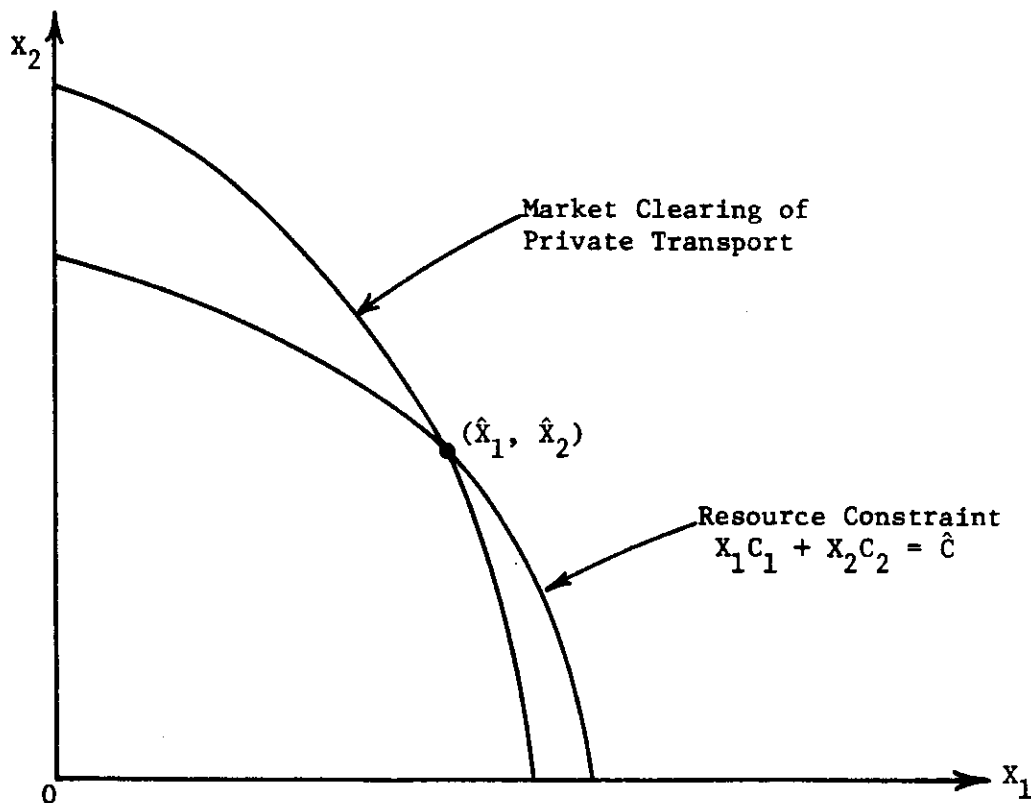


FIGURE 2. Resource and Market Clearing Constraint

the resource consumption is the same as at the free or uncontrolled market equilibrium. Notice that by construction the free market equilibrium  $(\hat{X}_1, \hat{X}_2)$  satisfies both constraints. Hence, unless there are non-convexities in the curves there will be no other resource allocation which can improve welfare and still satisfy both these constraints.

Shipman (1971) in developing a model to examine the consequences of not having a taxation instrument for private travellers was not concerned with the resource constraint which has been carried through our analysis here. For completeness, a variation on Shipman's model will be considered here, and the resource implications will be discussed. The argument is straightforward: since private transport is necessarily priced below its marginal social costs it may be better to price public transport below its marginal social cost.

The model will be richened slightly by including the possibility of cost interdependencies of the travel modes. The average costs functions will be written:

$$(15) \quad \begin{aligned} C_1 &= C_1(X_1, X_2) \\ C_2 &= C_2(X_1, X_2) . \end{aligned}$$

It is expected that the partial derivative of the average costs functions with respect to each argument will be positive. If, for example, the only form of public transport is rail, which has little or no interaction with the road network, then the differential of  $C_1$  with respect to  $X_1$  may be near zero, but on the positive side. The Shipman-type optimization problem may then be written:

$$(16) \quad \text{Max: } S = \int_0^{X_1} P_1(\omega, 0) d\omega + \int_0^{X_2} P_2(X_1, \omega) d\omega - X_1 \cdot C_1(X_1, X_2) - X_2 \cdot C_2(X_1, X_2)$$

$$\text{Subject to: } P_1(X_1, X_2) - C_1(X_1, X_2) = 0 .$$

The first order conditions may be obtained by forming the Lagrangian and differentiating with respect to  $X_1$ ,  $X_2$  and the Lagrangian multiplier and setting the resulting equations to zero. Solving these equations one obtains that the Lagrangian multiplier,  $\lambda$ , shown in (17).

$$(17) \quad \lambda = \frac{X_1 \frac{\partial C_1}{\partial X_1} + X_2 \frac{\partial C_2}{\partial X_1}}{\frac{\partial P_1}{\partial X_1} - \frac{\partial C_1}{\partial X_1}} .$$

Since,

$$\frac{\partial C_1}{\partial X_1}, \frac{\partial C_2}{\partial X_2} \geq 0$$

and 
$$\frac{\partial P_1}{\partial X_1} < 0$$

it follows that  $\lambda \leq 0$ . The inequality will hold strictly whenever the constraint is binding, that is, whenever there are any congestion externalities in the consumption of private transport. The  $\lambda$  measures the amount of surplus foregone in not being able to raise the price of private transport one dollar above its average private cost. One can also solve for the optimum price of public transport,



$$(18) \quad P_2 = \left( \frac{\partial C_1}{\partial X_2} - \frac{\partial P_1}{\partial X_2} \right) + X_1 \frac{\partial C_1}{\partial X_2} + \left( C_2 + X_2 \frac{\partial C_2}{\partial X_2} \right)$$

where  $(C_2 + X_2(\partial C_2/\partial X_2))$  is the marginal social cost of public transport,  $MC_2$ . Since,

$$(19) \quad \frac{\partial C_1}{\partial X_2} > 0 \quad \text{and} \quad \frac{\partial P_1}{\partial X_2} < 0 \quad (\text{substitutes})^*$$

it follows that so long as  $\partial C_1/\partial X_1$  is small, that is, the public transport user imposes only small additional costs on the private transport user then

$$(20) \quad P_2 < MC_2$$

and the proposition that is socially desirable to price public transport below its marginal cost has been established.

The solution of the problem in (16) is shown diagrammatically in Figure 3. The welfare gain results from the movement to a higher level set or contour of  $S$  when moving from the market solution  $(\hat{X}_1, \hat{X}_2)$  to the constrained optimum  $(X_1^*, X_2^*)$ . At the constrained optimum public transport is being subsidized, hence more public transport and less private transport are consumed. It is also true that more total resources are being consumed at  $(X_1^*, X_2^*)$  than were being consumed at  $(\hat{X}_1, \hat{X}_2)$ .

Thus far taxes and subsidies have been introduced into the analysis with little concern for how the subsidies might be raised or how net

---

\*An increase in consumption of  $X_2$  means the price of commodity two must have fallen. The demand curve for the substitute commodity one must fall towards the origin, so that at the unchanged level of consumption  $X_1$  the price must be lower.

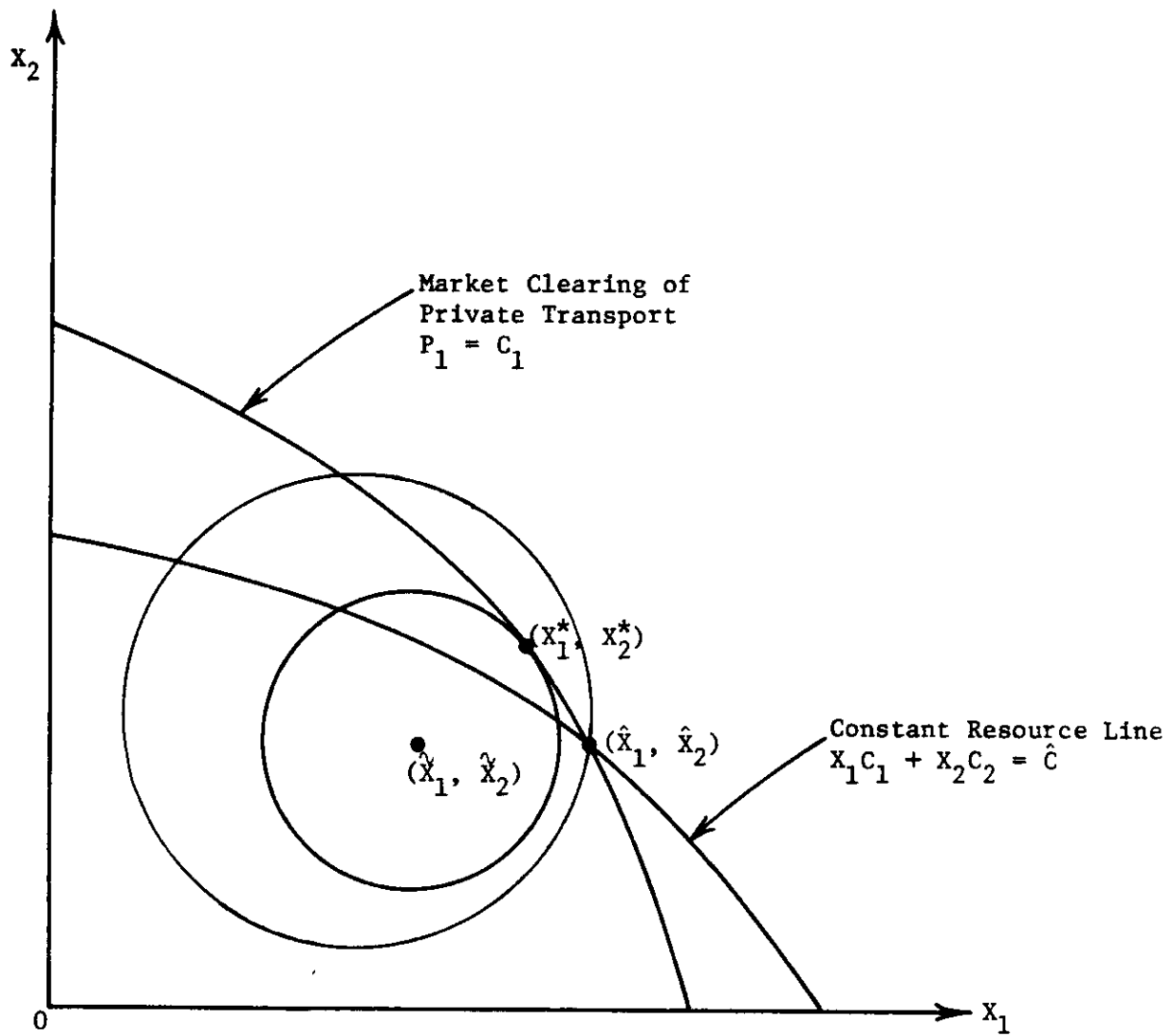


FIGURE 3. Optimum with Market Clearing Constraint on Private Transport

taxation revenue might be dispersed. Further, there has been no consideration of the possible distortionary effects of these taxes and subsidies in the rest of the economy. Financial constraints are, however, at the core of most public policy questions and the next section will go on to conceive the policy question as being to allocate resources in such

a way as to a welfare optimum subject to a financial constraint, as opposed to the resource constraints that have been considered up to this point.

#### Revenue Constrained Second-Best

At the level of the urban transport sector one rarely finds a situation where the public transport sector is given carte blanc to price its services as it pleases. Indeed, the public transport authority will typically have to make some target rate of profit (maybe more often than not a negative profit). There will be a wide variety of pricing policies available, for example, simple average cost pricing of public transport will satisfy the break-even constraint (if there is some level of output where the demand curve lies above the average cost curve). The problem is to find a pricing policy which meets the financial constraint at the least cost to efficiency. The question was first considered by Boiteux (1956), and has been analyzed in various road pricing contexts by Walters (1968) and Train (1977).

An excellent example of revenue constrained second-best pricing is to be found in the case of the Chicago Regional Transit Authority (RTA). The RTA has been given the power to levy a sales tax on gasoline sold within its administrative area for the purposes of covering any losses on its transit operations. Given the constraint on the RTA to break-even financially, it is interesting to ask what the optimum level of such a tax should be and what should be the optimum price to charge for public transport.

A formal optimization model can be constructed which yields interesting insights into the way in which the gasoline tax is determined.

Again, define  $X_1$  and  $X_2$  as the quantities of output of private and public transport respectively. Let the average cost of providing the services be  $C_1(X_1)$  and  $C_2(X_2)$  respectively. More sophistication could be gained by defining  $X_1$  and  $X_2$  in terms of geographical location and time of day, and the costs could be made a function  $C(X_1, X_2)$  as in some of the earlier discussions, but for the present these simple assumptions on costs will suffice to illustrate the chief features of the model. Let the inverse demand functions for private and public transport be represented  $P_1(X_1, X_2)$  and  $P_2(X_1, X_2)$  respectively, and let the tax be levied on  $X_1$  at a rate of  $t$  per unit. This tax on service units of private transport is an approximation to the RTA gasoline tax which amounts to something like  $t$  per vehicle mile, as opposed to per person mile. Since decision making is decentralized there is no control of entry into the private transport market and travellers will continue to use their automobiles up to the point where price, or willingness to pay, equals the marginal private cost of travel. Thus entry occurs to the point where

$$(21) \quad P_1(X_1, X_2) = C_1(X_1) + t .$$

The break-even constraint for the RTA requires that the sum of revenues from fare collections and from gasoline tax equal the cost of operating the transit system. That is:

$$(22) \quad X_2 \cdot P_2(X_1, X_2) + tX_1 = X_2 \cdot C_2(X_2) .$$

The constraint set can be illustrated with a diagram as seen in Figure

4. Higher rates of taxes on private transport will allow higher subsidies

to public transport and still satisfy the break-even constraint, hence the effect constraint line must be steeper than the free entry line.

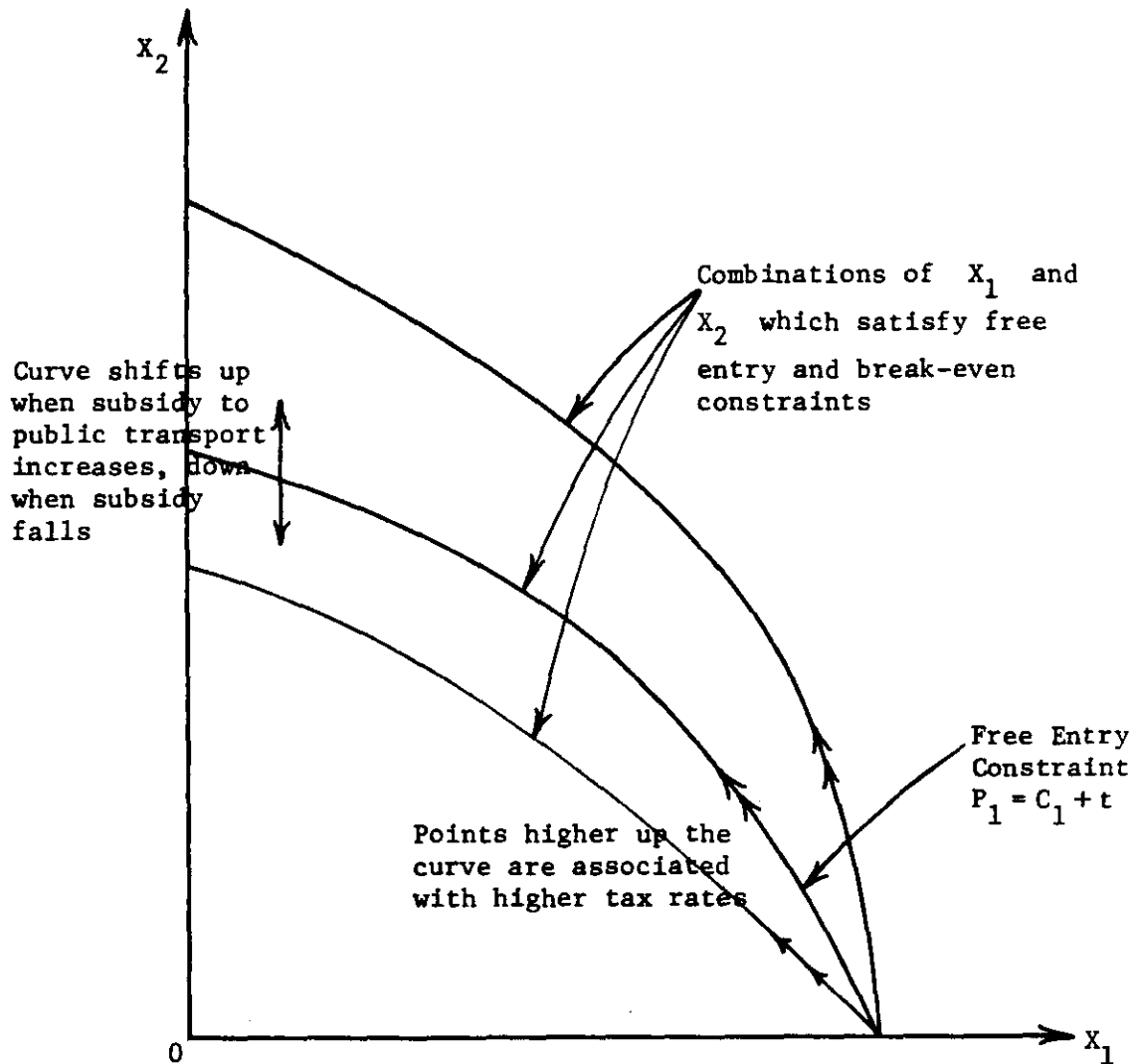


FIGURE 4. Constraint Set in RTA Pricing Model

The maximization problem is now formally written as one of maximizing the sum of producer and consumer surplus subject to the constraint set:

$$(23) \quad \text{Max}_{X_1, X_2, t} \int_0^{X_1} P_1(\omega, 0) d\omega + \int_0^{X_2} P_2(X_1, \omega) d\omega - X_1 \cdot C_1(X_1) - X_2 \cdot C_2(X_2)$$

$$\text{Subject to: } X_2 \cdot P_2(X_1, X_2) + t \cdot X_1 + X_2 \cdot C_2(X_2) = 0$$

$$P_1(X_1, X_2) - C_1(X_1) + t = 0 .$$

The Lagrangian can be formed for problem (23) and the first order conditions will be satisfied when the derivatives of the Lagrangian with respect to  $X_1$ ,  $X_2$  and  $t$ , and the two Lagrangian multipliers are set equal to zero. The resulting five equations are readily obtained and by appropriate substitution it can be shown that the optimum tax rate,  $t$ , will satisfy the condition,

$$(24) \quad t = \frac{-\lambda}{(1+\lambda)} \left[ X_1 \frac{\partial P_1}{\partial X_1} + X_2 \frac{\partial P_2}{\partial X_1} \right] + X_1 \frac{\partial C_1}{\partial X_1}$$

where  $\lambda$  is the Lagrangian multiplier on the financial constraint and it will be positive when the constraint on funds is in the nature of a shortage. The partial derivatives will be signed as follows:

$$\frac{\partial P_1}{\partial X_1}, \frac{\partial P_1}{\partial X_2} < 0 \quad \text{and} \quad \frac{\partial C_1}{\partial X_1} > 0 .$$

Equation (24) has excellent intuitive interpretation. First, the more inelastic the demand for private transport, that is, the larger is  $\partial P_1 / \partial X_1$ , the higher will be the gasoline tax. If demand is inelastic few units will be priced off the road by a higher tax hence only small welfare losses are involved. The higher the cross-elasticity of demand, the  $\partial P_1 / \partial X_2$  term, the lower the tax should be because taxation tends to distort the resource allocation between the two modes. The

final term in (24) is the component of the tax which corrects for the externality which the private road user imposes on the system.

### Conclusion

The pricing patterns derived from these partial equilibrium models are correct in their own terms but the assumptions for the models to be valid are quite stringent. It has been assumed that the market for urban transport is entirely isolated with no interrelation with the rest of the economy, either through substitution or income effects. While demand for urban transport may be fairly inelastic in the short-run, income effects and long-run production effects, such as altered land-use patterns, do exist.

The paper has reviewed two approaches to the constrained sub-optimization problem. The resource constrained approach takes the view that the links between sectors of the economy are complex and that the market by itself is not capable of making the appropriate allocations of resources to each sector, but rather such decisions primarily originate from some socio-political source, such as a government. So constrained, an attempt is made to reallocate the given resources in an optimal fashion. The resource constrained approach does not concern itself with the implications of the taxation or subsidy schemes which may be necessary to support, in a decentralized market, this optimum allocation. It is assumed that taxes and subsidies can be levied and dispersed in some non-distortionary fashion, such as lump-sum taxation, or that the distortions induced by having to raise tax revenues elsewhere in the economy are only small relative to the welfare gains that can be achieved from the reallocation of resources in the sector being manipulated.

The revenue constrained approach removes the question of the secondary effects of resource allocation, brought about by the need to raise taxes elsewhere in the economy, by requiring the sector in question to cover all its own financial costs. What is left aside is the question of how the urban transport sector fits into the rest of the economy, and, given the very complicated externalities involved in the transport-land-use interaction, whether the urban transport sector should be made larger or smaller relative to the rest of the economy.

The choice of the operative constraints rests on a value judgment regarding the best characterization of how the economy should be organized. This paper has identified some of these constraints and drawn out the implications of the different approaches to the problem.



## REFERENCES

- [1] Boiteux (1956), "Sur La Gestion des Monopoles Publics astreints a l'equilibre Budgetaire," Econometrica, translated as "On the Management of Public Monopolies subject to Budget Constraints," Journal of Economic Theory (1971).
- [2] Braeutigam, R. (1977), "Optimal Pricing with Intermodel Competition," American Economic Review, forthcoming.
- [3] Diamond, P. and J. S. Mirrlees (1971), "Optimal Taxation and Public Production," American Economic Review, Vol. 61 (March and June).
- [4] Engle, R. F., F. M. Fisher, J. R. Harris, and J. Rothenberg (1972), "An Econometric Simulation Model of Intra-Metropolitan Housing Location, Business, Transportation and Local Government," American Economic Review, Vol. 62, No. 2.
- [5] Hotelling, H. (1938), "The General Welfare in Relation to Problems of Taxation and of Railway Utility Rates," Econometrica, pp. 242-269.
- [6] Ingram, G. F., J. F. Kain, and J. R. Ginn (1972), The Detroit Prototype of the N.B.E.R. Urban Simulation Model, New York.
- [7] Kraus, M., H. Mohring, and T. Pinfold (1976), "The Welfare Costs of Nonoptimum Pricing and Investment Policies for Freeway Transportation," American Economic Review, Vol. 66, pp. 532-547.
- [8] Little, I. M. D. (1957), A Critique of Welfare Economics, second edition, Oxford University Press, Ch. 11.
- [9] Lowry, I. S. (1964), "A Mode- of a Metropolis," The RAND Corporation, Santa Monica.
- [10] Mishan, E. J. (1971), Cost-Analysis, London: Allen and Unwin.
- [11] Mohring, H. (1965), "Urban Highway Investments," in R. Dorfman, ed., Measuring Benefits of Government Investments, Brookings.
- [12] \_\_\_\_\_ (1970), "The Peak Load Problem with Increasing Returns and Pricing Constraints," American Economic Review, Vol. 60 (Sept.).
- [13] Neuburger, H. (1971), "User Benefits in the Evaluation of Transport and Land-Use Plans," Journal of Transport Economics and Policy, Vol. 5, No. 1.
- [14] Sherman, R. (1967), "A Private Ownership Bias in Transit Choice," American Economic Review, Vol. 57, No. 5.
- [15] \_\_\_\_\_ (1971), "Congestion Interdependence and Urban Transit Fares," Econometrica, Vol. 39 (May).

- [16] \_\_\_\_\_ (1972), "Subsidies to Relieve Urban Congestion," Journal of Transport Economics and Policy, Vol. 6, No. 1.
- [17] Shipman, W. (193 ), "Rail Passenger Subsidies and Benefit Cost Considerations," Journal of Transport Economics and Policy, Vol. 5, No. 1.
- [18] (Smeed Report) "Report Pricing: The Economic and Technical Possibilities," HMSO, London (1964).
- [19] Train, K. (1977), "Transit Prices under Increasing Returns to Scale and a Loss Constraint," Journal of Transport Economics and Policy, Vol. 9, No. 2.
- [20] Vickery, W. (1963), "Pricing in Urban and Suburban Transport," American Economic Review (May), Vol. 53.
- [21] \_\_\_\_\_ (1971), "Responsive Pricing of Public Utility Services," Bell Journal of Economics and Management Science, Vol. 2, No. 1.
- [22] Walters, A. (1968), "The Economics of Road User Charges," World Bank Occasional Paper No. 5, Baltimore, Ch. 4, Appendix.